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Quantum Renormalization Group and AdS/CFT

Sung-Sik Lee

Perimeter Institute, McMaster University
Canada

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Sung-Sik Lee
McMaster University
Perimeter Institute

AdS/CFT correspondence

[Maldacena]

- **Conjecture :**

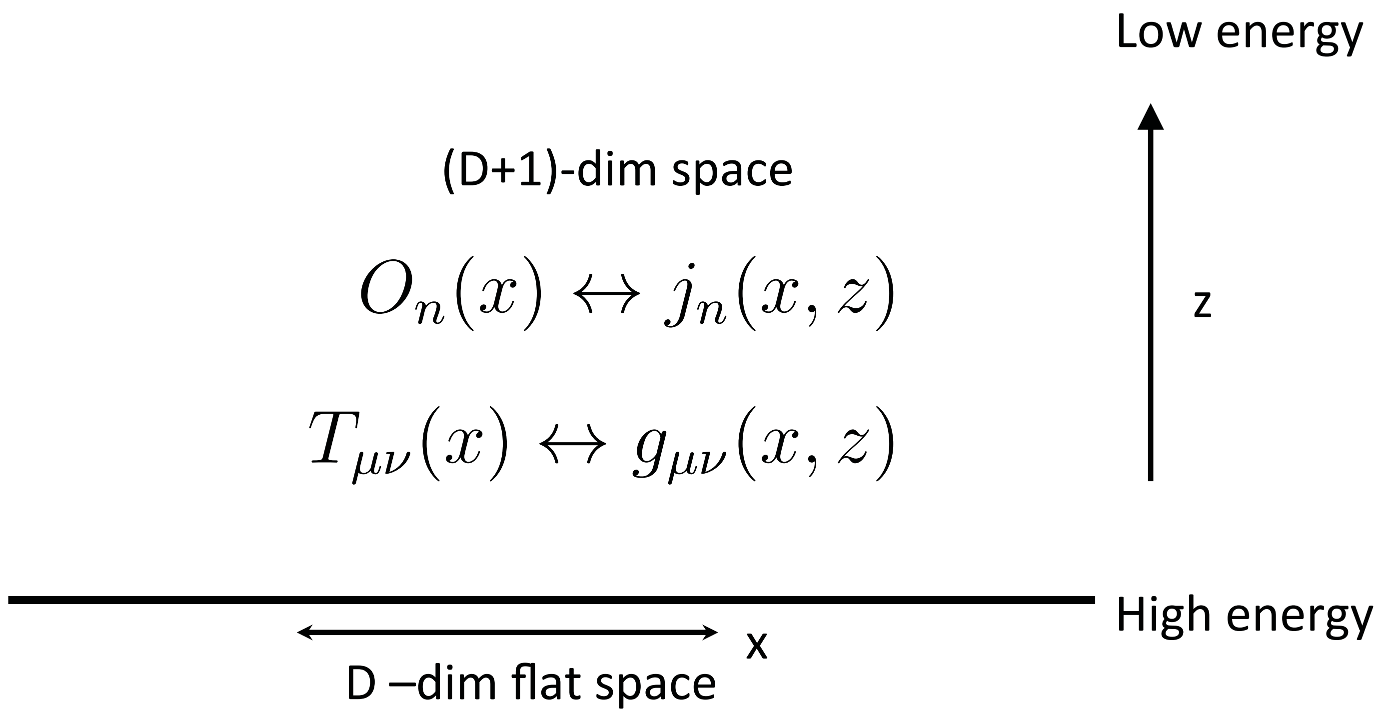
  D-dim QFT = (D+1)-dim quantum gravity

  – One can learn about QFT from Gravity
    - In large N limit, the gravity becomes classical
    - Classical gravity may capture non-trivial quantum fluctuations in QFT
    - Potential application to QCD, condensed matter systems

  – One can learn about Gravity from QFT
    - QFT as a non-perturbative definition of quantum gravity
    - QFT may provide new insights into gravity

AdS/CFT Dictionary

[Gubser, Klebanov, Polyakov; Witten]



$$O_n(x) \leftrightarrow j_n(x, z)$$

$$T_{\mu\nu}(x) \leftrightarrow g_{\mu\nu}(x, z)$$

$$\int D\phi(x) e^{iS_D[\phi(x)] + i\int J_n(x) O_n} = \int Dj(x, z) e^{iS_{D+1}[j(x,z)]}\Big|_{j_n(x,z=0)=J_n(x)}$$

$$\to e^{iS_{D+1}[\bar{j}(x,z)]}\Big|_{\bar{j}_n(x,z=0)=J_n(x)}$$

What is behind the correspondence?

