

Dark Radiation

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Outline:

- The Cosmic Neutrino Background (CNB)
- The Neutrino Effective Number (N_{eff})
- The Lensing Amplitude (A_L)
- Constraints before and after Planck on N_{eff} and A_L
- The CNB clustering parameters
- Some Dark radiation models
- Conclusions

The Cosmic Neutrino Background

When the rate of the weak interaction reactions, which keep neutrinos in equilibrium with the primordial plasma, becomes smaller than the expansion rate of the Universe, neutrinos decouple:

$$T_{dec} \approx 1MeV$$

After neutrinos decoupling, photons are heated by electrons-positrons annihilation. When also photons decouple, the ratio between the temperatures of photons and neutrinos will be fixed, despite the temperature decreases with the expansion of the Universe. We expect today a Cosmic Neutrino Background (CNB) at a temperature:

$$T_\nu = \left(\frac{4}{11} \right)^{1/3} T_\gamma \approx 1.945K \rightarrow kT_\nu \approx 1.68 \cdot 10^{-4} eV$$

With a number density of:

$$n_f = \frac{3}{4} \frac{\zeta(3)}{\pi^2} g_f T_f^3 \rightarrow n_{\nu_k, \bar{\nu}_k} \approx 0.1827 \cdot T_\nu^3 \approx 112 cm^{-3}$$

N_{eff}

The relativistic neutrinos contribute to the present energy density of the Universe:

$$\rho_{\text{rad}} = \rho_{\gamma} + \rho_{\nu} = g_{\gamma} \left(\frac{\pi^2}{30} \right) T_{\gamma}^4 + g_{\nu} \left(\frac{\pi^2}{30} \right) \left(\frac{7}{8} \right) T_{\nu}^4$$

$$\rho_{\text{rad}} = \left(1 + \left(\frac{7}{8} \right) \left(\frac{4}{11} \right)^{\frac{4}{3}} \left(\frac{g_{\nu}}{g_{\gamma}} \right) \right) \rho_{\gamma}$$

We can introduce the effective number of relativistic degrees of freedom:

$$\rho_{\text{rad}} = \left[1 + \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} N_{\text{eff}} \right] \rho_{\gamma}$$

The expected value is $N_{\text{eff}} = 3.046$, if we assume standard electroweak interactions and three active massless neutrinos. The 0.046 takes into account effects for the non-instantaneous neutrino decoupling and neutrino flavour oscillations. (Mangano et al. 2005)

N_{eff}

Measuring $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046$ we can constrain the dark radiation.

Increasing N_{eff} essentially increases the expansion rate H at recombination.

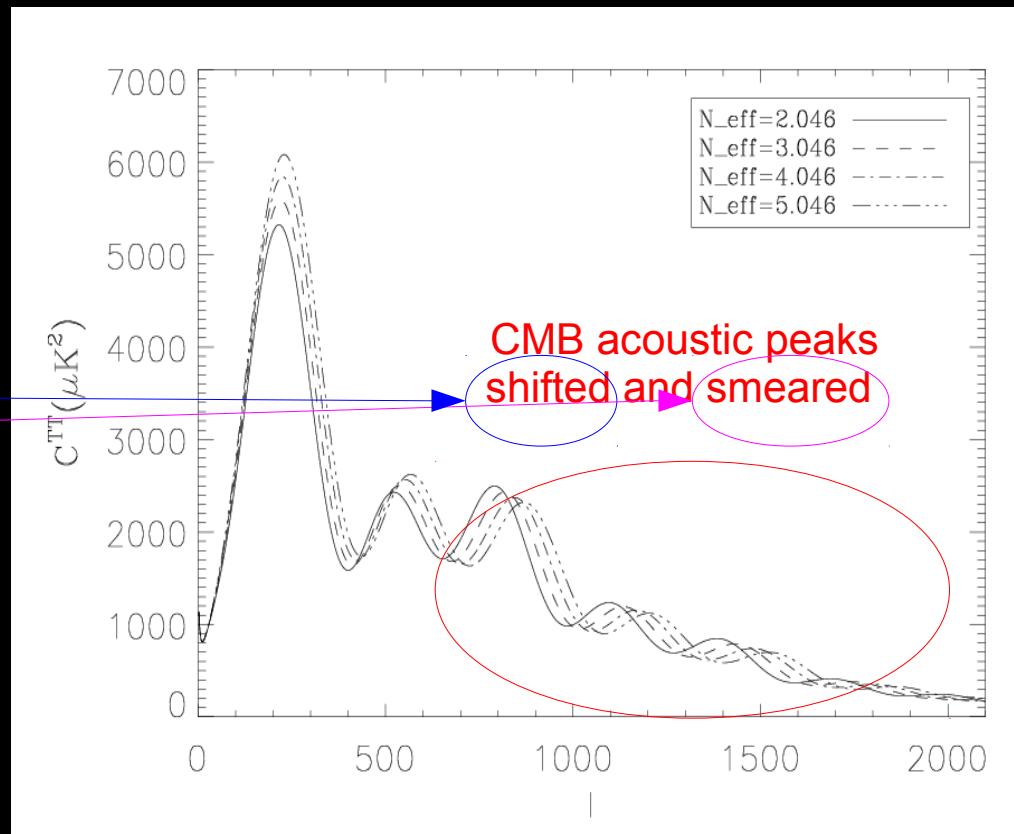
$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = H_0^2 \left(\frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda \right)$$

So decrease the sound horizon at recombination,

$$r_s = \int_0^{t_*} c_s dt / a = \int_0^{a_*} \frac{c_s da}{a^2 H}$$

and the diffusion distance (damping scale):

$$r_d^2 = (2\pi)^2 \int_0^{a_*} \frac{da}{a^3 \sigma_T n_e H} \left[\frac{R^2 + \frac{16}{15}(1+R)}{6(1+R^2)} \right]$$



The effect on the CMB power spectrum

Once the angular size of the sound horizon θ_s is fixed, we are fixing the angular scales of the acoustic peaks.

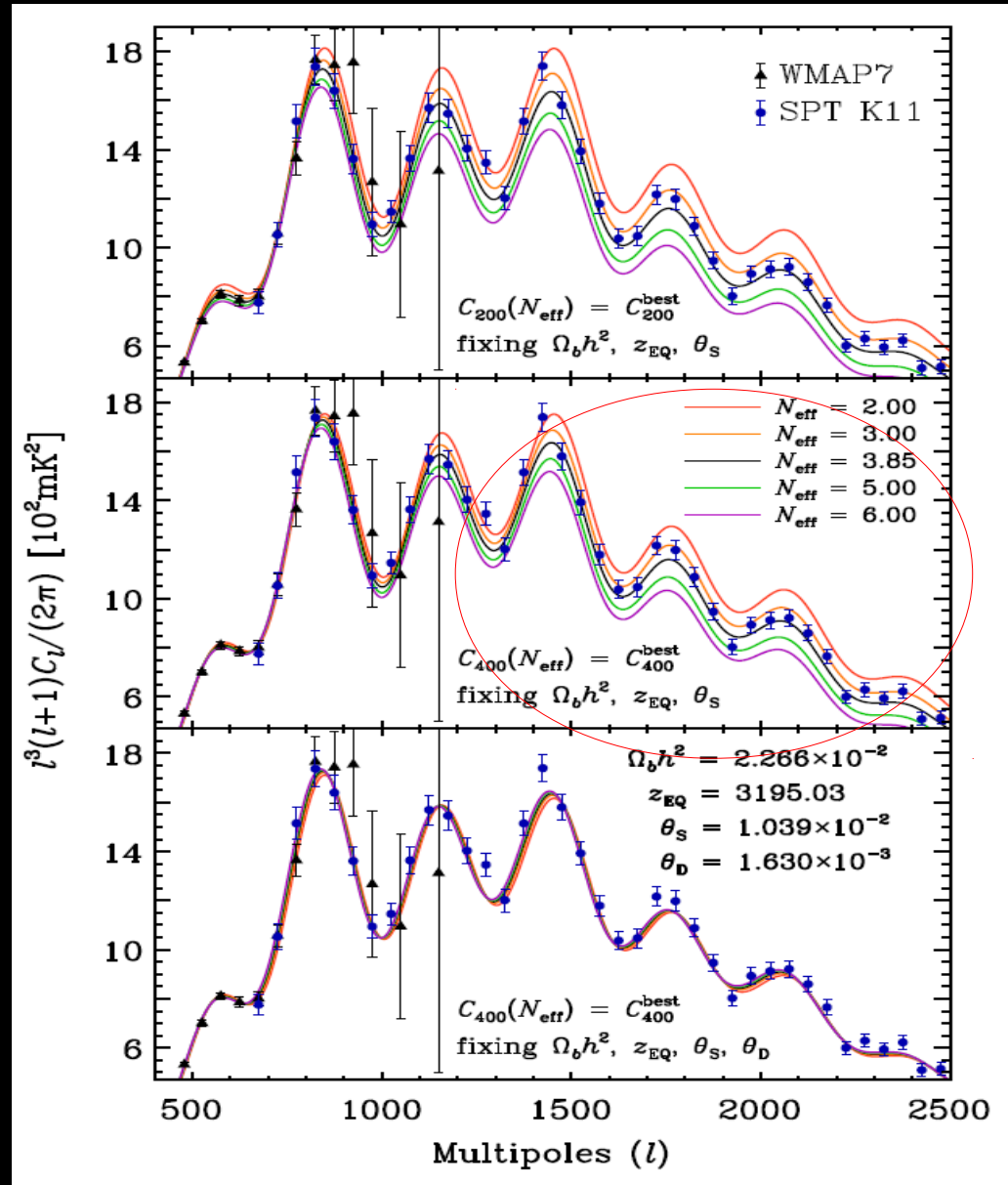
$$\theta_s = \frac{r_s}{D_A}$$

When we increase N_{eff} , we are increasing the angular scale of the diffusion length θ_d

