The Abdus Salam International Centre for Theoretical Physics

2469-2

Workshop and Conference on Geometrical Aspects of Quantum States in Condensed Matter

1 - 5 July 2013

Inflation from Chern-Simons terms

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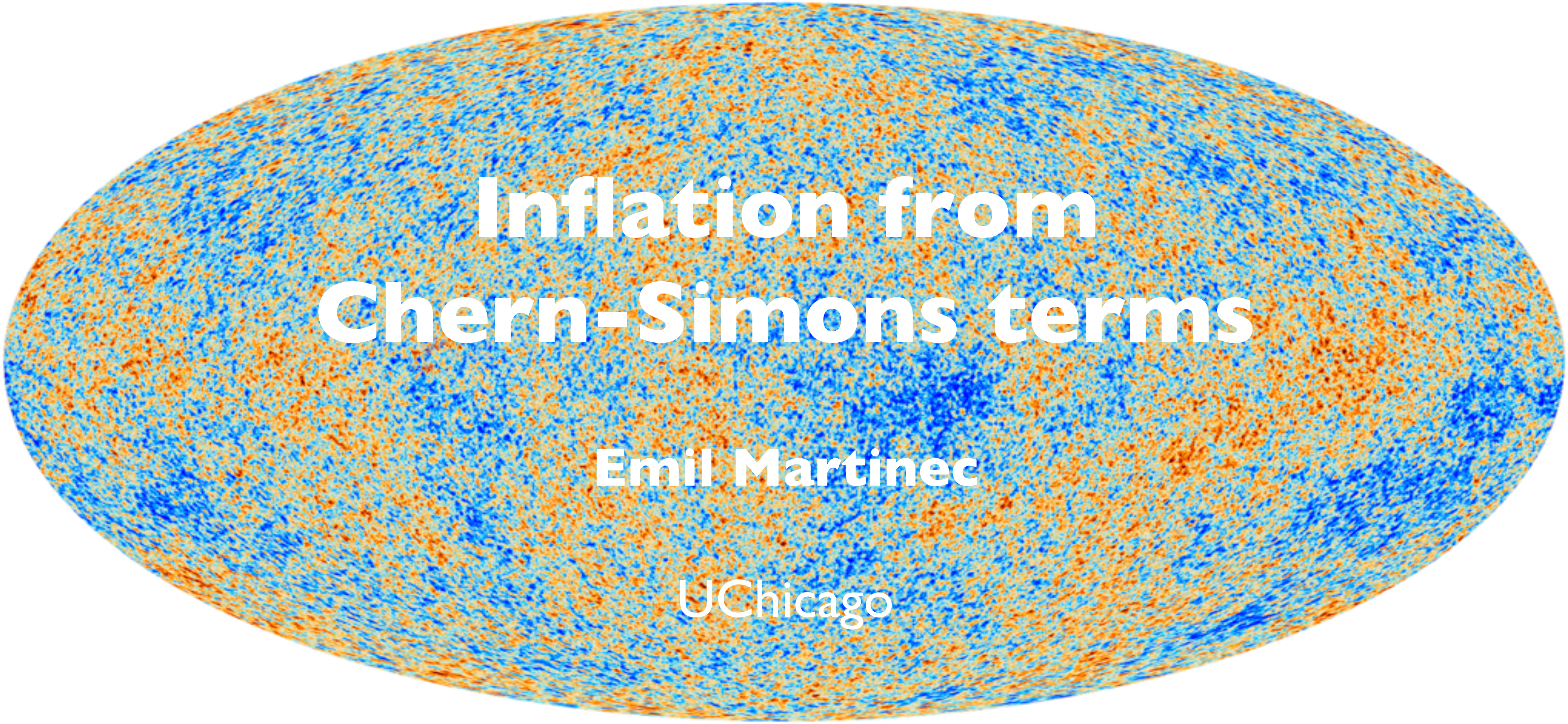


Inflation from Chern-Simons terms

Emil Martinec

UChicago

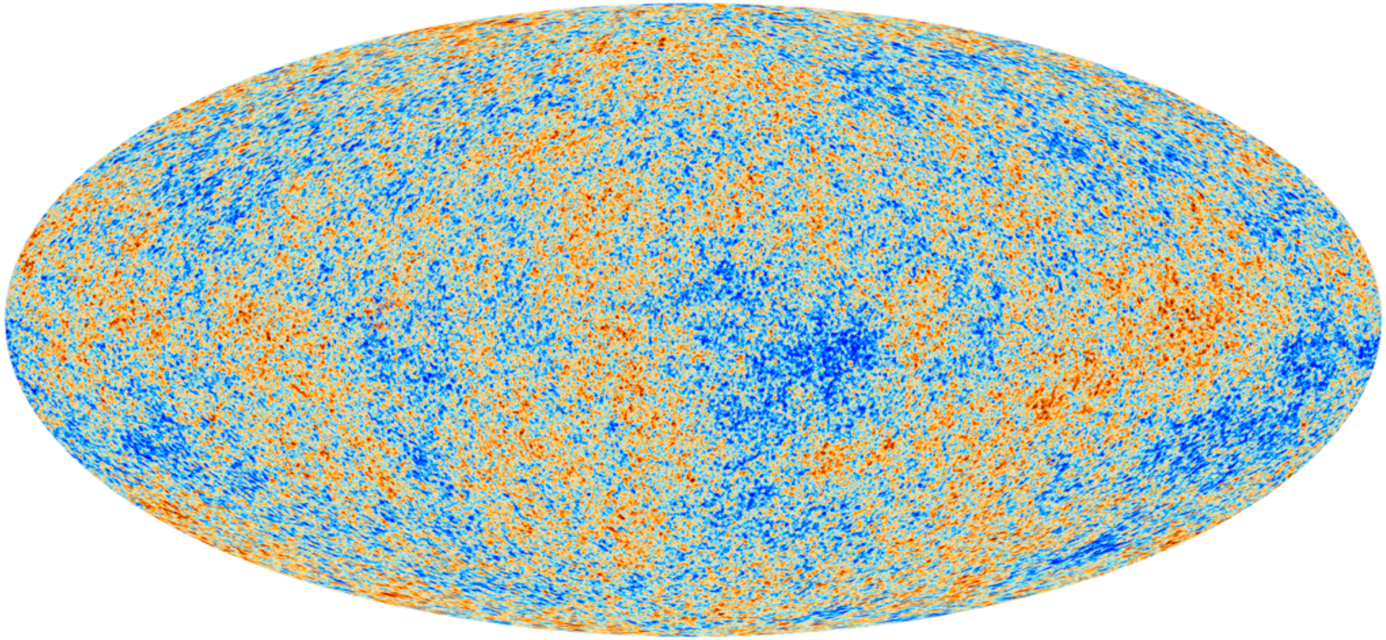
Semi-classical Cosmology
Geometrical Aspects of ^ ~~Quantum~~ States in ^ ~~Condensed Matter~~

Trieste, 4 July 2013

PRL: 108,261302 (2012) P. Adshead and M. Wyman
JHEP: 02, 027 (2013) with P. Adshead and M. Wyman
arXiv: 1301.2598, 1305.2930 with P. Adshead & M. Wyman

Inflation - why?

Fact 1: The universe is very homogeneous on large scales:

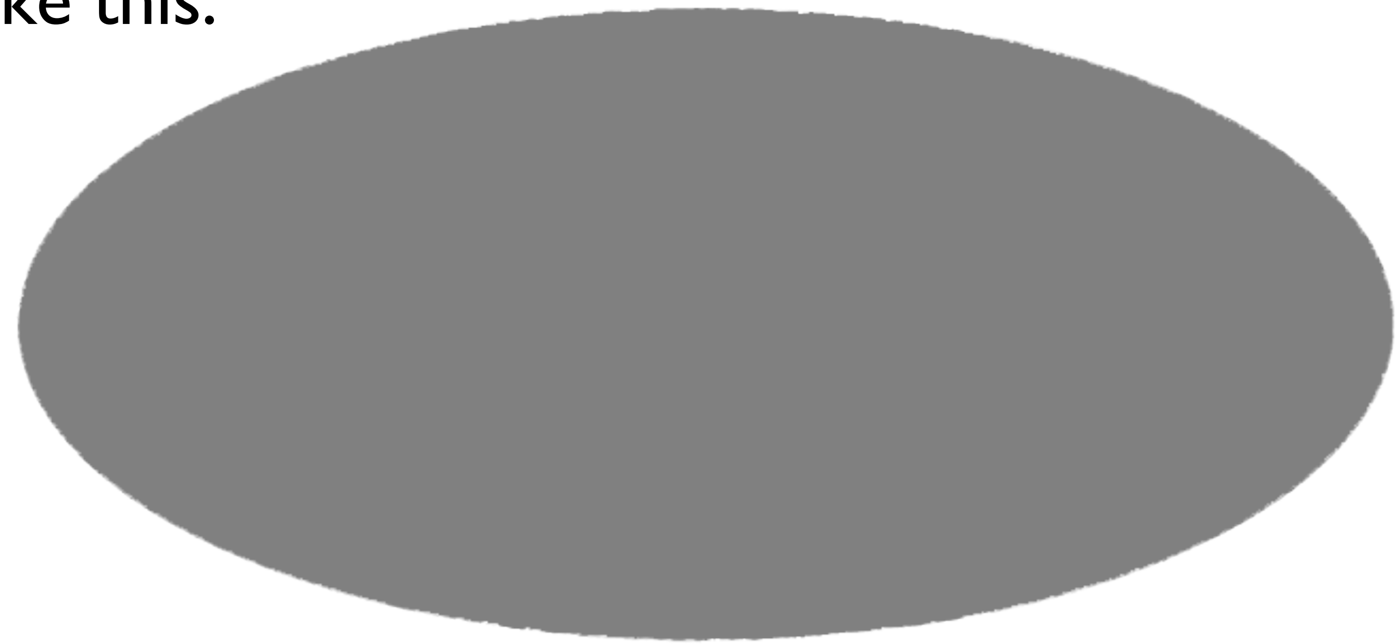


$T_{CMB} = 2.725 \pm (10^{-5})$

Suspiciously homogenous?

