CMB ISW-Lensing bispectrum in f(R) theory

based on Phys Rev D.88.024012 with Matarrese,Bartolo,Liguori

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Outline

- **f(R) gravity**

  Basic setup

  Background kinematics

  Perturbation dynamics

  PPF approach

  Bertschinger-Zukin parameterization

- **ISW and Lensing effect**
- **CMB constraint on f(R) gravity**

  Power spectrum

  Bispectrum

  Power spectrum-Bispectrum cross correlation

- **Conclusion**

f(R) gravity

Basic setup

Lagrangian (Jordan frame)

$$S = \int d^4x\sqrt{-g}\left\{\frac{R+f(R)}{2\mu^2}+\mathcal{L}_m\right\}\ ,\ \chi = \frac{df}{dR}\ ,\ \text{scalaron}$$

EoM (4th order derivatives)

B=$f_{RR}/(1+f_R)/H$

$$G_{\mu\nu} = \mu^2 T^m_{\mu\nu} + \left[f_R R_{\mu\nu} - (\frac{f}{2} - \Box f_R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R\right],$$

$$\Lambda g_{\mu\nu} = \frac{1}{\mu^2}\left[f_R R_{\mu\nu} - (\frac{f}{2} - \Box f_R)g_{\mu\nu} - \nabla_\mu\nabla_\nu f_R\right] \text{ (Mimic } \Lambda CDM)$$

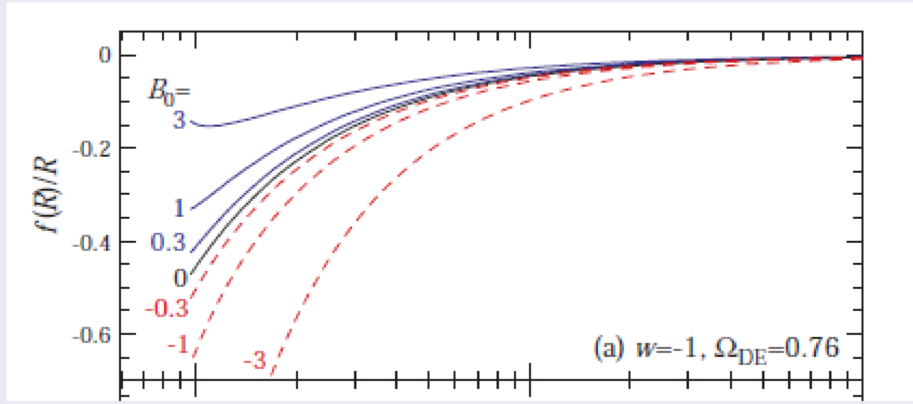
$$f''(t) + c_1(t)f'(t) + c_2(t)f(t) \sim \Lambda\ ,\ P_\pm = \frac{-7\pm\sqrt{73}}{4}$$

Decaying mode $\propto a^{P_-}$ $(\times)$ Growing mode $\propto B_0 a^{P_+} - 2\Lambda$ $(\surd)$

Astro-ph/0610532 Hu.et.al.

Engineering f(R) mimic LCDM background

one-parameter family solutions



$f(R)$ mimic given background. astro-ph/0610532 W.Hu *et.al.*

Analytic form

$$f(R) = -2\Lambda - \omega \left(\frac{\Lambda}{R-4\Lambda}\right)^{p_+-1} F_{2,1}[q_+, p_+ - 1, r_+, -\frac{1}{R-4\Lambda}]$$

HyperGeometric function

$$\mathrm{F}_{2,1}[a, b, c, z] = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 dt\; t^{b-1}(1-t)^{c-b-1}(1-z)^{-a}$$

$$\omega = \frac{D(R_0-4\Lambda)^{p_+}}{(p_+-1)\Lambda^{p_+-1}}\,, D \propto B_0\,, q_+ = \frac{1+\sqrt{73}}{12}\,, r_+ = 1 + \frac{\sqrt{73}}{6}\,, p_+ = \frac{5+\sqrt{73}}{12}$$

JH He, Phys. Rev. D86, 103505

$$f(R) = -2\Lambda - \omega \left(\frac{\Lambda}{R-4\Lambda}\right)^{p_+ - 1} F_{2,1}[q_+, p_+ - 1, r_+, -\frac{1}{R-4\Lambda}]$$

$$\mathrm{G}_{\mu\nu} = \mu^2 T^m_{\mu\nu} + \left[f_R R_{\mu\nu} - (\tfrac{f}{2} - f_R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R\right]$$

$$\mathrm{G}_{\mu\nu} = \mu^2 T^m_{\mu\nu} + \Lambda g_{\mu\nu} + \left[f_R R_{\mu\nu} - (\tfrac{f}{2} + \Lambda - f_R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R\right]$$

LCDM

$$\mathrm{T}^{f(R)}_{\mu\nu} = 0$$

CANNOT distinguish $f(R)$ from ΛCDM on background!

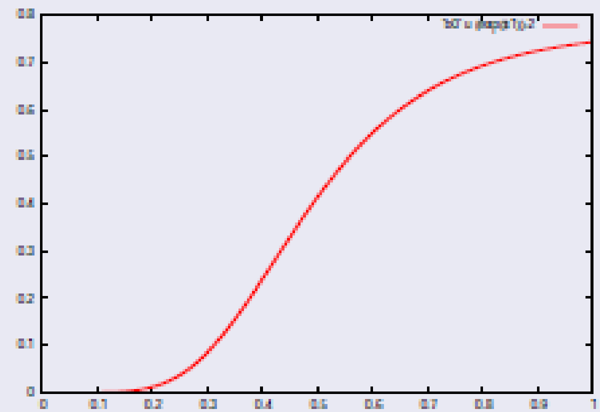
f(R) gravity

Perturbation dynamics

Super-horizon regime: shear $(\Phi + \Psi)$

$$ds^2 = -(1 + 2\Psi)dt^2 + a^2(1 + 2\Phi)dx^2$$

- GR: $\Phi = -\Psi$
- $f(R)$: $\Phi + \Psi = -B\frac{H'}{H} V_m$



Quasi-static regime: Newton constant

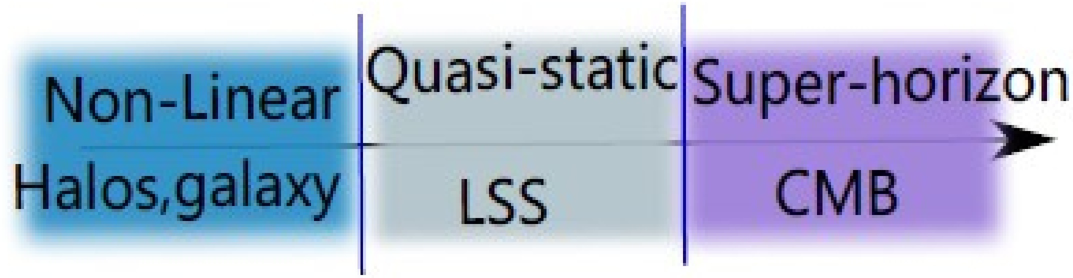
$$\text{Poisson}: \quad k^2 \frac{\Phi - \Psi}{2} = \frac{4\pi G}{1 + f_G} a^2 \rho_m \Delta_m$$