$$\theta_d = \frac{r_d}{D_A}$$

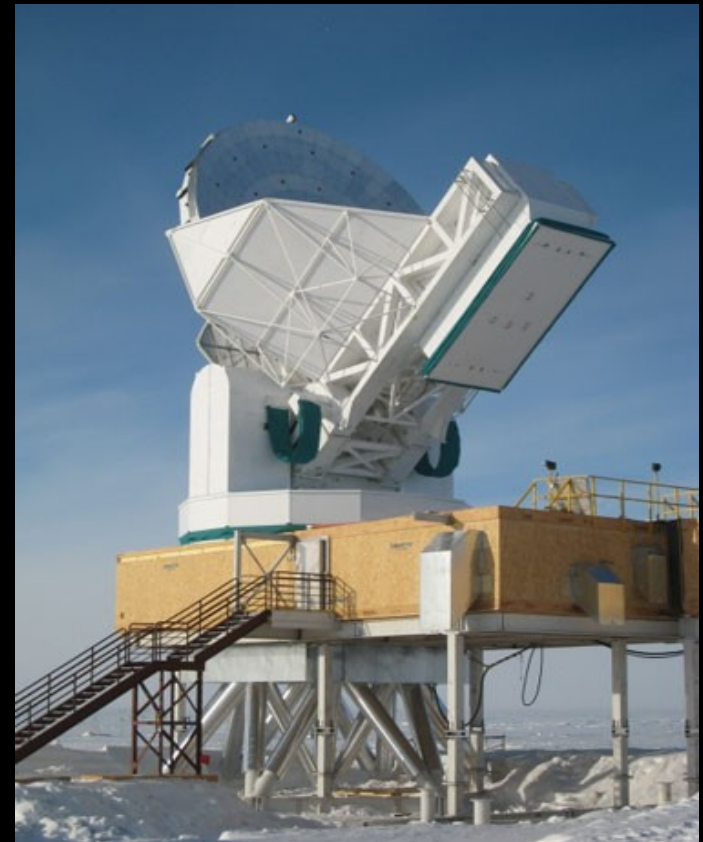
and the result is an increasing of the damping in the small angular scale anisotropy. (Hou, Keisler, Knox et al. 2013)

$$\theta_d = \frac{r_d}{r_s} \theta_s \simeq \frac{\sqrt{H}^{-1}}{H^{-1}} = \sqrt{H}$$



CMB constraints on N_{eff} before Planck (December 2012)

South Pole Telescope (+WMAP7)



(Z. Hou et al. 2012)

$$N_{\text{eff}} = 3.62 \pm 0.48$$

Atacama Cosmology Telescope (+WMAP7)



(J. L. Sievers et al. 2013)

$$N_{\text{eff}} = 2.78 \pm 0.55$$

The lensing amplitude

This tension between SPT and ACT was not related only to N_{eff} .

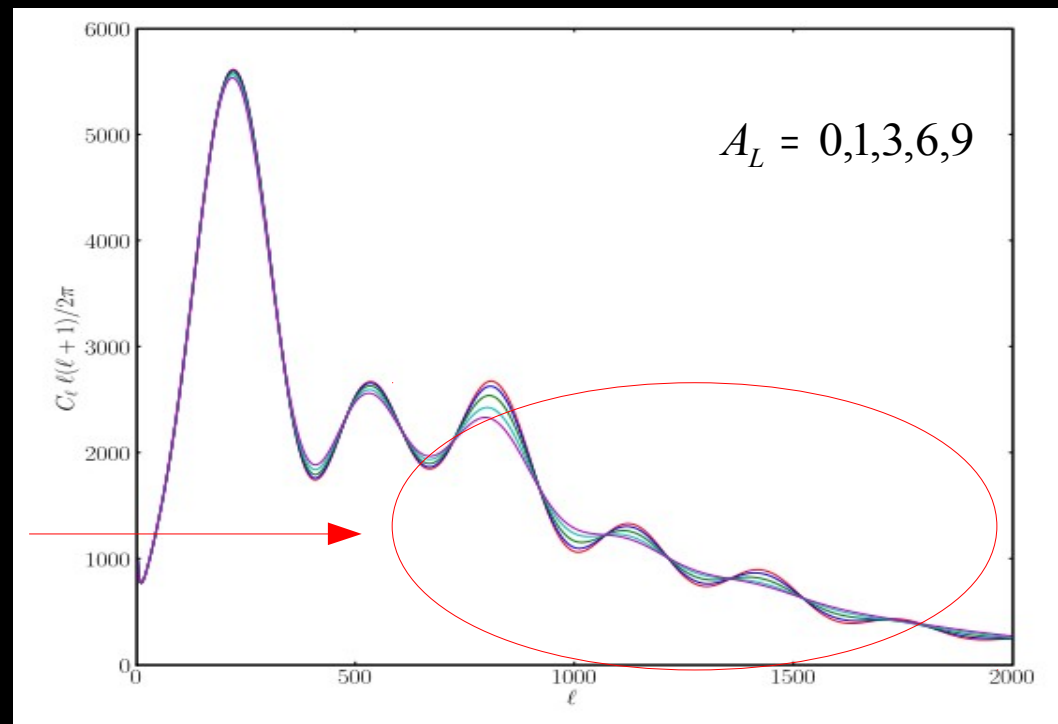
The lensing amplitude A_L parameterizes the rescaling of the lensing potential $\phi(n)$, then the power spectrum of the lensing field:

$$C_{\ell}^{\phi\phi} \rightarrow A_L C_{\ell}^{\phi\phi}$$

The gravitational lensing deflects the photon path by a quantity defined by the gradient of the lensing potential $\phi(n)$, integrated along the line of sight n , remapping the temperature field.

Its effect on the power spectrum is the smoothing of the acoustic peaks, increasing A_L .

If $A_L = 1$ then the theory is correct, else we have a new physics/systematics?



(E. Calabrese et al. 2008)

CMB constraints on A_L before Planck (December 2012)

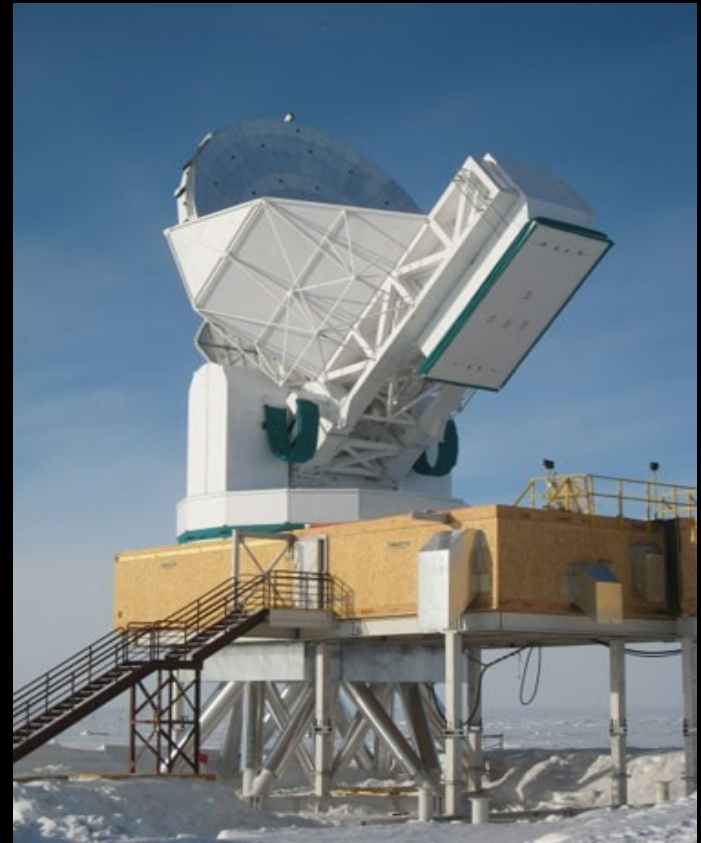
Atacama Cosmology Telescope (+WMAP7)



(J. L. Sievers et al. 2013)

$$A_L = 1.70 \pm 0.38$$

South Pole Telescope (+WMAP7)



(K. T. Story et al. 2012)

$$A_L = 0.86^{+0.15}_{-0.13}$$

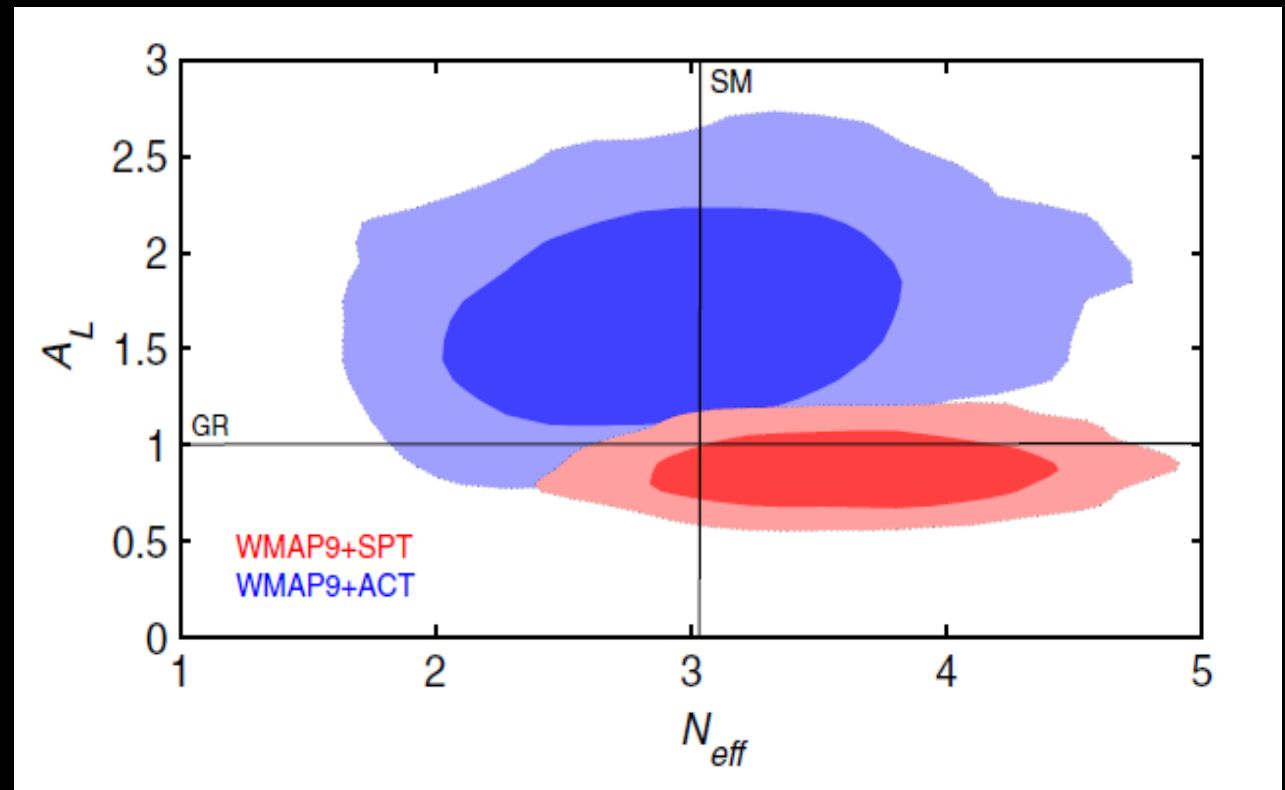
Pre-Planck constraints

Both N_{eff} and A_L affect the damping tail.

