Leonardo Senatore (Stanford and CERN)

The Effective Field Theory of Cosmological Large Scale Structures (up to 2-loops)

with Carrasco, Foreman and Green **to appear**
with Carrasco and Hertzberg **JHEP 2012**

Cosmological Non-linearities as an Effective Fluid

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

Where do we stand with non-Gaussianities Inflation?

☹ ~No detection ☹

With Smith and Zaldarriaga,
JCAP2009
JCAP2010

Optimal analysis of Planck data are ~ compatible with Gaussianity

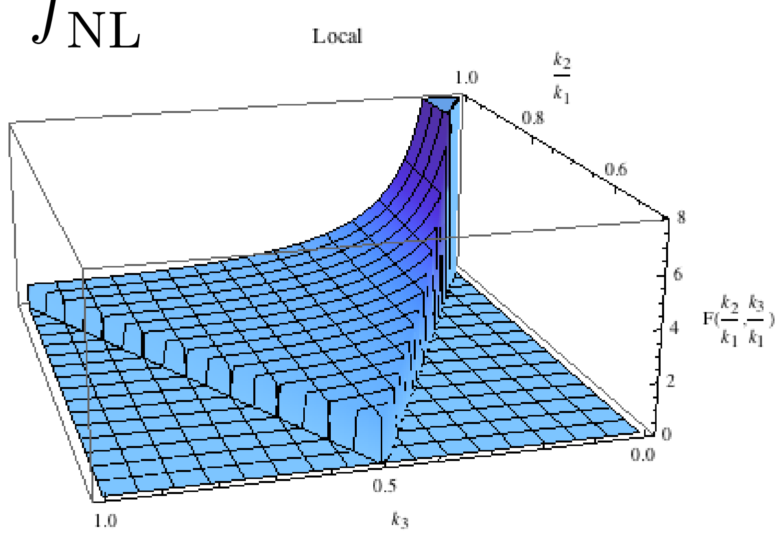
$$-1 < f_{NL}^{local} < 20 \quad \text{at } 95\% \text{ C.L.}$$

$$-187 < f_{NL}^{equil.} < 113 \quad \text{at } 95\% \text{ C.L.}$$

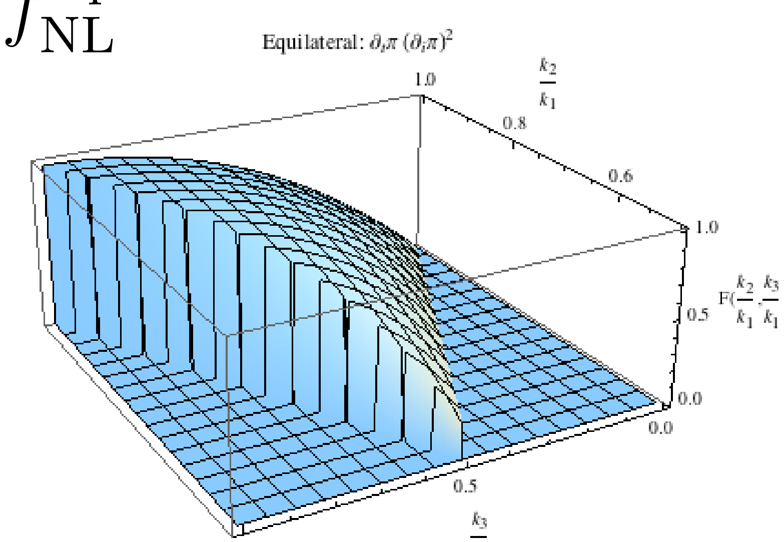
$$-124 < f_{NL}^{orthog.} < 32 \quad \text{at } 95\% \text{ C.L.}$$

Planck team 2013

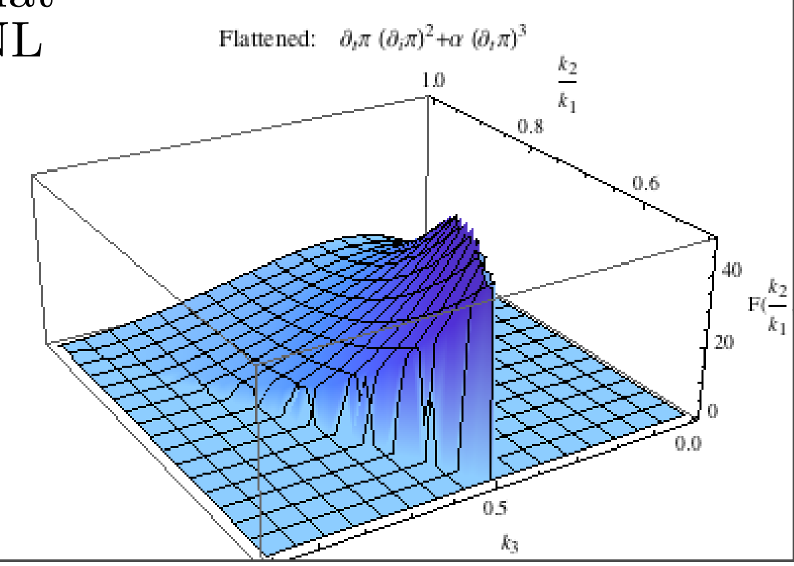
$f_{NL}^{loc.}$



$f_{NL}^{equil.}$

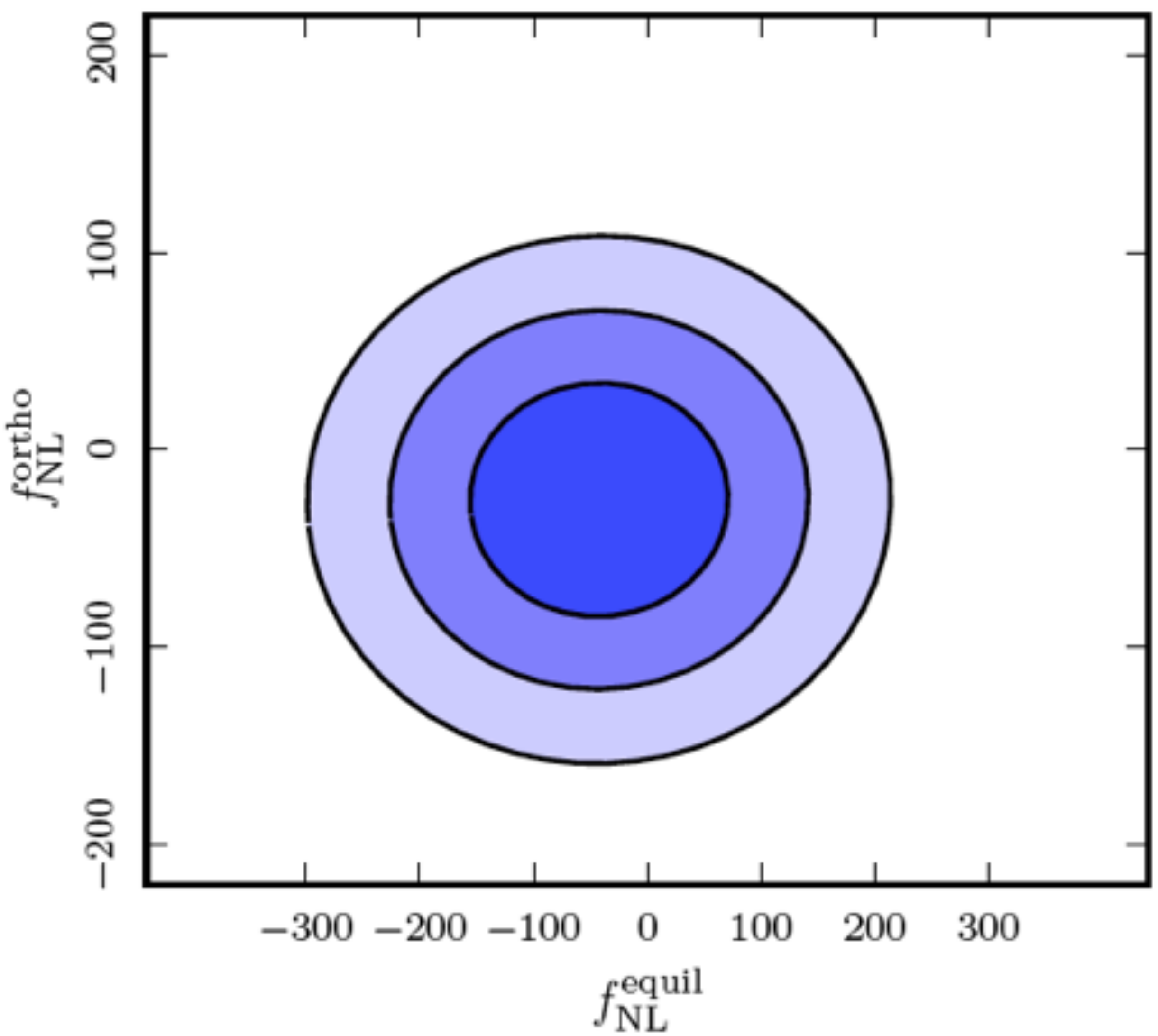


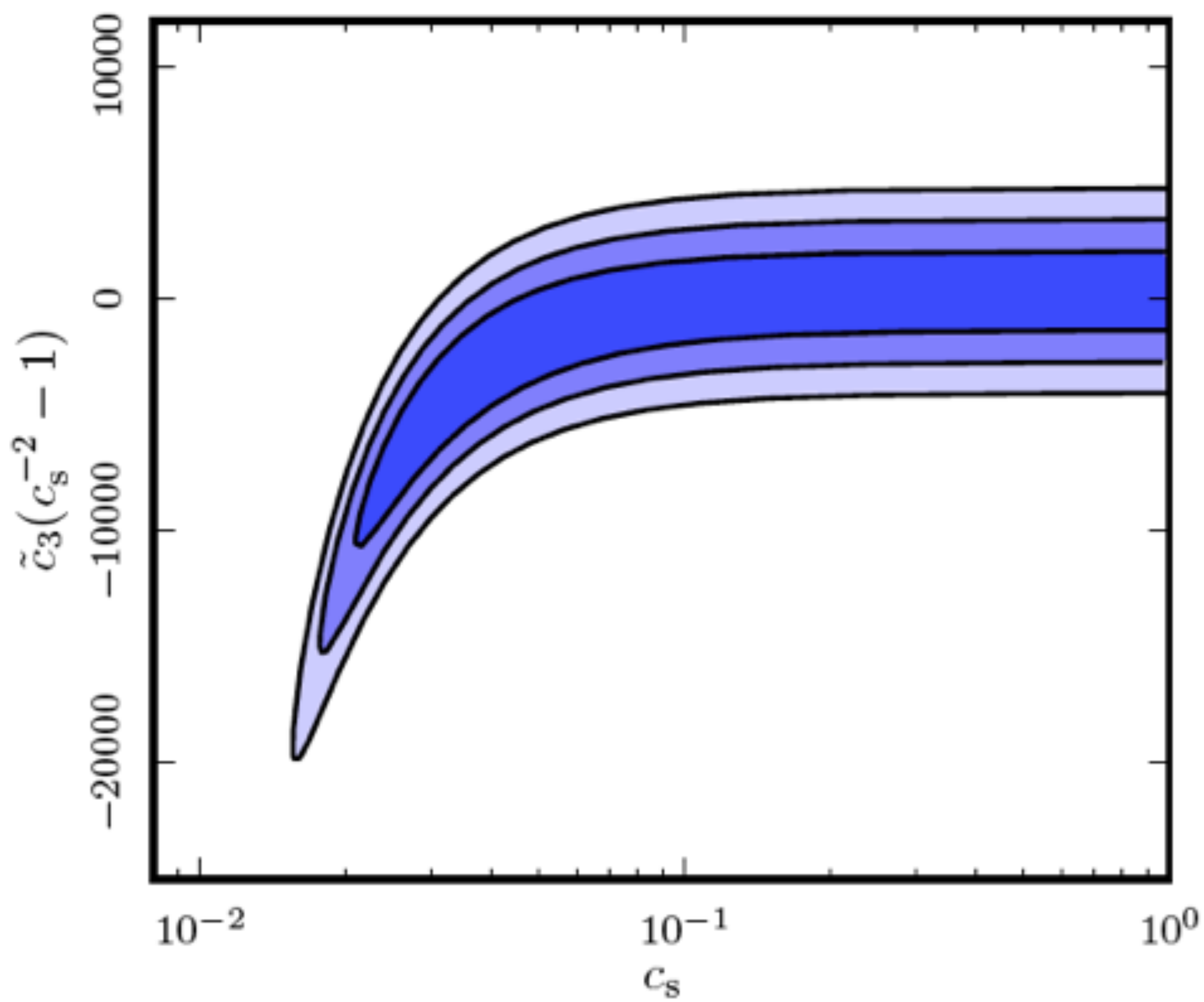
f_{NL}^{flat}



Limits in terms of parameters of a Lagrangian

- $$S = \int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot H}{c_s^2}\left(\dot\pi^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + (M_{\rm Pl}^2\dot H)\frac{1-c_s^2}{c_s^2}\left(\frac{\dot\pi(\partial_i\pi)^2}{a^2} + \frac{A}{c_s^2}\dot\pi^3\right) + \cdots\right]$$





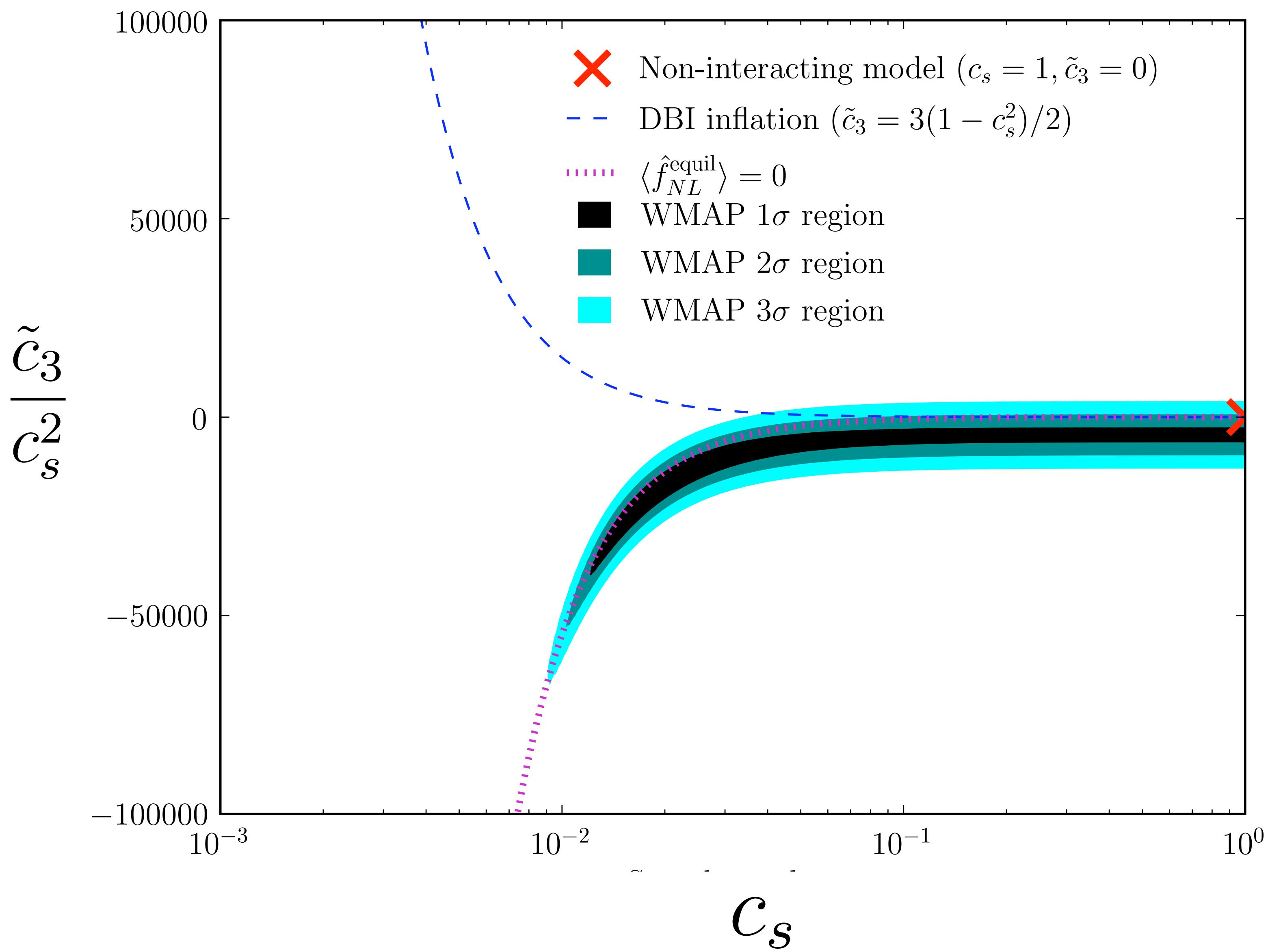
- These are contour plots of parameters of a fundamental Lagrangian
- Same as in particle accelerator Precision Electroweak Tests.
- Thanks to the EFT: A qualitatively new (and superior) way to use the cosmological data
- Universal limit $c_s \gtrsim 0.02$

with Smith and Zaldarriaga, **JCAP2010**
Planck Collaboration **2013**
see Barbieri, Giudice, Rattazzi ...

(Optimal) Limits on the parameters of the Lagrangian

$$S_\pi = \int d^4x\,\sqrt{-g}\left[M_{\rm Pl}^2\dot H\left(\dot\pi^2-(\partial_i\pi)^2\right)+M_2^4\left(\dot\pi^2+\dot\pi^3-\dot\pi(\partial_i\pi)^2\right)-M_3^4\dot\pi^3+\ldots\right]$$

- Limits on f_{NL}'s get translated into limits on the parameters



With Smith and Zaldarriaga, **JCAP2010**

Very similar in spirit to **Precision Electroweak Tests (Complete Connection to Particle Physics)**

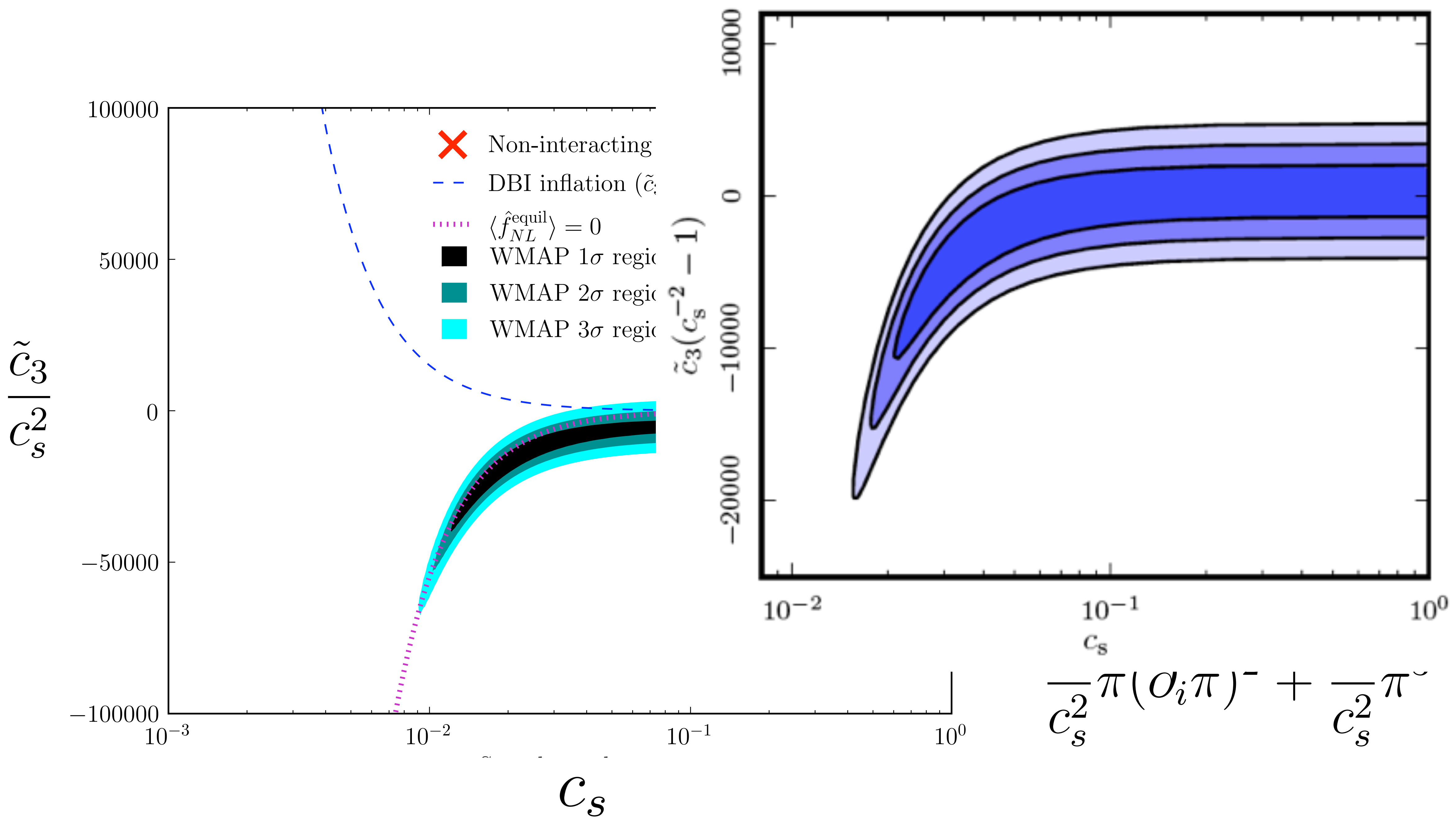
$$\frac{1}{c_s^2}\dot\pi(\partial_i\pi)^2+\frac{\tilde c_3}{c_s^2}\dot\pi^3$$

- Bound on speed of sound $c_s \gtrsim 0.011$!