CAN distinguish $f(R)$ from ΛCDM via perturbations!

Parameterized Post-Friedmann

0708.1190 W.Hu et. al.

- 3 regimes:



- Main Eqs:

$$\begin{cases} (1 + c_\Gamma^2 k_H^2)\left[\Gamma' + \Gamma + c_\Gamma^2 k_H^2(\Gamma - f_G \Phi_-)\right] = S \\ k^2[\Phi_- + \Gamma] = 4\pi G a^2 \rho_m \Delta_m \\ S = -\left[\frac{1}{g+1}\frac{H'}{H} + \frac{3}{2}\frac{H_m^2}{H^2 a^3}(1 + f_\zeta)\right]\frac{V_m}{k_H} + \left[\frac{g' - 2g}{g+1}\right]\Phi_- \end{cases}$$

- Need to provide: 1 coefficient and 3 functions

$$\left\{ c_\Gamma, g(\ln a, k), f_\zeta(\ln a), f_G(\ln a) \right\}$$

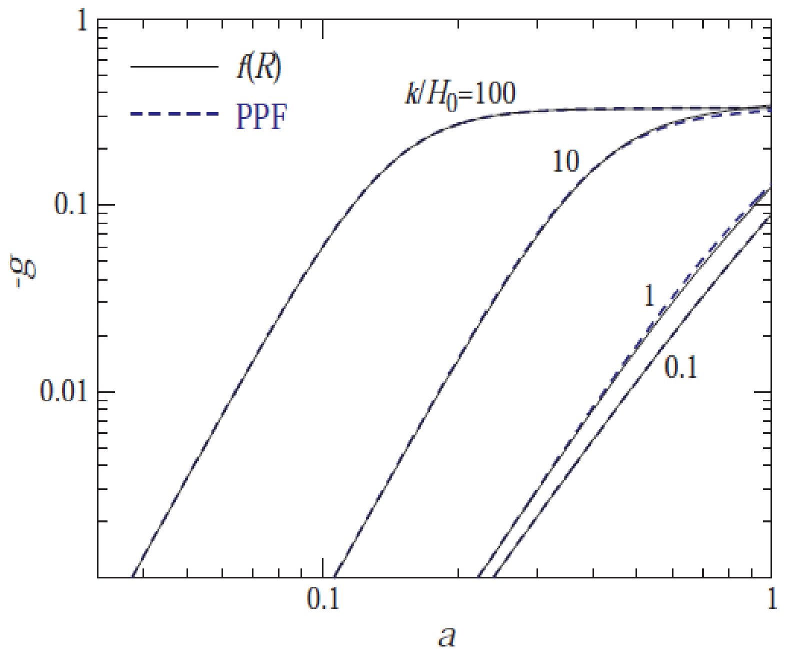
- For f(R) gravity, **W.Hu et. al. gives**

PPF implementation:

$$\begin{cases} c_\Gamma = 1 \ , \ f_\zeta = -\frac{1}{3} g \\ g(\ln a, k) = \frac{g_{SH} - \frac{1}{3}(c_g k_H)^2}{1+(c_g k_H)^2} \\ f_G = f_R \ , \ c_g = 0.71\sqrt{B} \end{cases}$$

Compton wavelength:

$$m_{f_R}^2 = \frac{1}{3}\left(\frac{1+f_R}{f_{RR}} - R\right) \ , \ \lambda_{f_R} = m_{f_R}^{-1} \ , \ \sqrt{B} \sim \frac{\lambda_{f_R}}{H^{-1}}$$



- Hu-Sawicki model 0705.1158

$$f(R) = -m^2 \frac{c_1 (R/m^2)^n}{1+c_2(R/m^2)^n} \ , \ m^2 = H_0^2 \Omega_m$$ 0705.1158 **Hu & Sawicki**

Cannot exact mimic Lambda CDM but deviation is very small,so we can embed Hu-Sawicki model into PPF

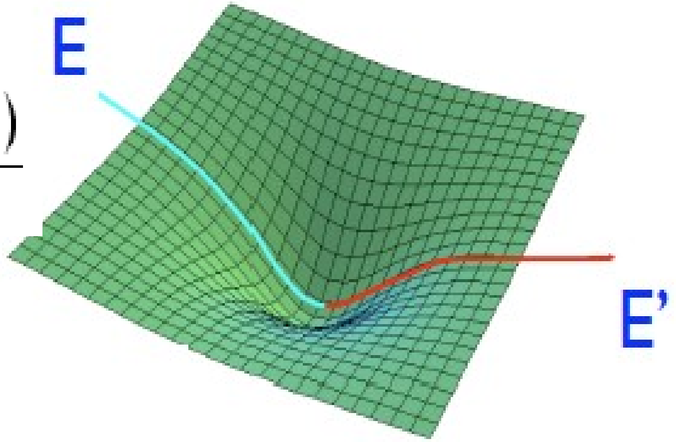
- Bertschinger-Zukin parameterization 0801.2431

Neglect time derivative term

Quasi-static regime

- Poisson Eq. $k^2\Psi = -\frac{a^2}{2M_p^2}\mu(a,k)\rho\Delta_m$, $\mu(a,k) = \frac{1+(2/3)B_0k^2a^s}{1+(1/2)B_0k^2a^s}$
- Anisotropic Eq. $\frac{\Phi}{\Psi} = \gamma(a,k)$, $\gamma(a,k) = \frac{1+(1/3)B_0k^2a^s}{1+(2/3)B_0k^2a^s}$, $s = 4, 3.5$

