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Geometry Topology and Entanglement in the FQHE

F. Duncan M. Haldane
Princeton University

Strada Costiera, 11 - 34151 - Trieste - Italy • Tel. +39 0402240111 • Fax. +39 040224163 • sci_info@ictp.it • www.ictp.it
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Geometry Topology and Entanglement in the FQHE

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- hidden geometry of the Laughlin state
- geometrodynamics of the FQHE
- geometry and entanglement

Laughlin state

- originally introduced as a "lowest Landau level wavefunction"

(I will explain why this is a misleading characterization)

$$\Psi_L^q(\{\boldsymbol{r}_i\}) = \prod_{i<j}(z_i - z_j)^q \prod_i e^{-\frac{1}{2}z_i^* z_i}$$

The most striking feature for theorists is that this is holomorphic!

- usual interpretation of z is

$$z = \frac{x + iy}{\sqrt{2\ell_B^2}}$$

magnetic area: $2\pi\ell_B^2$

(contains one flux quantum h/e)

- Laughlin explained that his wavefunction had a holomorphic factor because it was a lowest-Landau level wavefunction.
- I will explain why the holomorphic character has a quite different origin!
- This will explain why the Laughlin state can be found in systems unrelated to lowest Landau level systems
- It will also reveal the fundamental geometric degree of freedom of the FQHE state.

standard derivation

- non-relativistic Galileian-invariant Landau levels

$$H = \frac{|\vec{p} - e\vec{A}(\boldsymbol{r})|^2}{2m} = \tfrac{1}{2}\hbar\omega_c(a^\dagger a + a a^\dagger)$$

(Note isotropic effective mass)

- Landau level ladder operators (in the "symmetric gauge"):

$$a = \tfrac{1}{2}z + \frac{\partial}{\partial z^*} \qquad a^\dagger = \tfrac{1}{2}z^* - \frac{\partial}{\partial z} \qquad [a, a^\dagger] = 1$$

lowest Landau level wavefunctions

$$a\psi(\boldsymbol{r}) = 0 \longrightarrow \psi(\boldsymbol{r}) = f(z)e^{-\frac{1}{2}z^* z}$$

holomorphic function $\times$ Gaussian

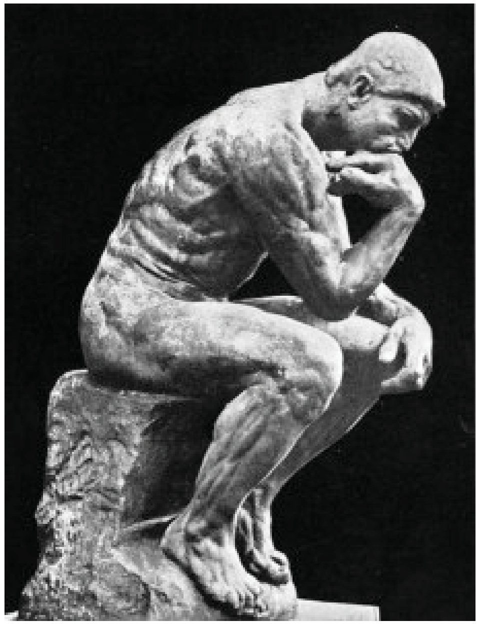
$$\Psi_L^q(\{\boldsymbol{r}_i\}) = \prod_{i<j}(z_i - z_j)^q \prod_i e^{-\frac{1}{2}z_i^* z_i}$$

- The $q = 3$ Laughlin state was confirmed (by numerical exact diagonalization studies) to be the essential description of the $1/3$ FQHE
- The holomorphic factor is incidentally noticed to be a cft correlator (conformal block) of the free boson cft with boson radius $R = \surd(2/q)$. (why?)

- So it is known to work, but **why?** (In my opinion, this question was never satisfactorily answered)

a common rationalization:

“Laughlin’s wavefunction cleverly lowers the Coulomb correlation energy by placing its zeroes at the locations of the particles”



we will see that this is an empty statement

problems with this

- The "explanation" of why the Laughlin state is correct are vague rationalizations, without quantitative content.
- The relation to cft is an empirical observation, and remains unexplained
- The $1/3$ FQHE state also occurs in the second Landau level and is described by the same Laughlin **state** (but not the same "wavefunction")
- It is recently also found on Chern-insulator lattice systems (by numerical diagonalization)

The physics of the FQHE in Landau levels is the physics of non-commuting "guiding centers" (quantum geometry) which cannot be described in terms of Schrodinger wavefunctions

$$\Psi \propto \prod_{i<j} (z_i - z_j)^q \prod_i e^{-\frac{1}{2} z_i^* z_i}$$

- $q = 1$ case is Slater determinant of filled lowest Landau level, uncorrelated, no topological order
- $q > 1$ case is highly correlated, topologically-ordered, related to a Jack polynomial

$$\prod_{i<j} (z_i - z_j)^q = \prod_{i<j} (z_i - z_j)^{q-1} J_\lambda^\alpha (z_1, \ldots, z_N)$$

$\alpha = -2$ Jack parameter

$\lambda = \{q(N-1), q(N-2), \ldots, q, 0\}$
padded partition of N parts

- Jack symmetric polynomials are:
- homogeneous and symmetric
- eigenfunctions of a Laplace-Beltrami operator

$$\alpha \sum_i (z_i \partial_{z_i})^2 J_\lambda^\alpha(z) + \sum_{i<j} (z_i - z_j)^{-1} (z_i \partial z_i - z_j \partial_{z_j}) J_\lambda^\alpha(z) = E_\lambda(\alpha) J_\lambda^\alpha(z)$$

$\alpha = -2$ Laughlin (and quasi holes)

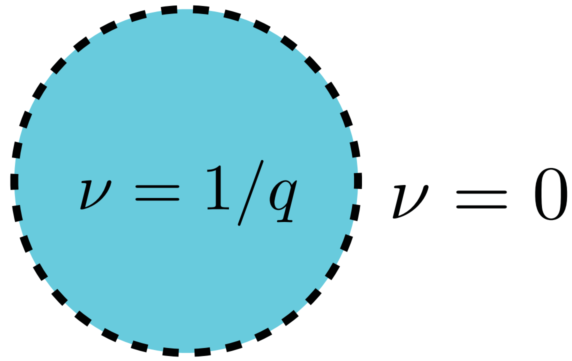
$\alpha = -3$ Moore-Read

$\alpha = -k, \quad k-1 = 3, 4, 5 \ldots$ Read-Rezayi Z_{k-1} parafermion states

- single particle states $\psi_m \propto z^m e^{-\frac{1}{2}z^*z}$
- radially ordered

- "root partition" of Jack:

$$n_m(\lambda) = \sum_i \delta_{m,\lambda_i}$$



"Fermi point"

(fuzzy) boundary of circular droplet

$$1111111111111|000000\ldots, \quad q = 1$$

$$1001001001001|000000\ldots, \quad q = 3$$

"not more than one particle in any group of q consecutive orbitals"

exclusion statistics ($q > 1$)

- "squeezing property" of Jacks

$$(z_1 - z_2)^3 = (z_1^3 z_2^0 - z_1^0 z_2^3) - 3(z_1^2 z_2^1 - z_1^1 z_2^2)$$

$$10010|000\ldots \qquad 01100|000\ldots$$

$$10010|000\ldots$$ root partition λ

$$01100|000\ldots$$ partition $\mu \prec \lambda$ dominated by root

a "squeeze"

- actual occupation of orbitals differs from root.

A Luttinger-type sum rule

$$N = \sum_m n_m^0 = \sum_m n_m$$

root occupation pattern

true occupation pattern after squeezing

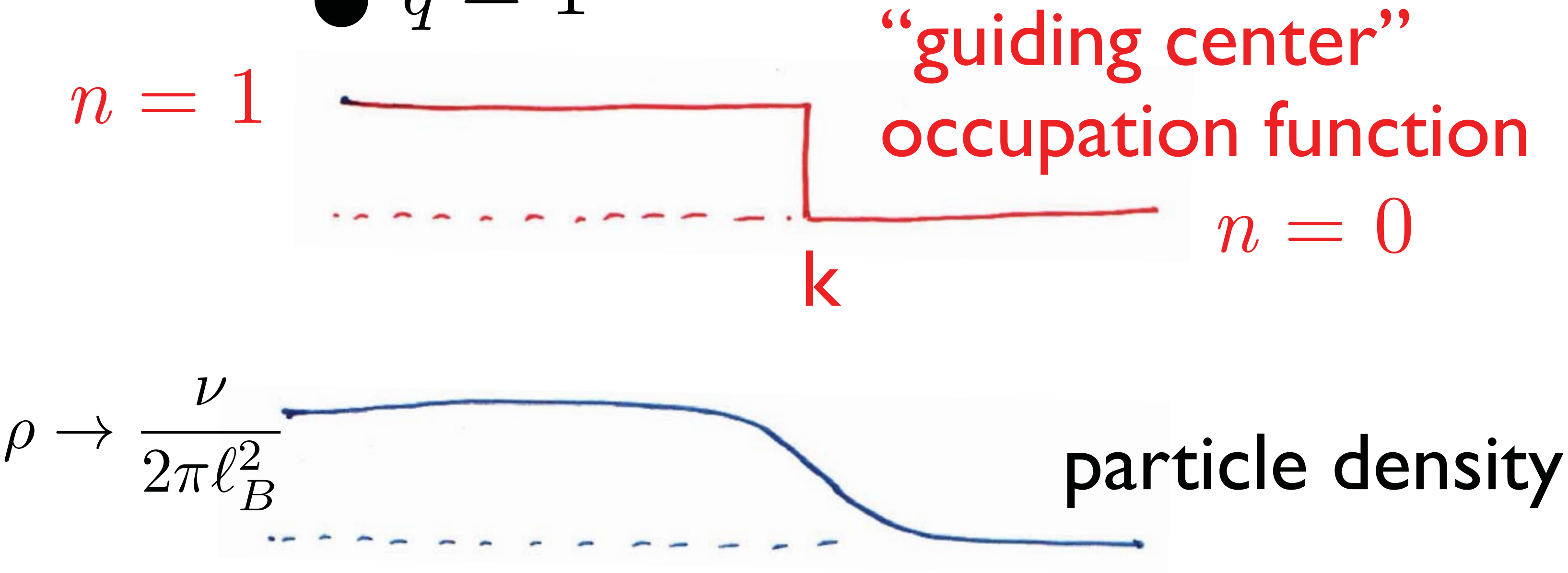
- in thermodynamic limit (map to cylinder, with circumference L, then infinite plane)

