



# Non-Gaussianity with Planck :

$f_{\text{NL}}$  and the smoothed bispectrum using the binned  
bispectrum estimator

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On behalf of the Planck collaboration

August 1st 2013

New Light in Cosmology  
from the CMB, Trieste

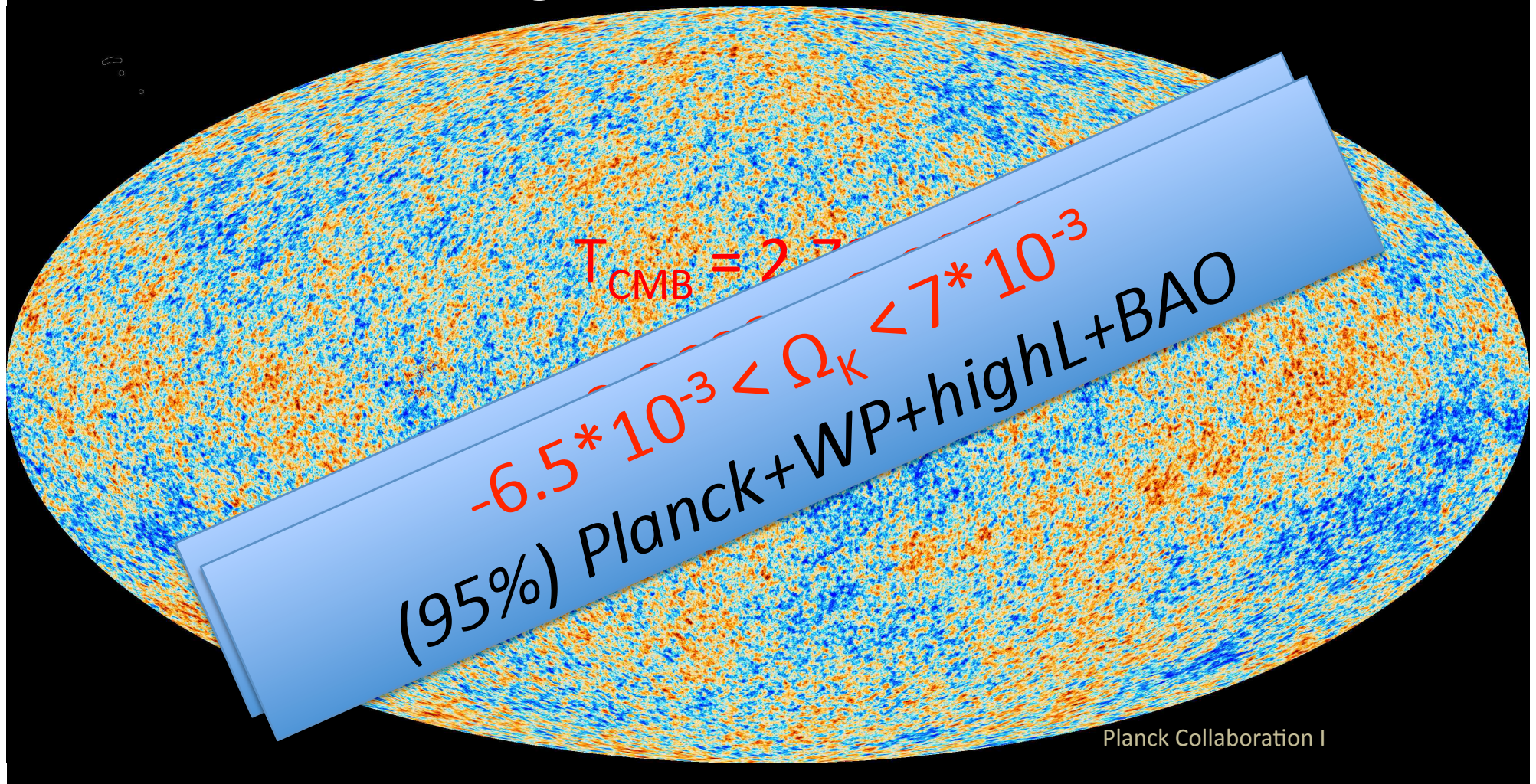
# Outline

- 1) Inflation
- 2) Non Gaussianity & Bispectrum
- 3) The binned bispectrum  $f_{\text{NL}}$  estimator
- 4) Results

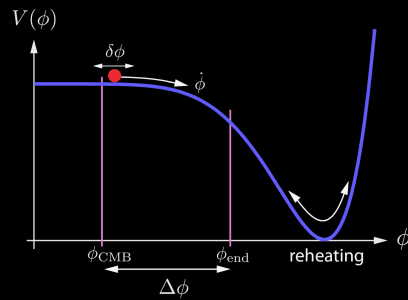
# 1. Inflation

# Results so far

- Almost homogeneous:







Cf. D.Bauman: 0907.5424

# “Simplest” Inflation

- ~~Single Field~~ **Local NG**
- Slow roll
- ~~Bunch Davies Vacuum~~ **Flattened NG**
- ~~Canonical Kinetic Term~~ **Equilateral NG**

Source of **nearly Gaussian** perturbations  $\phi(x) \Rightarrow$   
All statistical information is in **power spectrum**  $\langle \phi(k) \phi(k) \rangle$

## 2. Non Gaussianity & Bispectrum

# Observed bispectrum

3pt correlation function  $\leftarrow$  Fourier  $\rightarrow$  Bispectrum

$$B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} = \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle$$

Angular Bispectrum

$$B_{\ell_1 \ell_2 \ell_3} = \sqrt{\frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 1)}{4\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix} \sum_{m_1 m_2 m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3}$$

rotationally-invariant reduced Bispectrum

# Observed bispectrum estimator

3pt correlation function  $\leftarrow$  Fourier  $\rightarrow$  Bispectrum

$$\hat{B}_{\ell_1 \ell_2 \ell_3} = \int d\Omega T_{\ell_1}^{obs}(\Omega) T_{\ell_2}^{obs}(\Omega) T_{\ell_3}^{obs}(\Omega)$$

where

$$T_{\ell}^{obs}(\Omega) = \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\Omega)$$

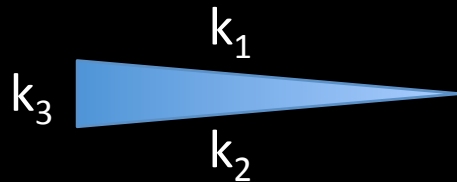


# Primordial bispectrum

$$\langle \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \phi(\mathbf{k}_3) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\phi(k_1, k_2, k_3)$$

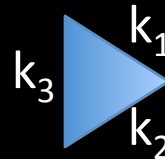
Primordial gravitational potential

$$B_\phi(k_1, k_2, k_3) = \sum_{i \in \text{shape}} f_{NL}^{(i)} F^{(i)}(k_1, k_2, k_3)$$

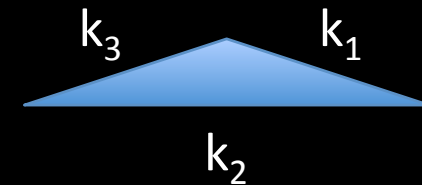


Squeezed / local: Multifields,  
curvaton, inhomogeneous  
reheating.