Inflation - why?

i.e. before we crank up the contrast, the actual fluctuation map looks like this:



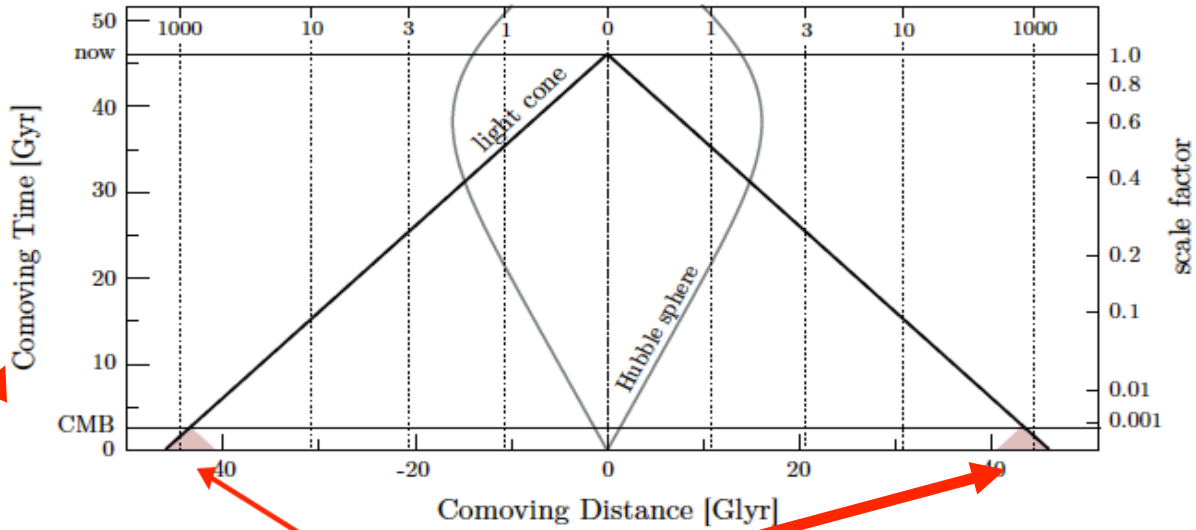
$T_{CMB} = 2.725 \pm (10^{-5})$

Suspiciously homogenous?

Inflation - why?

In standard hot big bang scenario, these large scales have never been in causal contact :

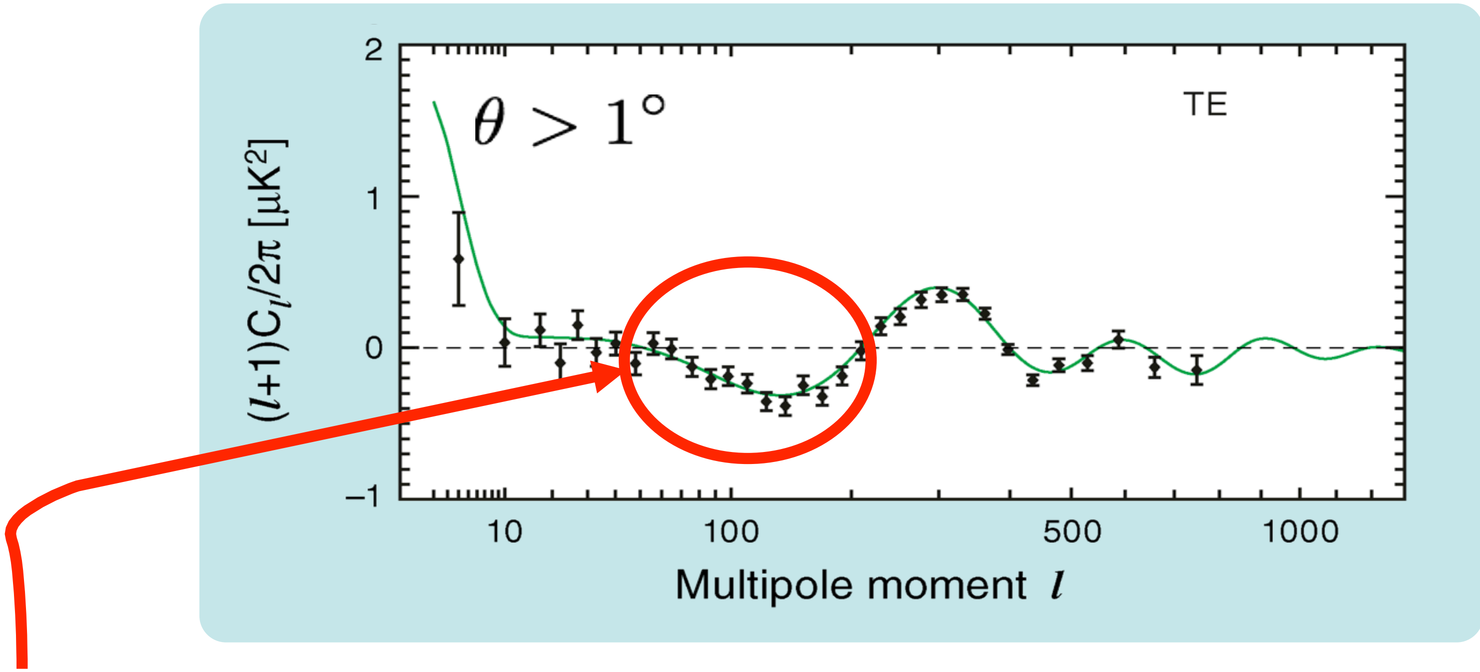
$$\tau = \int_{a=0}^{a=1} \frac{d \ln a}{aH}$$



Most spots on the CMB do not have overlapping past light cones, so why not O (1) fluctuations?

Inflation - why?

Fact 2: Fluctuations are correlated across apparently a-causal distances



Superhorizon (anti)correlation

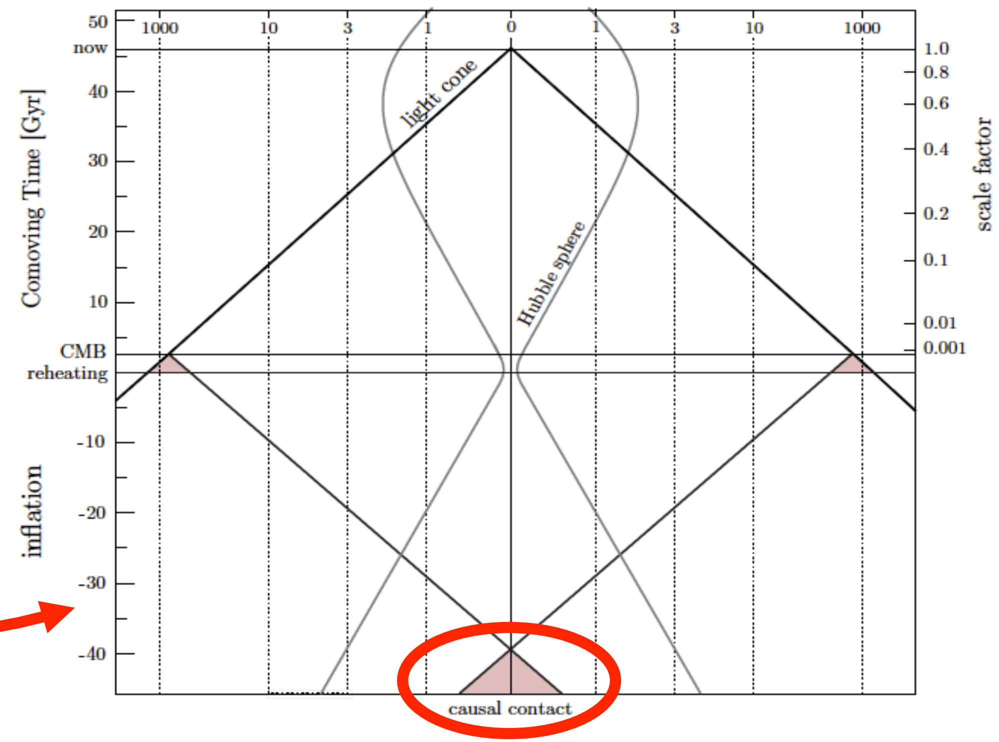
(WMAP)

Inflationary solution

A period of time where: $$\frac{d}{dt}(aH)^{-1} < 0$$

$$\tau = \int_{a=0}^{a=1} \frac{d\ln a}{aH}$$

Large contribution to τ at early times



Places all of CMB in causal contact – provides a physical mechanism to set up superhorizon correlations

Inflation - how?

Potential of slowly rolling scalar degree of freedom drives expansion (usual assumption)

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Eqs of motion for homogeneous (FRW) cosmology:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}\cdot d\mathbf{x}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

Inflation - how?

Highly overdamped scalar field motion results in quasi-de Sitter expansion

$$\underbrace{\ddot{\phi}}_{\approx 0} + 3H\dot{\phi} + V' = 0, \quad H^2 \approx \frac{V(\phi)}{3}, \quad a \approx e^{Ht}$$

Balancing these while maintaining $\frac{\dot{\phi}^2}{2} \ll V(\phi)$

over ~60 e-foldings of expansion of the scale factor *a(t)*

requires the *slow roll conditions* $\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \;\; \eta_V = \frac{V''}{V} \ll 1$

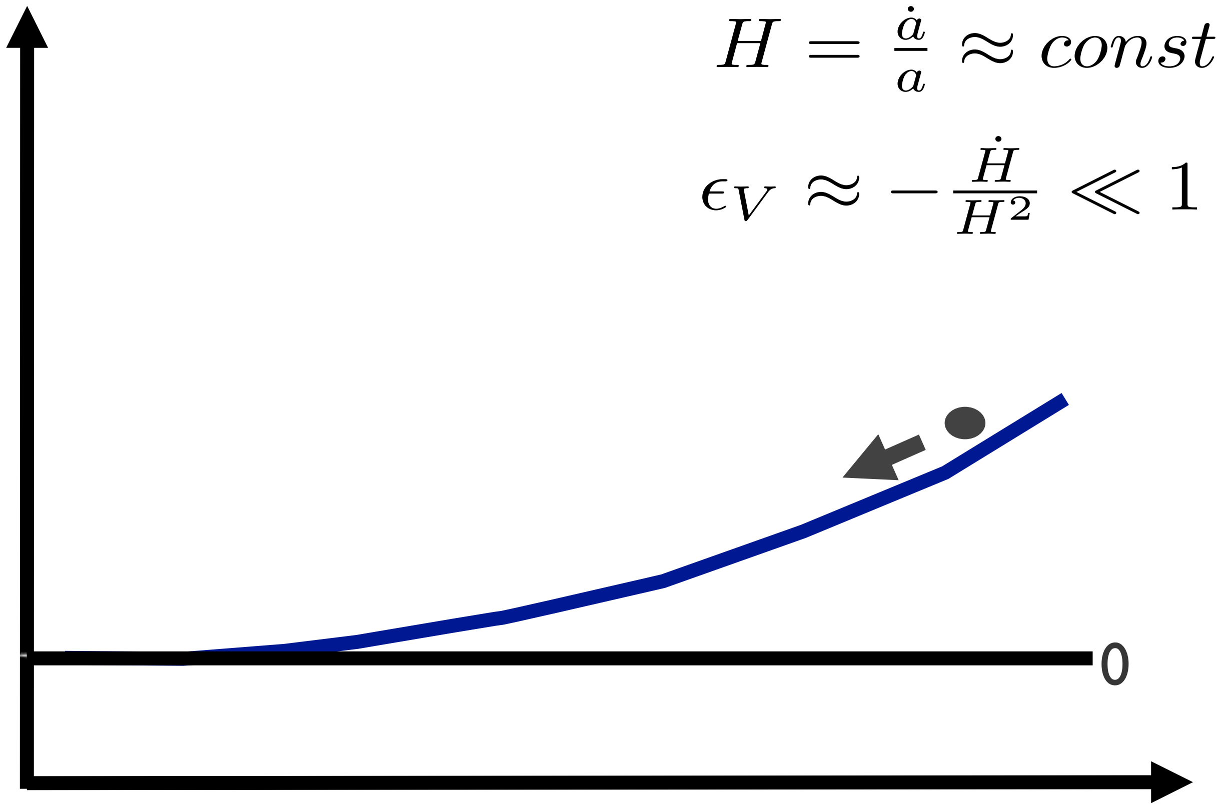
- Energy density dominated by slowly changing value of potential *i.e.* approximate cosmological constant