$$k = \frac{2\pi m}{L}$$

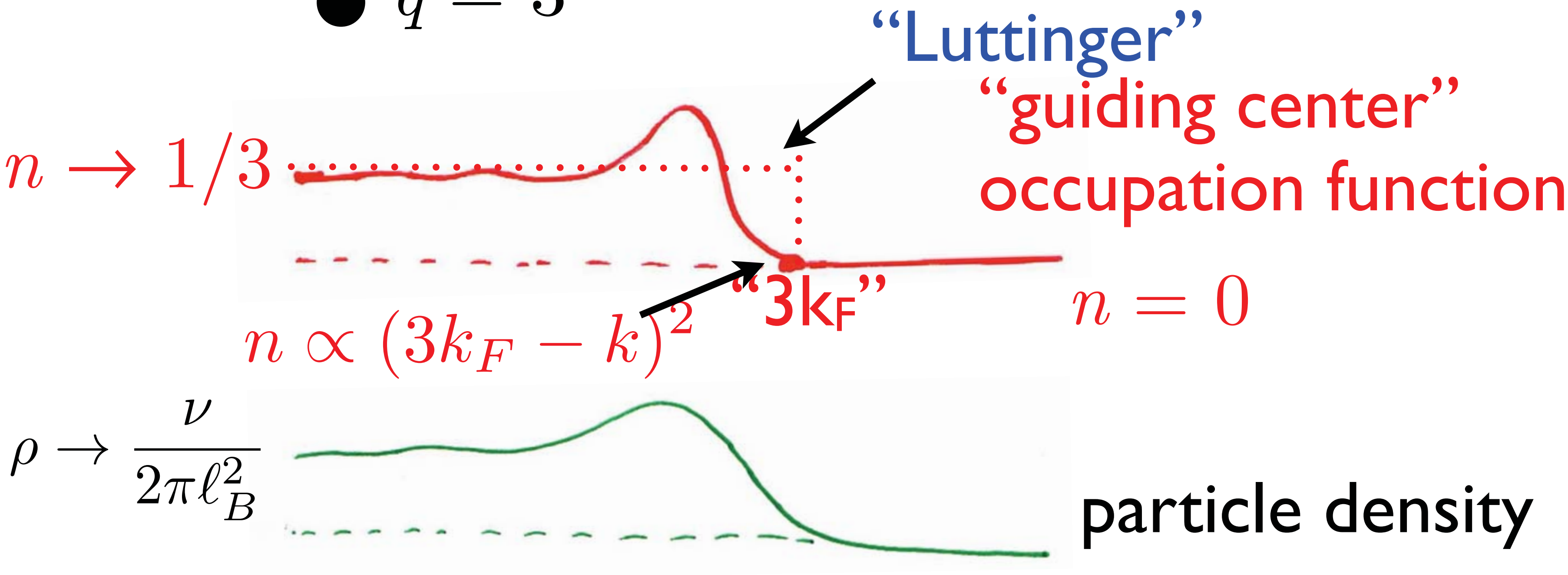
$$\int_{-\infty}^{\infty} dk\,(n(k) - n^0(k)) = 0$$

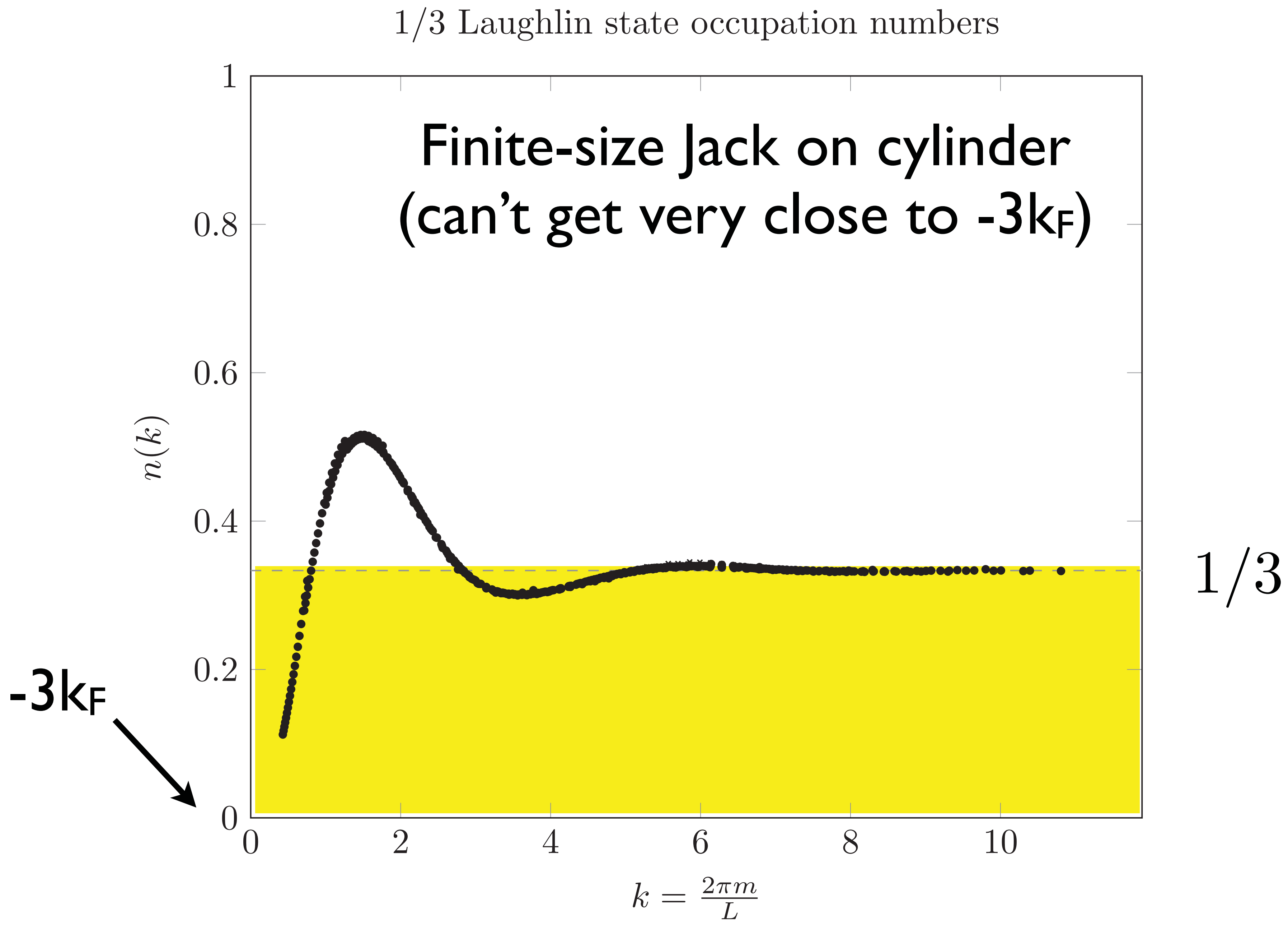
$$\begin{cases} n^0(k) = 1/q & k < qk_F \\ n^0(k) = 0 & k > qk_F \end{cases}$$

- $q = 1$


$n = 1$
"guiding center"
occupation function
$n = 0$
k
$\rho \to \frac{\nu}{2\pi\ell_B^2}$
particle density

- $q = 3$


"Luttinger"
"guiding center"
occupation function
$n \to 1/3$
"3kF"
$n = 0$
$n \propto (3k_F - k)^2$
$\rho \to \frac{\nu}{2\pi\ell_B^2}$
particle density



(cft shows small $k+3k_F$ behavior is quadratic)

- The occupation functions are highly structured, with generalized Fermi point singularities, and directly reflect the properties of the Jack polynomials, which are deeply related to conformal field theory.
- In contrast, the lowest-Landau electron densities derived from the interpretation of the Laughlin state as a Schroedinger wavefunction are rather smooth and featureless (WHY?)

In fact. all the non-trivial structure is present in the "guiding-center" degrees of freedom **without reference to Landau level structure**.

- The FQH is a correlated state of the non-commuting GUIDING CENTERS of quantized Landau orbits, obeying the algebra

$$[R^x, R^y] = -i\ell_B^2$$

- The classical coordinate of the electrion combines this with the Landau orbit radius

$$\left.\begin{aligned} \boldsymbol{r} &= \boldsymbol{R} + \tilde{\boldsymbol{R}} \\ [\tilde{R}^x, \tilde{R}^y] &= i\ell_B^2 \\ [\tilde{R}^x, R^y] &= 0 \end{aligned}\right\} \quad [r^x, r^y] = 0$$

- A Schroedinger wavefunction requires **<u>both</u>** degrees of freedom

$$\Psi(\boldsymbol{r}) = \langle \boldsymbol{r} | \left[|\psi_{\boldsymbol{R}}\rangle \otimes |\tilde{\psi}_{\tilde{\boldsymbol{R}}}\rangle \right]$$

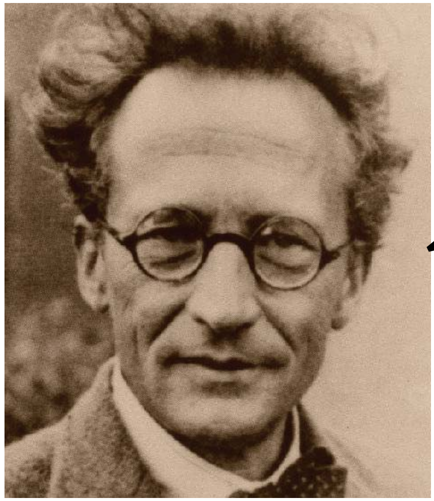
- The fundamental description of FQH states is a state $|\Psi_{\boldsymbol{R}_1,\ldots\boldsymbol{R}_N}\rangle$ in the many-guiding-center Hilbert space

- To make a wavefunction, with all particles in the same Landau level, we must "dress" it with a trivial state describing Landau orbits:

$$|\Psi_{\boldsymbol{R}_1,\ldots\boldsymbol{R}_N}\rangle \otimes \prod_{\otimes,i=1}^{N} |\psi_{\tilde{\boldsymbol{R}}_i}\rangle$$

- We can recover $|\Psi_{\boldsymbol{R}_1,\ldots\boldsymbol{R}_N}\rangle$ by "undressing" Laughlin's wavefunction.