Tension between ACT and SPT is more clear in the A_L vs N_{eff} plane.

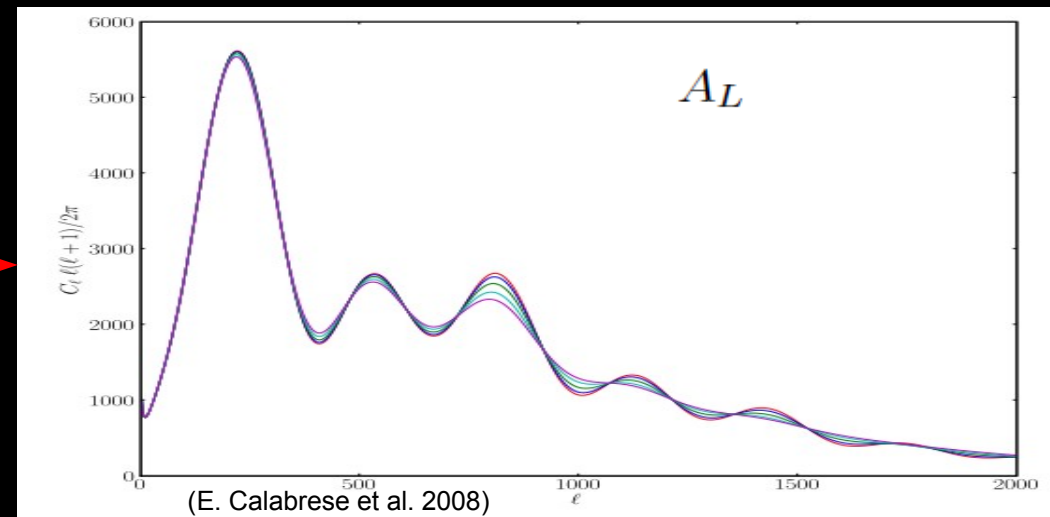
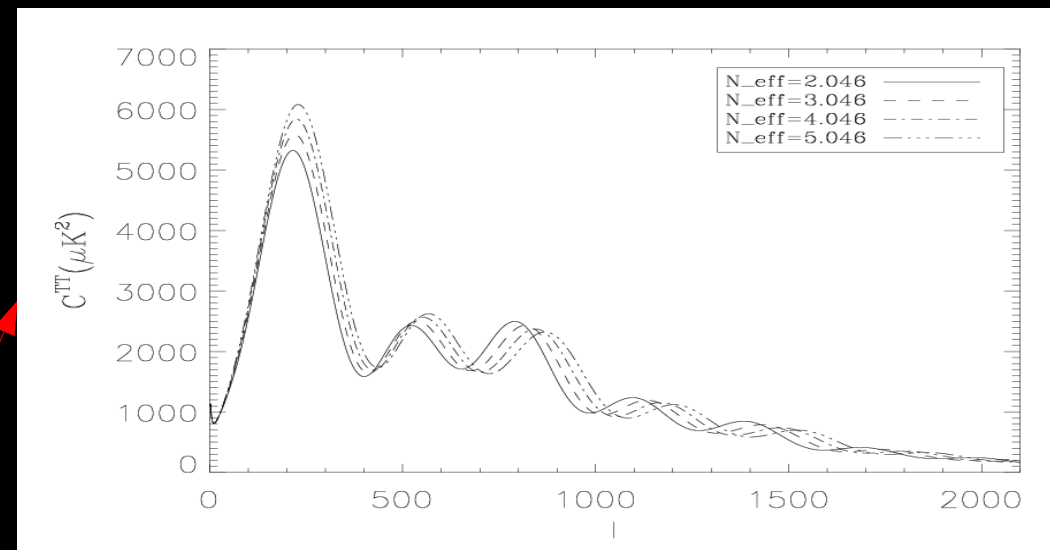
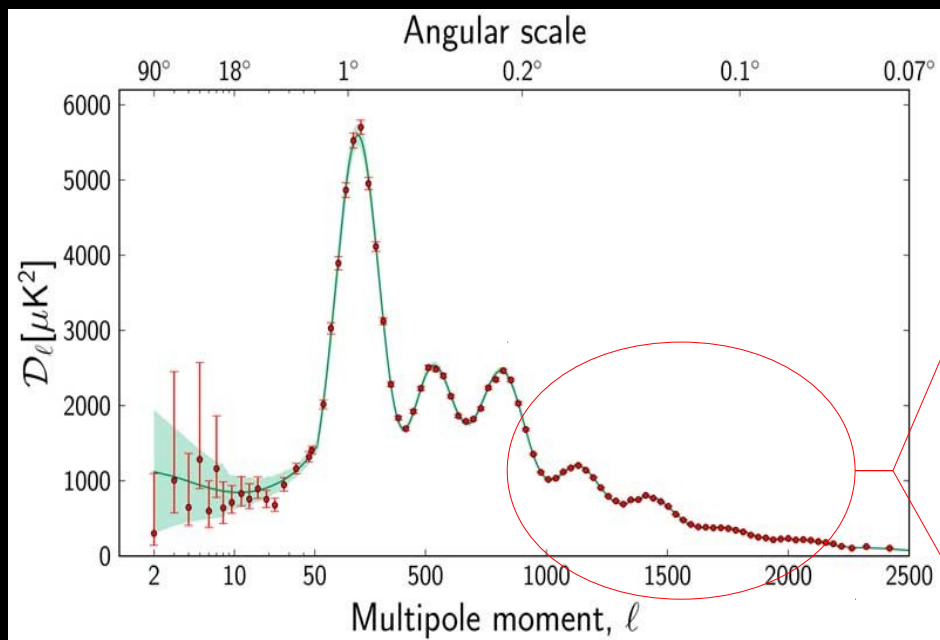
Parameter	WMAP9 + SPT
N_{eff}	3.72 ± 0.46
A_L	0.85 ± 0.13

Parameter	WMAP9 + ACT
N_{eff}	3.00 ± 0.61
A_L	1.70 ± 0.37



The damping tail

The Planck satellite detected with high precision the anisotropy damping tail, allowing to better constrain these two parameters of new physics.



What about Planck ?

Planck+WP 2013 result does not solve the issue!

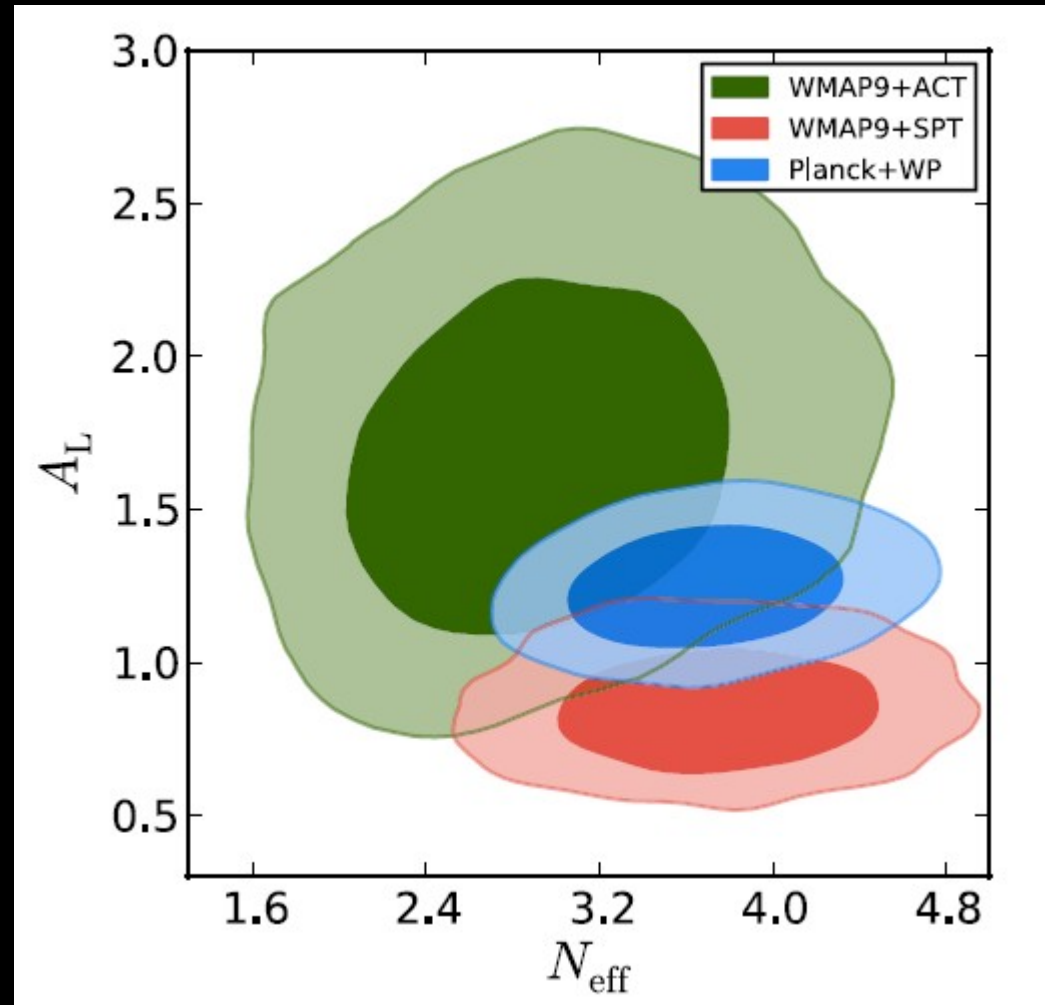
N_{eff} from Planck is in better agreement with SPT !