- Geometry is quasi-de Sitter

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}\cdot d\mathbf{x}$$

$$H = \frac{\dot{a}}{a} \approx const$$

$$\epsilon_V \approx -\frac{\dot{H}}{H^2} \ll 1$$



The ‘eta’ problem

Inflation as an EFT:

$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Theory is highly sensitive to Planck suppressed terms, e.g.

$$\mathcal{L} \supset R\phi^2$$

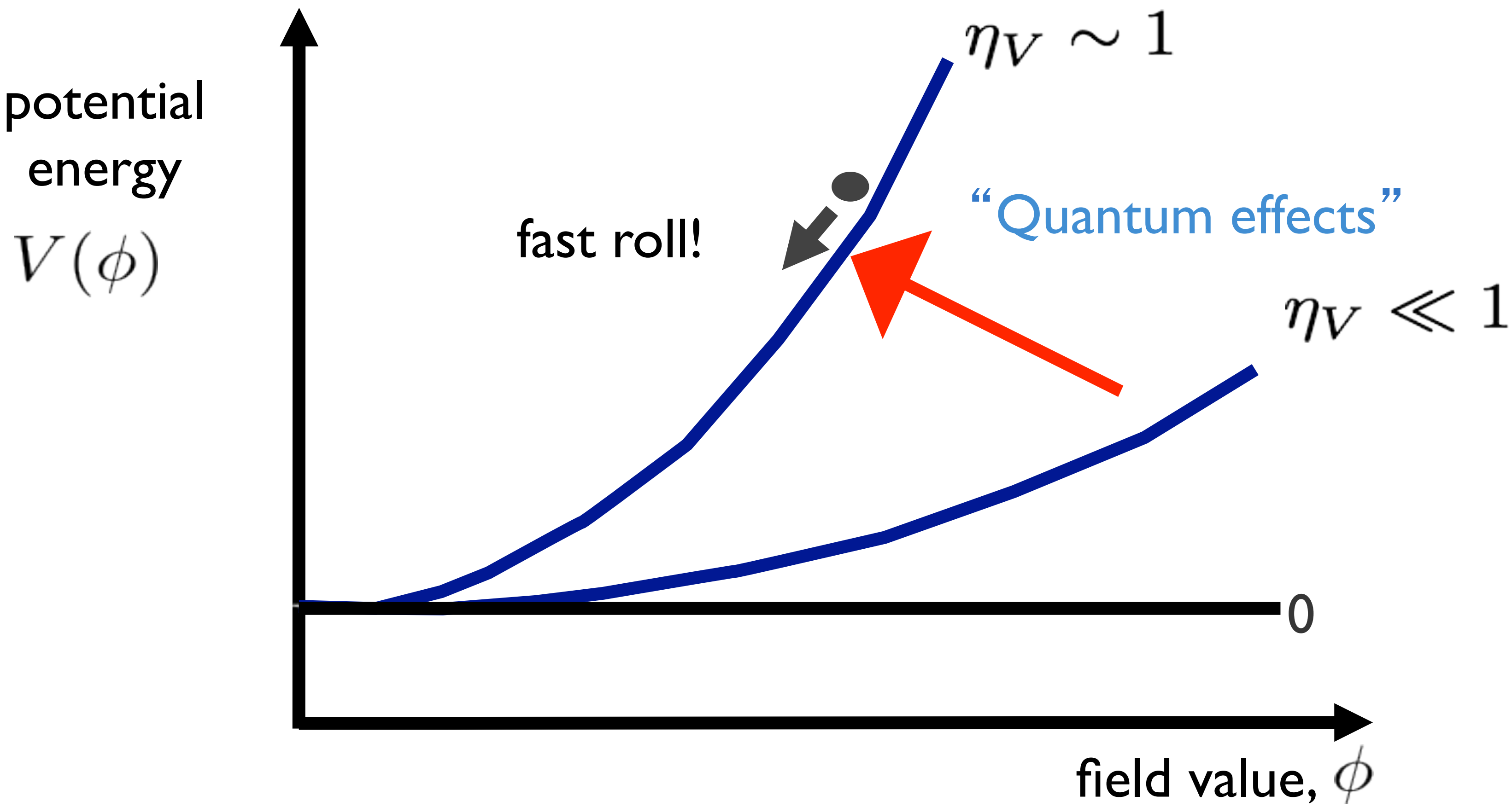
But, during inflation

$$R = H^2 \approx \frac{V}{3}$$

So, generic quantum effects

$$V \to V(1+\phi^2), \quad \eta_V = \frac{V''}{V} \ll 1 \to \mathcal{O}(1)$$

So there is an *inflationary* hierarchy problem.



Slowing the roll...

Ways around this:

- Symmetry! (natural inflation, axion monodromy...)

  Shift symmetry:

$$\phi \to \phi + c$$

-natural flat potential arises from non-perturbative effects,
or from leading order breaking

- Impose a speed limit (DBI)

$$\frac{1}{2}(\partial\phi)^2 \to \left(1 - \sqrt{1 - (\partial\phi)^2/T(\phi)}\right) T(\phi)$$

Slowing the roll...

Ways around this:

- Particle production/backreaction

$$\ddot{\phi} + 3H\dot{\phi} + V' = \alpha\langle\chi^2\rangle$$

(trapped infl., gauge prod.)

quantum average

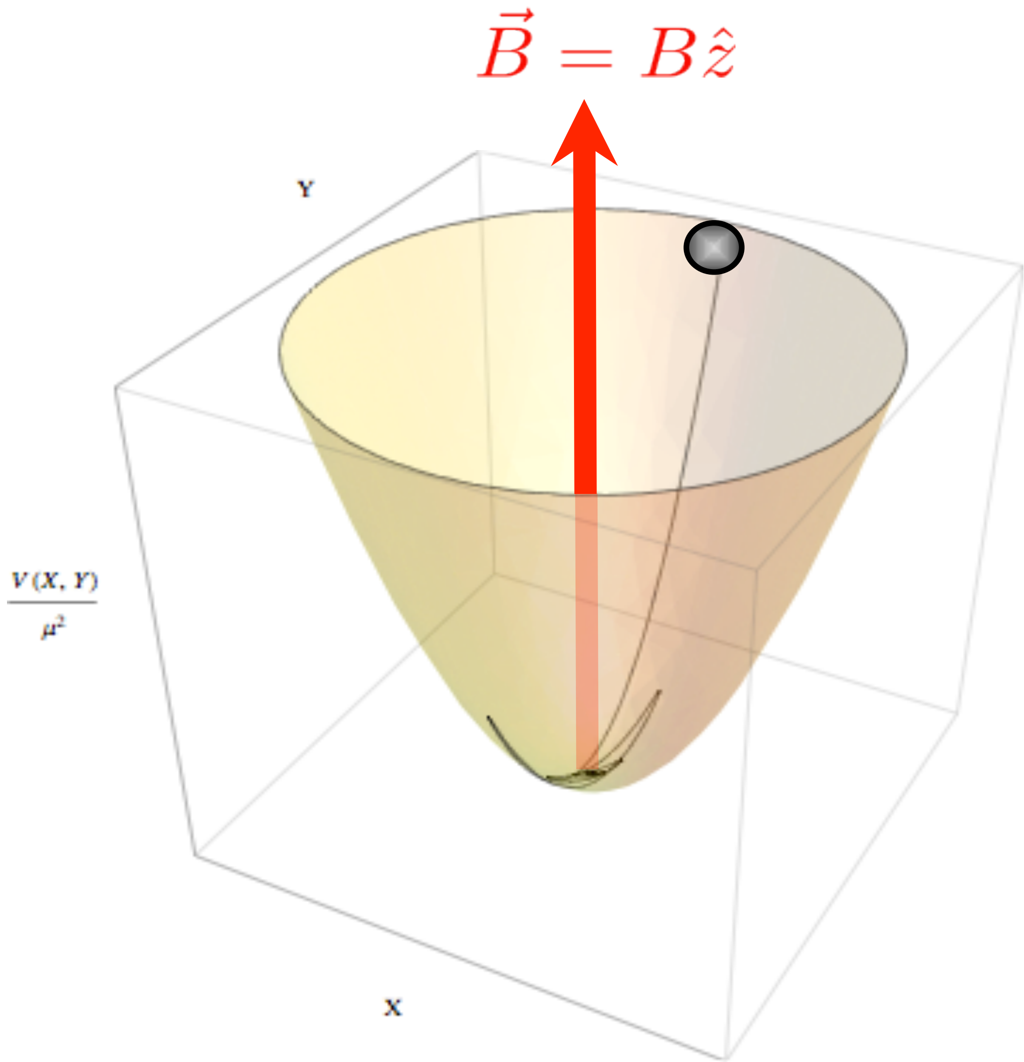
- Additional friction

  -decouple H and V' (Assisted inflation)

  -postulate more friction (warm inflation)

- Magnetic drift (our subject today)

The new ingredient is analogous to the Lorentz force:



potential force

$$\ddot{X} + H\dot{X} + \mu^2 X = +B\dot{Y}$$
$$\ddot{Y} + H\dot{Y} + \mu^2 Y = -B\dot{X}$$

ordinary friction

magnetic force

Normal Modes 1.