Schrödinger vs Heisenberg



$\Psi(\boldsymbol{r})$

wavefunction
in real space
(classical geometry)

$|\Psi\rangle$

state in
in Hilbert space

- resolution of conflict: the two formulations of QM are equivalent:

$$\Psi(\boldsymbol{r}) = \langle \boldsymbol{r} | \Psi \rangle$$

iff $\exists \; |\boldsymbol{r}\rangle$ s.t.

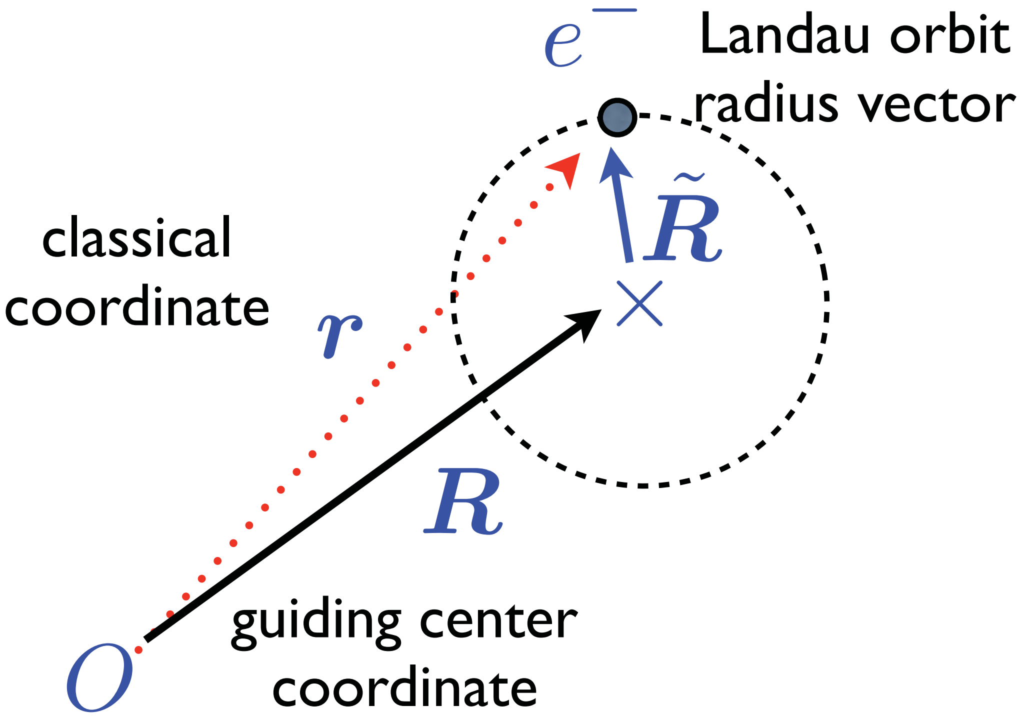
$$\langle \boldsymbol{r} | \boldsymbol{r}' \rangle = 0, \quad \boldsymbol{r} \neq \boldsymbol{r}'$$

requires an orthonormal basis in real space **obeying classical locality**

- classical locality (and Schrödinger-Heisenberg equivalence) fails after Landau quantization!

$$\boldsymbol{r} = \boldsymbol{R} + \tilde{\boldsymbol{R}}$$

$$\boldsymbol{r} = r^a \boldsymbol{e}_a$$
$$[r^a, r^b] = 0$$



non-commutative algebra

$$[\tilde{R}^a, \tilde{R}^b] = i\ell_B^2 \epsilon^{ab}$$
$$[R^a, R^b] = -i\ell_B^2 \epsilon^{ab}$$
$$[R^a, \tilde{R}^b] = 0$$

$$p_a - eA_a(\boldsymbol{r}) \equiv \epsilon_{ab}\hbar\tilde{R}^a/\ell_B^2 \qquad \ell_B^2 = \frac{\hbar}{eB} > 0$$

$$\boldsymbol{r} = \boldsymbol{R} + \tilde{\boldsymbol{R}}$$

eliminated by Landau quantization

- residual guiding center degrees of freedom are non-commutative

$$[R^a, R^b] = -i\ell_B^2 \epsilon^{ab}$$

- isomorphic to phase space, obeys uncertainty principle

guiding centers cannot be localized within an area less than $2\pi\ell_B^2$

- The Hamiltonian governing the residual guiding-center degrees of freedom:

$$H = \int \frac{d^2\boldsymbol{q}\ell_B^2}{2\pi} U(\boldsymbol{q}) \sum_{i<j} e^{i\boldsymbol{q}\cdot(\boldsymbol{R}_i - \boldsymbol{R}_j)}$$

$$U(\boldsymbol{q}) = \frac{\tilde{V}(\boldsymbol{q}) f(\boldsymbol{q}) f(-\boldsymbol{q})}{2\pi\ell_B^2}$$

$$\tilde{V}(q) = \int d^2\boldsymbol{r}_{ij}\ V(\boldsymbol{r}_{ij}) e^{i\boldsymbol{q}\cdot\boldsymbol{r}_{ij}}$$

Fourier transformed Coulomb interaction

$$f_n(q) = \langle\psi_n|e^{i\boldsymbol{q}\cdot\tilde{\boldsymbol{R}}}|\psi_n\rangle = L_n(u)e^{\frac{1}{2}u}$$

$$u = \frac{1}{2}|\boldsymbol{q}|^2\ell_B^2$$

Landau level form factor
(n = landau level index)

(depends on Landau orbit)

- in this limit, the state is an unentangled product of a non-trivial state of the guiding centers with a trivial state of the Landau orbits

$$|\Psi\rangle = |\Psi_{\boldsymbol{R}}\rangle \otimes |\Psi_{\tilde{\boldsymbol{R}}}\rangle$$

full state (which has a Schrödinger representation)

FQHE is here !

depends only on $U(\boldsymbol{q})$

Trivial state of Landau orbits

$$\prod_{\otimes} |\psi_n(\tilde{\boldsymbol{R}}_i)\rangle$$

characterized by form factor $f_n(\boldsymbol{q})$

- In what follows, I will regard the essential FQHE state as the purely-guiding center state defined by

$$H = \int \frac{d^2\boldsymbol{q}\ell_B^2}{2\pi} U(\boldsymbol{q}) \sum_{i<j} e^{i\boldsymbol{q}\cdot(\boldsymbol{R}_i - \boldsymbol{R}_j)}$$

$$[R^a, R^b] = -i\ell_B^2 \epsilon^{ab}$$

"quantum geometry*"

$$\rho(\boldsymbol{q}) = \sum_i e^{i\boldsymbol{q}\cdot\boldsymbol{R}}$$

$$[\rho(\boldsymbol{q}), \rho(\boldsymbol{q}')] = 2i\sin\left(\tfrac{1}{2}\epsilon^{ab} q_a q_b \ell_B^2\right)\rho(\boldsymbol{q} + \boldsymbol{q}')$$

GMP 1985

* "triple" {algebra,representation,Hamiltonian} satisfies Connes' definition

$$[R^a, R^b] = -i\ell_B^2 \epsilon^{ab}$$

- given a complex structure (Kähler form) one can define ladder operators

$$\omega_a^* \omega_b = \frac{1}{2}(g_{ab} - i\epsilon_{ab})$$

a Euclidean metric
det g = 1

2D antisymmetric
(Levi-Civita) symbol

$$\bar{a} = (\omega_a R^a)/\ell_B$$

$$[\bar{a}, \bar{a}^\dagger] = 1$$

- guiding-center "spin":

$$[L, \bar{a}^\dagger] = a^\dagger$$

$$L(g) = g_{ab}\Lambda^{ab}$$

$$\Lambda^{ab} = \frac{\{R^a, R^b\}}{4\ell_B^2}$$

generators of area-preserving linear deformations of the guiding centers

- New insight: the choice of the Euclidean metric g_{ab} is (so far) **arbitrary** (previous work always chose it as $\mathrm{diag}(1,1)$ to be congruent to the shape of the Landau orbits)

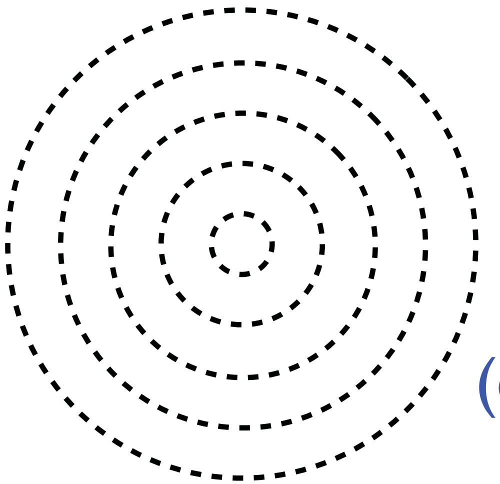
- The metric is a (hidden) variational parameter of the Laughlin **state,** and is the **fundamental physical degree of freedom** of FQHE states.