(Optimal) Limits on the parameters of the Lagrangian

$$S_\pi = \int d^4x\,\sqrt{-g}\left[M_{\rm Pl}^2\dot H\left(\dot\pi^2-(\partial_i\pi)^2\right)+M_2^4\left(\dot\pi^2+\dot\pi^3-\dot\pi(\partial_i\pi)^2\right)-M_3^4\dot\pi^3+\dots\right]$$

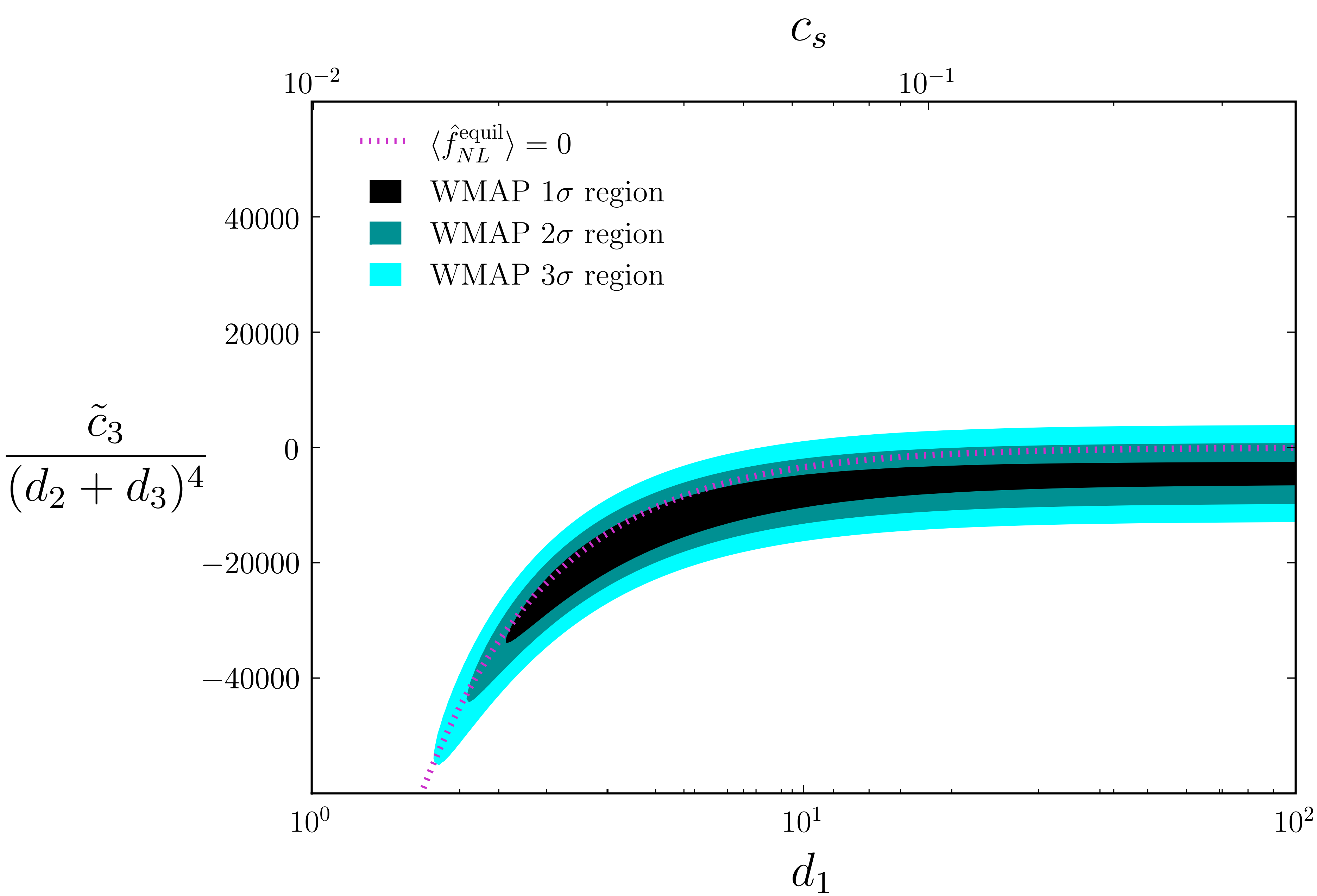
- Limits on f_{NL}'s get translated into limits on the parameters



- Bound on speed of sound $c_s \gtrsim 0.011$!

(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter. $d_1\,\delta g^{00}\delta K_i^i$
- Dispertion relation: $\omega^2 = c_s^2 k^2$ $\quad c_s^2 = d_1 \frac{H}{M} \ll 1$



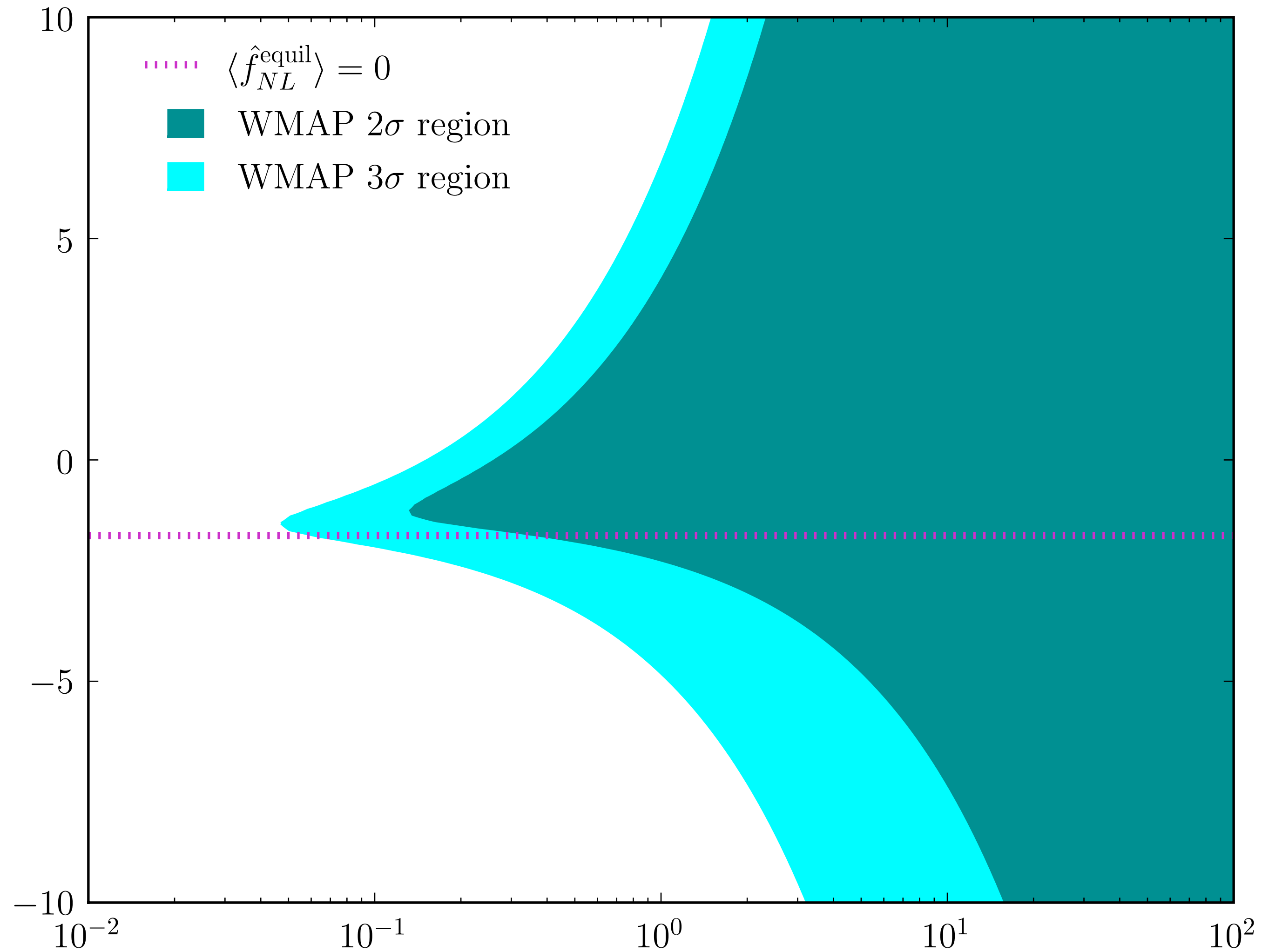
With Smith and Zaldarriaga, **JCAP2010**

Very similar in spirit to **Precision Electroweak Tests (Complete Connection to Particle Physics)**

(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter. $d_2\,\delta K_i^{i\,2}$
- Dispertion relation: $\omega^2 = (d_2 + d_3)\dfrac{k^4}{M^2}$

$$\frac{d_1}{(d_2+d_3)^{1/2}}$$



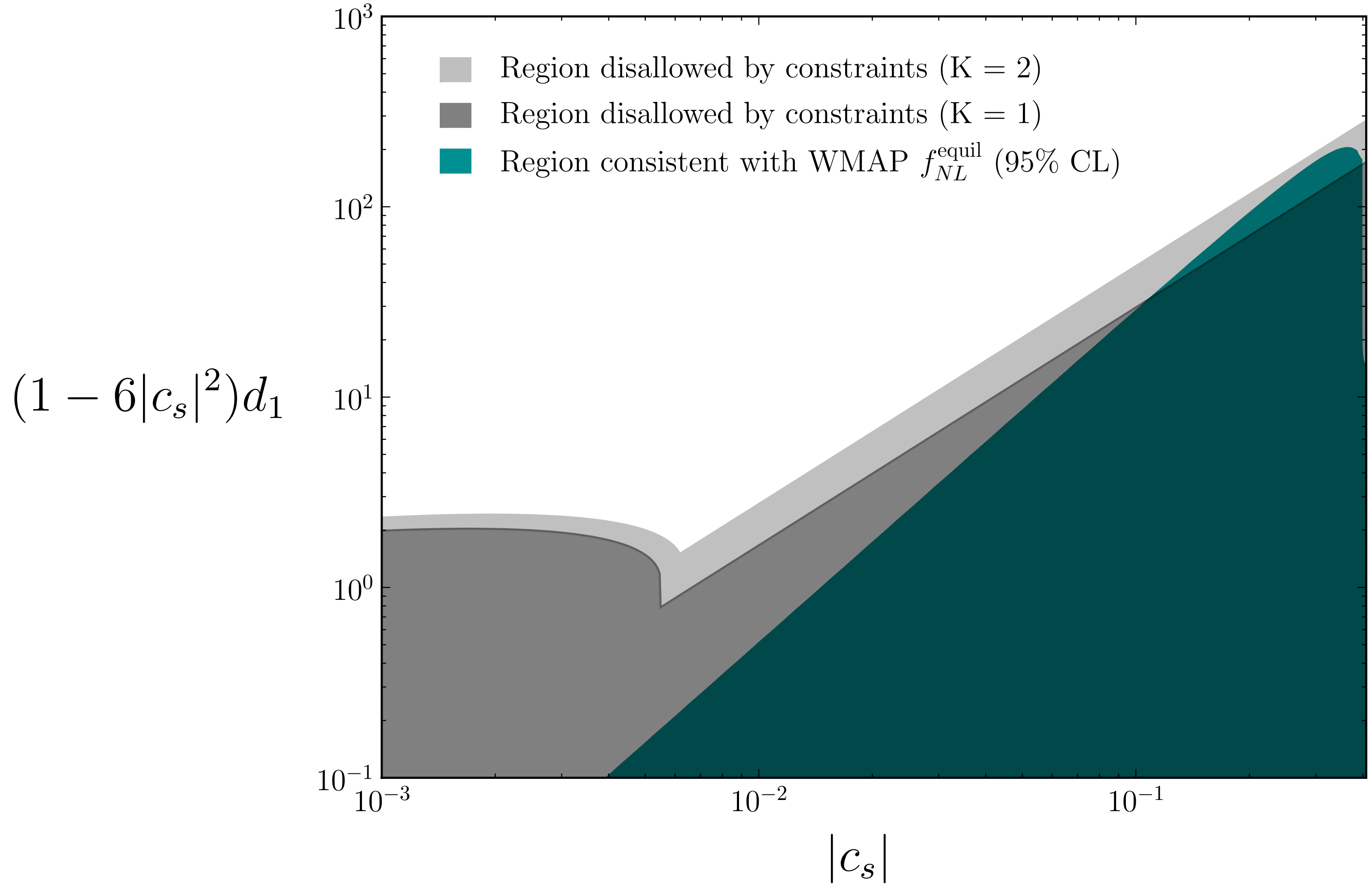
$(d_2 + d_3)$

With Smith and Zaldarriaga, **JCAP2010**

Very similar in spirit to **Precision Electroweak Tests** **(Complete Connection to Particle Physics)**

(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter.
- Negative c_s^2 due to $d_1 < 0$ $\qquad c_s^2 = d_1 \frac{H}{M} \ll 1$
- Ruled out at 95% CL.

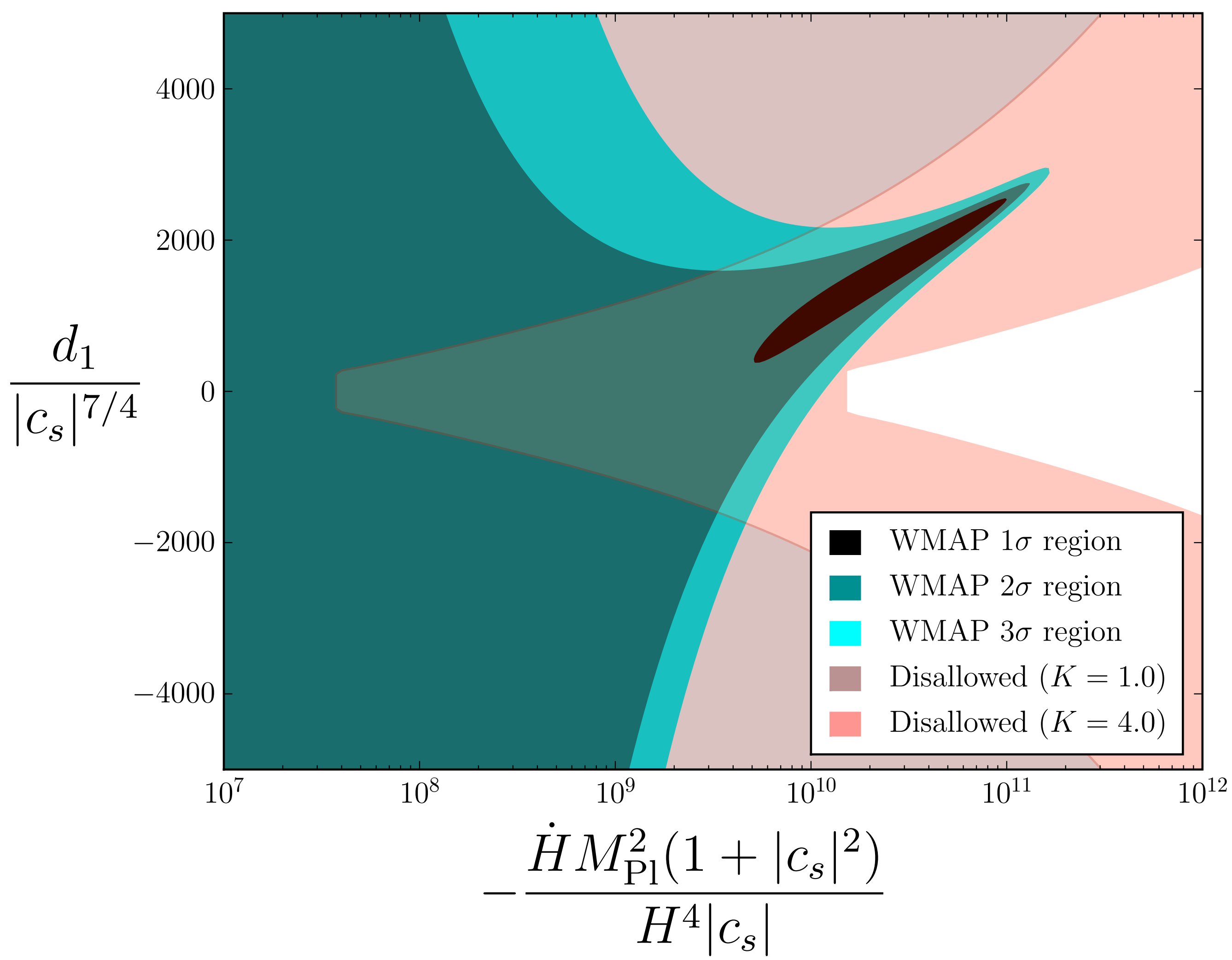


With Smith and Zaldarriaga, **JCAP2010**

Very similar in spirit to **Precision Electroweak Tests** **(Complete Connection to Particle Physics)**

(Optimal) Limits on the parameters of the Lagrangian

- Close to de Sitter.
- Negative c_s^2 due to $\dot{H} > 0$ $\qquad \dot{H} M_{\rm Pl}^2 (\partial_i \pi)^2$
- Ruled out at 95% CL.



