Cosmic strings and their non-Gaussianities after Planck 2013

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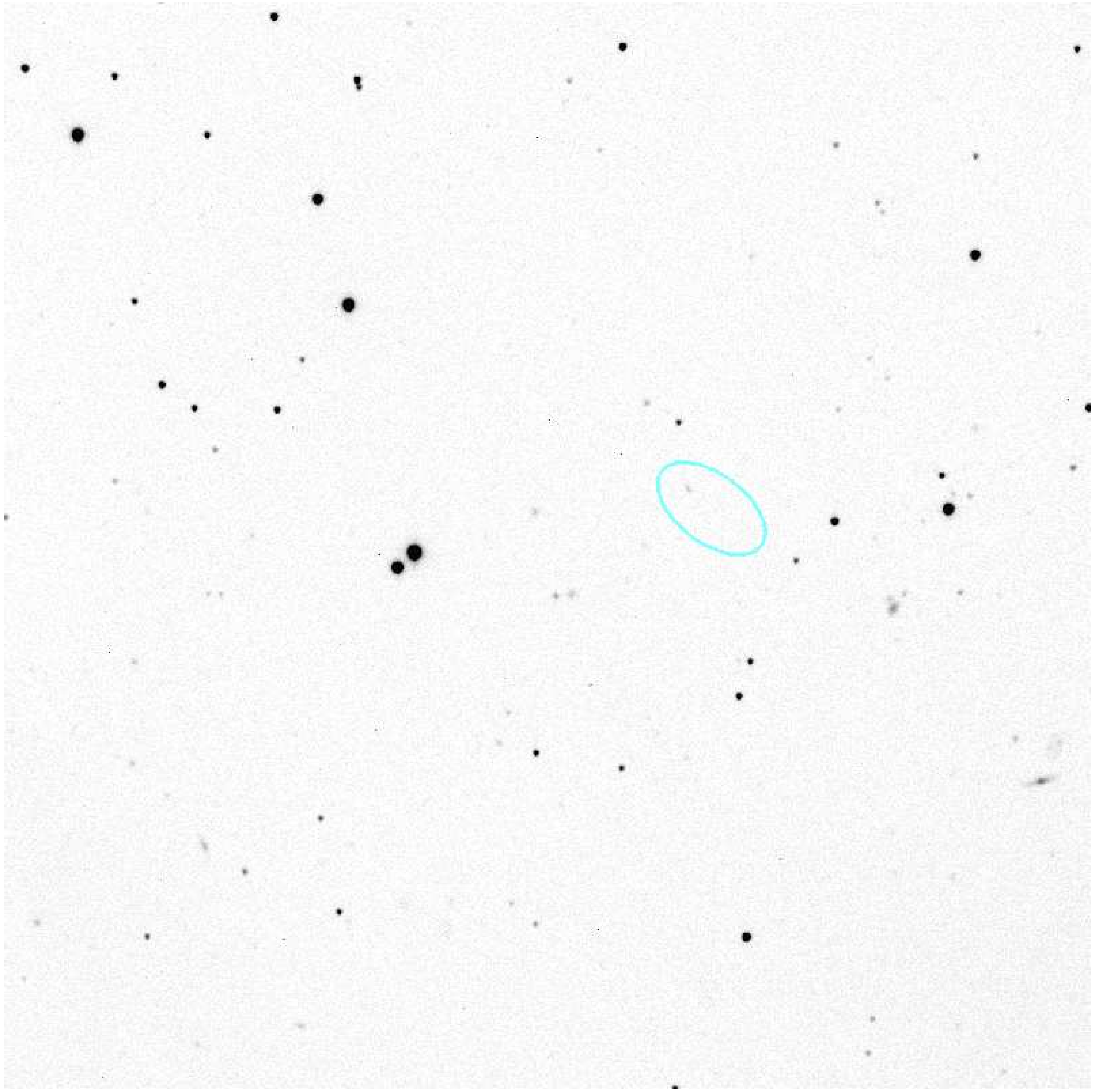
Outline

Planck 2013 results XXV: `arXiv:1303.5085`
CR, F. R. Bouchet: `arXiv:1204.5041`
CR: `arXiv:1005.4842`

Around the L2 point



Credit: V. Bouchet, S. de Visscher, C.R. (T62 Astroqueyras)

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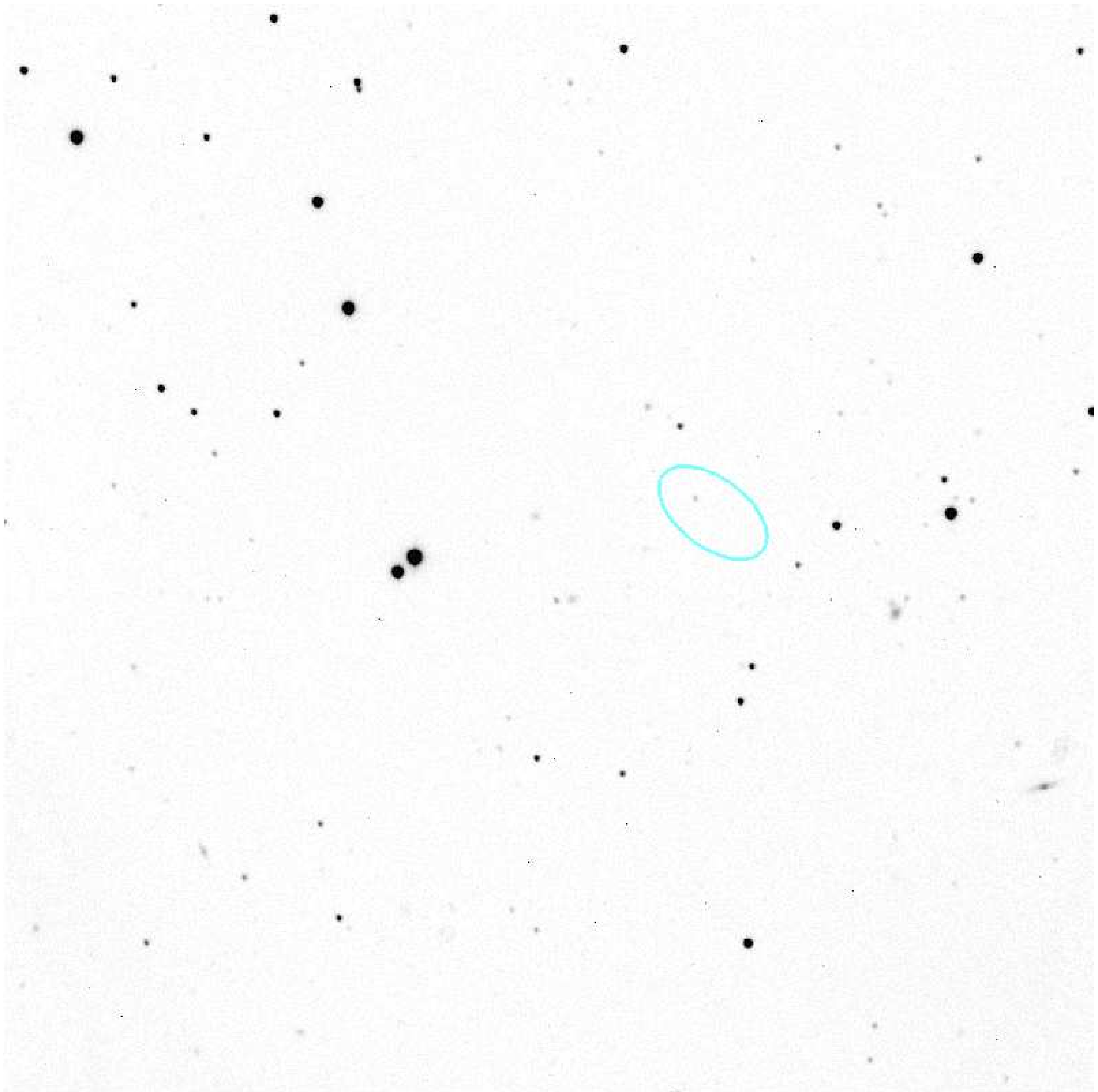