RG ≈ GR

- Radial direction in the bulk = length scale of QFT
- Bulk variables : scale dependent coupling functions
- Equations of motion in the bulk corresponds to the beta functions of QFT
- Radial evolution of the bulk fields correspond to the RG flow

RG ≈ GR

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- …..

However, the connection between RG and GR is incomplete

RG	GR
Non-dynamical coupling functions : Obey first-order beta functions	Bulk variables are dynamical : Bulk action has two-derivative term
RG flow is classical : Given initial condition, coupling functions are deterministic without uncertainty	Bulk variables have quantum fluctuations

This talk

Quantum RG = Quantum GR

Matrix model

$$Z[J(x)] = \int D\phi \; e^{i\int dx\mathcal{L}}$$

$$\mathcal{L} = J^n(x)O_n$$

- O_n : complete set of single-trace operators

e.g. $\mathrm{tr}[\phi^n],\ \mathrm{tr}[\phi\partial_\mu\partial_\nu\phi],\ \mathrm{tr}[\phi(\partial_{\mu_1}\partial_{\mu_2}...\partial_{\mu_i}\phi)...(\partial_{\nu_1}\partial_{\nu_2}...\partial_{\nu_i}\phi)], ..$

Renormalization Group

$$\Lambda \to \Lambda e^{-dz}$$

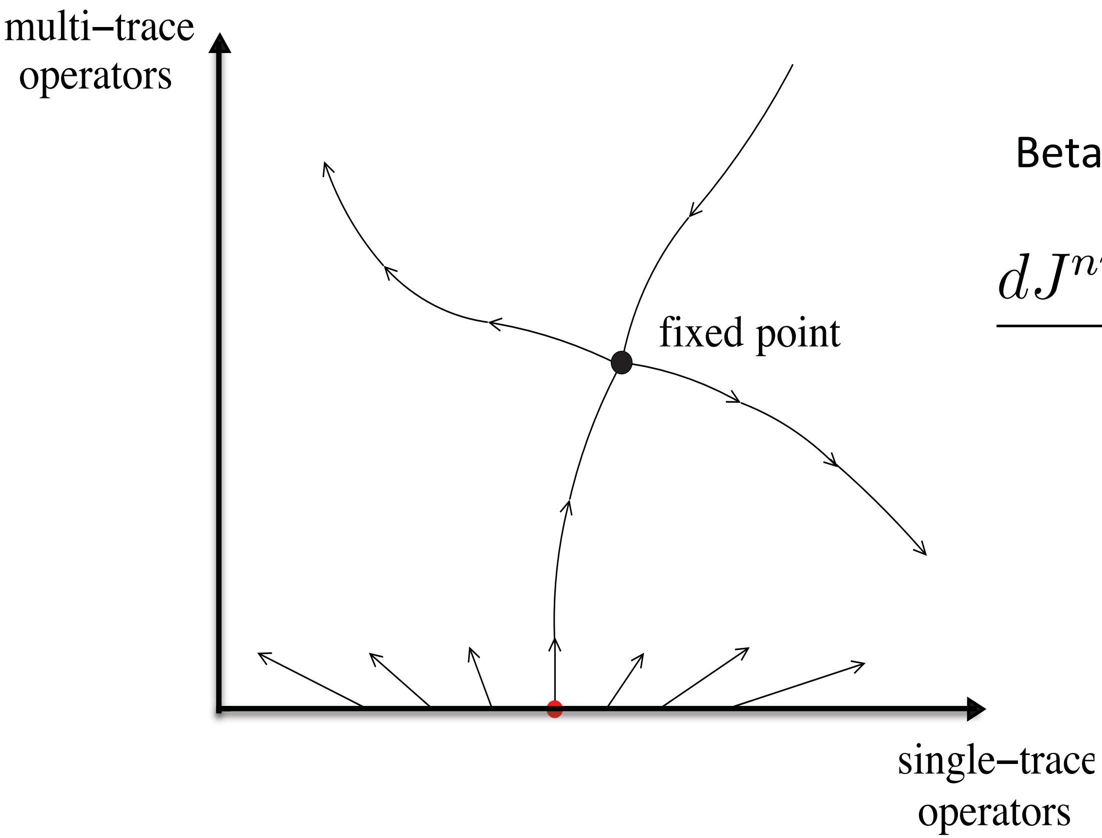
$$\mathcal{L} \to \mathcal{L}^{'} = \mathcal{L} + \delta\mathcal{L},$$

