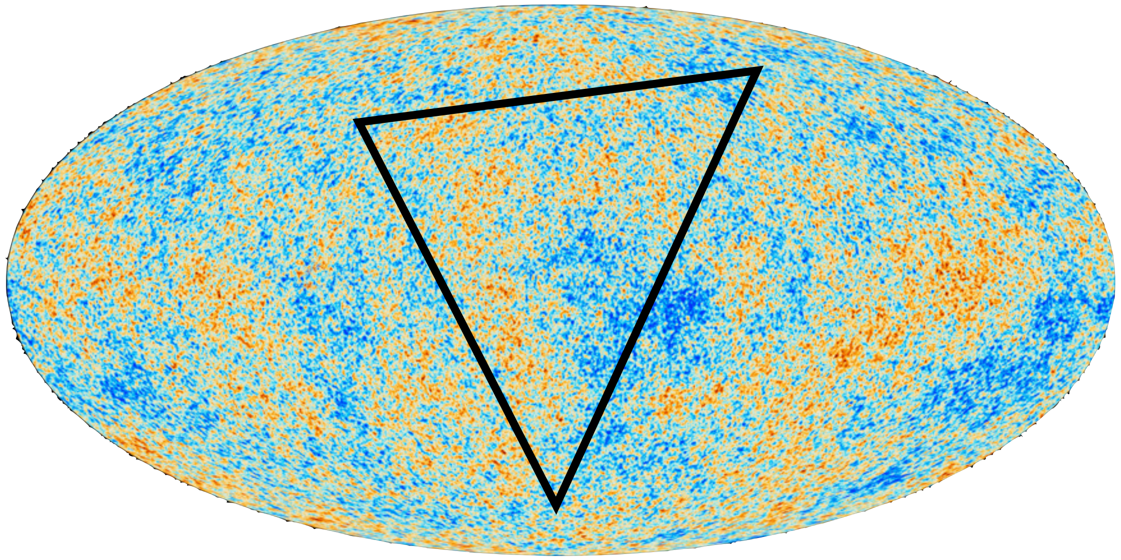
CMB bispectrum from second-order effects



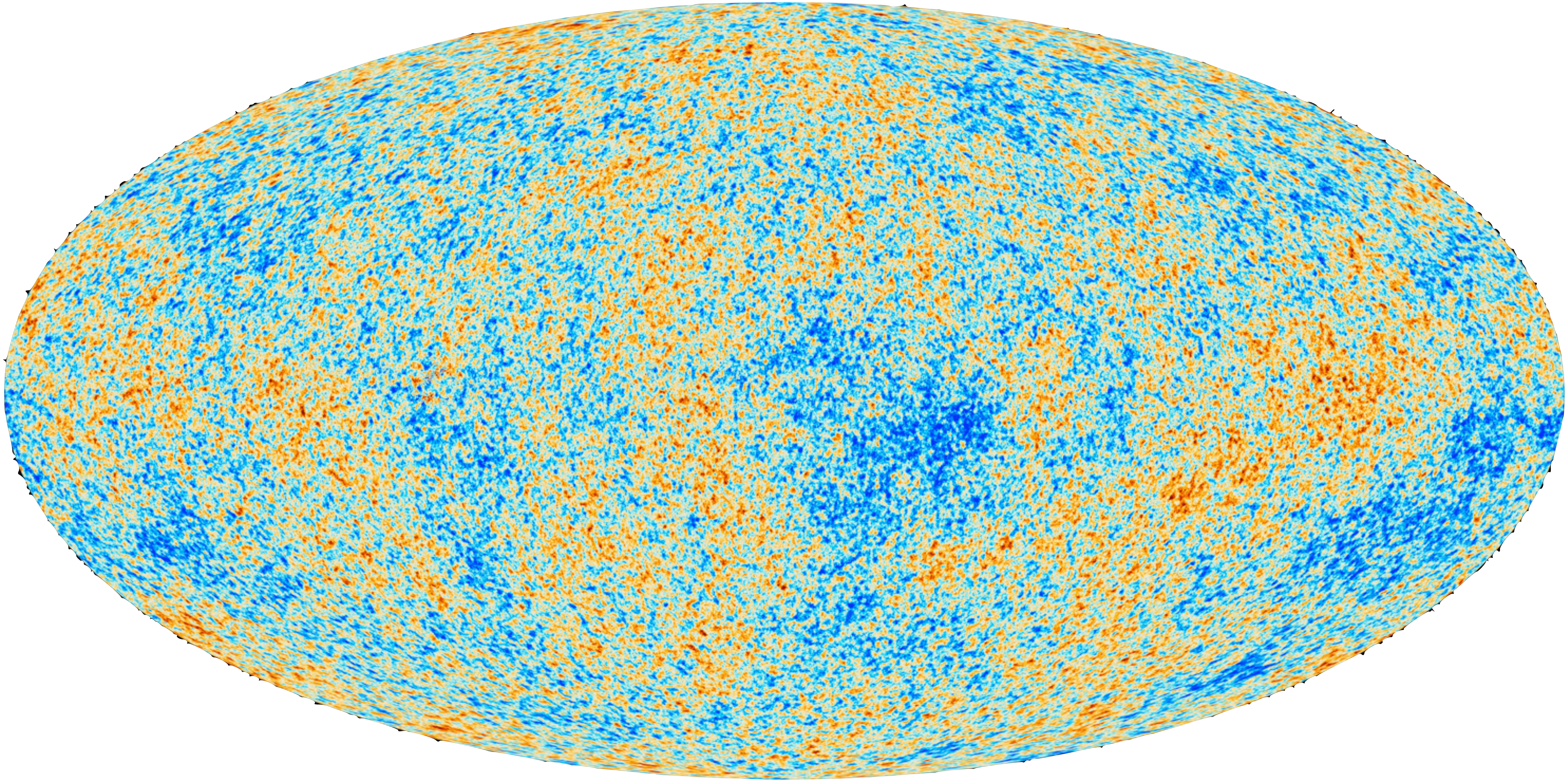
Filippo Vernizzi - IPhT, CEA Saclay

With Zhiqi Huang, PRL (1212.3573) + in preparation

Creminelli, Pitrou (1109.1822)

and Boubekeur, Creminelli, D'Amico, Noreña (0806.1016 and 0906.0980)

August 1, 2013 : ICTP - Trieste

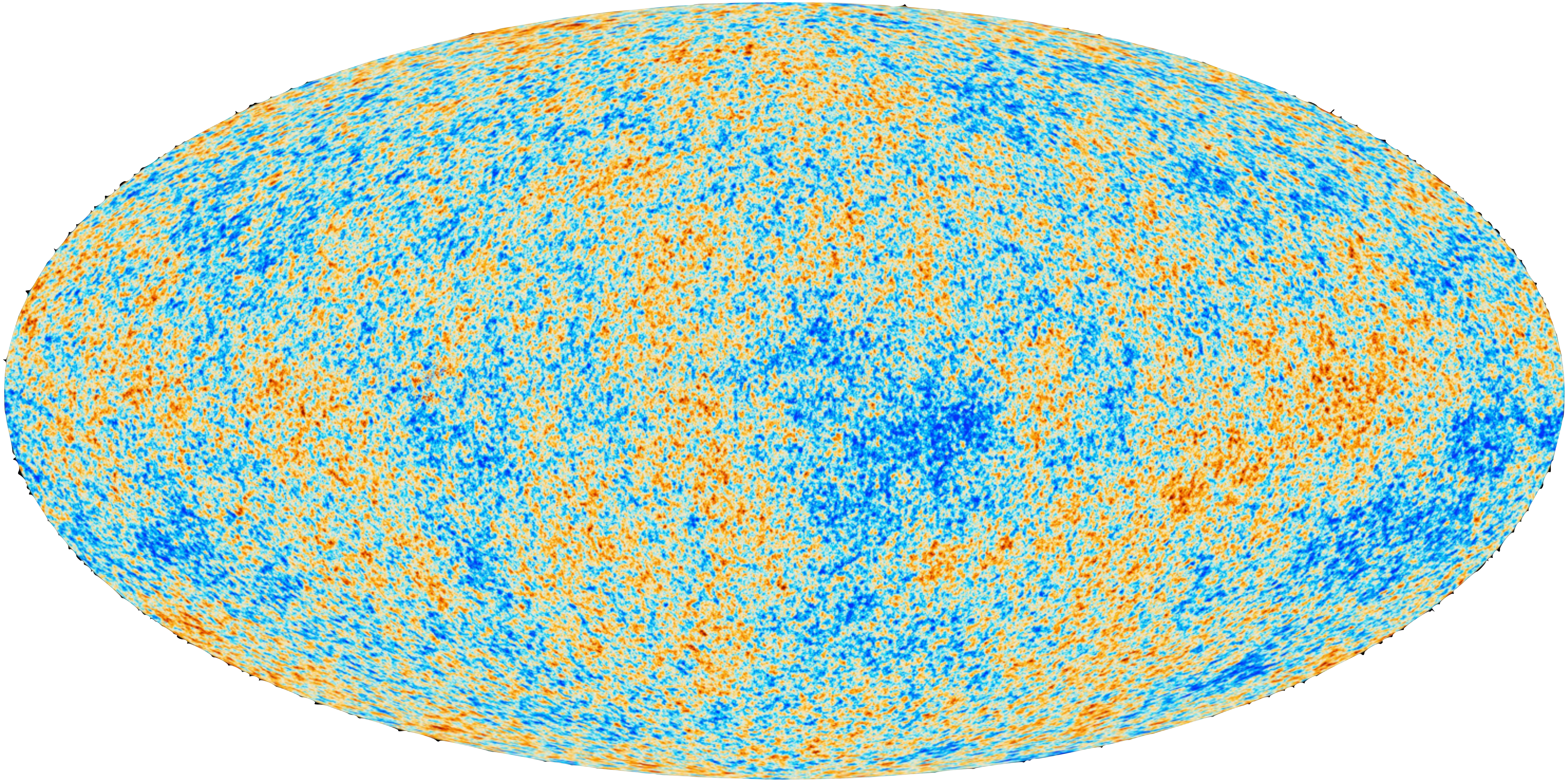


A CMB map contains ~10^6 pixels. Information could be hidden in higher-order statistics.

Bispectrum: angular 3-point function

$$\langle \frac{\Delta T}{T}(\hat{n}_1)\frac{\Delta T}{T}(\hat{n}_2)\frac{\Delta T}{T}(\hat{n}_3)\rangle \quad \Rightarrow \quad \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}\rangle = \mathcal{G}^{m_1 m_2 m_3}_{l_1 l_2 l_3} b_{l_1 l_2 l_3}$$

$$f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8$$ (Planck '13)



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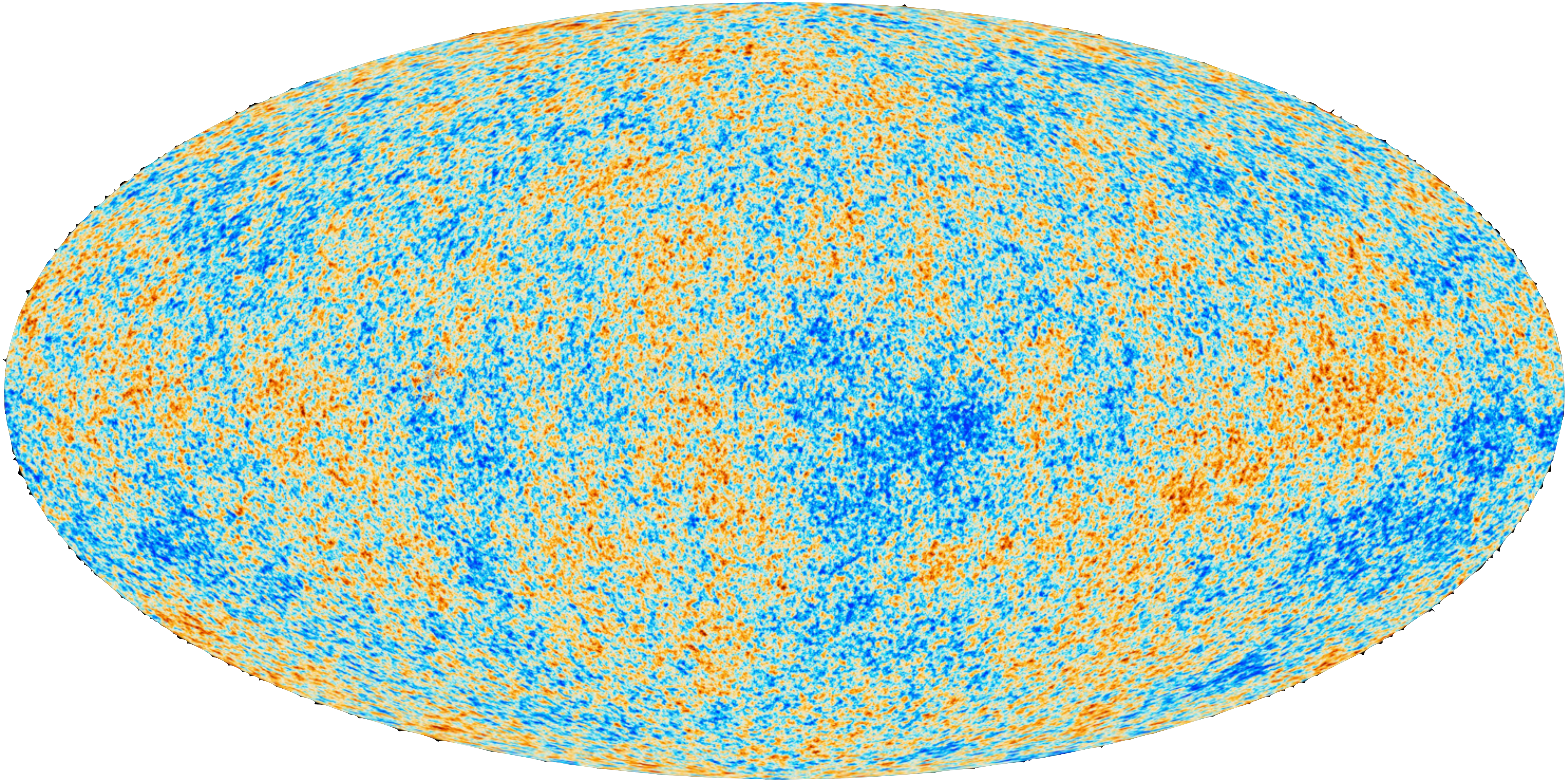
$$f_{\rm NL}^{\rm loc} \ll 1 \ ?$$