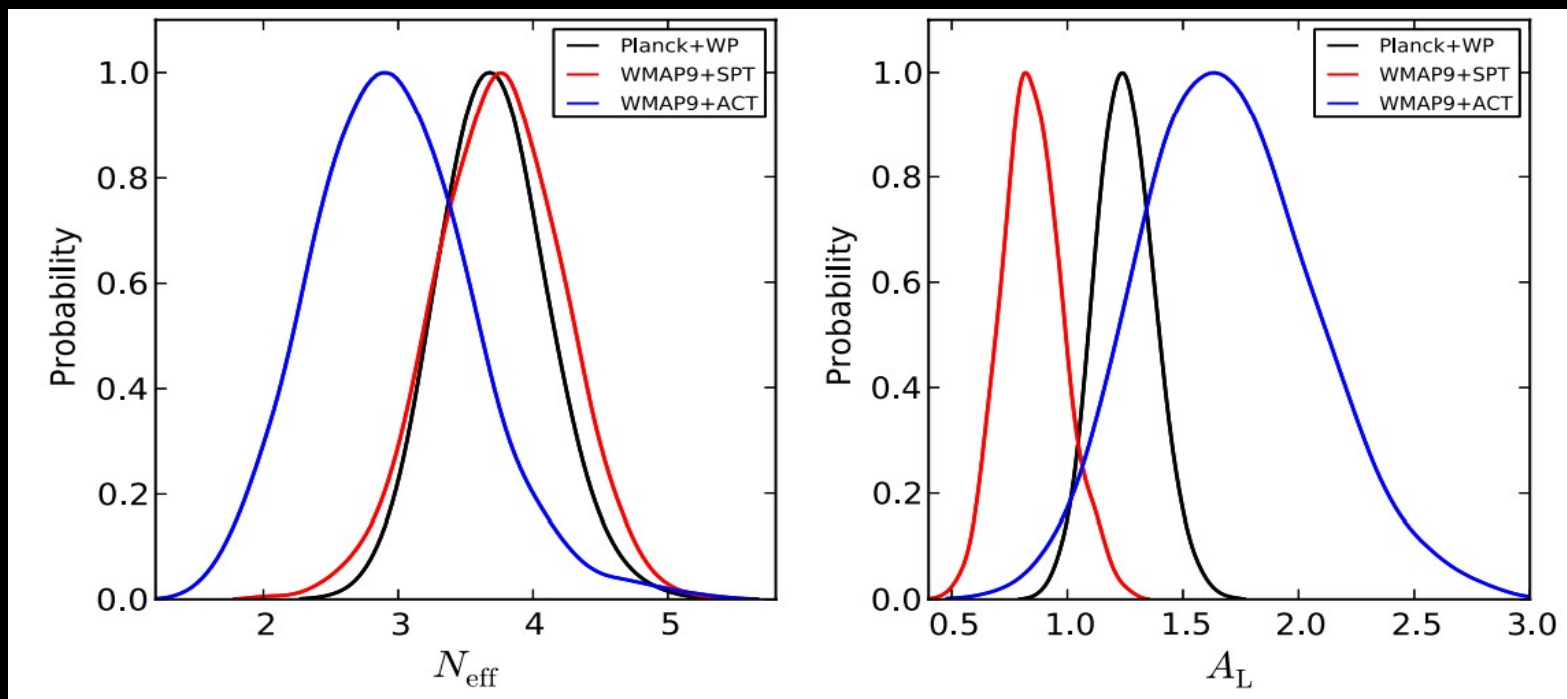
A_L from Planck is in better agreement with ACT !

$$N_{\text{eff}} = 3.71 \pm 0.40$$

$$A_L = 1.25 \pm 0.13$$

(68%; Planck+WP)





Parameter	Planck + WP	WMAP9 + SPT	WMAP9 + ACT
$\Omega_b h^2$	0.02306 ± 0.00051	0.02264 ± 0.00051	0.02295 ± 0.00052
$\Omega_c h^2$	0.1239 ± 0.0054	0.1232 ± 0.0080	0.112 ± 0.011
θ	1.04124 ± 0.00077	1.0415 ± 0.0012	1.0410 ± 0.0025
τ	0.095 ± 0.015	0.088 ± 0.014	0.090 ± 0.015
n_s	0.996 ± 0.018	0.982 ± 0.018	0.975 ± 0.019
$\log[10^{10} A_s]$	3.111 ± 0.034	3.169 ± 0.048	3.083 ± 0.044
N_{eff}	3.71 ± 0.40	3.72 ± 0.46	3.00 ± 0.61
A_L	1.25 ± 0.13	0.85 ± 0.13	1.70 ± 0.37
Ω_Λ	0.736 ± 0.022	0.736 ± 0.023	0.731 ± 0.025
$t_0[\text{Gyr}]$	13.08 ± 0.38	13.14 ± 0.43	13.74 ± 0.57
Ω_m	0.264 ± 0.022	0.264 ± 0.023	0.269 ± 0.025
$H_0[\text{km/s/Mpc}]$	74.9 ± 3.7	74.6 ± 3.7	70.9 ± 3.9

E. Di Valentino et al, Phys. Rev D, 88, 023501, 2013

N. Said, E. Di Valentino, M. Gerbino, Phys. Rev D, 88, 023513, 2013

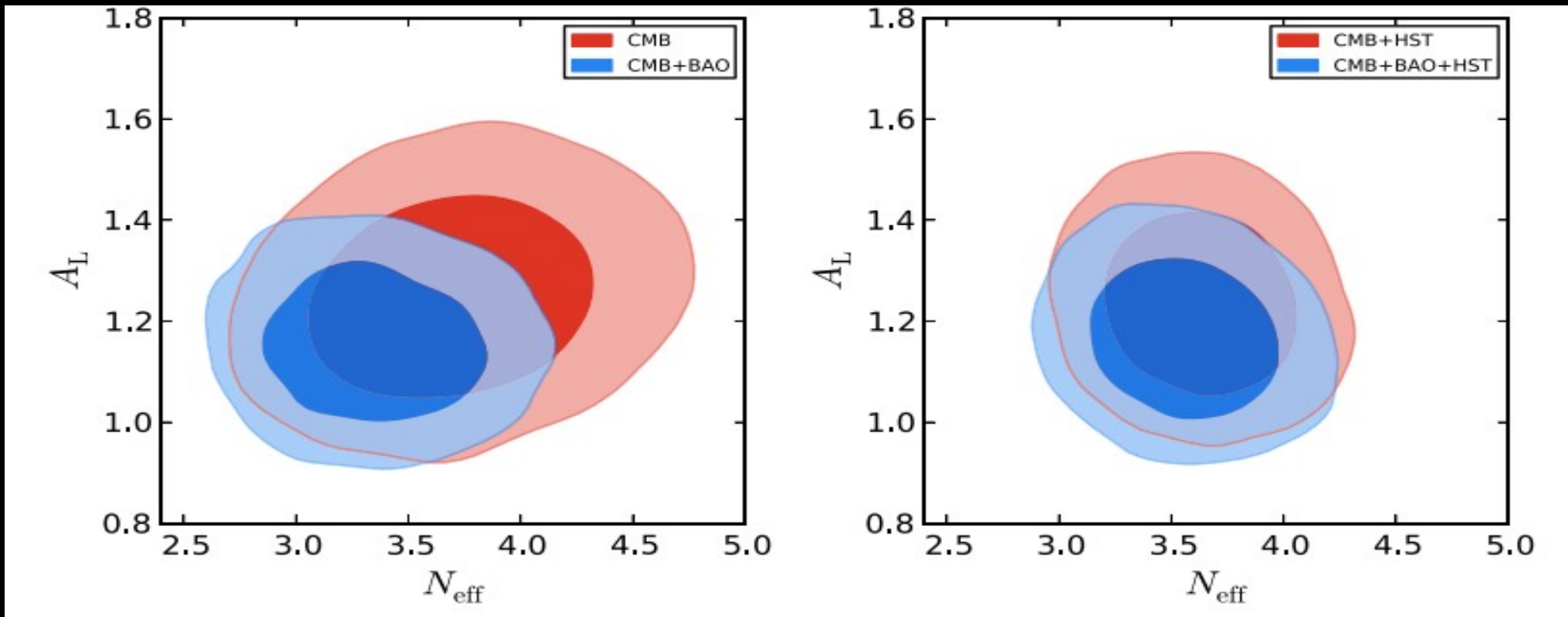
Current Constraints on Dark Radiation from CMB data

Parameter	WMAP7 + SPT [11]	WMAP9 + SPT [3]	WMAP7 + ACT [10]	WMAP9 + ACT [3]	Planck + WP [2]	PLANCK + WP
N_{eff}	3.62 ± 0.48	3.72 ± 0.46	2.78 ± 0.55	3.00 ± 0.61	3.51 ± 0.39	3.71 ± 0.40
A_L	1.00	0.85 ± 0.13	1.00	1.70 ± 0.37	1.00	1.25 ± 0.13
Parameter	WMAP7 + SPT [12]	WMAP9 + SPT [3]	WMAP7 + ACT [10]	WMAP9 + ACT [3]	Planck + WP [2]	PLANCK + WP
N_{eff}	3.046	3.72 ± 0.46	3.046	3.00 ± 0.61	3.046	3.71 ± 0.40
A_L	$0.86^{+0.15}_{-0.13}$	0.85 ± 0.13	1.70 ± 0.38	1.70 ± 0.37	$1.22^{+0.11}_{-0.13}$	1.25 ± 0.13

Allowing variations in A_L increases the mean value for N_{eff} by 5-10%.