$$\delta\mathcal{L} = dz\left\{\mathcal{L}_c[J,x) - \beta^n[J,x)O_n + G^{mn}[J,x)O_mO_n\right\}$$

- Under coarse graining, the original theory is mapped into another theory
- Although only a subset of operators are turned on at UV, all other symmetry allowed operators are generated at low energy
- Specifically, double-trace operators are generated out of single-trace operators to the linear order of dz

Conventional (Classical) RG

- One needs to keep track of the flow of all operators
- RG flow is deterministic
- Intractable for strongly coupled field theories



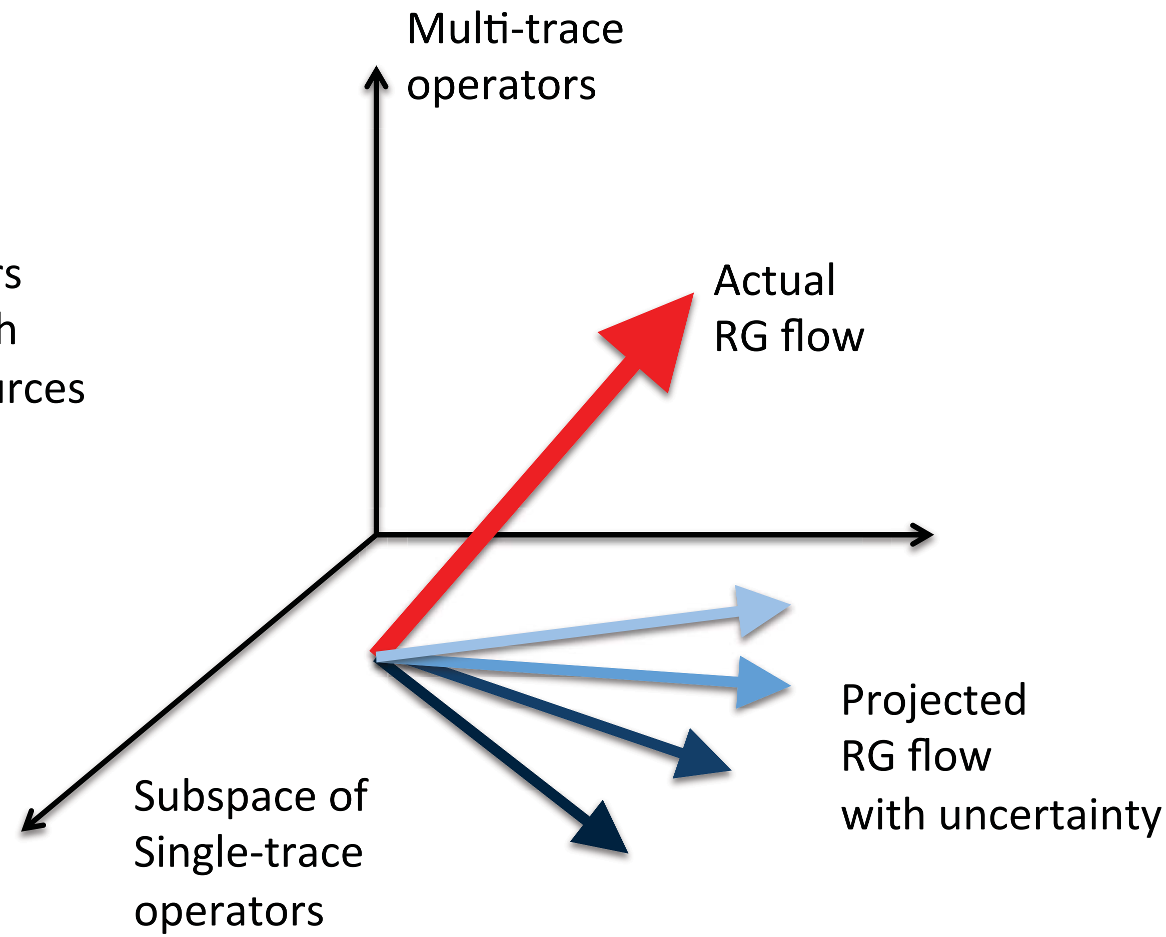
Beta function

$$\frac{dJ^{nm\ldots}(x,z)}{d\ z} = -\beta[J^n, J^{nm}, \ldots]$$

[Becchi, Guisto, Imbimbo (02)]

Quantum RG : projection creates uncertainty

- Theory with multi-trace operators can be mapped into a theory with single-trace operators whose sources are dynamical



Dynamical source and operator fields

$$Z = \int D\Phi e^{i \int \mathcal{L}'}$$

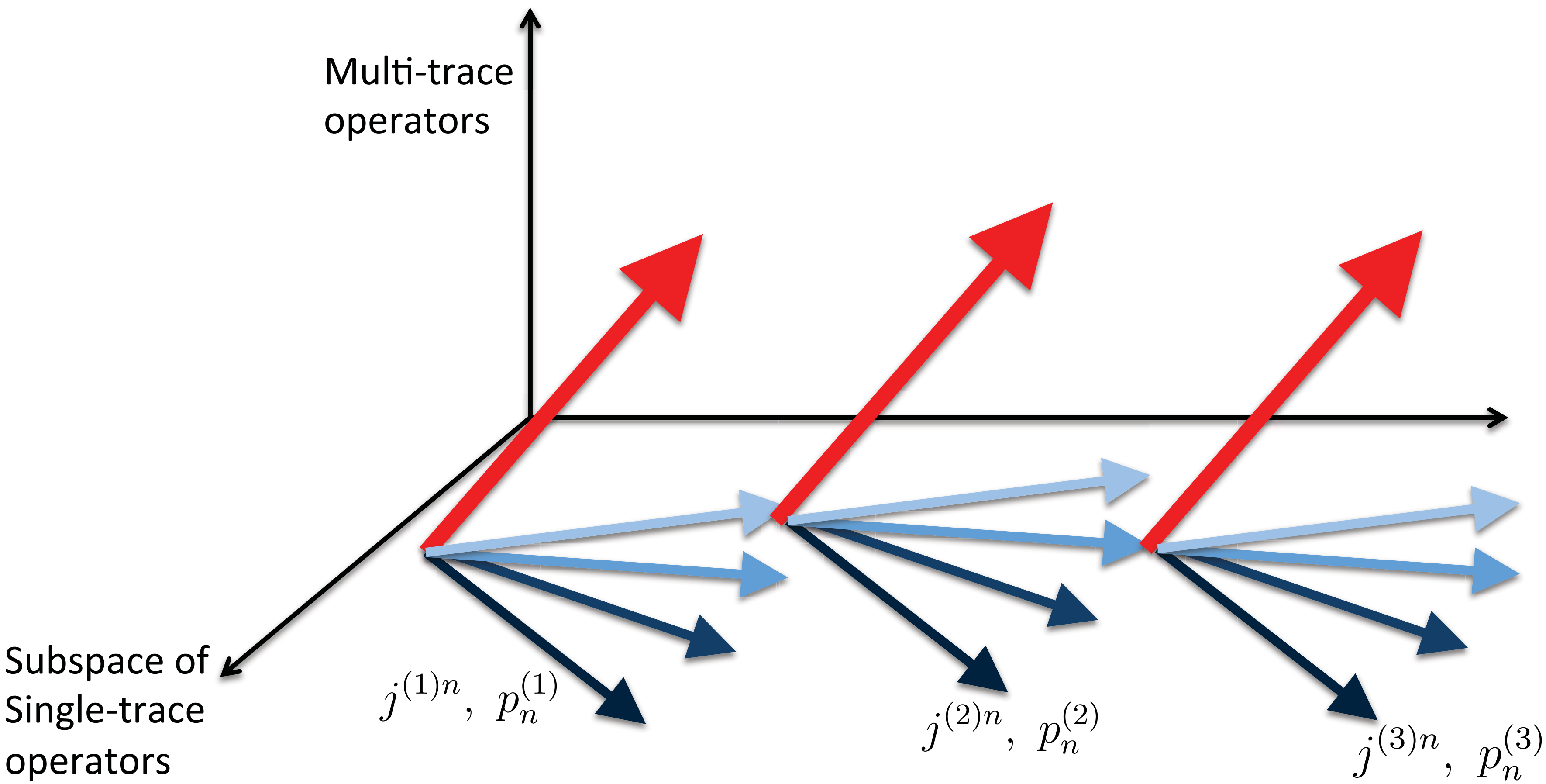
$$\mathcal{L}' = dz\mathcal{L}_c[J,x) + (J^n - dz\beta^n[J,x))\,O_n$$
$$+dzG^{mn}[J,x)O_mO_n$$