ISW effect $\Delta_T^{ISW}(\hat{n}) = 2\int dz \frac{d\phi(\hat{n},z)}{dz}$

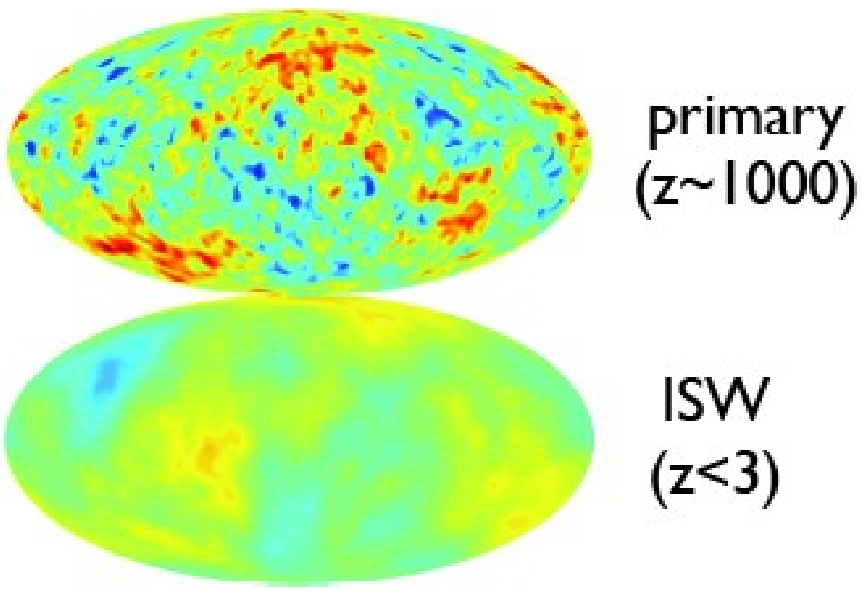


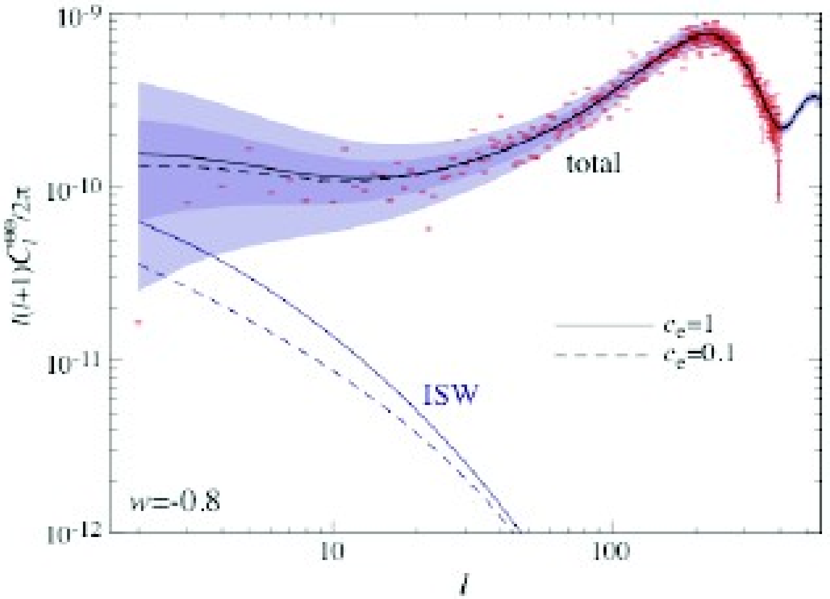
- Matter dominated epoch

  Φ=const. (linear growth = expansion rate) ⟶ No ISW

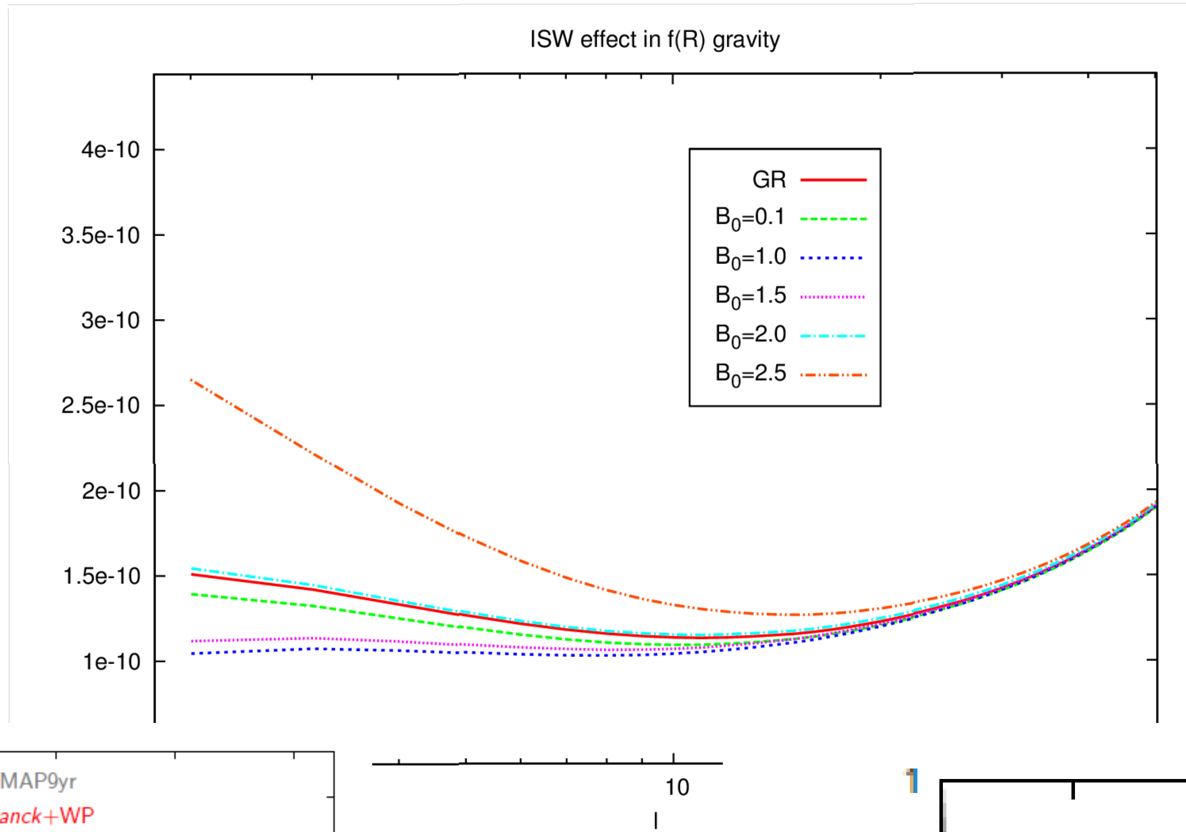
- Late accelerating epoch

  Φ decays (linear growth < expansion rate) ⟶ ISW (evidence DE)

