Anisotropic Primordial Universe

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ICTP, July 2013

Outline

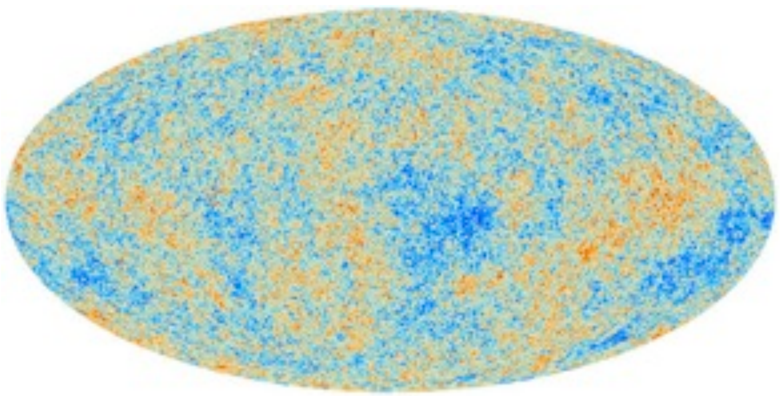
- Motivation for primordial anisotropy
- Anisotropic Inflation from gauge field dynamics
- δN formalism in anisotropic universe, power spectrum and bispectrum
- Dipole asymmetry from the long mode modulation
- Consistency conditions in dipole asymmetries
- Conclusion

Motivation

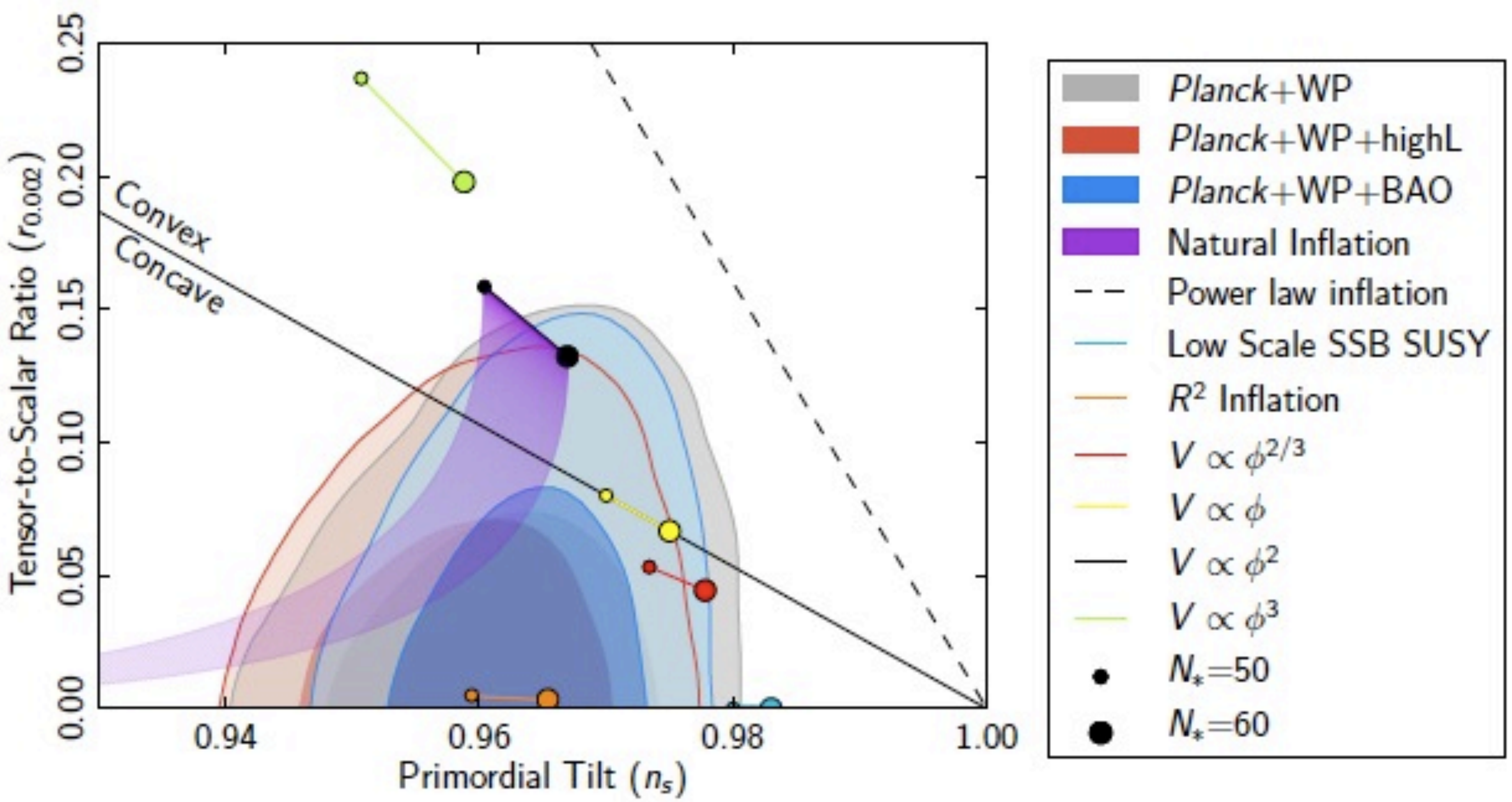
Inflation is the leading paradigm for early Universe and structure formations.