Late-time: ISW x lensing



Equilateral:  
non-standard kinetic  
terms



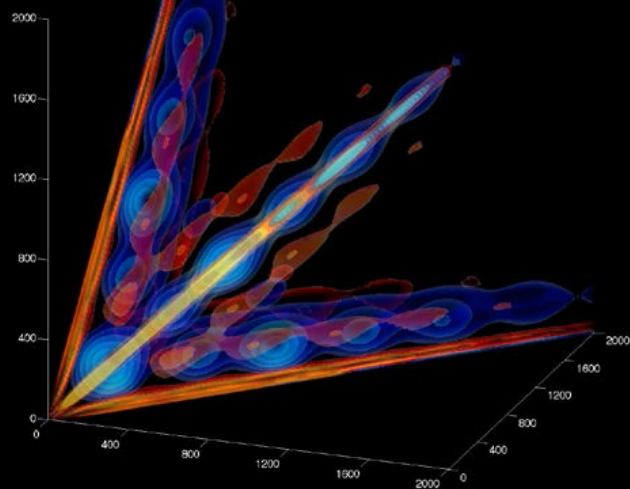
flattened:  
non Bunch-Davies  
vacuum

# Primordial bispectrum

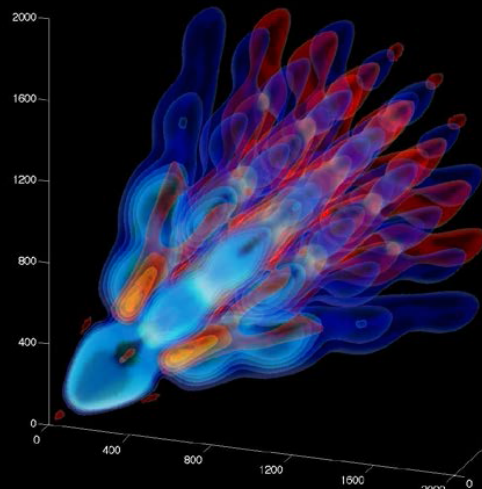
Transfer functions

$$B_{\ell_1 \ell_2 \ell_3} = \left(\frac{2}{\pi}\right)^3 \int_0^\infty x^2 dx \int dk_1 dk_2 dk_3 (k_1 k_2 k_3)^2 B_\phi(k_1, k_2, k_3) \Delta_{\ell_1}(k_1) \Delta_{\ell_2}(k_2) \Delta_{\ell_3}(k_3) \\ \times j_{\ell_1}(k_1 x) j_{\ell_2}(k_2 x) j_{\ell_3}(k_3 x) \int d\Omega Y_{\ell_1 m_1}(\Omega) Y_{\ell_2 m_2}(\Omega) Y_{\ell_3 m_3}(\Omega)$$