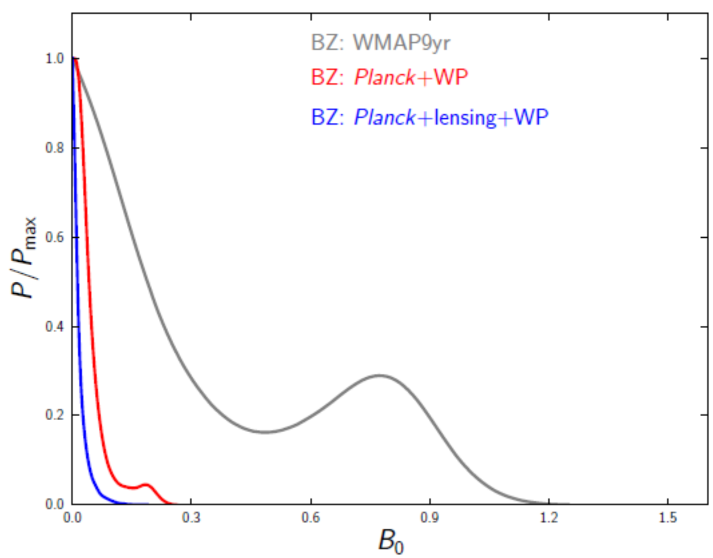




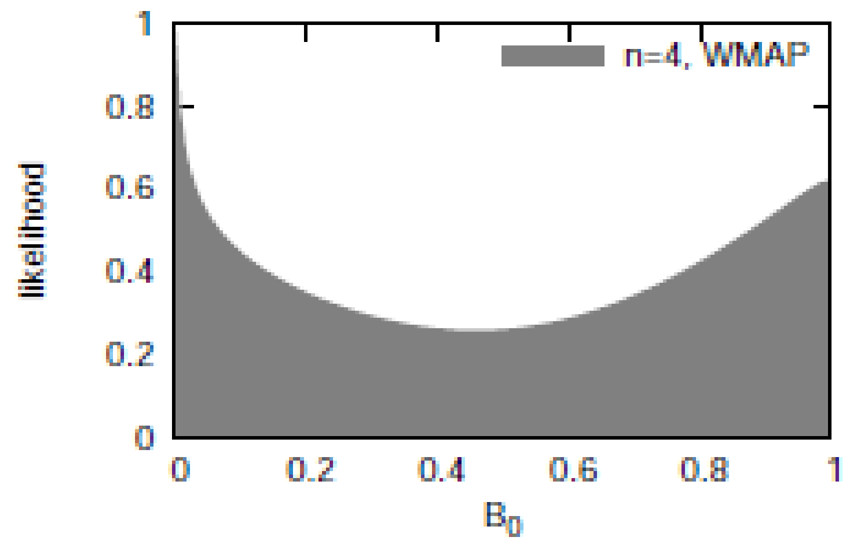
ISW in f(R) theory



BZ

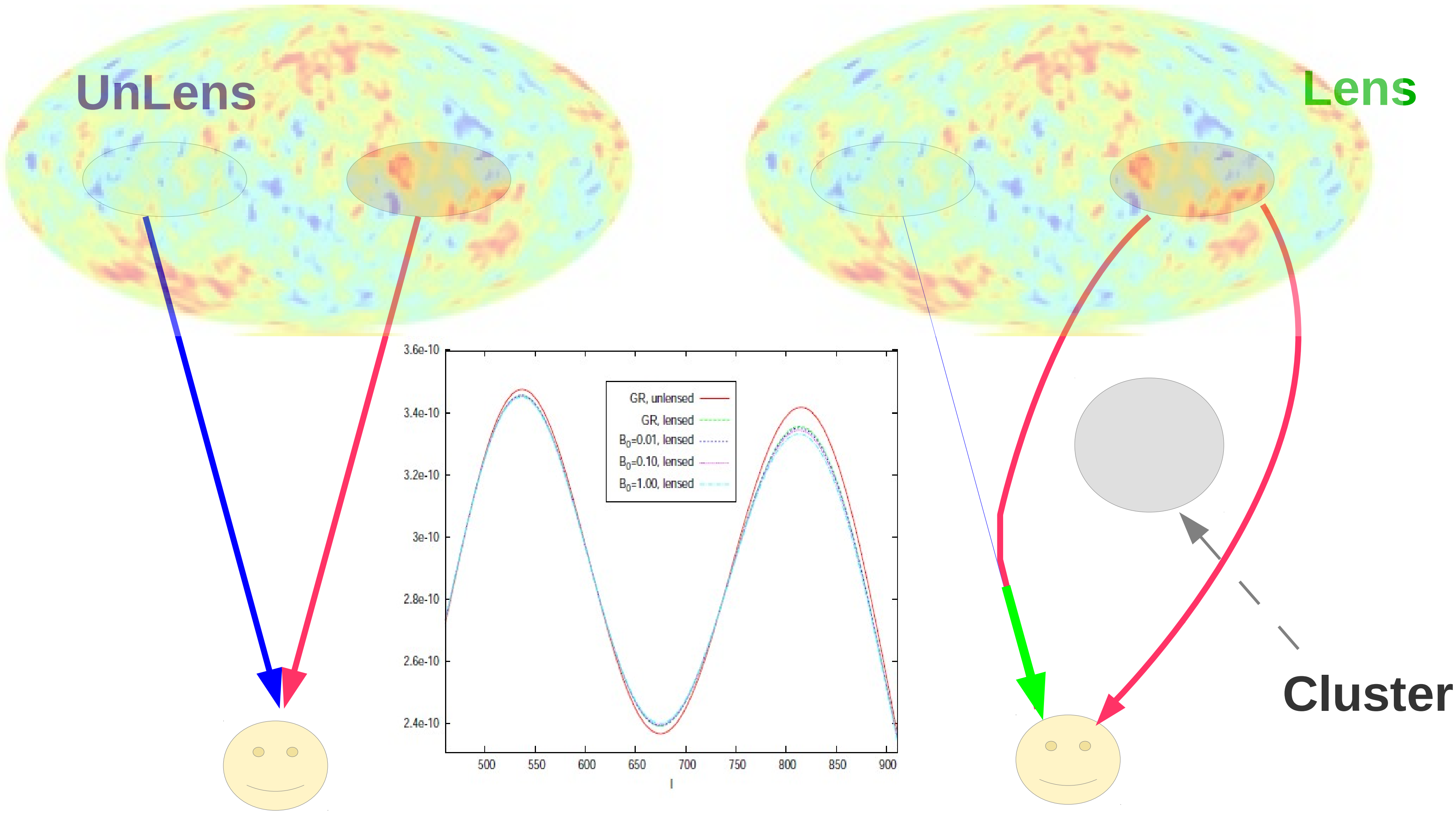


Hu-Sawicki



Weak lensing

$$\begin{aligned}\delta\tilde{T}(\hat{\mathrm{n}}) &= \delta T(\hat{\mathrm{n}} + \partial\phi) \\ &\simeq \delta T(\hat{\mathrm{n}}) + \Big[(\partial\phi)\cdot(\partial\delta T)\Big](\hat{\mathrm{n}})\end{aligned}$$



ISW-Lensing bispectrum

$$\delta\tilde{T}(\hat{\mathrm{n}}) = \delta T(\hat{\mathrm{n}} + \partial\phi)$$