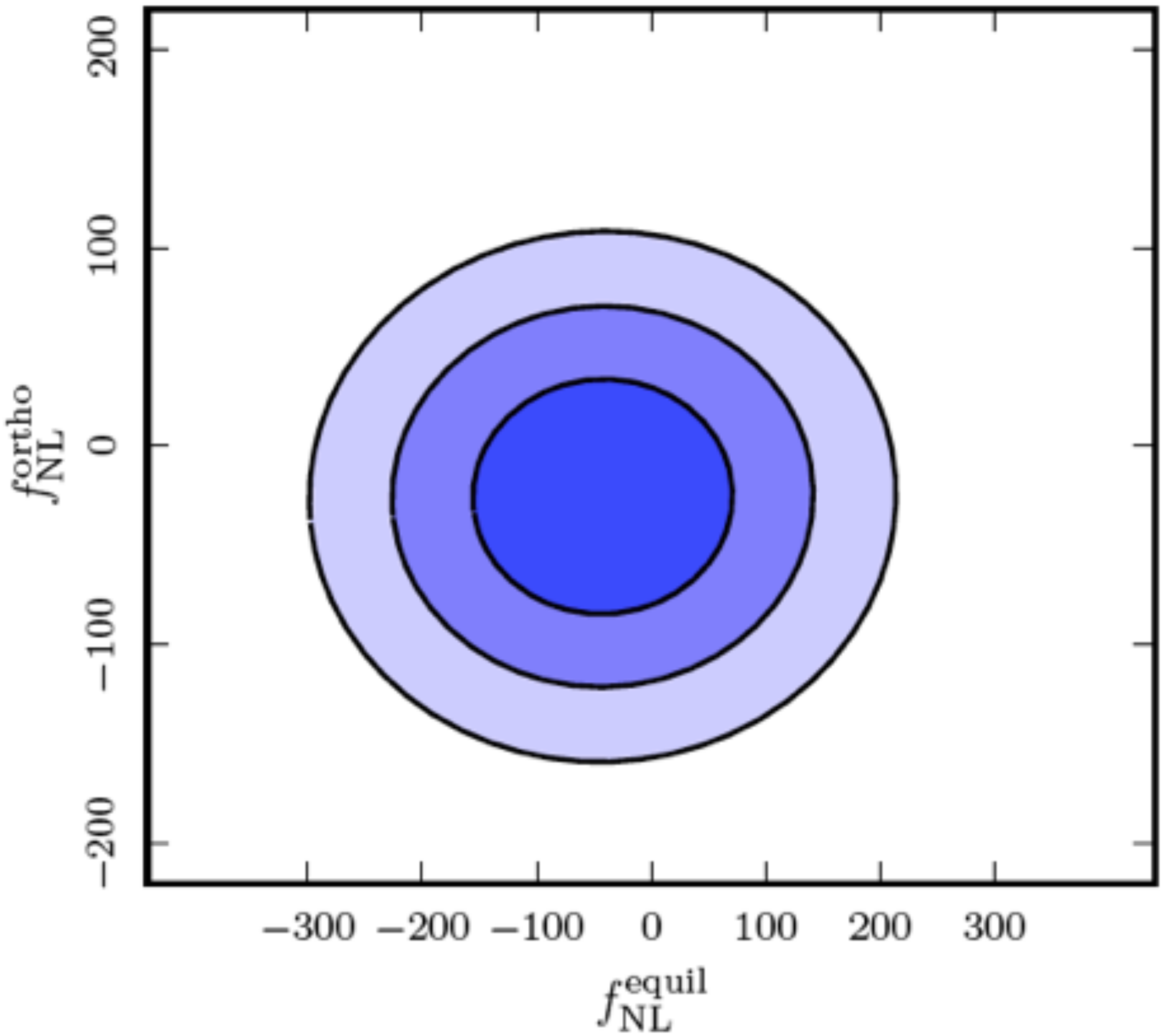
With Smith and Zaldarriaga, **JCAP2010**

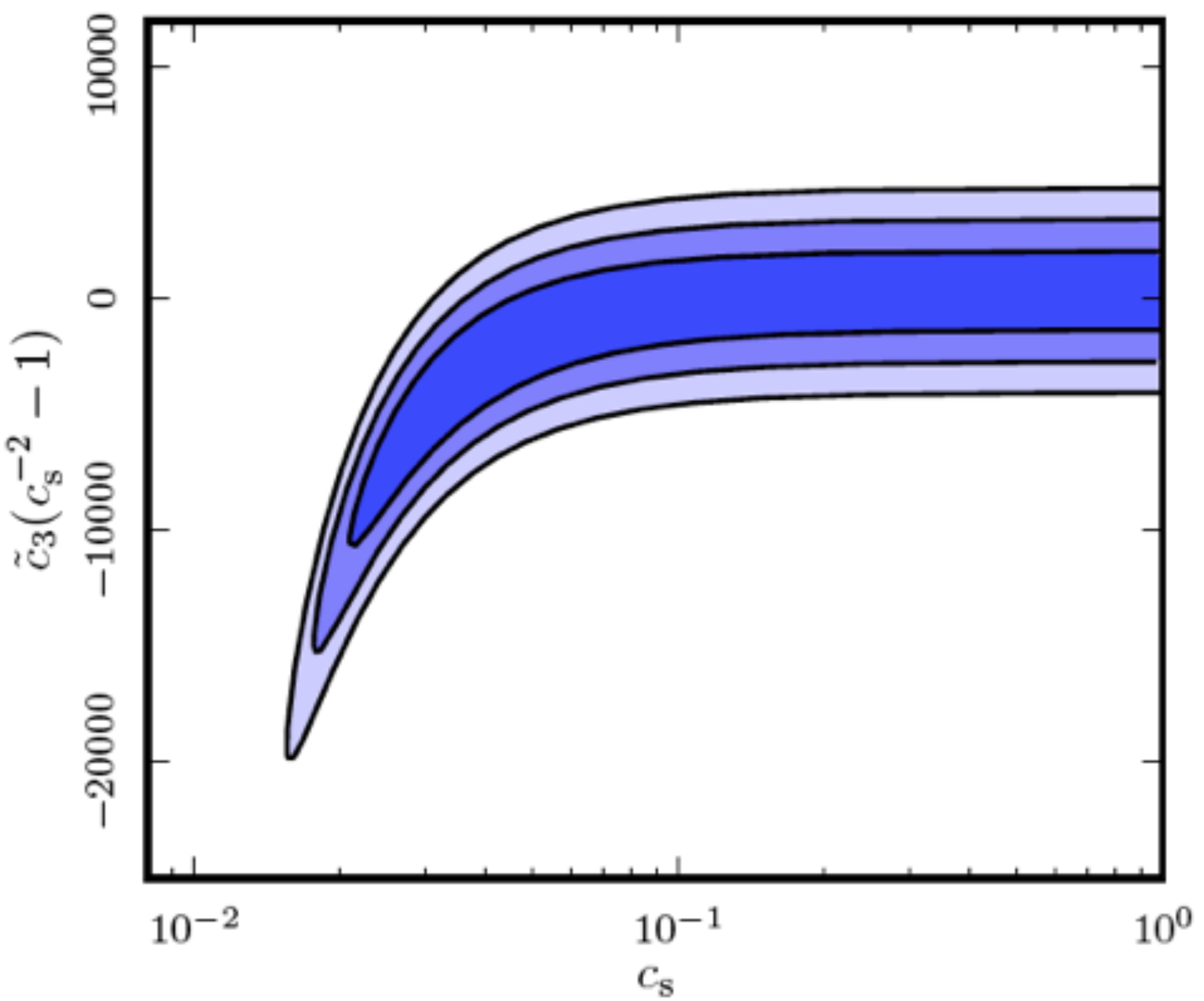
Very similar in spirit to **Precision Electroweak Tests (Complete Connection to Particle Physics)**

Limits in terms of parameters of a Lagrangian

- The Effective Field Theory of Inflation

$$S=\int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot H}{c_s^2}\left(\dot\pi^2-c_s^2\frac{(\partial_i\pi)^2}{a^2}\right)+(M_{\rm Pl}^2\dot H)\frac{1-c_s^2}{c_s^2}\left(\frac{\dot\pi(\partial_i\pi)^2}{a^2}+\frac{A}{c_s^2}\dot\pi^3\right)+\cdots\right]$$





with Smith and Zaldarriaga, **JCAP2010**
Planck Collaboration **2013**

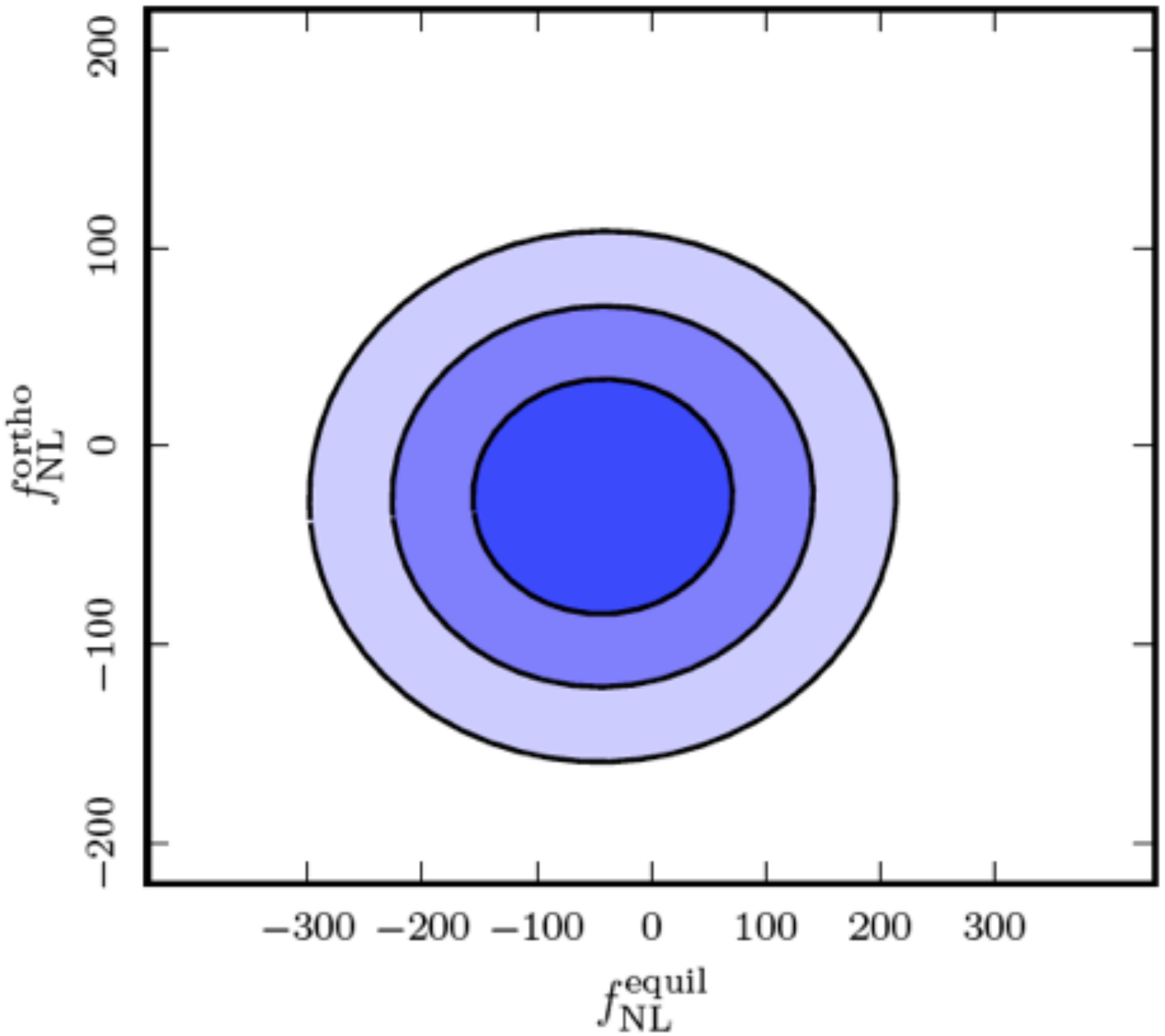
- This is great, but the phenomenology is reacher
- Cutoff $\quad \frac{\dot\pi_c^3}{\Lambda_U^2} \quad \Rightarrow \quad {\rm NG}\simeq f_{\rm NL}\zeta\sim\frac{H^2}{\Lambda_U^2}$

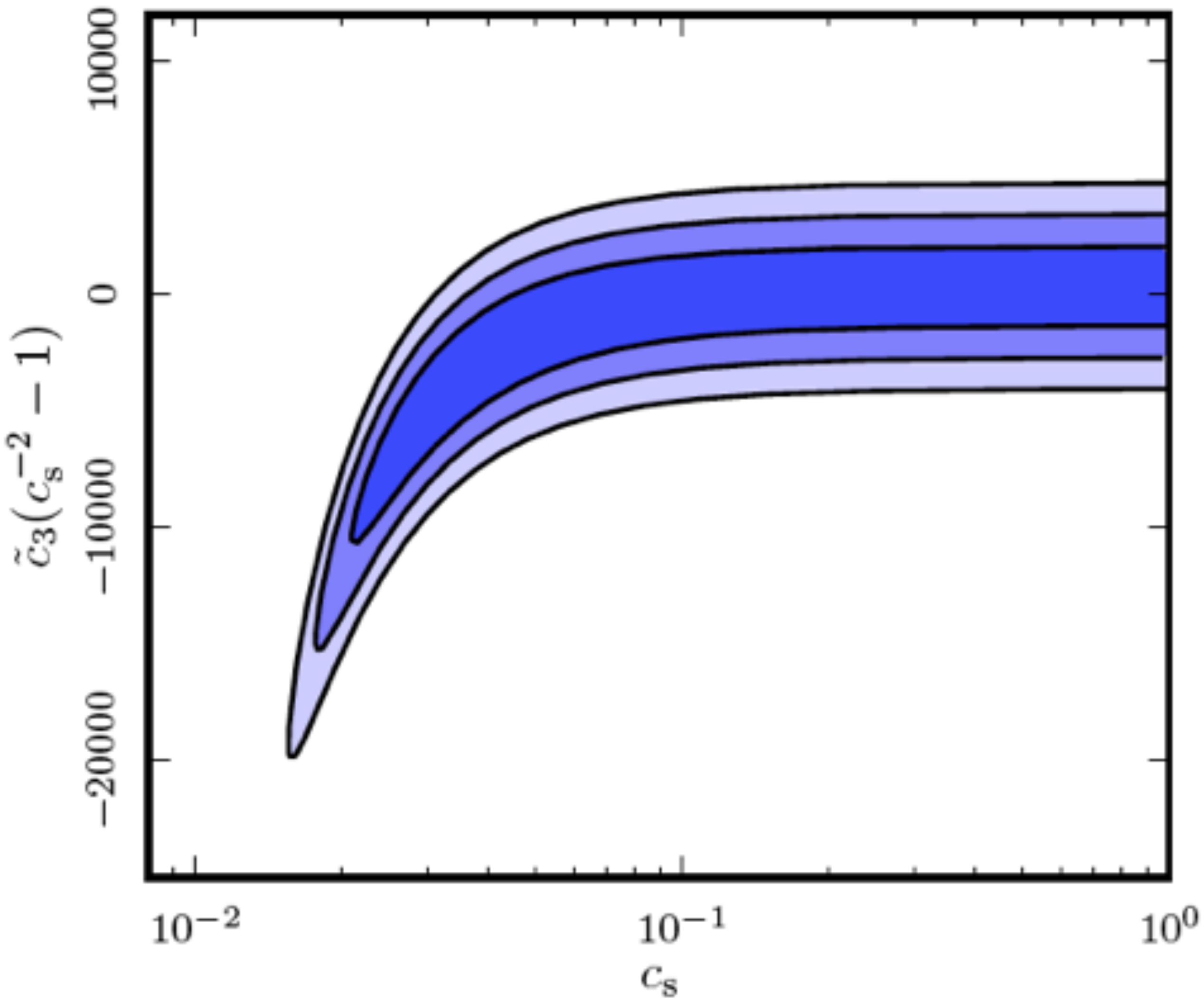
$$\Lambda_U^2\gtrsim\Lambda_{\rm min}^2\simeq 10^4H^2$$

Limits in terms of parameters of a Lagrangian

- The Effective Field Theory of Inflation

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) + (M_{\rm Pl}^2 \dot{H}) \frac{1-c_s^2}{c_s^2} \left(\frac{\dot{\pi}(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^3 \right) + \cdots \right]$$





with Smith and Zaldarriaga, **JCAP2010**
Planck Collaboration **2013**

- This is great, but the phenomenology is reacher
- Cutoff $\frac{\dot{\pi}_c^3}{\Lambda_U^2} \quad \Rightarrow \quad \text{NG} \simeq f_{\rm NL}\zeta \sim \frac{H^2}{\Lambda_U^2}$

$$\Lambda_U^2 \gtrsim \Lambda_{\rm min}^2 \simeq 10^4 H^2$$

What has Planck done to NG?

(that is to one of two main ways to test inflation)

Let us look at LHC

- Two thresholds for detection. Awesome!
- By unitarity of WW scattering

$$\Lambda_U \sim \frac{m_W}{g} \lesssim 1 \text{ TeV} \quad \Rightarrow \quad m_{\text{Higgs}} \sim g_{\text{weak}} \times 1 \text{ TeV} \ll 1 \text{ TeV}$$

  – Something was guaranteed

- If Higgs found, then tuning problem:

$$\delta m_{\text{Higgs, quantum}} \sim \Lambda_U^{\text{new}} \quad \Rightarrow \quad \text{New Physics (or new principle) guaranteed}$$

- So, with LHC (or SSC), huge learning guaranteed
  – 1 TeV is a threshold for discovery

Let us go to NG

- Threshold for detections $\qquad \dfrac{\dot\pi_c^3}{\Lambda_U^2} \quad \Rightarrow \quad \text{NG} \simeq f_{\rm NL}\zeta \sim \dfrac{H^2}{\Lambda_U^2}$

$$\Lambda_U \lesssim \Lambda_{U,\,\text{threshold}} \quad \Rightarrow \quad f_{\rm NL} \gtrsim \frac{H^2}{\Lambda^2_{U,\,\text{threshold}}}$$

- We do not have a compelling threshold (we just make them possible!)
- We have lower bound: $\Lambda_{U,\,\text{threshold}} \gtrsim H \quad \Rightarrow \quad f_{\rm NL} \lesssim 10^5$
  - This is the only correct prediction of Inflation on NG: weakly coupled field theory
- Minimal size of NG: from gravity

Maldacena
JCAP2003

$$f_{\text{NL, minimal}} \sim \epsilon \sim 10^{-2} \ll 10 \sim f_{\text{NL, Planck}}$$

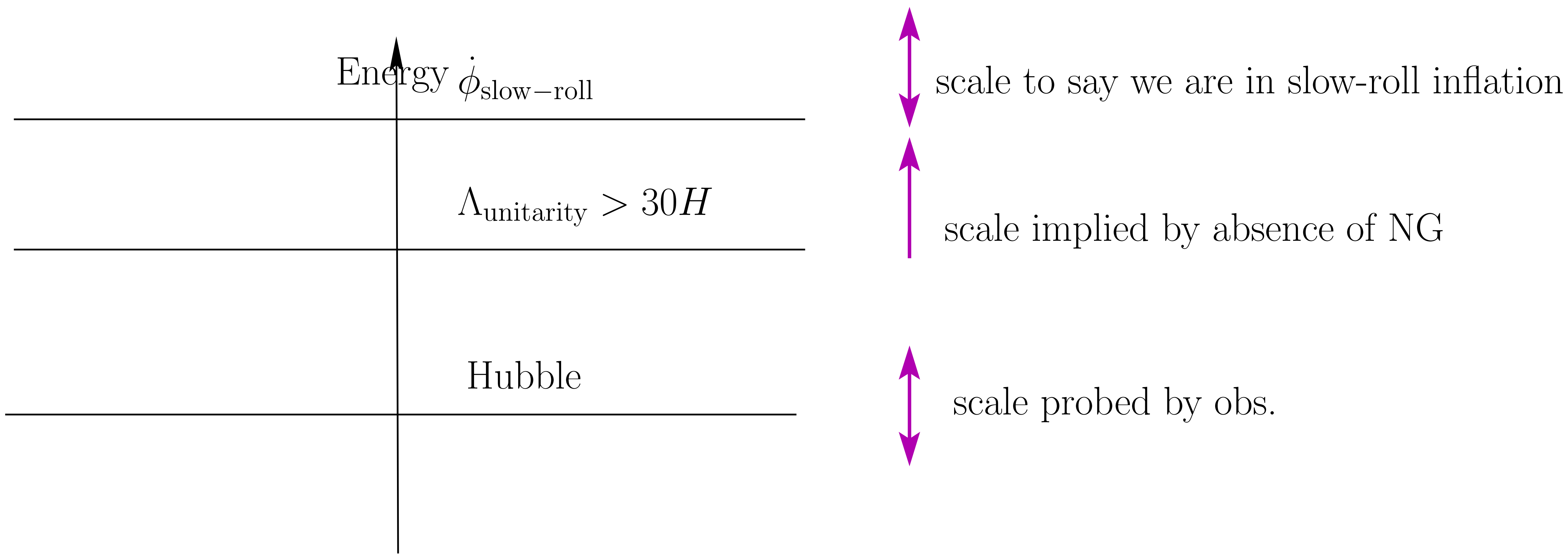
- Another threshold is

$$f_{\rm NL}^{\text{equil., orthog.}} \sim 1 \quad \Rightarrow \quad \Lambda_U^4 \gtrsim \dot H M_{\rm Pl}^2 \sim \dot\phi^2_{\text{slow}-\text{roll}}$$

  - With this we would be allowed to glue the EFT to slow-roll inflation
    - the bottom-up `verification' of slow-roll inflation (with assumption)
  - this is more than a factor of 10 far away.