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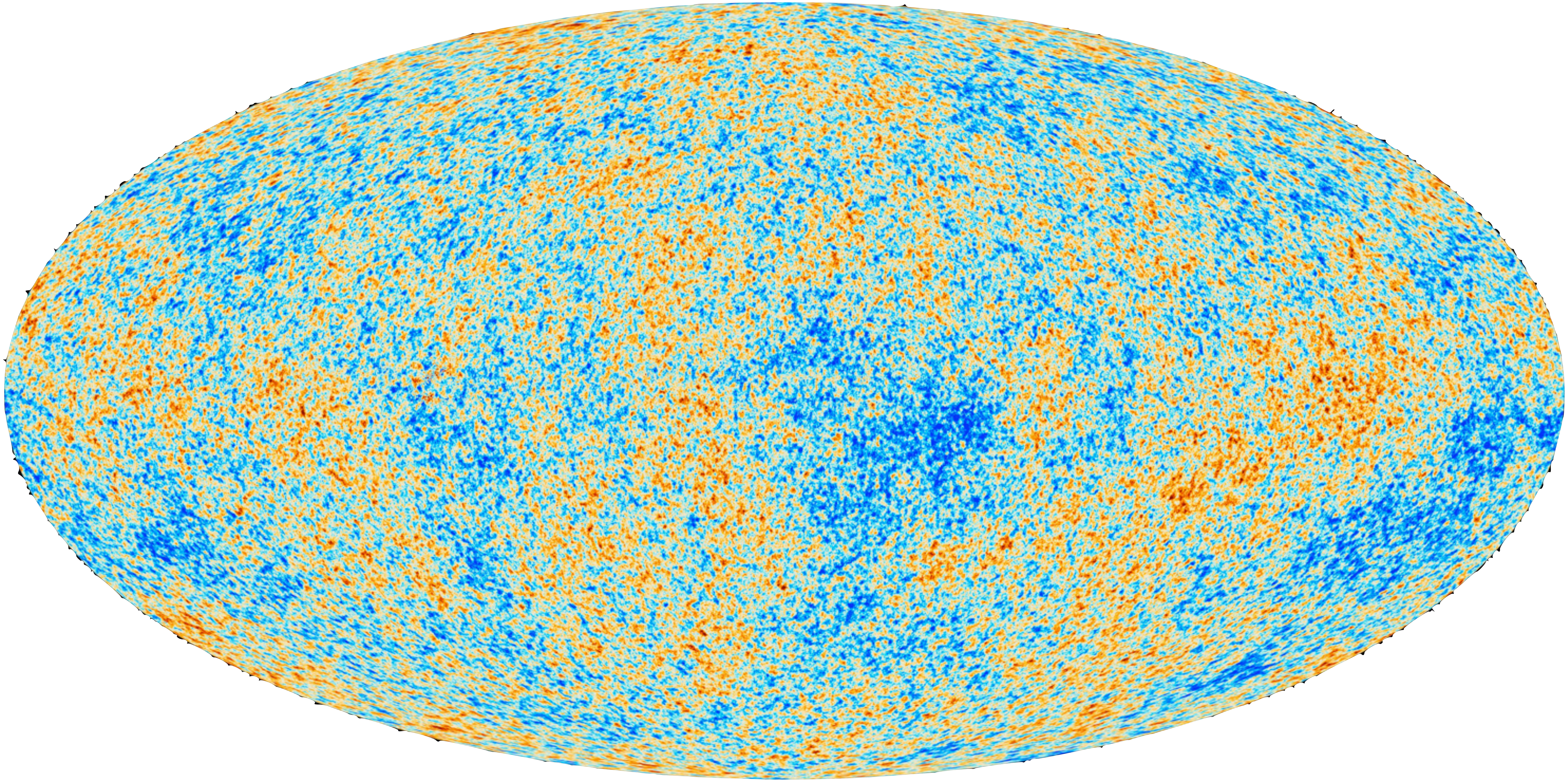
$$f_{\rm NL}^{\rm loc} \sim \text{few ?}$$



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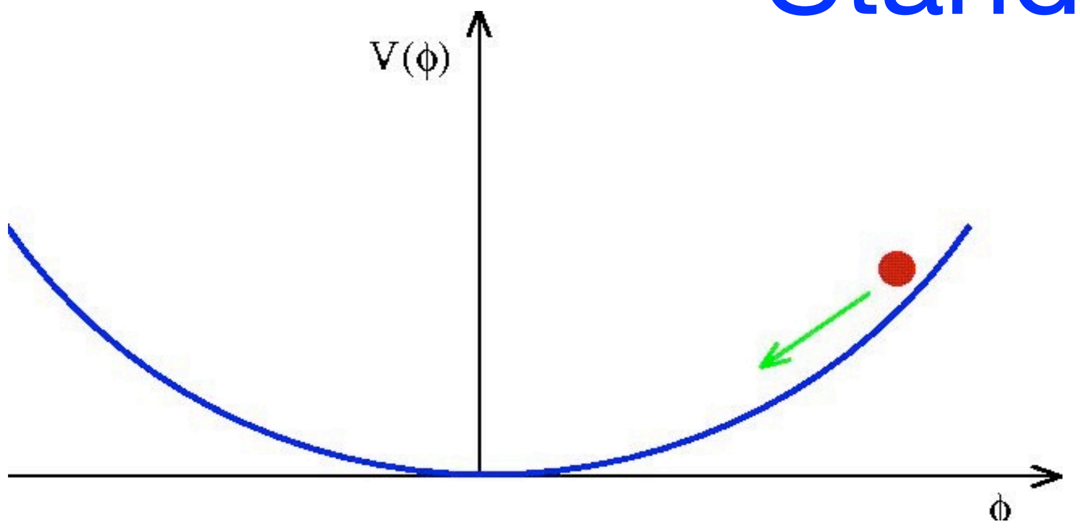
Any little improvement of the current constraints may be very important!

Standard picture



$$S=\int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R-\frac{1}{2}(\partial\phi)^2-V(\phi)\right]$$

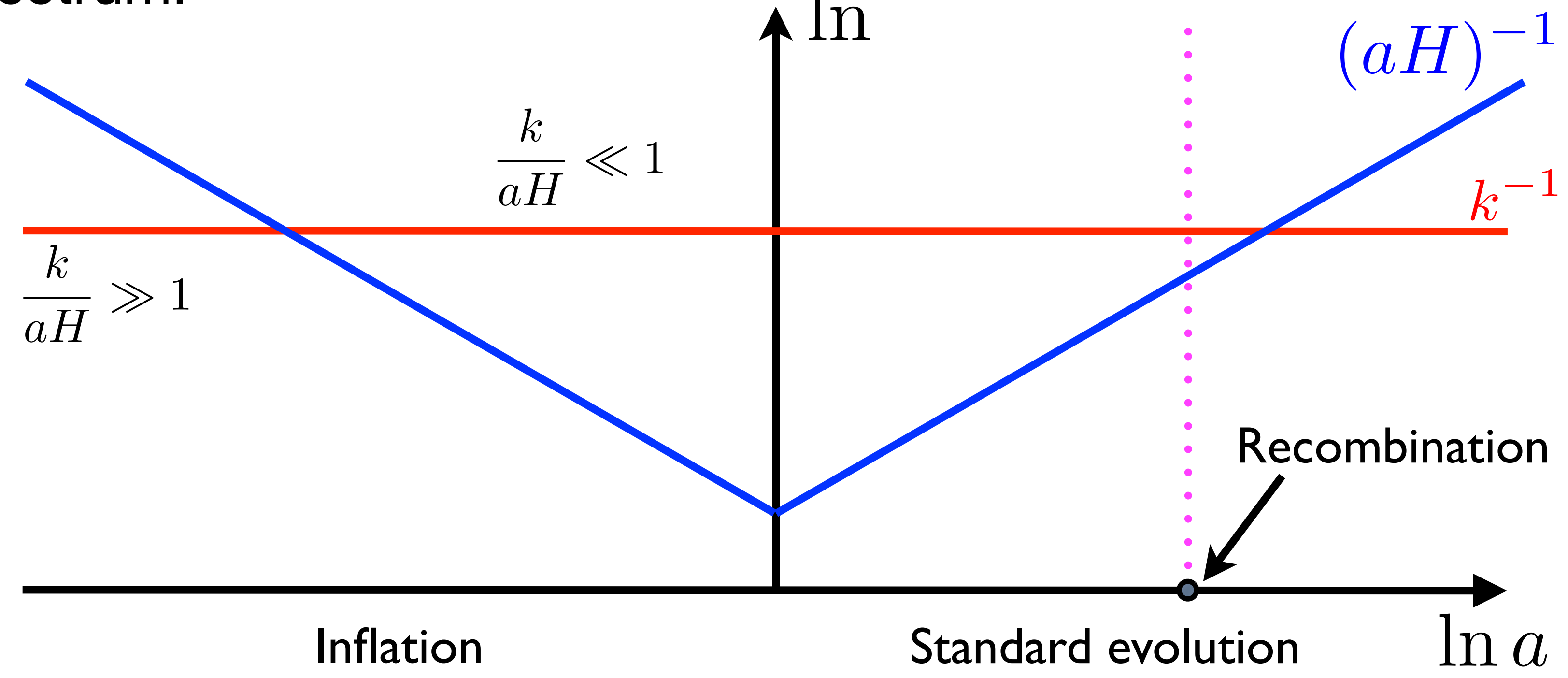
Curvature fluctuations of a constant inflaton field hypersurface:

$$\phi=\phi(t)$$

$$g_{ij}=a^2(t)e^{2\zeta}\delta_{ij}$$

$$S=\frac{1}{2}\int dtd^3xa^3\frac{\dot{\phi}^2}{H^2}\left[\dot{\zeta}^2-\frac{1}{a^2}(\partial\zeta)^2\right]$$

Each mode exits the Hubble radius, $k/a(t)\ll H$, and get frozen out the horizon with an almost scale-invariant spectrum:



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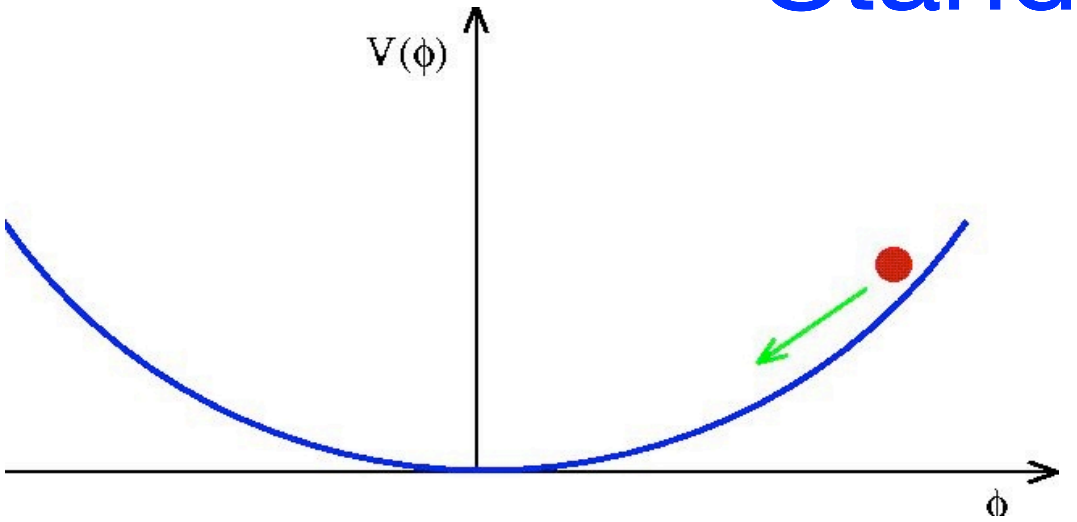

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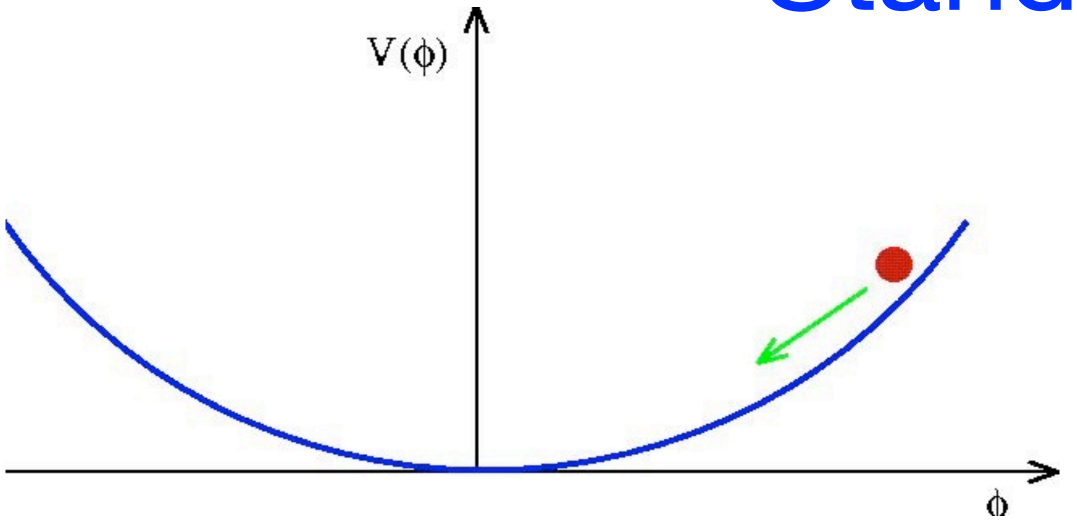
$$\langle\zeta_{\vec k}\zeta_{\vec k'}\rangle = (2\pi)^3\delta(\vec k + \vec k')P_\zeta(k)$$

$$k^3P_\zeta(k) = \left.\frac{H^4}{2\dot\phi^2}\right|_{k=aH} = A_\zeta\left(\frac{k}{k_*}\right)^{n_s-1}$$

$$n_s - 1 \simeq -0.04$$

These modes re-enter the horizon and we observe them today...