There is a small, but not negligible correlation between the two parameters.

What about including HST or BAO data?



Results:

- ✓ HST brings $N_{\text{eff}} > 3.046$ at more than 95% c.l.
- ✓ BAO brings $N_{\text{eff}} = 3.046$ in between 68% c.l.
- ✓ BAO+HST gives $N_{\text{eff}} > 3.046$ at about 95% c.l.

(HST wins over BAO)

We consider recent **HST** measurements (Riess et al. 2011) of $H_0 = (73.8 \pm 2.4)$ km/s/Mpc;

and for **BAO** surveys we include:

- **SDSS-DR7** at redshift $z=0.35$
- **SDSS-DR9** at $z=0.57$
- **WiggleZ** at $z=0.44, 0.60$, and 0.73 .

A test for the CNB: Perturbations

The CNB can be further checked by considering two additional parameters:

the sound speed in the CNB rest frame c_{eff}^2 and the neutrino viscosity c_{vis}^2 .

Modelling the CNB as a Generalized Dark Matter (GDM) component (W. Hu 1998), the background evolution is given by the conservation equation:

$$\frac{\dot{\rho}_g}{\rho_g} = -3(1 + w_g)\frac{\dot{a}}{a}$$

where $w_g = p_g/\rho_g$ is the equation of state.

The perturbations evolution has only two degrees of freedom. We introduce c_{eff}^2 that describes pressure fluctuations respect to density perturbations:

$$w_g \Gamma_g = (c_{\text{eff}}^2 - c_g^2) \delta_g^{(\text{rest})}$$

and c_{vis}^2 that parameterizes the relationship between the anisotropic stress and the metric shear:

$$w_g \left(\dot{\pi}_g + 3 \frac{\dot{a}}{a} \pi_g \right) = 4c_{\text{vis}}^2 (k v_g - \dot{H}_T)$$

A test for the CNB: Perturbations

A combination of these three parameters specifies the clustering properties of GDM, helping to determine its nature from the observational constraints.

We have:

- a scalar-field dark matter $\longrightarrow (w_g, c_{eff}^2, c_{vis}^2) = (w_g, 1, 0)$

- CDM $\longrightarrow (w_g, c_{eff}^2, c_{vis}^2) = (0, 0, 0)$

- Radiation $\longrightarrow (w_g, c_{eff}^2, c_{vis}^2) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

So, for standard neutrinos we expect $\longrightarrow c_{eff}^2 = c_{vis}^2 = \frac{1}{3}$

A test for the CNB: Perturbations

Indeed, we consider the set of Boltzmann equations that describes perturbations in massless neutrino (Archidiacono et al. 2011), to set constraints on the clustering parameters:

$$\dot{\delta}_\nu = \frac{\dot{a}}{a} (1 - 3c_{\text{eff}}^2) \left(\delta_\nu + 3 \frac{\dot{a}}{a} \frac{q_\nu}{k} \right) - k \left(q_\nu + \frac{2}{3k} \dot{h} \right)$$

$$\dot{q}_\nu = k c_{\text{eff}}^2 \left(\delta_\nu + 3 \frac{\dot{a}}{a} \frac{q_\nu}{k} \right) - \frac{\dot{a}}{a} q_\nu - \frac{2}{3} k \pi_\nu$$

$$\dot{\pi}_\nu = 3 c_{\text{vis}}^2 \left(\frac{2}{5} q_\nu + \frac{8}{15} \sigma \right) - \frac{3}{5} k F_{\nu,3}$$

$$\frac{2l+1}{k} \dot{F}_{\nu,l} - l F_{\nu,l-1} = -(l+1) F_{\nu,l+1} \quad l \geq 3$$

CNB Perturbations: Results

Assuming $N_{\text{eff}}=3.046$, Planck+WP suggests a higher value of the viscosity parameter c_{vis}^2 , in tension with the standard value at about 1.5 standard deviations, and a lower value of the sound speed c_{eff}^2 , ruling out the standard value at more than 95% c.l.:

Parameter	ΛCDM	$+c_{\text{vis}}^2 + c_{\text{eff}}^2$
$100 \Omega_b h^2$	2.206 ± 0.028	2.118 ± 0.047
$\Omega_c h^2$	0.1199 ± 0.0027	0.1157 ± 0.0038
100θ	1.0413 ± 0.0006	1.0412 ± 0.0014
$\log[10^{10} A_S]$	3.089 ± 0.025	3.173 ± 0.052
τ	0.090 ± 0.013	0.089 ± 0.013
n_S	0.9606 ± 0.0073	0.998 ± 0.018
A_L	$\equiv 1$	$\equiv 1$
c_{vis}^2	$\equiv 0.33$ 	0.60 ± 0.18
c_{eff}^2	$\equiv 0.33$ 	0.304 ± 0.013
$H_0^{(a)}$	67.3 ± 1.2	68.0 ± 1.3

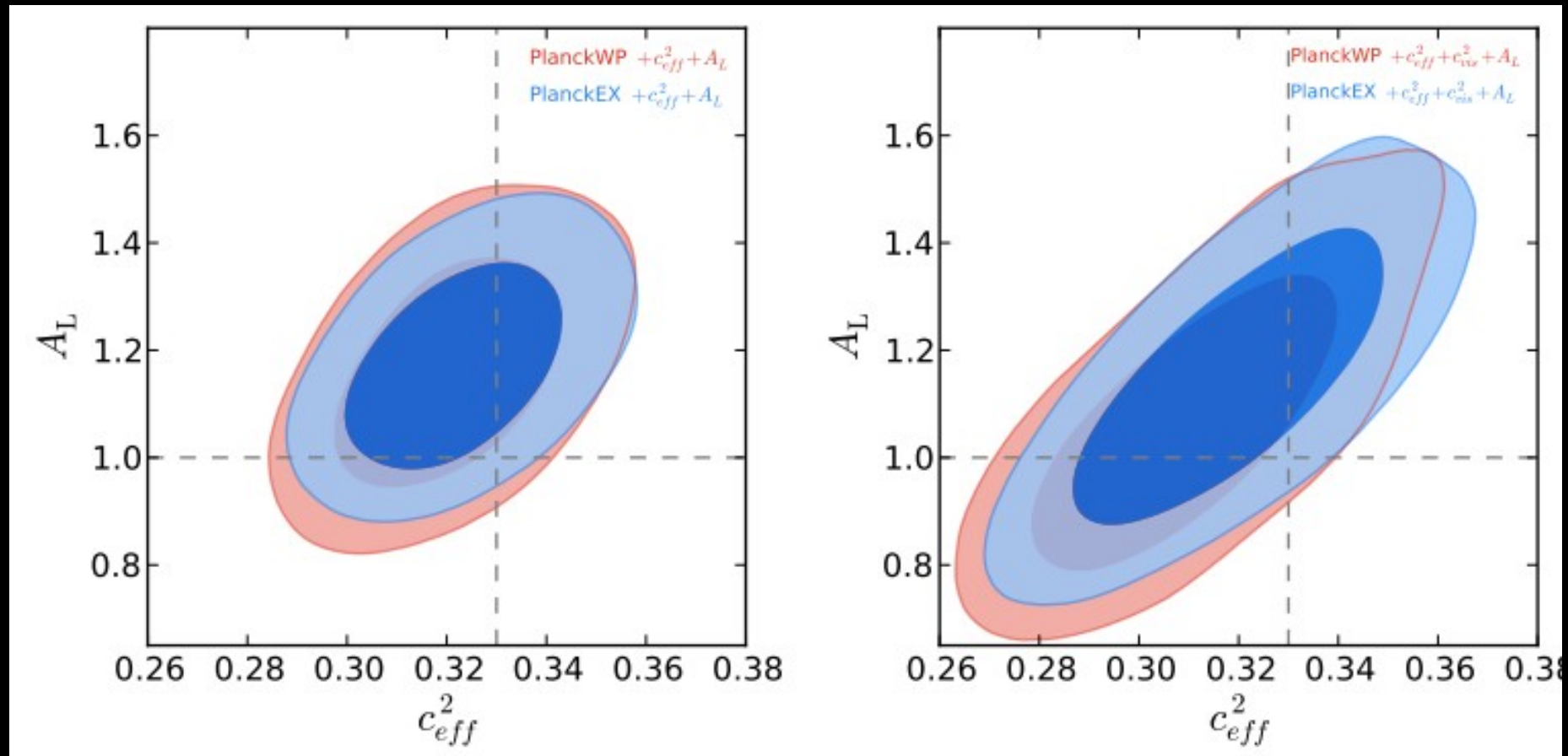
CNB Perturbations: Results

We find a correlation between the neutrino parameters and the lensing amplitude:

Parameter	Λ CDM	$+c_{\text{eff}}^2 + c_{\text{vis}}^2 + A_L$
$100 \Omega_b h^2$	2.206 ± 0.028	2.162 ± 0.095
$\Omega_c h^2$	0.1199 ± 0.0027	0.1159 ± 0.0036
100θ	1.0413 ± 0.0006	1.0420 ± 0.0020
$\log[10^{10} A_S]$	3.089 ± 0.025	3.141 ± 0.078
τ	0.090 ± 0.013	0.089 ± 0.014
n_S	0.9606 ± 0.0073	0.989 ± 0.023
A_L	$\equiv 1$	1.08 ± 0.18
c_{vis}^2	$\equiv 0.33$	0.51 ± 0.22
c_{eff}^2	$\equiv 0.33$	0.311 ± 0.019
$H_0^{(a)}$	67.3 ± 1.2	68.6 ± 1.7