- Consider the dynamics in the large field limit B >> μ , with no friction
- Motion has two normal modes:

1. Angular mom. anti-parallel to field: $\vec{B} \parallel -\vec{L}$

-Lorentz force balances inertia: $\vec{v} \times \vec{B} \approx -\frac{v^2}{r}\hat{r}$

-Circular motion at the Larmor freq.: $\omega_- \sim B$

Fast mode for large fields $B \gg \mu$

Normal Modes 2.

- Consider the dynamics in the large field limit B >> μ, with no friction
- Motion has two normal modes:

2. Angular mom. parallel to field: $\vec{B} \parallel \vec{L}$

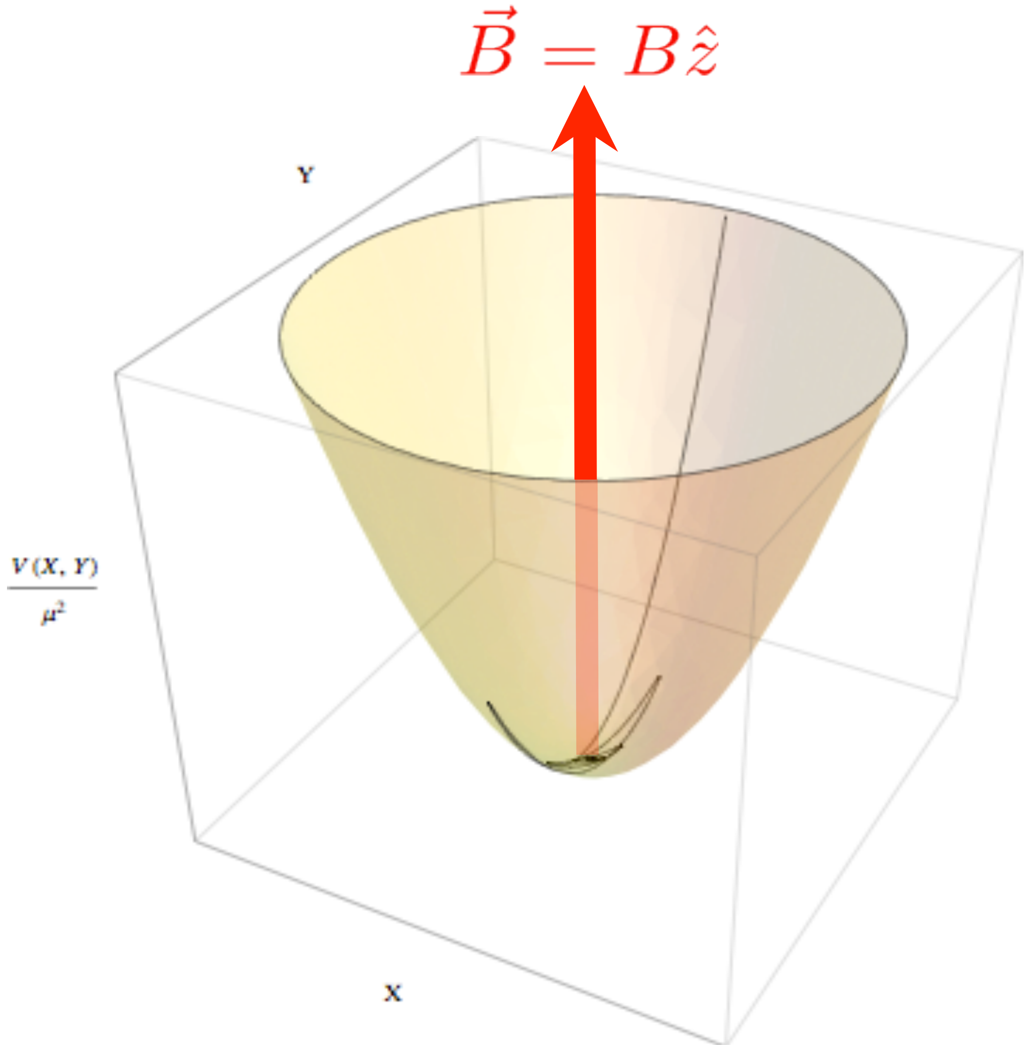
-Lorentz force opposes potential force: $\vec{v} \times \vec{B} \approx -\nabla \mathcal{V}$

-Orbital freq.: $\mu^2 r_+ \approx B v_+ \Rightarrow \omega_+ \approx \frac{\mu^2}{B}$

-Smaller than Larmor freq. by μ^2/B^2

Slow mode for large fields $B \gg \mu$

At low field strength...

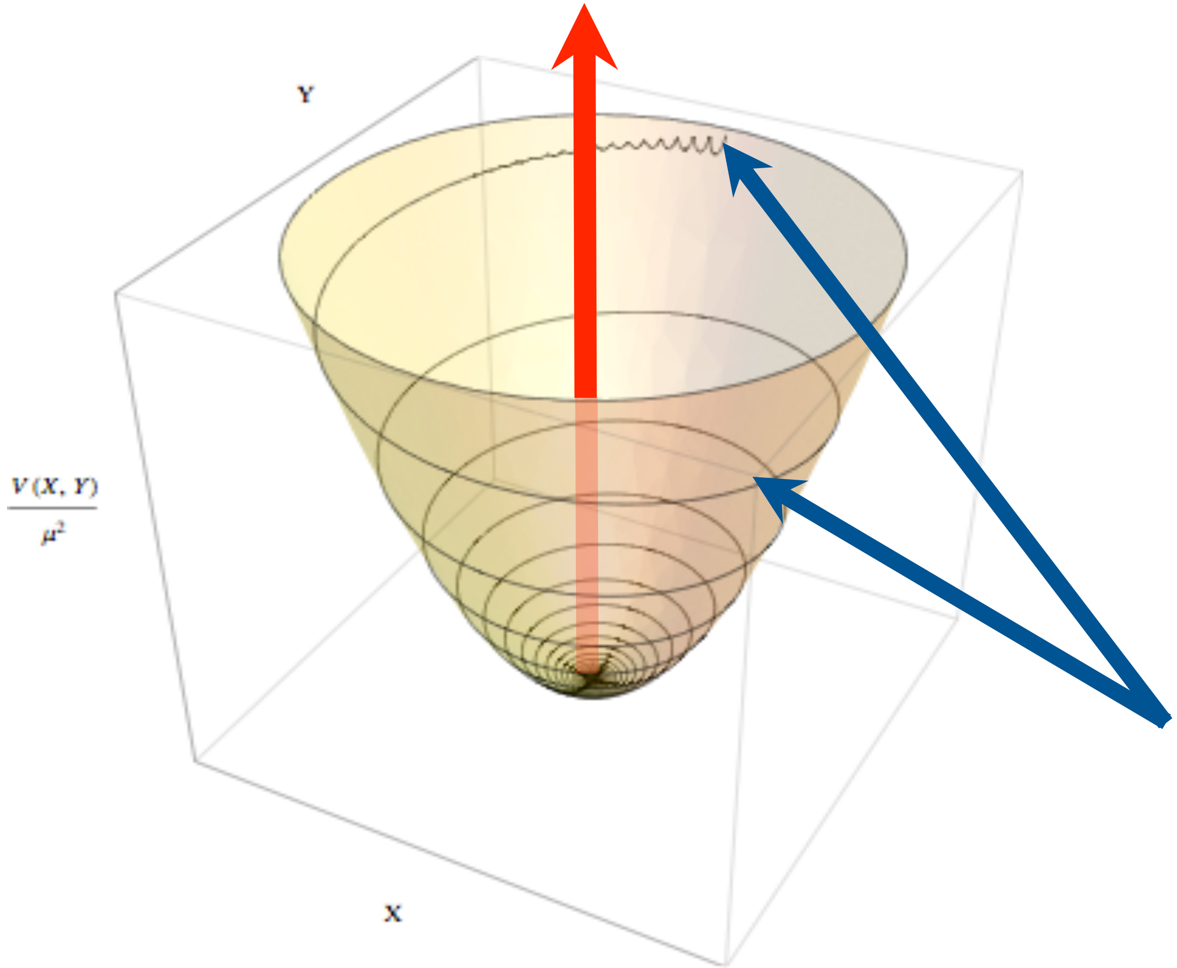


$$B = 0.1\mu$$

$$H = \frac{\mu^2}{\sqrt{3}}$$

At high field, magnetic drift:

$\vec{B} = B\hat{z}$



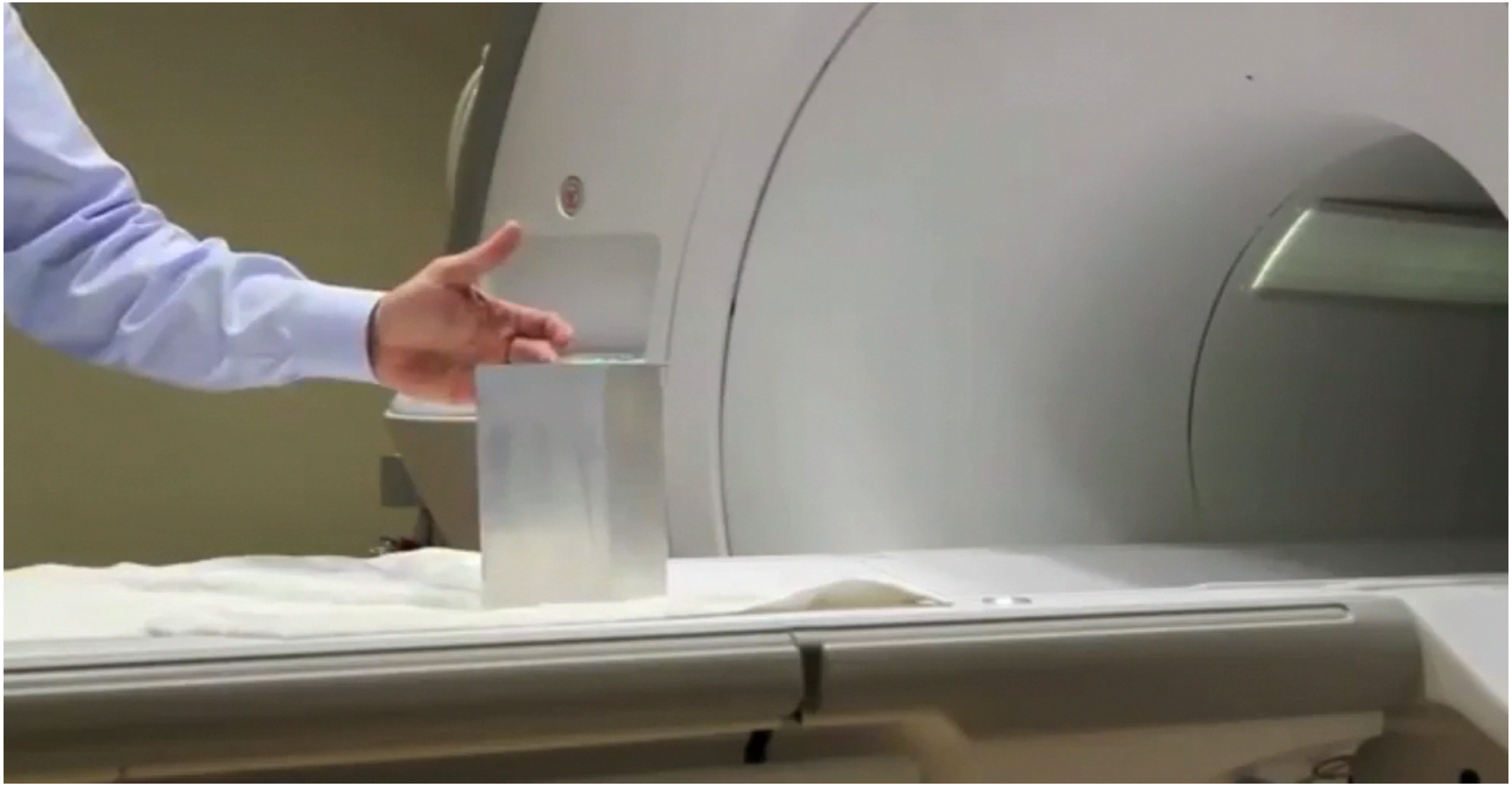
$$B = 10\mu$$

$$H = \frac{\mu^2}{\sqrt{3}}$$

Fast mode rapidly damps leaving slow magnetic drift mode.

Long **slow** spiral down the potential!

Here is some recent experimental data from the high field magnet group at U. Tube:



Building a new mechanism:

$$\mathcal{L} = \dot{\mathbf{x}}^2 - \mathcal{V}(\mathbf{x}) + \mathcal{A}(\mathbf{x}) \cdot \dot{\mathbf{x}}$$

$$\downarrow$$

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{2}\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right] - \frac{\lambda}{4f}\mathcal{X}\frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}}\mathrm{Tr}\left[F_{\mu\nu}F_{\alpha\beta}\right]\right]$$

Start with the basics...

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{2}\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right] - \frac{\lambda}{4f}\mathcal{X}\frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}}\mathrm{Tr}\left[F_{\mu\nu}F_{\alpha\beta}\right]\right]$$

Usual inflationary action.

with something like $V(\mathcal{X}) = \mu^4\left(1 + \cos\left(\frac{\mathcal{X}}{f}\right)\right)$

"Natural Inflation" - Freese, Frieman and Olinto '90

add gauge fields,

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{2}\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right] - \frac{\lambda}{4f}\mathcal{X}\frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}}\mathrm{Tr}\left[F_{\mu\nu}F_{\alpha\beta}\right]\right]$$

Action for a vector (gauge) field theory.

and let them interact.

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{2}\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right] - \frac{\lambda}{4f}\mathcal{X}\frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}}\mathrm{Tr}\left[F_{\mu\nu}F_{\alpha\beta}\right]\right]$$

Interaction

- Dimension 5 operator
- Chern-Simons term (quasi-topological)
- First order in time derivatives ("magnetic field" in field space)

Consider a classical, *homogeneous* field:

Discussed circa 1980 for SU(2) fields,

$$A_0^a = 0 \qquad A_i^a = \psi(t)a(t)\delta_i^a$$

solves the non-Abelian gauge field equations of motion on an FRW background.

$$E_{\mathrm{chromo}} \propto \dot{\psi} + H\psi \qquad B_{\mathrm{chromo}} \propto \tilde{g}\psi^2$$

This is 'Chromo-Natural' Inflation.

- Equations of motion:

$$\ddot{\mathcal{X}} + 3H\dot{\mathcal{X}} + V'(\mathcal{X}) = -3\tilde{g}\frac{\lambda}{f}\psi^2(\dot{\psi} + H\psi)$$

$$\ddot{\psi} + 3H\dot{\psi} + (\dot{H} + 2H^2)\psi + 2\tilde{g}^2\psi^3 = \tilde{g}\frac{\lambda}{f}\psi^2\dot{\mathcal{X}},$$

Ordinary friction

Potential force

'Magnetic' force

P.Adshead and M. Wyman, PRL: 108,261302 (2012), PRD 86, 043530 (2012)

Magnetic drift leads to slow roll.

- In the slow-roll, large λ limit, the system simplifies.

$$\dot{\mathcal{X}} = \frac{f}{\lambda}\left(2\tilde{g}\psi + \frac{2H^2}{\tilde{g}\psi}\right)$$

$$\dot{\psi} = -H\psi + \frac{f}{3\tilde{g}\lambda}\frac{V'(\mathcal{X})}{\psi^2}$$

- To a good approximation

  - Gauge field quasi-static: $\psi = \left(\frac{V'}{3\tilde{g}\lambda H}\right)^{1/3}$

  - Axion velocity independent of V' : $\frac{\dot{\mathcal{X}}}{f} \sim \frac{H}{\lambda}$

(EJM, P.Adshead, M.Wyman 2012)

Sufficient Inflation?

- Total number of e-foldings determined from

$$N(\mathcal{X}_0) = \int_{\mathcal{X}_{\text{end}}}^{\mathcal{X}_0} \left(\frac{d\mathcal{X}}{dN} \right)^{-1} d\mathcal{X}$$

$$= \int_{\pi}^{\mathcal{X}_0/f} \frac{\frac{1}{2}\left(3\tilde{g}^2 \lambda^4 \mu^4 (1+\cos x)^2 \sin x\right)^{1/3}}{\left(\lambda^2 \mu^4 (1+\cos x)^4\right)^{1/3} + (3\tilde{g}^2 \sin x)^{2/3}} dx$$

- Inflation duration maximized for $\dfrac{\tilde{g}^2}{\lambda} = \dfrac{\mu^4}{3}$

- Max e-foldings $N_{\max} \approx \dfrac{3}{5}\lambda \Rightarrow \lambda \sim \mathcal{O}(100)$

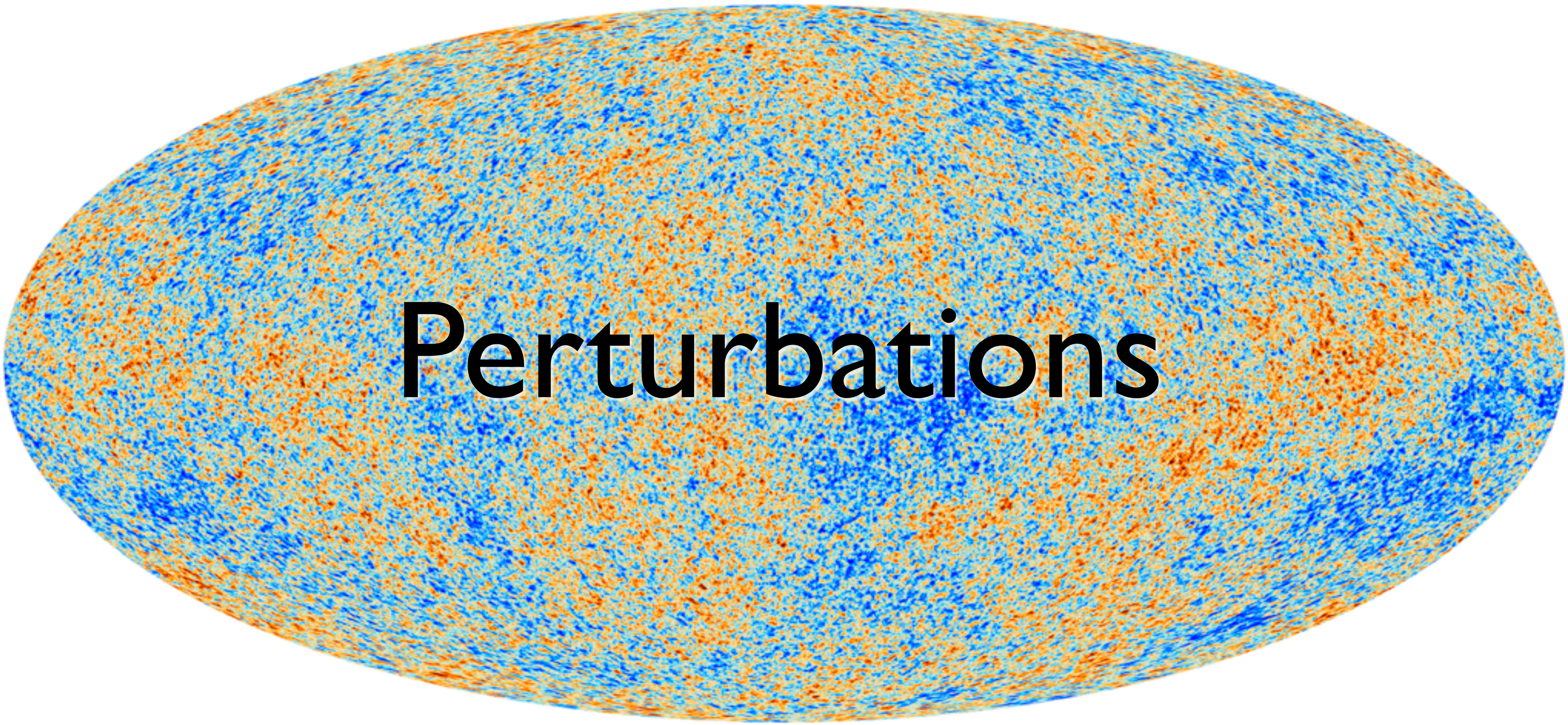
Possible concerns

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}(\partial\mathcal{X})^2 - V(\mathcal{X}) - \frac{1}{2}\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu}\right] - \frac{\lambda}{4f}\mathcal{X}\frac{\epsilon^{\mu\nu\alpha\beta}}{\sqrt{-g}}\mathrm{Tr}\left[F_{\mu\nu}F_{\alpha\beta}\right]\right]$$

- We require the tuning of a parameter in the Lagrangian to be order 100; but really this is just a ratio of scales
- Effective cutoff of the theory is lowered to f/λ
- Typical energy scale probed $H \sim \mu^2/M_{pl}$

