THE LEVELS OF STATISTICAL ANISOTROPY IN THE POWER SPECTRUM OF THE CURVATURE PERTURBATION ARE SCALE-DEPENDENT: APPLICATION TO GAUGE-FLATION AND HAIRY INFLATION

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Questions

- Is the Universe experiencing anisotropic expansion?
- What is the relation between anisotropic expansion and the properties of the field correlators?
- Is the Universe statistically anisotropic?
- How many levels of statistical anisotropy are there?: just one?
- How many preferred directions do there exist?: just one?
- In particular, if the expansion is isotropic, what is the preferred direction?

Questions

- Are the levels of statistical anisotropy scale-dependent?
- What is going on with Hairy inflation? (WKS,2009)
- What is going on with Gauge-flation? (MS-J,2011)

Facts (and more questions)

- The usual parameterization of an statistically anisotropic power spectrum of the curvature perturbation (ACW, 2007) employs one level of statistical anisotropy and one preferred direction:

$$P_\zeta(\mathbf{k}) = P_\zeta^{\rm iso}(k)\left[1 + g_\zeta(\hat{\mathbf{k}}\cdot\hat{\mathbf{d}})^2\right]$$

Why just one?

- The level of statistical anisotropy is scale-independent. Why?

- WMAP 5-year data analysis is made assuming a scale-independent level of statistical anisotropy (GAWE, 2010).

Facts (and more questions)

- In the Hairy inflation model (the $f(\Phi)F^2$ model) (WKS, 2009) the cosmological perturbation theory is made assuming that the preferred direction is the same as the one in which the Universe is experiencing anisotropic expansion. Why should it be that way?
- Current analysis of the Gauge-flation model (the Non-Abelian Gauge inflation model) (MS-J, 2011) show that the power spectrum of the curvature perturbation is statistically isotropic. But this is unlikely, so…??

Our main purpose

- To show that, asking for internal theoretical consistency, the levels of statistical anisotropy must be, in general, scale-dependent.
- To call for a unification, among data analyst cosmologists and the different groups of theoretical cosmologists, of the parameterization employed when investigating statistical anisotropy in the power spectrum of the curvature perturbation.

Byproducts

- We postulate the conjecture that anisotropic expansion is equivalent to having at least one of the field perturbations non-invariant under spatial rotations.
- We find new levels of statistical anisotropy and new preferred directions when scalar and vector fields play a role during inflation.
- We find non-zero levels of statistical anisotropy in the Gauge-flation model, in contrast to (MS-J, 2011), that rules out the model in view of the observations.

How do we reach our purpose?

By answering one by one the previously stated questions.

Questions/Answers

Question

+ Is the Universe experiencing anisotropic expansion?

Answer

+ No definitive answer yet, but the current constraint is given by (CCFM,2011)

$$\left|\frac{\Sigma}{H}\right| < 0.012.$$

Questions/Answers

Question

- What is the relation between anisotropic expansion and the properties of the field correlators?

Answer

- If the expansion is anisotropic, the field perturbations live in an anisotropic metric, so there is no reason to think that all the field perturbations are rotationally invariant.
- If the expansion is isotropic, the perturbed field equations in momentum space only involve ***k*** in the field perturbation variables, so all the field perturbations should be rotationally invariant.

Questions/Answers

Question

- What is the relation between anisotropic expansion and the properties of the field correlators?

Answer

- So, we conjecture that *anisotropic expansion* is equivalent to *having at least one of the field correlators non-invariant under field rotations.*

↓

- *Statistical anisotropy in the field perturbations.*

Questions/Answers

Question

- Is the Universe statistically anisotropic?

Answer

- No definitive answer yet, but the current constraint is

$$|g_\zeta| \lesssim 0.03.$$

↓

- No scale-dependence employed! (GAWE, 2010)

Questions/Answers

Question

- How many levels of statistical anisotropy are there?: just one?

Answer

- ACW employ just one level, but that corresponds to the simplest case when just one degree of freedom is present during inflation.
- By employing the δN formalism extended to the case of (in general) anisotropic expansion and scalar and vector fields involved during inflation (V-TRBA, 2011), we find…

Questions/Answers

Question

- How many levels of statistical anisotropy are there?: just one?

Answer

- Case I - Isotropic Expansion: there are as many levels of statistical anisotropy as the number of vector fields present:

$$g_\zeta^a = \frac{(r_{long}^a - 1)(N_i^a)^2 P_+^a(k_1)}{(N_\phi^I)^2 P_{\delta\phi}^I(k_1) + (N_i^a)^2 P_+^a(k_1)}.$$

Questions/Answers

Question

+ How many levels of statistical anisotropy are there?: just one?