CNB Perturbations: Results

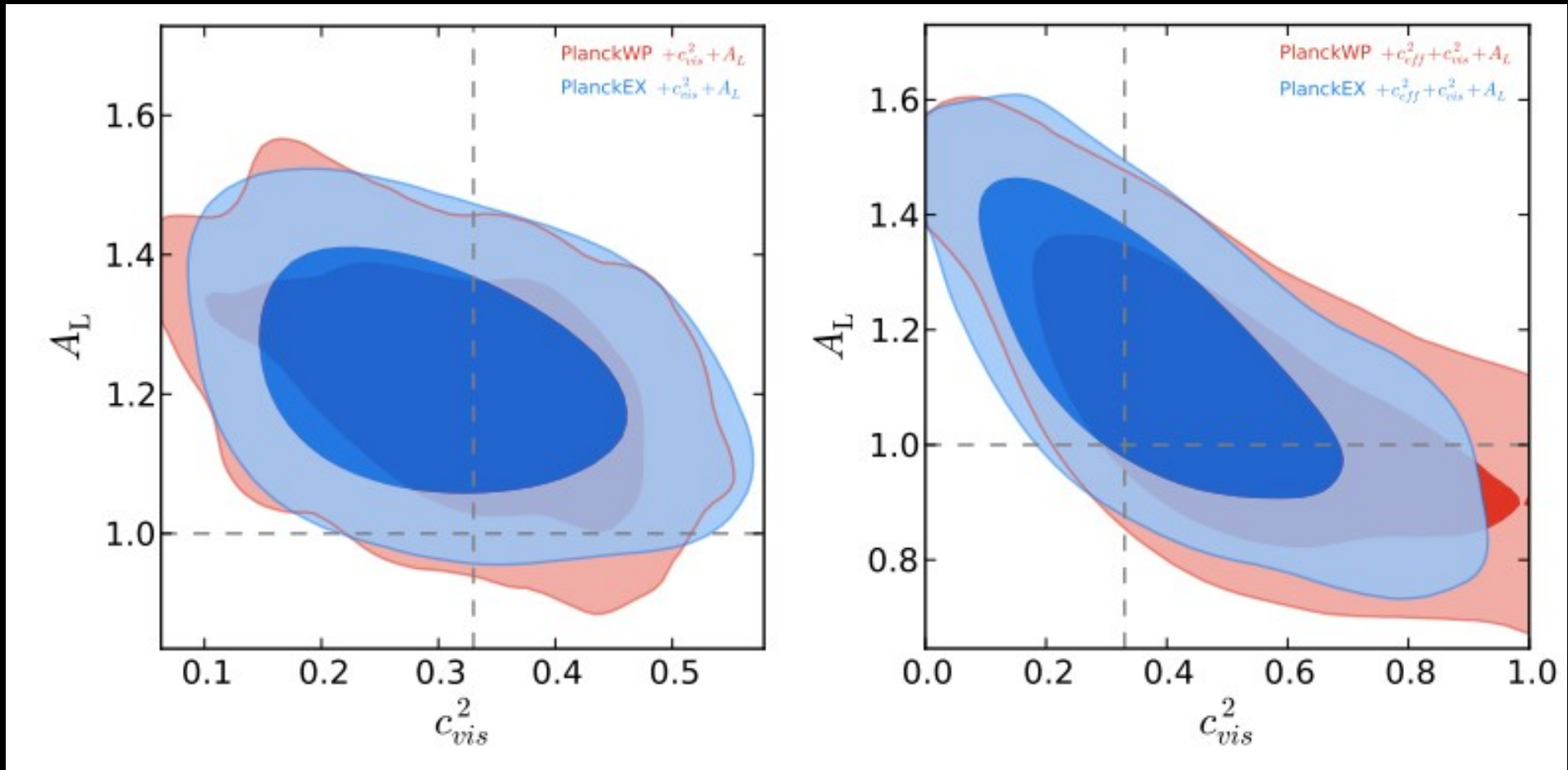
We find a correlation between the neutrino parameters and the lensing amplitude:



$A_L=1$ can be in more agreement with data with a lower c_{eff}^2 .

CNB Perturbations: Results

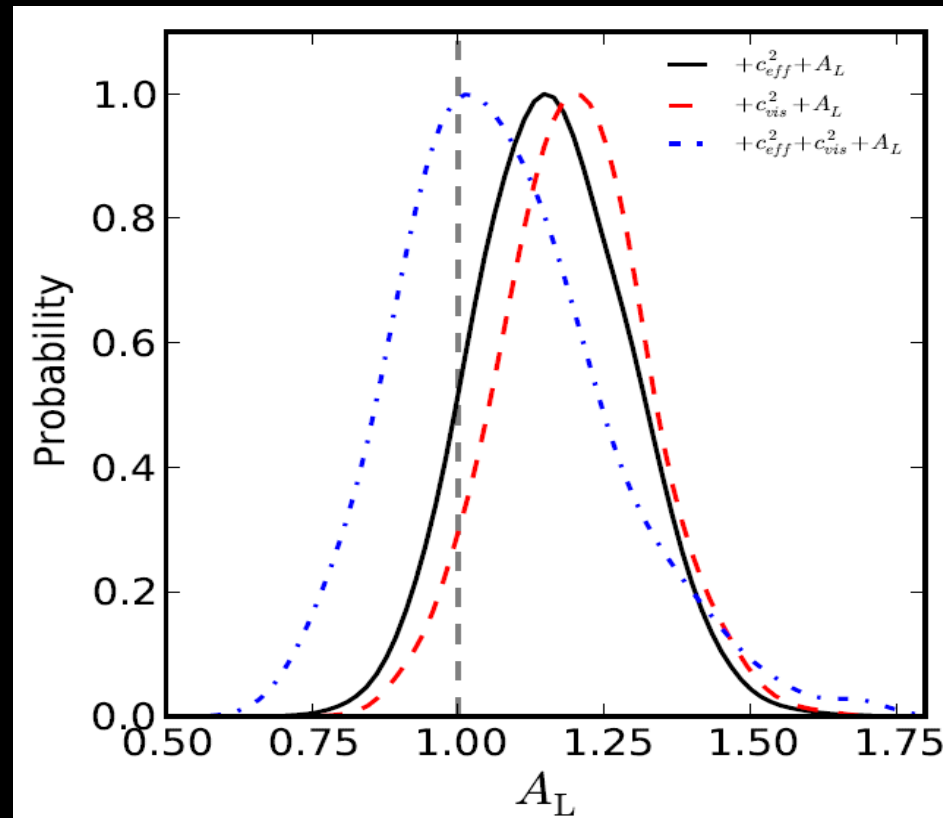
We find a correlation between the neutrino parameters and the lensing amplitude:



$A_L=1$ can be in more agreement with data with a higher c_{vis}^2 .

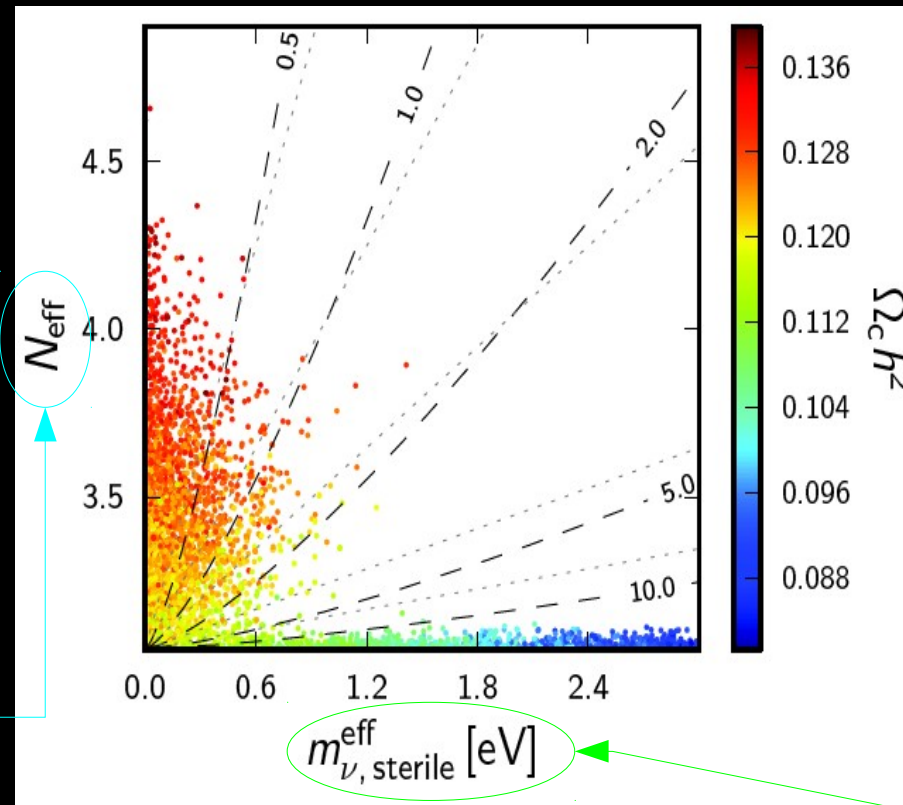
CNB Perturbations: Results

If we allow the CNB clustering parameters to vary, the anomalous large value of A_L measured by Planck disappears..



The effective sterile neutrino mass

We can constrain simultaneously the effective sterile neutrino mass and N_{eff} . Their relationship is strongly model dependent, but fixed the model we can infer the physical mass of the particle.



Contribution of the sterile neutrino when it is massless.