Basics predictions of inflation: The power spectrum are

- Nearly scale-invariant
- Nearly Gaussian
- Nearly adiabatic

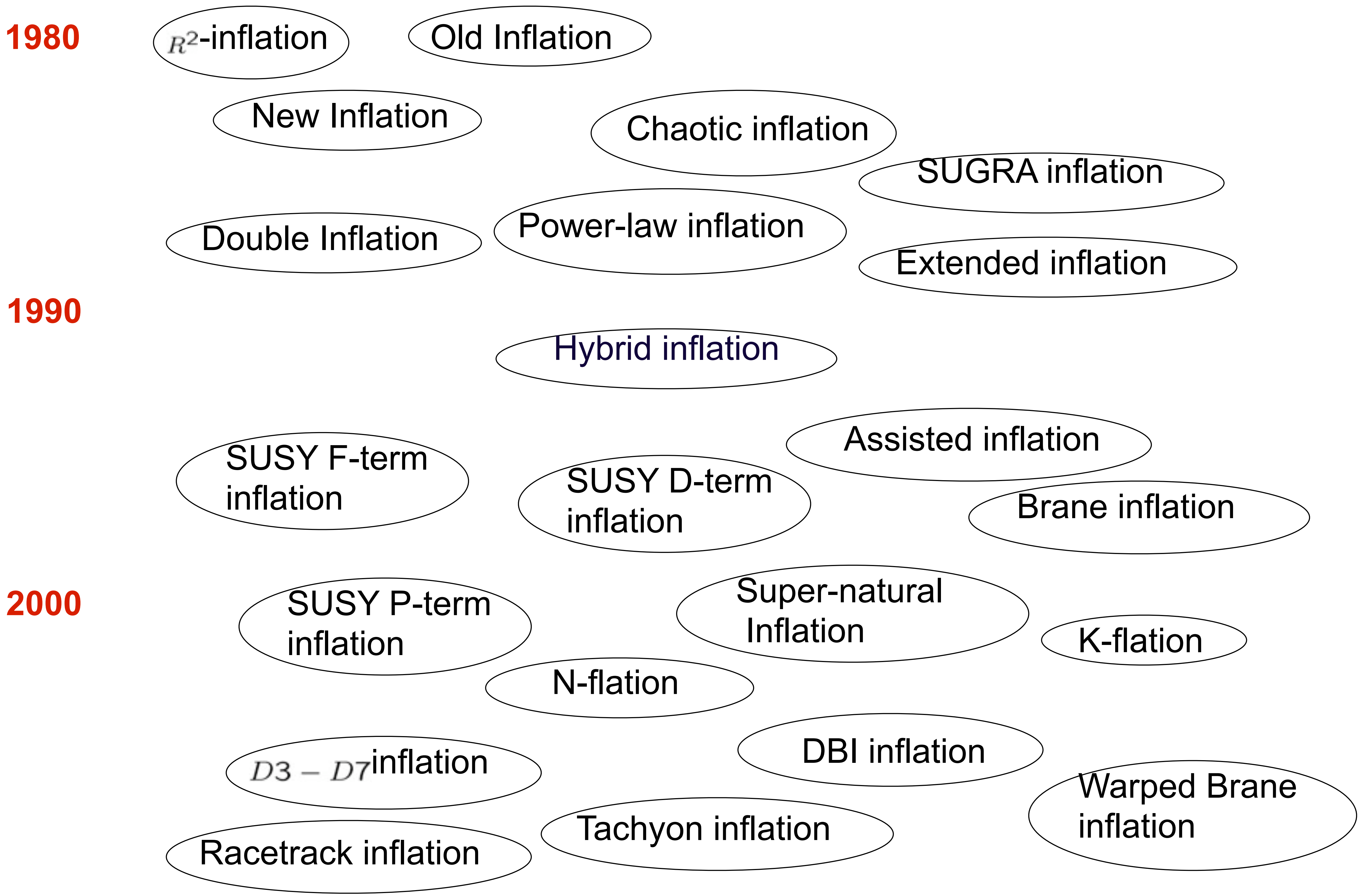


These predictions are in good agreement with the PLANCK data.



PLANCK 2013

Inflation in the context of ever changing fundamental theory



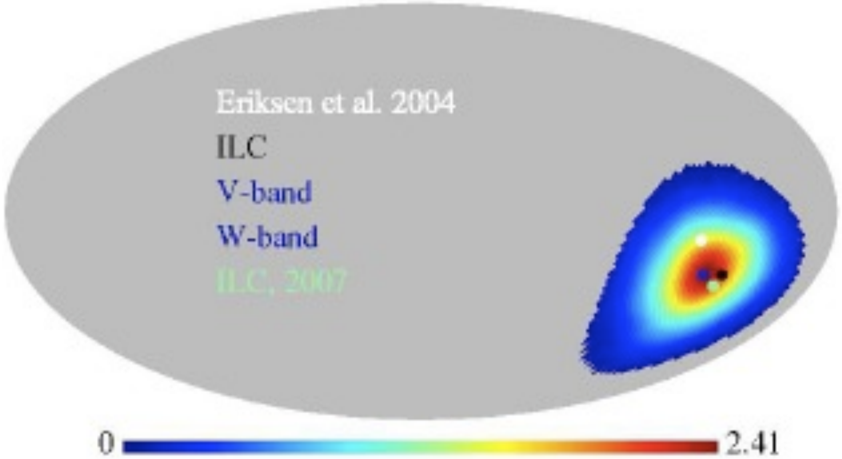
Borrowed from Lev Kofman

A Fundamental Question: Is the Universe isotropic?

- PLNCAK has reported anisotropies on CMB map
- Power Asymmetry: the power spectrum measured in one direction is different than the opposite direction
- A brief History:
  - After the release of WMAP first year data, Eriksen et al (2004) suggested the existence of hemispherical asymmetry. They found that the Southern hemisphere has more power than the Northern hemisphere.
  - Gordon et al (2005) proposed a dipolar modulation of hemispherical asymmetry

    $$\Delta T(\hat{\mathbf{n}}) = \overline{\Delta T(\hat{\mathbf{n}})}(1 + A\,\hat{\mathbf{n}}.\hat{\mathbf{p}})$$

    in which $\hat{\mathbf{p}}$ is the preferred dipole direction.
  - Eriksen et al (2007) fit this model with the WMAP 3-year data to get $A = 0.114$.
- PLANCK : $A = 0.07 \pm 0.02$ for $2 \ll \ell \lesssim 64$ with $(l, b) = (227^\circ, -21^\circ)$



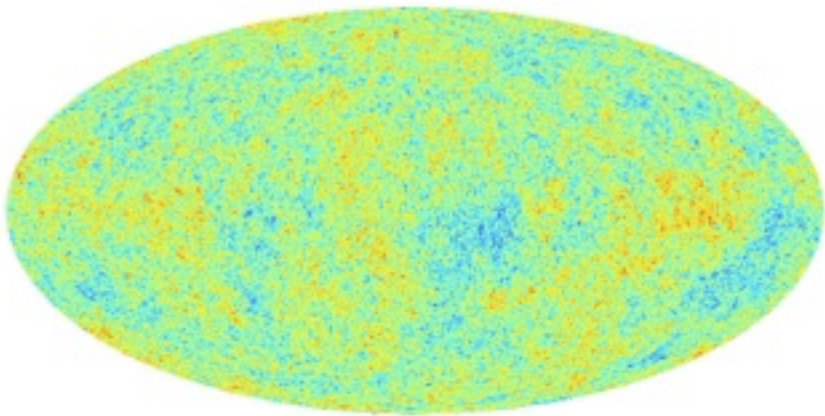
See also this morning talk by David Maino

Hoftuft et al, 2009.

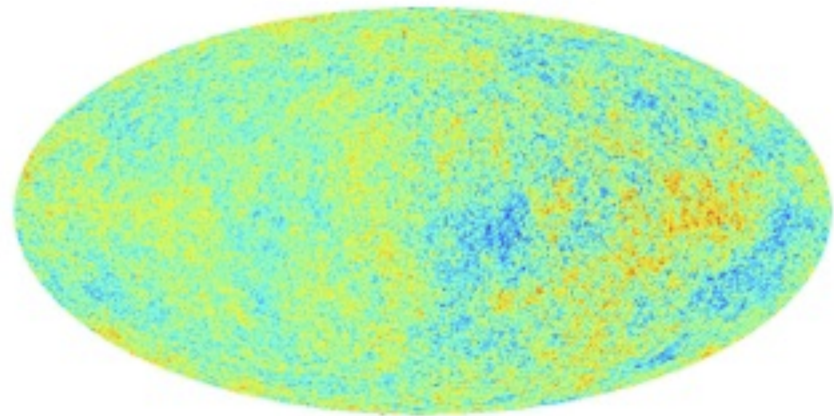
Anisotropic Power Spectrum

The simplest ansatz to model the hemispherical asymmetry in CMB power spectrum is the dipole modulation

$$\Delta T(\hat{n}) = \Delta T(\hat{\mathbf{n}})(1 + A\,\hat{\mathbf{n}}.\hat{\mathbf{p}})$$