$$\simeq \delta T(\hat{\mathrm{n}}) + \Big[(\partial\phi)\cdot(\partial\delta T)\Big](\hat{\mathrm{n}})$$

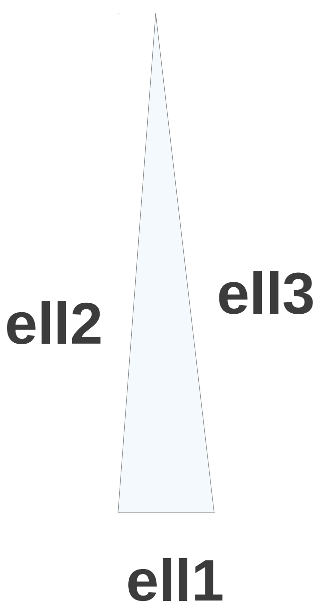
- ISW $\frac{\delta T}{T}|_{ISW} = \int d\chi (\Phi - \Psi)_{,\tau}(\hat{n}, \chi)$
- Lensing $\phi(\hat{n}) = -\int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi}(\Phi - \Psi)(\hat{n}, \chi)$

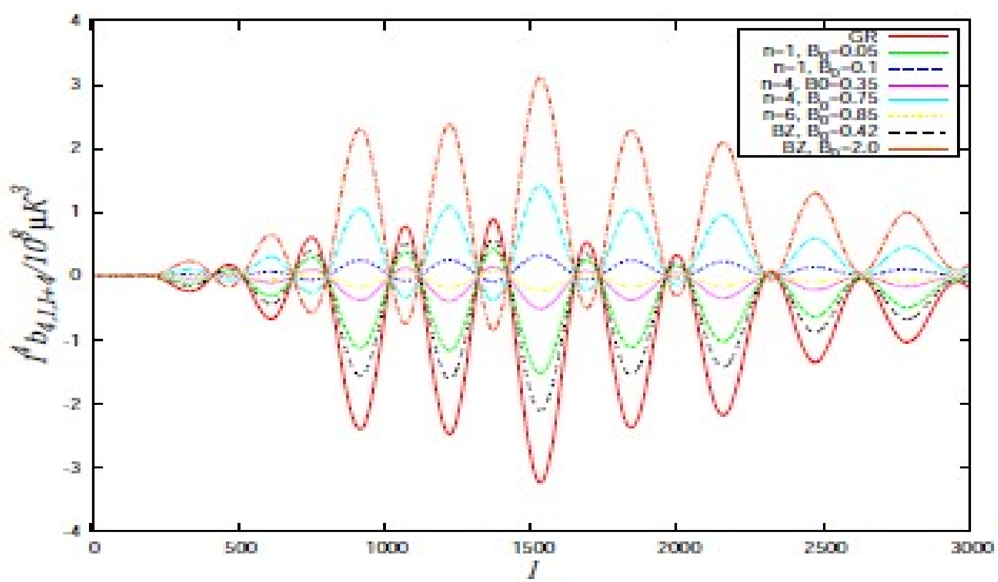
Same source

- Secondary bispectrum (Even without primordial bispectrum)

$$b^{ISW-L}_{l_1 l_2 l_3} = \left[\frac{-l_1(l_1+1) + l_2(l_2+1) + l_3(l_3+1)}{2} C^T_{l_2} C^{\phi T}_{l_3} + 5 \text{ perm.}\right]$$