(the metric is fixed as $\mathrm{diag}(1,1)$ in the "Laughlin wavefunction")

- “symmetric gauge” basis Landau level states

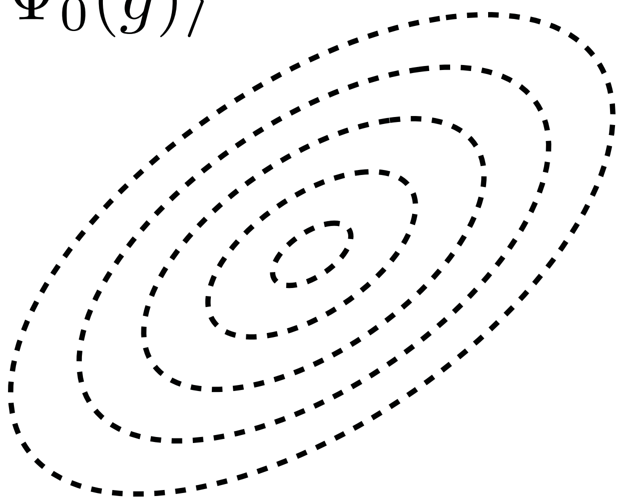
- basis of Landau level states with general metric

$$|\psi_m(g)\rangle = \frac{(\bar{a}^\dagger)^m}{\sqrt{m!}}|\Psi_0(g)\rangle$$



$$\bar{a}|\psi_0(g)\rangle = 0$$

(central coherent state)



wavefunctions in lowest Landau level:

$$\langle \boldsymbol{r}|\psi_0\rangle \propto e^{-\frac{1}{2}z^* z}$$

$$\langle \boldsymbol{r}|\psi_0\rangle \propto e^{\frac{1}{2}\gamma z^2} e^{-\frac{1}{2}z^* z}$$

$$|\gamma| < 1$$

- in the original Schroedinger/lowest Landau level language

$$a = \frac{1}{2}z + \frac{\partial}{\partial z^*}$$

$$a^\dagger = \frac{1}{2}z^* - \frac{\partial}{\partial z}$$

Landau levels

$$\bar{a} = \frac{1}{2}\bar{z} + \frac{\partial}{\partial \bar{z}^*}$$

$$\bar{a}^\dagger = \frac{1}{2}\bar{z}^* - \frac{\partial}{\partial \bar{z}}$$

Guiding centers

- general relation

$$\bar{z} = \alpha z^* + \beta z \qquad (|\alpha|^2 - |\beta|^2 = 1)$$

- original ("Laughlin wavefunction") relation

$$\bar{z} = z^* \longrightarrow$$

$$\bar{a} e^{-\frac{1}{2}zz^*} = a e^{-\frac{1}{2}z^* z} = 0$$

$$\bar{a}^\dagger f(z) e^{-\frac{1}{2}z^* z} = z f(z) e^{-\frac{1}{2}z^* z}$$

- one can now write the Heisenberg form of the Laughlin state, liberated from any dependence on the Landau orbit geometry

$$|\Psi_L^q(g)\rangle = \prod_{i<j} \left(\omega_a^*(R_i^a - R_j^a)\right)^q |\Psi_0(g)\rangle$$

$$\omega_a R_i^a |\Psi_0(g)\rangle = 0 \qquad \omega_a^* \omega_b = \tfrac{1}{2}(g_{ab} - i\epsilon_{ab})$$

- It is the exact zero-energy ground state of the "pseudopoential" model with

$$U(\boldsymbol{q}; g) = \sum_{m<q} V_m L_m(q_g^2 \ell_B^2) e^{-\frac{1}{2} q_g^2 \ell_B^2}$$

$$V_m > 0 \qquad q_g^2 \equiv g^{ab} q_a q_b$$

- coherent state basis

$$\bar{a}|\bar{z}\rangle = \bar{z}|\bar{z}\rangle \quad |\bar{z}\rangle = e^{\bar{z}\bar{a}^\dagger - \bar{z}^*\bar{a}}|0\rangle$$

$$S(\bar{z}, \bar{z}^*; \bar{z}', \bar{z}'^*) = \langle \bar{z}|\bar{z}'\rangle = e^{\bar{z}^*\bar{z}' - \frac{1}{2}(\bar{z}'^*\bar{z}' + \bar{z}^*\bar{z})}$$

- non-null eigenstates of the overlap define an orthonormal basis

$$\int \frac{d\bar{z}' d\bar{z}'^*}{2\pi} S(\bar{z}, \bar{z}^*; \bar{z}', \bar{z}'^*)\psi(\bar{z}', \bar{z}'^*) = \lambda\psi(\bar{z}, \bar{z}^*)$$

- non-null eigenstates are degenerate with $\lambda = 1$

$$\psi(\bar{z}, \bar{z}^*) = f(\bar{z}^*)e^{-\frac{1}{2}\bar{z}^*\bar{z}}$$

holomorphic!

"accidentally" coincide with lowest-Landau level wavefunctions if $\bar{z} = z^*$!!!

- This is the true origin of holomorphic functions in the theory of the FQHE
- NOTHING to do with lowest Landau level states, derives from overlaps between states in a non-orthogonal overcomplete basis!
- Has obvious parallels in theory of flat-band Chern insulators, where the projected lattice-site basis is non-orthogonal and overcomplete

$$|\Psi_L^q\rangle = \prod_i \int \frac{d\bar{z}_i^* d\bar{z}_i}{2\pi} \prod_{i<j} (\bar{z}_i^* - \bar{z}_j^*)^q \prod_i e^{-\frac{1}{2}\bar{z}_i^* \bar{z}_i} |\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_N\rangle$$

"Laughlin wavefunction"

many-particle coherent state

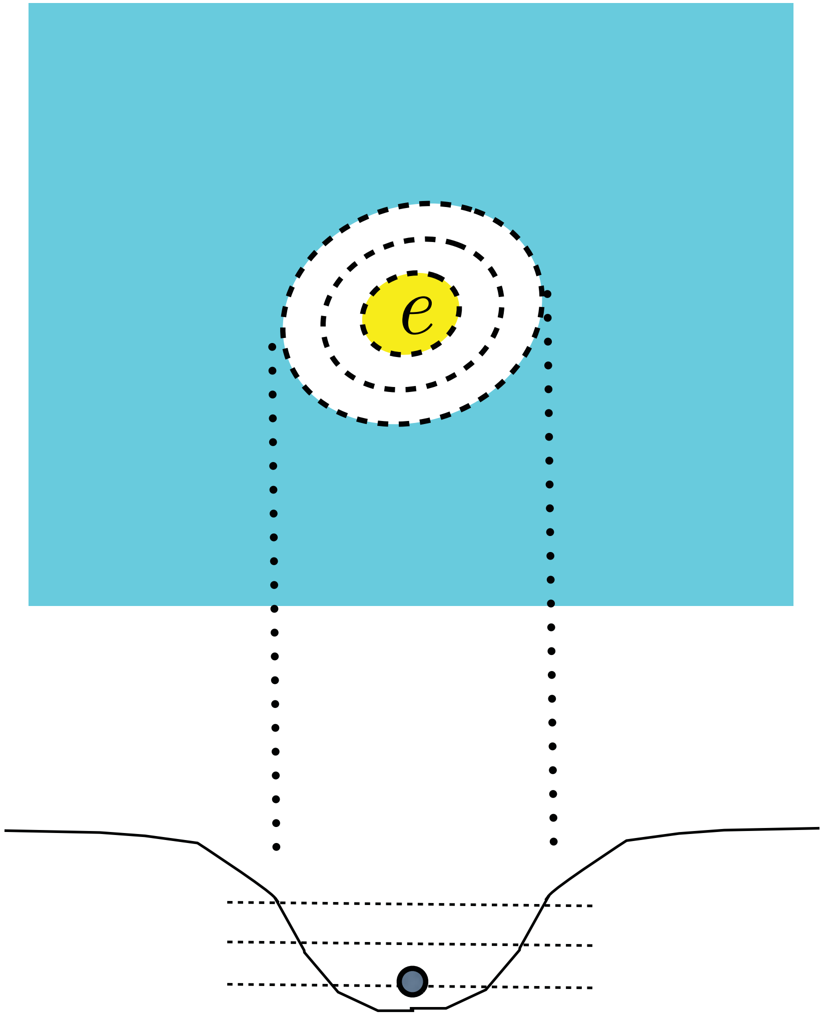
$$\bar{a}_i |\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_N\rangle = \bar{z}_i |\bar{z}_1, \bar{z}_2, l \ldots, \bar{z}_N\rangle$$

- The metric is a physical degree of freedom that characterizes the **shape** of the correlation hole surrounding a particle in the Laughlin state
- The $1/q$ Laughlin state can be characterized as describing a "condensate" of "composite bosons" formed by "attaching" q "flux quanta" (orbitals) to the particles.
- more generally, the composite boson is formed by attaching q "flux quanta" to p particles.

The metric describes the shape of the composite boson

$1/3$ Laughlin state

If the central orbital is filled, the next two are empty



The composite boson has inversion symmetry about its center

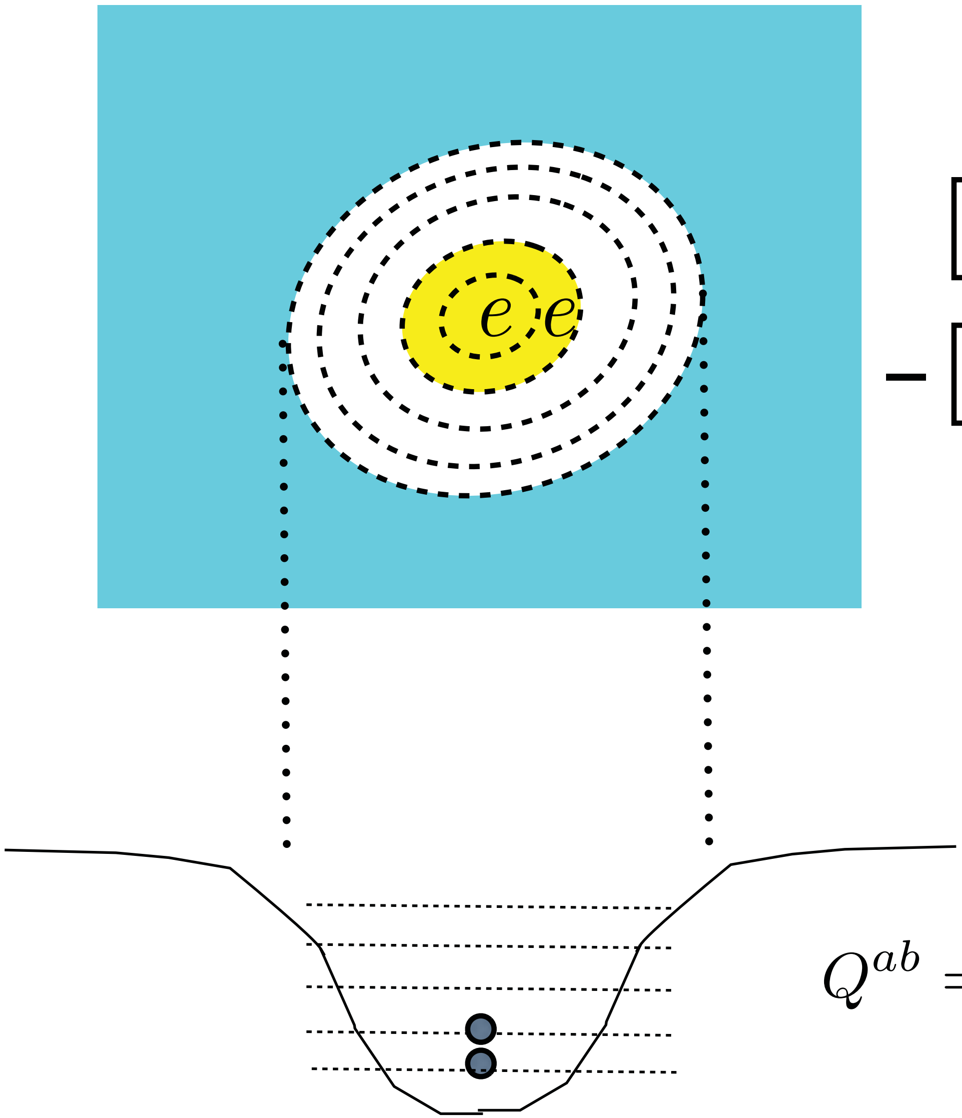
It has a "spin"