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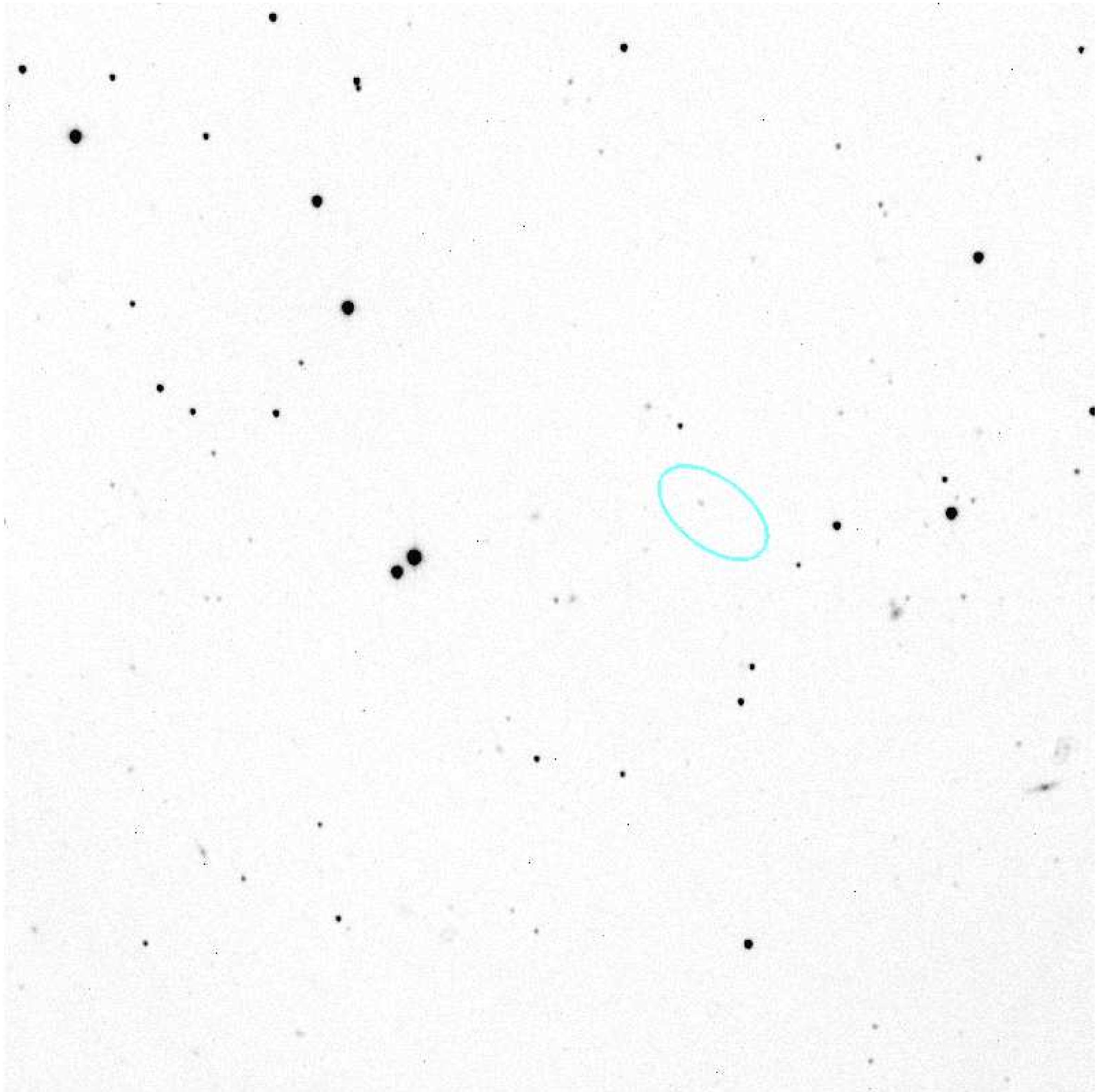

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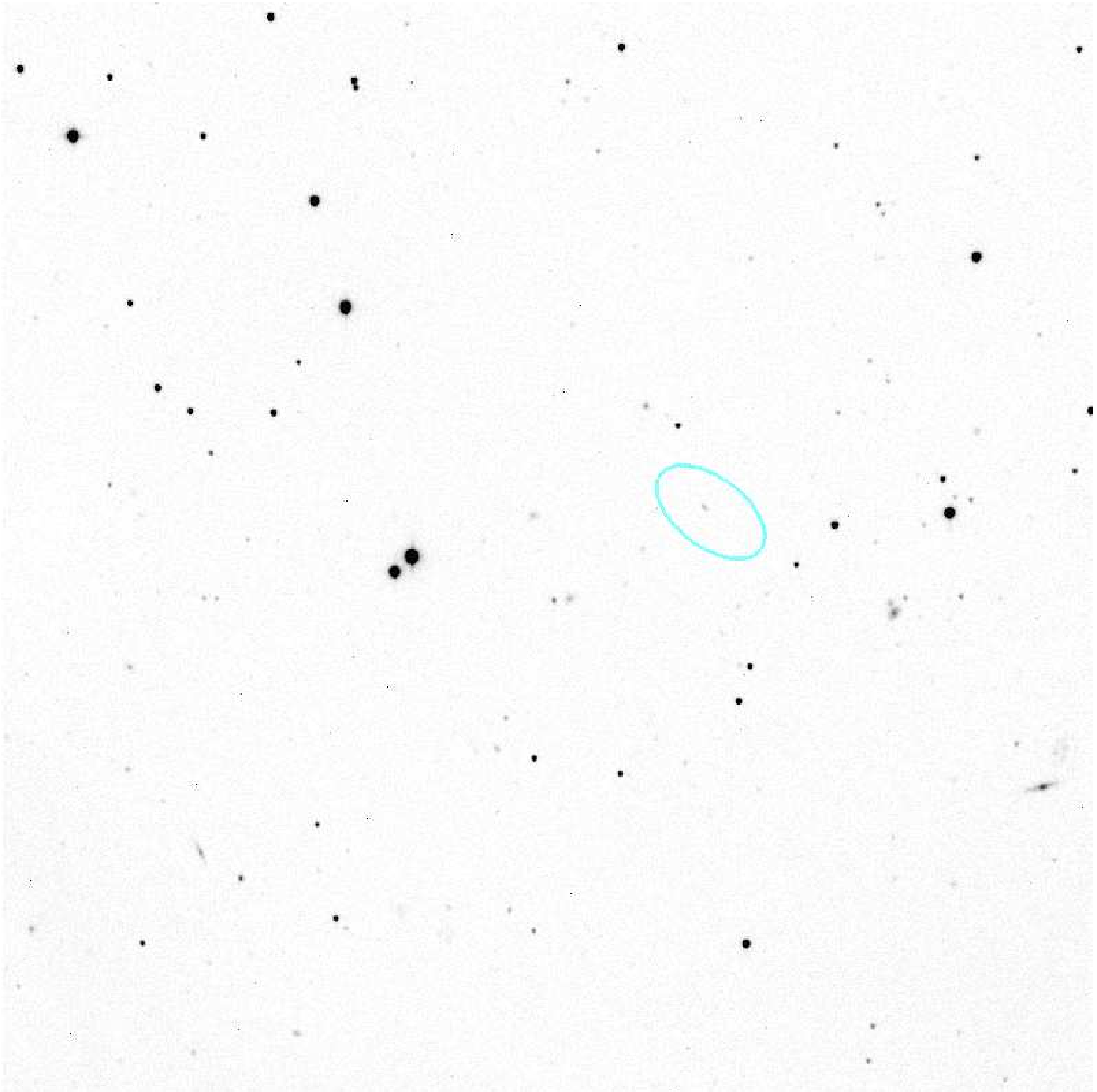

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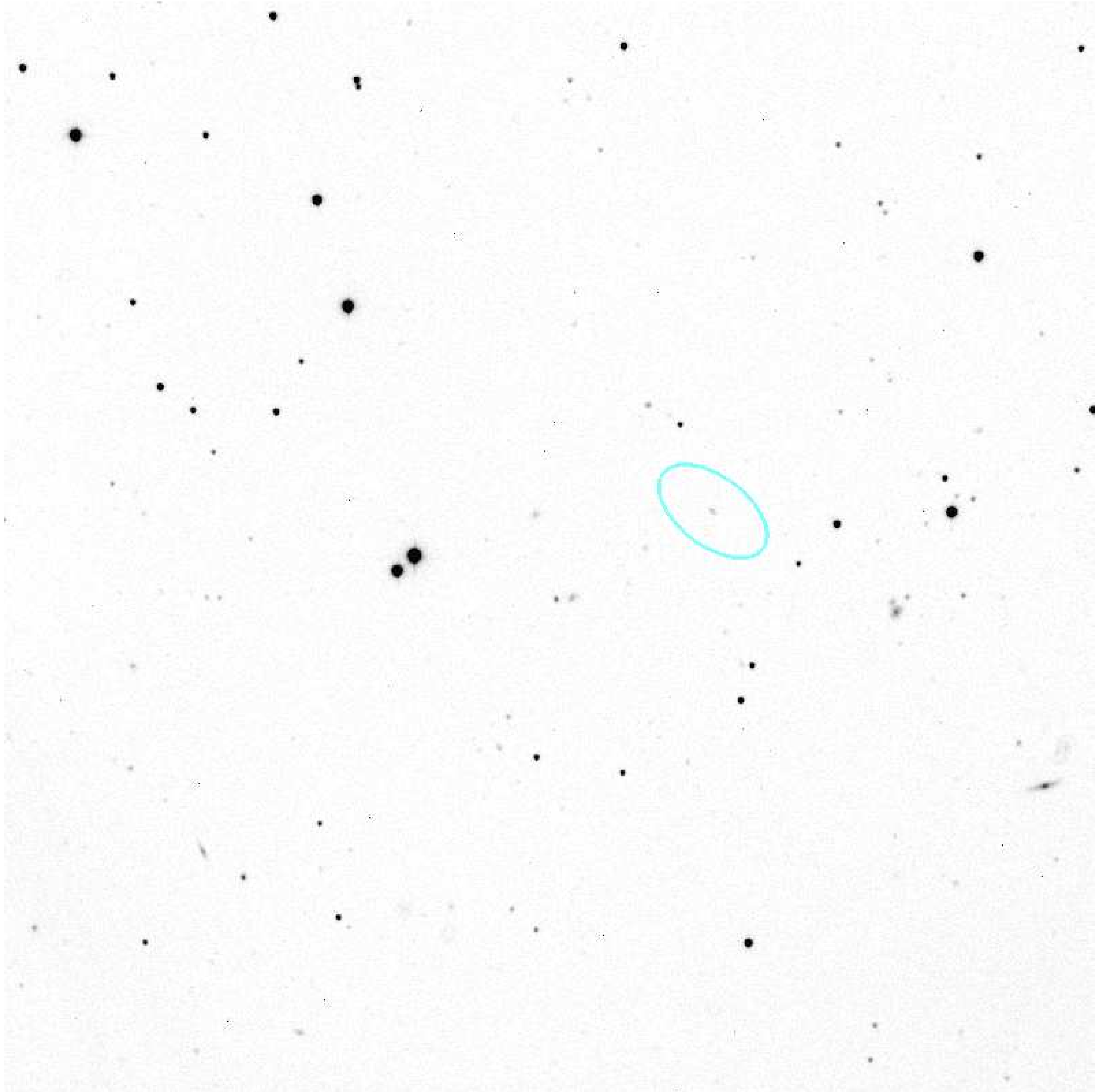