Energy Scales to probe

- .

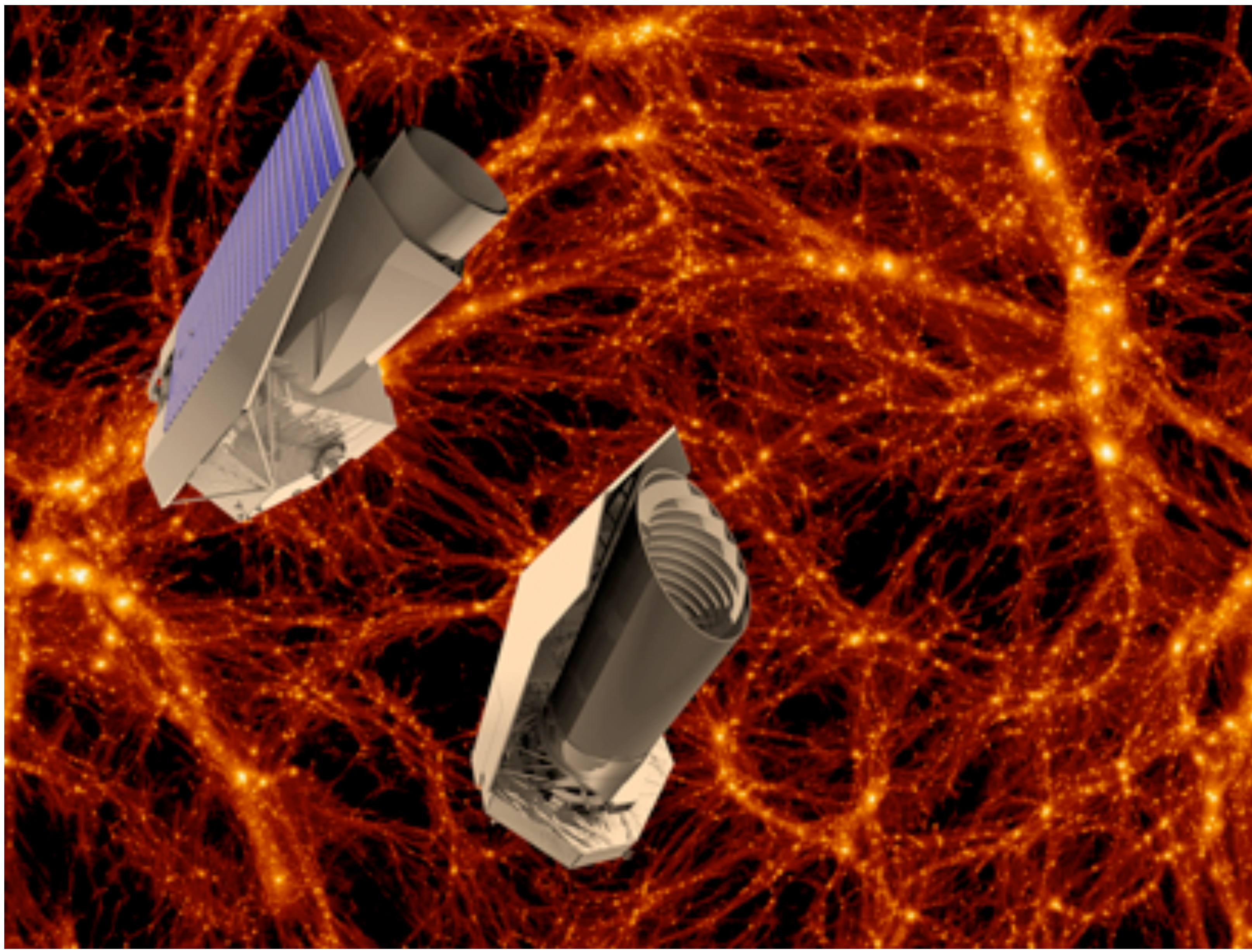


What has Planck done to theory?

- Planck improve limits wrt WMAP by a factor of ~3.
- Since $$\mathrm{NG} \sim \frac{H^2}{\Lambda_U^2} \quad \Rightarrow \quad \Lambda_U^{\mathrm{min,\,Planck}} \simeq 2\,\Lambda_U^{\mathrm{min,\,WMAP}}$$
- Given the absence of known or nearby threshold, this is not much.
- Planck is great
- but Planck is not good enough
  - not Plank's fault, but Nature's faults
    - Please complain with Nature
- Planck was an opportunity for a detection, not much an opportunity to change the theory in absence of detection
- On theory side, little changes
  - contrary for example to LHC, where any result is changing the theory

What is next?

- Plank will increase by a factor of less than 2.
- Next are Large Scale Structures
- Like moving from LEP to LHC



What is next?

- Forecasts

$$\Delta f_{\rm NL}^{\rm equil.,\,orthog.}({\rm Planck}) \sim 75$$
$$\Delta f_{\rm NL}^{\rm equil.,\,orthog.}({\rm Euclid}) \sim 10$$

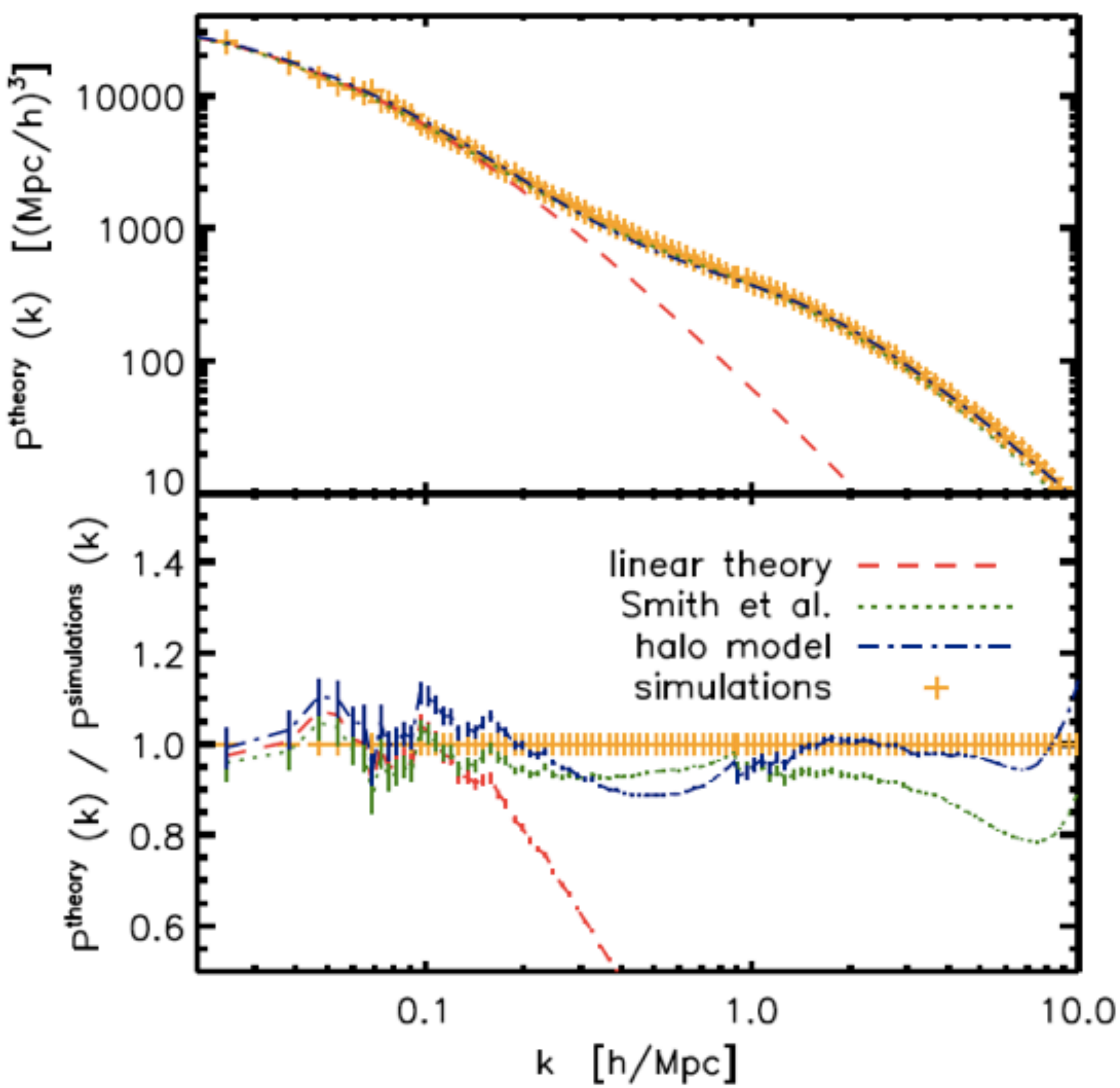
$$\text{Improvement} \sim 7$$

- They use

$$k_{\rm max} \simeq 0.1\, h\, {\rm Mpc}^{-1}$$

- But the theory is probably wrong
  - (to me)

Sefusatti and Komatsu **2007**



Leonardo Senatore (Stanford and CERN)

The Effective Field Theory of Cosmological Large Scale Structures

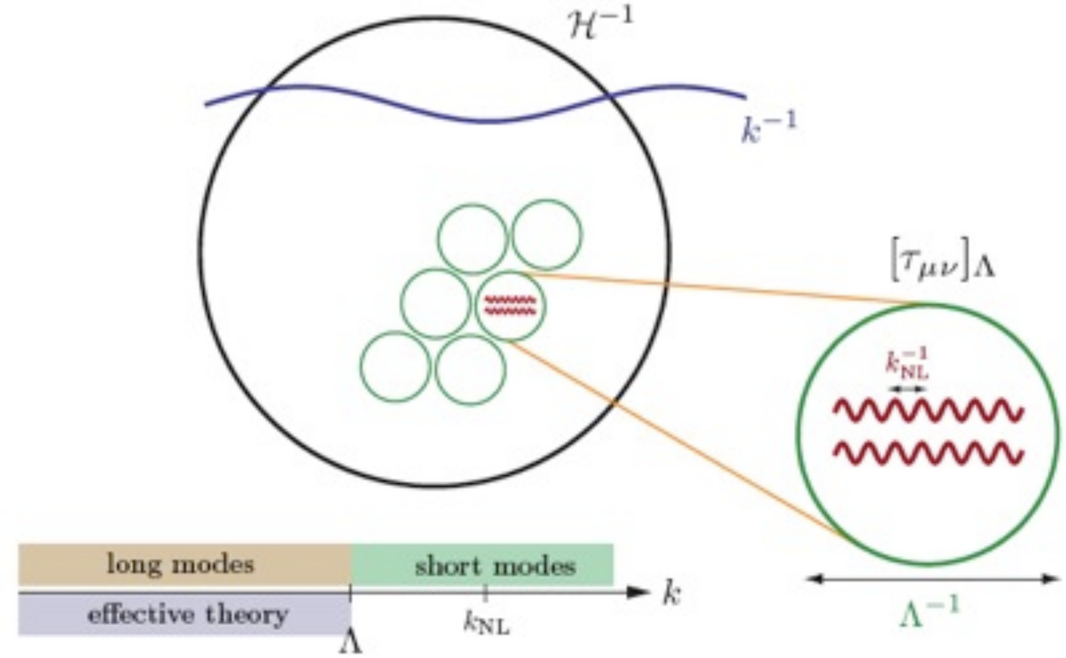
with Carrasco and Hertzberg **JHEP 2012**
with Carrasco, Foreman and Green

Cosmological Non-linearities as an Effective Fluid

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

Our Universe as a Chiral Lagrangian

- How does our universe looks like?

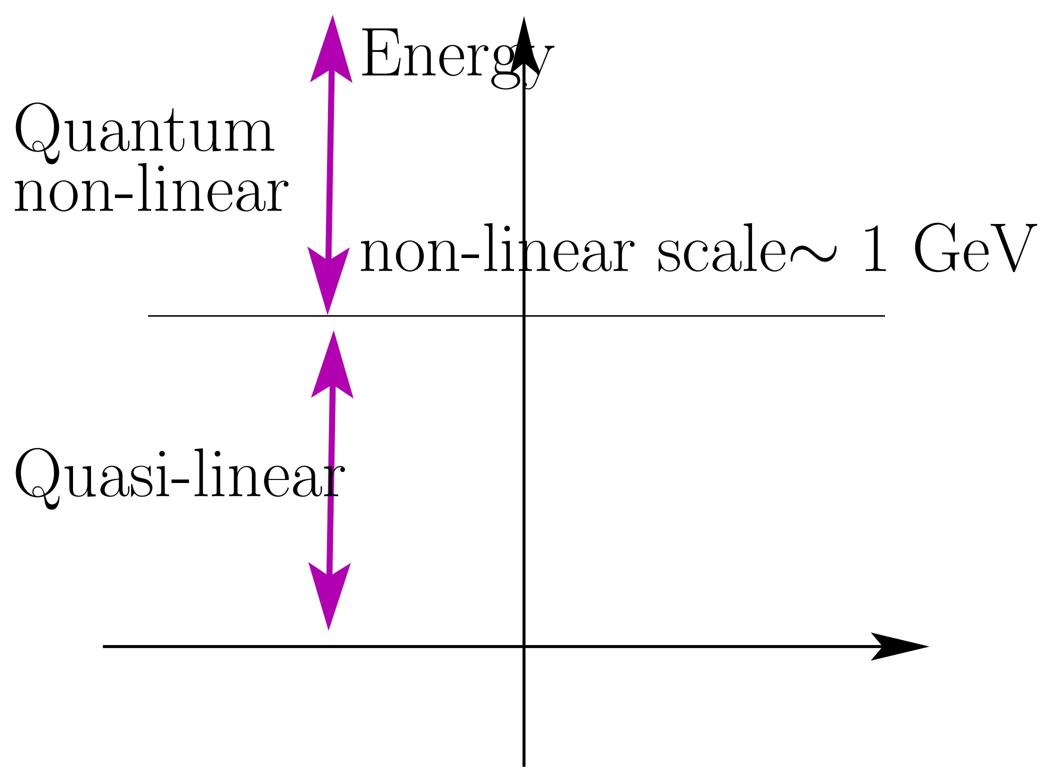


- Non-linear on short scales $\lambda_{NL} \sim 1 - 10$ Mpc $\delta\rho/\rho \gg 1$
- Linear on large-scales

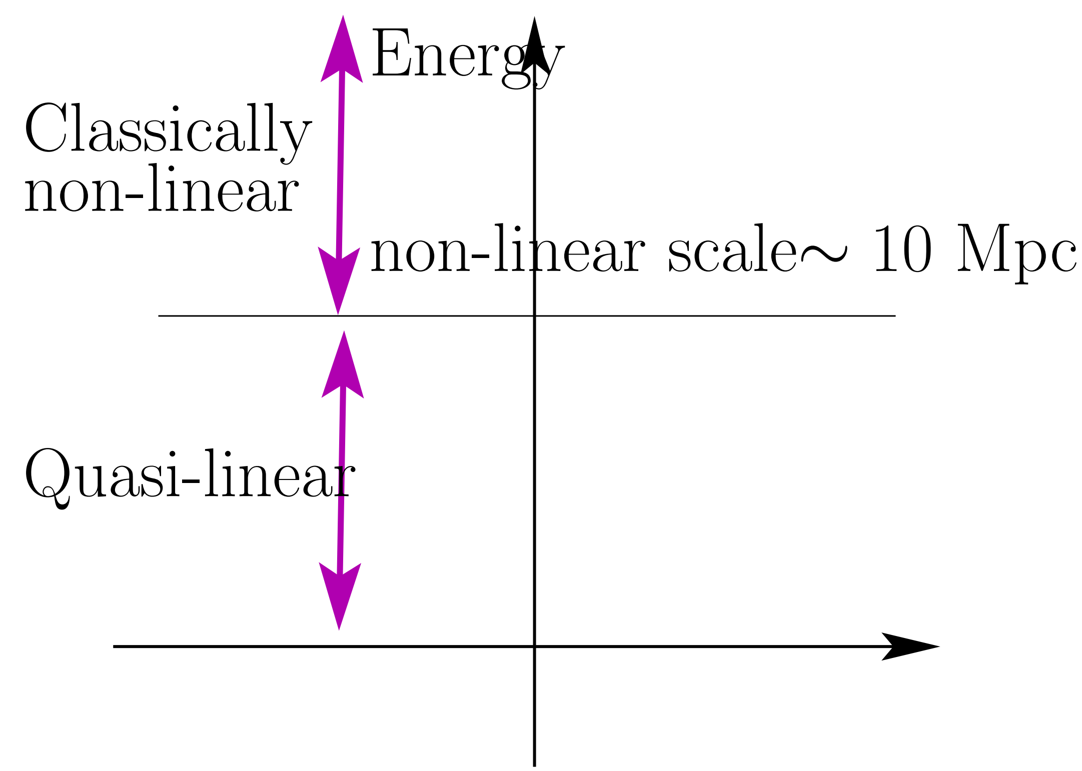
$H^{-1} \sim 14000$ Mpc $\qquad \delta\rho/\rho \ll 1$

- Similar to Chiral Lagrangian

Chiral Lagrangian



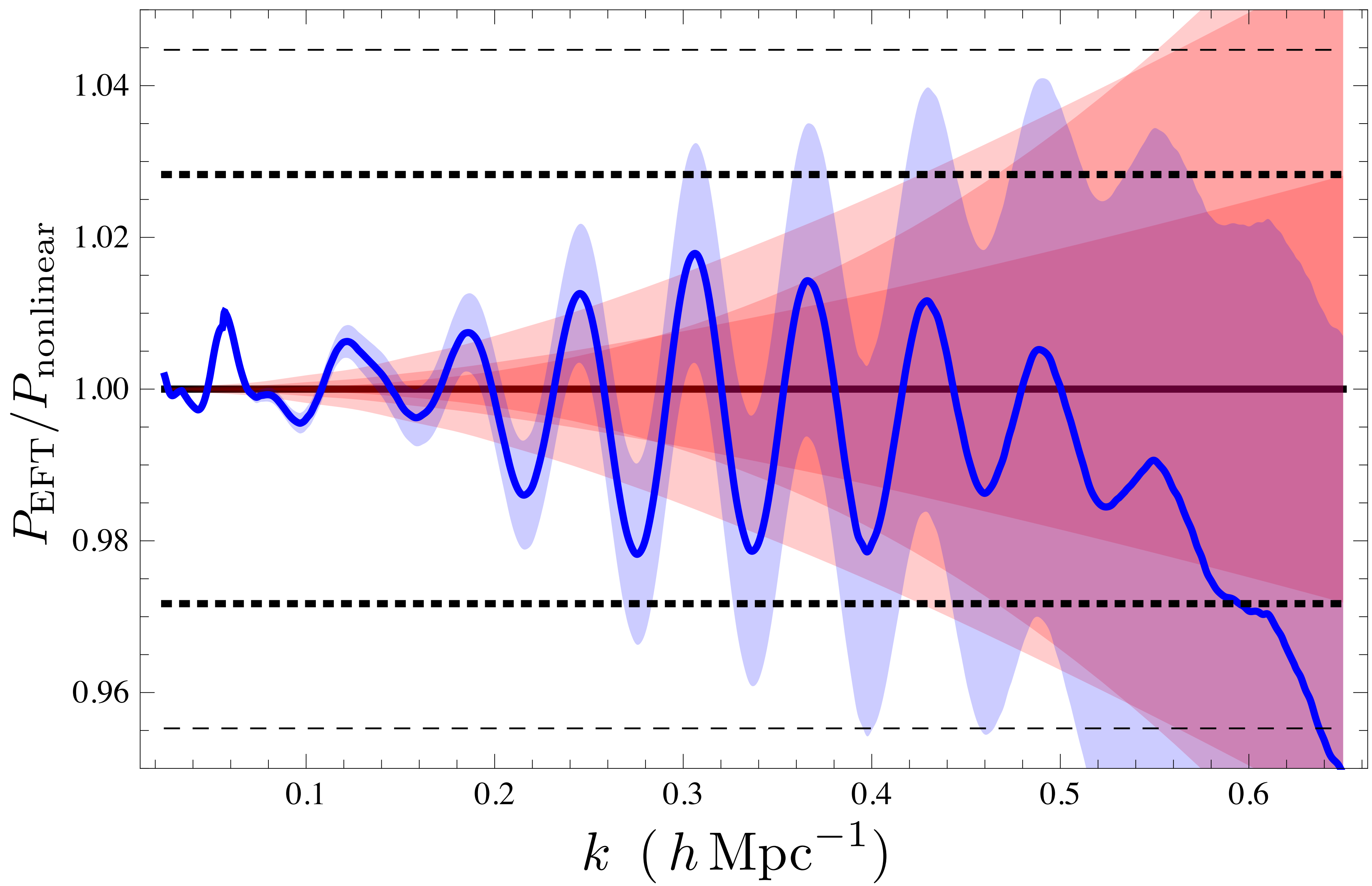
Universe



- Universe as an Effective Fluid with higher derivative stress-tensor in expansion in $k/k_{\rm NL}$

A much higher kmax

- So far predictions studied with the wrong theory
- At 2.5 loops (using loops, counterterms, matching, etc. on astro scales!!)