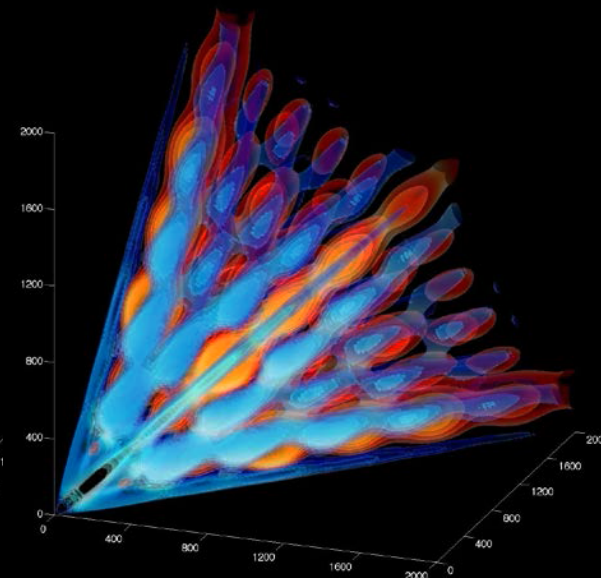
Temperature  
bispectrum



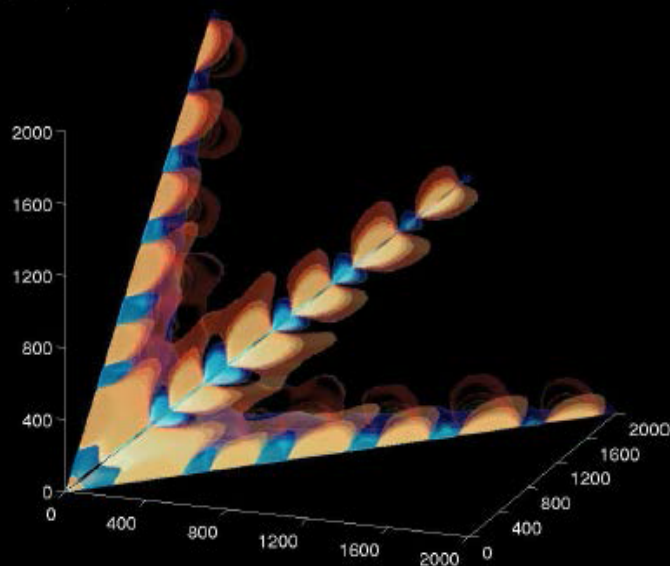
Local



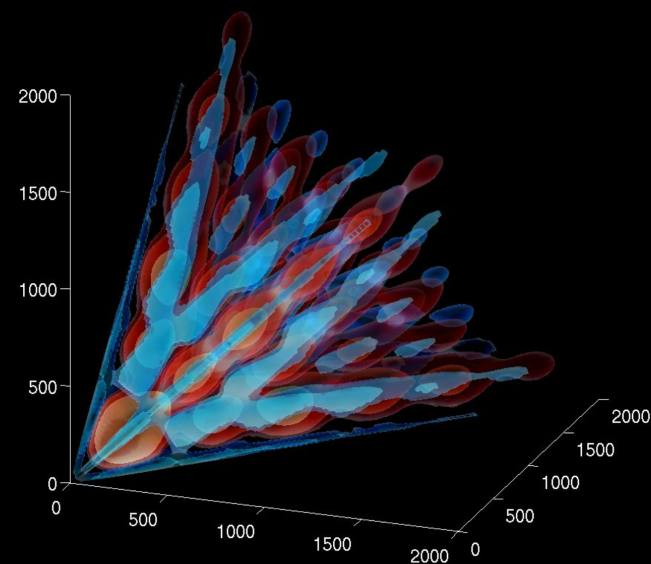
Equilateral



Orthogonal



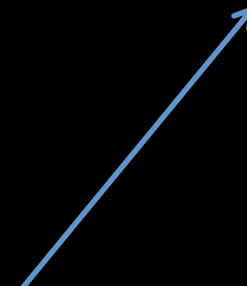
Lensing x ISW



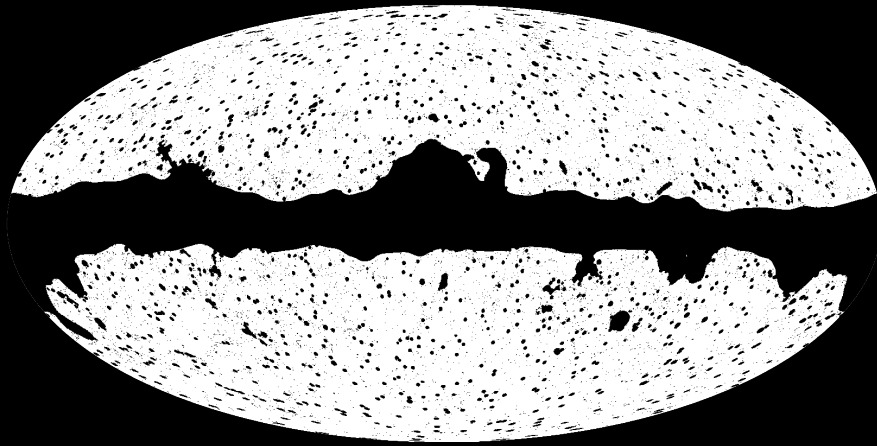
Unresolved point sources

$$f_{NL}$$

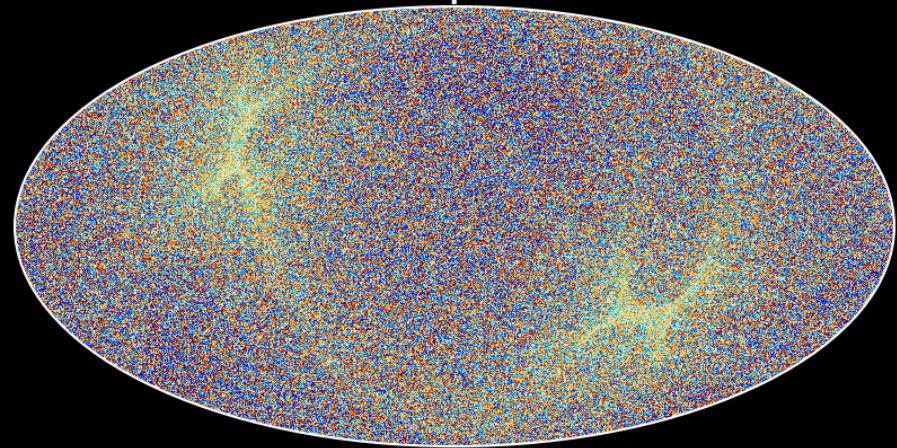
$$\hat{f}_{NL} = \frac{1}{N} \sum_{\ell_1 \ell_2 \ell_3 = \ell_{min}}^{\ell_{max}} \frac{B_{\ell_1 \ell_2 \ell_3}^{obs} B_{\ell_1 \ell_2 \ell_3}^{th} (f_{NL}=1)}{\text{Var}_{\ell_1 \ell_2 \ell_3}}$$

$$\text{Var}_{\ell_1 \ell_2 \ell_3} \simeq (b_{\ell_1}^2 C_{\ell_1} + N_{\ell_1})(b_{\ell_2}^2 C_{\ell_2} + N_{\ell_2})(b_{\ell_3}^2 C_{\ell_3} + N_{\ell_3})$$


# Anisotropic noise and mask



Anisotropic Noise



# Linear correction

$$B_{\ell_1 \ell_2 \ell_3}^{\text{lin}} = \sum_p \left[ T_{\ell_1}^{\text{obs}} \langle T_{\ell_2}^{\text{G}} T_{\ell_3}^{\text{G}} \rangle + T_{\ell_2}^{\text{obs}} \langle T_{\ell_1}^{\text{G}} T_{\ell_3}^{\text{G}} \rangle + T_{\ell_3}^{\text{obs}} \langle T_{\ell_1}^{\text{G}} T_{\ell_2}^{\text{G}} \rangle \right]$$


Averaged on gaussian maps with noise, beam and mask as in observed maps ( typically  $O(100)$  )

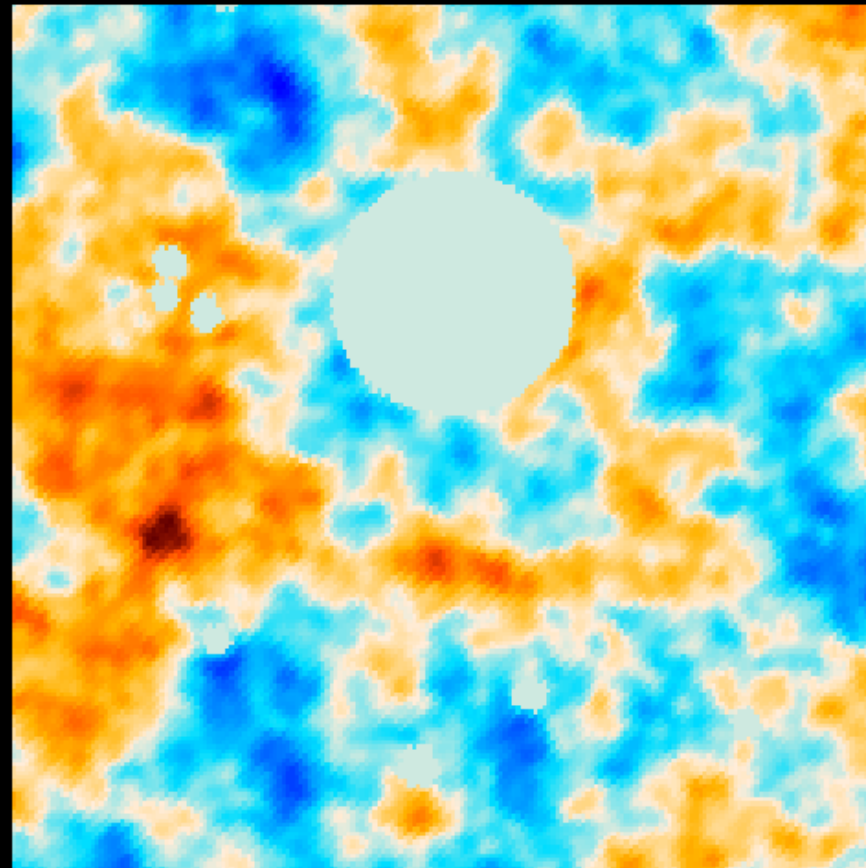
$$B_{\ell_1 \ell_2 \ell_3}^{\text{obs}} \rightarrow B_{\ell_1 \ell_2 \ell_3}^{\text{obs}} - B_{\ell_1 \ell_2 \ell_3}^{\text{lin}}$$

# Inpainting

0 iterations

Diffusive  
inpainting:  
iterative, average  
over 8  
neighboring  
pixels

1.5 ' /pix, 200x200 pix



(28,55)



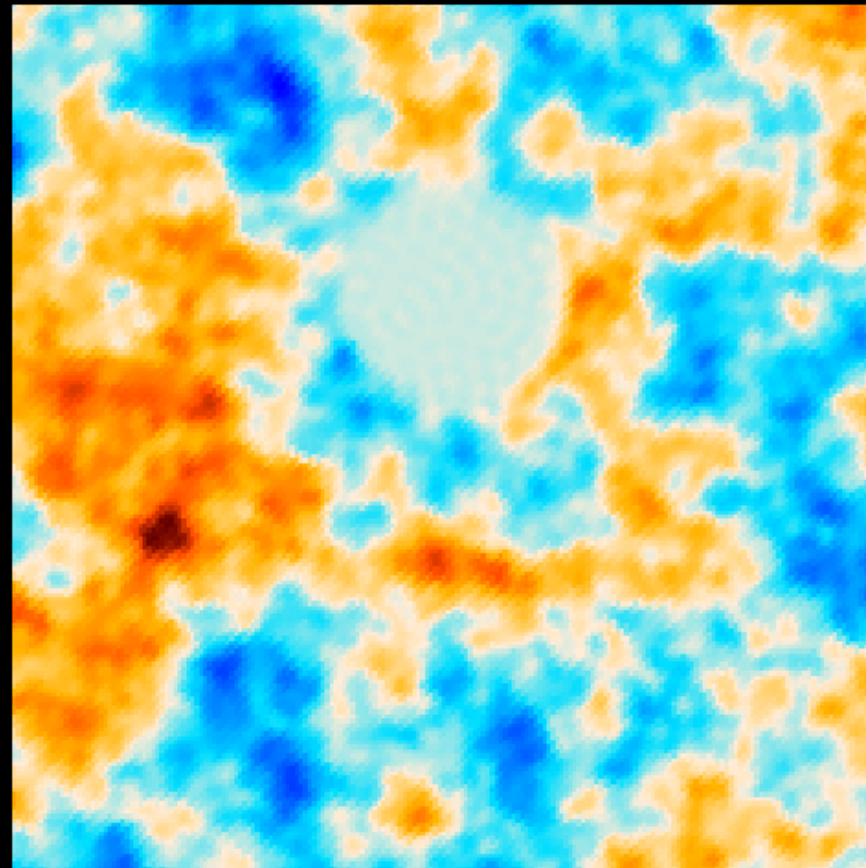


# Inpainting

5 iterations

Diffusive  
inpainting:  
iterative, average  
over 8  
neighboring  
pixels

1.5 '/pix, 200x200 pix



(28,55)