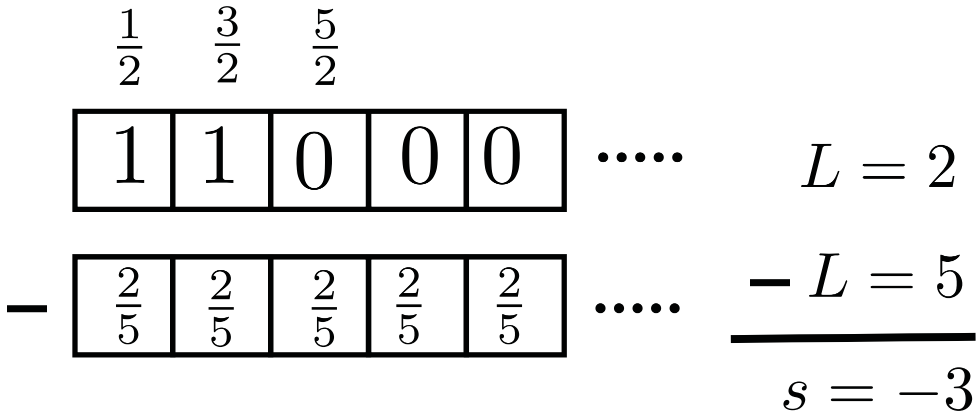
$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$		
1	0	0	.....	$L = \frac{1}{2}$
$-\ \frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	.....	$-\ L = \frac{3}{2}$

$$s = -1$$

the electron excludes other particles from a region containing 3 flux quanta, creating a potential well in which it is bound

$2/5$ state





$$L = \frac{g_{ab}}{2\ell_B^2} \sum_i R_i^a R_i^b$$

$$Q^{ab} = \int d^2r\, r^a r^b \delta\rho(r) = s\ell_B^2 g^{ab}$$

second moment of neutral
composite boson
charge distribution

- The composite boson behaves as a neutral particle because the Berry phase (from the disturbance of the the other particles as its “exclusion zone” moves with it) cancels the Bohm-Aharonov phase

- It behaves as a boson provided its statistical spin cancels the particle exchange factor when two composite bosons are exchanged

p particles	$(-1)^{pq} = (-1)^p$	fermions
q orbitals	$(-1)^{pq} = 1$	bosons

- The shape of the composite boson is determined by minimizing the sum of the correlation energy and the background potential energy.
- If there is no background potential, the metric is flat and the charge density is uniform
- If there is a background potential $g_{ab}(\boldsymbol{r})$ varies with position to give a charge density fluctuation

Gaussian curvature of metric

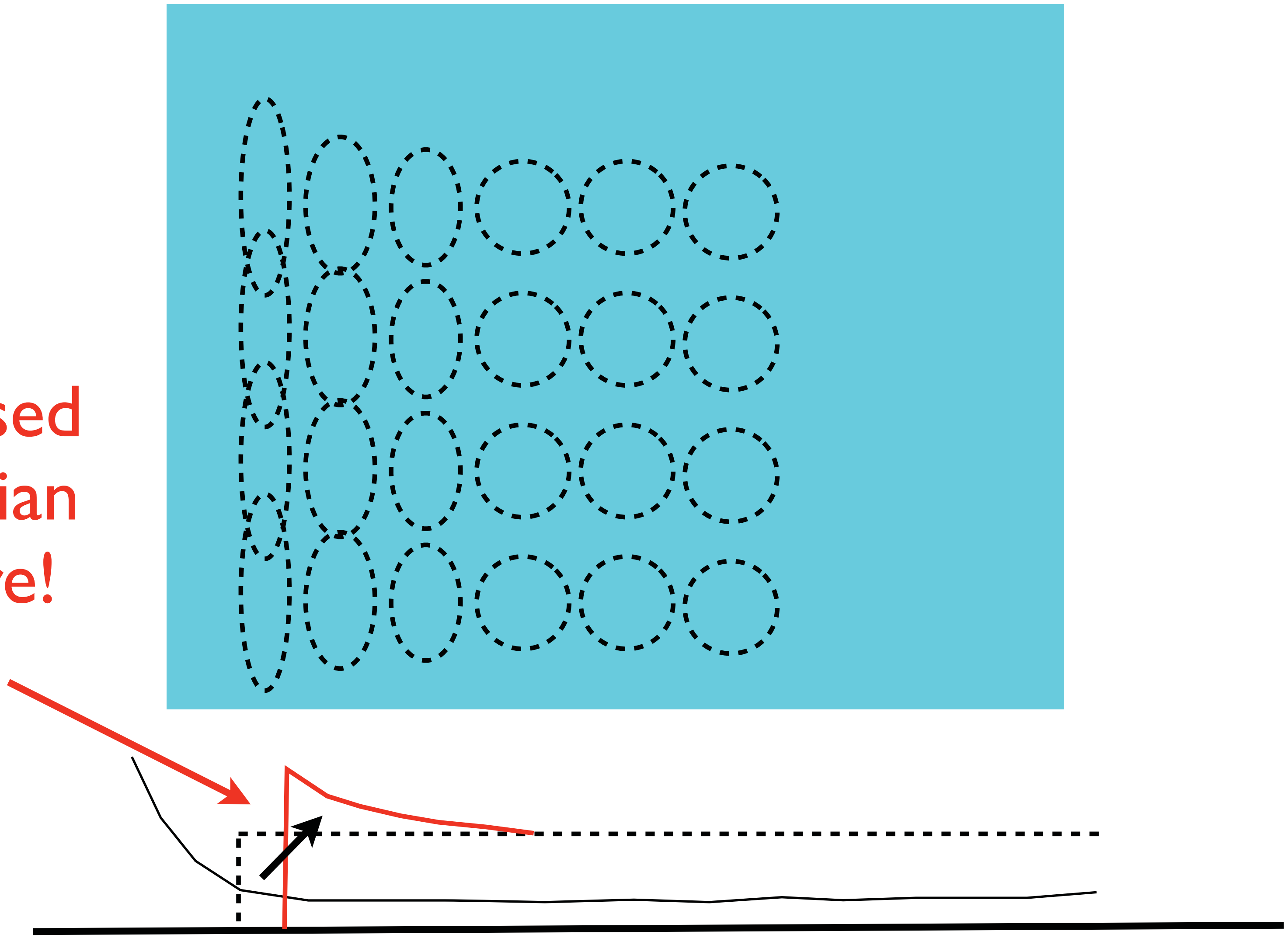
$$\delta\rho(\boldsymbol{r}) = esK(\boldsymbol{r})$$

"spin"

$$K(r) = \underbrace{\tfrac{1}{2}\partial_a\partial_b g^{ab}}_{\text{from variation of second moment of charge distribution}} + \underbrace{\tfrac{1}{8} g_{ab}\epsilon_{cd}\epsilon^{ef}\partial_e g^{ac}\partial_f g^{bd}}_{\text{from Berry phase associated with shape change}}$$

- metric deforms (preserving $\det\ g = 1$)in presence of non-uniform electric field

fluid compressed by Gaussian curvature!



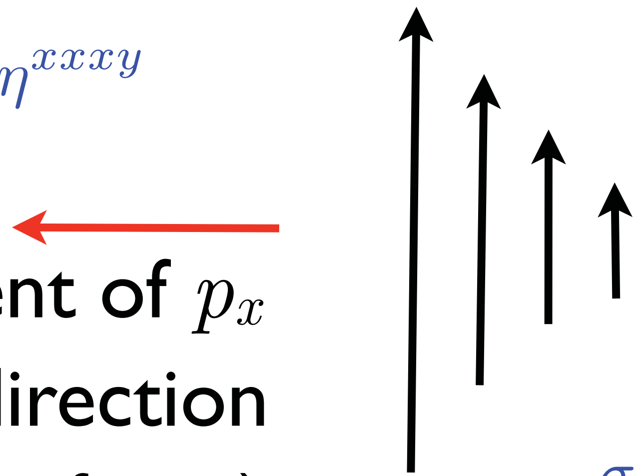
potential near edge

produces a dipole momemt

- Hall viscosity $\eta^{abcd} = \frac{eBs}{4\pi q} \frac{1}{2} \left(g^{ac} \epsilon^{bd} + g^{bd} \epsilon^{ac} + a \leftrightarrow b \right)$

(plus a similar term from the Landau orbit degrees of freedom (Avron et al))

η^{xxxy}



current of p_x
in x-direction
(stress force)

$\frac{dv^y}{dx}$

$\sigma^a_b = \epsilon_{be} \eta_H^{aecf} \epsilon_{cf} \partial_c v^d$

Hall viscosity determines a dipole moment per unit length at the edge of the fluid

- Total guiding center angular momentum of a fluid disk of N elementary droplets

$$L_{\mathrm{gc}} = \frac{1}{2\ell_B^2} g_{ab} \sum_i R^a R_i^b = \tfrac{1}{2} pq \bar{N}^2 + s_{\mathrm{gc}} \bar{N}$$

statistical (conformal) spin → $\tfrac{1}{2} pq \bar{N}^2$

geometric (guiding-center) spin → $s_{\mathrm{gc}} \bar{N}$

(dipole at edge)

$$P_b = B\epsilon_{ab} p^b$$

momentum → P_b

electric dipole → p^b

n(k)