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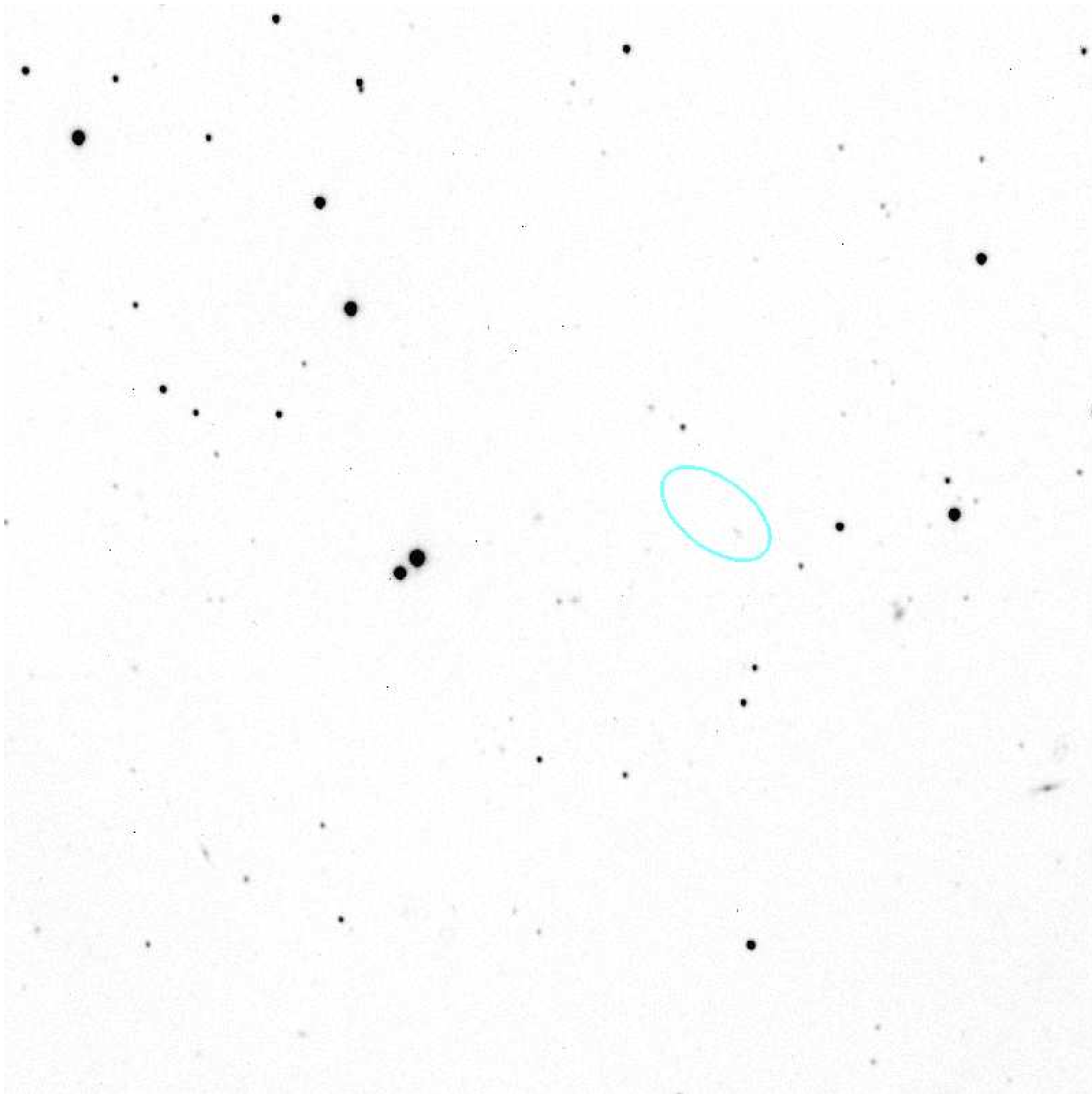

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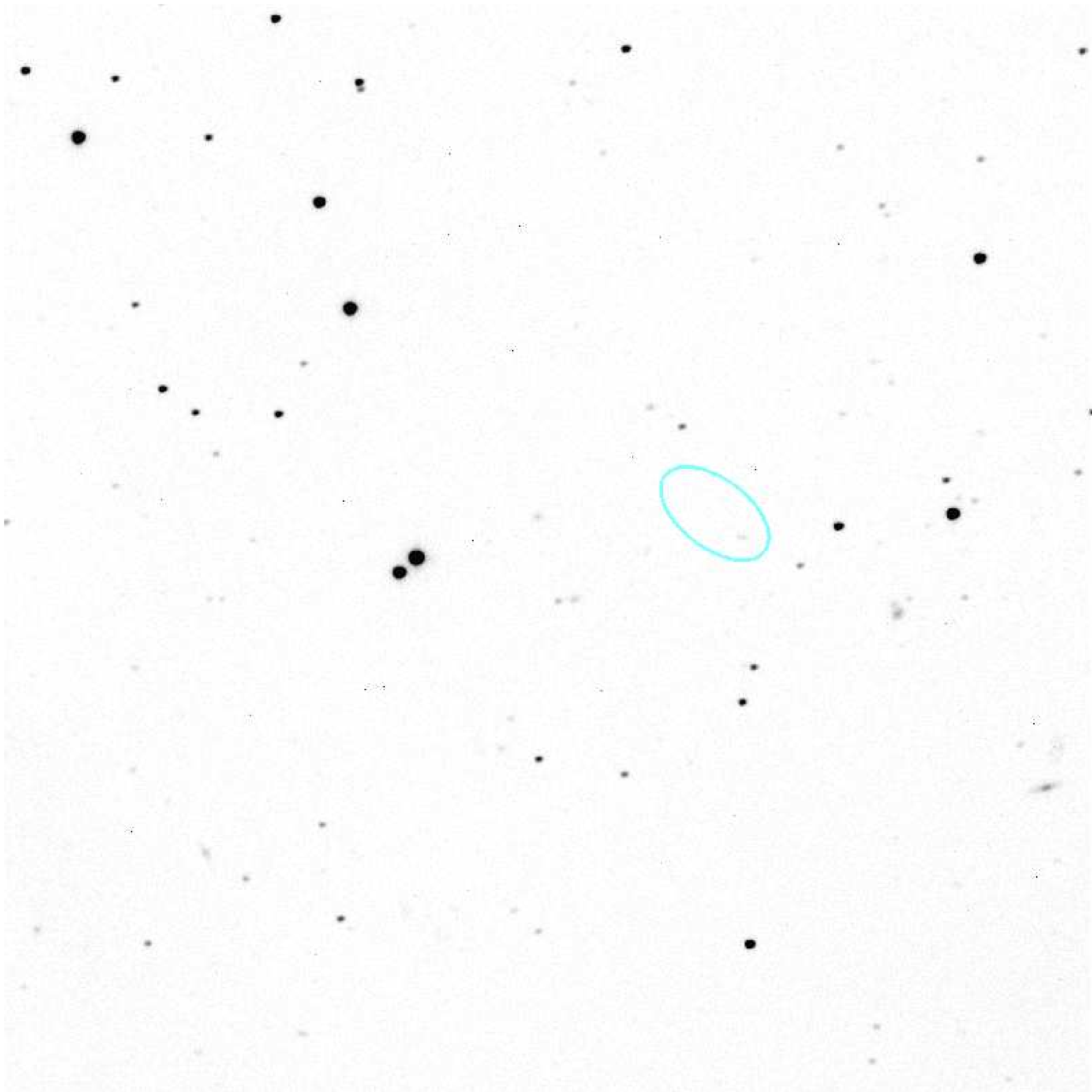