A=0



A=0.3

Seljebotn, 2010

The correspond primordial curvature perturbation power spectrum is

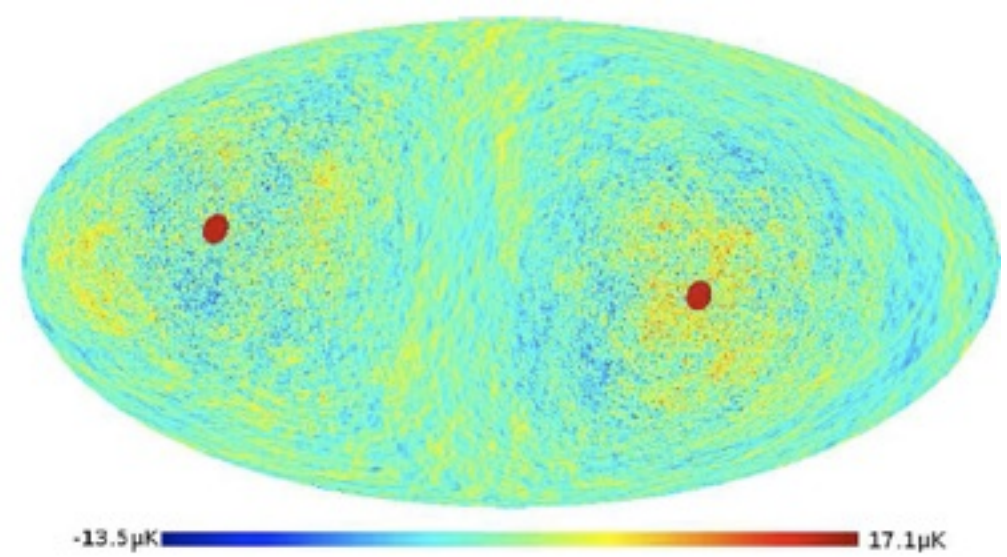
$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}^{(0)}(1 + A\,\hat{\mathbf{n}}.\hat{\mathbf{p}})$$

Generalization:

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}(1 + \sum_{LM} g_{LM} Y_{LM}(\hat{\mathbf{k}}))$$

$L = 1$, dipole asymmetry

$L = 2$, quadrupole asymmetry $\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}_{\mathcal{R}}^{(0)}(1 + g_* (\hat{\mathbf{p}}.\hat{\mathbf{k}})^2)$



Groeneboom & Eriksen, 2008 $g^*=0.14$

Anisotropic Inflation from Gauge Field Dynamics

The model contains a $U(1)$ gauge field minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{f^2(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\right]$$

Here $1/f(\phi)$ is the time-dependent gauge kinetic coupling.

We turn on the background gauge field $A_\mu = (0, A_x(t), 0, 0)$
The background metric is

$$\begin{aligned} ds^2 &= -dt^2 + e^{2\alpha(t)}\left(e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2 + dz^2)\right) \\ &= -dt^2 + a(t)^2dx^2 + b(t)^2(dy^2 + dz^2) \end{aligned}$$

In this view $H \equiv \dot{\alpha}$ is the average Hubble expansion rate and

$$H_a \equiv \frac{\dot{a}}{a} \quad , \quad H_b \equiv \frac{\dot{b}}{b}$$

The anisotropy in the system is measured by

$$\frac{\dot{\sigma}}{H} \equiv \frac{H_b - H_a}{H}$$

The background equations are too complicated to be solved !

It is instructive to compare the gauge field energy density to the total energy density

$$R \equiv \frac{\rho_A}{V} = \frac{\dot{A}^2 f(\phi)^2 e^{-2\alpha}}{2V} = \frac{p_A^2}{2V} f^{-2} e^{-4\alpha}$$

In the absence of the conformal factor we see that the gauge field energy density decays exponentially like $e^{-4\alpha} = 1/a^4$.

We choose $f(\phi)$ such that R becomes small but constant

$$a \propto \exp\left[-\int d\phi \frac{V}{V_\phi}\right] .$$

So if one chooses

$$f \propto \exp\left[-n \int d\phi \frac{V}{V_\phi}\right]$$

this yields $f \propto a^n$

$$V = \frac{1}{2} m^2 \phi^2 \qquad \rightarrow \qquad f(\phi) = \exp\left(\frac{c\phi^2}{2M_P^2}\right) = \left(\frac{a}{a_f}\right)^{-2c}$$

with c a constant very close to unity.

The attractor solution

With this choice of $f(\phi)$ one makes sure that the energy density of the gauge field is sub-dominant but non-decaying
$\rightarrow$ The Attractor Solution

One can show that during the attractor solution

$$R = \frac{c-1}{2c}\epsilon_H = \frac{1}{2}I\epsilon_H \quad , \quad I \equiv \frac{c-1}{c}$$

Back-reaction of the gauge field:

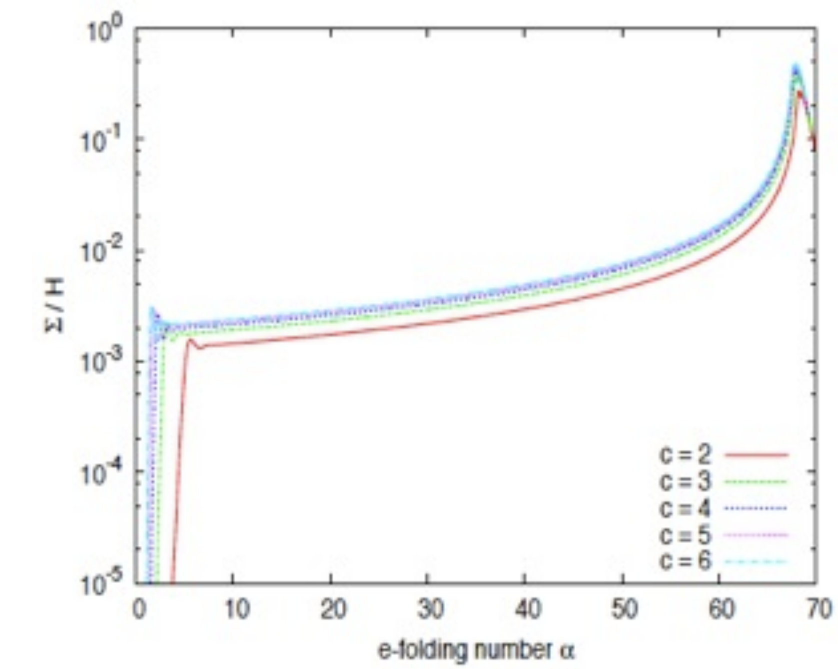
$$M_P^{-2}\frac{d\phi}{d\alpha} \simeq -\frac{V_\phi}{V} + \frac{c-1}{c}\frac{V_\phi}{V} \; .$$