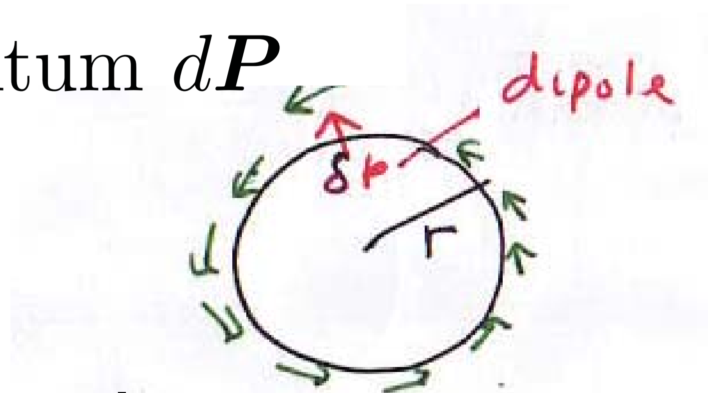
orbital occupation

The dipole at a segment of the edge has a momentum

$$dP_a = \frac{\hbar}{e\ell_B^2}\epsilon_{ab}dp^b$$

momentum dipole

momentum $d\boldsymbol{P}$



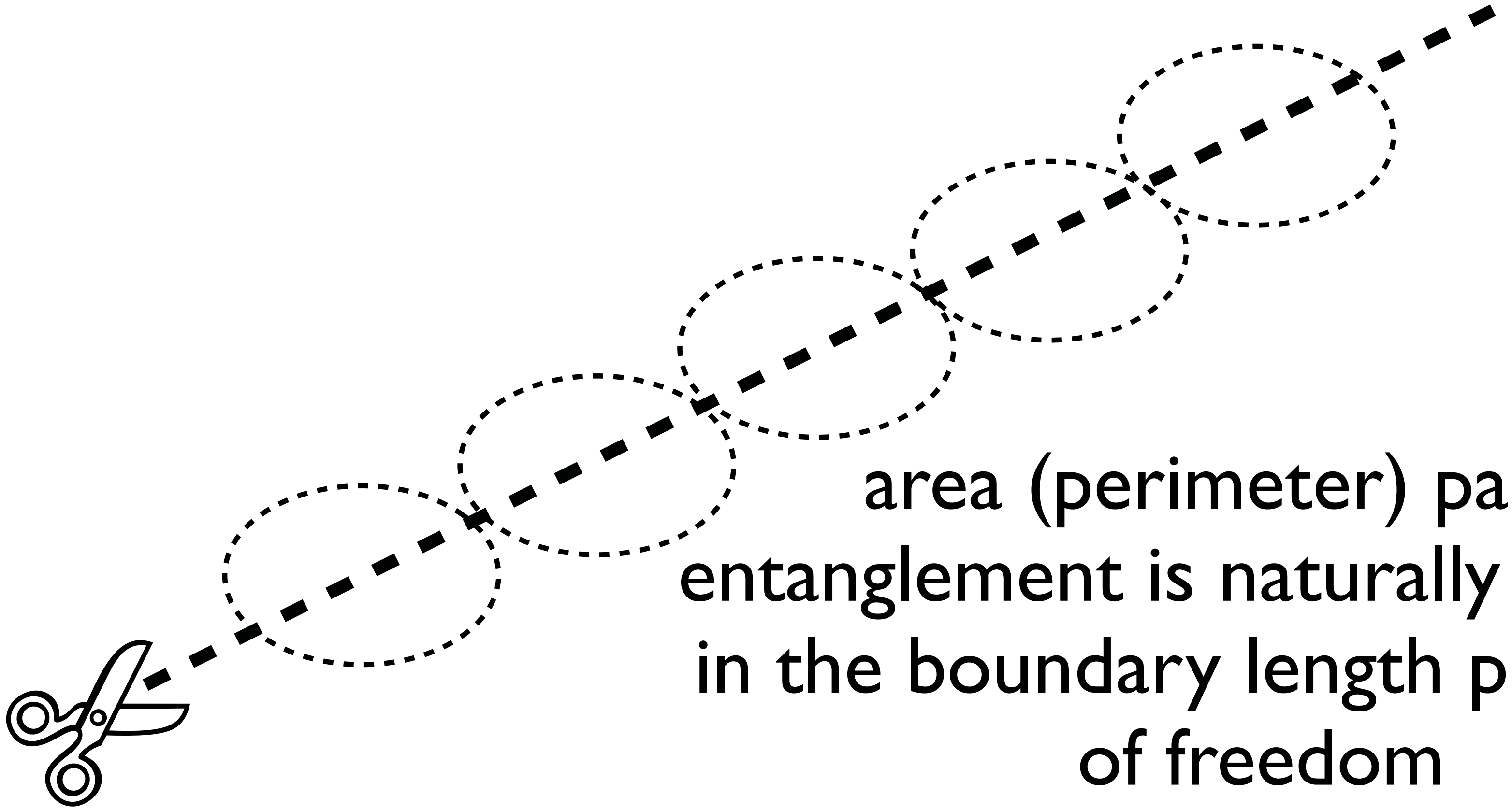
circular droplet

doesn't contribute to total momentum:

$$\oint dP_a = 0$$

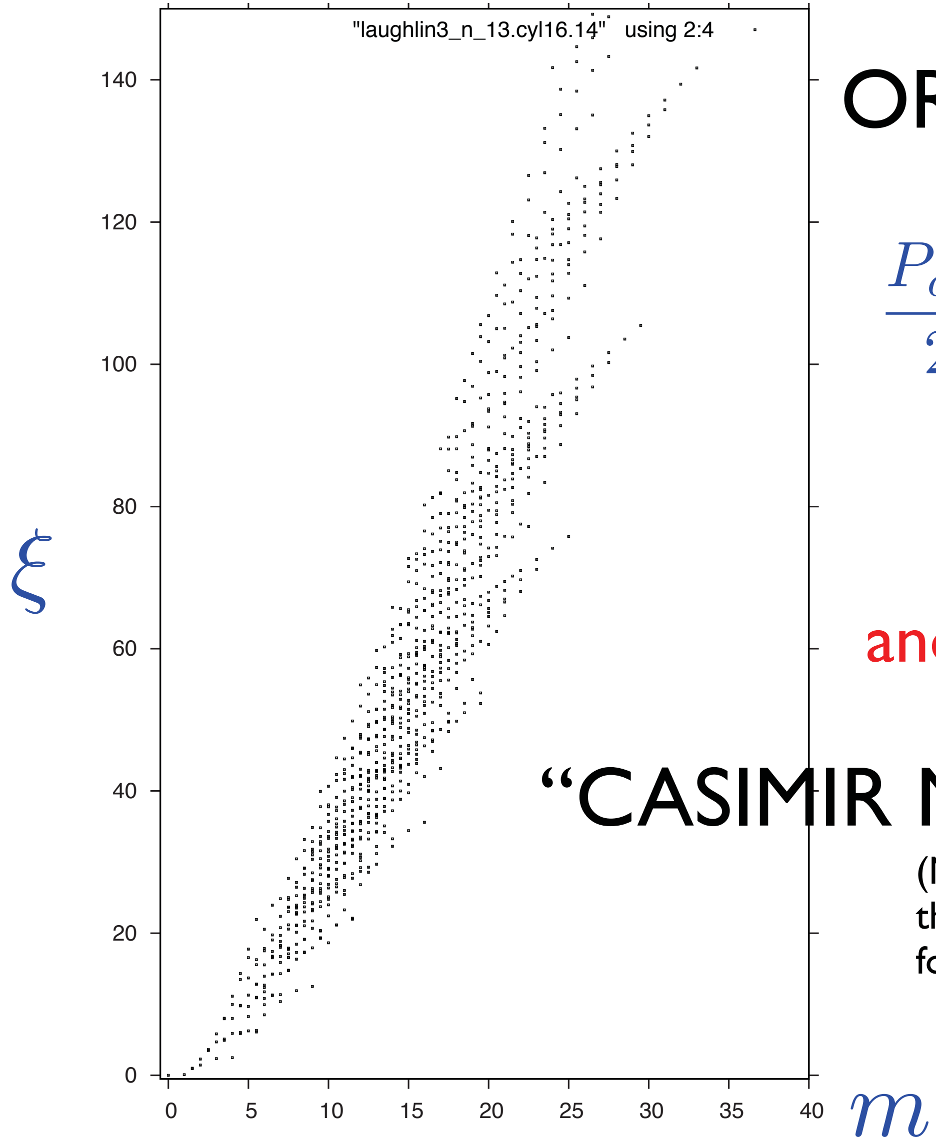
it <u>does</u> contribute an extra term to total angular momentum:

$$\Delta L^z(\boldsymbol{g}) = \hbar \oint \epsilon^{ab} g_{bc} r^c dP_a \neq 0$$



area (perimeter) part of
entanglement is naturally measured
in the boundary length per degree
of freedom

measured in diameters of composite bosons



ORBITAL CUT

$$\frac{P_a L^a}{2\pi} = \frac{\sum_\alpha m_\alpha e^{-\xi_\alpha}}{\sum_\alpha e^{-\xi_\alpha}} = \eta_H^{cd}\epsilon_{ac}\epsilon_{bd}\frac{L^a L^b}{2\pi \ell_B^2} + \frac{1}{24}(\tilde{c} - \nu) - h$$

signed conformal anomaly (chiral stress-energy anomaly)