$$Z = \int D\Phi D j^{(1)n} D p_n^{(1)} e^{i \int \mathcal{L}''}$$

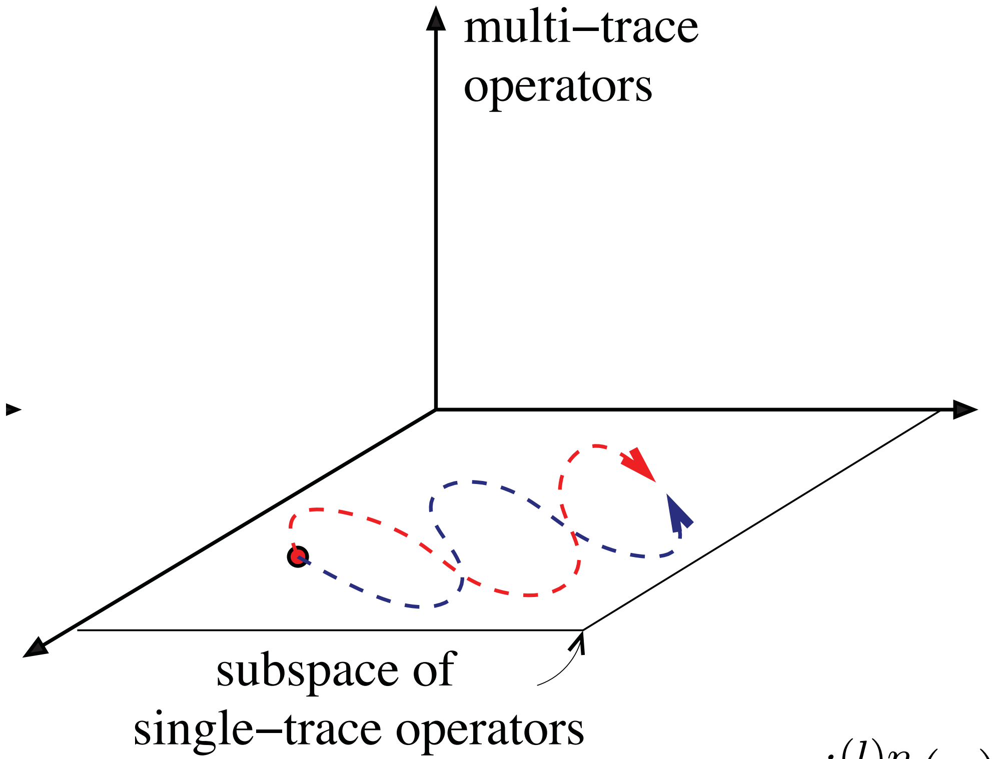
$$\mathcal{L}'' = dz\mathcal{L}_c[J,x) + j^{(1)n}O_n + p_n^{(1)}(j^{(1)n} - J^n)$$
$$+dz\beta^n[J,x)p_n^{(1)} + dzG^{mn}[J,x)p_m^{(1)}p_n^{(1)}$$