Thermally distributed

$$m_{\nu, \text{sterile}}^{\text{eff}} = (T_s/T_\nu)^3 m_{\text{sterile}}^{\text{thermal}} = (\Delta N_{\text{eff}})^{3/4} m_{\text{sterile}}^{\text{thermal}}$$

Distributed proportionally to active neutrinos (Dodelson-Widrow model)

$$m_{\nu, \text{sterile}}^{\text{eff}} = \chi_s m_{\text{sterile}}^{\text{DW}}$$

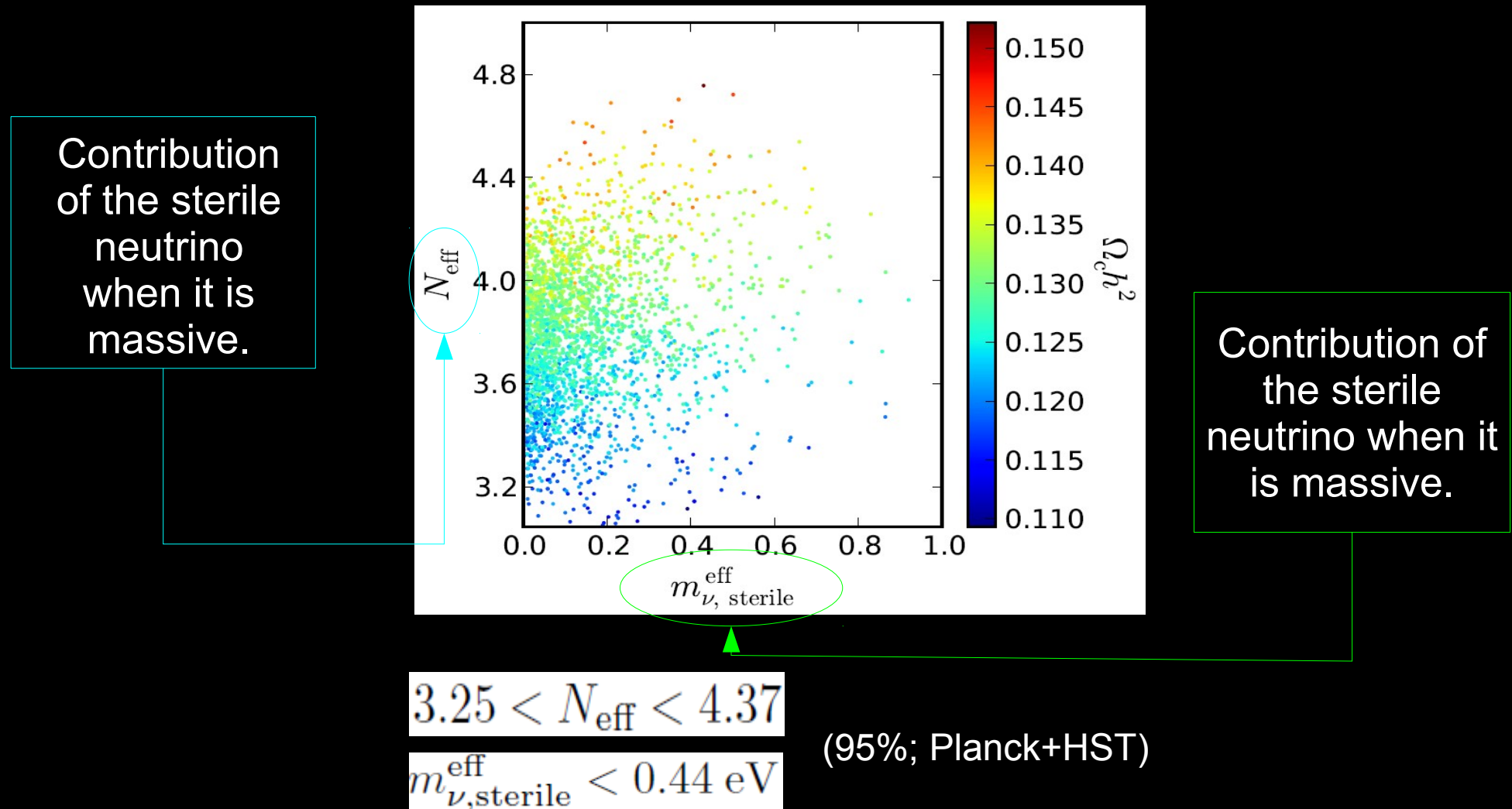
$$\Delta N_{\text{eff}} = \chi_s$$

Contribution of the sterile neutrino when it is massive.

$$\left. \begin{array}{l} N_{\text{eff}} < 3.91 \\ m_{\nu, \text{sterile}}^{\text{eff}} < 0.59 \text{ eV} \end{array} \right\} (95\%; \text{CMB for } m_{\text{sterile}}^{\text{thermal}} < 10 \text{ eV})$$

The effective sterile neutrino mass

Adding the HST measurements we obtain stronger bounds on these parameters, and the standard value $N_{\text{eff}}=3.046$ is excluded at more than 2 sigma.



Dark radiation models

When we combine the Planck Satellite data with HST measurements we have at 95% c.l.:

$$N_{\text{eff}} = 3.83 \pm 0.54$$

And when low multipole polarization measurements from the WMAP 9 years data release and high multipole CMB data from both ACT and SPT are added in the analysis, we find a 95% c.l.:

$$N_{\text{eff}} = 3.62^{+0.50}_{-0.48}$$

These bounds indicate the presence of an extra dark radiation $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046$ at the $\sim 2.4\sigma$ confidence level and can be exploited to set limits on any model containing extra dark radiation species.

Light Sterile Neutrino Model

The simplest way to explain the extra dark radiation is to include extra sterile neutrino species.

We focus here on the so-called **(3+1) neutrino mass model**.

It never reaches a complete thermalization, so its abundance is much lower than the thermal one and depends dramatically on the flavour mixing processes operating at the decoupling period.

We consider small mixing both between the active and heavy sectors and between the sterile and light neutrino sectors. In other words, given the flavor neutrinos ν_α ($\alpha = e, \mu, \tau, s$) and the massive base ν_i ($i = 1, 2, 3, 4$) related through a 4×4 unitary matrix:

$$\nu_\alpha = U_{\alpha i} \nu_i$$

The sterile neutrino contributes in this way to the energy density of the Universe (A. Melchiorri et al. 2008):

$$\Omega_s h^2 \simeq 7 \times 10^{-5} \left(\frac{\Delta m_{41}^2}{eV^2} \right) \sum_a \frac{g_a}{\sqrt{C_a}} \left(\frac{U_{a4}}{10^{-2}} \right)^2$$

and to ΔN_{eff} :

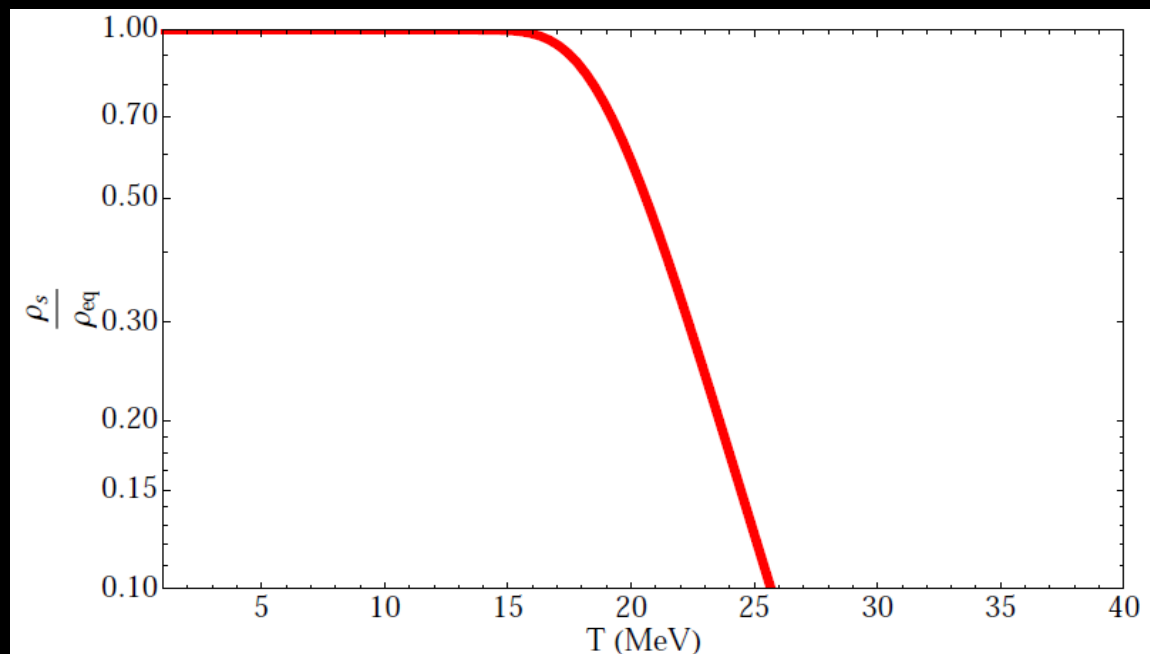
$$\Delta N_{\text{eff}} = \frac{\Omega_s h^2}{\frac{7}{8} \left(\frac{4}{11} \right)^{\frac{4}{3}} \Omega_\gamma h^2}$$

(3+1) neutrino mass model

We can compute the sterile neutrino abundance at decoupling from $T=100$ MeV up to $T = 1$ MeV, as a function of the temperature:

$$\left(\frac{\partial \rho}{\partial T}\right) = -\frac{1}{HT} (i [H_m + V_{\text{eff}}, \rho] - \{\Gamma, (\rho - \rho_{\text{eq}})\})$$

We have used for the active neutrino mixing parameters, the global fit from [M. C. Gonzales-Garcia et al. \(2012\)](#); and for the sterile neutrino mixing parameters the global fit from neutrino oscillation data ([J. M. Conrad et al. 2013](#)): $U_{e4} = 0.14$, $U_{\mu 4} = 0.17$ and $\Delta m^2_{14} = 0.93 \text{ eV}^2$. We have assumed a normal hierarchy scheme with $m_1=0$, the sum of the active neutrino masses equal to 0.056, and normalized the sterile neutrino abundance to the equilibrium distribution.