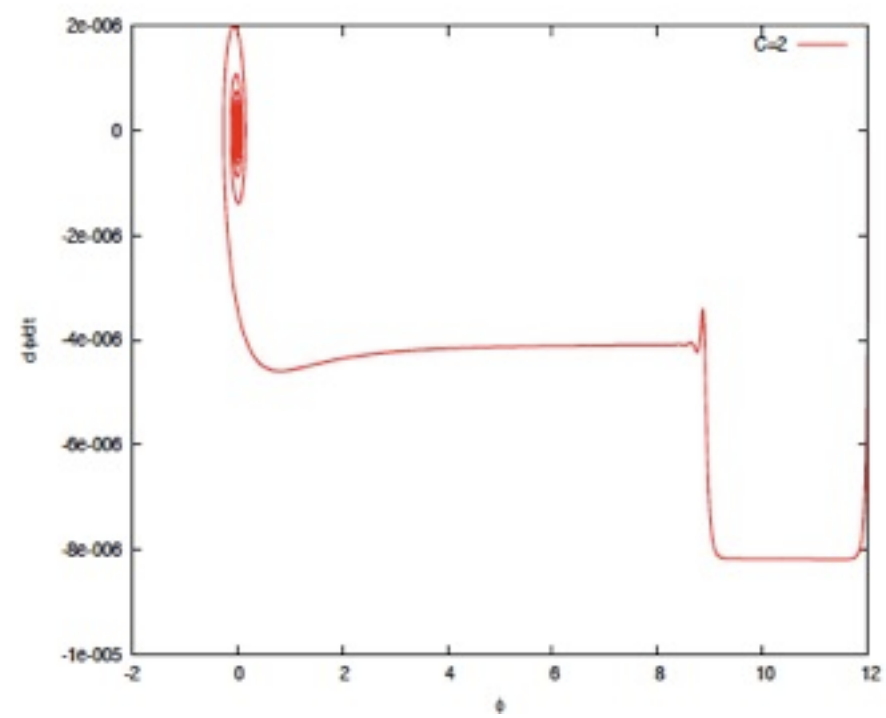
This means that the back-reactions of the gauge field on the inflation field change the effective mass of the inflaton $m_{eff}^2 = \frac{m_{eff}^2}{c}$.

The inflaton trajectory is given by

$$\phi_e^2 - \phi^2 = 4M_P^2\alpha(1-I)$$

Watanabe, Kanno, Soda, 09





Perturbations in anisotropic inflation

There are two methods to study perturbations in anisotropic inflation background:

Bartolo, Matarrese, Peloso, Ricciardone, 2012
See talk by Angelo Ricciardone

- In-In formalism: more rigorous but difficult
- δN formalism: simpler but can be quite tricky!

We start with the δN formalism.

δN is a very powerful method in cosmological perturbation theory. δN allows to calculate the curvature perturbations to all order in perturbation theory.

Advantage:

we only need to solve the number of e-folds as a function of background fields, $N = N(\phi_i)$.

$$\begin{aligned} \mathcal{R} &= \delta N \\ &= N_{\phi_i}\delta\phi_i + \frac{1}{2}N_{\phi_i\phi_j}\delta\phi_i\delta\phi_j \end{aligned} \tag{1}$$

Disadvantage:

It is applicable only on superhorizon scales. It can not take into effect the intearctions which happen inside the horizon or around the time of horizon crossing. Example: DBI model

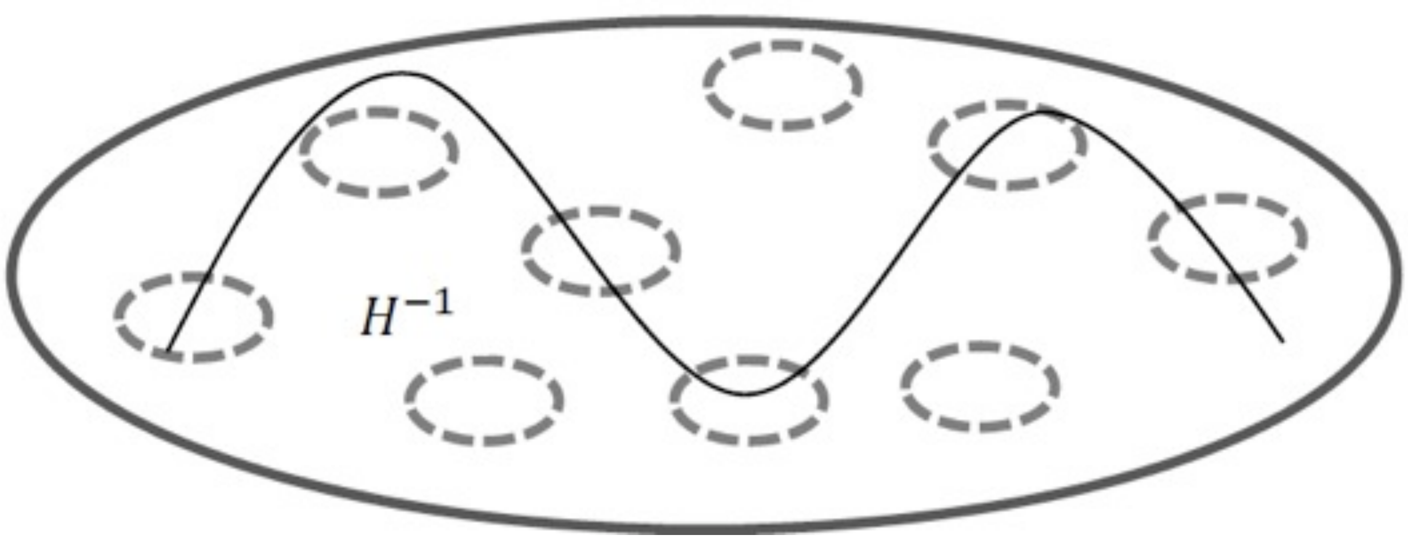
Separate Universe Approach:

It is assumed that inside each small patch the universe behaves like an FRW Universe with

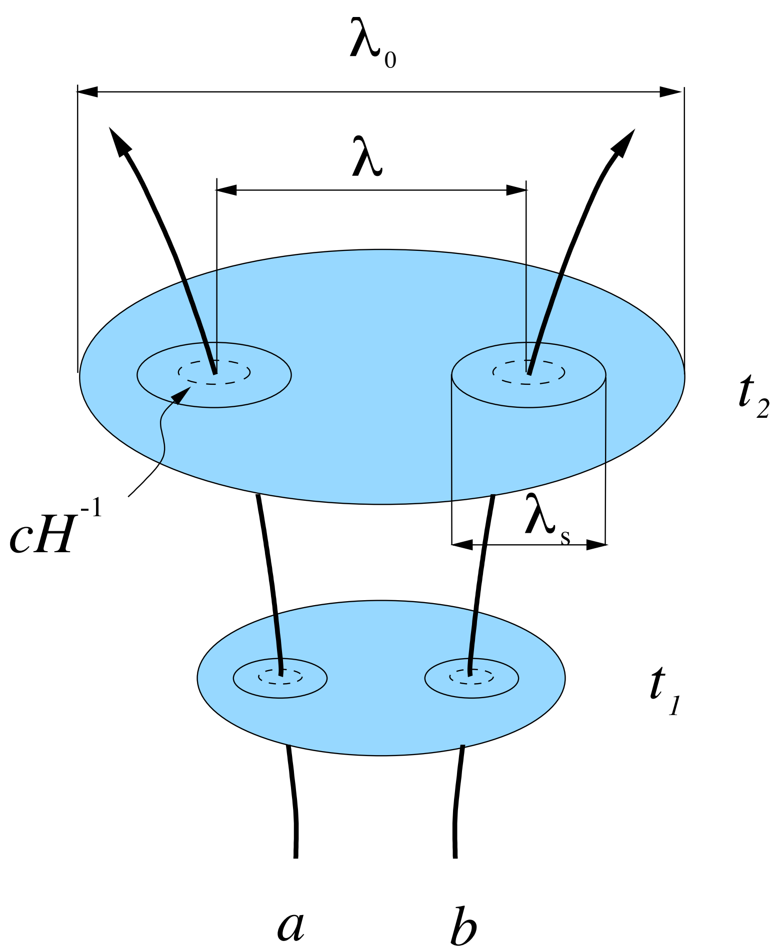
$$a(t,\mathbf{x}) = a(t)e^{\psi(t,\mathbf{x})}$$
$$\rho = \rho(t,\mathbf{x}) \quad , \quad p = p(t,\mathbf{x})$$

In δN formalism $\mathcal{R} = \delta N$.