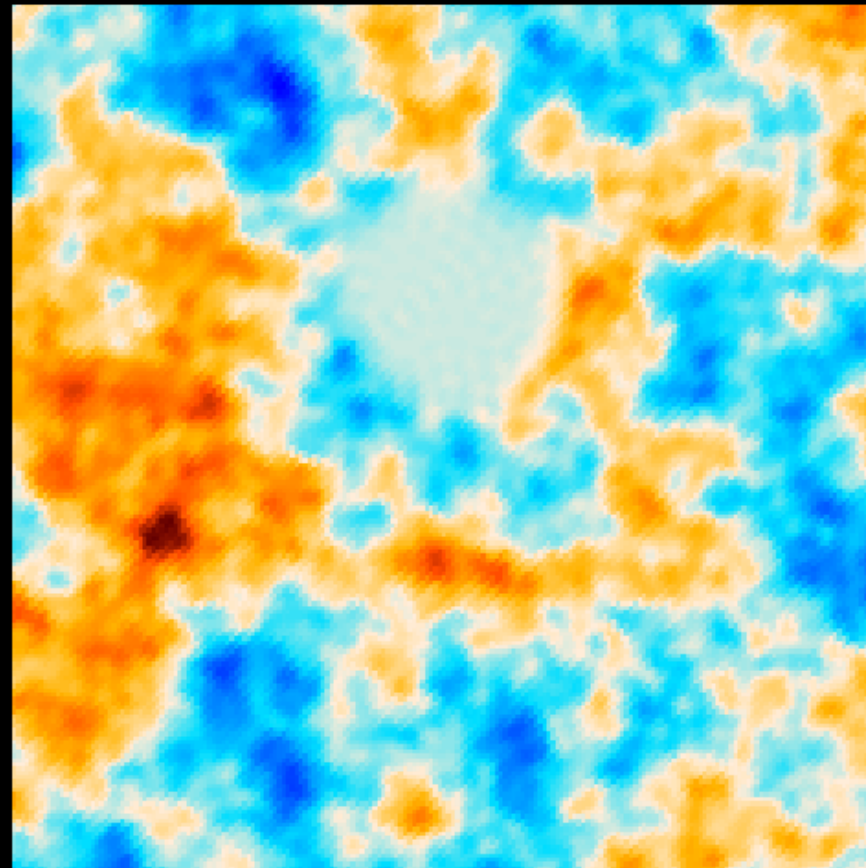


# Inpainting

10 iterations

Diffusive  
inpainting:  
iterative, average  
over 8  
neighboring  
pixels

1.5 '/pix, 200x200 pix



(28,55)

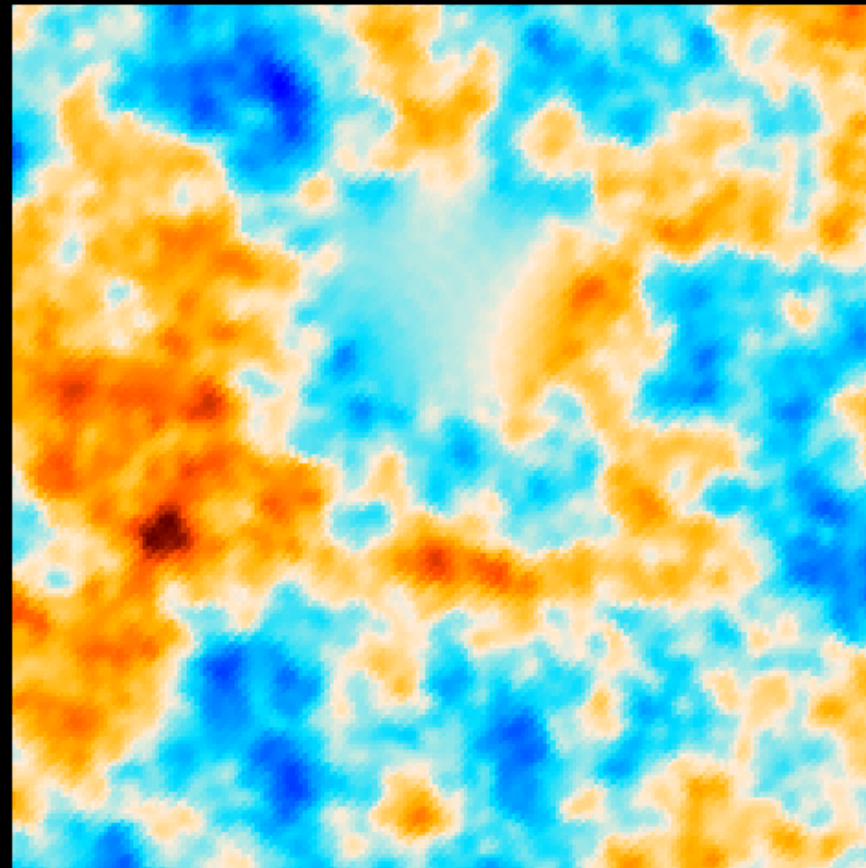


# Inpainting

100 iterations

Diffusive  
inpainting:  
iterative, average  
over 8  
neighboring  
pixels

1.5 '/pix, 200x200 pix



(28,55)

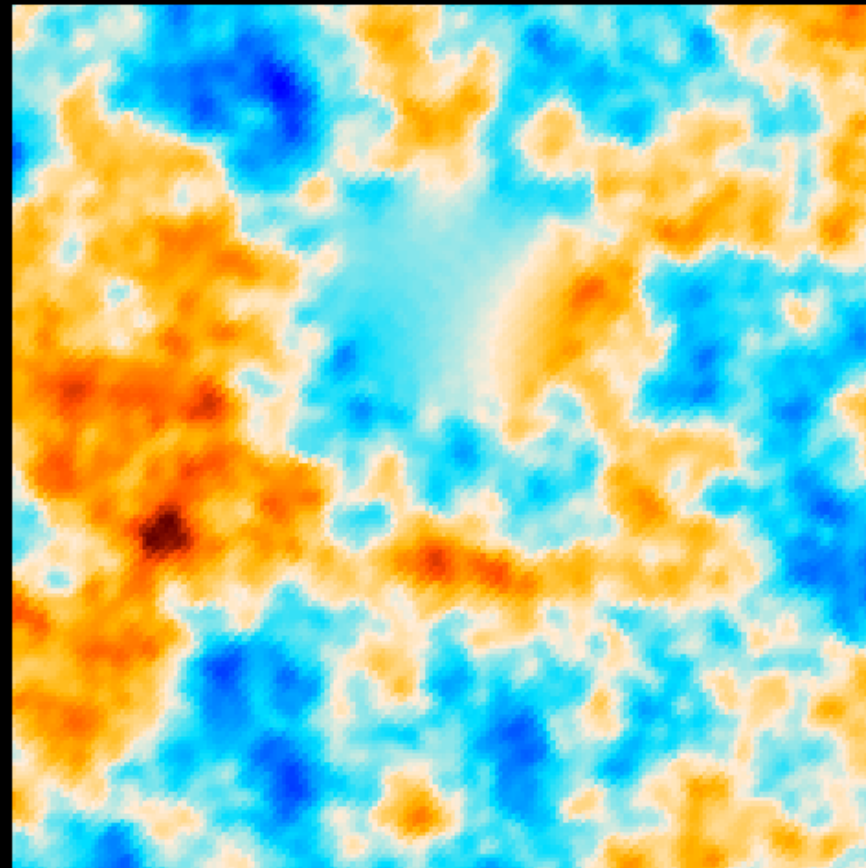


# Inpainting

2000 iterations

Diffusive  
inpainting:  
iterative, average  
over 8  
neighboring  
pixels

1.5 '/pix, 200x200 pix



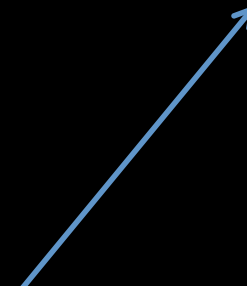
(28,55)



$$f_{NL}$$

Computationally impossible with current CPUs  
(2500\*200 Gb= 500 Tb)

$$\hat{f}_{NL} = \frac{1}{N} \sum_{\ell_1 \ell_2 \ell_3 = \ell_{min}}^{\ell_{max}} \frac{(B_{\ell_1 \ell_2 \ell_3}^{obs} - B_{\ell_1 \ell_2 \ell_3}^{lin}) B_{\ell_1 \ell_2 \ell_3}^{th(f_{NL}=1)}}{Var_{\ell_1 \ell_2 \ell_3}}$$