Squeezed limit



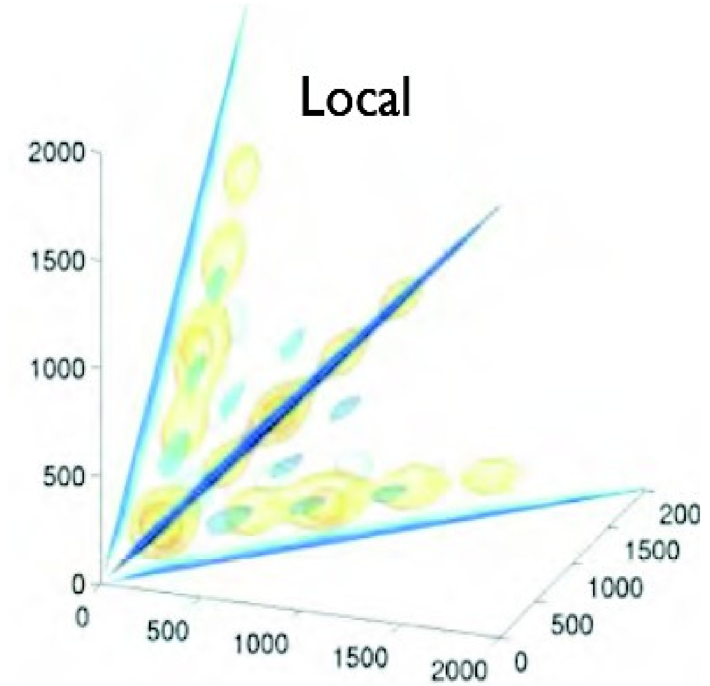


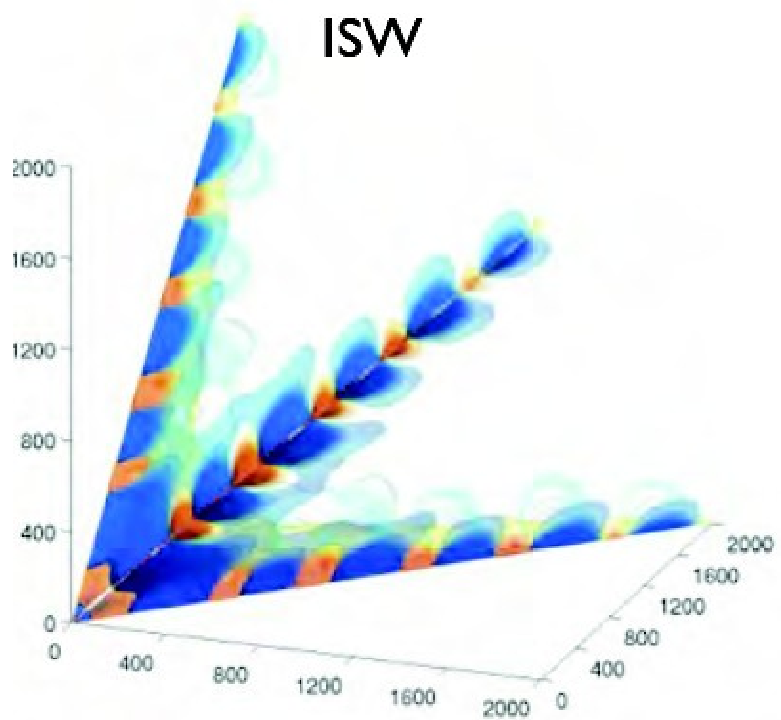
Contamination

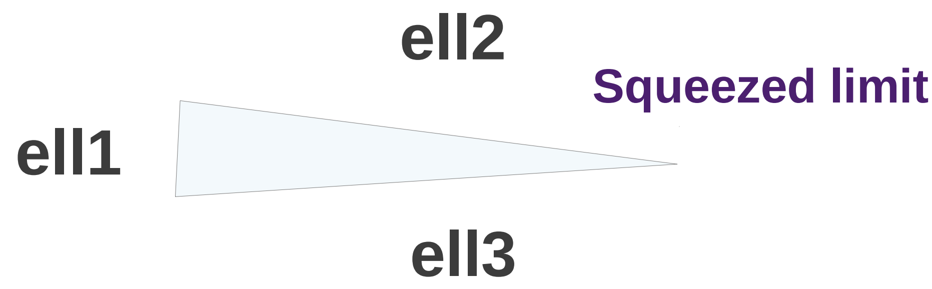
- Contamination of ISW-L

Bisp on local f_NL ~ 9.3

(Hanson et.al. 0905.4732)





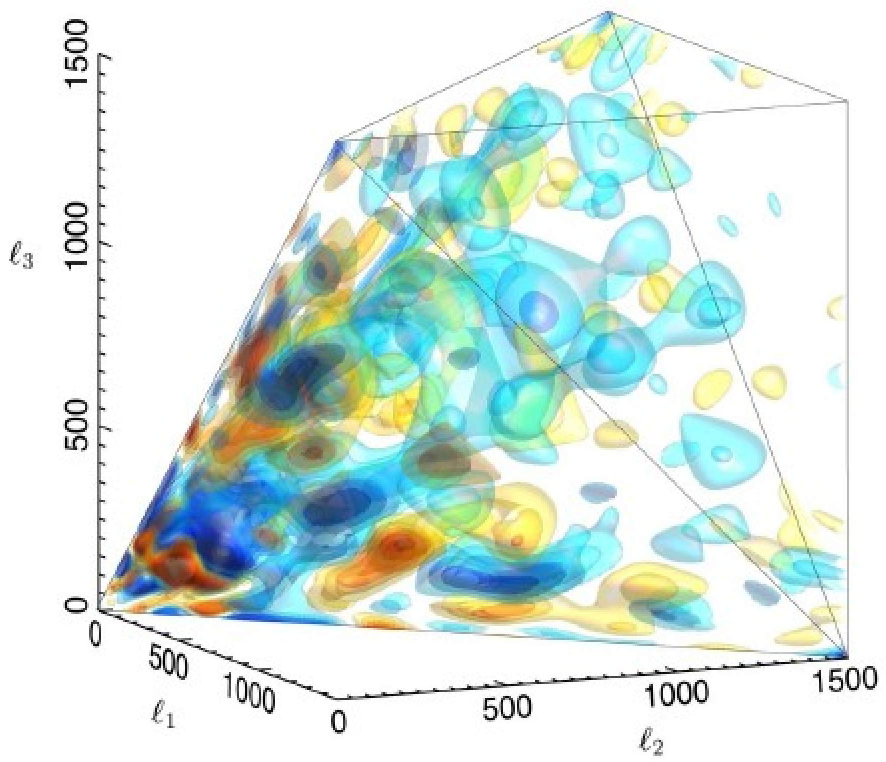


Planck results

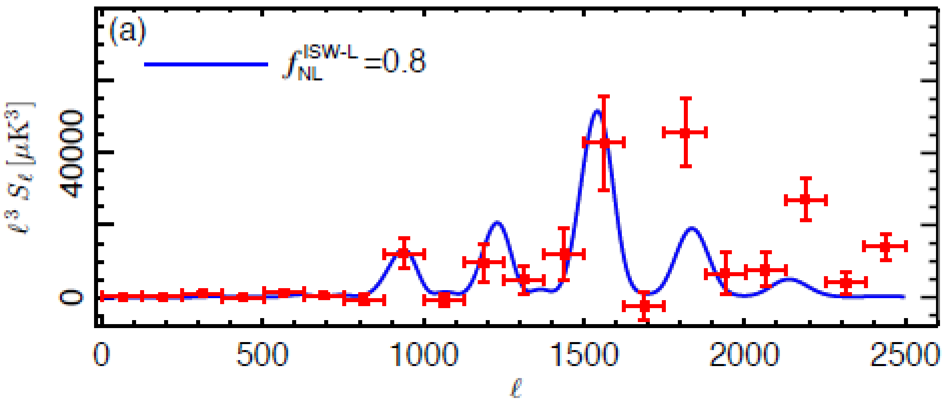
- Contamination to f_NL ~ 7.1

	Independent KSW	ISW-lensing subtracted KSW
SMICA		
Local	9.8 ± 5.8	**2.7 ± 5.8**
Equilateral	−37 ± 75	**−42 ± 75**
Orthogonal	−46 ± 39	**−25 ± 39**