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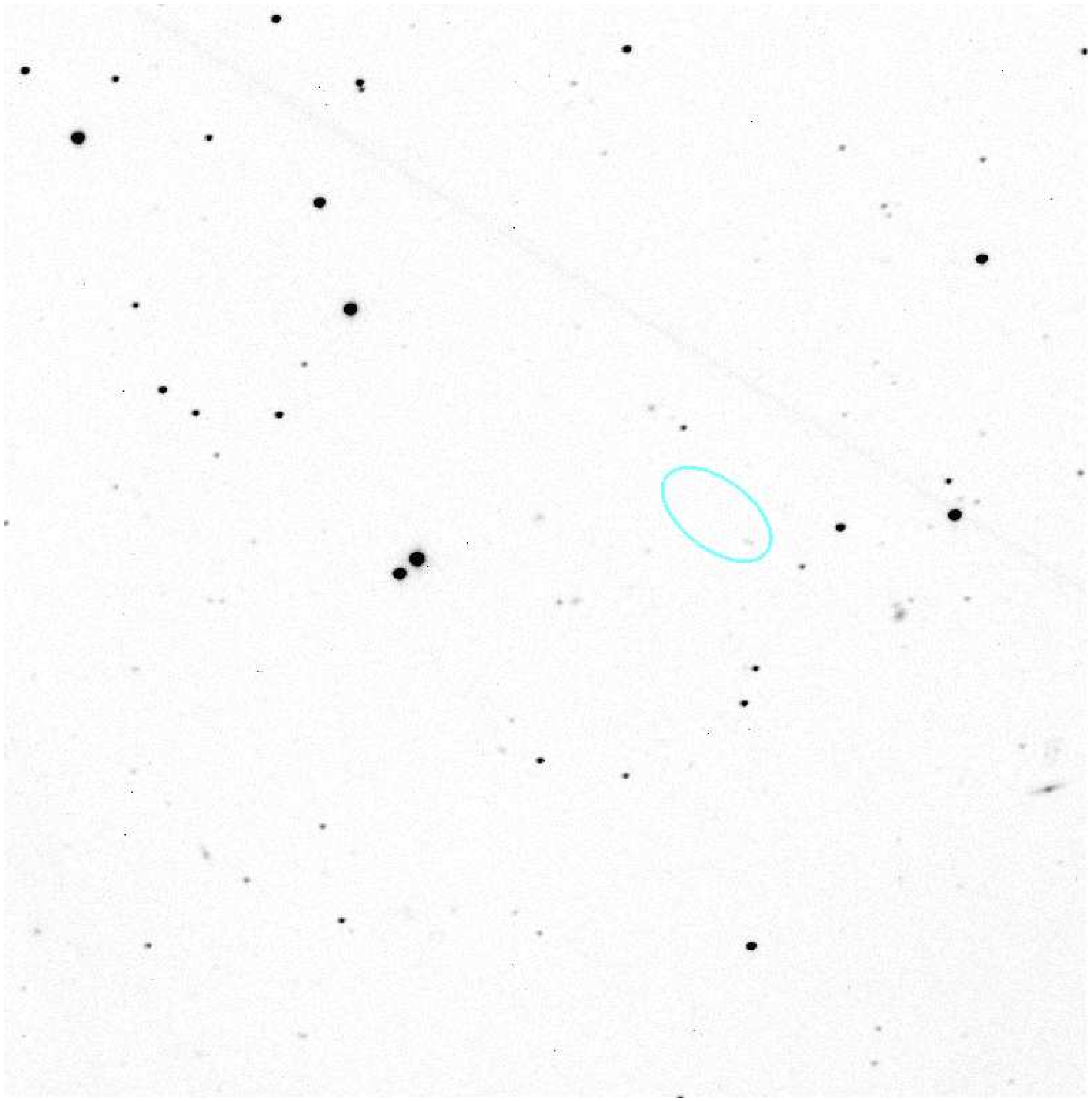

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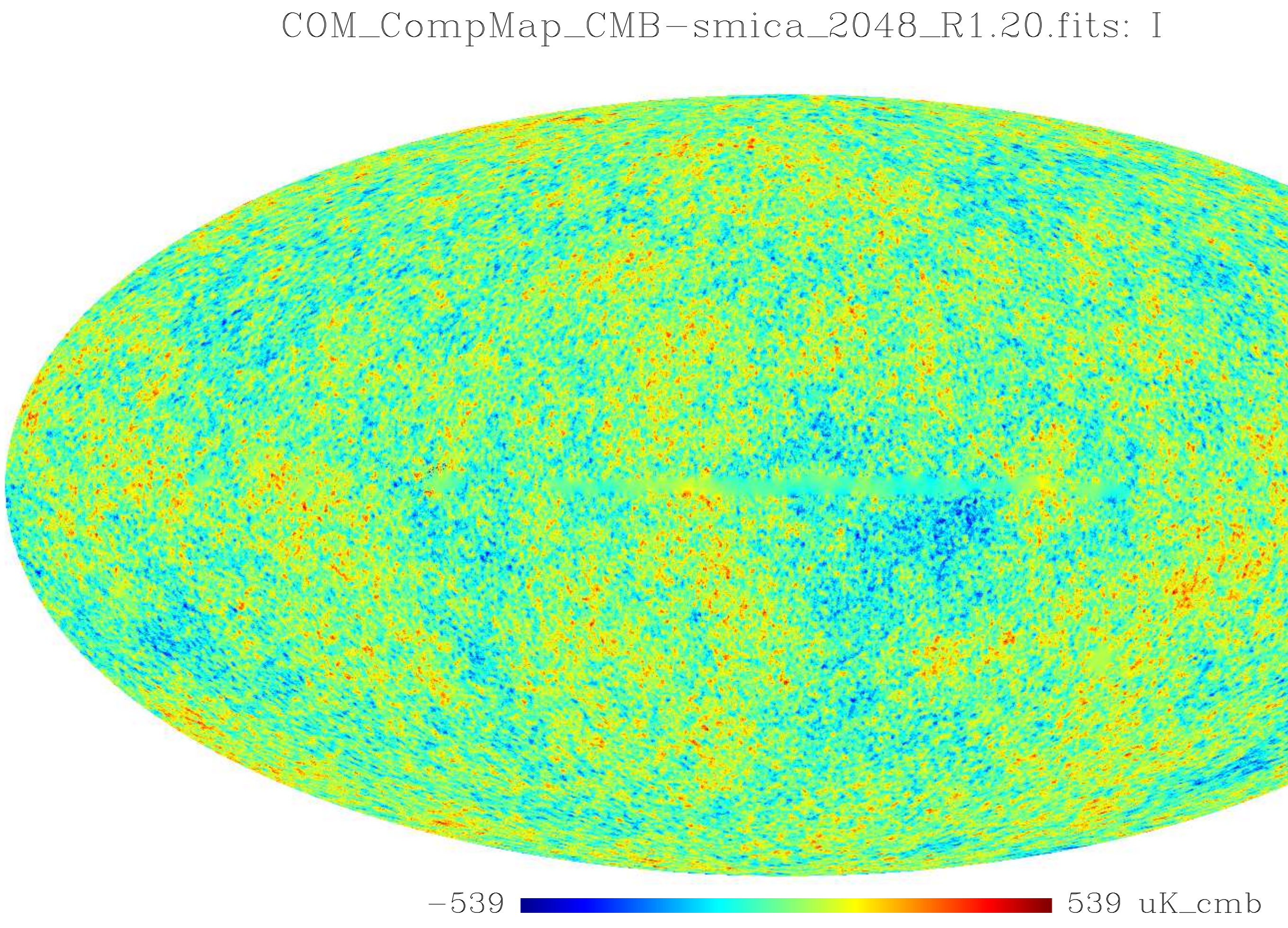
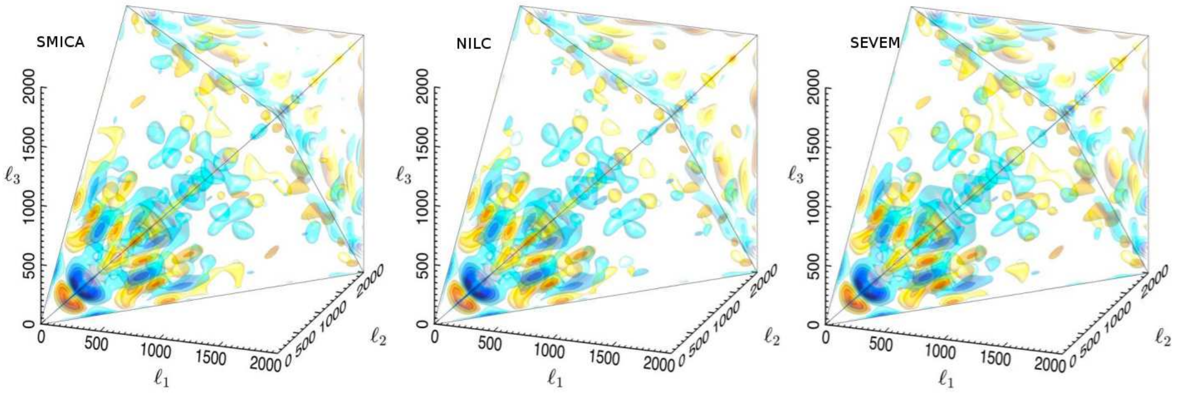
Typical Planck image

- CMB map with 50 000 000 pixels

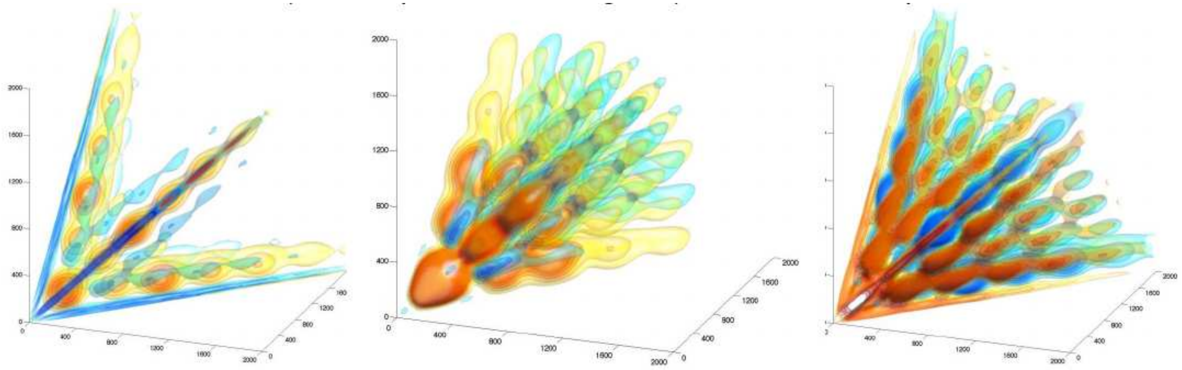


Testing for primordial non-Gaussianities

- Planck 2013 XXIV `arXiv:1003.5084`: CMB Bispectrum



- Primordial non-Gaussianities (local, equilateral, orthogonal)