- We reach $$k_{\rm max} \simeq 0.6\, h\,{\rm Mpc}^{-1}$$

Consequences

Big Improvement!

- So far predictions studied with the wrong theory
- Next are Large Scale Structures

$$\Delta f_{\rm NL}^{\rm equil.,\,orthog.}({\rm Planck}) \sim 75$$
$$\Delta f_{\rm NL}^{\rm equil.,\,orthog.}({\rm Euclid}) \sim 10$$

Sefusatti and Komatsu **2007**

$$\text{Improvement} \sim 7$$

- If we use

$$k_{\rm max} \simeq 0.6\, h\, {\rm Mpc}^{-1}$$

- and I rescale by $\left(\frac{{k_{\rm max}}^{\rm EFT}}{{k_{\rm max}}^{\rm old}}\right)^{\frac{3}{2}} \sim \left(\frac{0.6}{0.1}\right)^{\frac{3}{2}} \simeq 16$
- We get $\text{New Improvement} \simeq 7 \quad \to \quad 110$

- And this is good. This is a lot

Big Improvement!

- So far predictions studied with the wrong theory
- Next are Large Scale Structures

$$\Delta f_{\rm NL}^{\rm equil.,\, orthog.}({\rm Planck}) \sim 75$$
$$\Delta f_{\rm NL}^{\rm equil.,\, orthog.}({\rm Euclid}) \sim 10$$

Sefusatti and Komatsu **2007**

$$\text{Improvement} \sim 7$$

- They use

$$k_{\rm max} \simeq 0.6\, h\, {\rm Mpc}^{-1}$$

- If I rescale by $\left(\frac{{k_{\rm max}}^{\rm EFT}}{{k_{\rm max}}^{\rm old}}\right)^{\frac{3}{2}} \sim \left(\frac{0.6}{0.1}\right)^{\frac{3}{2}} \simeq 16$
- We get $\text{New Improvement} \simeq 7 \quad \to \quad 110$
- And this is good. This is a lot

Big Improvement!

- With $\text{New Improvement} \simeq 7 \quad \to \quad 110$
- We get
  - With no detection:
    - . $f_{\rm NL}^{\rm loc.} \lesssim 1$
      - Good for testing multifield: practically ruled out
    - . $f_{\rm NL}^{\rm equil.,\, orthog.} \lesssim 1 \quad \Rightarrow \quad c_s^2 \simeq 1$
      - Making the speed of sound order 1
      - Making $\Lambda_U \sim \dot H M_{\rm Pl}^2 \sim \dot\phi^2_{\rm slow\ \ roll}$
        - » We would be allowed to believe in slow-roll
- And most importantly,
  - A very decent shot at a detection!
  - which of course is revolutionary
- With this, we improve even with DES, HEDTEX, that are happening now.

A challenge for the astro theorists!

- This is the potential $\text{New Improvement} \simeq 2.5 \quad \to \quad 15$
  - If not more.
- The problem of Dark Matter clustering is being successfully addressed
  - Thanks to the EFT of LSS
- Can we manage the other ASTRO problems:
  - Halo Bias, Galaxy Bias, non-local Bias, Finger of God, Baryons, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc, etc.
- A lot to understand, but this is what is at stakes.
- It an opportunity
  - and a challenge

Leonardo Senatore (Stanford and CERN)

The Effective Field Theory of Cosmological Large Scale Structures up to 2-loops

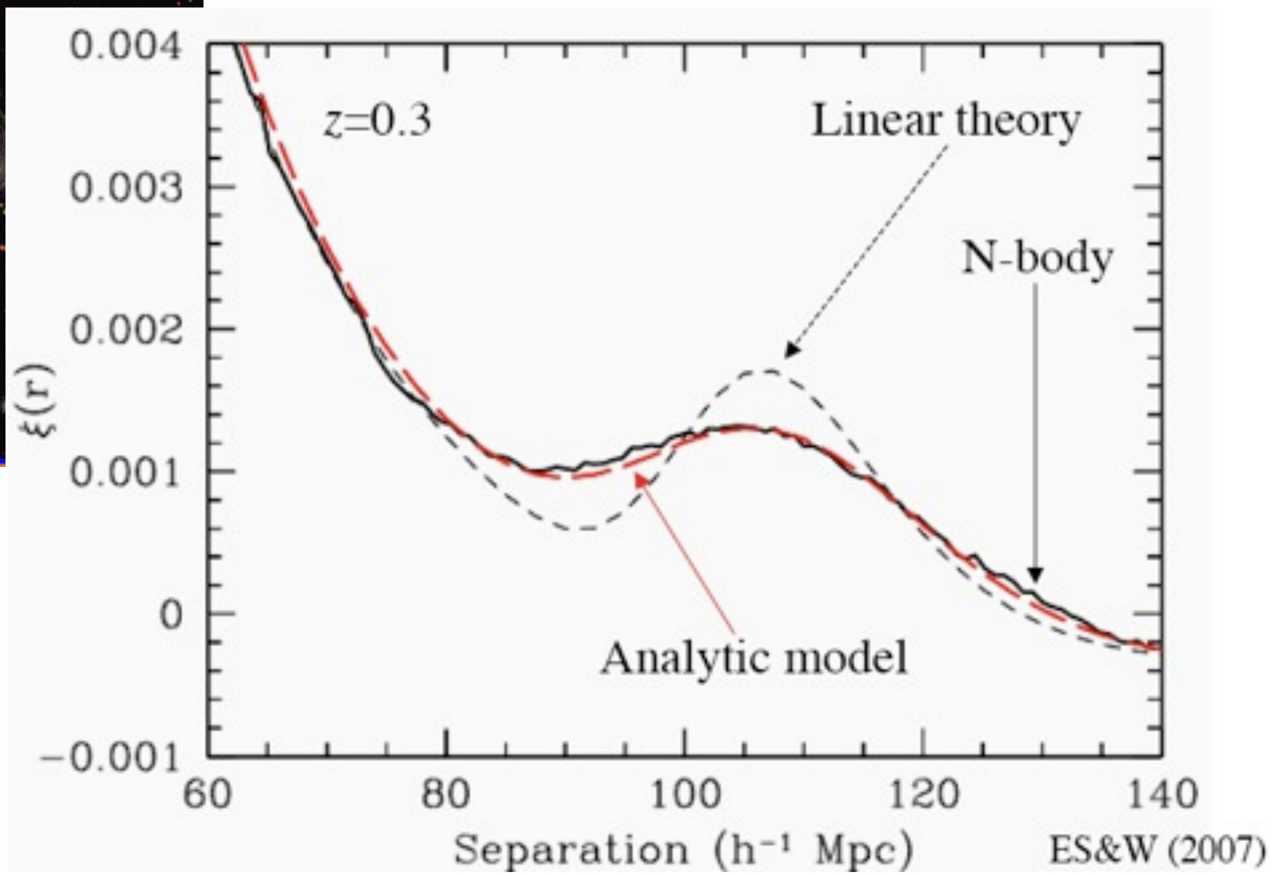
with Carrasco and Hertzberg **JHEP 2012**
with Carrasco, Foreman and Green

Cosmological Non-linearities as an Effective Fluid

with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

A well defined perturbation theory for LSS Surveys

- Observe the correlation of Galaxies

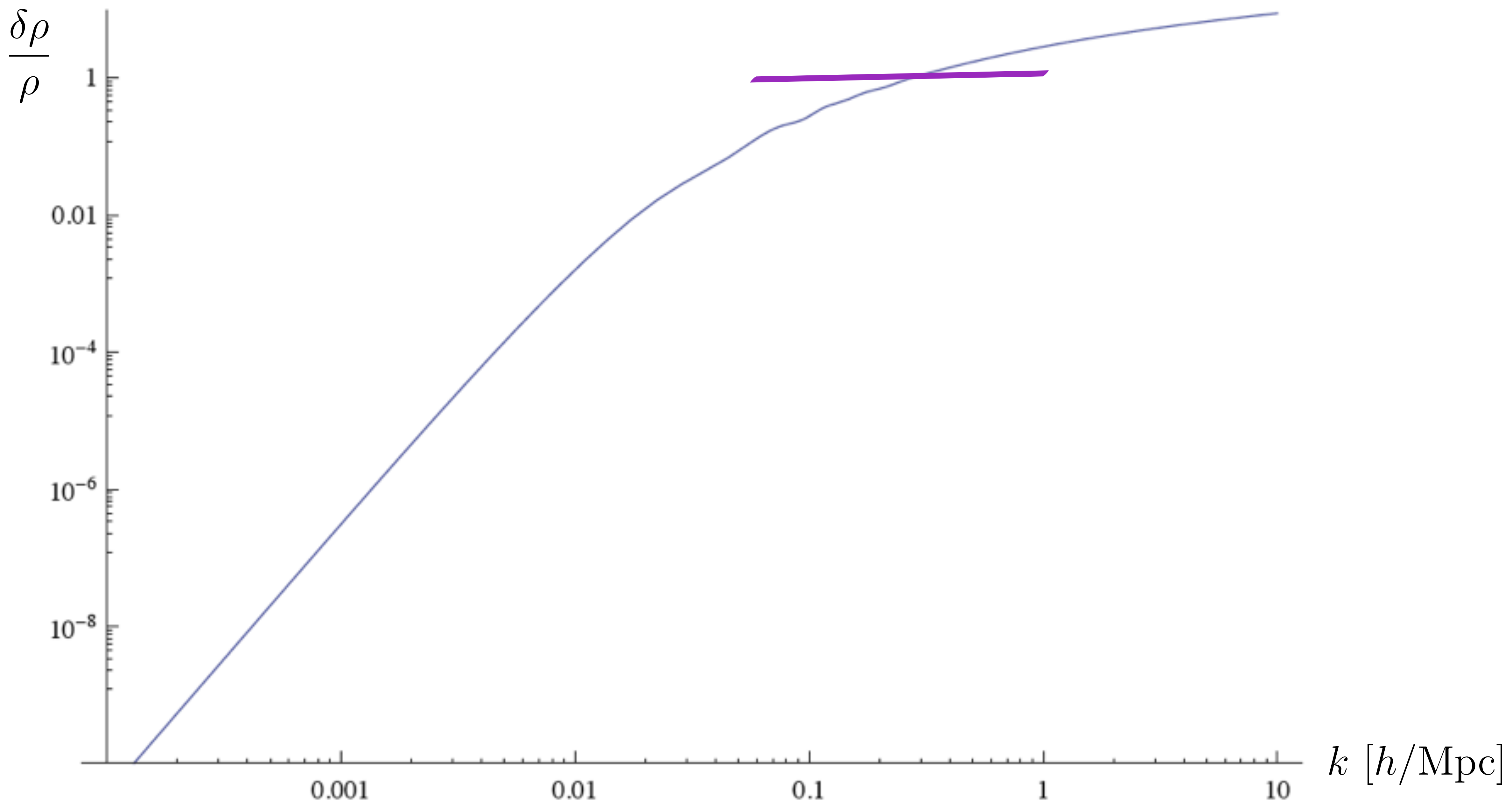


Analogous of CMB peaks

- Information about Dark Energy, Non-Gauss, ...

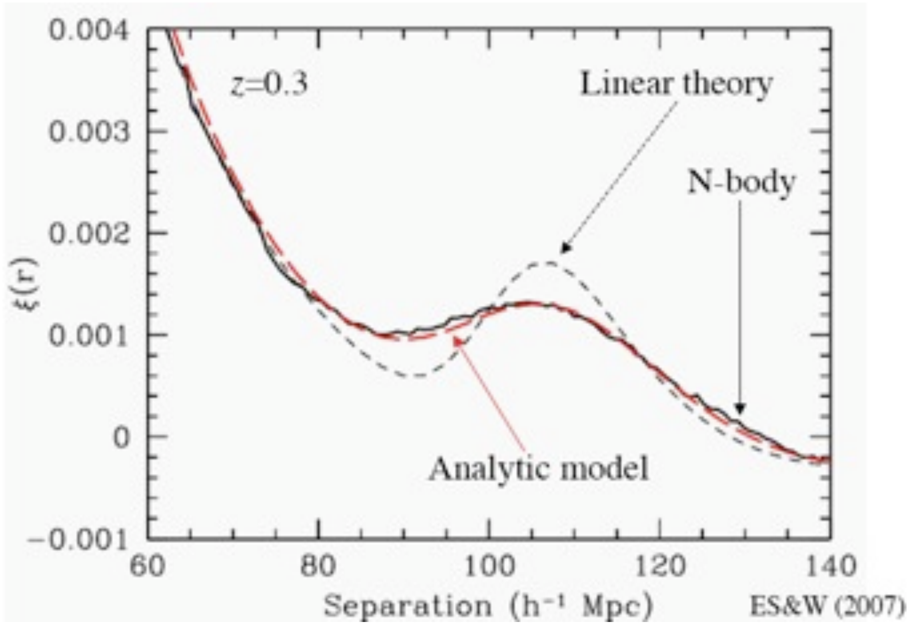
A well defined perturbation theory

- Non-linearities at short scale



A well defined perturbation theory

- Baryon Acoustic Oscillations scale is close to non-linear scale (factor of ~10)

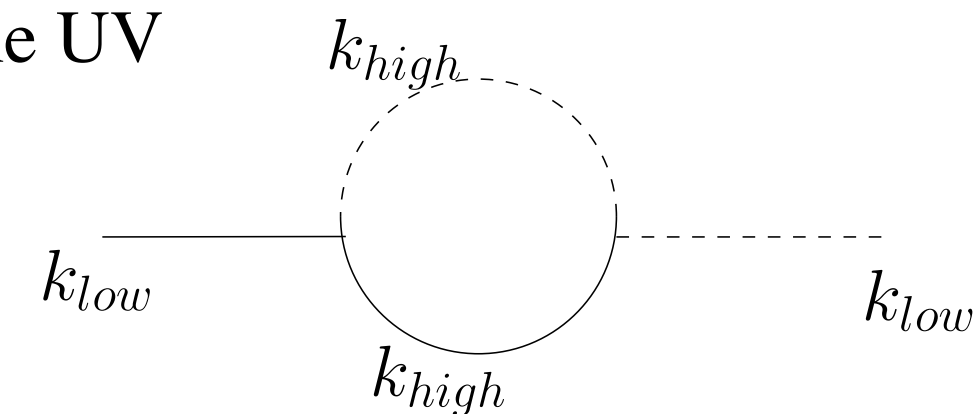


- It is very unclear if current perturbation theory is well defined (at 1% level ?!)

- Standard techniques
  - perfect fluid $\dot{\rho} + \partial_i \left(\rho v^i\right) = 0 \ ,$
  - expand in $\delta \sim \frac{\delta\rho}{\rho}$ and solve

$$\delta^{(n)} \sim \int \text{GreenFunction} \ \times \ \text{Source}^{(n)} \left[\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(n-1)}\right]$$

- Perturbative equations break in the UV
  - $\delta \sim \frac{k}{k_{NL}} \gg 1 \quad \text{for} \quad k \gg k_{NL}$
  - no perfect fluid if you truncate



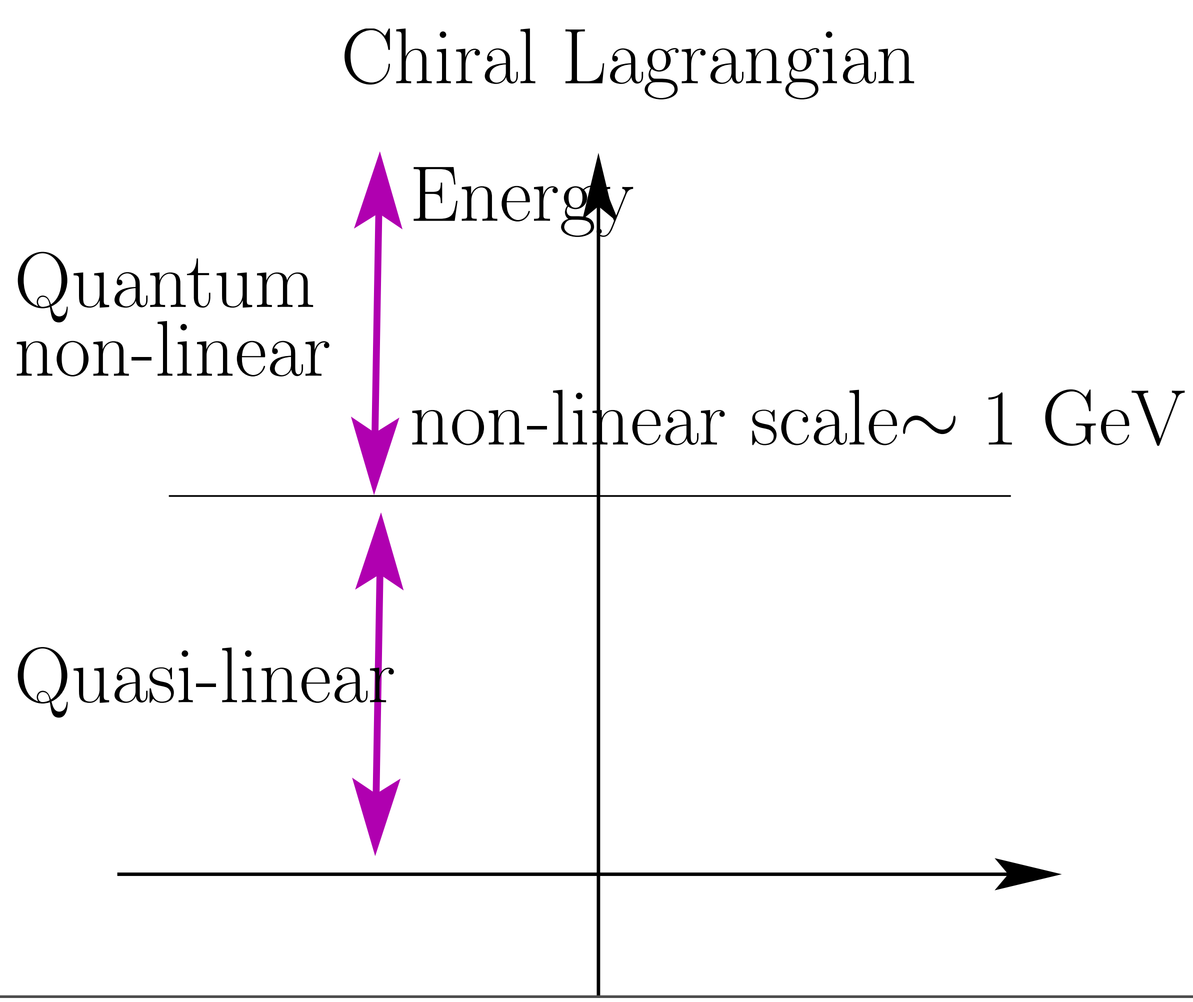
Idea of the
Effective Field Theory

QCD Chiral Lagrangian Reminder

- Pions are described by

$$S = \int d^4x \left[(\partial\pi)^2 + \frac{1}{F_\pi^2}\pi^2(\partial\pi)^2 + \frac{1}{\tilde{F}_\pi^2}(\partial\pi)^4 + \dots \right]$$