Quantum RG

- Iterate these steps
  1) Coarse grain starting from single-trace action
  2) Project to the subspace



Quantum fluctuations in RG path



- Only single-trace operators are included
- Generating function is given by a sum over all RG paths
- The weight of each path is determined by a (D+1)-dimensional action

$$j^{(l)n}(x),\ p_n^{(l)}(x) \rightarrow j^n(x,z),\ p_n(x,z)$$

$$Z = \int Dj(x,z)Dp(x,z)\ \ e^{iS^{D+1}[j,p]}$$

SL, NPB (2011); JHEP (2012)

Holographic Action

$$S^{D+1} = N^2 \int dz \int d^D x \left\{ p_n(\partial_z j^n) + \mathcal{L}_c(x;j] + \beta^m(x;j]p_m + \frac{G^{mn}(x;j]}{2} p_m p_n \right\}$$

- Casimir energy, beta functions of single-trace operators and double-trace operators on the subspace of single-trace operators completely specify the (D+1)-dimensional action
- j and p are canonical conjugate to each other with respect to the RG `time' evolution
  - Casimir energy : potential energy for j
  - Double-trace operator : kinetic energy for p

Scale-Reversal Symmetry

$$S^{D+1} = N^2 \int dz \int d^D x \left\{ p_n(\partial_z j^n) + \mathcal{L}_c(x;j] + \beta^m(x;j]p_m + \frac{G^{mn}(x;j]}{2} p_m p_n \right\}$$

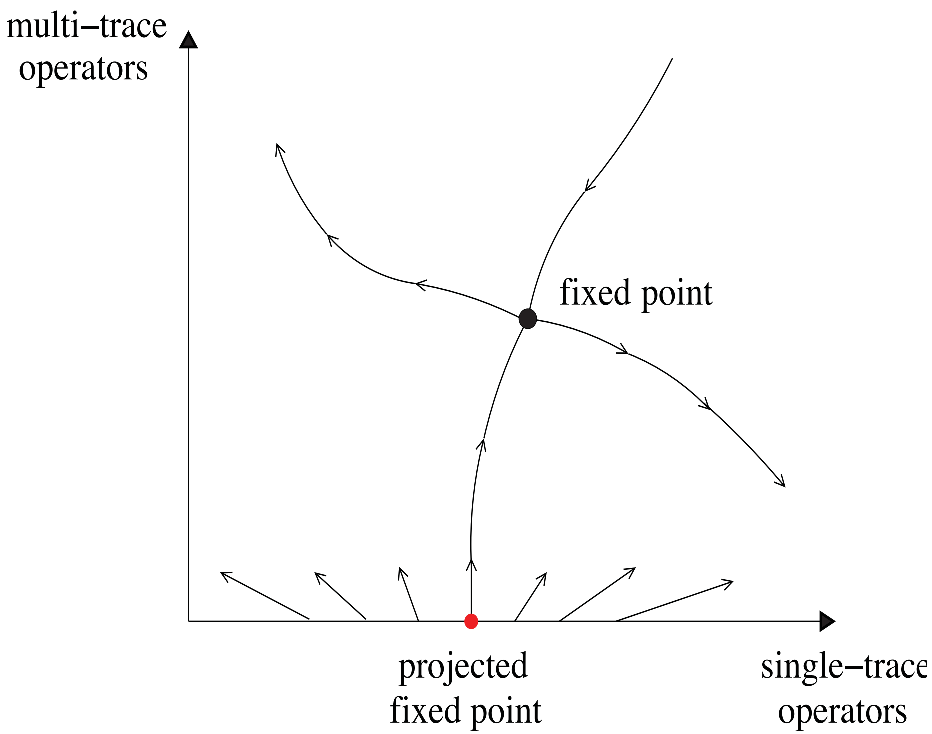
- Generically, one expects that the bulk action breaks the Scale-Reversal (SR) symmetry under which z goes to -z because RG flow is irreversible
- However, irreversible RG flow can be still described by SR symmetric bulk action because of the boundary at UV cut-off, which explicitly breaks the SR symmetry
- On the other hand, all known holographic duals respect the SR symmetry in the bulk
- If this is indeed the case for all QFTs, what is implications ?
  - It turns out that the SR symmetry in the bulk can be maintained if the beta function for the single-trace operators is a gradient flow with respect to the metric given by the beta function for the double-trace operators