- With $\langle \phi\phi\phi \rangle \simeq f_{\rm NL}\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)F(k_1,k_2,k_3)$

$$f_{\rm NL}^{\rm loc} = 2.7 \pm 5.8, \quad f_{\rm NL}^{\rm eq} = -42 \pm 75 \quad f_{\rm NL}^{\rm orth} = -25 \pm 39$$

Cosmic strings basics

- Line-like remnants of the early universe that should still be present
  - Solitons created during cosmological phase transitions $\mathcal{G} \to \mathcal{H}$ (unavoidable if $\pi_1(\mathcal{G}/\mathcal{H}) \neq 1$) [Kibble 76]
  - Cosmologically streched objects from String Theory [Witten 85]
  - Generically formed at the end of inflation [Sarangi 02; Jeannerot 03]
- Prototypical model: Nambu–Goto string networks: 1 parameter U
  - Numerical simulations shows that they relax towards a self-similar configuration = scaling (see movie)
  - Energy density of long strings and loops evolves as radiation/matter instead of $\rho \propto a^{-2}$

$$\rho_{\mathrm{inf}} \frac{d_{\mathrm{h}}^2}{U}\bigg|_{\mathrm{mat}} = 28.4 \pm 0.9, \qquad \rho_{\mathrm{inf}} \frac{d_{\mathrm{h}}^2}{U}\bigg|_{\mathrm{rad}} = 37.8 \pm 1.7.$$

Nambu–Goto strings dynamics

- Two-dimensional worldsheet surface located at $x^\mu = X^\mu(\xi^a)$.
- Lorentz invariance along the string ($\tau \equiv \xi^0 \; \sigma \equiv \xi^1$)

$$S = -U \int \mathrm{d}\tau \mathrm{d}\sigma \sqrt{-\gamma}, \quad \gamma_{ab} = g_{\mu\nu} X^\mu_{,a} X^\nu_{,b} \text{ (induced metric)}$$

- String motion in FLRW (TT gauge: $X^0 = \tau$, $\dot{\boldsymbol{X}} \cdot \acute{\boldsymbol{X}} = 0$)

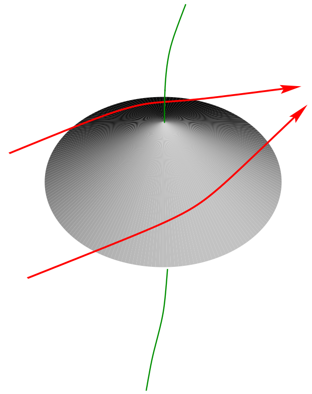
$$\ddot{\boldsymbol{X}} + 2\mathcal{H}\left(1 - \dot{\boldsymbol{X}}^2\right) - \frac{1}{\varepsilon}\left(\frac{\acute{\boldsymbol{X}}}{\varepsilon}\right)' = 0, \quad \dot{\varepsilon} + 2\mathcal{H}\varepsilon\dot{\boldsymbol{X}}^2 = 0, \quad \varepsilon = \sqrt{\frac{\acute{\boldsymbol{X}}^2}{1 - \dot{\boldsymbol{X}}^2}}$$

- Hubble damped propagation of left- right-moving waves

$$\mathcal{H} = 0 \quad \Rightarrow \quad \acute{\boldsymbol{X}}(\tau, \sigma) = \frac{1}{2}\left[\vec{p}(\sigma + \tau) + \vec{q}(\sigma - \tau)\right]$$

Induce non-Gaussian CMB distorsions

- vacuum tubes $\Rightarrow$ no static gravitational effects $(T = U)$
- Do have General Relativity effects on light and thus on CMB! (Gott-Kaiser-Stebbins)



- ISW from Nambu–Goto stress tensor + Einstein equations: [Hindmarsh 94, Stebbins 95]

$$\Theta(\hat{\boldsymbol{n}}) \equiv \frac{\delta T}{T_{\mathrm{CMB}}} = -4GU \int_{\boldsymbol{X}\,\cap\,\boldsymbol{x}_\gamma} \left[\boldsymbol{u}(\hat{\boldsymbol{n}}) \cdot \frac{\boldsymbol{X}_\perp}{\boldsymbol{X}_\perp^2}\right] \left(1 + \hat{\boldsymbol{n}} \cdot \dot{\boldsymbol{X}}\right) \mathrm{d}\sigma$$

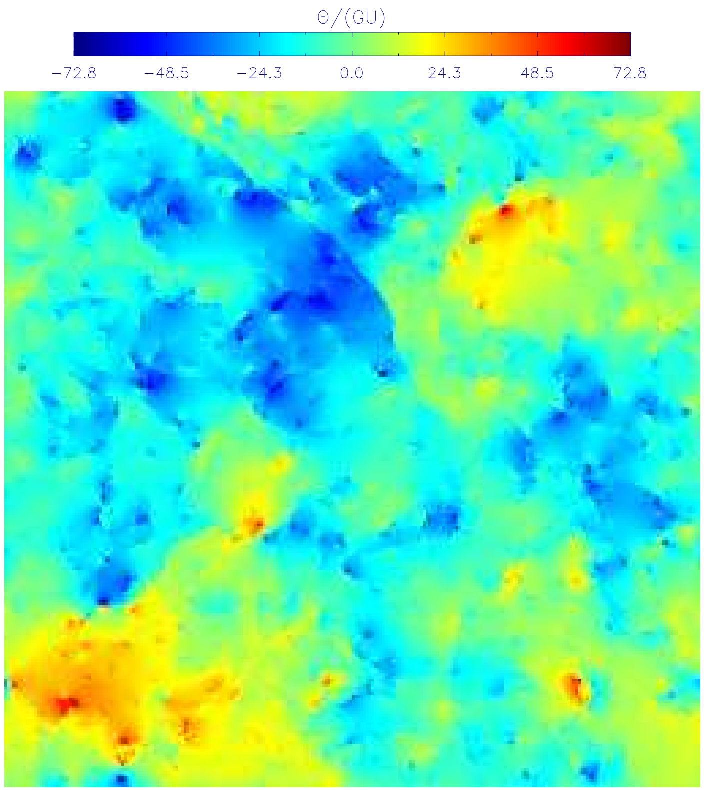
$$\boldsymbol{u} = \dot{\boldsymbol{X}} - \frac{(\hat{\boldsymbol{n}} \cdot \boldsymbol{X}') \cdot \boldsymbol{X}'}{1 + \hat{\boldsymbol{n}} \cdot \dot{\boldsymbol{X}}} \qquad \boldsymbol{X}_\perp \equiv X\hat{\boldsymbol{n}} - \boldsymbol{X}$$

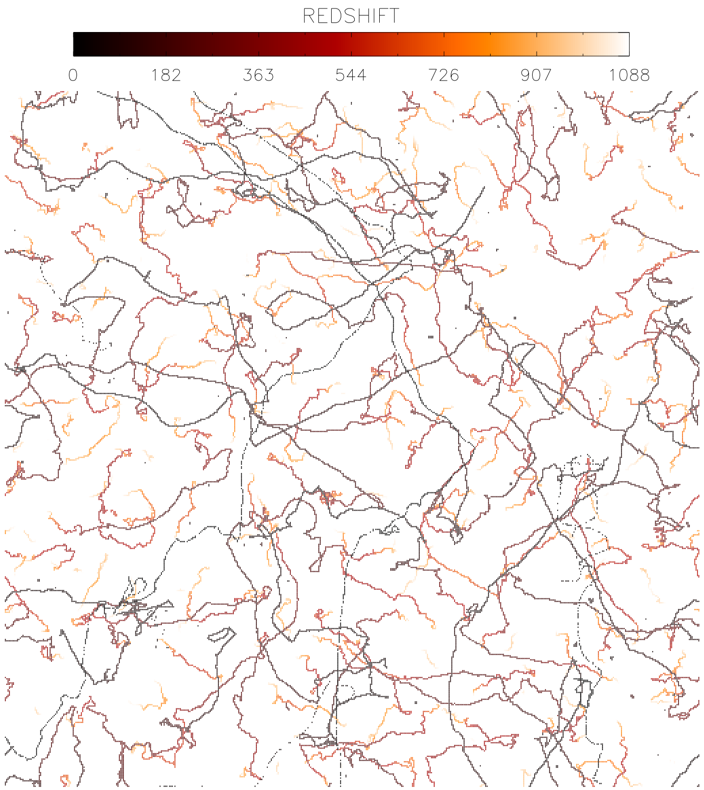
Small angles and flat sky limit

- At small angular scales, in 2D transverse Fourier space ($\boldsymbol{k} \cdot \hat{\boldsymbol{n}} \simeq 0$):

$$\Theta \simeq \frac{8\pi i\, GU}{\boldsymbol{k}^2} \int_{\boldsymbol{X}\,\cap\,\boldsymbol{x}_\gamma} (\boldsymbol{u} \cdot \boldsymbol{k})\, e^{-i\,\boldsymbol{k}\cdot\boldsymbol{X}}\, \mathrm{d}\sigma$$

- Flat sky simulation over 7.2° [Fraisse, CR, Spergel, Bouchet 07]