Standard picture



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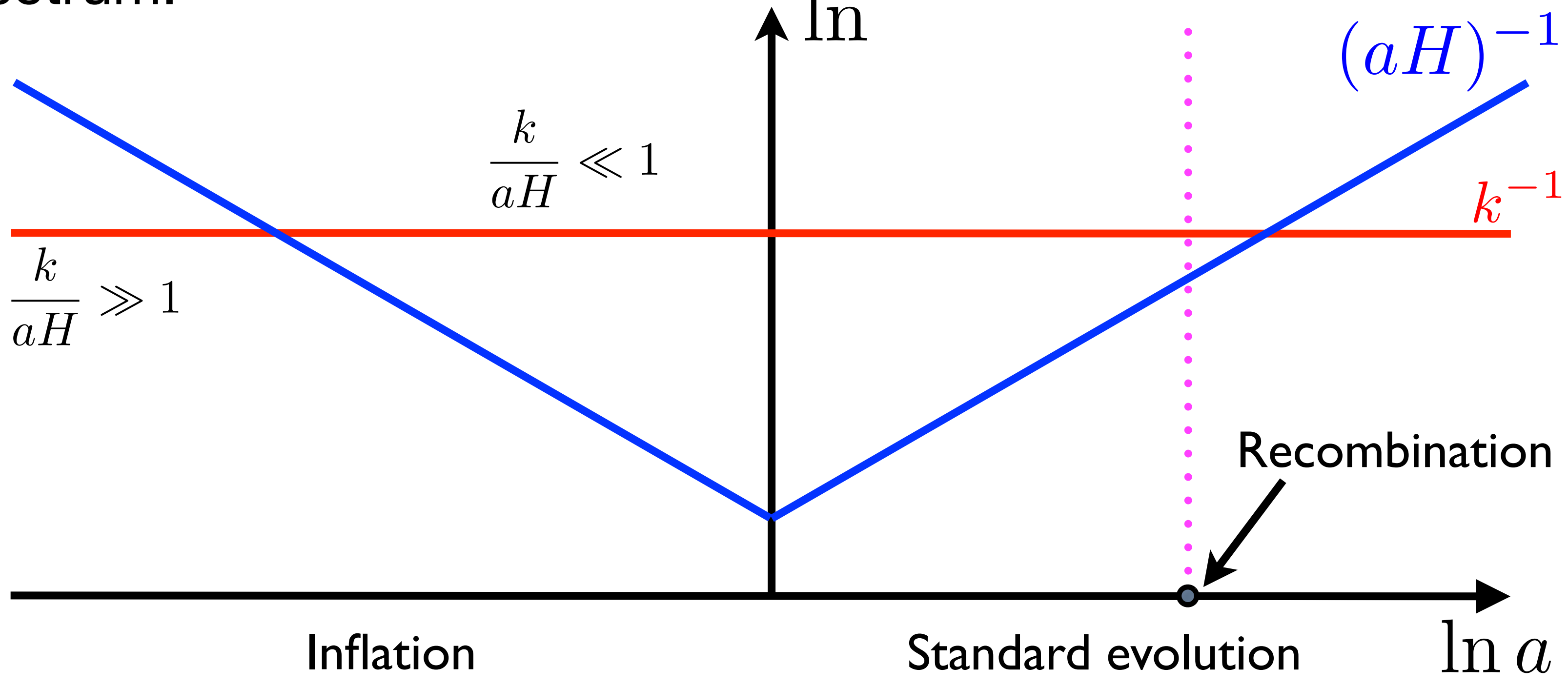
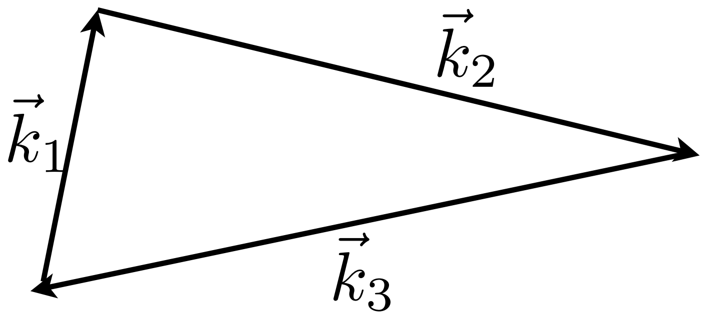


Beyond Gaussianity

Are there correlations between these modes?

$$\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)F_\zeta(k_1,k_2,k_3)$$



Gravity induces interactions which are suppressed by slow-roll:

$$S = \int d^4x a^3 \frac{\dot{\phi}^2}{2H^2}\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial\zeta)^2 + \frac{2}{H}\frac{\partial_i}{\partial^2}\dot{\zeta}\partial_i\zeta\frac{\partial^2}{a^2}\zeta + \dots\right]$$

Maldacena ’02

$$F_\zeta(k_1,k_2,k_3) \sim \mathcal{O}(\epsilon,\eta)P(k_1)P(k_2) + \dots \qquad \Rightarrow \qquad \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^{\frac{3}{2}}} \sim 0.01\times 10^{-5}!!$$

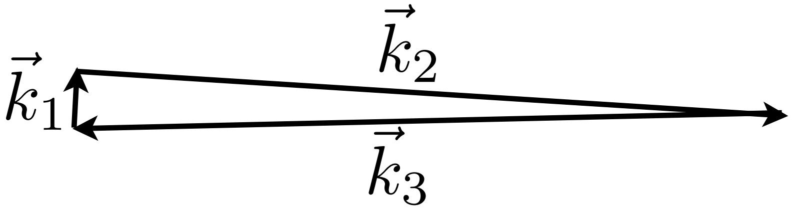
Any modification from this standard picture enhances non-Gaussianity:

- ➡ Modified inflaton Lagrangian: higher-derivative terms, DBI, ghost inflation, etc...
- ➡ Multi-field inflation; curvaton or varying decay rate reheating
- ➡ Alternative to inflation

$F_\zeta(k_1,k_2,k_3)$ potentially contains a wealth of information about the source of perturbations

Squeezed limit

$$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$$



Maldacena’s consistency relation (for single-field models) in the squeezed limit:

$$\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle = -(n_s - 1) P(k_L) P(k_S)$$

Maldacena ’02

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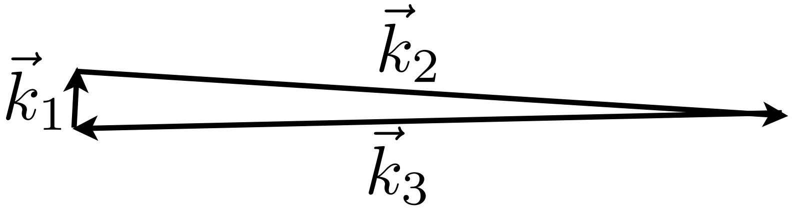

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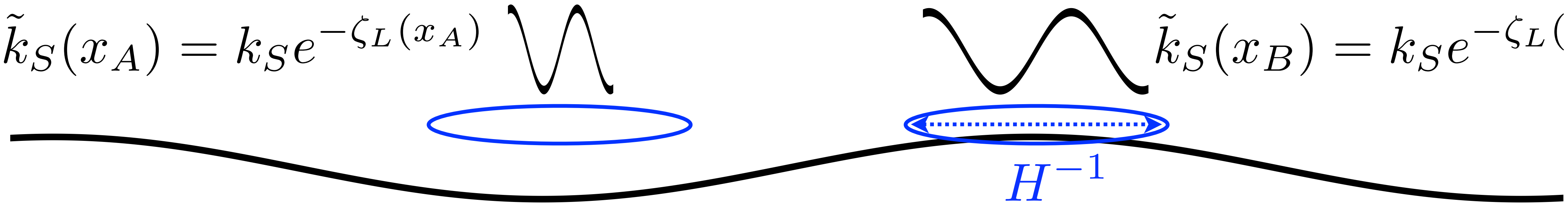
Maldacena ’02

The effect of the long mode translates into a rescaling of the momenta:

$$g_{ij} dx^i dx^j = a^2(t) e^{2\zeta_L(\vec{x})} d\vec{x}^2 = a^2(t) d\tilde{\vec{x}}^2 \quad \Rightarrow \quad \tilde{k} = k e^{-\zeta_L}$$

$$\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle = \langle \zeta_{\vec{k}_L} \langle \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle_{\zeta_L} \rangle \approx \langle \zeta_{\vec{k}_L} P(\tilde{k}_S) \rangle = -\frac{d \ln (k_S^3 P_{k_S})}{d \ln k_S} P(k_S) P(k_L)$$

$$\tilde{k}_S(x_A) = k_S e^{-\zeta_L(x_A)} \qquad \tilde{k}_S(x_B) = k_S e^{-\zeta_L(x_B)}$$



Flat spectrum n$_s$-1=0

Squeezed limit

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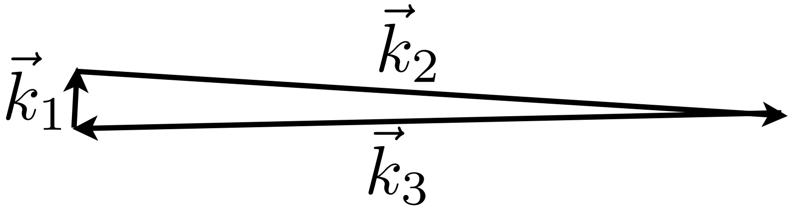

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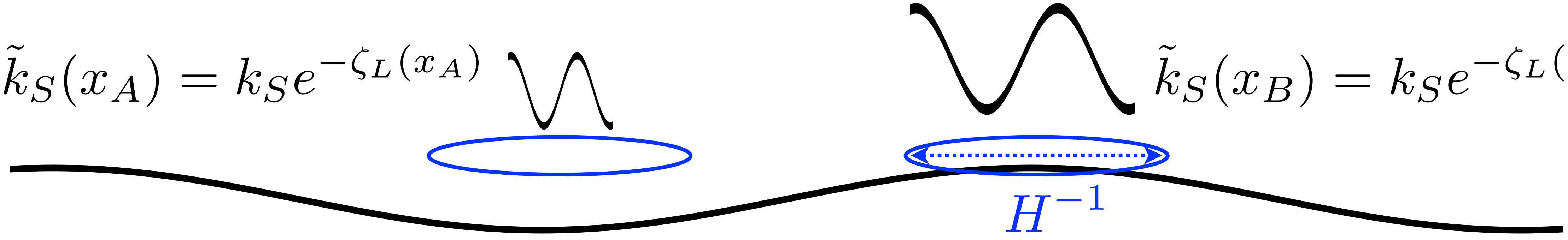
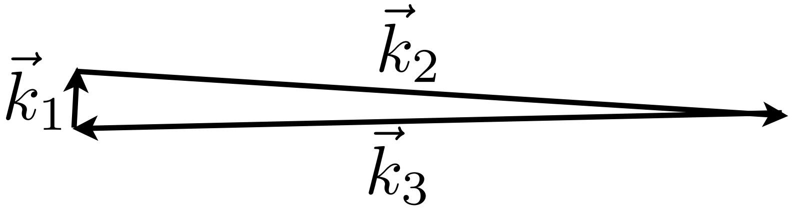


Red spectrum n_s-1<0

Squeezed limit

$$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$$



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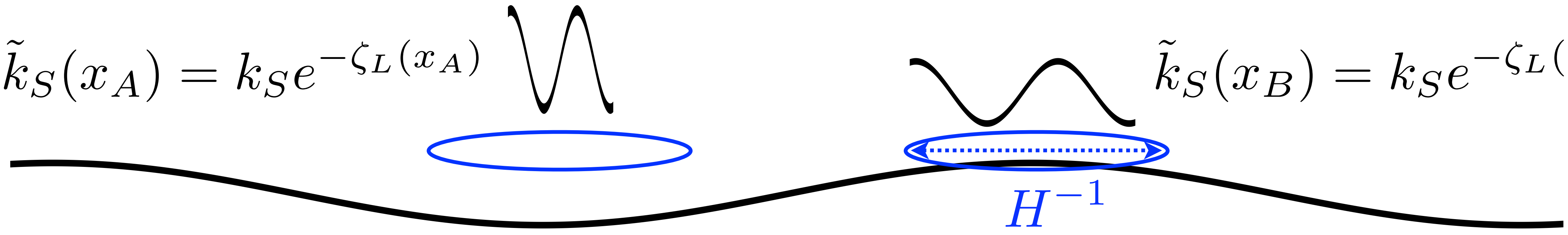