Relatively easy to arrange for $H \ll f/\lambda$

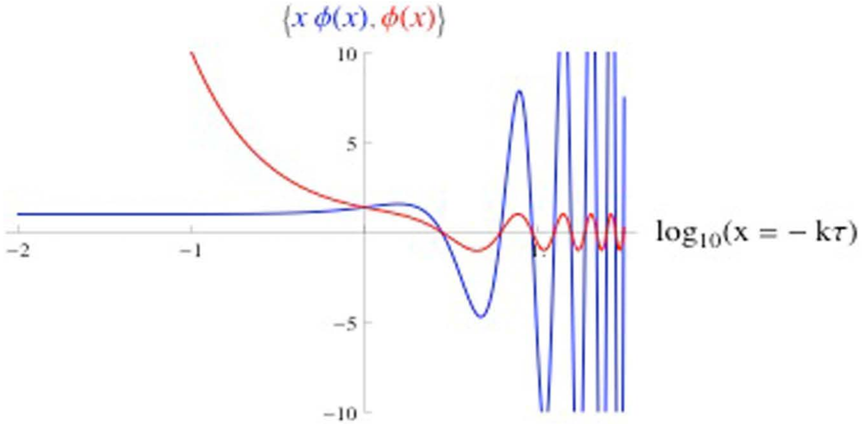
Perturbations



Baseline story for scalar modes

The CMB fluctuation spectrum has its origins in vacuum fluctuations of the inflaton. For a scalar inflaton in slow roll coupled only to gravity, these are fluctuations of a nearly massless free scalar field in background quasi-de Sitter geometry:

$$\langle \phi_k \phi_{-k} \rangle = \left(\frac{H}{2\pi}\right)^2$$



Time

$$\langle \mathcal{R}_k \mathcal{R}_{-k} \rangle$$

Einstein eqs

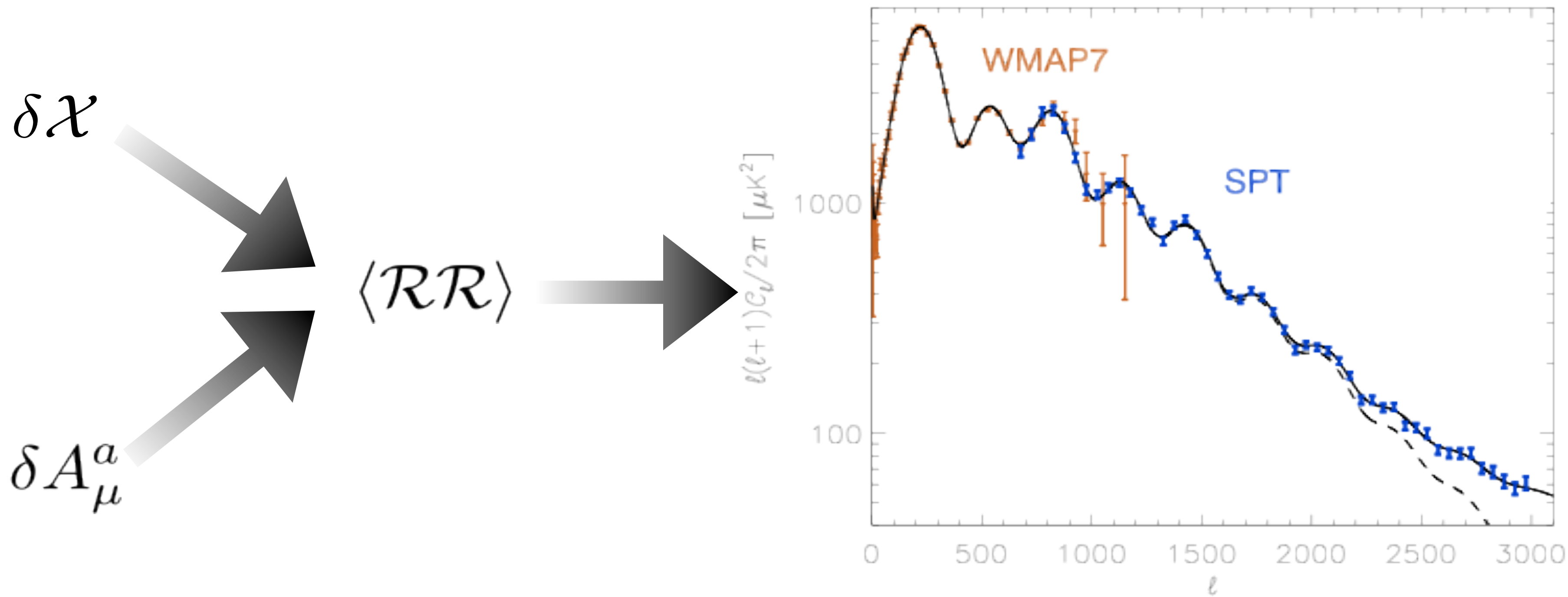
Observables: Overall power $O(10^{-10})$, and rate of change n_s of curvature flucts wrt time or scale

Gauge fields provide new physical consequences.

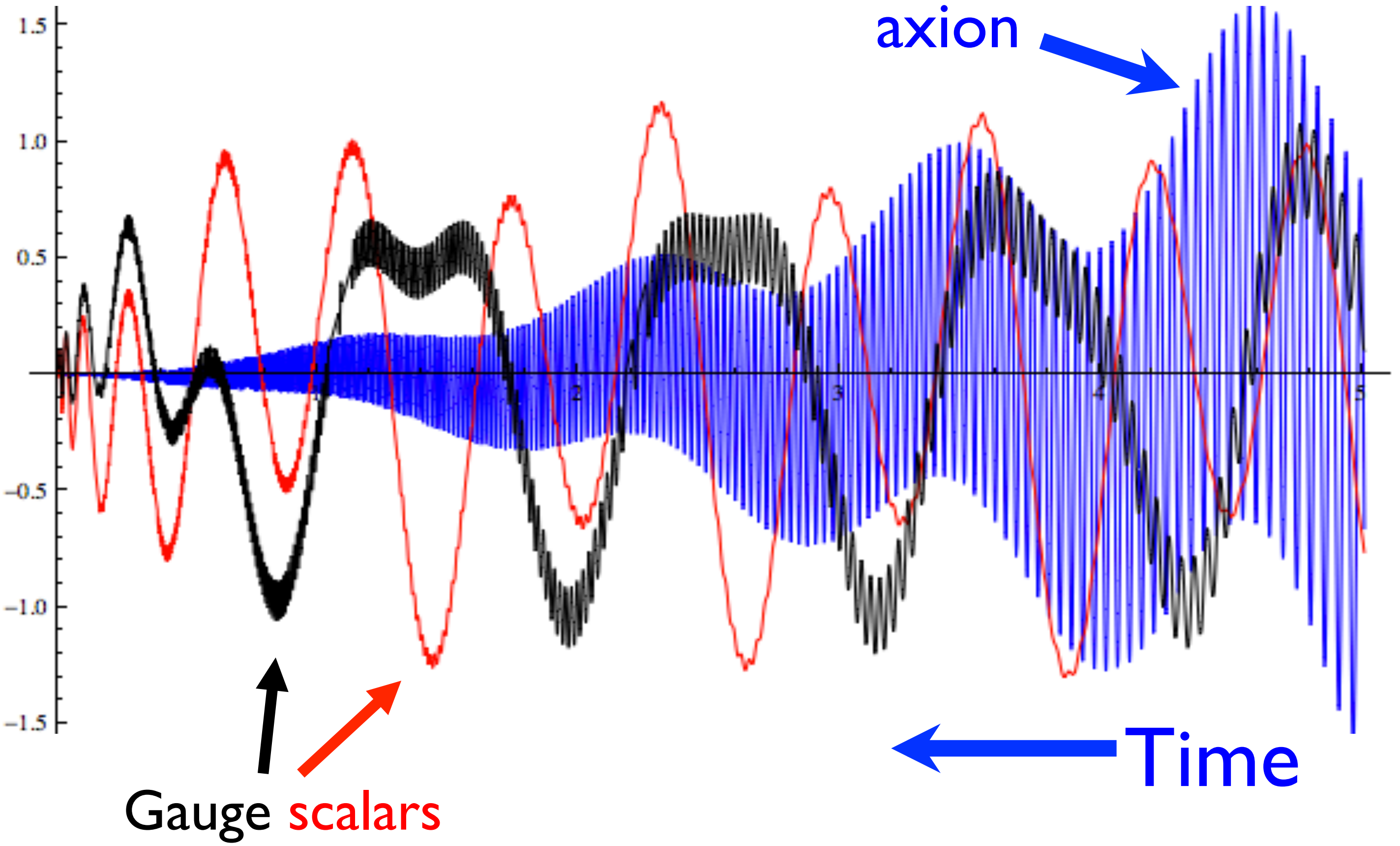
χ	A_μ^a
↓	↓
1 scalar	2 scalars 1 vector 1 tensor

New degrees of freedom = new observational handles.