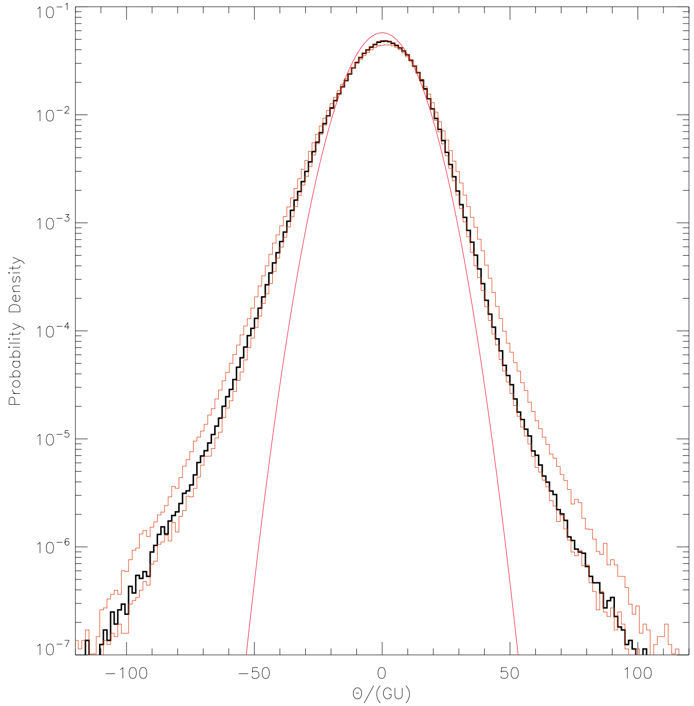


Real space non-Gaussian signals

- One-point functions

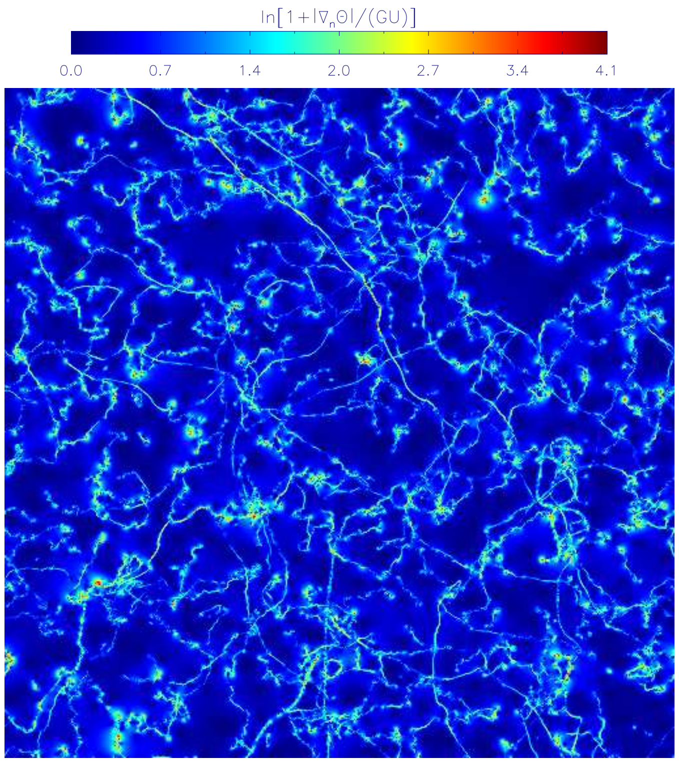
$$g_1 \equiv \left\langle \frac{\overline{(\Theta - \bar{\Theta})^3}}{\sigma^3} \right\rangle \simeq -0.22 \pm 0.12$$

$$g_2 \equiv \left\langle \frac{\overline{(\Theta - \bar{\Theta})^4}}{\sigma^4} \right\rangle - 3 \simeq 0.69 \pm 0.29.$$



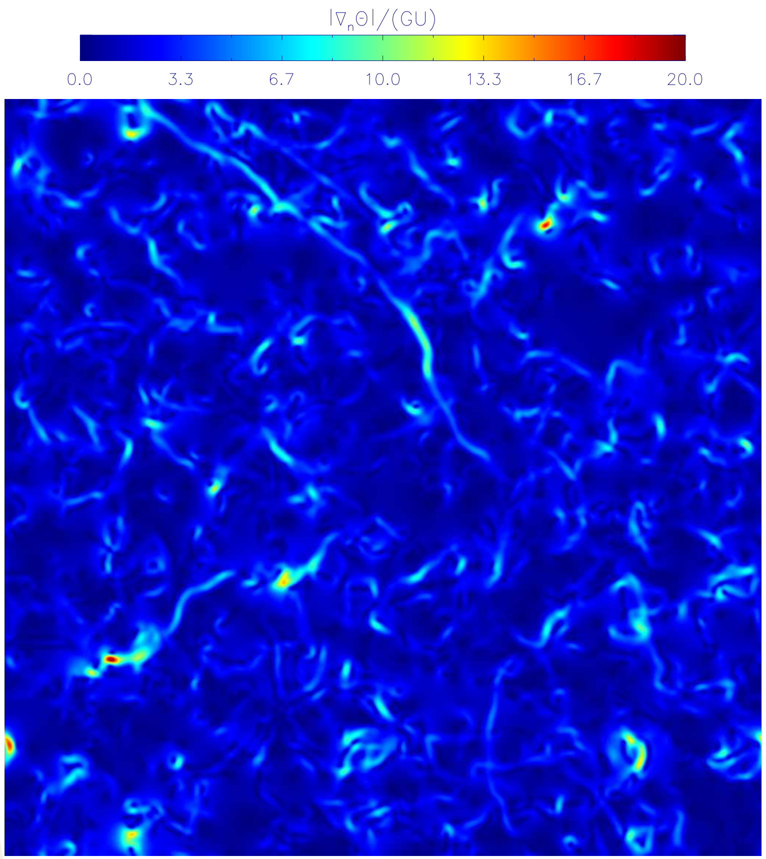
- Gradient magnitude

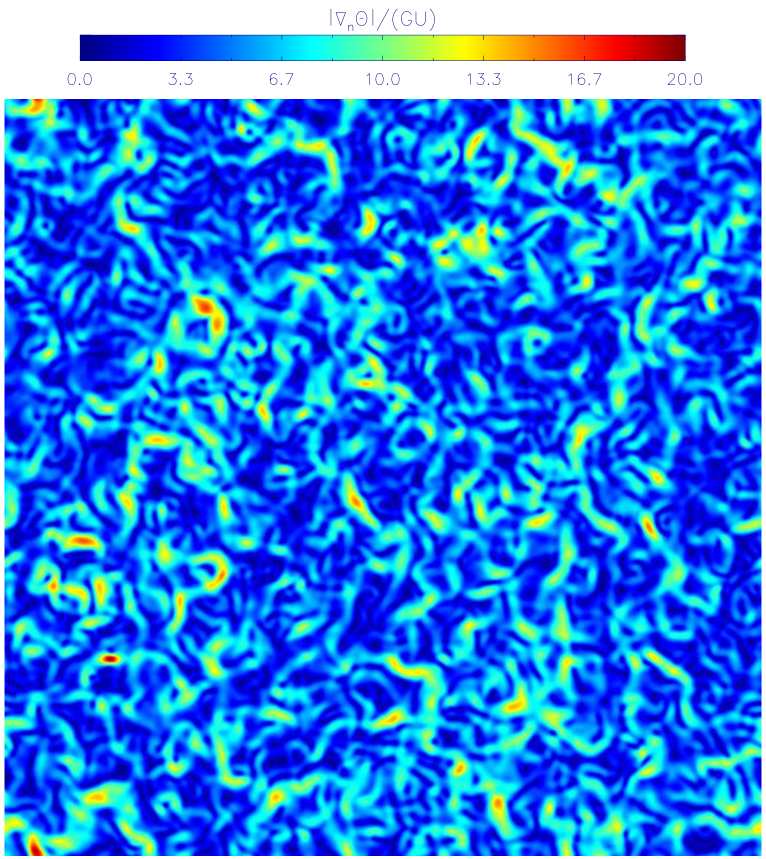
$$|\nabla\Theta| \equiv \sqrt{\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\alpha}\right)^2 + \left(\frac{\mathrm{d}\Theta}{\mathrm{d}\beta}\right)^2}$$



Are these features detectable with PLANCK?

- Experimental beam damps the signal: $5'$ Gaussian beam
  - Hidden in secondary anisotropies $\Rightarrow$ depends on GU
  - Gradient magnitude is sensitive to all: inf + SZ + stgs (7×10^{-7})





Analytical CMB bispectrum at small scales

- Bispectrum in the small angle limit: $\kappa_{ab} \equiv \boldsymbol{k}_a \cdot \boldsymbol{k}_b \gg 1$ (l.c. gauge)

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = i\epsilon^3 \frac{1}{\mathcal{A}} \frac{k_{1A} k_{2B} k_{3C}}{k_1^2 k_2^2 k_3^2} \int \mathrm{d}\sigma_1 \mathrm{d}\sigma_2 \mathrm{d}\sigma_3 \left\langle \dot{X}_1^A \dot{X}_2^B \dot{X}_3^C e^{i\delta^{ab} \boldsymbol{k}_a \cdot \boldsymbol{X}_b} \right\rangle$$

with $\dot{X}_a^A = \dot{X}^A(\sigma_a)$, $a, b \in \{1, 2, 3\}$, $\epsilon = 8\pi G U$

- Assuming $\dot{\boldsymbol{X}}$ and $\acute{\boldsymbol{X}}$ are Gaussian random variables [Hindmarsh, CR, Suyama 09]

$$B(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = -\epsilon^3 \pi c_0 \frac{\bar{v}^2}{\bar{t}^4} \frac{L\hat{\xi}}{\mathcal{A}} \frac{1}{\hat{\xi}^2} \frac{1}{k_1^2 k_2^2 k_3^2} \left[\frac{k_1^4 \kappa_{23} + k_2^4 \kappa_{31} + k_3^4 \kappa_{12}}{(\kappa_{23}\kappa_{31} + \kappa_{12}\kappa_{31} + \kappa_{12}\kappa_{23})^{3/2}} \right]$$