$$\tilde{k}_S(x_A) = k_S e^{-\zeta_L(x_A)}$$

$$\tilde{k}_S(x_B) = k_S e^{-\zeta_L(x_B)}$$

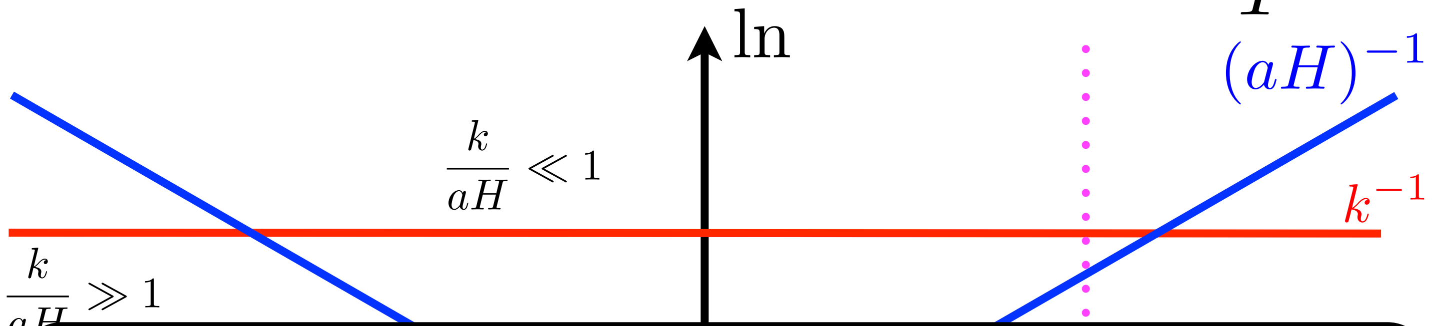
Blue spectrum n_s-1>0

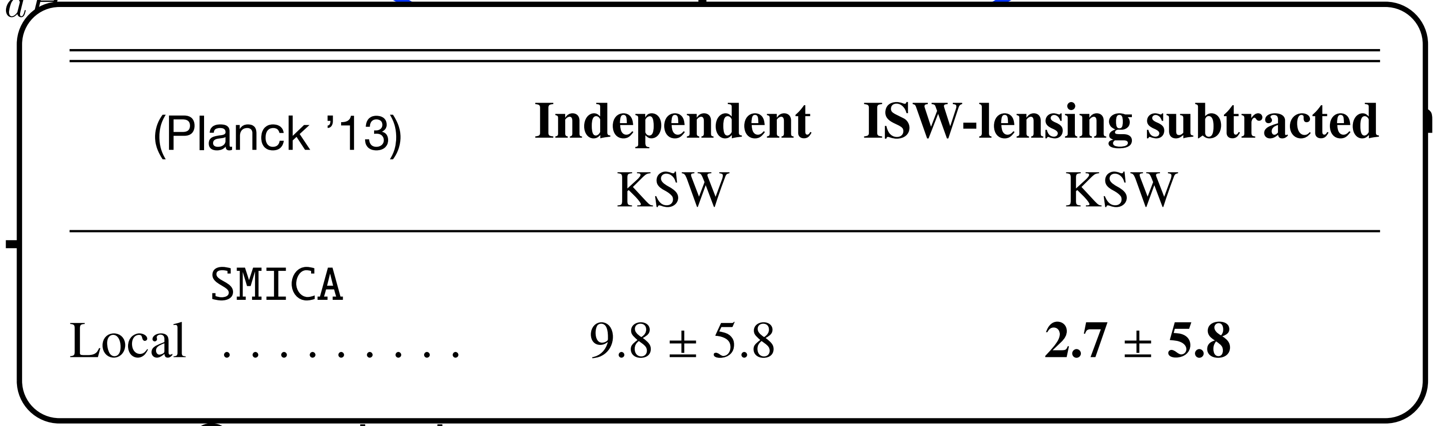
All single-field models predict negligible NG in the squeezed limit: a detection of **local** NG rules out all single-field models!

Intrinsic nonlinear effects

Even in the absence of primordial non-Gaussianity, $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = 0$, the CMB is non-Gaussian!

$$\zeta \quad \overset{\text{Nonlinear relation}}{\Longrightarrow} \quad \Theta = \frac{\delta T}{T}$$





(Planck '13)	**Independent** KSW	**ISW-lensing subtracted** KSW
`SMICA`		
Local	9.8 ± 5.8	**2.7 ± 5.8**

2nd-order effects induce non-Gaussianity:

- late time: ISW-lensing; Goldberg, Spergel, '99 $f_{\rm NL}^{\rm loc} = 7.1$ **Detected by Planck!**
- at recombination: 2nd-order perturbations in the fluid + GR nonlinearities.

$$\delta = \delta^{(1)} + \delta^{(2)} \quad \Rightarrow \quad \begin{array}{l} D[\delta^{(1)}] = 0 \\ D[\delta^{(2)}] = S[\delta^{(1)2}] \end{array} \quad \Rightarrow \quad f_{\rm NL} \sim \frac{\langle \delta^{(2)} \delta^{(1)} \delta^{(1)} \rangle}{\langle \delta^{(1)} \delta^{(1)} \rangle^2} \sim \text{few}$$

All these effects are order ~ few, such as current Planck constraints

Intrinsic nonlinear effects

- Full Boltzmann code:

Based on many contributions:

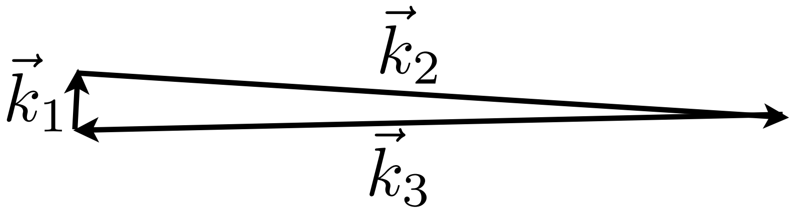
Bartolo, Matarrese, Riotto ’04, ’06; Bernardeau, Pitrou, Uzan ’08; Pitrou ’08 (CMBquick2); Bartolo, Riotto ’08; Khatri, Wandelt ’08; Senatore, Tassev, Zaldarriaga ’08; Nitta et al. ’09, Boubekeur, Creminelli, D’Amico, Norena, ’09, Beneke and Fidler ’10,...

- Bernardeau, Pitrou, Uzan ’08 (CMBquick2)
- Khatri, Wandelt ’08 (perturbed rec.)
- Senatore, Tassev, Zaldarriaga ’08 (perturbed recombination)
- **Huang, Vernizzi ’12 (CosmoLib2nd)**
- Su, Lim, Shellard ’12
- Pettinari, Fidler, Chrittenden, Koyama, Wands ’13 (SONG)

- Squeezed limit:

- Creminelli, Zaldarriaga ’04
- **Creminelli, Pitrou, Vernizzi ’11**
- Bartolo, Matarrese, Riotto ’11
- Lewis ’12
- Pajer, Schmidt, Zaldarriaga ’13

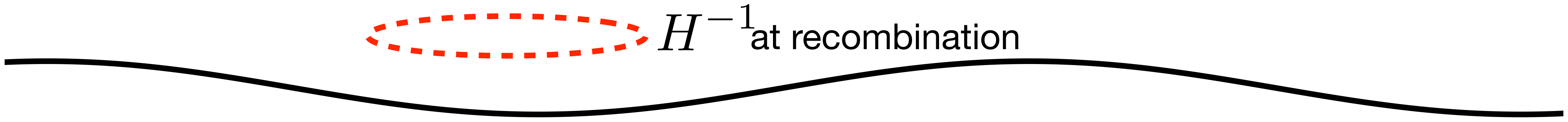
$$k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3$$



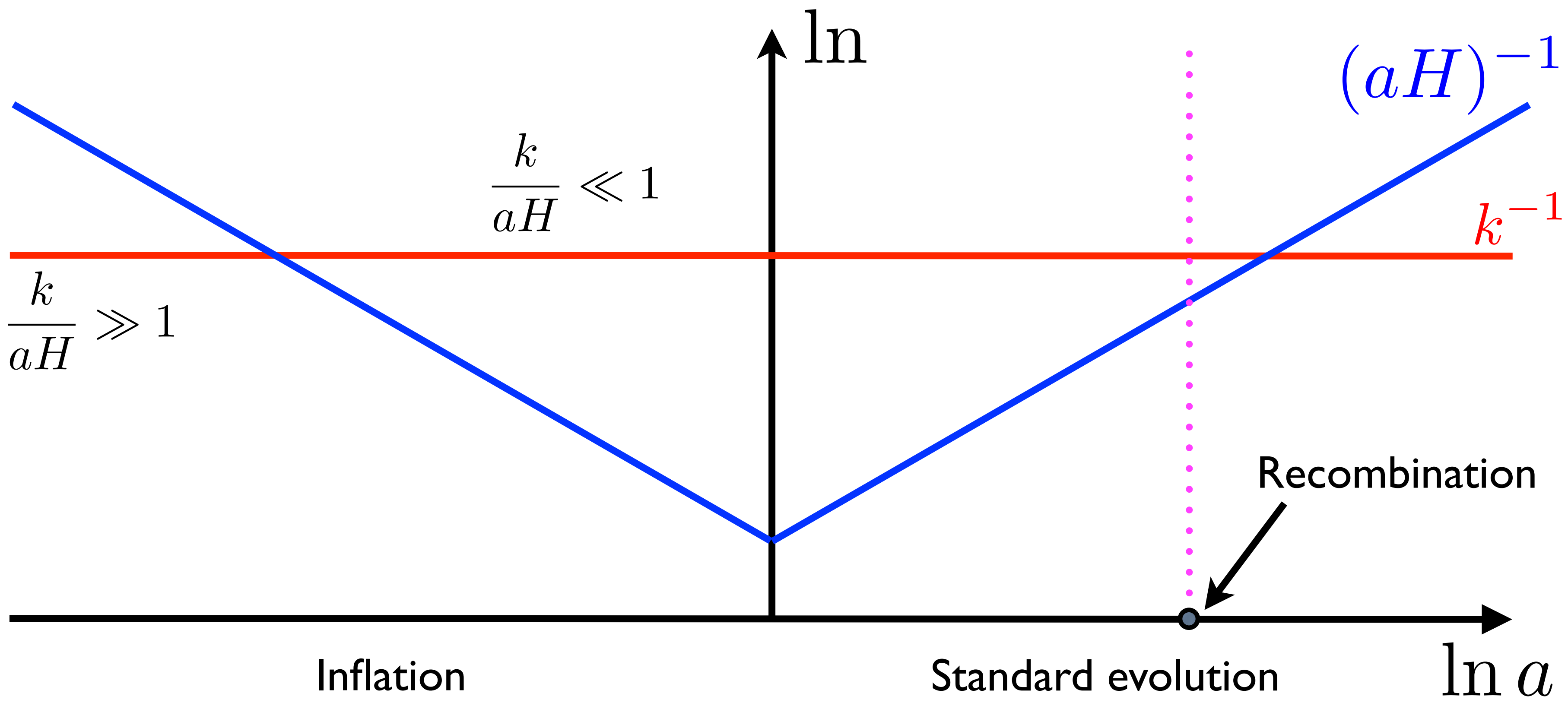
The squeezed limit can be used as a consistency check of the full calculation, i.e. of 2nd-order Boltzmann codes:

Particular squeezed limit

One of the angles must subtend a scale longer than Hubble radius at recombination (but smaller than Hubble radius today):



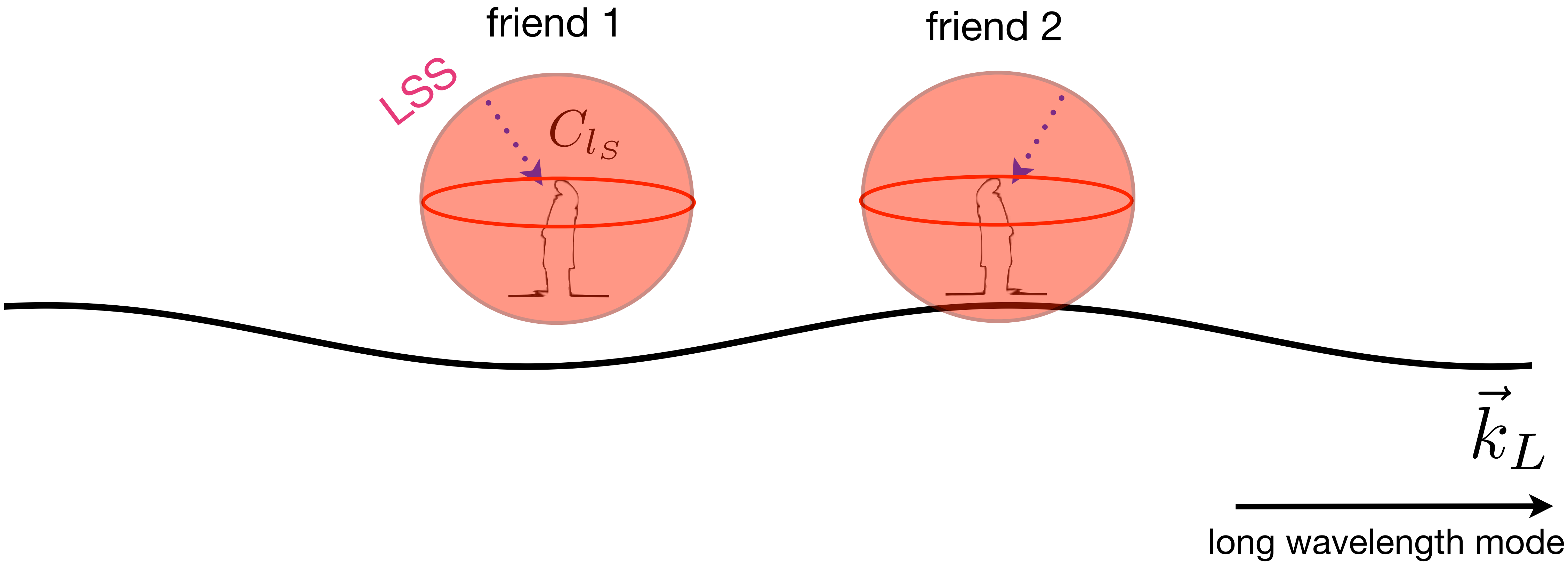
For the bispectrum: $b_{l_1 l_2 l_3}$, $l_1 \ll l_2 \simeq l_3$ & $l_1 \lesssim l_H \sim 110$