- Significance of A_nl ~ 2.5σ



	SMICA	NILC	SEVEM	C-R
KSW	0.81 ± 0.31	0.85 ± 0.32	0.68 ± 0.32	0.75 ± 0.32
Binned	0.91 ± 0.37	1.03 ± 0.37	0.83 ± 0.39	0.80 ± 0.40
Modal	0.77 ± 0.37	0.93 ± 0.37	0.60 ± 0.37	0.68 ± 0.39



Can ISW-L bisp do late-time parameter estimation ?

- B0 cross-correlation coefficients with standard CP (fisher matrix)

$$\mathbf{F}_{ij} = \sum_l \sum_{X,Y} \frac{\partial C_{Xl}}{\partial p_i} \left(\mathrm{Cov}_l\right)^{-1}_{XY} \frac{\partial C_{Yl}}{\partial p_j} ,$$

$$w_{X,c} = \left(\sigma_{X,c}\theta_{\mathrm{beam}}\right)^{-2} , \qquad \bar{w}_X = \sum_c w_{X,c}$$

$$\bar{B}_l = \frac{1}{\bar{w}_X} \sum_c w_{X,c} B_{c,l} , \quad X \in (T,P)$$

$$B_{c,l} \simeq \exp\left\{ \frac{-l(l+1)\theta^2_{\mathrm{beam}}}{8\ln 2} \right\} .$$

$$\left(\mathrm{Cov}_l\right)_{\mathrm{TT}} = \frac{2}{(2l+1)f_{\mathrm{sky}}} \left[C_{Tl} + \bar{w}_T^{-1}\bar{B}_l^{-2}\right]^2 ,$$

$$\left(\mathrm{Cov}_l\right)_{\mathrm{EE}} = \frac{2}{(2l+1)f_{\mathrm{sky}}} \left[C_{El} + \bar{w}_P^{-1}\bar{B}_l^{-2}\right]^2 ,$$

$$\left(\mathrm{Cov}_l\right)_{\mathrm{TE}} = \frac{2}{(2l+1)f_{\mathrm{sky}}} C^2_{Cl} ,$$

TABLE IX. Cross-correlation coefficients of the standard cosmological parameters with B_0 in Hu-Sawicki model ($n = 4$)

B_0-CP	cross-correlation coefficients
A_s	-7.9×10^{-2}
n_s	-0.12
τ	-8.0×10^{-2}
$\Omega_b h^2$	-0.18
$\Omega_m h^2$	-4.7×10^{-2}
h	8.2×10^{-2}

Quadratic estimator

- Power spectrum likelihood

$$\chi^2_{\rm P} = \sum_{l=2}^{3000} \frac{\left[C_l^{th} - C_l^{fid}\right]^2}{\sigma^2_{{\rm P},l}}$$

$$\sigma^2_{{\rm P},l} = \frac{2}{(2l+1)}\left(b_l^2 C_l^T + N_l\right)^2$$

- Bispectrum likelihood

Planck blue book

TABLE VII. WMAP and Planck experiment

Experiment	Frequency	$\theta_{\rm beam}$	σ_T	σ_P
WMAP:	94	12.6	49.9	70.7
	60	21.0	30.0	42.6
	40	28.2	17.2	24.4
Planck:	217	5.0	13.1	26.8
	143	7.1	6.0	11.5
Junk:2012qt	100	10.0	6.8	10.9
	70	14.0	12.8	18.3

[a] Frequencies in GHz. Beam size $\theta_{\rm beam}$ is the FWHM in arcminutes. Sensitivities σ_T and σ_P are in μK per FWHM beam.

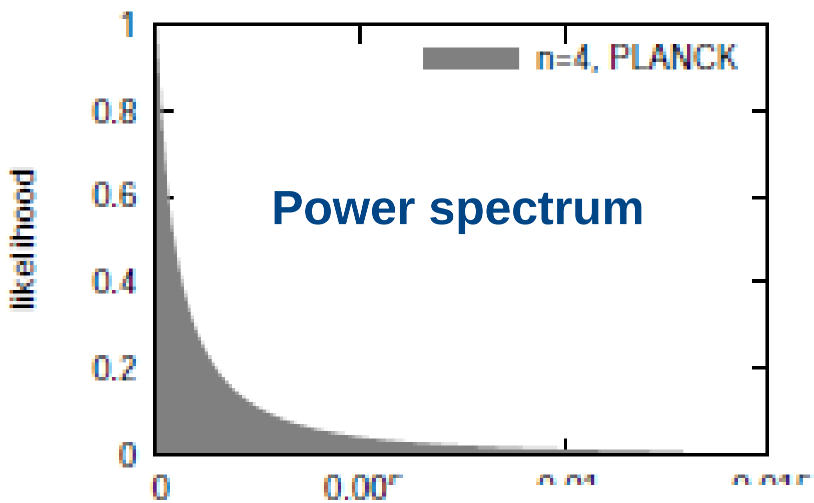
$$\chi^2 = \sum_l \frac{\left[B_{l_1 l_2 l_3}(B_0, \Omega_b, \cdots) - B_{l_1 l_2 l_3}(Fid)\right]^2}{2\sigma^2}, \sigma^2 = \Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}$$

$$\mathrm{C}_l = b_l C_l + N_l \quad \Delta_{l_1 l_2 l_3} = \begin{cases} 6\,, & (l_1 = l_2 = l_3)\,, \\ 2\,, & (\text{two identical})\,, \\ 1\,, & (\text{all different})\,. \end{cases}$$