- Leading order measures the projected string statistics

$$\Gamma(\sigma_{ab}) \equiv \left\langle [\boldsymbol{X}(\sigma_a) - \boldsymbol{X}(\sigma_b)]^2 \right\rangle \sim \bar{t}^2 \sigma_{ab}^2$$

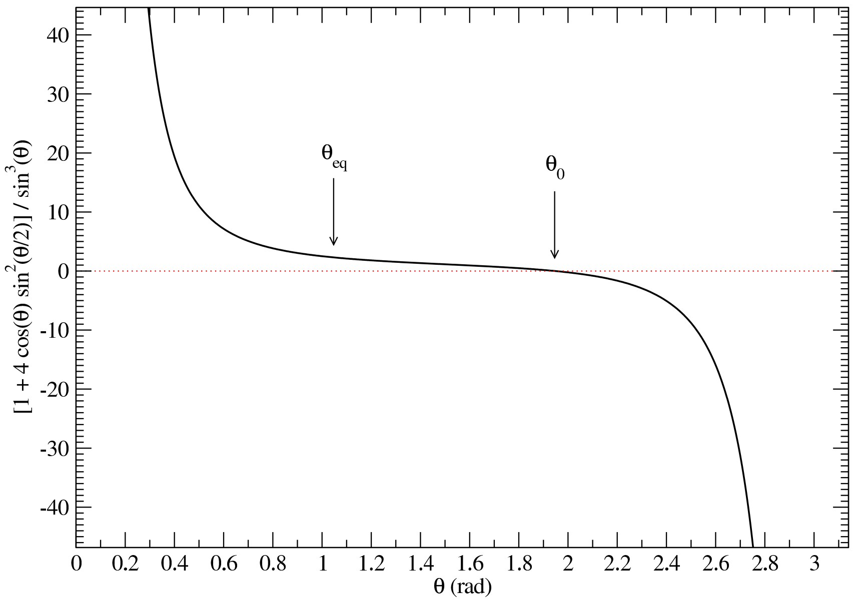
$$\Pi(\sigma_{ab}) \equiv \left\langle [\boldsymbol{X}(\sigma_a) - \boldsymbol{X}(\sigma_b))] \cdot \dot{\boldsymbol{X}}(\sigma_b) \right\rangle \sim \frac{1}{2} \frac{c_0}{\hat{\xi}} \sigma_{ab}^2$$

Example: isoscele triangle configurations

- Wavenumbers such that $k_1 = k_2 = k$ and $k_3 = 2k\sin(\theta/2)$

$$B_{\ell\ell\theta}(k,\theta) = -\epsilon^3 \pi c_0 \frac{\bar{v}^2}{\bar{t}^4} \frac{L\hat{\xi}}{\mathcal{A}} \frac{1}{\hat{\xi}^2 k^6} \frac{1 + 4\cos\theta \sin^2(\theta/2)}{\sin^3\theta}$$

- Amplified on elongated triangles; $\pm$ at $\theta_0 = 2\arccos\dfrac{\sqrt{3-\sqrt{3}}}{2}$



Analytical CMB trispectrum at small scales

- Geometrical factor scales as $k^{-\rho}$: $\rho = 6 + 1/(1+\chi)$
- Sensitive to higher order terms in the correlators ($0 < \chi < 1$, $c_1 > 0$)

$$\text{Polchinski–Rocha model} \Rightarrow \delta_{AB} \left\langle \acute{X}^A_a \acute{X}^B_b \right\rangle \simeq \bar{t}^2 - c_1 \left(\frac{\sigma_{ab}}{\hat{\xi}} \right)^{2\chi}$$

- Window onto the string microstructure!
  - ✦ NG: power-law exponent of the loop distribution
  - ✦ Other strings: related to the mean square velocity
- In details [Hindmarsh, CR, Suyama 09; Regan, Shellard 09]

$$T_\infty(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \simeq \epsilon^4 \frac{\bar{v}^4}{\bar{t}^2} \frac{L\hat{\xi}}{\mathcal{A}} \left(c_1 \hat{\xi}^2\right)^{-1/(2\chi+2)} f(\chi) g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$

$$f(\chi) = \frac{\pi}{\chi+1} \Gamma\left(\frac{1}{2\chi+2}\right) \left[4(2\chi+1)(\chi+1)\right]^{1/(2\chi+2)}$$

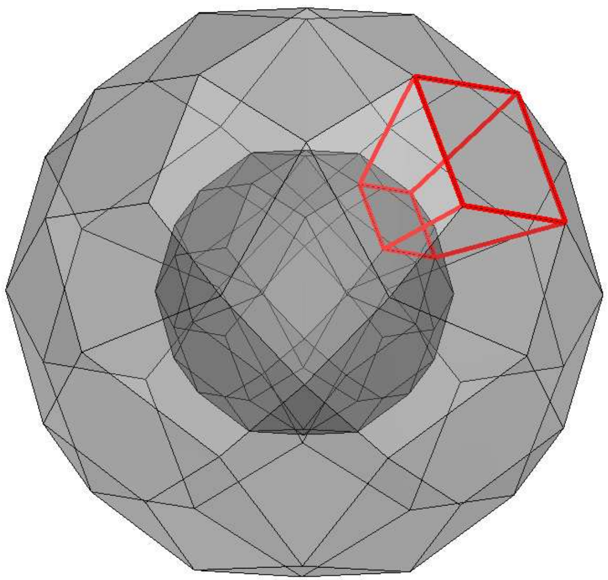
$$g(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \frac{\kappa_{12}\kappa_{34} + \kappa_{13}\kappa_{24} + \kappa_{14}\kappa_{23}}{k_1^2 k_2^2 k_3^2 k_4^2} \left[Y^2\right]^{-1/(2\chi+2)}$$

$$Y^2(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \equiv -\kappa_{12} \left(k_3^2 k_4^2 - \kappa_{34}^2\right)^{\chi+1} + \circlearrowleft,$$

But PLANCK is a full sky experiment...

Filling the transparent universe with strings

- Searching for string NG with Planck requires full sky $\Rightarrow$ simulations
  - ✦ Each simulation is a box of initial resolution 2000^3 (movie box)
  - ✦ Have to be stacked to fill 13 billion light years (HEALpix)



- This can be done with 3072 CS runs
- In which we propagate the CMB. . .

Massively parallel ray tracing method

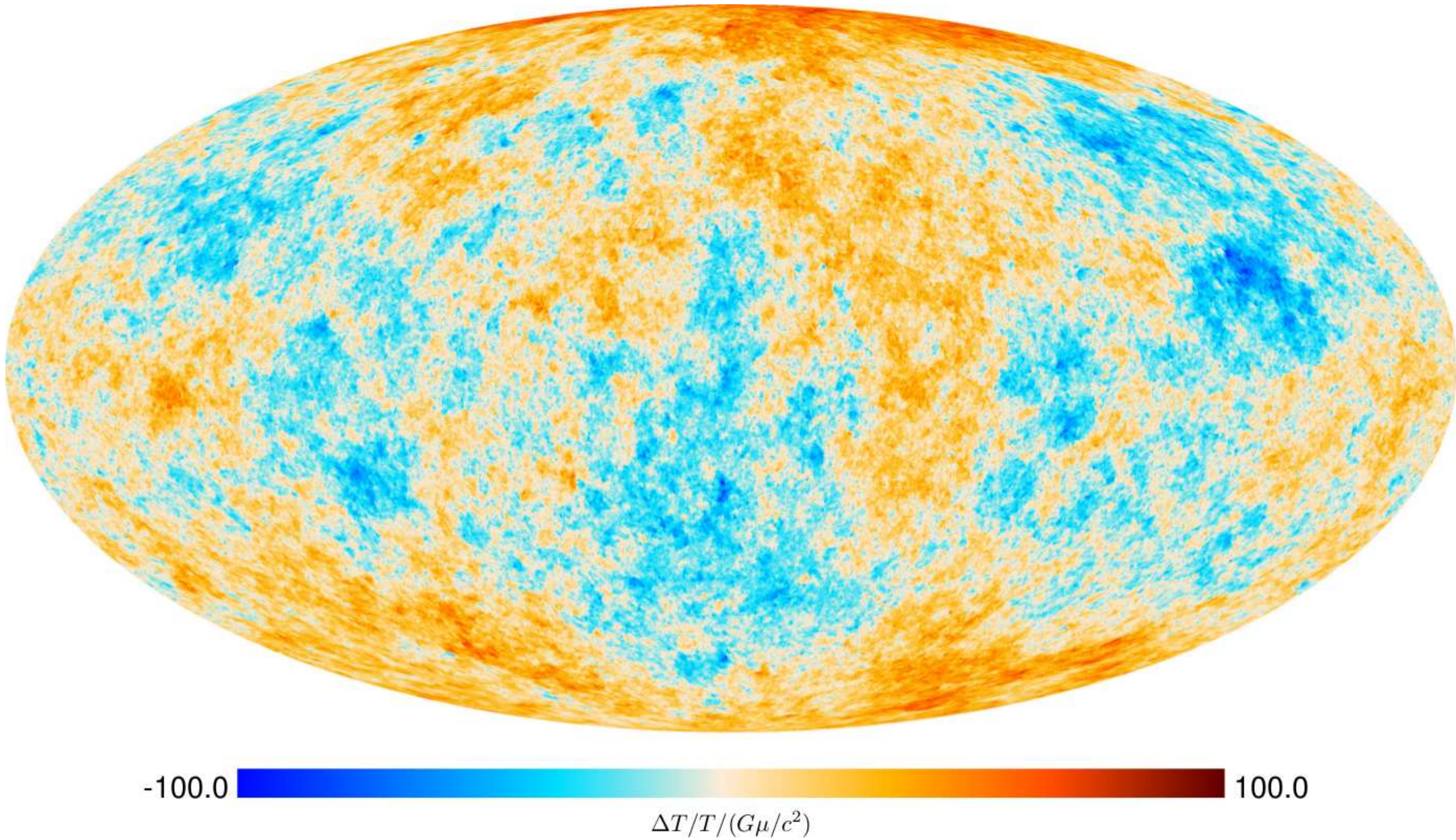
- Sky pixelized with $200\,000\,000$ lines of sight (4 times Planck maps)
  - Each direction receives cumulative contributions from all CS
  - Account for roughly 10^{17} iterations
- Parallelization implementation
  - `MPI` over the 3072 boxes + reduction
  - `OpenMP` over the $200\,000\,000$ pixels
  - `Vectorization` of the most inner loop (string segments)
- Code development performed on the `CP3-cosmo` cluster (100 cores)
- Reasonable computing time demands a 100 TeraFlops computer :-/
  - The Planck collaboration has a few. . . (thanks to J. Borrill) :)