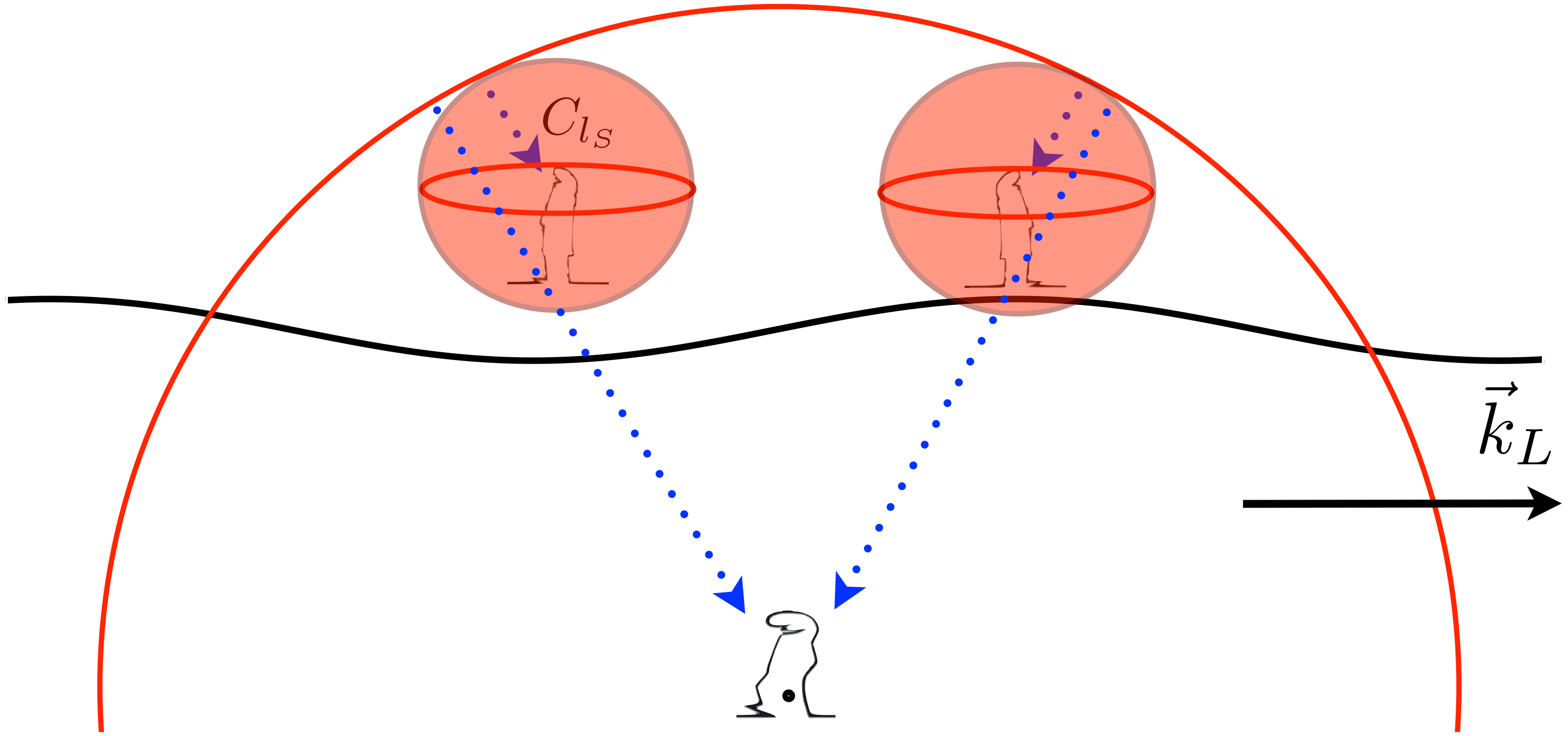
Physical argument

Creminelli, Zaldarriaga, ’04

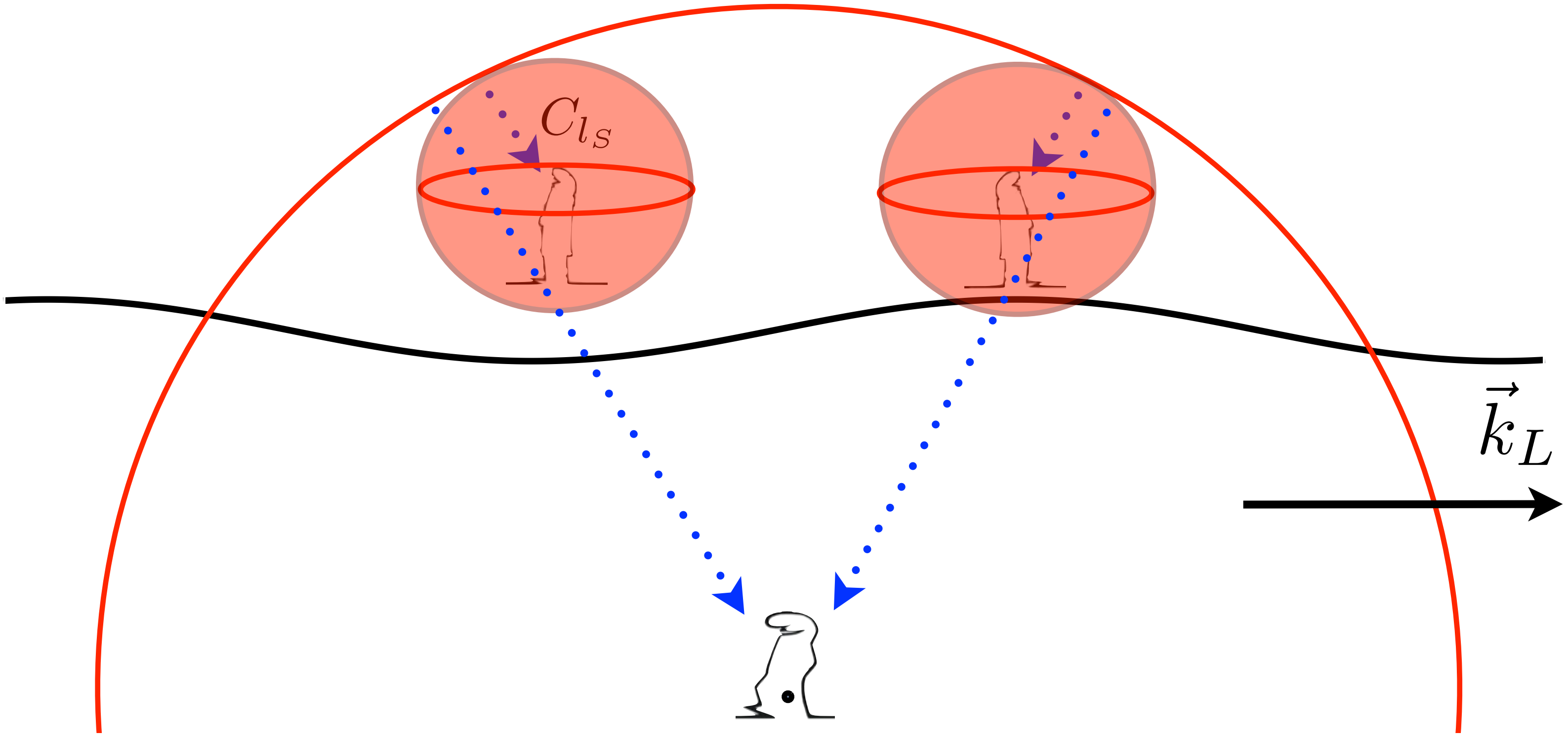
Single-field inflation: 1 clock, e.g. everything is determined by T.



Local physics is identical in Hubble patches that differ only by super-horizon modes: two observers in different places on LSS will see exactly the same CMB anisotropies (at given T).



The long mode is inside the horizon and I can compare different patches. Will see a modulation of the 2-point function due to large scale T:



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1) The long mode changes the local average temperature:

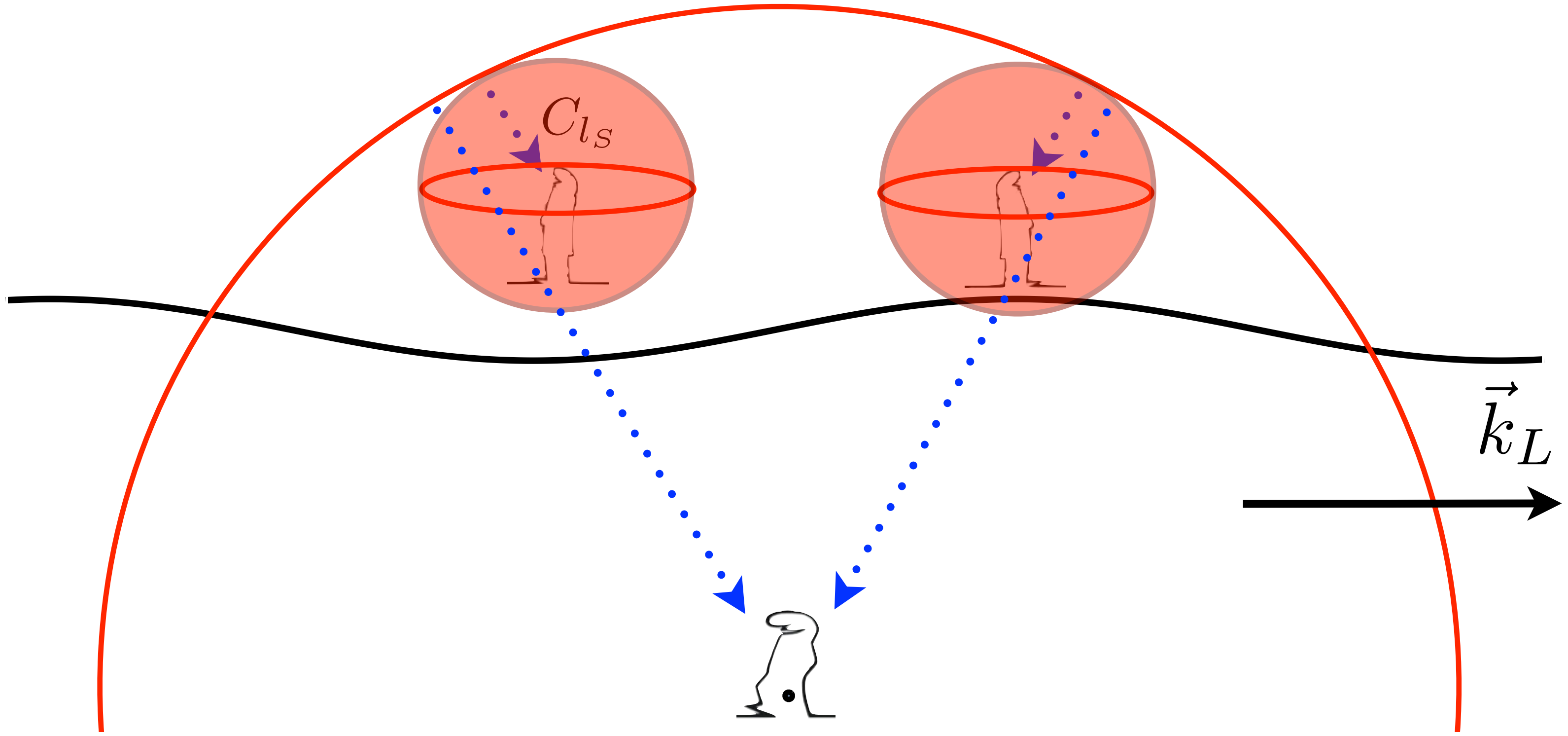
Gaussian variable (on large scales and squeezed limit)

$$a(t) \to a(t)e^{\zeta} \qquad T \propto \frac{1}{a} \quad \Rightarrow \quad T \equiv \bar{T}(t)e^{\tilde{\Theta}}$$

$$\Theta \equiv \frac{\delta T}{\bar{T}} = \tilde{\Theta} + \frac{1}{2}\tilde{\Theta}^2$$

- Local effect absorbable with a nonlinear change of variable: $f_{\rm NL}^{\rm loc} \sim -\frac{1}{6}$

$$b_{l_1 l_2 l_3} = C_{l_1}C_{l_2} + C_{l_1}C_{l_3} + C_{l_2}C_{l_3} + \tilde{b}_{l_1 l_2 l_3}$$



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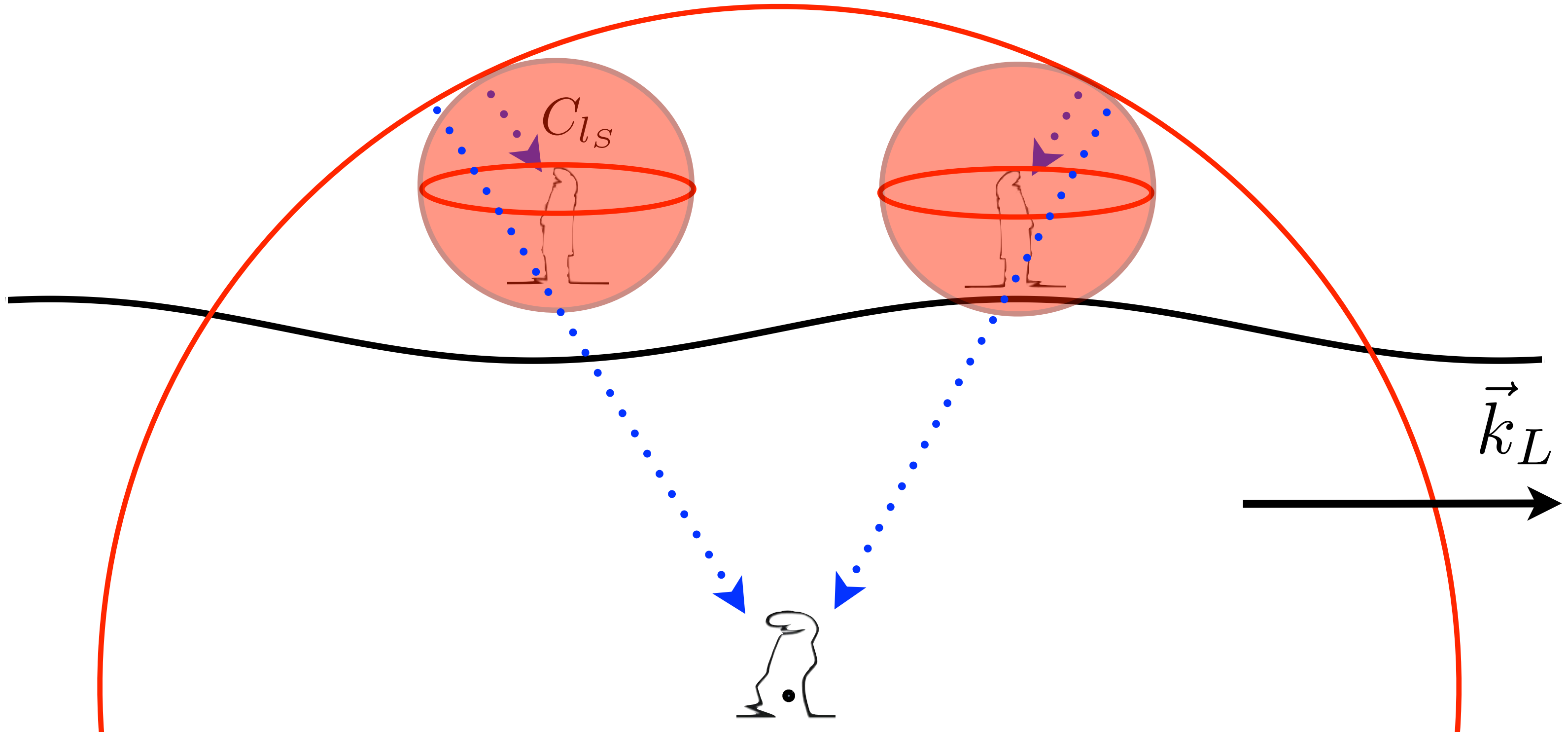
2) Transverse rescaling of spatial coords ⇒ rescaling of angles:

$$C_l \to C_l + \zeta(\hat{n}\cdot\nabla_{\hat{n}} C_l) \quad \Rightarrow \quad \tilde{b}_{l_1 l_2 l_3} = -\frac{1}{2} C^{T\zeta}_{l_1}\left(C_{l_2}\frac{d\ln(l_2^2 C_{l_2})}{d\ln l_2} + C_{l_3}\frac{d\ln(l_3^2 C_{l_3})}{d\ln l_3}\right)$$

$$l_1 \ll l_2, l_3$$

$$l^2 C_l \simeq A_T \left(\frac{l}{l_*}\right)^{n_s-1}, \qquad l \ll 100$$



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$$l_1 \ll l_2, l_3$$

- Squeezed limit consistency relation:

$$b_{l_1 l_2 l_3} = C_{l_1} C_{l_2} + C_{l_1} C_{l_3} + C_{l_2} C_{l_3} - \frac{1}{2} C_{l_1}^{T\zeta} \left(C_{l_2} \frac{d \ln(l_2^2 C_{l_2})}{d \ln l_2} + C_{l_3} \frac{d \ln(l_3^2 C_{l_3})}{d \ln l_3} \right)$$

with Creminelli, Pitrou '11; Bartolo, Matarrese, Riotto; '11, Lewis '12

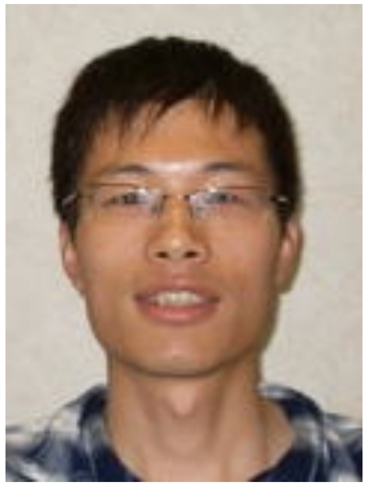
This relation can be used as consistency check of Boltzmann codes based on a physical limit

CosmoLib2$^{\text{nd}}$: the Boltzmann code

- Solve Boltzmann and Einstein equations up to 2$^{\text{nd}}$ order:

$$\frac{df_I}{d\eta} = C_I[f_I]\ , \qquad I = \gamma, \nu, b, \mathrm{CDM}$$

$$\& \qquad G_{ij} = 8\pi G \sum_I T^{(I)}_{ij}$$



Zhiqi Huang

- Integrate the photon temperature along the line of sight
- No lensing and time delay (will be included soon!)

- Comparison with the previous code CMBquick2 by Cyril Pitrou:

★ Fortran, no license, faster and parallelized

★ Full-sky bispectrum

★ Much (10^5) more accurate

★ Perturbed recombination: RECAST consistently perturbed with metric fluctuations. Boltzmann solutions past many tests (squeezed limit, analytic sols.).

★ Better scheme to integrate photon distribution along the line of sight

CosmoLib2nd: the Boltzmann code

- Solve Boltzmann and Einstein equations up to 2nd order:

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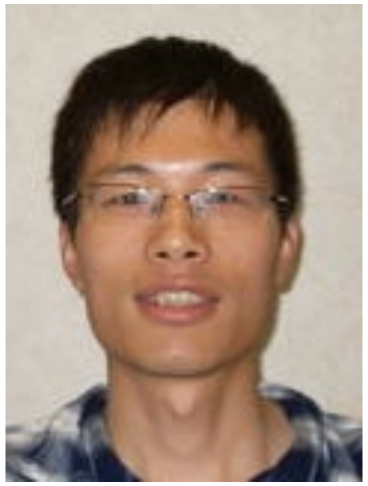

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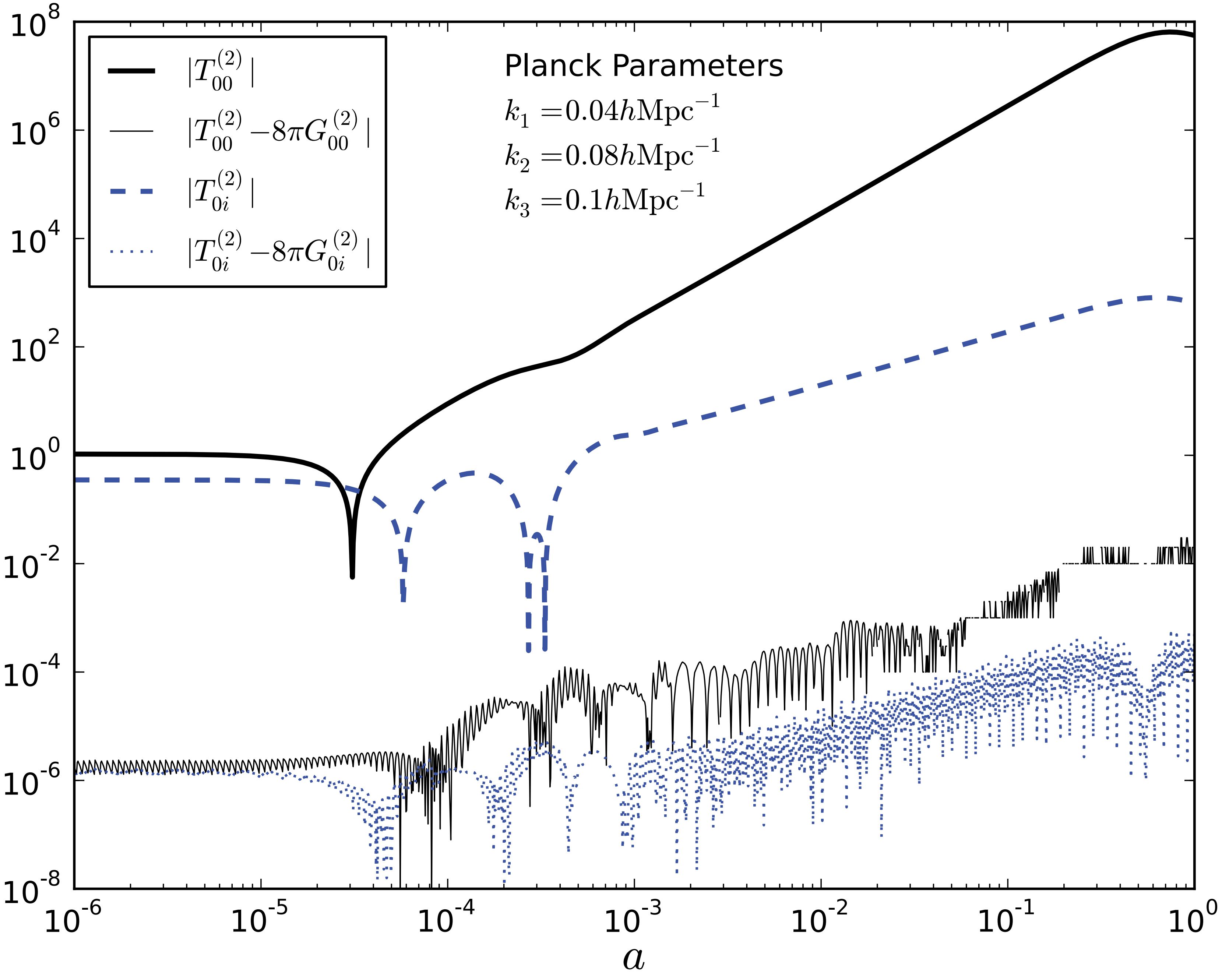
CosmoLib2$^{\text{nd}}$ accuracy

- The accuracy can be estimated by evaluation of energy and momentum constraint equations

⇒ 10^{-6} accurate



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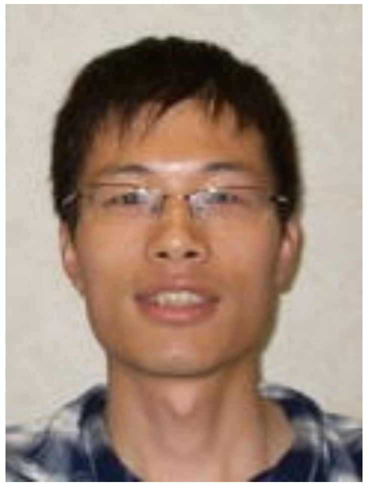

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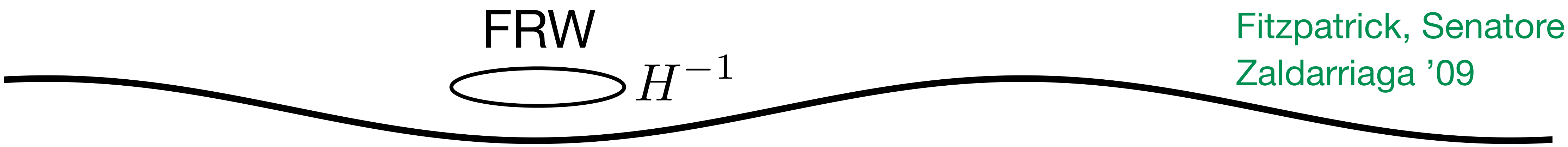
2^{nd}-order evolution as a coord change

Maldacena ’02; Weinberg ’03; Fitzpatrick, Senatore and Zaldarriaga ’09



Locally, possible to rewrite a perturbed FRW metric as an unperturbed one by reabsorbing the long mode with a coordinate transformation. Ex, in matter dominance:

$$ds^2 = a^2(\eta)\left[-(1+2\Phi_L)d\eta^2 + (1-2\Phi_L)dx^2\right] \quad \Rightarrow \quad ds^2 = a^2(\tilde{\eta})\left[-d\tilde{\eta}^2 + d\tilde{x}^2\right]$$

$$\begin{aligned}\tilde{\eta} &= \eta(1-\zeta/5) \\ \tilde{x}^i &= x^i(1+\zeta)\end{aligned} \qquad \text{with } \zeta = -\frac{3}{5}\Phi_L$$

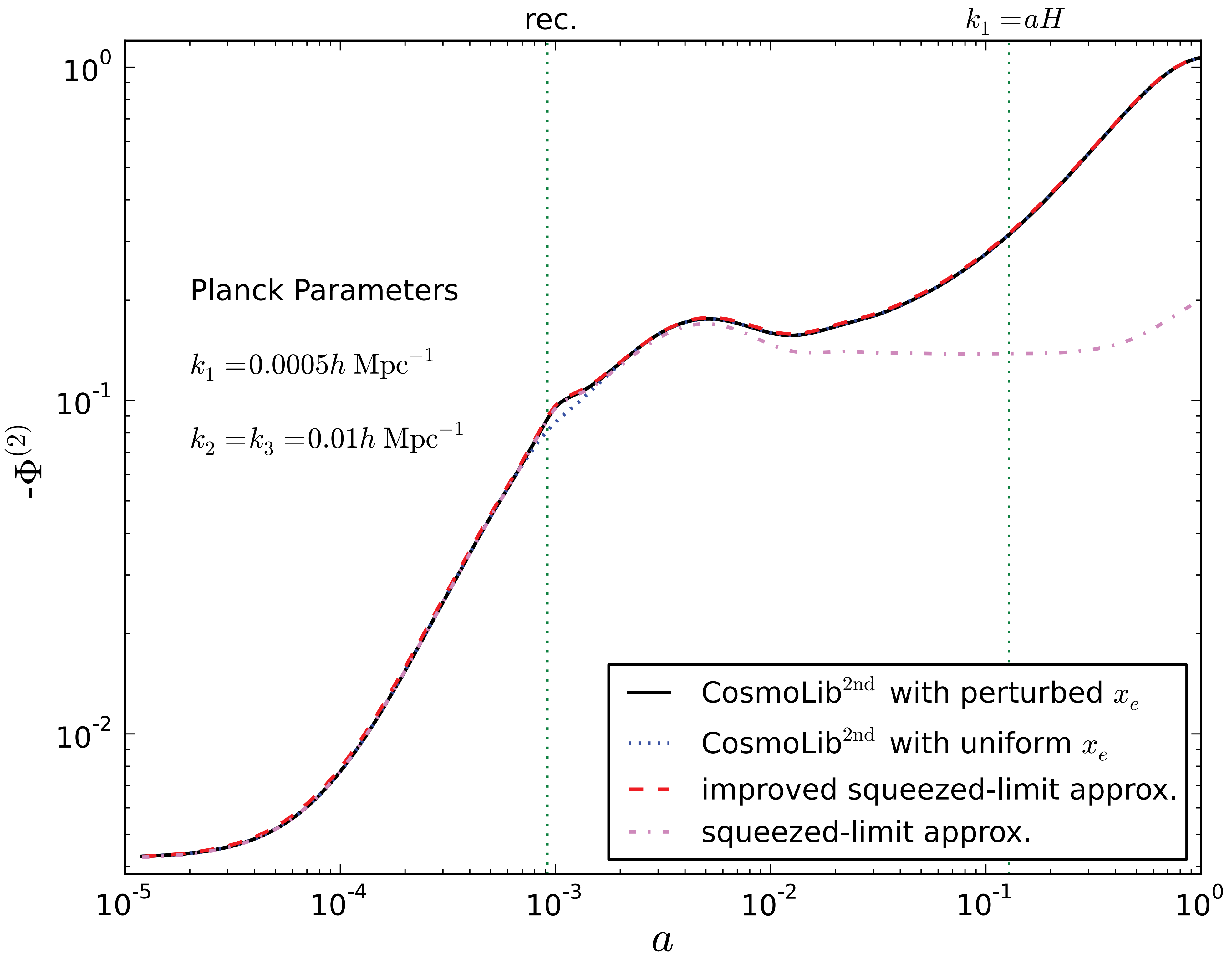
Conversely, start from a perturbed metric at 1^{st}-order and “generate” 2^{nd}-order couplings between short and long modes by the inverse coordinate transformation:

$$ds^2 = a^2(\tilde{\eta})\left[-(1+2\tilde{\Phi}_S)d\tilde{\eta}^2 + (1-2\tilde{\Psi}_S)d\tilde{x}^2\right] \Rightarrow ds^2 = a^2(\eta)\left[-e^{2\Phi}d\eta^2 + e^{2\Psi}dx^2\right]$$

$$\Phi = \tilde{\Phi}_S + \Phi_L - \frac{1}{5}\zeta\frac{\partial\tilde{\Phi}_S}{\partial\ln\eta} + \zeta x^i\frac{\partial\tilde{\Phi}_S}{\partial x^i}$$