Answer

+ Case II - Anisotropic Expansion: there are *I + 2a* levels of statistical anisotropy:

$$g_N^a = \frac{(r_{long}^a - 1)(N_i^a)^2 P_+^{a\,iso}(k_1)}{(N_\phi^I)^2 P_{\delta\phi}^{I\,iso}(k_1) + (N_i^a)^2 P_+^{a\,iso}(k_1)}.$$

$$g_{\delta\phi}^I = \frac{\tilde{g}_{\delta\phi}^I(k_1)(N_\phi^I)^2 P_{\delta\phi}^{I\,iso}(k_1)}{(N_\phi^I)^2 P_{\delta\phi}^{I\,iso}(k_1) + (N_i^a)^2 P_+^{a\,iso}(k_1)}.$$

$$g_+^a = \frac{\tilde{g}_+^a(k_1)(N_i^a)^2 P_+^{a\,iso}(k_1)}{(N_\phi^I)^2 P_{\delta\phi}^{I\,iso}(k_1) + (N_i^a)^2 P_+^{a\,iso}(k_1)}.$$

Questions/Answers

Question

- How many preferred directions do there exist?: just one?

Answer

- Case I - Isotropic Expansion: there are as many preferred directions as the number of vector fields present:

$$P_\zeta(\mathbf{k}_1) = P_\zeta^{iso}(k_1)\left[1 + g_\zeta^a(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{N}}^a)^2\right] .$$

- Case II - Anisotropic Expansion: there are *I + 2a* preferred directions:

$$P_\zeta(\mathbf{k}_1) = P_\zeta^{iso}(k_1)\left[1 + g_{\delta\phi}^I(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{d}}_{\delta\phi}^I)^2 + g_+^a(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{d}}_+^a)^2 + g_N^a(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{N}}^a)^2\right] .$$

Questions/Answers

Question

- In particular, if the expansion is isotropic, what is the preferred direction?

Answer

- Answer in the form of question:

Why should we always choose the preferred direction as the one in which the Universe is expanding anisotropically if there clearly exist examples of isotropic expansion and preferred direction(s)?

- ✓ Don’t mix up “preferred direction for anisotropic expansion” with “preferred direction for statistical anisotropy”.

Questions/Answers

Question

- Are the levels of statistical anisotropy scale-dependent?

Answer

- The levels of statistical anisotropy are, in general, scale-dependent. They could be scale-independent for some specific models.

- ✓ However, if we insist on having *just one* preferred direction, the resultant level of statistical anisotropy turns out to be scale-dependent (not on the wavenumber but on the wavevector):

$$P_\zeta(\mathbf{k}_1) = P_\zeta^{iso}(k_1)\left[1 + g_\zeta(\mathbf{k}_1)(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{d}})^2\right].$$

Reflection

- Data analysts have obtained bounds on the level of statistical anisotropy and the preferred direction assuming in turn that the level of statistical anisotropy is scale-independent.
- Theoreticians (Hairy inflation) have assumed that the preferred direction is the one given by the direction in which the Universe expands anisotropically.
- Theoreticians (Gauge-flation) have ignored the fact that isotropic expansion doesn't mean statistical isotropy.
- We call for an urgent unification among experimental and theoretical cosmologists about the parameterization employed for the level of non-gaussianity and the preferred direction.

Questions/Answers

Question

+ What is going on with Hairy inflation? (WKS,2009)

Answer

+ The model:

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{4}f(\phi)F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right].$$

+ The model employs one Abelian gauge field and one scalar field and leads to anisotropic expansion.

Questions/Answers

Question

- What is going on with Hairy inflation? (WKS,2009)

Answer

- Several groups have found statistical anisotropy for this model employing different techniques.
- The preferred direction is always identified as the one in which the Universe expands anisotropically.

Questions/Answers

Question

- What is going on with Hairy inflation? (WKS,2009)

Answer

- Our results show that there exist three preferred directions which are actually the same and equal to the direction in which the Universe expands anisotropically.
- The level of statistical anisotropy (and its scale-dependence) seems to be slightly corrected by the anisotropic expansion.

Questions/Answers

Question

- What is going on with Gauge-flation? (MS-J,2011)

Answer

- The model:

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + \frac{\kappa}{384}(\epsilon^{\mu\nu\lambda\sigma}F^a_{\mu\nu}F^a_{\lambda\sigma})^2\right].$$

- The model employs a triad of non-Abelian SU(2) gauge fields only and leads to isotropic expansion.

Questions/Answers

Question

- What is going on with Gauge-flation? (MS-J,2011)

Answer

- No statistical anisotropy is found by (MS-J,2011) (cosmological perturbation theory is employed).
- Our results show that there exist three levels of statistical anisotropy, all of them equal to one.
- The model would be ruled out by observations since it generates too much statistical anisotropy.

Thanks!!