1. The scalar system generates observable scalar curvature perturbations…

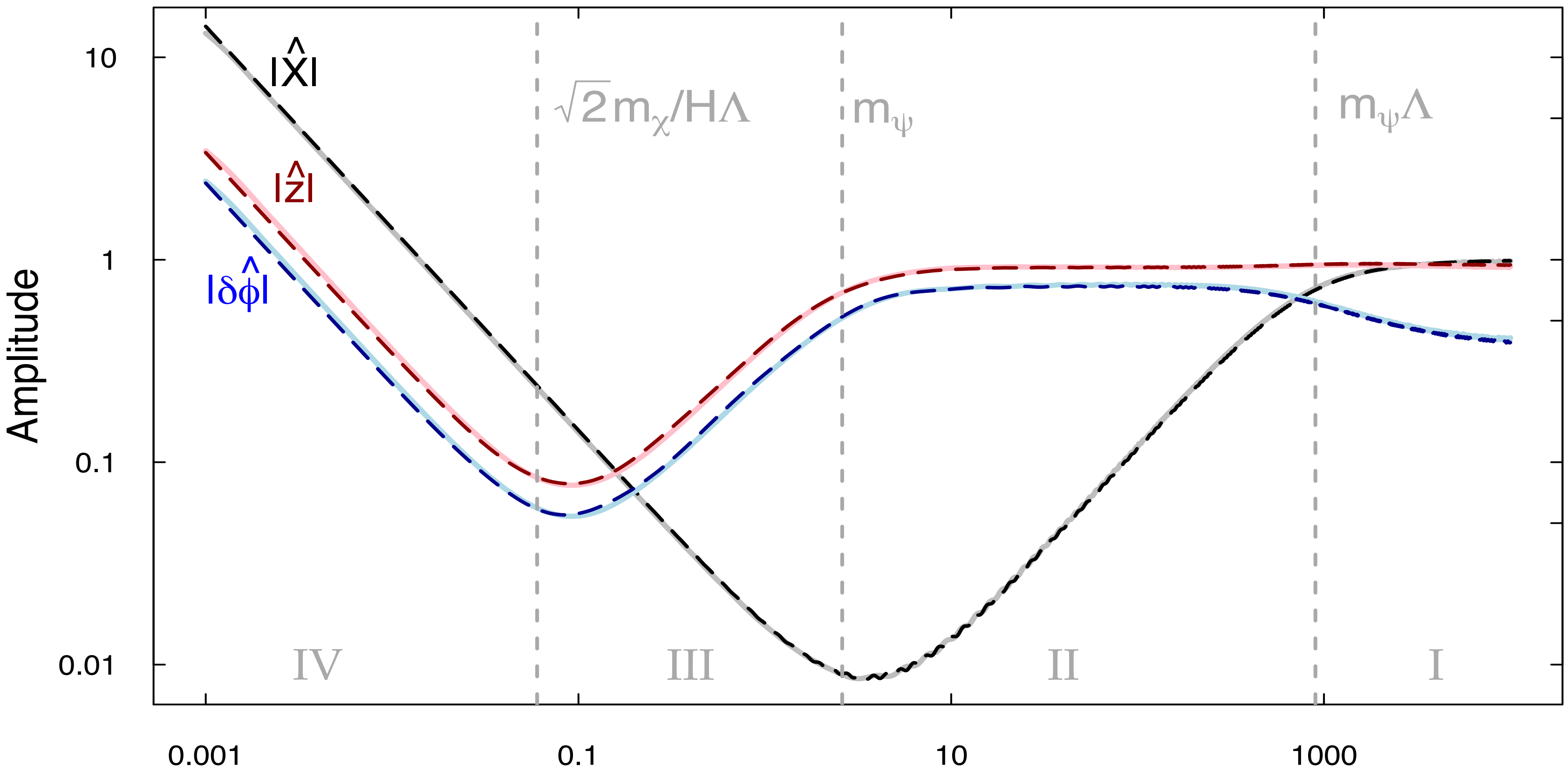


But it's a bit complicated…



…until you realize it's magnetic drift in a harmonic well!

There are several regimes of slow (magnetic drift) mode behavior…



$$m_\psi = \frac{g\psi}{H}$$

$$\Lambda = \frac{\lambda\psi}{f}$$

Time

…but in the end it's the familiar story

- Standard single-field inflation formulae apply

$$\mathcal{R} \approx \frac{\delta \mathcal{X}}{\dot{\mathcal{X}}/H}$$

- However, both numerator and denominator are smaller by a factor λ due to magnetic drift

Caveat:

- The background gauge field sets an effective mass scale for the gauge fields through the $g^2 [A,A]^2$ term in the action
- Physics depends on $m_\psi = \frac{g\psi}{H}$
- For $m_\psi < \sqrt{2}$ one of the gauge scalars goes unstable around $|k\tau| \approx m_\psi \Lambda$

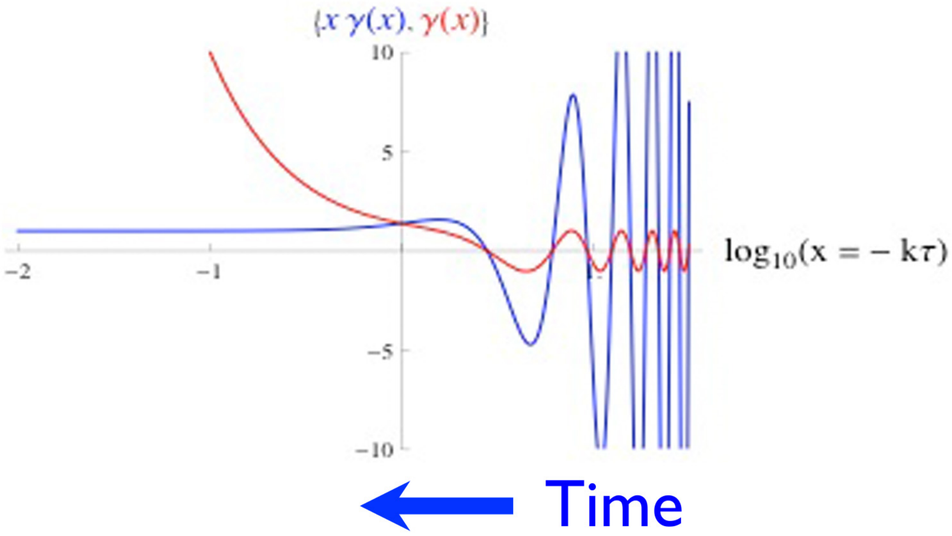
2. The tensor sector has new features.

$$ds^2 = -dt^2 + a^2 e^{\gamma_{ij}} dx^i dx^j$$

$$A_\mu = (0, a(t)\psi(t)\delta_i^a + t_i^a(t, \mathbf{x}))\,\frac{\sigma_a}{2}$$

Baseline story for tensor modes

Spin two modes of graviton $\gamma^{\pm}$ obey same wave equation as massless scalar in quasi-de Sitter background, leading to much the same analysis; one finds an approximately scale-invariant spectrum of primordial gravitons



$$\langle \gamma^{\pm}_{k} \gamma^{\pm}_{-k} \rangle = \left(\frac{H}{2\pi}\right)^2$$

In this case, the metric fluctuations directly give tensor curvature perturbation

Potential observable: ratio $r \sim O(\varepsilon)$ of tensor-to-scalar power

Gauge and gravity tensors mix.

- Equations of motion in helicity basis

$$\hat{\gamma}^{\pm\prime\prime} + \left(k^2 - \frac{2}{\tau^2}\right)\hat{\gamma}^{\pm} = \mathcal{C}t^{\pm}$$

$$\hat{t}^{\pm\prime\prime} + \left(k^2 + \frac{g\psi}{H}\frac{\lambda}{f}\dot{\mathcal{X}}\frac{1}{\tau^2}\right)\hat{t}^{\pm} \mp k\left(\frac{\lambda}{f}\dot{\mathcal{X}} + 2\frac{g\psi}{H}\right)\frac{1}{\tau}\hat{t}^{\pm} = \mathcal{C}\hat{\gamma}^{\pm}$$

- Gauge field spin-2 modes mix linearly with the graviton

-Linear mixing due to $\mathrm{Tr}[F_{ij}\delta F_{kl}]g^{ik}\delta g^{jl}$

(P.Adshead, EJM, M.Wyman, arXiv: 1301.2598, 1305.2930)

Gauge and gravity tensors mix.

usual equation

$$\hat{\gamma}^{\pm\prime\prime} + \left(k^2 - \frac{2}{\tau^2}\right)\hat{\gamma}^{\pm} = \mathcal{C}t^{\pm}$$

$$\hat{t}^{\pm\prime\prime} + \left(k^2 + \frac{g\psi}{H}\frac{\lambda}{f}\dot{\mathcal{X}}\frac{1}{\tau^2}\right)\hat{t}^{\pm} \mp k\left(\frac{\lambda}{f}\dot{\mathcal{X}} + 2\frac{g\psi}{H}\right)\frac{1}{\tau}\hat{t}^{\pm} = \mathcal{C}\hat{\gamma}^{\pm}$$

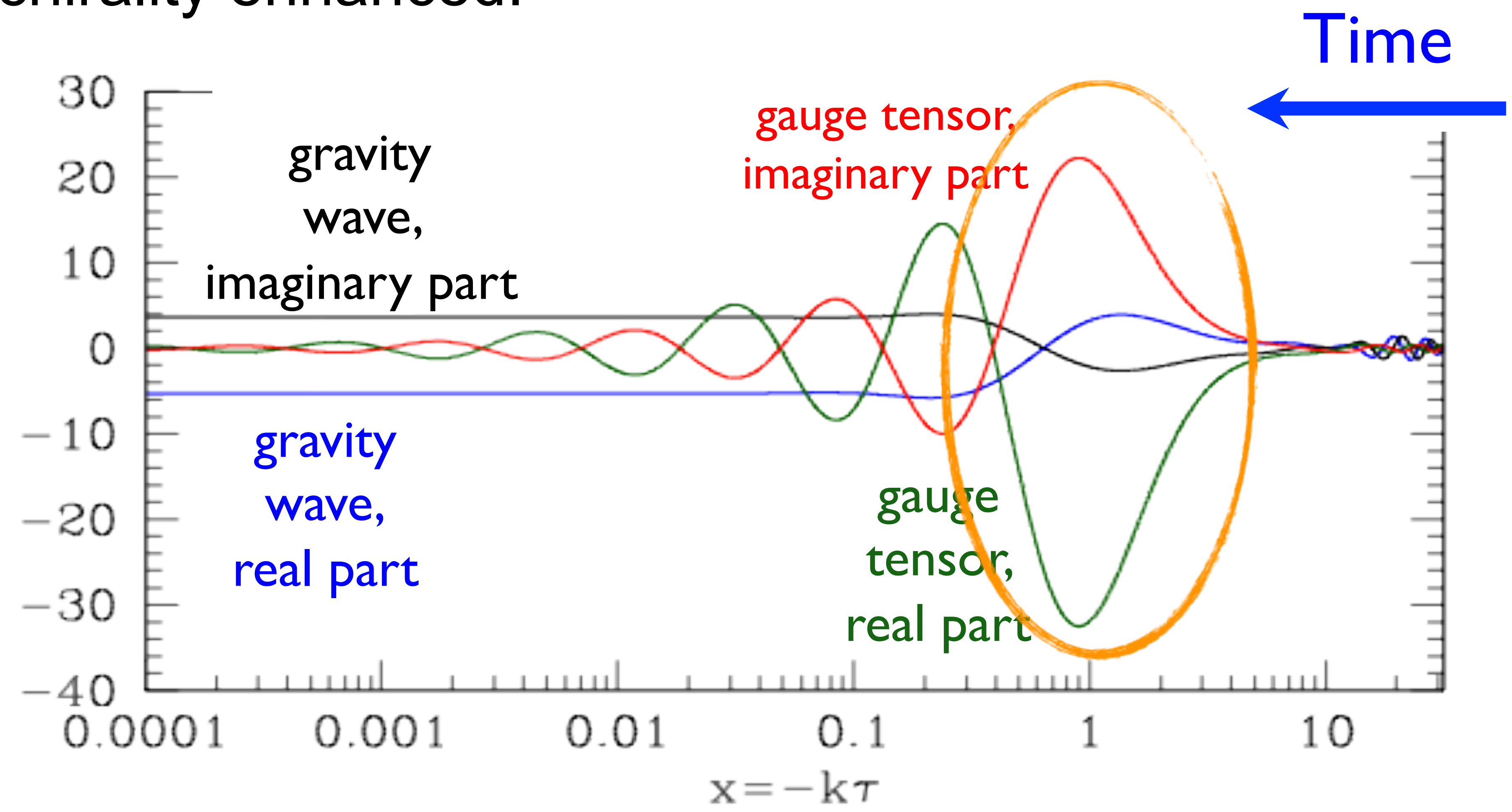
Gauge tensors *split;* one is **amplified.**
Enhancement grows roughly as $\exp[c(m_{\psi} - \sqrt{3})]$

- Parity is *spontaneously broken* by the background.