CosmoLib2$^{\mathrm{nd}}$ checks

- We can use the squeezed limit to directly check the solutions of the Boltzmann code:



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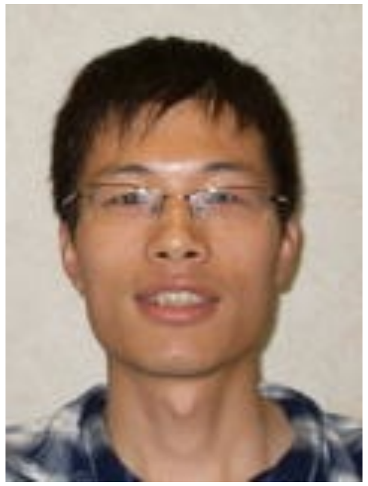

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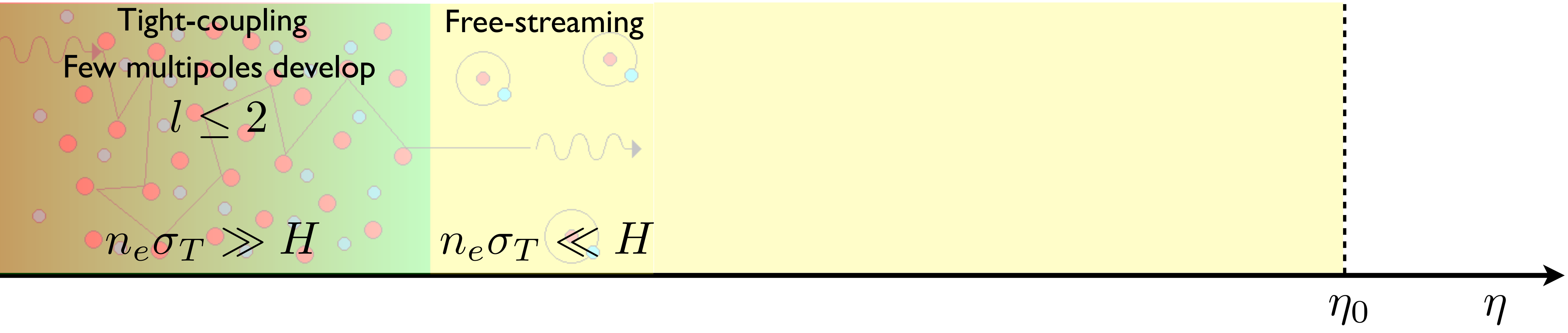
Line-of-sight treatment

with Z. Huang, ’12



Tight-coupling

Few multipoles develop

$l \leq 2$

$n_e \sigma_T \gg H$

Free-streaming

$n_e \sigma_T \ll H$

η_0

η

- Photon temperature equation:

$$\frac{d}{d\eta}(\Theta + \Phi) - \Theta(\dot{\Psi} - \Phi_{,i} n^i) - E = (\bar{n}_e \sigma_T a)(1 + \delta_e + \Phi) F$$

temperature fluctuations

integrated effects

collision term

$$E \equiv (\dot{\Phi} + \dot{\Psi}) - \dot{\omega}_i n^i - \dot{\chi}_{ij} n^i n^j / 2$$

ISW (+ RS), vector and tensor contributions

$$F \equiv \Theta_{00} - \Theta - \frac{1}{2}\sqrt{\frac{4\pi}{5^3}} \sum_m \Theta_{2m} Y_{2m}(\hat{n}) + \hat{n} \cdot \vec{v}$$

$$+ 7(\hat{n} \cdot \vec{v})^2/4 - v^2/4 + \hat{n} \cdot \vec{v} \left(\Theta + 3\Theta_{00} - \frac{1}{2}\sqrt{\frac{4\pi}{5^3}} \sum_m \Theta_{2m} Y_{2m}(\hat{n}) + i\sqrt{\frac{\pi}{3}} \sum_m \Theta_{1m} Y_{1m}(\hat{n}) \right)$$

$$+ 2\pi v \sqrt{\frac{2}{15}} \sum_{m,M} \begin{pmatrix} 1 & 1 & 2 \\ m & M & -m-M \end{pmatrix} \Theta_{2,m+M} Y_{1m}(\hat{n}) Y_{1M}(\hat{v}) (-1)^{m+M} + i\sqrt{\frac{\pi}{3}} v \sum_m \Theta_{1m} Y_{1m}(\hat{v})$$

from Tassev, Senatore, Zaldarriaga, ’08

Line-of-sight treatment

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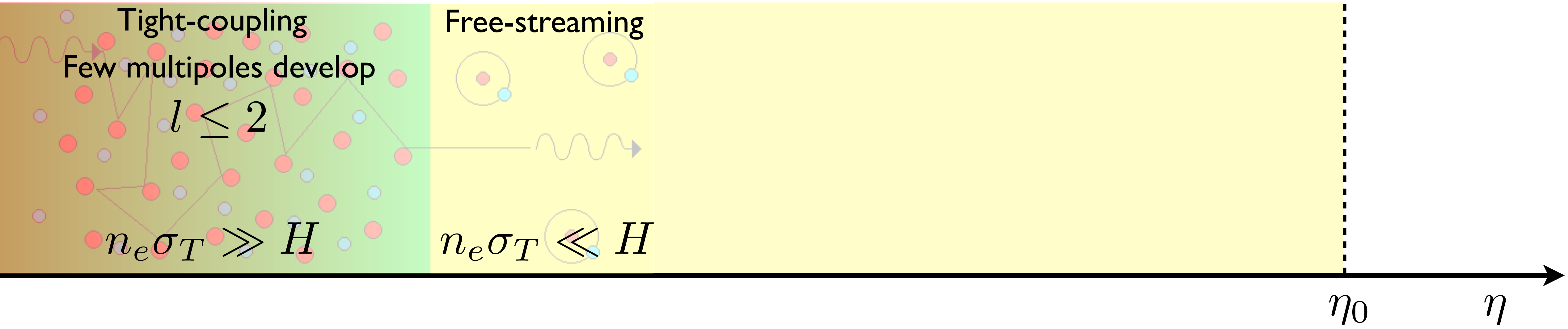

- Photon temperature equation:

$$\frac{d}{d\eta}(\Theta + \Phi) - \Theta(\dot{\Psi} - \Phi_{,i} n^i) - E = (\bar{n}_e \sigma_T a)(1 + \delta_e + \Phi) F$$

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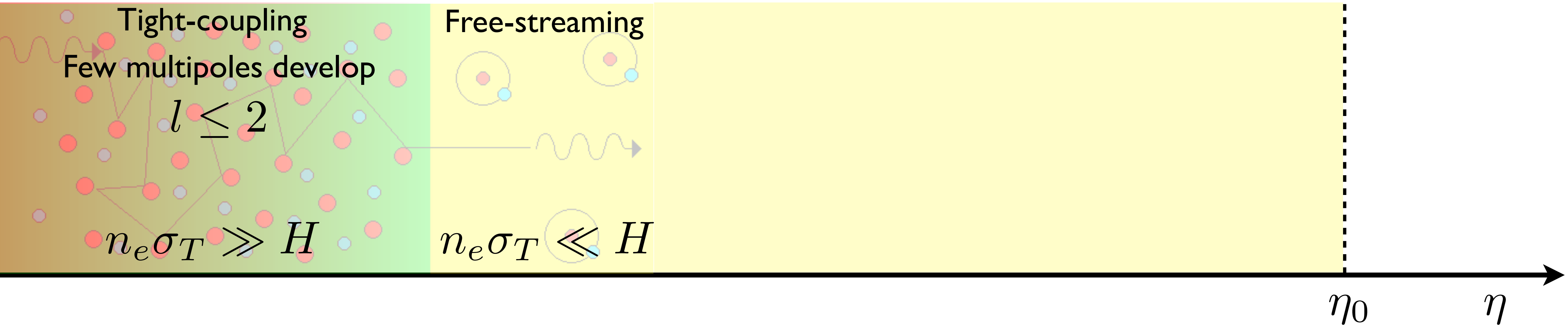


- Photon temperature equation:

$$\frac{d}{d\eta}(\tilde{\Theta} + \Phi) - E = (\bar{n}_e \sigma_T a)(1 + \delta_e + \Phi)\tilde{F}$$

temperature fluctuations

integrated effects

Gaussian collision term

$$T = \bar{T} e^{\tilde{\Theta}} \ , \quad \tilde{\Theta} \equiv \Theta - \frac{1}{2}\Theta^2$$

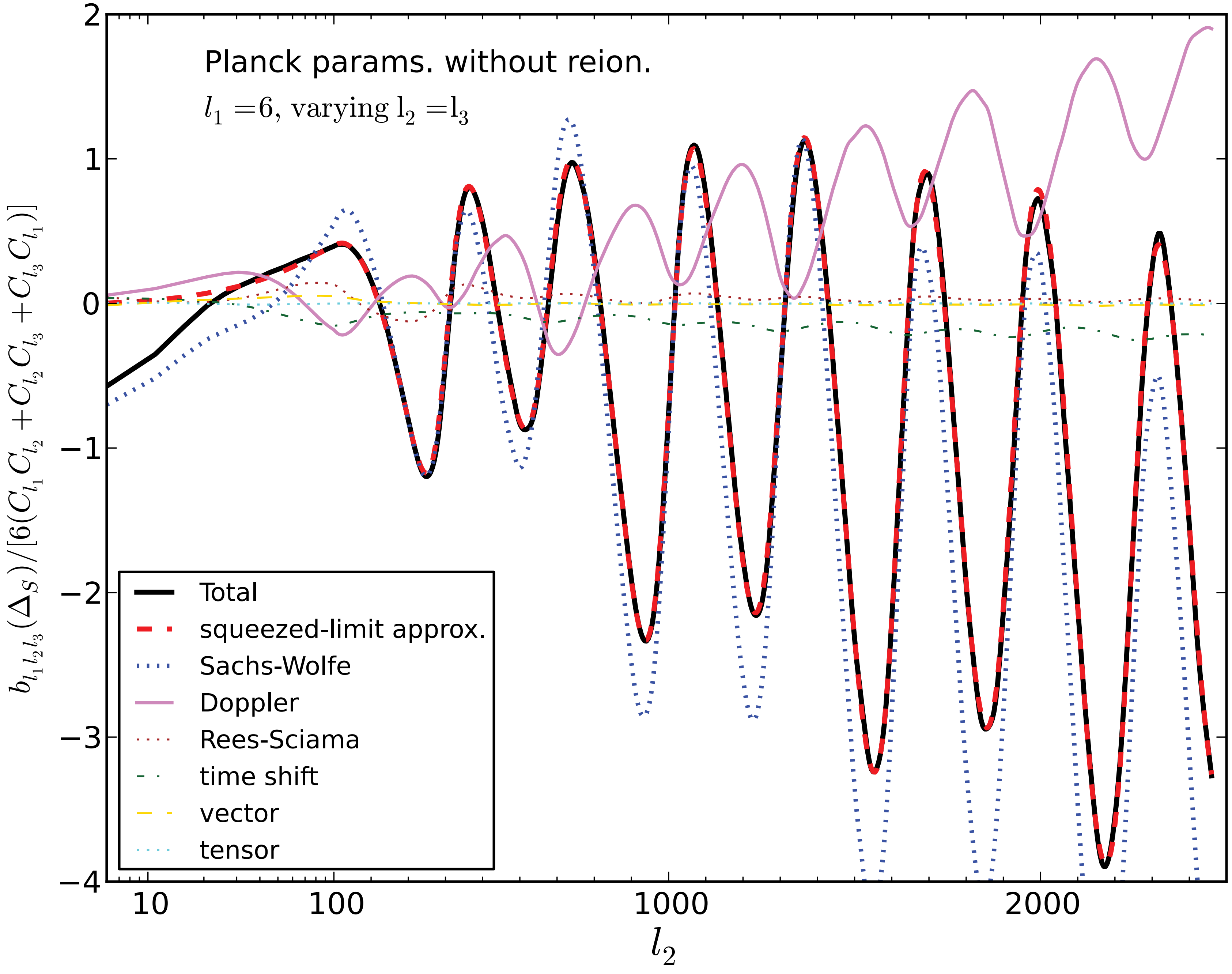
$$b_{l_1 l_2 l_3} = C_{l_1} C_{l_2} + C_{l_1} C_{l_3} + C_{l_2} C_{l_3} + \tilde{b}_{l_1 l_2 l_3}$$