chiral anomaly

virasoro level of sector

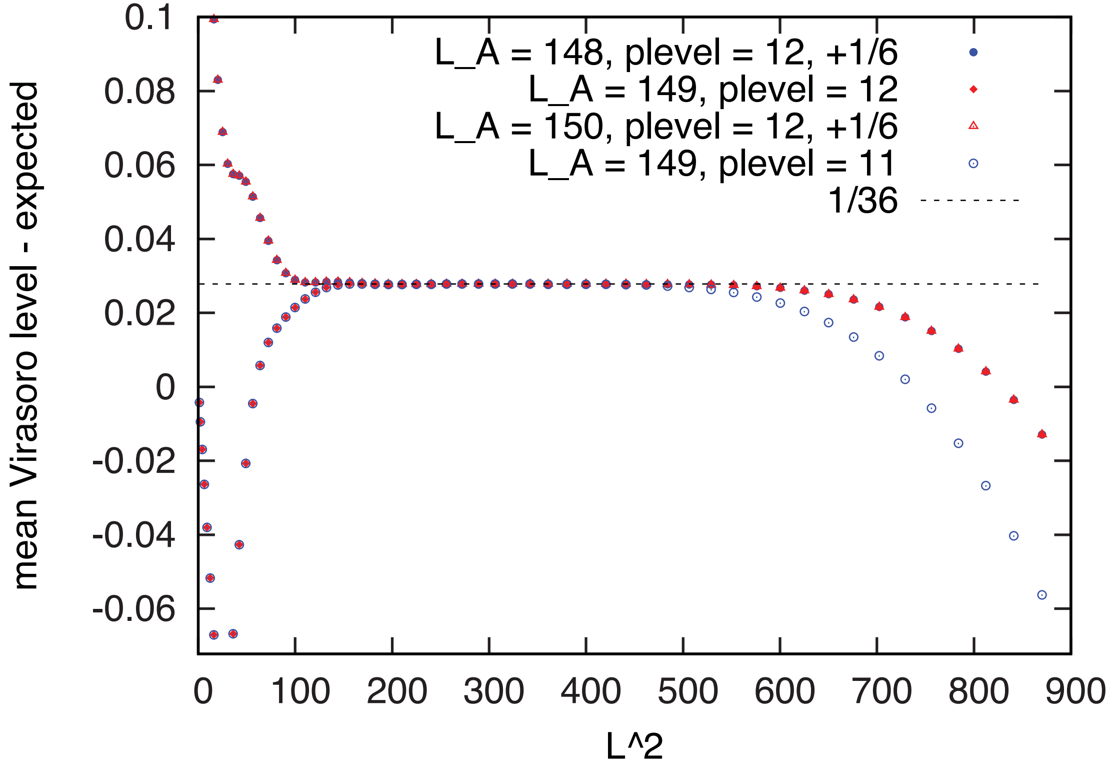
“CASIMIR MOMENTUM”

(NOT “real-space cut” which requires the Landau orbit degrees of freedom and their form factor to be included

- Hall viscosity gives “thermally excited” momentum density on entanglement cut, relative to “vacuum”, at von Neumann temperature T = 1

Yeje Park, Z Papic, N Regnault

Laughlin ν=$1/3$ state: topologically conserved "chiral central charge" is explicitly seen to be $\tilde{c} = 1$



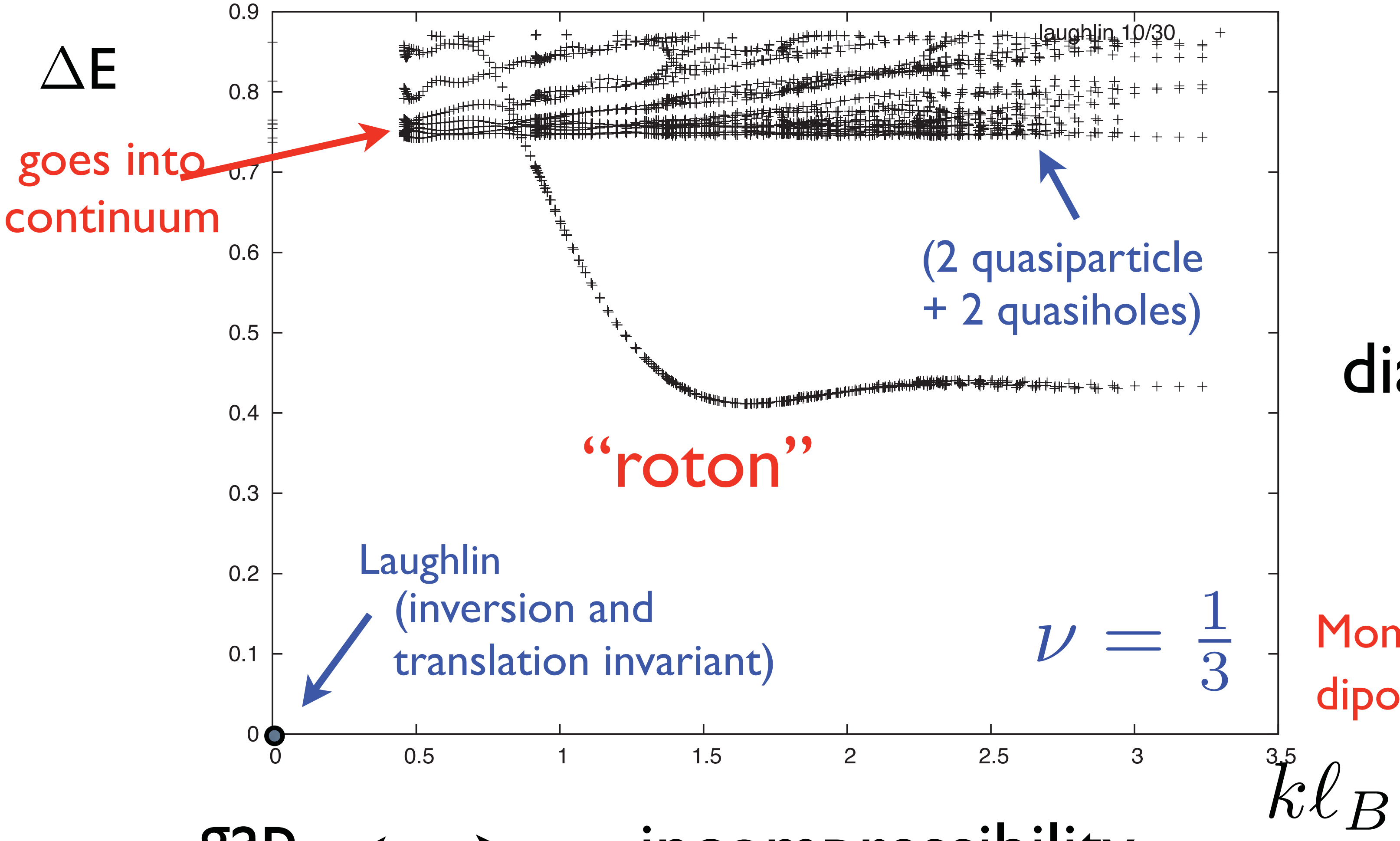
$$\frac{1}{36}$$

$$\frac{1}{24}(\tilde{c} - \nu) = \frac{1}{24}(1 - \frac{1}{3}) = \frac{1}{36}$$

MPS calculation: ("plevel" is Virasoro level at which auxilliary space is truncated, which causes errors at large L)

- other consequences of geometry
- long-wavelength of GMP mode is “graviton”
- q4 behavior of guiding-center structure factor is due to zero-point fluctuations f metric (components do not commute, determinant is Casimir)
- Conformal field theory has a fixed metric; geometrodynamics is like extension of special relativity to GR!

unfortunately, long-wavelength limit of "graviton" collective mode is hidden in "two-roton continuum"



numerical finite-size diagonalization

Momentum $\propto$ dipole moment

gap ⟷ incompressibility

Gap for tangential electric polarization (no dielectric screening)

single-mode approx. $E(\boldsymbol{q})s(\boldsymbol{q}) \leq \frac{1}{2}G^{abcd}q_a q_b q_c q_d \ell_B^2.$

- Geometric action

(after Chern-Simons fields are integrated over)

electromagnetic gauge potentials

spin connection of metric

$$S = \int d^3x \mathcal{L}_0 - \mathcal{H}_0$$

$$\mathcal{L}_0 = \frac{1}{4\pi pq\hbar}\epsilon^{\mu\nu\lambda}\left(peA_\mu - s\Omega^g_\mu\right)\partial_\nu\left(peA_\lambda - \hbar s\Omega^g_\lambda\right)$$

(reduces to electromagnetic Chern-Simons action when s = 0 (integer QHE))

$$\mathcal{H}_0 = J^0 U(J^0 g) \qquad J^0 = \frac{1}{2\pi pq\hbar}\left(peB - \hbar s J^0_g\right)$$

Gaussian curvature

$$J^\mu_g = \epsilon^{\mu\nu\lambda}\partial_\nu\Omega^g_\lambda$$

correlation energy density

energy function

composite-boson density

Geometric distortion energy

correlation energy density

$$\mathcal{H}_0 = (\det G)^{1/2} U(G) = J^0 U(J^0 g)$$

geometric chemical potential (of composite bosons)

$$\mu_g = U(G) + G_{ab} \frac{\partial U}{\partial G_{ab}}$$

shear-stress tensor (traceless)

$$\sigma^a_b = 2 G_{bc} \frac{\partial U}{\partial G_{ac}} - \delta^a_b G_{cd} \frac{\partial U}{\partial G_{cd}}$$

$$\sigma^a_a = 0$$

$$\sigma^a_c(x)\epsilon^{bc} = \sigma^{bc}(x)\epsilon^{ac}$$

$$\sigma^a_c(x) g^{bc}(x) = \sigma^{bc}(x) g^{ac}(x)$$

both expressions are symmetric in $a \leftrightarrow b$

Stress tensor is traceless because the gapped quantum incompressible fluid does not transmit pressure

(unlike incompressible limit of classical incompressible fluid, which has speed of sound $v_s \to \infty$

Euler equation

- action is minimized by Hall viscosity condition

$$J^0 \sigma^a_b(G) = \eta^{ac}_{bd}(G) \nabla^g_c J^d$$

composite boson current

Traceless stress-tensor

covariant spatial gradient of $J^a = J^0 v^a$

fluid flow-velocity

Hall viscosity

$$\eta^{ac}_{bd}(G) = \tfrac{1}{2}\hbar s \epsilon_{be}\epsilon_{df} J^0 \Gamma^{aecf}_H(g)$$

$$\Gamma^{abcd}_H(g) = \tfrac{1}{2}\left(\epsilon^{ac}g^{bd} + \epsilon^{ad}g^{bc} + \epsilon^{bc}g^{ad} + \epsilon^{bd}g^{ac}\right)$$

$$\eta^{ac}_{bd} = -\eta^{ca}_{db}$$

dissipationless

$$\eta^{ab}_{ac} = \eta^{ba}_{ca} = 0$$ incompressible

- composite boson current

Gaussian curvature density and current

$$J^0 = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab} peB - \hbar s J_g^0\right)$$

$$J^a = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab}(peE_b - \partial_b \mu_g) - \hbar s J_g^a\right)$$

responds to gradient of geometric chemical potential as well as electric field

$$peE_a J^a \neq 0$$

Energy flow from electromagnetic field to FQH fluid

$$pe(J^0 E_a + \epsilon_{ab} J^a B) \neq 0$$

tangential momentum flow from electromagnetic field to FQH fluid

- Action gives gapped spin-2 (graviton-like) collective mode that coincides at long wavelengths with the "single-mode approximation" of Girvin-MacDonald and Platzman.