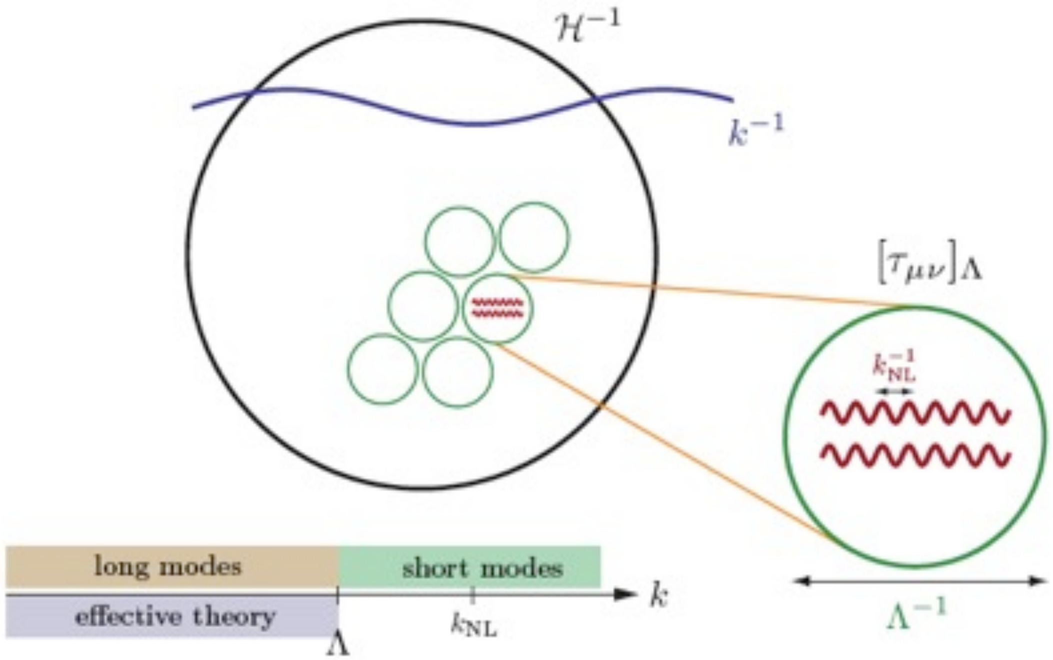
- For $m_\pi \lesssim E \lesssim 4\pi F_\pi$
- Perturbative expansion in $\dfrac{E}{4\pi F_\pi} \ll 1$



Our Universe as a Chiral Lagrangian

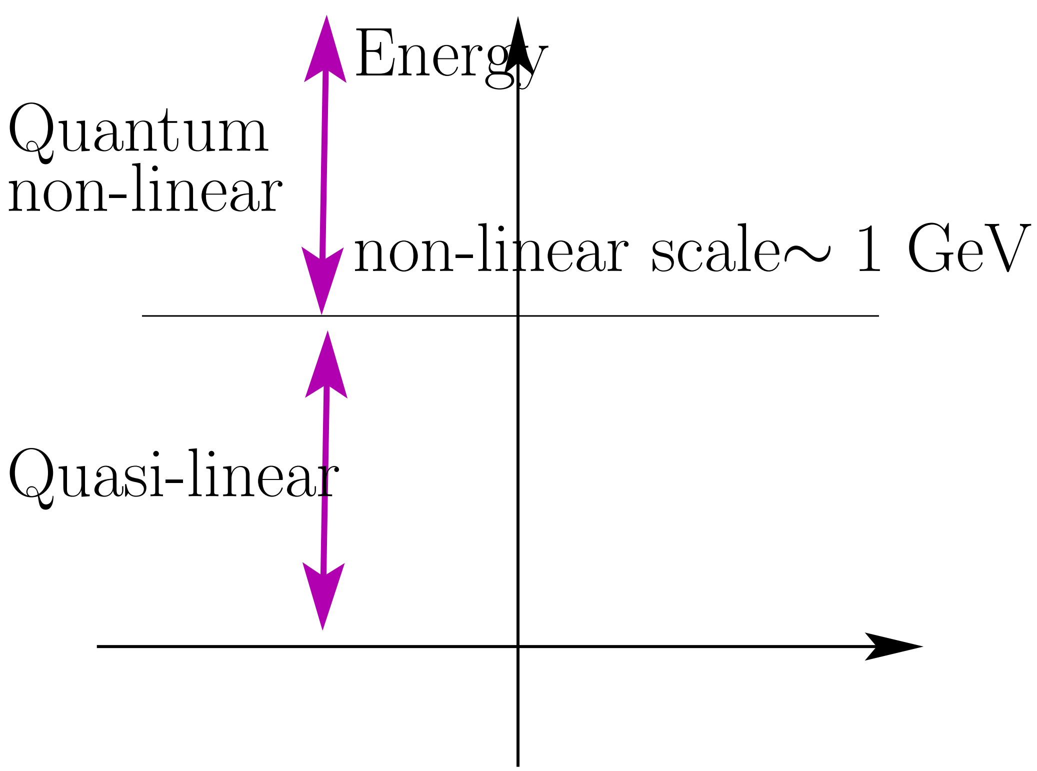
- How does our universe looks like?
- Non-linear on short scales $\lambda_{NL} \sim 1 - 10$ Mpc
  $\delta\rho/\rho \gg 1$
- Linear on large-scales

  $H^{-1} \sim 14000$ Mpc $\quad\quad \delta\rho/\rho \ll 1$

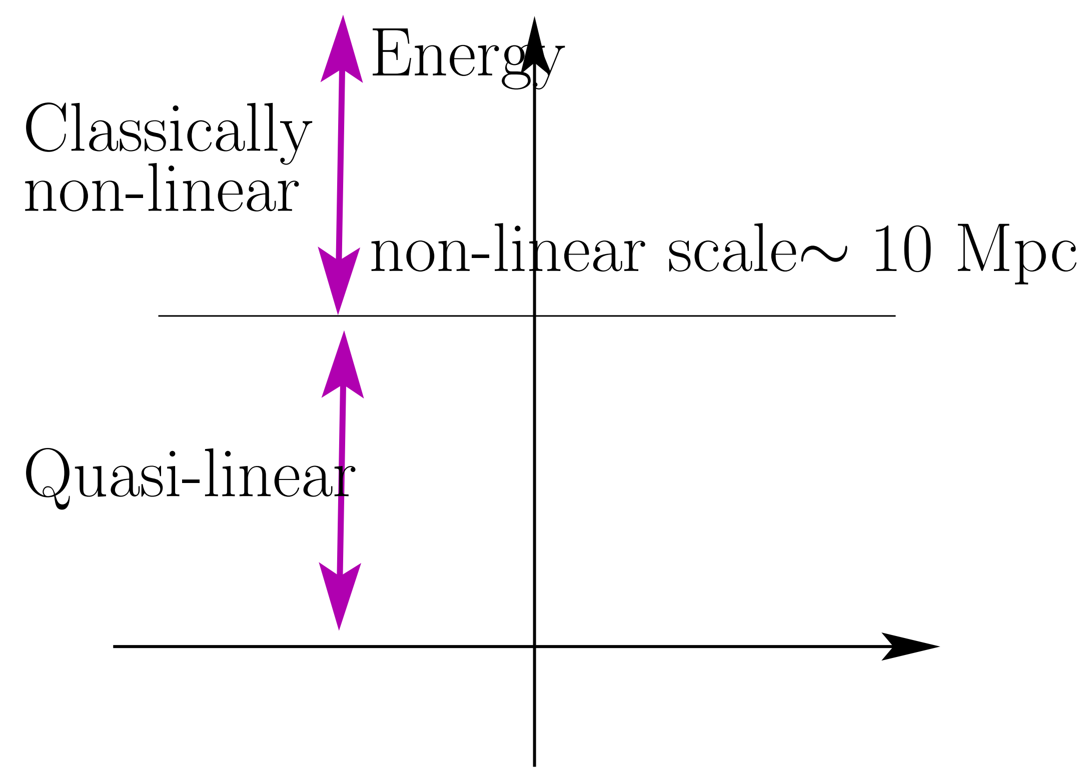


- Similar to Chiral Lagrangian

Chiral Lagrangian



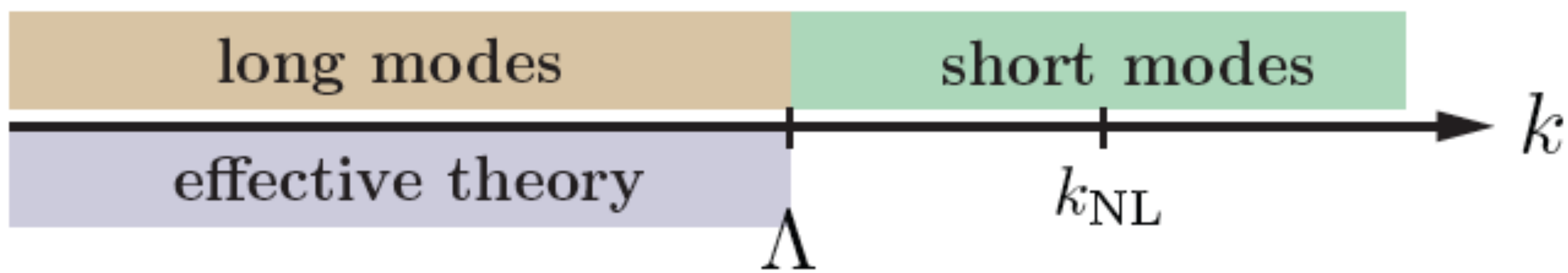
Universe



- Universe as an Effective Fluid with higher derivative stress-tensor in expansion in $k/k_{\rm NL}$

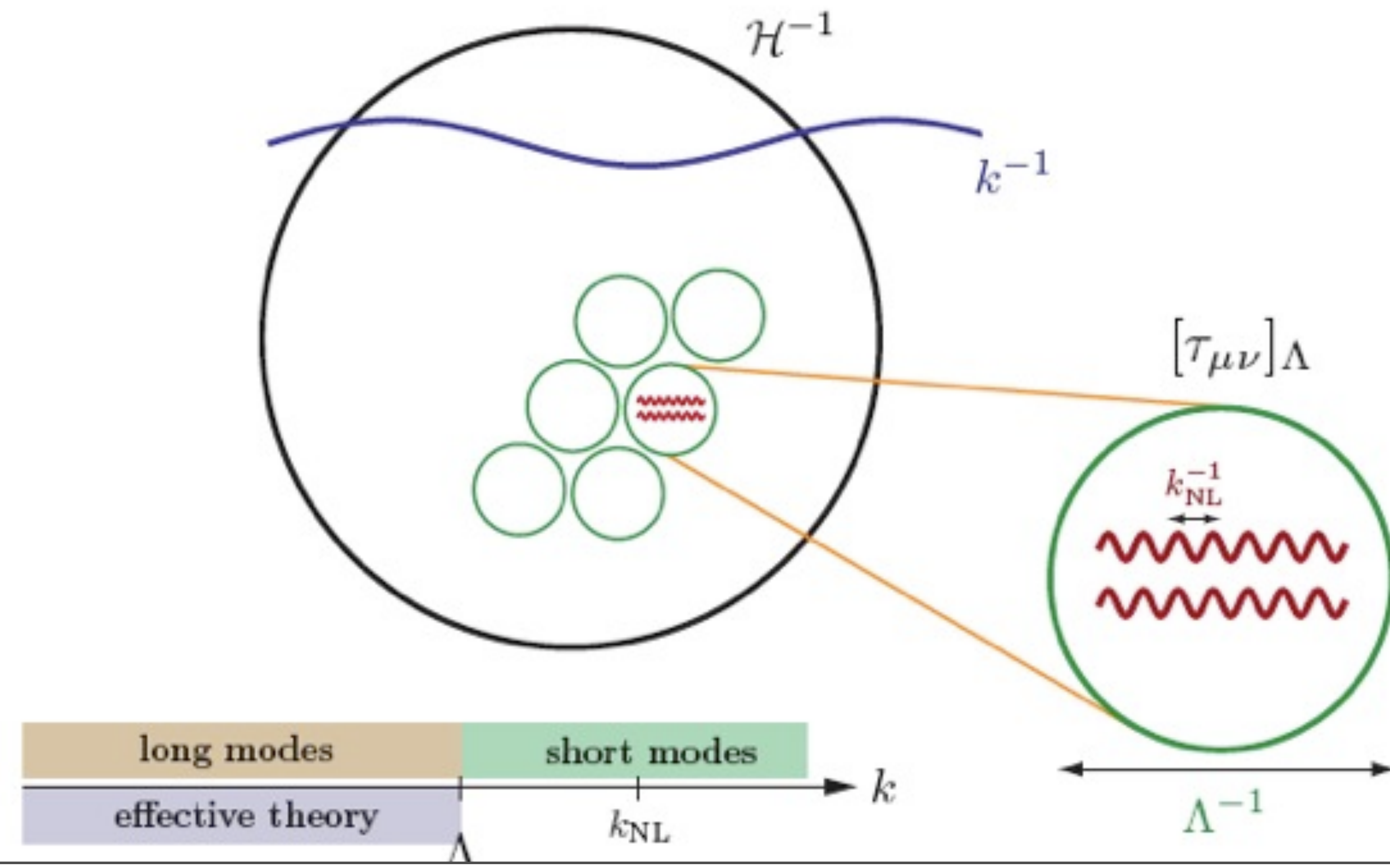
A well defined perturbation theory

- We will define a manifestly convergent perturbation theory



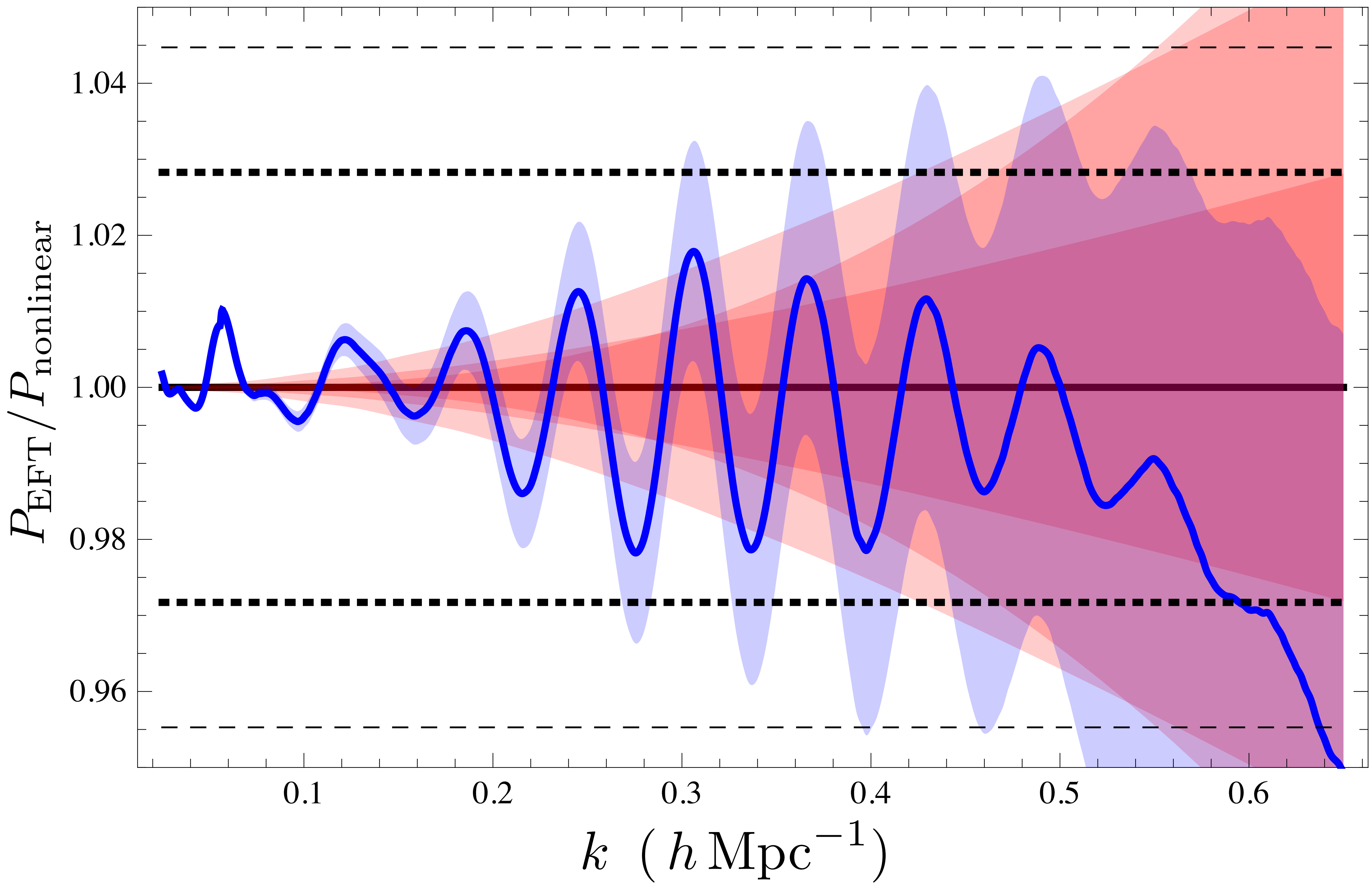
  - where the ingredient is
    an fluid-like system with

$$\delta_\ell, v_\ell, \Phi_\ell \ll 1$$



Bottom line result

- 2-loop in the EFT



- Data go as k_{max}^3

Construction of the Effective Field Theory: from UV to IR

From Dark Matter Particles to Cosmic Fluid

- UV
- Dark Matter described by distribution $f(\vec{x},\vec{p}) = \sum \delta^{(3)}(\vec{x}-\vec{x}_n)\delta^{(3)}(\vec{p}-m\,a\,\vec{v}_n)$
- Boltzmann equation $$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{ma^2}\cdot\frac{\partial f}{\partial \vec{x}} - m\sum_{n,\bar{n};\bar{n}\neq n}\frac{\partial \phi_{\bar{n}}}{\partial \vec{x}}\cdot\frac{\partial f_n}{\partial \vec{p}} = 0\ .$$
- and Newtonian gravity $\partial^2\phi = 4\pi G a^2\,(\rho-\rho_b)$
- Smoothing the fields $$W_\Lambda(\vec{x}) = \left(\frac{\Lambda}{\sqrt{2\pi}}\right)^3 e^{-\frac{1}{2}\Lambda^2 x^2}$$ $$\mathcal{O}_l(\vec{x},t) = [\mathcal{O}]_\Lambda(\vec{x},t) = \int d^3x'\,W_\Lambda(\vec{x}-\vec{x}')\mathcal{O}(\vec{x}')$$
- Smooth Boltzmann equation $$\left[\frac{Df}{Dt}\right]_\Lambda = \frac{\partial f_l}{\partial t} + \frac{\vec{p}}{ma^2}\cdot\frac{\partial f_l}{\partial \vec{x}} - m\sum_{n,\bar{n},n\neq\bar{n}}\int d^3x' W_\Lambda(\vec{x}-\vec{x}')\frac{\partial \phi_n}{\partial \vec{x}'}(\vec{x}')\cdot\frac{\partial f_{\bar{n}}}{\partial \vec{p}}\ .$$
- and take moments $$\int d^3p\, p^{i_1}\dots p^{i_n}\left[\frac{Df}{Dt}\right]_\Lambda(\vec{x},\vec{p}) = 0\ ;$$
- Boltzmann hierarchy perturbative by powers of $\frac{k}{k_{NL}}\ ,\qquad \frac{1}{k_{MFP}} \sim v_{DM}H^{-1} \sim \frac{1}{k_{NL}}$
  - This naively would not be a fluid (plus subtlety about time-hierarchy)

From Dark Matter Particles to Cosmic Fluid

- We get a fluid!
- First two moments:

Continuity

$$\dot{\rho}_l + 3H\rho_l + \frac{1}{a}\partial_i(\rho_l v_l^i) = 0 \ ,$$

Momentum

$$\dot{v}_l^i + H v_l^i + \frac{1}{a} v_l^j \partial_j v_l^i + \frac{1}{a}\partial_i \phi_l = -\frac{1}{a\rho_l}\partial_j \left[\tau^{ij}\right]_\Lambda$$

- Short distance fluctuations appear as enhanced stress tensor for long modes

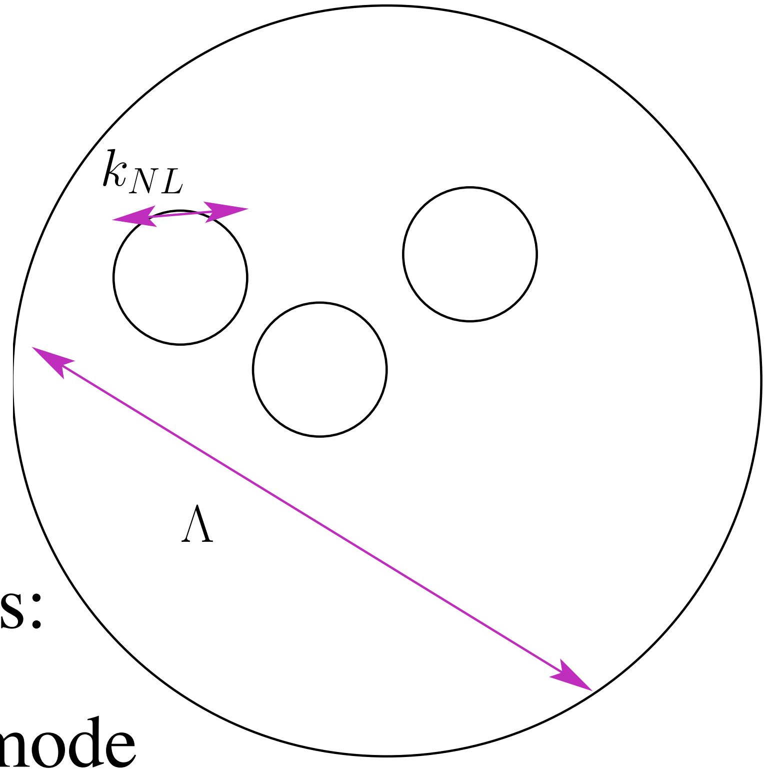
$\left[\tau^{ij}\right]_\Lambda = \kappa_l^{ij} + \Phi_l^{ij}$ ~ kinetic + potential : $\kappa \sim \rho v_s^2 \ ,$ $\Phi \sim \rho_s \Phi_s$

» So far, this theory still contains short distance fluctuations:

» this is not yet a long wavelength, well defined, EFT.