- Change of variable improves the convergence of the expansions

$\Rightarrow$ $\tilde{F}$ becomes Gaussian on super-Hubble scales

The squeezed limit

with Z. Huang, ’12

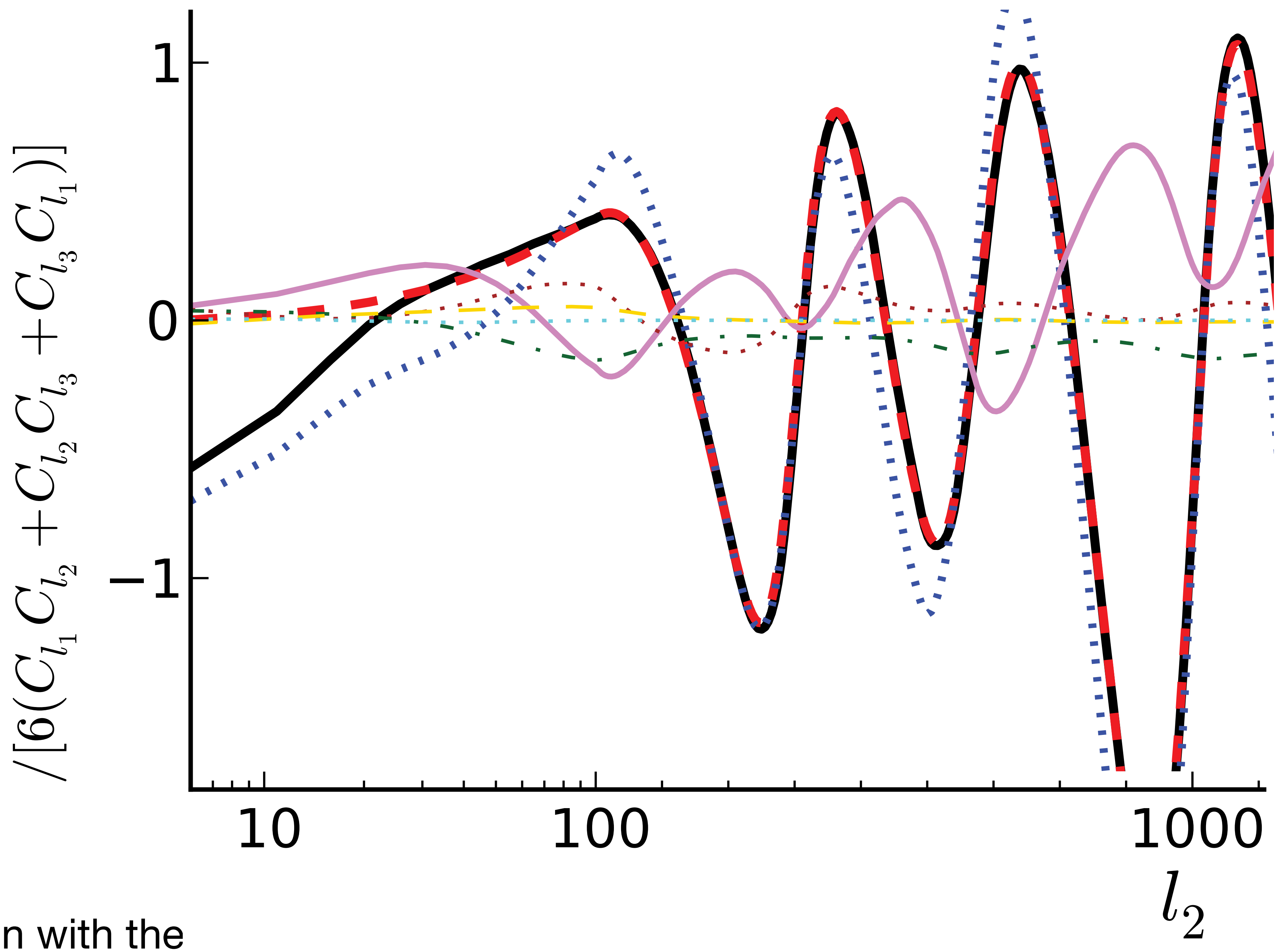


- Comparison with the analytic formula:

$$\tilde{b}_{l_1 l_2 l_3} = -\frac{1}{2} C^{T\zeta}_{l_1} \Big(C_{l_2} \frac{d \ln(l_2^2 C_{l_2})}{d \ln l_2} + C_{l_3} \frac{d \ln(l_3^2 C_{l_3})}{d \ln l_3} \Big) \qquad l_1 \ll l_2, l_3$$

The squeezed limit

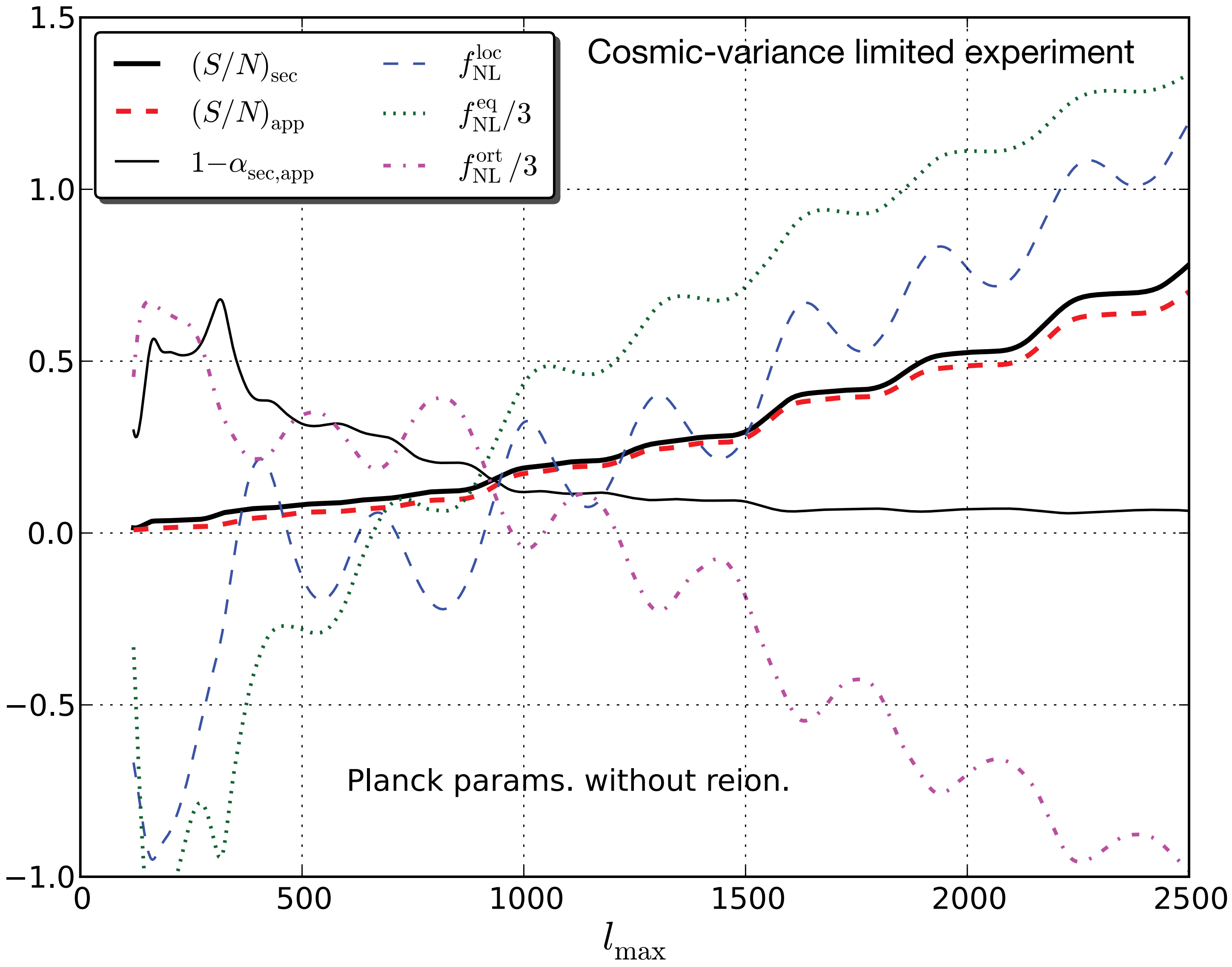
with Z. Huang, ’12



- Comparison with the

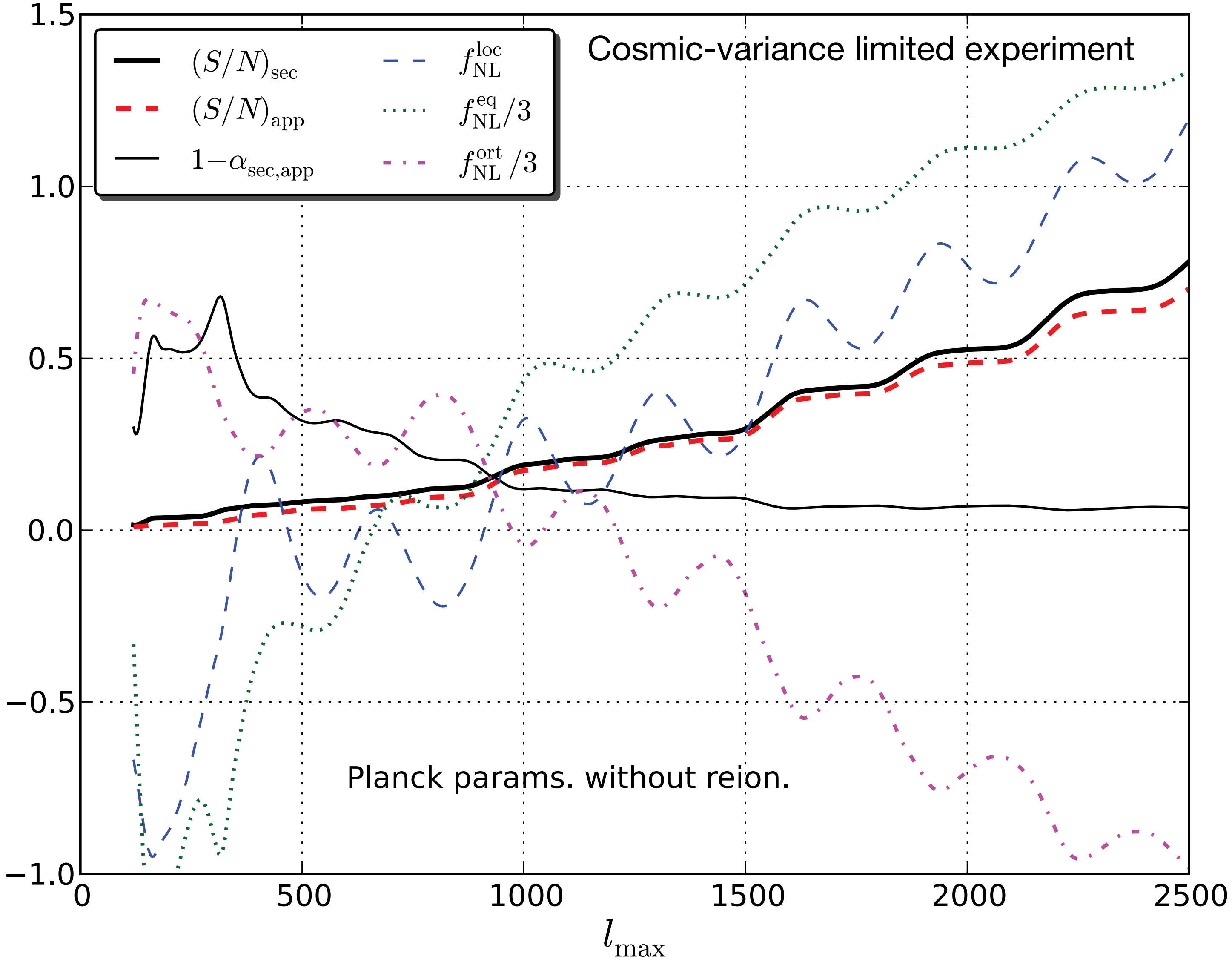
$$\tilde{b}_{l_1 l_2 l_3} = -\frac{1}{2} C_{l_1}^{T\zeta} \left(C_{l_2} \frac{d\ln(l_2^2 C_{l_2})}{d\ln l_2} + C_{l_3} \frac{d\ln(l_3^2 C_{l_3})}{d\ln l_3} \right) \left[1 + \mathcal{O}(l_1/l_2)^2\right]$$

Observability and contamination



	ISW-lensing subtracted
	KSW
`SMICA`	
Local	**2.7** ± **5.8**
Equilateral	**−42** ± **75**
Orthogonal	**−25** ± **39**

Observability and contamination



(Planck '13)	**ISW-lensing subtracted** KSW
`SMICA`	
Local	**2.7** ± **5.8**
Equilateral	**−42** ± **75**
Orthogonal	**−25** ± **39**

- Comparison with other references for $l_{\rm max} = 2000$:

agrees with Senatore, Tassev, Zaldarriaga '08 for $l_{\rm min}$=100:

with Creminelli, Pitrou, '11,
Bartolo, Matarrese, Riotto, '11 $f_{\rm NL}^{\rm loc} = 0.94$

Su, Lim, Shellard '12: $S/N = 0.69; \quad f_{\rm NL}^{\rm loc} = 0.88;$

Pettinari, Fidler, Chrittenden, Koyama, Wands '13: $S/N = 0.47; \quad f_{\rm NL}^{\rm loc} = 0.57$

Conclusion

Zhiqi Huang

We did a very long calculation and found nothing!

Chinese wisdom...

Conclusion

- **Second order effects are finally under control!**

- In the squeezed limit (one mode longer than horizon at recombination), it is possible to compute the CMB bispectrum exactly. with Creminelli, Pitrou, '11

- Relation valid for adiabatic (single clock) perturbations. Already takes into account NG from single-field models. It is a consistency relation on the observable (CMB temperature) in the squeezed limit.

- Full calculation, on all scales, of bispectrum from nonlinear effects at recombination with CosmoLib2nd. with Z. Huang, '12

- Perfect agreement with consistency relation and previous literature. Small contamination to local primordial non-Gaussianity: $f_{\rm NL}^{\rm loc} = 0.82$. Sizable effect, $S/N = 0.47$, but not enough for detection. Larger $l_{\rm max}$ and polarization?

- In any case, full exploitation of Planck data requires detailed knowledge of **all** nonlinear effects:

Let's include them in the next analysis!