Global fitting of B0

CP	Fiducial value[a]
A_s	2.43×10^{-9}
n_s	0.963
τ	0.088
$\Omega_b h^2$	0.0226
$\Omega_m h^2$	0.1109
h	0.70
$B_0(n=4)$[b]	0
$B_0(n=1)$	0

- Grid B0 (0~1), fixing other CP best-fit value of WMAP7



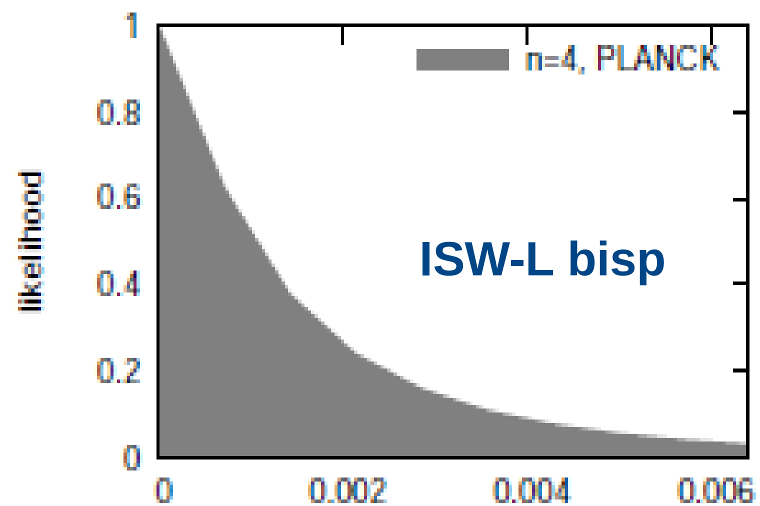


TABLE VI. B_0 error bars (68% CL) summary for Planck experiment

Model	Pow	Bisp	Joint
1	6.68×10^{-5}	2.90×10^{-4}	5.85×10^{-5}
4	2.06×10^{-3}	1.89×10^{-3}	6.90×10^{-4}
6	1.51×10^{-2}	6.29×10^{-3}	2.84×10^{-3}
BZ	1.10×10^{-2}	3.85×10^{-2}	7.87×10^{-3}

Power spectrum-bispectrum cross correlation

$$b^{ISW-L}_{l_1 l_2 l_3} \sim C^T_l C^{T\phi}_l \,, \Delta \equiv (C^{unlen}_l, B_{l_1 l_2 l_3}) \,, \mathcal{L} \propto \exp(\Delta \cdot CoV^{-1} \cdot \Delta^T)$$

$$\mathrm{CoV}^{-1} \equiv \frac{1}{\sigma^2_{\mathrm{P}}\sigma^2_{\mathrm{B}} - (\sigma^2_{\mathrm{PB}})^2} \begin{bmatrix} \sigma^2_{\mathrm{B},l_1 l_2 l_3} & -\sigma^2_{\mathrm{PB},l_1 l_2 l_3 l} \\ -\sigma^2_{\mathrm{PB},l l_1 l_2 l_3} & \sigma^2_{\mathrm{P},l} \end{bmatrix} .$$

$$\sigma^2_{\mathrm{PB},l l_1 l_2 l_3} \equiv \left\langle \Delta C^T_l \Delta B^{\mathrm{ISW-L}}_{l_1 l_2 l_3} \right\rangle \sim C^T_l C^{T\phi}_l C^T_l$$

$$\chi^2_{PB} \ll \chi^2_p, \chi^2_B \,,\ \chi^2 \approx \chi^2_{\mathrm{P}} + \chi^2_{\mathrm{B}} + \chi^2_{\mathrm{PB}} \,, \chi^2_{\mathrm{PB}} = \frac{-2\sigma^2_{\mathrm{PB},l l_1 l_2 l_3} \Delta C^T_l \Delta B^{\mathrm{ISW-L}}_{l_1 l_2 l_3}}{\sigma^2_{\mathrm{P},l}\sigma^2_{\mathrm{B},l_1 l_2 l_3}}$$

$$\chi^2_{PB}/\chi^2_P < 0.03$$

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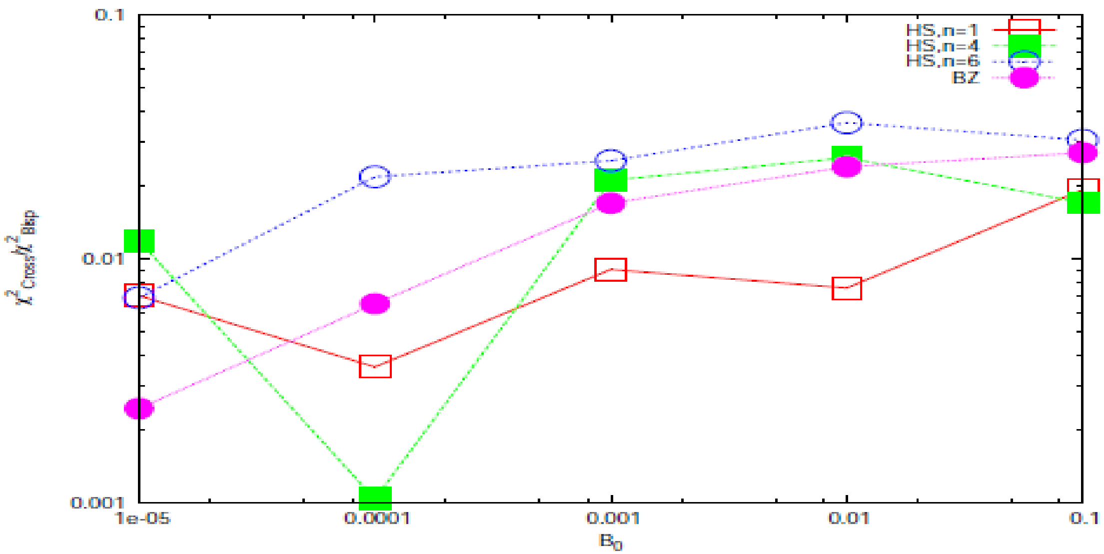

FIG. 10. Likelihood ratio between cross-correlation and bispectrum $\chi^2_{\mathrm{PB}}/\chi^2_{\mathrm{B}}$.

Conclusion

- Pow-Bisp cross correlation is negligible

- ISW-Lensing bisp is robust on constraining f(R) gravity, the joint analysis can improve the power spectrum result on B0 bound by a factor 2~3

Thank you