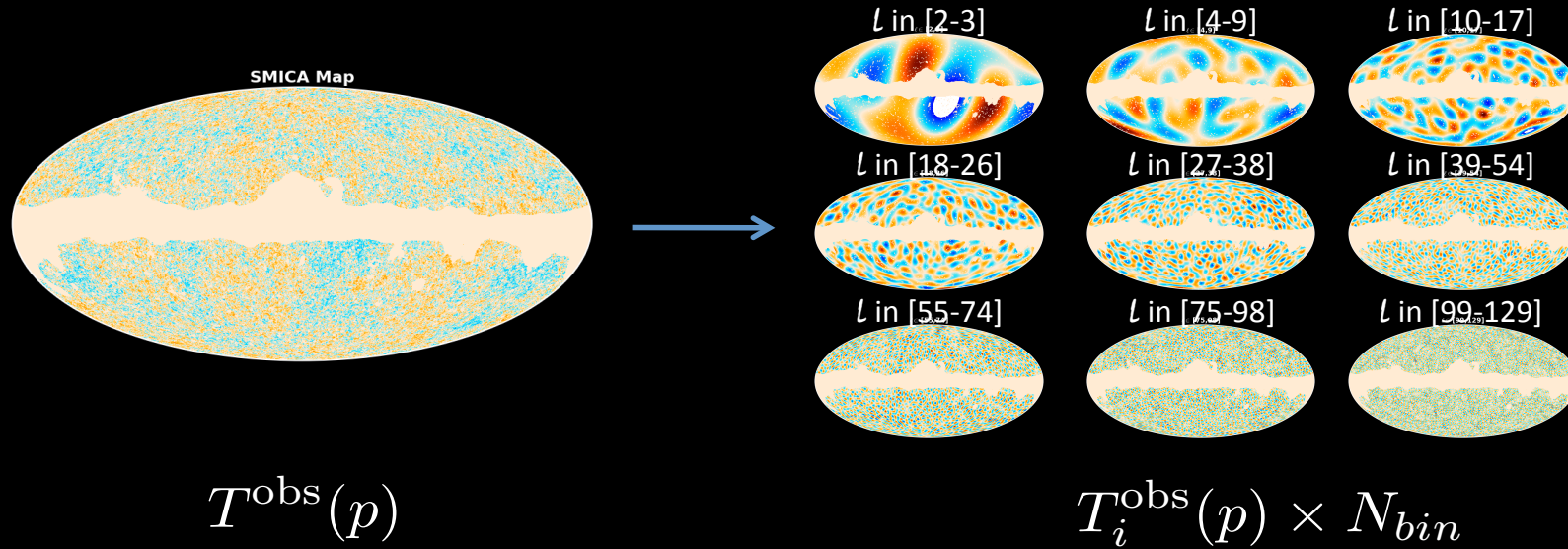
$$Var_{\ell_1 \ell_2 \ell_3} = (b_{\ell_1}^2 C_{\ell_1} + N_{\ell_1})(b_{\ell_2}^2 C_{\ell_2} + N_{\ell_2})(b_{\ell_3}^2 C_{\ell_3} + N_{\ell_3})$$


# 3. Binned Bispectrum

See: M.Bucher, B.Van Tent and C.S.Carvalho : 2010, MNRAS, 407, 2193, arXiv:0911.1642  
Other methods : - Komatsu, Spergel, Wandelt : 2005, APJ, 634, 14, astro-ph: 0305189  
- Modal: Fergusson, Liguori, Shellard : 2010, PRD, 82, 2, arXiv:0912.5516

# Binned Bispectrum

$$T_{\ell}^{\text{obs}}(\Omega) \longrightarrow T_i^{\text{obs}}(\Omega) = \sum_{\ell \in \text{bin } i} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\Omega)$$



$$B_{i_1 i_2 i_3}^{\text{obs}} \propto \sum_p T_{i_1}^{\text{obs}}(p) T_{i_2}^{\text{obs}}(p) T_{i_3}^{\text{obs}}(p)$$



# Linear correction

$$B_{i_1 i_2 i_3}^{\text{lin}} = \sum_p \left[ T_{i_1}^{\text{obs}} \langle T_{i_2}^{\text{G}} T_{i_3}^{\text{G}} \rangle + T_{i_2}^{\text{obs}} \langle T_{i_1}^{\text{G}} T_{i_3}^{\text{G}} \rangle + T_{i_3}^{\text{obs}} \langle T_{i_1}^{\text{G}} T_{i_2}^{\text{G}} \rangle \right]$$


Averaged on gaussian maps with noise, beam and mask as in observed maps ( $\sim 200$ )

$$B_{i_1 i_2 i_3}^{\text{obs}} \rightarrow B_{i_1 i_2 i_3}^{\text{obs}} - B_{i_1 i_2 i_3}^{\text{lin}}$$

# Binned Bispectrum

$$B_{i_1 i_2 i_3}^{th} = \sum_{\ell_1 \in \text{bin } i_1} \sum_{\ell_2 \in \text{bin } i_2} \sum_{\ell_3 \in \text{bin } i_3} B_{\ell_1 \ell_2 \ell_3}^{th}$$

$$\hat{f}_{NL} = \frac{1}{N} \sum_{i_1 i_2 i_3 = i_{min}}^{i_{max}} \frac{(B_{i_1 i_2 i_3}^{obs} - B_{i_1 i_2 i_3}^{lin}) B_{i_1 i_2 i_3}^{th}(f_{NL}=1)}{\text{Var}_{i_1 i_2 i_3}}$$

In the  $f_{NL}$  analysis we used 51 bins, that were optimized to get the weakest variance increase compared to non-binned case.