(PA, E. Martinec, M. Wyman, arXiv: 1301.2598)

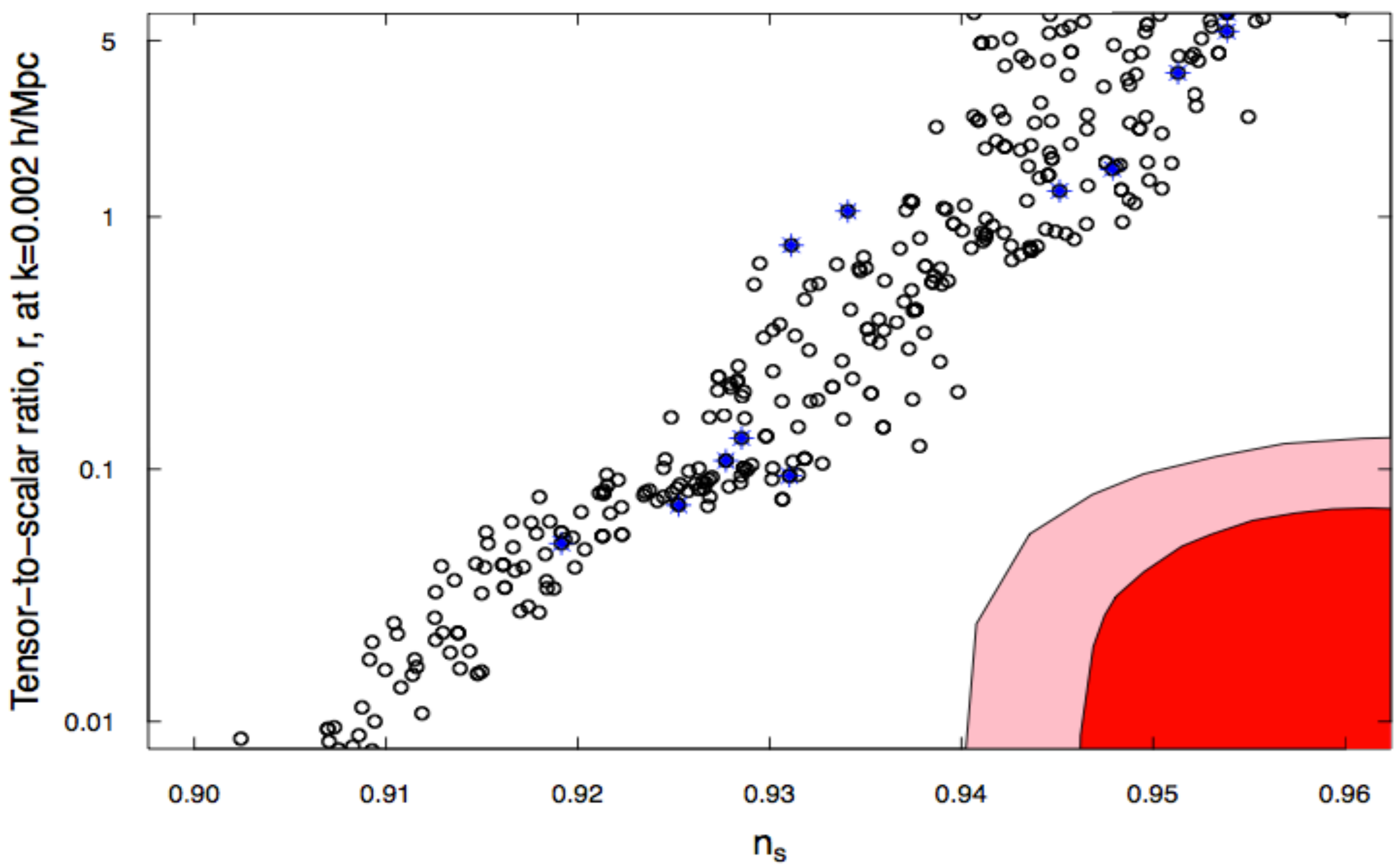
The gauge field feeds the gravitational waves

One chirality enhanced:



But this gravitational wave enhancement causes problems!

Tensor-to-scalar ratio



spectral tilt

Dynamics is caught between two instabilities:

- Gauge mass too small leads to scalar instability
- Gauge mass too large leads to tensor instability

But system is ultimately stable, just nonlinear

$$\ddot{\mathcal{X}} + 3H\dot{\mathcal{X}} + V'(\mathcal{X}) = -\frac{\lambda}{f}\mathrm{Tr}[\mathbf{E}\cdot\mathbf{B}]$$

$$\dot{\mathbf{E}} + H\mathbf{E} - \nabla\times\mathbf{B} = \frac{\lambda}{f}(\nabla\mathcal{X}\times\mathbf{E} + \dot{\mathcal{X}}\mathbf{B})$$

$$\nabla\cdot\mathbf{E} = \frac{\lambda}{f}\nabla\mathcal{X}\times\mathbf{B}$$

Summary

- Magnetic drift physics ➔ Slow roll inflation
- Shape of potential rendered irrelevant
- Mediated by Chern-Simons interactions
- 4D Chromo-Natural inflation model, but string theory CS terms give others
- ~1% (but technically natural) tuning in choosing a large λ
- Chiral gravitational waves!

Future directions.

- Modifications for consistency w/Planck data
- How will string theory versions differ?
- Are the scalar perturbations non-gaussian?
- Other connections to particle physics?

  e.g. Baryo/Lepto-genesis?

Invitation

- Another model of inflation - why do I care?

  -Rather than a new model, a new mechanism

Replace: $\eta_V = M_{\rm Pl}^2 \frac{V''}{V} \ll 1$

With: $M_{\rm Pl}^2 \frac{V''}{V} \sim 1$ and (e.g.) $\mathrm{Tr}[F^2]$

Generic potential   Naturally light

- Definite testable predictions

  e.g. Parity violation and chiral gravitational waves…

More generally, can embed in SU(N)

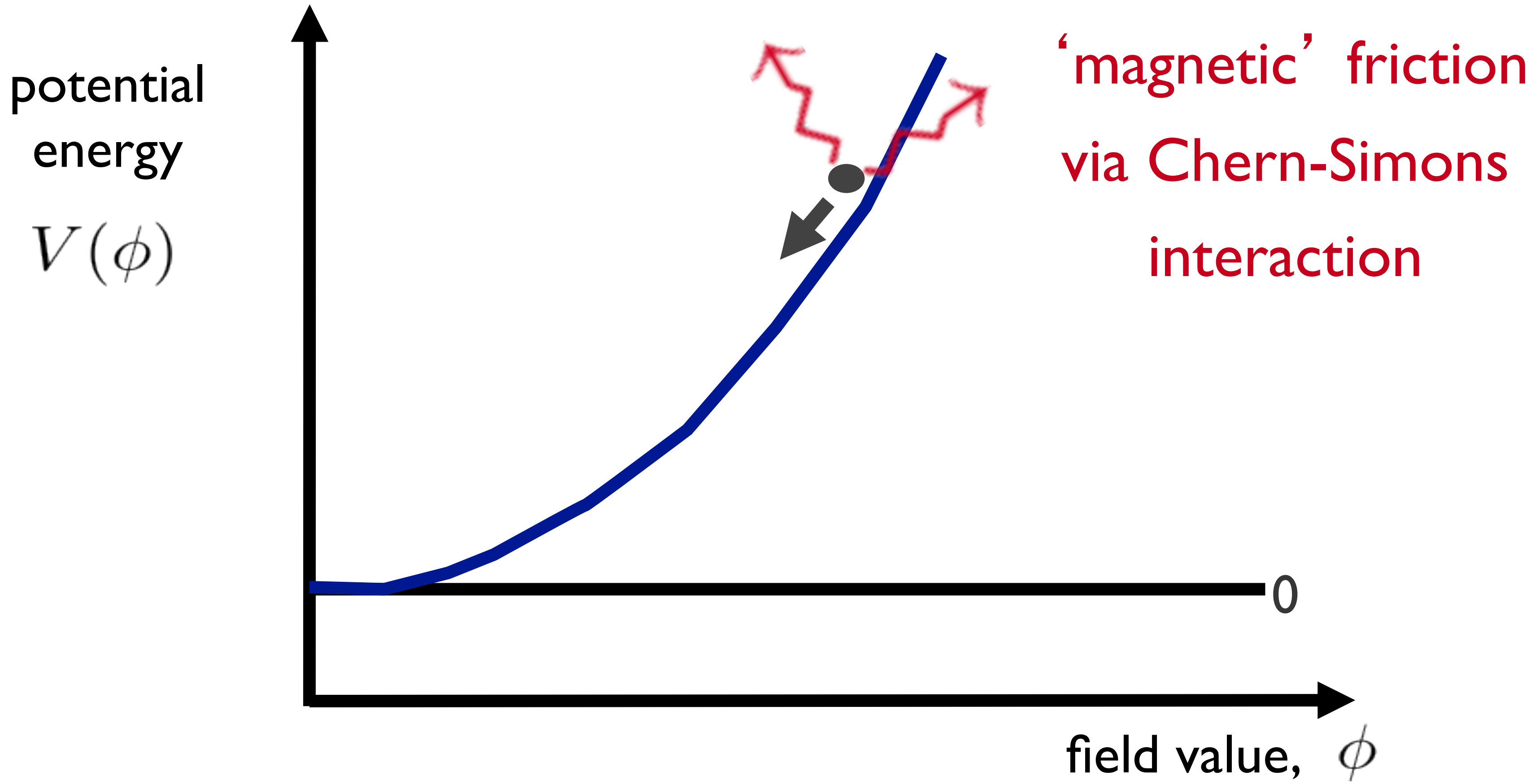
$$A_i(t) = \psi(t) a(t) J_i$$

where J_i generate SU(2) in an N-dimensional representation.

For large N, the gauge field configuration describes a 'fuzzy sphere'

$$J^2 = \frac{N(N^2-1)}{4} \mathbb{1}$$

Today, a way to avoid this hierarchy problem.



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