Integrate out
UV modes

Integrating out UV modes



- Integrate out short modes: i.e. solve equations of motion
- This is true realization by realization
- Good approximation:
  - For effect on large scales $\Lambda \ll k_{NL}$, take first two moments:
    - . $\langle [\tau_s^{\mu\nu}]_\Lambda \rangle_{\phi_\ell}$ space-dependence from background long-mode
    - . $\mathrm{Var}([\tau_{\mu\nu}]_\Lambda) \equiv \langle [\tau_{\mu\nu}]_\Lambda^2 \rangle - \langle [\tau_{\mu\nu}]_\Lambda \rangle^2$: random statistical fluctuations (check later)
- Taylor expand: $\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = \langle [\tau^{ij}]_\Lambda \rangle_0 + \left. \frac{\partial \langle [\tau^{ij}]_\Lambda \rangle_{\delta_l}}{\partial \delta_l} \right|_0 \delta_l + \ldots \ .$
- Obtain function of long-wavelength 2-derivatives gravitational long modes

$$\langle \frac{1}{\rho_{\rm b}} \tau^{ij} \rangle_{\phi_l} = \delta_{ij} p + \int d\tau' \kappa_1(\tau, \tau')\, \delta^{ij} \partial^2 \phi(\tau', \vec{x}_{\rm fl}) + \int d\tau' \kappa_2(\tau, \tau')\, \partial^i \partial^j \phi(\tau', \vec{x}_{\rm fl}) + \cdots \ ,$$

  - no time hierarchy between short and long mode $\Longrightarrow$ the EFT is non-local in time

- Now effective theory has only long-wavelength modes. We made it!
- Similar to Chiral Lagrangian F_π : UV physics in higher derivative terms

Perturbation Theory with the EFT

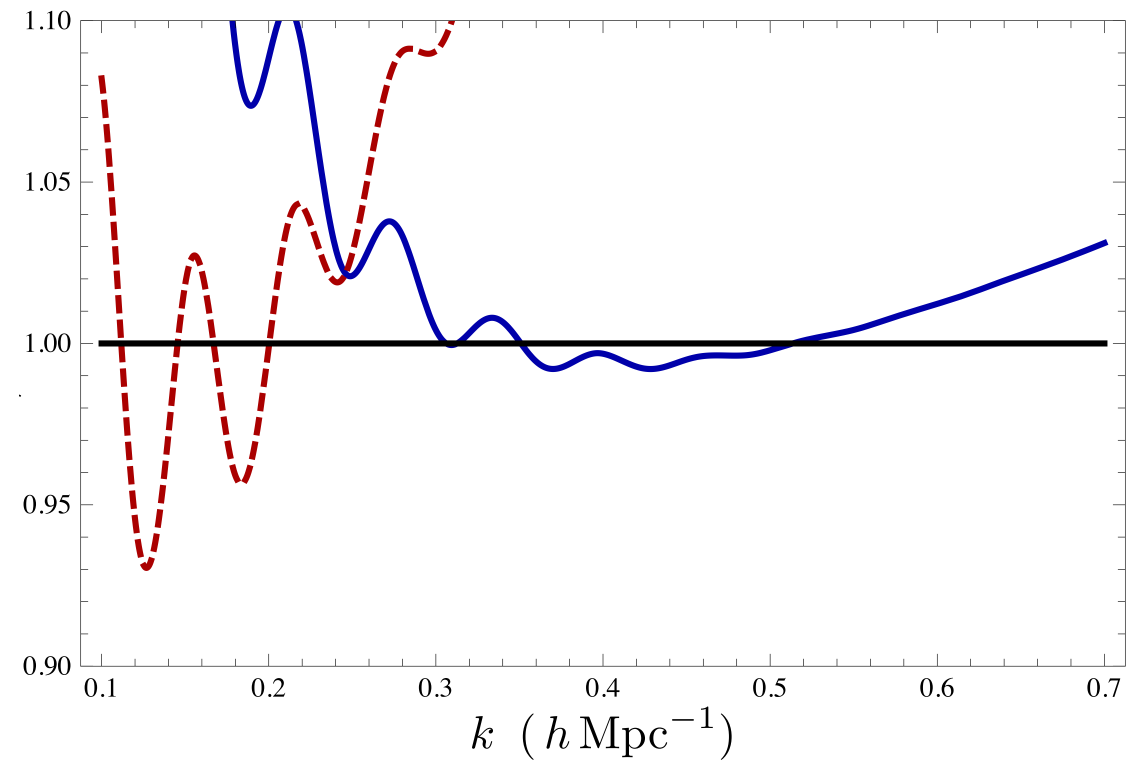
Perturbation Theory within the EFT

- In the EFT we can solve iteratively (loop expansion) $\delta_\ell, v_\ell, \Phi_\ell \ll 1$

$$\nabla^2\phi_l = \frac{3}{2}H_0^2\,\Omega_{\rm m}\frac{a_0^3}{a}\delta_l \ ,$$
$$\dot\delta_l = -\frac{1}{a}\partial_i\left([1+\delta_l]v_l^i\right) \ ,$$
$$\dot v_l^i + H v_l^i + \frac{1}{a}v_l^j\partial_j v_l^i + \frac{1}{a}\partial^i\phi_l = -\frac{1}{a\rho_l}\,\partial_j\tau^{ij}\Big|_{\phi_l}$$

- Approximate as piecewise scaling universe
  - estimates

$$P_{11}(k) = (2\pi)^3\begin{cases}\frac{1}{k_{\rm NL}^3}\left(\frac{k}{k_{\rm NL}}\right)^{-2.1} & \text{for } k > k_{\rm tr}\ ,\\ \frac{1}{\tilde k_{\rm NL}^3}\left(\frac{k}{\tilde k_{\rm NL}}\right)^{-1.7} & \text{for } k < k_{\rm tr}\ ,\end{cases}$$



$k_{\rm NL} = 4.6\,h\,{\rm Mpc}^{-1}$ $\quad k_{\rm tr} = 0.25\,h\,{\rm Mpc}^{-1}$ $\quad \tilde k_{\rm NL} = 1.8\,h\,{\rm Mpc}^{-1}$

Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis: $n = -3/2$,

$$P^{\rm I}_{\text{2-loop}} = (2\pi)\left[c_0^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)^2 \left(\frac{k}{k_{\rm NL}}\right)^1 P_{11} + c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)^1 \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_2^\Lambda \log\left(\frac{k}{\Lambda}\right) \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + c_1^{1/\Lambda} \left(\frac{k}{\Lambda}\right)^1 \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading finite terms in } \frac{k}{\Lambda}\right]$$

Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis: $n = -3/2$,

$$P^{\rm I}_{\text{2-loop}} = (2\pi)\left[c_0^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)^2 \left(\frac{k}{k_{\rm NL}}\right)^1 P_{11} + c_1^\Lambda \left(\frac{\Lambda}{k_{\rm NL}}\right)^1 \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}\right.$$
$$+ c_2^\Lambda \log\left(\frac{k}{\Lambda}\right)\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$
$$\left. + c_1^{1/\Lambda}\left(\frac{k}{\Lambda}\right)^1\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading finite terms in } \frac{k}{\Lambda}\right]$$

  - absence of counterterm

Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis: $n=-3/2$,

$$P^{\rm I}_{\text{2-loop}} = (2\pi)\Bigg[c_0^\Lambda\left(\frac{\Lambda}{k_{\rm NL}}\right)^2\left(\frac{k}{k_{\rm NL}}\right)^1 P_{11} + c_1^\Lambda\left(\frac{\Lambda}{k_{\rm NL}}\right)^1\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$

$$+c_2^\Lambda\log\left(\frac{k}{\Lambda}\right)\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$

$$+c_1^{1/\Lambda}\left(\frac{k}{\Lambda}\right)^1\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11} + \text{subleading finite terms in } \frac{k}{\Lambda}\Bigg]$$

  - absence of counterterm
  - One divergent term $\Longrightarrow$ $$P_{\text{2-loop counter}} = (2\pi)c^\Lambda_{\rm counter}\left(\frac{\Lambda}{k_{\rm NL}}\right)\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$

$$c^\Lambda_{\rm counter} = -c_1^\Lambda + \delta c_{\rm counter}\left(\frac{k_{\rm NL}}{\Lambda}\right)$$

  - Sum up and $\Lambda\to\infty$.

$$P^{\rm I}_{\text{2-loop}} + P_{\text{2-loop counter}} = (2\pi)\delta c_{\rm counter}\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + (2\pi)c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$

Calculable terms in the EFT

- Has everything being lost?

$$P^{\rm I}_{\text{2-loop}} + P_{\text{2-loop counter}} = (2\pi)\delta c_{\rm counter}\left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + (2\pi)c_1^{\rm finite}\left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$

  – to make result finite, we need to add a counterterm with finite part

    - need to fit to data (like a coupling constant)

Calculable terms in the EFT

- Had everything being lost?

$$P^{\mathrm{I}}_{\text{2-loop}} + P_{\text{2-loop counter}} = (2\pi)\delta c_{\text{counter}} \left(\frac{k}{k_{\mathrm{NL}}}\right)^2 P_{11} + (2\pi) c_1^{\text{finite}} \left(\frac{k}{k_{\mathrm{NL}}}\right)^3 P_{11}$$

  – to make result finite, we need to add a counterterm with finite part
    - need to fit to data (like a coupling constant)

  – the subleading finite term is not degenerate with a counterterm.
    - it cannot be changed
    - it is calculable by the EFT
      – so it predicts an observation $c_1^{\text{finite}} = 0.044$

Lesson

- Each loop-order L contributed a finite, calculable term of order

$$P^{\rm I}_{\text{L-loops finite}} \sim \left(\frac{k}{k_{\rm NL}}\right)^{(3+n)L} \left(\frac{k}{k_{\rm NL}}\right)^{n}$$

  – each higher-loop is smaller and smaller

- This happen after canceling the divergencies with counterterms

$$P^{\rm I}_{\text{L-loops diverg.}} \sim \left(\frac{\Lambda}{k_{\rm NL}}\right)^{(3+n)L-2} \left(\frac{k}{k_{\rm NL}}\right)^{2} \left(\frac{k}{k_{\rm NL}}\right)^{n} + \text{subleading divergences}$$

  – at each higher loop one needs to adjust the lower order counterterms

    - by this is not a new fit, this is calculable

Example

- At 1-loop, we add a counterterm

$$P_{\text{EFT-1-loop}} = P_{11} + P_{\text{1-loop}} - 2\,(2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\text{NL}}} P_{11}$$

- $c^2_{s(1)}$ is chosen by fitting to data so that

$$P_{\text{1-loop}}(k = k_{\text{ren}})_{\Lambda\to\infty} = P_{\text{NL}}(k_{\text{ren}}) \quad \Rightarrow \quad c^2_{s(1)}(k_{\text{ren}}) = \text{number} = (-3.36 \pm 0.020) \times \frac{1}{2\pi}\left(\frac{k_{\text{NL}}}{h\,\text{Mpc}^{-1}}\right)^2$$

- At 2-loop, there is a divergency that requires the same counterterm.

$$P^{\text{I}}_{\text{2-loop}} = (2\pi)\left[c_1^{\Lambda}\left(\frac{\Lambda}{k_{\text{NL}}}\right)^1\left(\frac{k}{k_{\text{NL}}}\right)^2 P_{11} + c_1^{\text{finite}}\left(\frac{k}{k_{\text{NL}}}\right)^3 P_{11}\right]$$

  – Adjust $c^2_{s(1)} \quad \to \quad c^2_{s(1)} + c^2_{s(2)}$ in a known way (without looking again at the data)

$$c^2_{s(2)}(k_{\text{ren}}) = \frac{P_{\text{2-loop}}(k_{\text{ren}}) + c^2_{s(1)}(k_{\text{ren}}) P^{(c_s)}_{\text{1-loop}}(k_{\text{ren}})}{(k^2_{\text{ren}}/k^2_{\text{NL}}) P_{11}(k_{\text{ren}})} + [c^2_{s(1)}(k_{\text{ren}})]^2 \frac{k^2_{\text{ren}}}{k^2_{\text{NL}}}$$

Calculation up to 2-loops

- We need to add all terms whose finite contribution is larger than the 3-loop term
- Candidates

$$P_{\rm counter} \sim (2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\rm NL}} \left(2P_{11}(k) + 2P^{\rm finite}_{\text{1-loop}}(k)\right) + (2\pi c^2_{s(1)})^2 \frac{k^4}{k^4_{\rm NL}} P_{11}(k) + 2(2\pi) c^2_{s(2)} \frac{k^2}{k^2_{\rm NL}} P_{11}(k) + (2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\rm NL}} P^{\rm finite}_{\text{2-loop}}(k) + 2\kappa \frac{k^2}{k^2_{\rm NL}} P^{\rm finite}_{\text{1-loop}}(k) + 2(2\pi)\lambda \frac{k^4}{k^4_{\rm NL}} P_{11}(k) + \ldots \ .$$

Calculation up to 2-loops

- We need to add all terms whose finite contribution is larger than the 3-loop term
- Candidates

$$P_{\text{counter}} \sim (2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\text{NL}}} \left(2P_{11}(k) + 2P^{\text{finite}}_{\text{1-loop}}(k)\right) + (2\pi c^2_{s(1)})^2 \frac{k^4}{k^4_{\text{NL}}} P_{11}(k) + 2(2\pi) c^2_{s(2)} \frac{k^2}{k^2_{\text{NL}}} P_{11}(k) +$$

$$(2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\text{NL}}} P^{\text{finite}}_{\text{2-loop}}(k) + 2\kappa \frac{k^2}{k^2_{\text{NL}}} P^{\text{finite}}_{\text{1-loop}}(k) + 2(2\pi)\lambda \frac{k^4}{k^4_{\text{NL}}} P_{11}(k) + \dots \; .$$

new coefficients

Calculation up to 2-loops

- We need to add all terms whose finite contribution is larger than the 3-loop term
- Candidates

$$P_{\text{counter}} \sim (2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\text{NL}}} (2P_{11}(k) + 2P^{\text{finite}}_{\text{1-loop}}(k)) + (2\pi c^2_{s(1)})^2 \frac{k^4}{k^4_{\text{NL}}} P_{11}(k) + 2(2\pi) c^2_{s(2)} \frac{k^2}{k^2_{\text{NL}}} P_{11}(k) +$$
$$(2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\text{NL}}} P^{\text{finite}}_{\text{2-loop}}(k) + 2\kappa \frac{k^2}{k^2_{\text{NL}}} P^{\text{finite}}_{\text{1-loop}}(k) + 2(2\pi)\lambda \frac{k^4}{k^4_{\text{NL}}} P_{11}(k) + \dots \ .$$

- Estimate:
  - when 3-loop is important $\quad \frac{P_{\text{3-loop finite}}}{P_{\text{non-linear}}}\Big|_{k=0.5\, h\, \text{Mpc}^{-1}} \sim 0.03$

$\Longrightarrow$ We should include all terms that are larger than 3-L before $k \sim 0.5\, h\, \text{Mpc}^{-1}$

Calculation up to 2-loops

- We need to add all terms whose finite contribution is larger than the 3-loop term
- Candidates

$$P_{\text{counter}} \sim (2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}(2P_{11}(k)+2P^{\text{finite}}_{\text{1-loop}}(k)) + (2\pi c^2_{s(1)})^2\frac{k^4}{k^4_{\text{NL}}}P_{11}(k) + 2(2\pi)c^2_{s(2)}\frac{k^2}{k^2_{\text{NL}}}P_{11}(k) + (2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P^{\text{finite}}_{\text{2-loop}}(k) + 2\kappa\frac{k^2}{k^2_{\text{NL}}}P^{\text{finite}}_{\text{1-loop}}(k) + 2(2\pi)\lambda\frac{k^4}{k^4_{\text{NL}}}P_{11}(k) + \dots\ .$$

- Estimate:
  - when 3-loop is important $\left.\frac{P_{\text{3-loop finite}}}{P_{\text{non-linear}}}\right|_{k=0.5\,h\,\text{Mpc}^{-1}} \sim 0.03$

We should include all terms larger before $k \sim 0.5\,h\,\text{Mpc}^{-1}$

  - check:

$$\frac{4\pi c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{\text{1-loop finite}}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 6.8\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{0.2}$$

$$\frac{(2\pi c^2_{s(1)})^2\frac{k^4}{k^4_{\text{NL}}}P_{11}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 1.5\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{1.3}$$

$$\frac{4\pi c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{\text{2-loop finite}}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 0.92\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{1.3}$$

$$\alpha\frac{2\kappa\frac{k^2}{k^2_{\text{NL}}}P_{\text{1-loop finite}}}{P^{\text{R}}_{\text{3-loop finite}}} \sim 0.20\,\kappa\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{0.2}$$

$$\frac{4\pi\lambda\frac{k^4}{k^4_{\text{NL}}}P_{11}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 0.16\,\lambda\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{0.2}$$

Calculation up to 2-loops

- We need to add all terms whose finite contribution is larger than the 3-loop term
- Candidates

$$P_{\text{counter}} \sim (2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}(2P_{11}(k)+2P^{\text{finite}}_{\text{1-loop}}(k)) + (2\pi c^2_{s(1)})^2\frac{k^4}{k^4_{\text{NL}}}P_{11}(k) + 2(2\pi)c^2_{s(2)}\frac{k^2}{k^2_{\text{NL}}}P_{11}(k) + (2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P^{\text{finite}}_{\text{2-loop}}(k) + 2\kappa\frac{k^2}{k^2_{\text{NL}}}P^{\text{finite}}_{\text{1-loop}}(k) + 2(2\pi)\lambda\frac{k^4}{k^4_{\text{NL}}}P_{11}(k) + \dots\ .$$