- charge fluctuations relative to the back-ground charge density fixed by the magnetic flux are given by the Gaussian curvature

$$J_g^0 = -\frac{1}{2}\partial_a\partial_b g^{ab} + \frac{1}{8}g_{ac}\epsilon_{bd}\epsilon^{ef}\left(\partial_e g^{ab}\right)\left(\partial_f g^{cd}\right)$$

$$\delta J_e^0 = \frac{e^* s}{2\pi} J_g^0$$

second derivative of metric

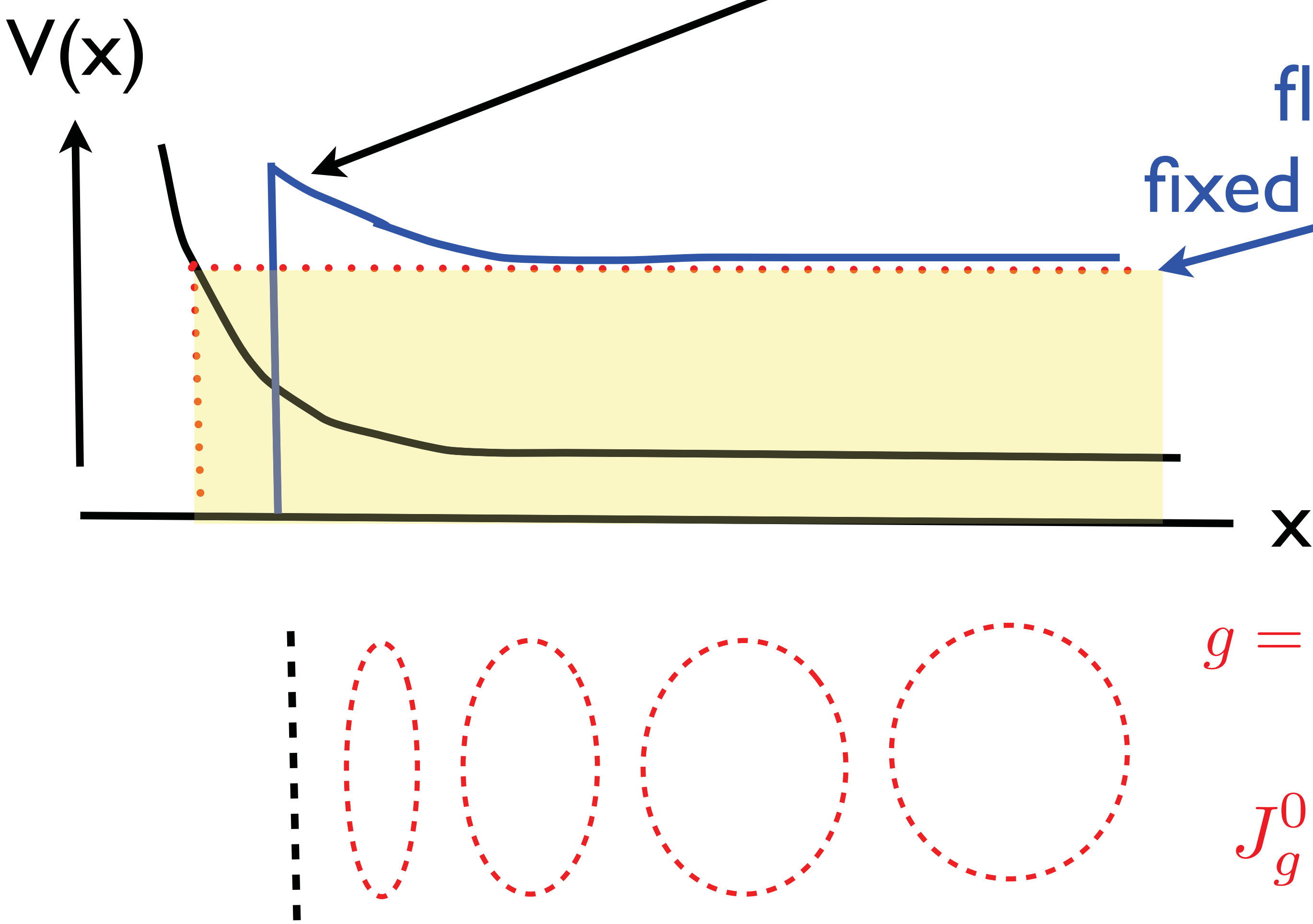
zero-point fluctuations of gaussian curvature give quantitatively correct $O(q^4)$ structure factor

- near edges:

fluid is compressed at edges by creating Gaussian curvature

$$\delta J_e^0 = \frac{e^* s}{2\pi} J_g^0$$

fluid density fixed by flux density



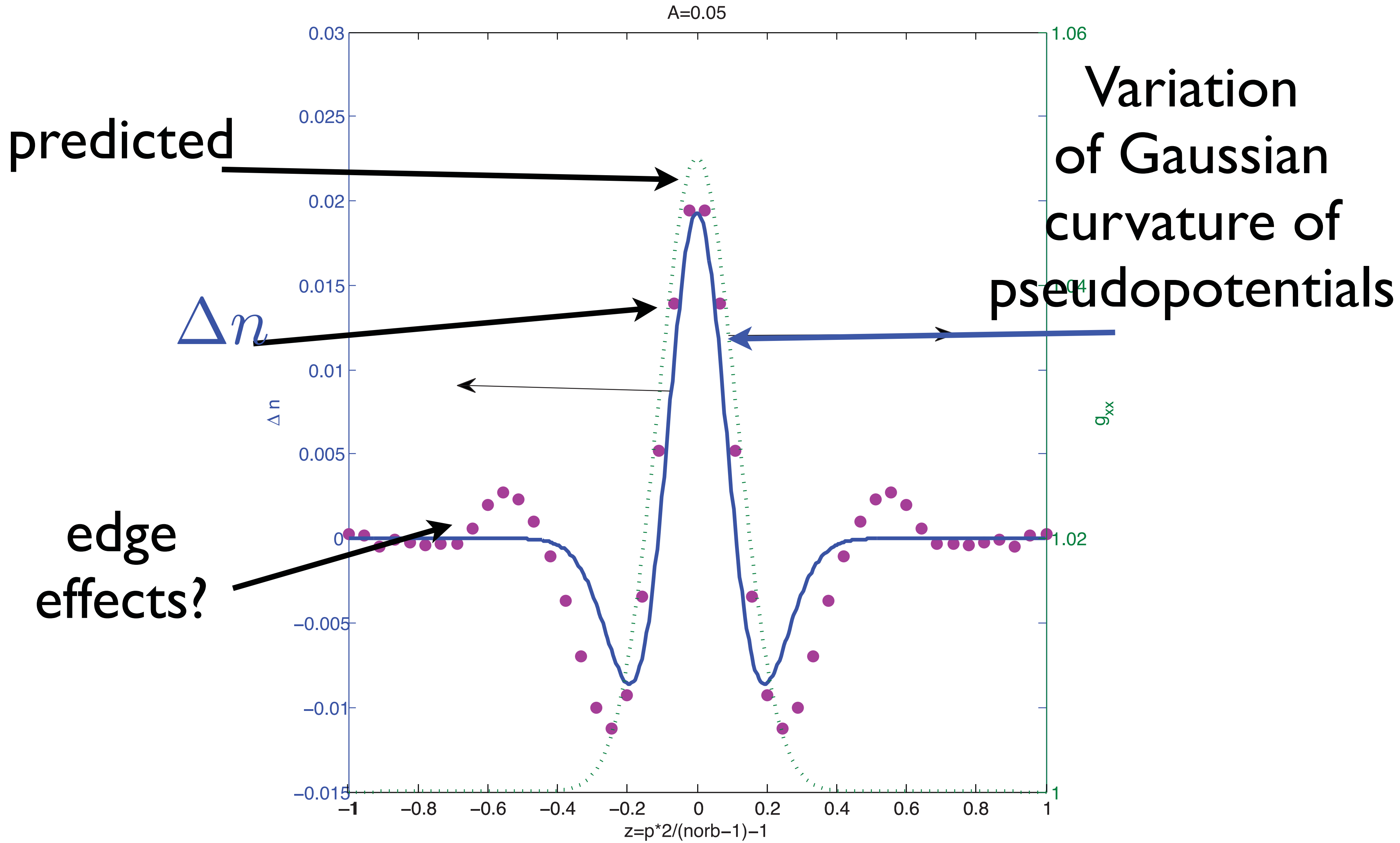
$$g = \begin{pmatrix} \alpha(x) & 0 \\ 0 & \frac{1}{\alpha(x)} \end{pmatrix}$$

$$J_g^0 = -\frac{1}{2}\frac{d^2}{dx^2}\frac{1}{\alpha(x)}$$

For larger s, fluid becomes more compressible
(less distortion needed for a given density change)

initial numerical study of Laughlin 1/3 on cylinder with edges

(with Zlatko Papic and Sonika Johri)



SUMMARY

- New collective geometric degree of freedom leads to a description of the origin of incompressibility in FQHE in a continuum "geometric field theory"
- many new relations: guiding-center spin characterizes coupling to Gaussian curvature of intrinsic metric, stress in fluid, guiding-center structure-factors, etc.

http://wwwphy.princeton.edu/~haldane

Can be also be accessed through Princeton University Physics Dept home page
(look for Research:condensed matter theory)

also see arXiv (search for author=haldane)