To use this prescription the initial surface has to be the flat surface, $\psi = 0$. The final surface has to be the surface of constant energy, $\delta\rho = 0$.



Bazrafshan, 2011



Wands et al, 2000.

We would like to extend δN formalism to anisotropic Universe

$$ds^2 = -dt^2 + a_1(t)^2 dx^2 + a_2(t)^2 dy^2 + a_3(t)^2 dz^2 \,. \qquad (2)$$

The background equations are

$$\begin{aligned}
3\mathcal{H}^2 &\equiv \sum_{i>j} \bar{H}_i \bar{H}_j = \frac{\bar{\rho}}{M_P^2} \\
\bar{T}^0{}_i &= \bar{q}_i = 0 \\
M_P^2 \dot{\bar{H}}_i &= -3M_P^2 \bar{H}\bar{H}_i + \frac{1}{2}(\bar{\rho} - \bar{p}) + \bar{\pi}^i{}_i \\
-u_\mu \nabla_\nu T^{\mu\nu} &= \dot{\bar{\rho}} + 3\bar{H}(\bar{\rho} + \bar{p}) + \bar{H}_j \bar{\pi}^i{}_j \delta_i^j = 0.
\end{aligned}$$

In which the Hubble parameters are defined via

$$\bar{H}_i(t) = \frac{\dot{a}_i}{a_i} \quad , \quad \bar{H} \equiv \frac{1}{3}\sum_{i=1}^{3} \bar{H}_i$$

Note that $\bar{H} \neq \mathcal{H}$.

Let us define the gradient expansion perturbation parameter ϵ

$$\epsilon \equiv \frac{k}{aH}$$

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$$\epsilon \equiv \frac{k}{aH}$$

Locally, inside in Hubble size patch, the $(0,0)$ fields equations become

$$3M_P^2\mathcal{H}^2(\mathbf{x},t) = \rho(\mathbf{x},t) + O(\epsilon^2),$$

$$M_P^2\frac{dH_i(\mathbf{x},t)}{d\tau} = -3M_P^2H(\mathbf{x},t)H_i(\mathbf{x},t) + \frac{1}{2}\left(\rho(\mathbf{x},t) - p(\mathbf{x},t)\right) + {\pi^i}_i(\mathbf{x},t) + \mathcal{O}(\epsilon^2)$$

$$\frac{d\rho(\mathbf{x},t)}{d\tau} + 3H(\mathbf{x},t)\left(\rho(\mathbf{x},t) + p(\mathbf{x},t)\right) + \sum_i {\pi^i}_i(\mathbf{x},t)H_i(\mathbf{x},t) = \mathcal{O}(\epsilon^2)\,.$$

in which

$$\mathcal{H}^2(\mathbf{x},t) \equiv \frac{1}{3}\sum_{i>j} H_i(\mathbf{x},t)H_j(\mathbf{x},t)\,,$$

$$H(\mathbf{x},t) \equiv \frac{1}{3}\sum_i H_i(\mathbf{x},t)$$

Conclusion: The separate universe recipe works and it is enough to replace any background function $f(t)$ by its local form $f(\mathbf{x},t)$ and also using new local directional Hubble parameters $H_i(\mathbf{x},t)$.

Application to Anisotropic Inflation

In model of anisotropic inflation with U(1) gauge field we have

$$V = \frac{1}{2} m^2 \phi^2 \qquad \rightarrow \qquad f(\phi) = \exp\left(\frac{c\phi^2}{2M_P^2}\right)$$

During the attractor phase

$$\dot{A}^2 f^2 e^{-2\alpha} = I\, \epsilon_H V$$

Furthermore, the back-reaction of the gauge field on the inflaton dynamics changes the effective mass of inflaton

$$M_P^{-2} \frac{d\phi}{d\alpha} \simeq -\frac{V_\phi}{V} + \frac{c-1}{c} \frac{V_\phi}{V}$$

This yields

$$\phi_e^2 - \phi^2 = 4M_P^2 \alpha (1 - I)$$

Playing with the above equations yields

$$\delta N \simeq -\frac{\phi}{2M_P^2} \delta\phi + 2IN \frac{\delta \dot{A}_x}{\dot{A}}$$

This is our result for δN to linear order in terms of $\delta\phi$ and $\delta\dot{A}$.

The gauge field fluctuations are given by

$$\delta A_i = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\overrightarrow{k}.\overrightarrow{x}} \vec{\epsilon}_\lambda(k) \frac{\widehat{V}_i}{f} ,$$

in which

$$\widehat{V} = a_\lambda(\overrightarrow{k}) V_\lambda(k) + a^\dagger_\lambda(-\overrightarrow{k}) V^*_\lambda(k) .$$

They satisfy

$$V_\lambda(k)'' + \left(k^2 - \frac{f''}{f}\right) V_\lambda(k) = 0 ,$$

Deep inside the horizon

$$V_\lambda(k) \simeq \frac{1+ik\eta}{\sqrt{2}k^{3/2}\eta} e^{-ik\eta} .$$

As a result

$$\frac{\delta\dot{\vec{A}}}{\dot{A}} = \sum_\lambda \vec{\epsilon}_\lambda \frac{\sqrt{3}H}{\sqrt{2I\epsilon_H k^3}} \qquad (k > aH)$$

In particular, we see that on super-horizon scale, $\delta\dot{A}_x/\dot{A}_x$ is a constant.