# Smoothed Binned Bispectrum

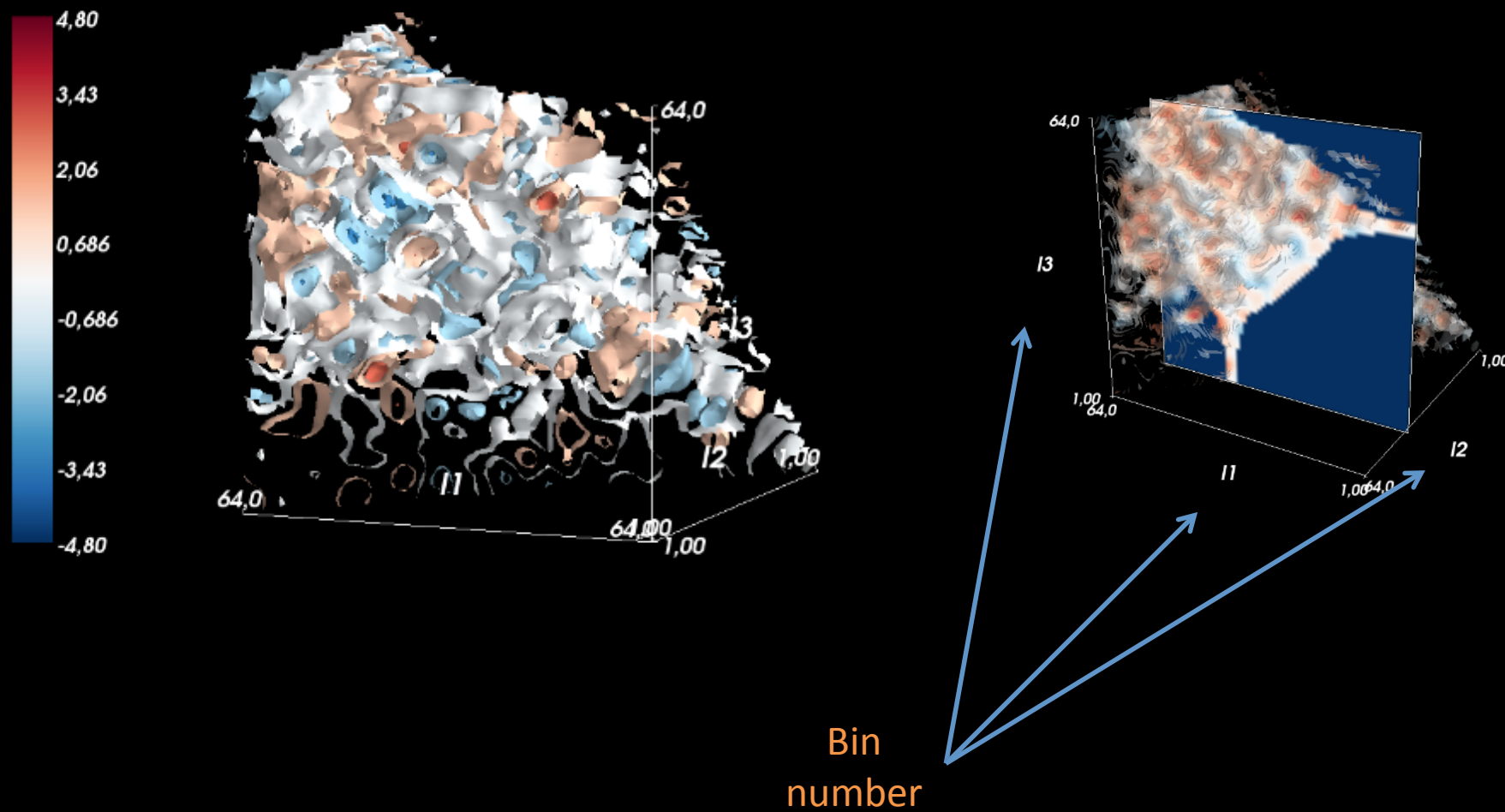
- We want to look for significant features in the bispectrum of the data without theoretical prior on the expected NG signal.
- We look at the SNR in the binned bispectrum :

$$\frac{(B_{i_1 i_2 i_3}^{\text{obs}} - B_{i_1 i_2 i_3}^{\text{lin}})}{\sqrt{\text{Var}_{i_1 i_2 i_3}}}$$

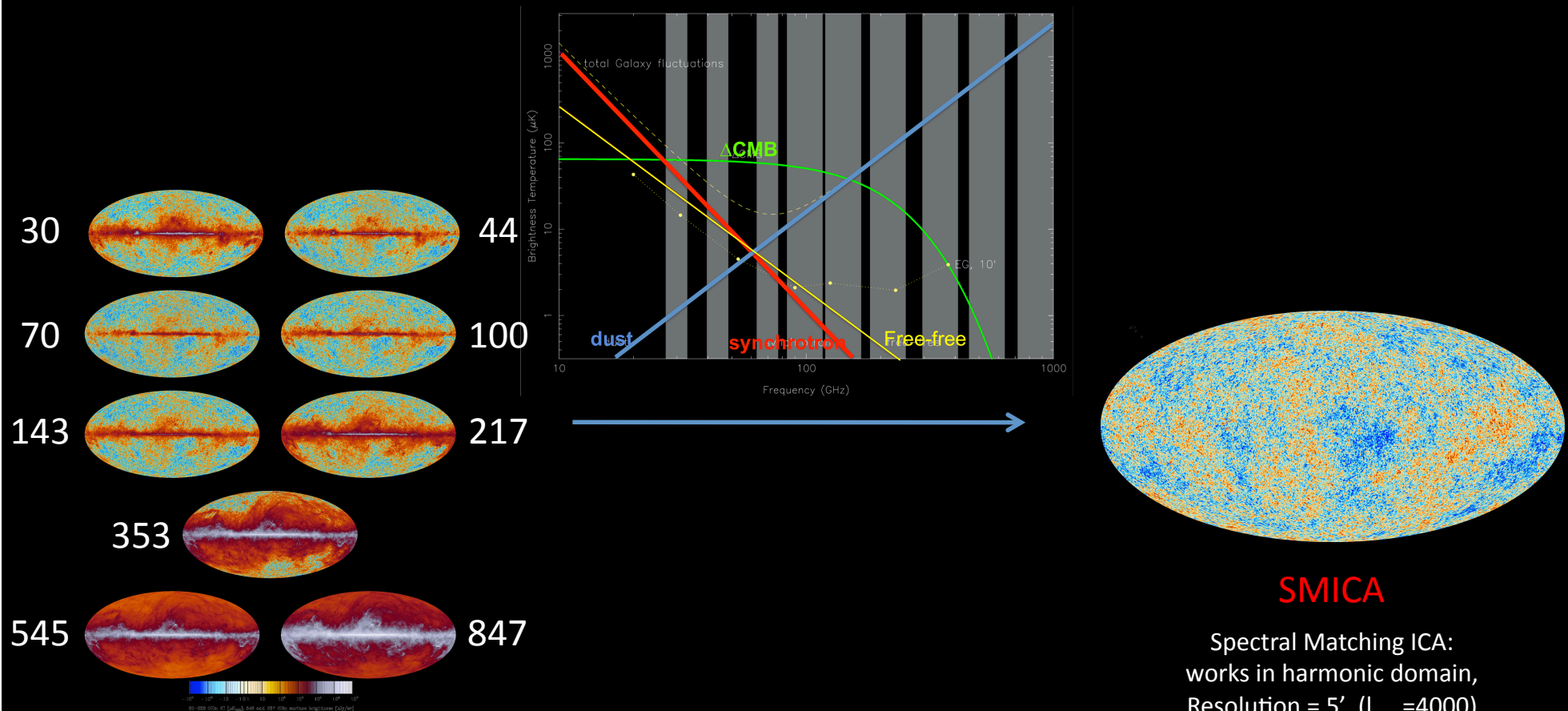
- We smooth the bispectrum in bin space with a gaussian kernel to make visible a signal that is coherent between bins.