- Estimate:
  - when 3-loop is important $\frac{P_{\text{3-loop finite}}}{P_{\text{non-linear}}}\Big|_{k=0.5\,h\,\text{Mpc}^{-1}} \sim 0.03$

We should include all terms larger before $k \sim 0.5\,h\,\text{Mpc}^{-1}$

- check:

Only these are larger: the old ones

$$\frac{4\pi c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{\text{1-loop finite}}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 6.8\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{0.2}$$

$$\frac{(2\pi c^2_{s(1)})^2\frac{k^4}{k^4_{\text{NL}}}P_{11}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 1.5\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{1.3}$$

$$\frac{4\pi c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{\text{2-loop finite}}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 0.92\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{1.3}$$

$$\alpha\frac{2\kappa\frac{k^2}{k^2_{\text{NL}}}P_{\text{1-loop finite}}}{P^{\text{R}}_{\text{3-loop finite}}} \sim 0.20\,\kappa\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{0.2}$$

$$\frac{4\pi\lambda\frac{k^4}{k^4_{\text{NL}}}P_{11}(k)}{P^{\text{R}}_{\text{3-loop finite}}} \sim 0.16\,\lambda\left(\frac{k}{0.5\,h\,\text{Mpc}^{-1}}\right)^{0.2}$$

Calculation up to 2-loops

- We need to add all terms whose finite contribution is larger than the 3-loop term
- Candidates

$$P_{\rm counter} \sim (2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\rm NL}}(2P_{11}(k)+2P^{\rm finite}_{\rm 1\text{-}loop}(k)) + (2\pi c^2_{s(1)})^2\frac{k^4}{k^4_{\rm NL}}P_{11}(k) + 2(2\pi)c^2_{s(2)}\frac{k^2}{k^2_{\rm NL}}P_{11}(k) +$$

$$(2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\rm NL}}P^{\rm finite}_{\rm 2\text{-}loop}(k) + 2\kappa\frac{k^2}{k^2_{\rm NL}}P^{\rm finite}_{\rm 1\text{-}loop}(k) + 2(2\pi)\lambda\frac{k^4}{k^4_{\rm NL}}P_{11}(k) + \dots \ .$$

No needed

- Only 1-parameter to fit up to $k \simeq 0.5\, h\, {\rm Mpc}^{-1}$

Summary

- Do 1-loop calculation

$$P_{\text{EFT-1-loop}} = P_{11} + P_{\text{1-loop}} - 2\,(2\pi) c^2_{s(1)} \frac{k^2}{k^2_{\text{NL}}} P_{11}$$

- Fit $c^2_{s(1)}$
  - we fit in the range $k \sim 0.15 - 0.25\, h\,\text{Mpc}^{-1}$

$$c^2_{s(1)} = (1.62 \pm 0.033) \times \frac{1}{2\pi} \left(\frac{k_{\text{NL}}}{h\,\text{Mpc}^{-1}} \right)^2$$

- Do 2-loop calculation with no additional fitting

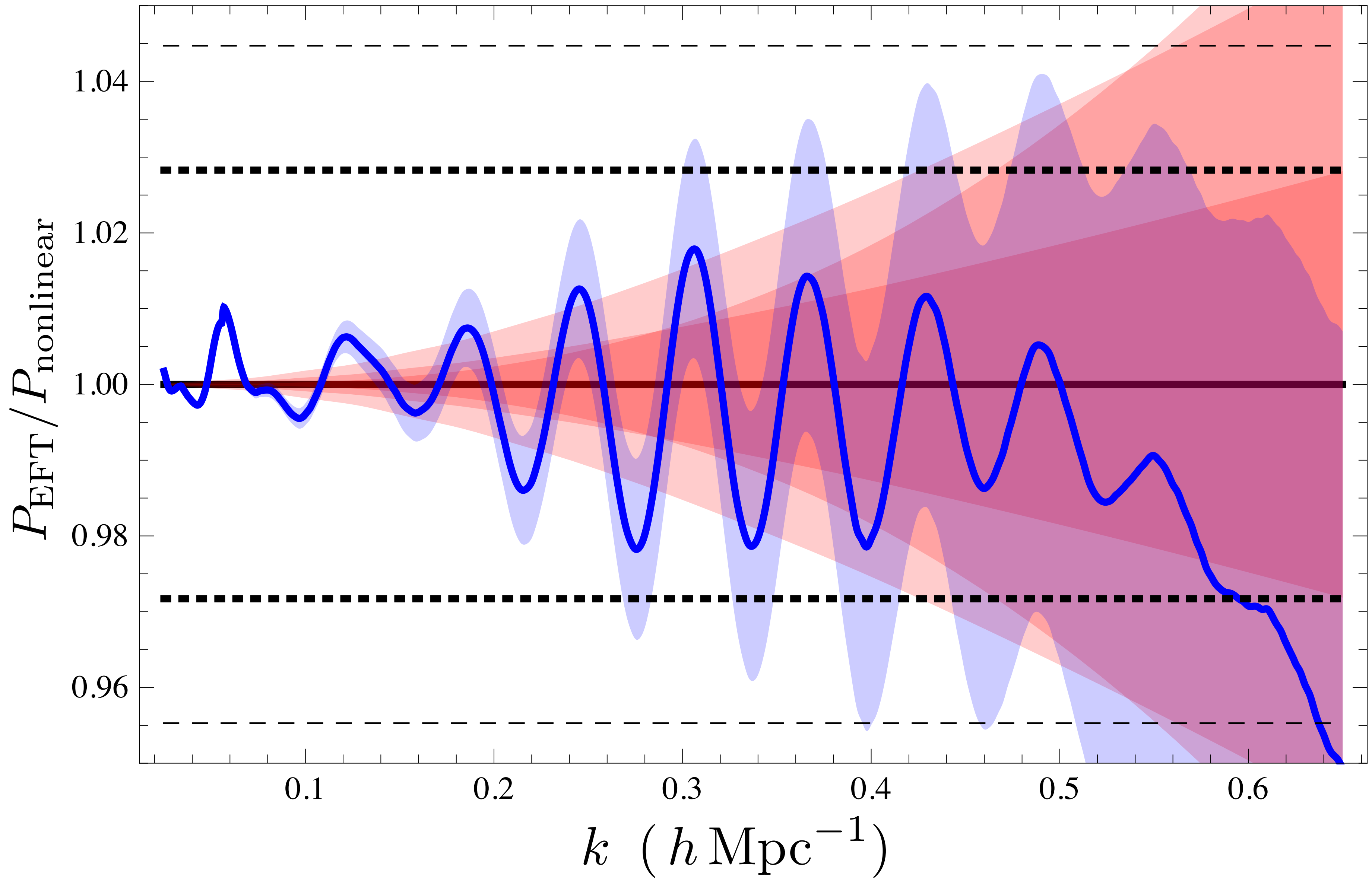
$$P_{\text{EFT-2-loop}} = P_{11} + P_{\text{1-loop}} + P_{\text{2-loop}} - 2\,(2\pi)(c^2_{s(1)} + c^2_{s(2)}) \frac{k^2}{k^2_{\text{NL}}} P_{11} + (2\pi) c^2_{s(1)} P^{(c_s,p)}_{\text{1-loop}} + (2\pi)^2 c^4_{s(1)} \frac{k^4}{k^4_{\text{NL}}} P_{11}$$

  - just adjust counterterm as calculable

$$c^2_{s(2)} = (-3.36 \pm 0.020) \times \frac{1}{2\pi} \left(\frac{k_{\text{NL}}}{h\,\text{Mpc}^{-1}} \right)^2$$

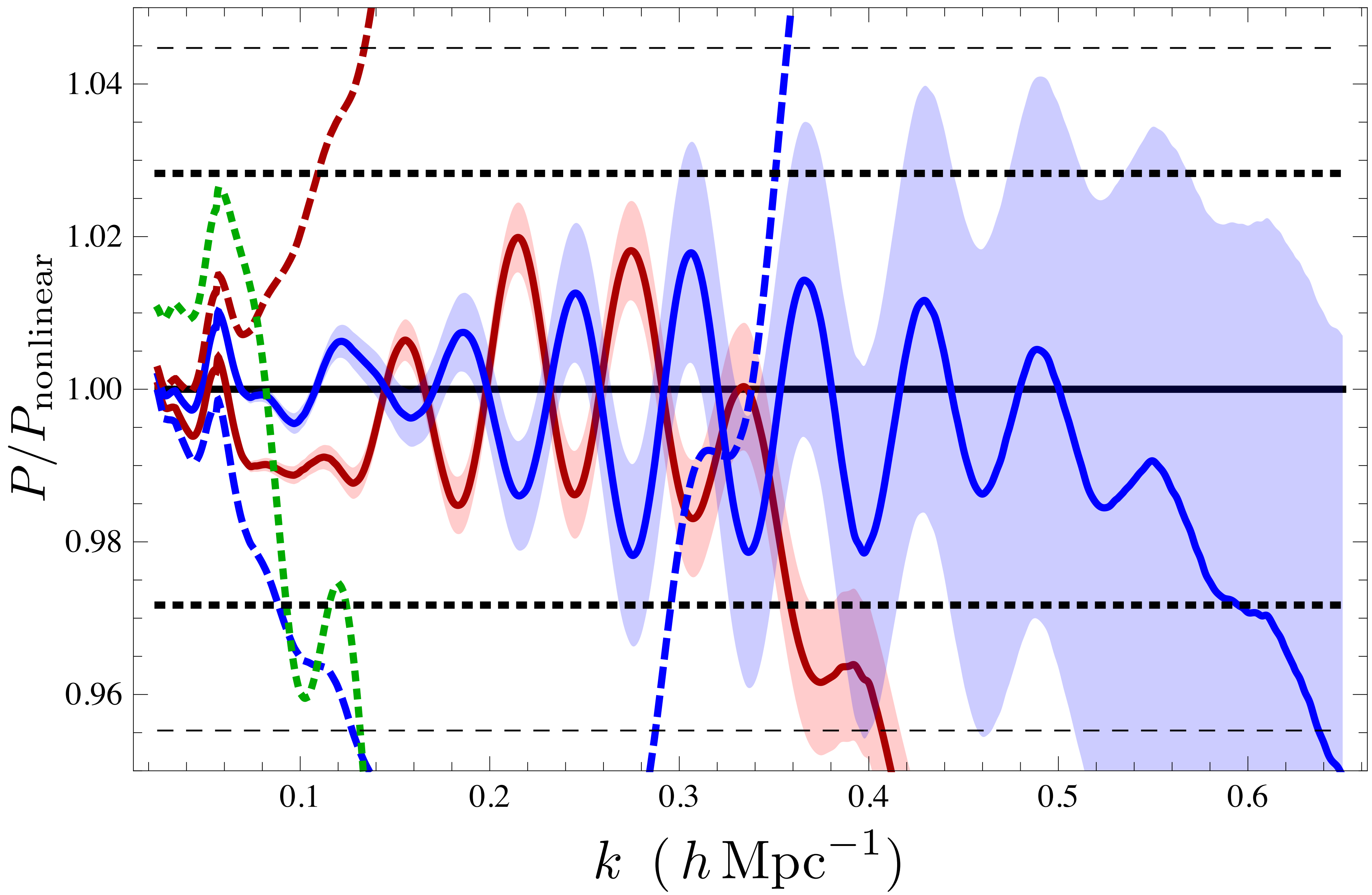
Results

EFT of Large Scale Structures



- Well defined and manif. converg. $\left(\frac{k}{k_{NL}}\right)^N$
- we fit until $k_{\rm max} \simeq 0.6\, h\, {\rm Mpc}^{-1}$, as where we should stop fitting
  - there are 200 more quasi linear modes than previously believed!

EFT of Large Scale Structures

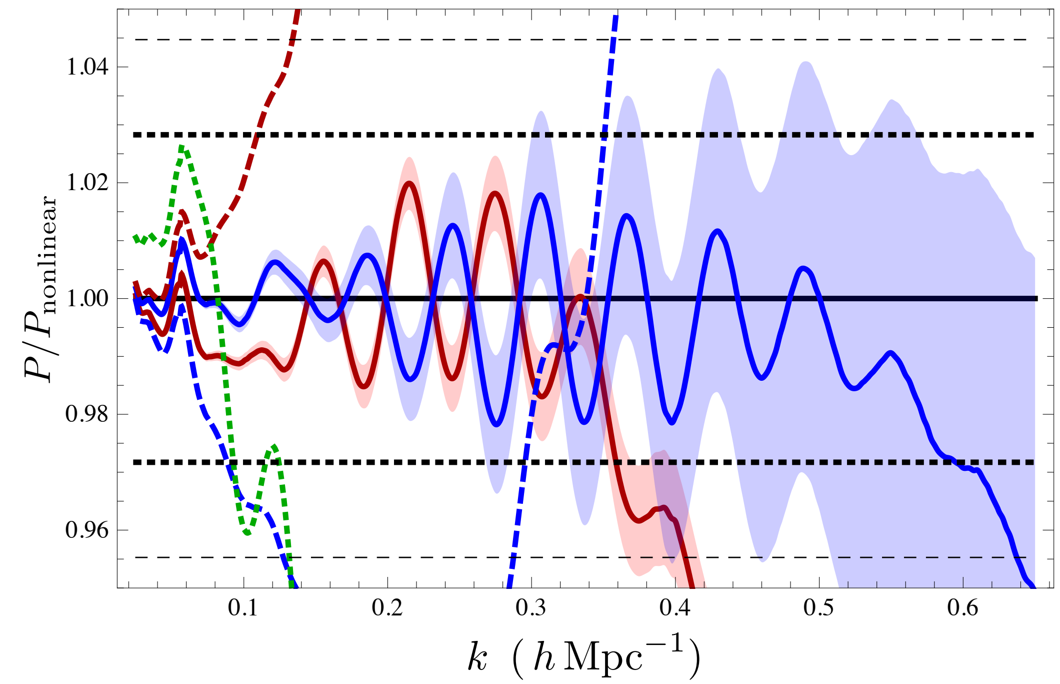


- Comparison with SPT
- Change from 1-loop to 2-loop predicted

$$P_{\text{EFT-2-loop}} = P_{11} + P_{\text{1-loop}} + P_{\text{2-loop}} - 2\,(2\pi)(c^2_{s(1)} + c^2_{s(2)})\frac{k^2}{k^2_{\rm NL}}P_{11} + (2\pi)c^2_{s(1)}P^{(c_s,p)}_{\text{1-loop}} + (2\pi)^2 c^4_{s(1)}\frac{k^4}{k^4_{\rm NL}}P_{11}$$

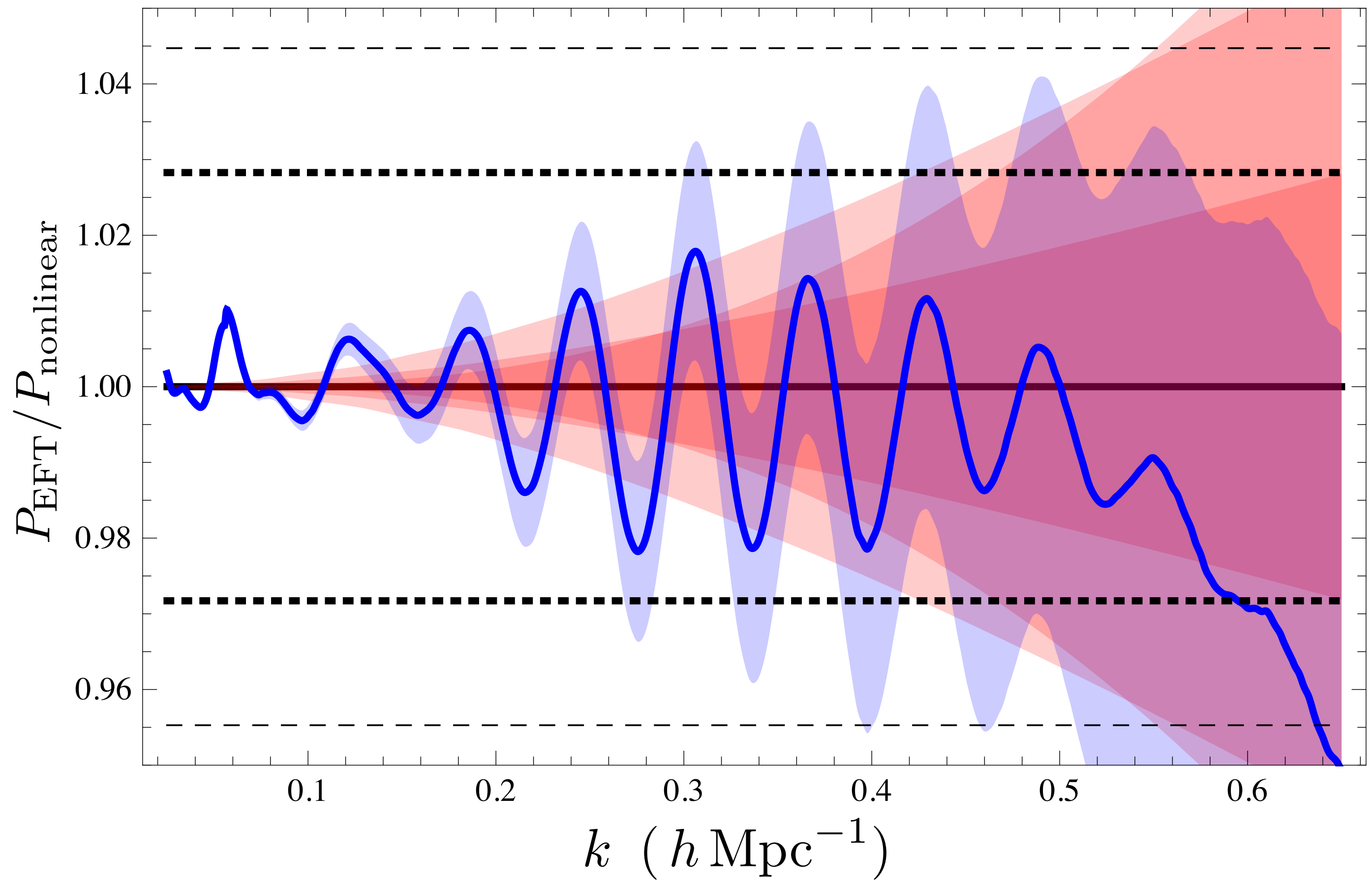
  – the other new terms are clearly important

EFT of Large Scale Structures



- We are fitting parameters
  - only 1-parameter
  - nature chooses the coupling constants
- Loops are big:
  - in Particle Physics loops are infinite
    - sum of loop+counterterm needs to be small
      - Nobel Prize in 1965
- You are fitting to high k (so overfitting):
  - We fit in $k \sim 0.15 - 0.25\, h\, {\rm Mpc}^{-1}$, just because numerical data are not good enough
  - The prediction up to 0.6 is clearly independent of the fit, so no overfittingg
- How do you know it is right:
  - it is manifestly right: we are integrating out well known UV physics at long distance
      - Nobel Prize in 1982

EFT of Large Scale Structures



- A manifestly convergent perturbation theory $\left(\frac{k}{k_{\rm NL}}\right)^L$
- we fit until $k_{\rm max} \simeq 0.6\, h\,{\rm Mpc}^{-1}$, as where we should stop fitting
  – there are 200 more quasi linear modes than previously believed!
  – huge impact on possibilities for $f_{\rm NL}^{\rm equil.,\, orthog.} \lesssim 1$
- Can all of us handle it?! This is an opportunity and a challenge for us
  – Primordial Cosmology can still have a bright future

EFT of Large Scale Structures

*``It would be fantastic to have
a perturbation theory that works"*

Uros Seljak, Trieste, July 2013

EFT of Large Scale Structures

``It would be fantastic to have a perturbation theory that works”

Uros Seljak, Trieste, July 2013