$$\beta^m(x;j] = G^{mn}(x;j] \frac{\delta c[j]}{\delta j^n(x)}$$

  - In this case, one can shift p to trade SR-odd term with boundary terms and a mass term that is proportional to the scaling dimension

[SL, 1305.3908]

Can one show that quantum gravity emerges from a QFT ?



- Consider a projected fixed point of a D-dim. Matrix field theory
- Suppose that only energy-momentum tensor has finite scaling dimension for the projected RG flow
- This implies that only energy-momentum tensor and multi-trace operators made of the energy-momentum tensor will appear along the RG flow

D-dim QFT on a curved background

$$Z[g^{(0)}] = \int D\Phi \quad e^{iS_1[\Phi;g^{(0)}(x)]}$$

- We deform the projected fixed point by adding single trace deformation of energy-momentum tensor
- This is equivalent to putting the theory on a curved background metric
- We assume that the theory is regularized respecting the D-dim. Diffeomorphism invariance

Coarse graining

$$Z[g^{(0)}] = \int D\Phi \quad e^{iS_1[\Phi;g^{(0)}(x)]+i\delta S'[T^{\mu\nu};g^{(0)}]}$$

spacetime dependent speed of RG [Osborn]

$$\delta S'[T^{\mu\nu};g^{(0)\mu\nu}] = dzN^2 \int d^Dx \ N^D(x)\Big\{\sqrt{|g^{(0)}|}\left(C_0 + C_1^D \mathcal{R}(x;g^{(0)}]\right)$$

$$-\beta_{\mu\nu}T^{\mu\nu} + \frac{B_{\mu\nu;\rho\sigma}}{2}T^{\mu\nu}T^{\rho\sigma} + ..\Big\}$$

Change of scale :
Warping factor

Double-trace operators

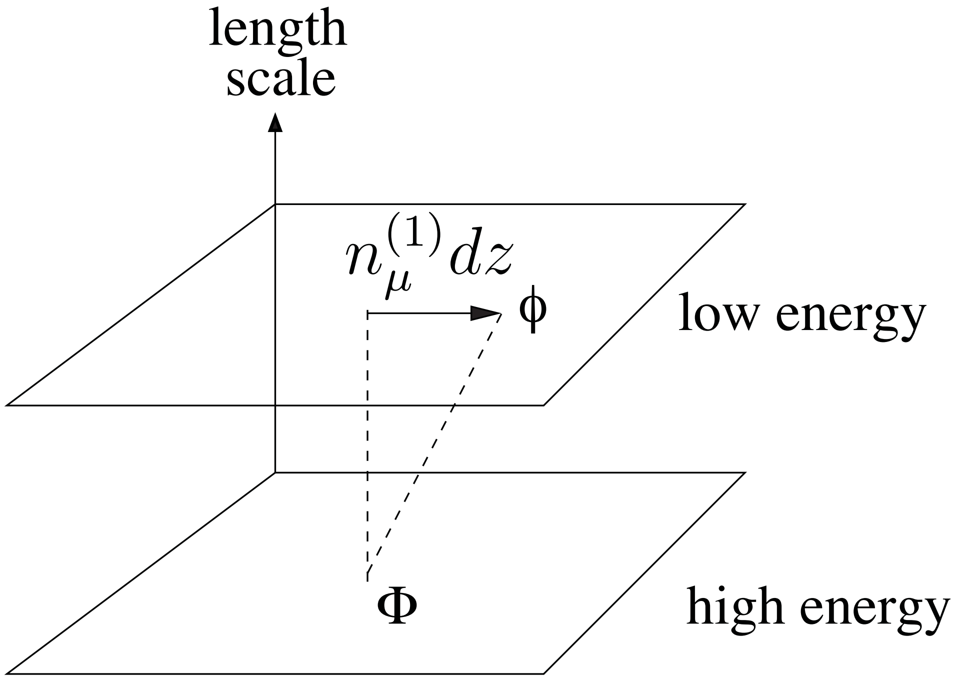
Higher derivative terms

Casimir energy
[Sakharov]