64 bins  $\ell \in [2, 2000]$

We have a 3D bispectrum, but slices are easier to study



# Component separation (snapshot)



Using the Planck's frequency coverage, we can separate foregrounds from the CMB

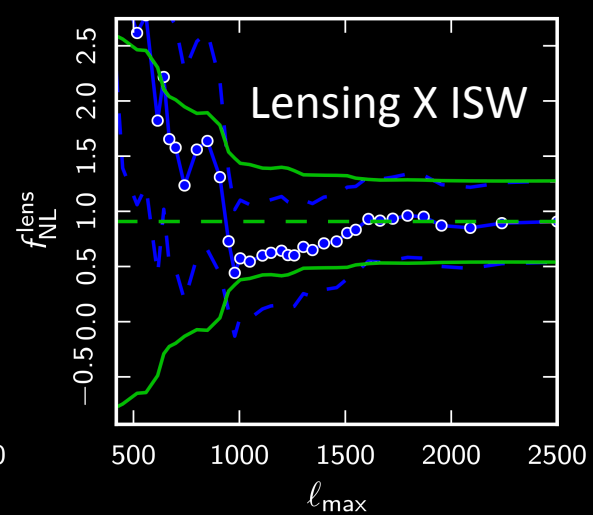
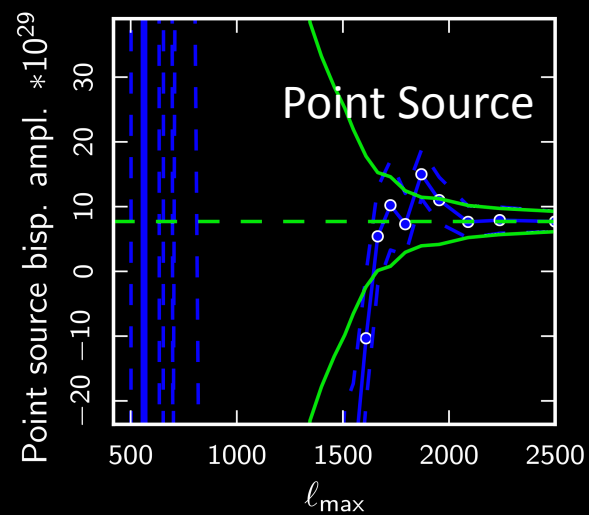
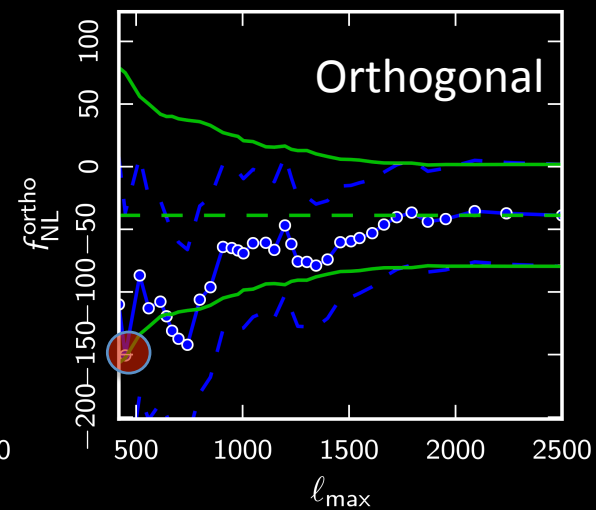
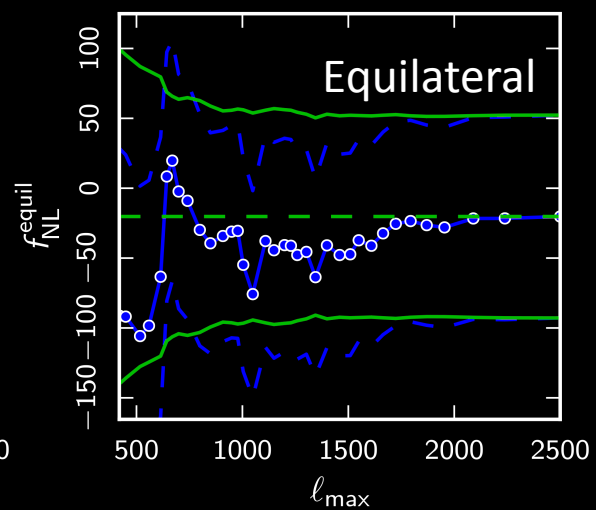
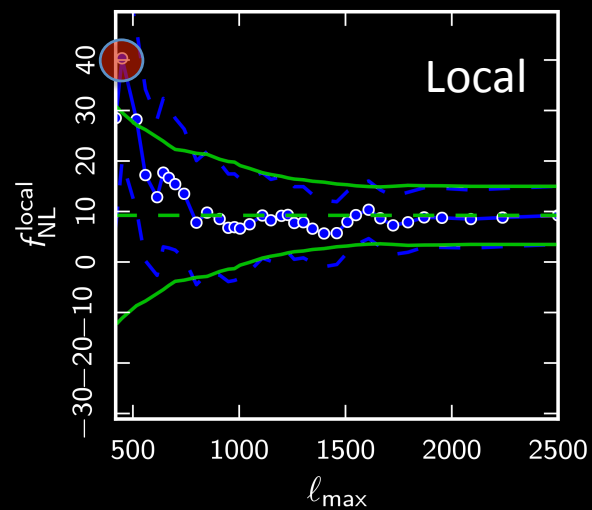
## 5. Results

# Binned Bispectrum $f_{NL}$

|                                           |           | Independent |           |           |                  | ISW-lensing subtracted |           |  |
|-------------------------------------------|-----------|-------------|-----------|-----------|------------------|------------------------|-----------|--|
|                                           | KSW       | Binned      | Modal     |           | KSW              | Binned                 | Modal     |  |
| SMICA                                     |           |             |           |           |                  |                        |           |  |
| Local . . . . .                           | 9.8 ± 5.8 | 9.2 ± 5.9   | 8.3 ± 5.9 | . . . . . | <b>2.7 ± 5.8</b> | 2.2 ± 5.9              | 1.6 ± 6.0 |  |
| Equilateral . . . . .                     | -37 ± 75  | -20 ± 73    | -20 ± 77  | . . . . . | <b>-42 ± 75</b>  | -25 ± 73               | -20 ± 77  |  |
| Orthogonal . . . . .                      | -46 ± 39  | -39 ± 41    | -36 ± 41  | . . . . . | <b>-25 ± 39</b>  | -17 ± 41               | -14 ± 42  |  |
|                                           |           |             |           |           |                  |                        |           |  |
| Diff point sources (x10 <sup>29</sup> ) : | 7.7 ± 1.5 | 7.7 ± 1.6   |           |           |                  |                        |           |  |
| Lensing x ISW :                           | 0.81±0.31 | 0.91 ± 0.37 |           |           |                  |                        |           |  |
| « detection »                             |           |             |           |           |                  |                        |           |  |