Now we can calculate the power spectrum

A.A. Abolhasani, R. Emami, J. Taghizadeh, H. F., 2013

$$\mathcal{R} = \delta N = -\frac{\phi}{2M_P^2}\delta\phi + 2IN\frac{\delta\dot{A}_x}{\dot{A}}.$$

The isotropic and the anisotropic parts are $\mathcal{P}_\mathcal{R} \equiv \mathcal{P}_0 + \Delta\mathcal{P}$ in which

$$\mathcal{P}_0 = \frac{H^2}{8\pi^2 M_P^2 \epsilon_H}\,.$$

To calculate the anisotropic power spectrum we note that $\delta\phi$ and $\delta\dot{A}$ are mutually uncorrelated so $\langle\delta\phi\delta\dot{A}\rangle|_* = 0$. As a results

$$\begin{aligned}\Delta\mathcal{P} &= \frac{k_1^3}{2\pi^2}4I^2N^2\left\langle\frac{\delta\dot{A}_x(k_1)}{A_x}\frac{\delta\dot{A}_x(k_2)}{A_x}\right\rangle \\ &= \frac{k_1^3}{2\pi^2}\frac{6IH^2}{\epsilon_H k_1^3}N^2\sin^2\theta = 24\,IN^2\mathcal{P}_0\sin^2\theta\end{aligned}$$

in which the angle θ is defined via $\cos\theta = \hat{n}.\hat{k}$. Now comparing this with the anisotropy factor g_* defined via

$$\mathcal{P}_\mathcal{R}(\vec{k}) = \mathcal{P}_0\left(1 + g_*(\hat{k}.\hat{n})^2\right)\,.$$

we obtained

$$g_* = -24IN^2$$

The observational constraints from CMB and LSS require $|g_*| < 0.3$. With $N = 60$ we obtain $I < 10^{-5}$.

The IR Problem: From the formula $g_* = -24IN^2$ we see that anisotropies diverges with N^2.

This is because once the gauge field excitations leave the horizon they become classical. However, δA_i are anisotropic so as the gauge field excitations leaves the horizon they renormalize the background value $A_x(t)$.

This indicate a fine-tuning in the problem: N has to be large enough to solve the flatness and the horizon problem, but not too large to destroy the isotropy.

As we shall see this is not restricted to power spectrum, this problem also shows up in bispectrum and trispectrum.

Bispectrum

A.A. Abolhasani, R. Emami, J. Taghizadeh, H. F., 2013

The second oder δN is

$$\delta N = N_\phi \delta\phi + N_{\dot{A}} \delta\dot{A} + \frac{N_{\phi\phi}}{2}\delta\phi^2 + \frac{N_{\dot{A}\dot{A}}}{2}\delta\dot{A}^2 + N_{\phi\dot{A}}\delta\phi\delta\dot{A}_x$$

in which to leading order in I

$$N_\phi \simeq -\frac{\phi}{2M_P^2} \quad , \quad N_{\phi\phi} \simeq \frac{2f_{,\phi}^2}{f^2} + \frac{2f_{,\phi\phi}}{f} + \frac{\phi^2}{M_P^4} + \frac{4\phi}{M_P^2}\frac{f_{,\phi}}{f}$$

and

$$N_{,\dot{A}} \simeq \frac{2IN}{\dot{A}} \quad , \quad N_{,\dot{A}\dot{A}} \simeq \frac{2IN}{\dot{A}^2} \quad , \quad N_{,\phi\dot{A}} \simeq \frac{4IN}{\dot{A}}\frac{f_\phi}{f}$$

The leading contribution in bispectrum comes from $N_{\dot{A}\dot{A}}$ term and

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle$$
$$\simeq 4I^3 N(k_1)N(k_2)N(k_3)\int\frac{d^3p}{(2\pi)^3}\langle\delta\dot{A}_x(\vec{k}_1)\delta\dot{A}_x(\vec{k}_2)\delta\dot{A}_i(\vec{p})\delta\dot{A}_i(\vec{k}_3-\vec{p})\rangle + 2\text{perm.}$$

The bispectrum is

$$\langle\zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle \simeq 288 I N_{k_1}N_{k_2}N_{k_3}\left(C(\vec{k}_1,\vec{k}_2)P_0(k_1)P_0(k_2) + 2\text{perm.}\right)(2\pi)^3\,\delta^3(\sum_i \vec{k}_i)$$

in which the momentum shape function $C(\vec{k}_1,\vec{k}_2)$ is defined via

$$C(\vec{k}_1,\vec{k}_2) \equiv \left(1-(\widehat{k}_1.\widehat{n})^2-(\widehat{k}_2.\widehat{n})^2+(\widehat{k}_1.\widehat{n})\,(\widehat{k}_2.\widehat{n})\,(\widehat{k}_1.\widehat{k}_2)\right)$$

In the squeezed limit we get

$$\begin{aligned} f_{NL} &= 240 I N(k_1) N(k_2)^2 C(\vec{k}_1, \vec{k}_2) \qquad (k_1 \ll k_2 \simeq k_3) \\ &\simeq 10 N \, |g_*| \, C(\vec{k}_1, \vec{k}_2) \end{aligned}$$

Similarly, calculating the trispectrum $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle$ we obtain

$$\begin{aligned} \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle &\simeq 3456 I N_{k_1} N_{k_2} N_{k_3} N_{k_4} \Big(D(\vec{k}_3, \vec{k}_4, \vec{k}_1 + \vec{k}_3) P(k_3) P(k_4) P(|\vec{k}_1 + \vec{k}_3|) \\ &\quad + 11 \text{perm.} \Big) (2\pi)^3 \, \delta^3 \left(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4 \right) , \end{aligned}$$

in which

$$\begin{aligned} D(\vec{k}_3, \vec{k}_4, \vec{k}_1 + \vec{k}_3) &= 1 - (\widehat{k}_4 . \widehat{n})^2 - (\widehat{k}_3 . \widehat{n})^2 - (\widehat{k_1 + k_3} . \widehat{n})^2 + (\widehat{k}_3 . \widehat{n})(\widehat{k}_4 . \hat{n})(\widehat{k}_3 . \widehat{k}_4) \\ &\quad + (\widehat{k}_4 . \widehat{n})(\widehat{k_1 + k_3} . \widehat{n})(\widehat{k_1 + k_3} . \widehat{k}_4) + (\widehat{k}_3 . \widehat{n})(\widehat{k_1 + k_3} . \widehat{n})(\widehat{k_1 + k_3} . \widehat{k}_3) \\ &\quad - (\widehat{k}_3 . \widehat{n})(\widehat{k}_4 . \widehat{n})(\widehat{k_1 + k_3} . \widehat{k}_3)(\widehat{k_1 + k_3} . \widehat{k}_4) . \end{aligned}$$

In the collapsed limit $\vec{k}_1 + \vec{k}_3 = \vec{k}_2 + \vec{k}_4 = 0$

$$\tau_{NL}(k_1, k_2, k_3, k_4) \simeq 3456 I N(k_3)^2 N(k_4)^2 D(\vec{k}_3, \vec{k}_4, \vec{k}_1 + \vec{k}_3) .$$

Our result is in exact agreement with the results obtained from in-in formalism!