512 nodes / 12K cores runs at NERSC

- National Energy Research Scientific Computing Center (Berkeley U.S.)
- The "`Hopper`" Cray XE6 machine (world rank 8 in Nov 2011)
  - ✦ More than 6000 nodes with Dual processor 24 cores
  - ✦ 3D Cray Gemini: Maximum injection bandwidth per node 20 GB/s

After a million of cpu-hours

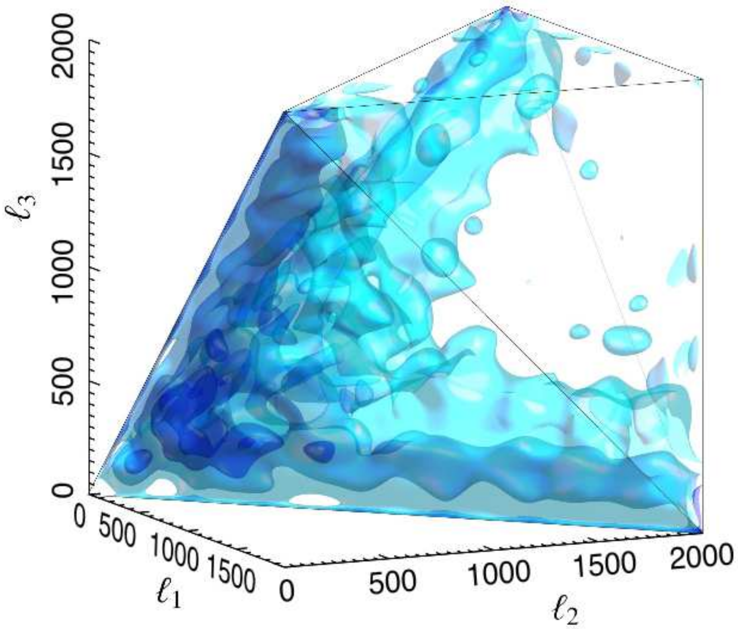
- Full sky synthetic string map of 2×10^8 pixels



- $\times 4$ for tests and string challenges

Non-Gaussian searches for cosmic strings

- Full sky bispectrum



✦ Different methods
- Modal bispectrum
- Wavelets
- Minkowski functionals

- Planck constraints on cosmic strings non-Gaussianities

$$f_{\mathrm{NL}}^{\mathrm{strg}} = 0.30 \pm 0.21 \Rightarrow GU < 8.8 \times 10^{-7}$$

$$\text{Real space} \Rightarrow GU < 7.8 \times 10^{-7}$$

- Very robust (ISW only) but slightly weaker than power spectrum bounds $GU < 1.3 \times 10^{-7} \to 3.2 \times 10^{-7}$

Conclusion

- no strings non-Gaussianities $\Rightarrow GU < 7.8 \times 10^{-7}$ (not easy to get)
- Possible improvements
  - Finding string induced non-Gaussianities? $\Rightarrow$ window on their nature (trispectrum)
  - Going further than the ISW contribution
  - Next Planck data release + polarization + small scales experiments (BB [Seljak 06])
- Other observables than CMB: signal $\propto (GU)^{2,3,4}$
  - Galaxy surveys
  - 21 cm

Observable string correlators

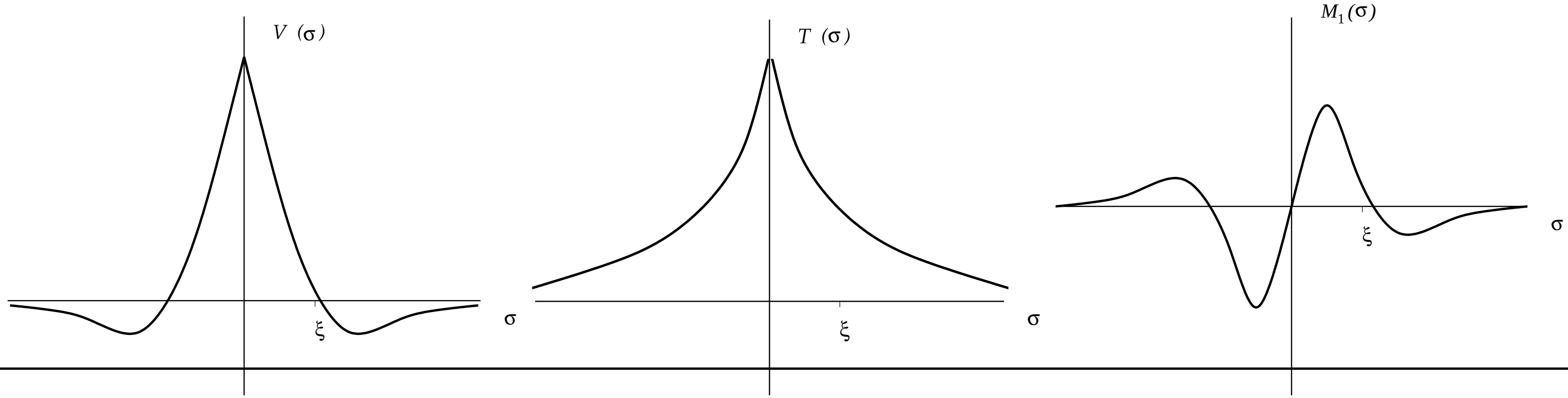
- Wicks theorem ⇒ bispectrum depends on

$$\Gamma(\sigma-\sigma') = \int_{\sigma'}^{\sigma} \mathrm{d}\sigma_1 \int_{\sigma'}^{\sigma} \mathrm{d}\sigma_2 T(\sigma_1-\sigma_2), \quad \Pi(\sigma-\sigma') = \int_{\sigma'}^{\sigma} \mathrm{d}\sigma_1 M(\sigma_1-\sigma')$$

$$\left\langle \dot{X}_a^A \dot{X}_b^B \right\rangle = \frac{\delta^{AB}}{2} V(\sigma_{ab}), \left\langle \dot{X}_a^A \acute{X}_b^B \right\rangle = \frac{\delta^{AB}}{2} M(\sigma_{ab}), \left\langle \acute{X}_a^A \acute{X}_b^B \right\rangle = \frac{\delta^{AB}}{2} T(\sigma_{ab})$$

- Sensitive to the (averaged projected) small scales $\sigma \to 0$:

$$V(\sigma) \sim \bar{v}^2, \qquad \Gamma(\sigma) \sim \bar{t}^2\sigma^2, \qquad \Pi(\sigma) \sim \frac{1}{2}\frac{c_0}{\hat{\xi}}\sigma^2 \quad [\hat{\xi} \equiv \Gamma'(\infty)]$$



String bispectrum and cosmic expansion

- Proportional to $c_0 \equiv \hat{\xi} \left\langle \acute{\acute{\boldsymbol{X}}} \cdot \dot{\boldsymbol{X}} \right\rangle \neq 0$?
  - Light cone gauge + FLRW + $\dot{\boldsymbol{X}}$, $\acute{\boldsymbol{X}}$ Gaussian random variables

$$\left\langle \acute{\acute{\boldsymbol{X}}} \cdot \dot{\boldsymbol{X}} \right\rangle = \bar{\mathcal{H}} \left(\left\langle \dot{\boldsymbol{X}}^2 \right\rangle \left\langle \acute{\boldsymbol{X}}^2 \right\rangle - \left\langle \dot{\boldsymbol{X}} \cdot \acute{\boldsymbol{X}} \right\rangle^2 \right) = \bar{\mathcal{H}} \bar{v}^2 \bar{t}^2$$

  - For $\bar{\mathcal{H}} > 0 \Rightarrow c_0 > 0$: breaking of time reversal invariance

- String bispectrum exists only in an expanding universe
  - Gives a negative skewness by integration
  - This is the CMB temperature bispectrum (what you see!)
    - As opposed to primordial (f_{NL})

Tested against simulated maps

- Estimator [Spergel 99; Aghanim 03; Komatsu 03]: $\Theta_u(\boldsymbol{x}) \equiv \int \frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^2} \hat{\Theta}_{\boldsymbol{k}} W_u(k)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$

$$B_{k_1 k_2 k_3} = \frac{\left\langle \int \Theta_{k_1}(\boldsymbol{x}) \Theta_{k_2}(\boldsymbol{x}) \Theta_{k_3}(\boldsymbol{x}) \mathrm{d}\boldsymbol{x} \right\rangle}{\int \frac{\mathrm{d}\boldsymbol{p}\mathrm{d}\boldsymbol{q}}{(2\pi)^4} W_{k_1}(p)\, W_{k_2}(q)\, W_{k_3}(|\boldsymbol{p}+\boldsymbol{q}|)}$$

- Power-law and dependency in θ recovered