# $\ell_{\max}$ dependence

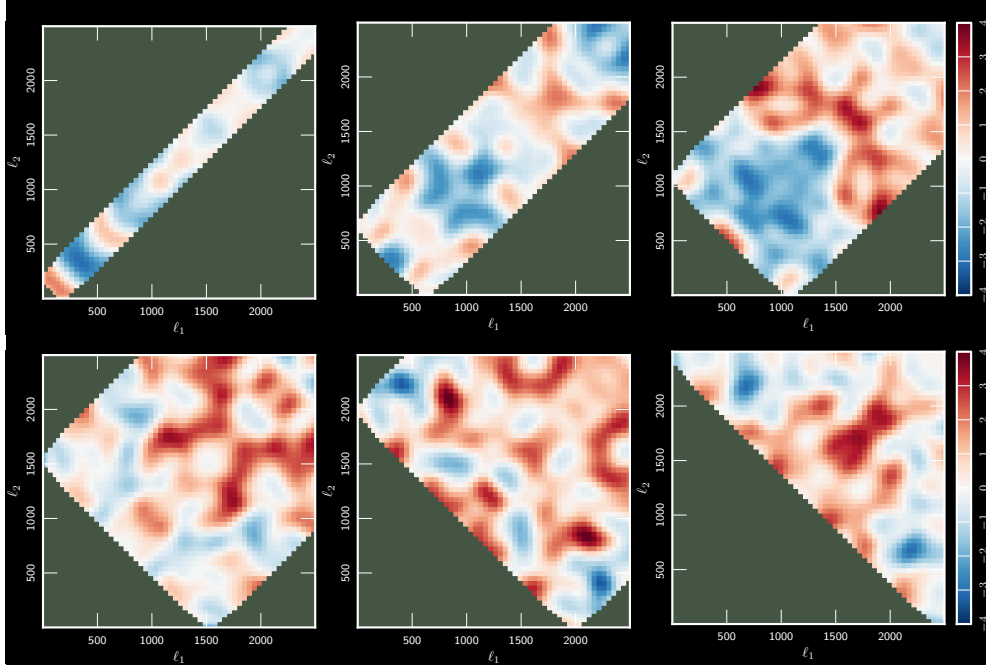
WMAP9

 $37.2 \pm 19.9$  $51 \pm 136$  $-245 \pm 100$ 

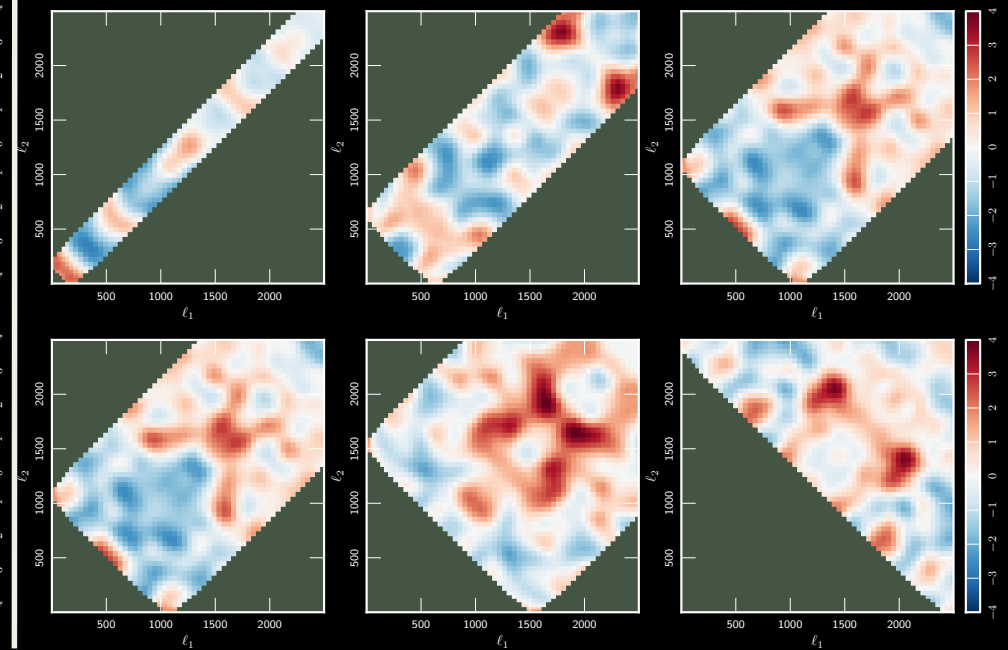


# Smoothed Binned Bispectrum

SMICA smoothed bispectrum



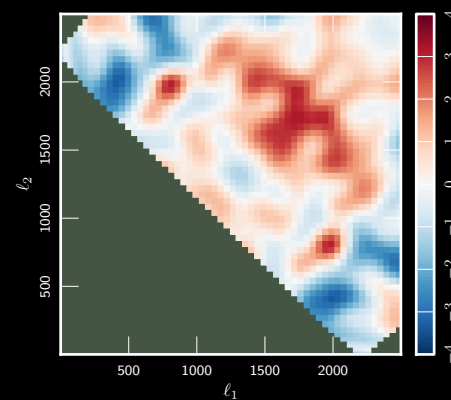
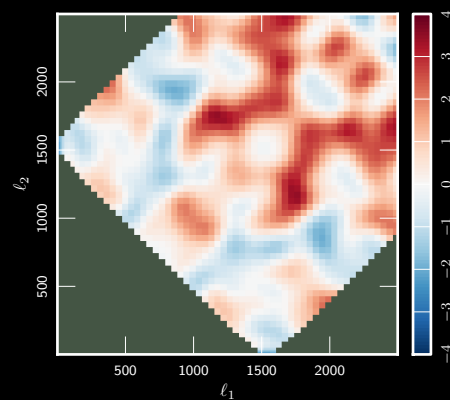
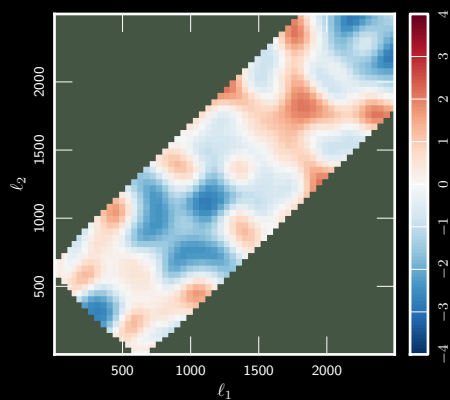
Raw 143Ghz smoothed bispectrum



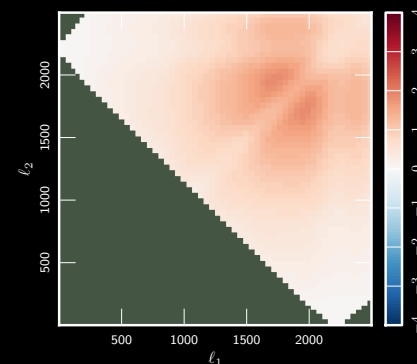
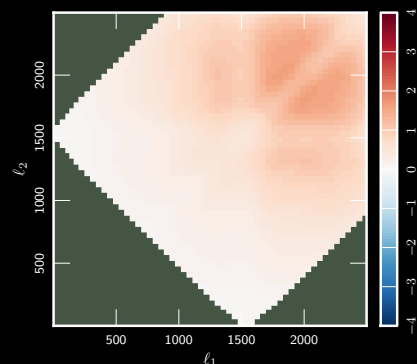
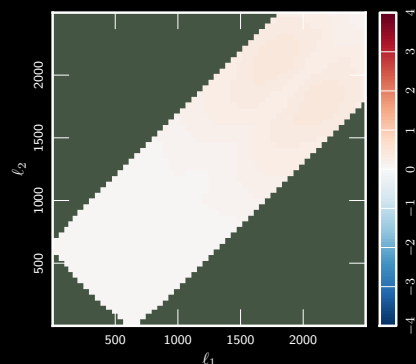
Red: positive bispectrum  $\rightarrow S/N < +4$

Blue: Negative bispectrum  $\rightarrow S/N > -4$

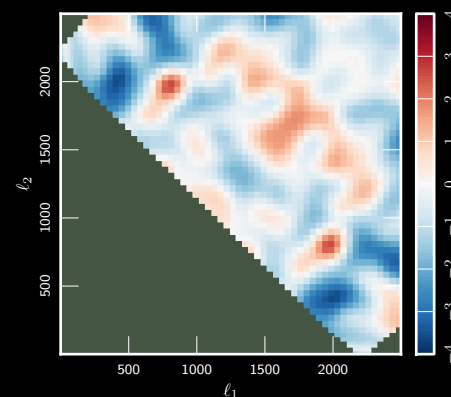
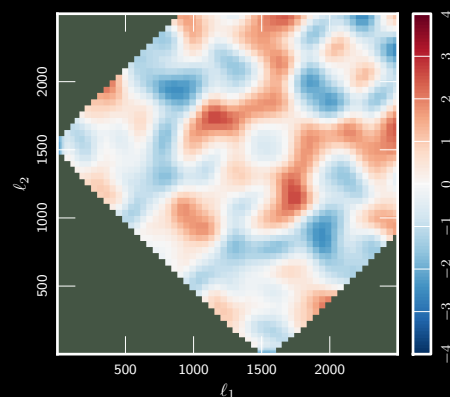
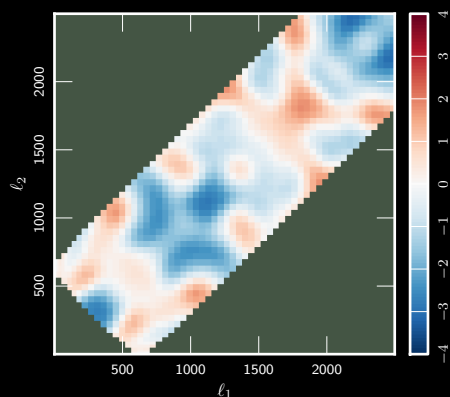
SMICA  
bispectrum



point source  
bispectrum



point source bispectrum  
removed



$\ell \in [610, 654]$

$\ell \in [1510, 1554]$

$\ell \in [2185, 2229]$

# Conclusion

- Binned Bispectrum allows a **parametric** ( $f_{NL}$ ) and **blind** analysis of NG.
- **No detection** of local, equilateral and orthogonal NG, in agreement with other methods.
- Diffuse point source and lensing x ISW NG **detected**.
- We see the point sources NG at high  $l$ .



# planck



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Deutsches Zentrum  
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