Bartolo, Matarrese, Peloso, Ricciardone, 2012

See talk by Angelo Ricciardone

PLANCK has reported hemispherical asymmetry.
A good way to parametrize the hemispherical asymmetry is via dipole modulation

$$\Delta T(\hat{n}) = \overline{\Delta T(\hat{\mathbf{n}})}(1 + A\,\hat{\mathbf{n}}.\hat{\mathbf{p}})$$

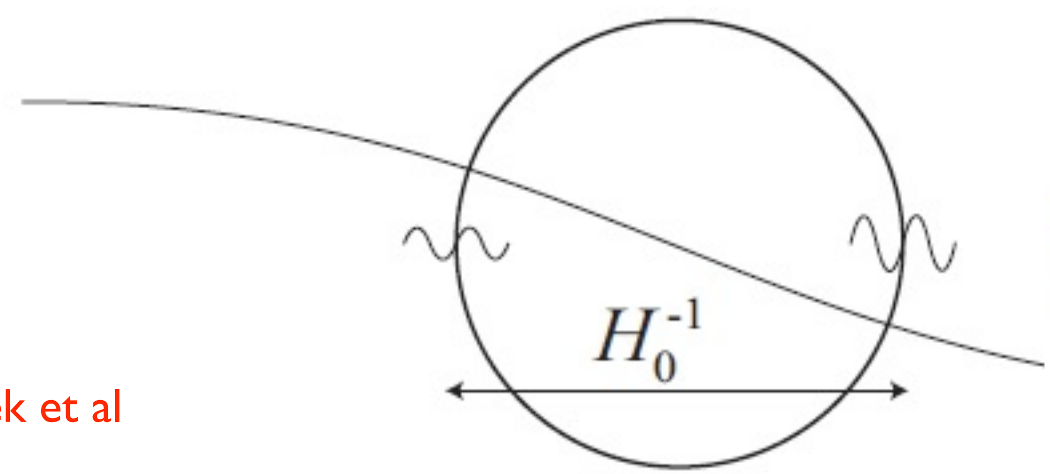
in which $\hat{\mathbf{p}}$ is the preferred dipole direction. From Planck, we have $A = 0.07 \pm 0.02$ for $\ell < 64$.

For the primordial curvature perturbation this translates into

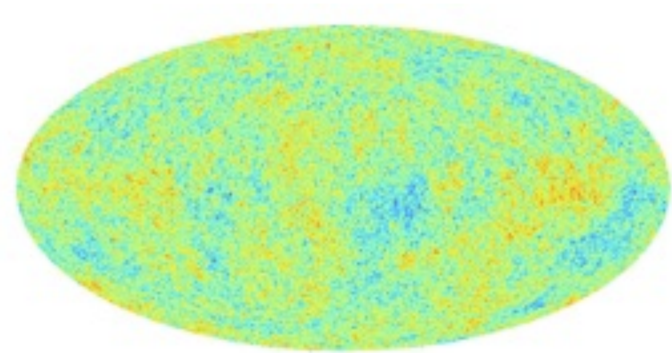
$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}^{(0)}(1 + 2A\,\hat{\mathbf{n}}.\hat{\mathbf{p}}) \quad \rightarrow \quad \frac{\nabla P_{\mathcal{R}_k}}{P_{\mathcal{R}_k}} \simeq \frac{2A\hat{\mathbf{p}}}{x_{CMB}}$$

Question: How to model this dipole modulation?

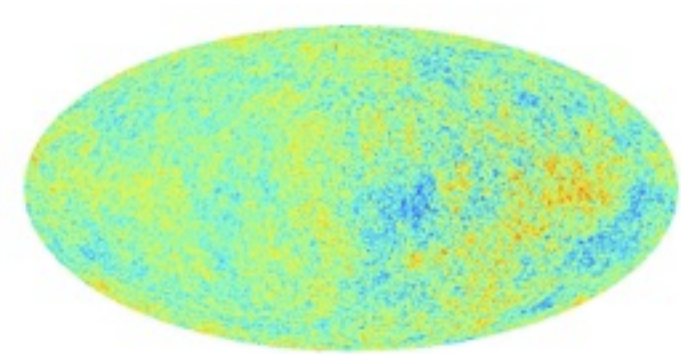
Long Mode Modulation: Suppose there exists a very long super-horizon mode from pre-inflationary epoch which can modulate the smaller CMB-scale modes (Erickcek, Kamionkowski, Carroll (2008) , also Grishchuk and Zeldovich (1978))



Erickcek et al



A=0



A=0.3

A Consistency Condition relating A to f_{NL}:

Namjoo, Baghram, H.F. arXiv: 1305.0813

Assumption: inflation perturbation has single source.

Consider a single, long wavelength, mode $k_L \gg k_{CMB}$

$$\mathcal{R}_L = \mathcal{R}_{k_L} \sin(\mathbf{k}_L.\mathbf{x}) = {\mathcal{P}_{\mathcal{R}}}_L^{1/2} \sin(\mathbf{k}_L.\mathbf{x}) \, .$$

$$\begin{aligned}\left\langle \mathcal{R}(\mathbf{k_L})\mathcal{R}(\mathbf{k_1})\mathcal{R}(\mathbf{k_2})\right\rangle &\simeq \left\langle \mathcal{R}(\mathbf{k_L}) \left\langle \mathcal{R}(\mathbf{k_1})\mathcal{R}(\mathbf{k_2})\right\rangle_{\mathcal{R}(\mathbf{k_L})}\right\rangle \\ &\simeq \left\langle \mathcal{R}(\mathbf{k_L}) \left(\mathcal{R}(\mathbf{k_L}) \frac{\partial}{\partial \mathcal{R}(\mathbf{k_L})} \left\langle \mathcal{R}(\mathbf{k_1})\mathcal{R}(\mathbf{k_2})\right\rangle \Big|_{\mathcal{R}(\mathbf{k_L})=0} \right) \right\rangle\end{aligned}$$

This leads to

$$\frac{1}{P_{\mathcal{R}_k}} \nabla P_{\mathcal{R}_k} = \frac{12}{5} f_{NL} \nabla \mathcal{R}_L \, .$$

Definition $\frac{\nabla P_{\mathcal{R}_k}}{P_{\mathcal{R}_k}} \simeq \frac{2A\,\hat{\mathbf{p}}}{x_{CMB}}$ and noting that we started with a single sin mode yields

$$A(k) = \frac{6}{5} \left(k_L x_{CMB} \, \mathcal{P}^{1/2}_{\mathcal{R}_{k_L}} \right) f_{NL}$$

This is our consistency condition, relating the amplitude of dipole asymmetry to the amplitude of non-Gaussianity in the squeezed limit.

See also D. Lyth, arXiv: 1304. 1270.

We have found

$$A(k) = \frac{6}{5} \left(k_L x_{CMB} \mathcal{P}^{1/2}_{\mathcal{R}_{k_L}} \right) f_{NL}$$

To obtain interesting bound from this formula, we nee some constraints on $\left(k_L x_{CMB} \mathcal{P}^{1/2}_{\mathcal{R}_{k_L}} \right)$.

The strongest constrains on the effects of long mode modulation coms from the octupole $Q_3 = 3\sqrt{C_3} \lesssim 2.7 \times 10^{-3}$:

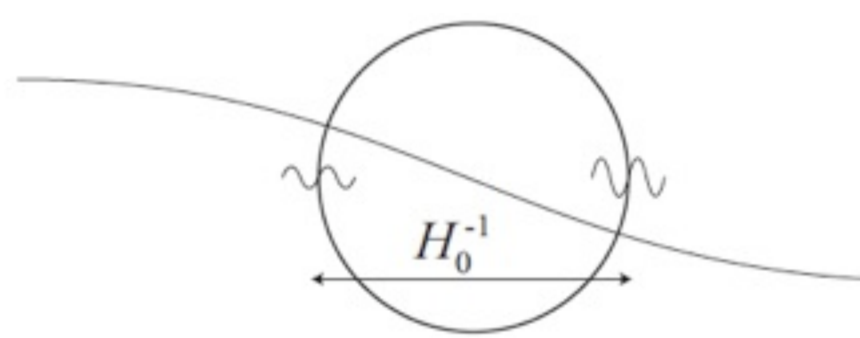
$$(k_L x_{CMB})^3 \mathcal{P}_{\mathcal{R}_{k_L}} \lesssim 53 Q_3 \qquad (\textit{Erickcek et al}, (2008))$$

On the other hand, to trust our perturbation treatment we require

$$\mathcal{P}_{\mathcal{R}_L} < 1$$

Combining these two equations yield

Erickcek et al



$$|A| \lesssim 10^{-1} |f_{NL}| \,.$$

To explain the observed anisotropy we require $|A(k)| = 0.07 \pm 0.02$

Namjoo, Baghram, H.F. arXiv: 1305.0813

Examples:

- Single field slow-roll inflation
  One has $f_{NL} \sim 1 - n_s$ so we obtain $|A| \lesssim 10^{-1}(1 - n_s) \sim 10^{-3}$. This is too small to address the observed dipole asymmetry.