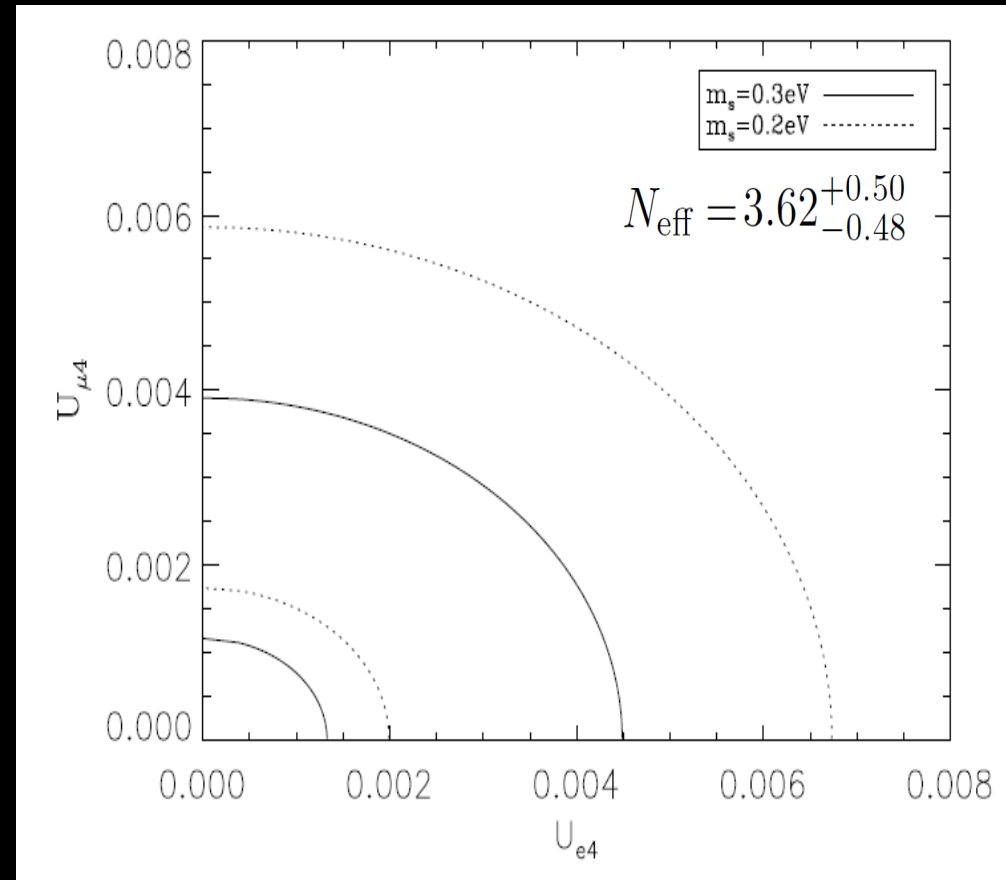
(E. Di Valentino,
A. Melchiorri and
O. Mena,
arXiv:1304.5981)

(3+1) neutrino mass model

It is possible to set constraints on the sterile neutrino mixing parameters from the recent ΔN_{eff} measurements, for a given value of the sterile neutrino mass $m < \sim 0.3$ eV, setting $U_{\tau 4} = 0$.

We find that the relatively large values of the sterile neutrino mixing parameters preferred by short baseline oscillation data are excluded for $0.2 < m_s < 0.3$ eV. For instance, the best fit point to appearance short baseline data in (3 + 1) model (J. M. Conrad et al. 2013) is found at $\Delta m^2_{14} = 0.15$ eV², being $U_{e4} = 0.39$ and $U_{\mu 4} = 0.39$.

For lower sterile neutrino masses $m_s < 0.1$ eV we have higher mixing parameters, but this mass is highly disfavored by oscillation analyses. Larger values of the sterile neutrino mass will not be relativistic at decoupling and they can not be tested exploiting the value of N_{eff} measured by Planck experiment.



(E. Di Valentino, A. Melchiorri and O. Mena, arXiv:1304.5981)

Extended dark sector models

These contain a dark sector, with relativistic degrees of freedom, that eventually decouple from the standard model sector contributing to N_{eff} .

We consider the so-called **asymmetric dark matter scenario**, in which the extra degrees of freedom are produced by the annihilations of the thermal symmetric dark matter component. The dark sector contains both light (g_ℓ) and heavy (g_h) degrees of freedom at the temperature of decoupling T_d from the standard model .

For higher $T_d > \text{MeV}$, the contribution to N_{eff} will be (M. Blennow et al. 2012):

$$\Delta N_{\text{eff}} = \frac{13.56}{g_{\star S}(T_D)^{\frac{4}{3}}} \frac{(g_\ell + g_h)^{\frac{4}{3}}}{g_\ell^{\frac{1}{3}}}$$

While for lower temperatures ($T_d < \text{MeV}$), if the dark sector couples to neutrinos, we have:

$$N_{\text{eff}} = \left(3 + \frac{4}{7} \frac{(g_h + g_\ell)^{\frac{4}{3}}}{g_\ell^{\frac{1}{3}}}\right) \left(\frac{3 \times \frac{7}{4} + g_H + g_h + g_\ell}{3 \times \frac{7}{4} + g_h + g_\ell}\right)^{\frac{4}{3}}$$

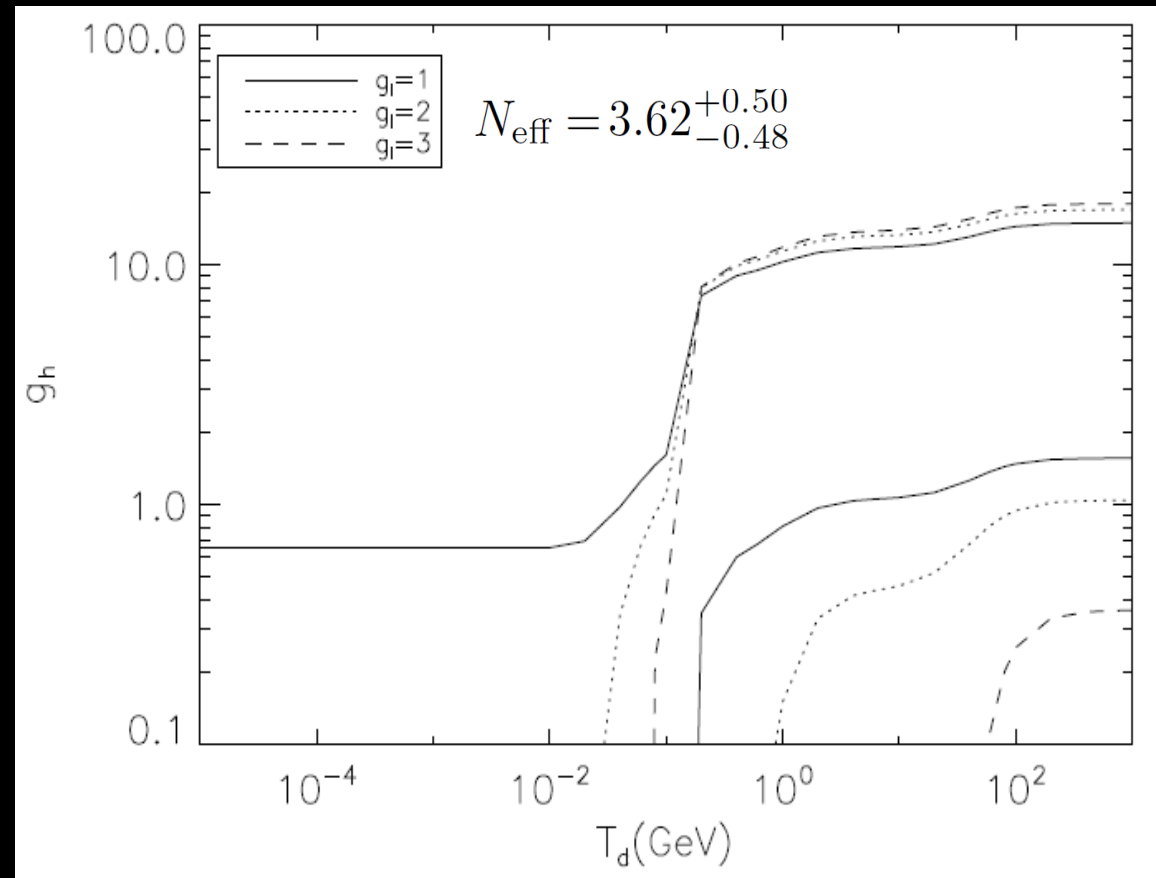
where g_H the number of degrees of freedom that become non relativistic between BBN and the dark sector decoupling period.

Asymmetric dark matter scenario

We can use the measured value of N_{eff} to find the required g_h that heat the dark radiation plasma as a function of T_d , for a fixed value of g_ℓ , for $g_H = 0$.

For $T_d > \text{MeV}$, in order to have a larger value of N_{eff} , the standard model relativistic degrees of freedom will be heated, requiring therefore heating in the dark sector, then the larger values of g_h .

On the other hand, at lower decoupling temperatures, the required g_h decrease as N_{eff} does, so the extra heavy degrees of freedom are disfavoured.



Conclusions:

- ✓ The Planck experiment doesn't solve the tension between the ACT and SPT data:

Parameter	Planck + WP	CMB + HST	CMB + BAO	CMB + BAO + HST
N_{eff}	3.71 ± 0.40	3.63 ± 0.27	3.35 ± 0.31	3.56 ± 0.27
A_L	1.25 ± 0.13	1.24 ± 0.12	1.16 ± 0.10	1.17 ± 0.10

- ✓ The large anomalous value of the A_L can be explained with the CNB clustering parameters.

Parameter	$+c_{\text{eff}}^2 + c_{\text{vis}}^2 + A_L$
A_L	1.08 ± 0.18
c_{vis}^2	0.51 ± 0.22
c_{eff}^2	0.311 ± 0.019

- ✓ Adding the HST data we obtain an evidence at more than 2 sigma for the dark radiation, and we can constrain a couple of models.

Thank you!

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