Shift

$$Z[g^{(0)}] = \int D\Phi \;\; e^{iS_1[\Phi;g^{(0)}(x)]+i\delta S'[T^{\mu\nu};g^{(0)}]+i\delta S''[T^{\mu\nu};g^{(0)}]}$$

$$\delta S''[T^{\mu\nu};g^{(0)\mu\nu}] \; = \; dzN^2\int d^Dx(\nabla^{(1)}_\mu n^{(1)}_\nu + \nabla^{(1)}_\nu n^{(1)}_\mu)T^{\mu\nu}$$



Shift of the coordinate of the low energy field relative to the coordinate of the high energy field

Auxiliary fields

$$Z[g^{(0)}] \;=\; \int Dg^{(1)}_{\mu\nu} D\pi^{(1)\mu\nu} D\Phi \quad e^{iN^2 \int d^D x \;\; \pi^{(1)\mu\nu}(g^{(1)}_{\mu\nu} - g^{(0)}_{\mu\nu})}$$

$$\times e^{i\delta S' [\pi^{(1)\mu\nu}, g^{(0)}] + i\delta S'' [\pi^{(1)\mu\nu}, g^{(0)}]} e^{iS_1[\Phi; g^{(1)}]}$$

- A pair of auxiliary fields are introduced to remove the double-trace operator of $T^{\mu\nu}$
- δS becomes quadratic in $\pi^{(1)\mu\nu}$: this provides a Gaussian width for $g^{(1)}_{\mu\nu}$, which becomes a genuine fluctuating metric

Einstein Gravity

[SL, 1305.3908]

$$S_{D+1} = \frac{N^2}{2\kappa^2}\int dz \int d^D x \left[\pi_{\mu\nu}\partial_z g^{\mu\nu} - N^D \mathcal{H} - N^\mu \mathcal{H}_\mu\right]$$
$$= \frac{N^2}{2\kappa^2}\int d^{D+1}X \sqrt{|G|}\left(-\Lambda + {}^{(D+1)}\mathcal{R} + ..\right).$$

Casimir energy

Beta function of $T^{\mu\nu}T^{\rho\sigma}$

$$\mathcal{H} = -\sqrt{g}\left[C_0 + R^D + \frac{g^{-1}}{2}\left(\frac{\pi^2}{D-1} - \pi^{\mu\nu}\pi_{\mu\nu}\right) + ..\right]$$
$$\mathcal{H}^\mu = -2\nabla_\nu \pi^{\mu\nu}$$

This form is fixed by D-dimensional diff. inv. and the gauge invariance associated with the choice of local RG scheme

First-class constraints

- Independence of partition function on RG schemes (speed of RG and shifts) ➔ (D+1)-constraints

$$< \mathcal{H}_M(x,z) >= \frac{1}{Z}\frac{\delta Z}{\delta N^M(x,z)} = 0 \qquad \mathcal{H} = 0, \quad \mathcal{H}_\mu = 0$$

M=0, 1, 2, …, (D-1), D $\quad N^D(x,z) \equiv \alpha(x,z)$ and $\mathcal{H}_D \equiv \mathcal{H}$

- The (D+1)-constraints are (classically) first-class

$$\frac{\partial}{\partial z} < \mathcal{H}_M(x,z) >= \int d^D y\, N^{M'}(y,z) \langle \{\mathcal{H}_M(x,z), \mathcal{H}_{M'}(y,z)\}\rangle = 0$$

$$\{\mathcal{H}_M(x,z), \mathcal{H}_{M'}(y,z)\} = 0$$

Summary

- Quantum RG provides a general/microscopic understanding of AdS/CFT correspondence
- Quantum RG is an alternative way of organizing RG, which is tractable for some large N strongly coupled QFT
- Gradient RG flow = SR symmetry in holographic duals
- The Einstein gravity can be derived