- Curvaton Model (Erickcek et al, 2008, Lyth, 2013)
  In standard curvaton model, f_{NL} is independent of n_s so f_{NL} can be as large as allowed by PLANCK $f_{NL} = 2.7 \pm 5.8$ (68 % CL) . As a result one can easily obtain $A = 0.07$ to address the dipole asymmetry.

- non-attractor model
  In non-attractor model, $\mathcal{R}$ is not frozen on super-horizon scales and during the non-attractor phase

$$f_{NL} = \frac{5(1 + c_s^2)}{4c_s^2}$$

  One can easily get large enough f_{NL} to get $A = 0.07$.

  Advantage: The dipole asymmetry disappears on smaller CMB scales, from the constraints on quazar studies (Hirata, 2009). Therefore, one should consider a scale-dependence f_{NL}. This is naturally constructed in non-attractor model.

Consistency condition for asymmetry in gravitational waves

Consider a general operator $\mathcal{O}$ with the single-source perturbations. Define

$$\mathcal{P}_{\mathcal{O}}(k) \simeq \mathcal{P}^{iso}_{\mathcal{O}}(k)\left(1 + 2A_{\mathcal{O}}(k)\hat{\mathbf{p}}.\mathbf{x}/x_{CMB}\right) ,$$

$$\langle \mathcal{R}(\mathbf{k}_L)\mathcal{O}(\mathbf{k_1})\mathcal{O}(\mathbf{k_2})\rangle \equiv (2\pi)^3\delta(\mathbf{k_L} + \mathbf{k_2} + \mathbf{k_3})\left(\frac{12}{5} f_{NL}^{\mathcal{R}\mathcal{O}}\right) P_{\mathcal{R}}(k_L)P_{\mathcal{O}}(k_1)$$

Following the same steps as before, we obtain

$$A_{\mathcal{O}} \simeq \frac{6}{5} f_{NL}^{\mathcal{R}\mathcal{O}} \left(x_{CMB}k_L\mathcal{P}^{1/2}_{\mathcal{R}_L}\right) .$$

Example : $\mathcal{O} = h_{ij}$:

$$\frac{A_T}{A_{\mathcal{R}}} \simeq \frac{f_{NL}^{\mathcal{R}h}}{f_{NL}^{loc}} .$$

A. Abolhasani, S. Baghram, H. F., M. H. Namjoo, arXiv: 1305.0813

On the other hand, from Maldacena's analysis we have

$$\frac{12}{5} f_{NL}^{loc} = -(1 - n_s) \qquad , \qquad \frac{12}{5} f_{NL}^{\mathcal{R}h} = -n_T .$$

therefore

$$\left|\frac{A_T}{A_{\mathcal{R}}}\right| \simeq \frac{n_T}{n_s - 1} = \frac{r}{8(1 - n_s)} .$$

Observationally this is a very interesting result! With $n_s = 0.96, r \lesssim 0.13, A = 0.07$ this yields A_T

Multiple Fields Models

The situation becomes much easier in multiple fields models

Example: Asymmetry from the inhomogeneous created at the end of inflation :

$$V = \frac{\lambda}{4}\left(\chi^2 - \frac{M^2}{\lambda}\right)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2 + \frac{\gamma^2}{2}\chi^2\sigma^2 .$$

The surface of end of inflation is given by

$$\phi_e^2 + \frac{\gamma^2}{g^2}\sigma^2 = \phi_c^2 \qquad \phi_c \equiv \frac{M}{g} .$$

The curvature perturbation is

$$\mathcal{R}_e = \frac{3}{\alpha}\left[\frac{\delta\phi_*}{\phi_*} + \frac{\mathcal{F}}{1-\mathcal{F}}\frac{\delta\sigma}{\sigma} + \frac{\mathcal{F}(1+\mathcal{F})}{2(1-\mathcal{F})^2}\left(\frac{\delta\sigma}{\sigma}\right)^2\right] + \dots .$$

in which we have defined

$$\alpha \equiv \frac{m^2}{H^2} = \frac{12\lambda m^2 M_P^2}{M^4} \qquad \mathcal{F} \equiv \frac{\gamma^2\sigma_*^2}{g^2\phi_c^2} .$$

Let us define the weight of the σ field in $\mathcal{P}_{\mathcal{R}}$

$$w_\sigma \equiv \frac{N_\sigma^2 \mathcal{P}_\sigma}{\mathcal{P}_{\mathcal{R}}} = \frac{\mathcal{F}^2 \phi_*^2}{\mathcal{F}^2 \phi_*^2 + (1-\mathcal{F})^2}$$

The resulting non-Gaussianity is

$$f_{NL} = w_\sigma^2 f_{NL}^\sigma \qquad , \qquad f_{NL}^\sigma = \frac{\alpha\,(1+\mathcal{F})}{6\mathcal{F}}$$

Plugging this in the formula yields

$$|A| \lesssim \frac{|f_{NL}|}{10\,\sqrt{w_\sigma}}\,.$$

Alternatively, in terms of τ_{NL} this yields

$$A \lesssim \frac{\sqrt{\tau_{NL}}}{12}\,.$$

It is very interesting that the upper bound on A is independently controlled by τ_{NL}.

With the upper bound $\tau_{NL} < 2800$ (95 % CL) from Planck data one obtains $A \lesssim 4.4$.

Conclusion

- There are strong evidences for hemispherical asymmetry in CMB.
- a good way to parametrize the hemispherical asymmetry is via dipole modulation. PLANCK data shows $A \simeq 0.07$
- The modulation from a long large amplitude mode may be behind the CMB dipole asymmetry. Simple single field models of inflation can not generate large enough dipole asymmetry.
- There is a consistency relation between the amplitude of tensor perturbation asymmetry and the scalar perturbation asymmetry $A_T/A_{\mathcal{R}} \sim r/8(1-n_s)$. It is interesting to see if dipole asymmetry can be observed in CMB B-mode polarizations.
- Another natural way to produce primordial anisotropy is to turn on $U(1)$ gauge fields. With the appropriate form of the conformal factor $f(\phi)$ one can generate attractor solutions.
- The level of quadrupole anisotropy is given by $g_* = -24IN^2$, diverging with N.
- There are large orientation-dependence bispectrum and trispectrum in these models which can be tested by CMB and LSS.