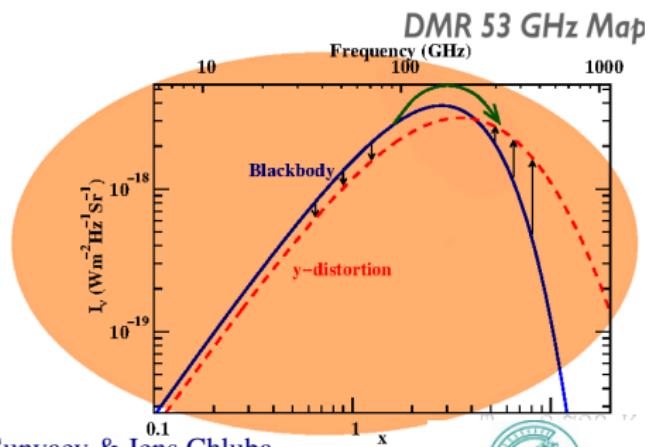
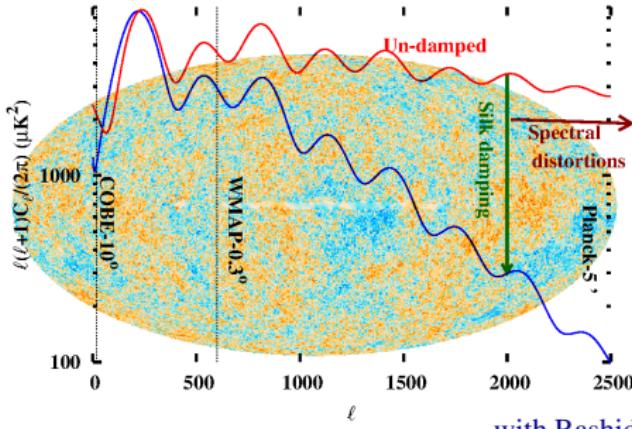


# After Planck: The road to observing 17 e-folds of inflation

Rishi Khatri



with Rashid Sunyaev & Jens Chluba

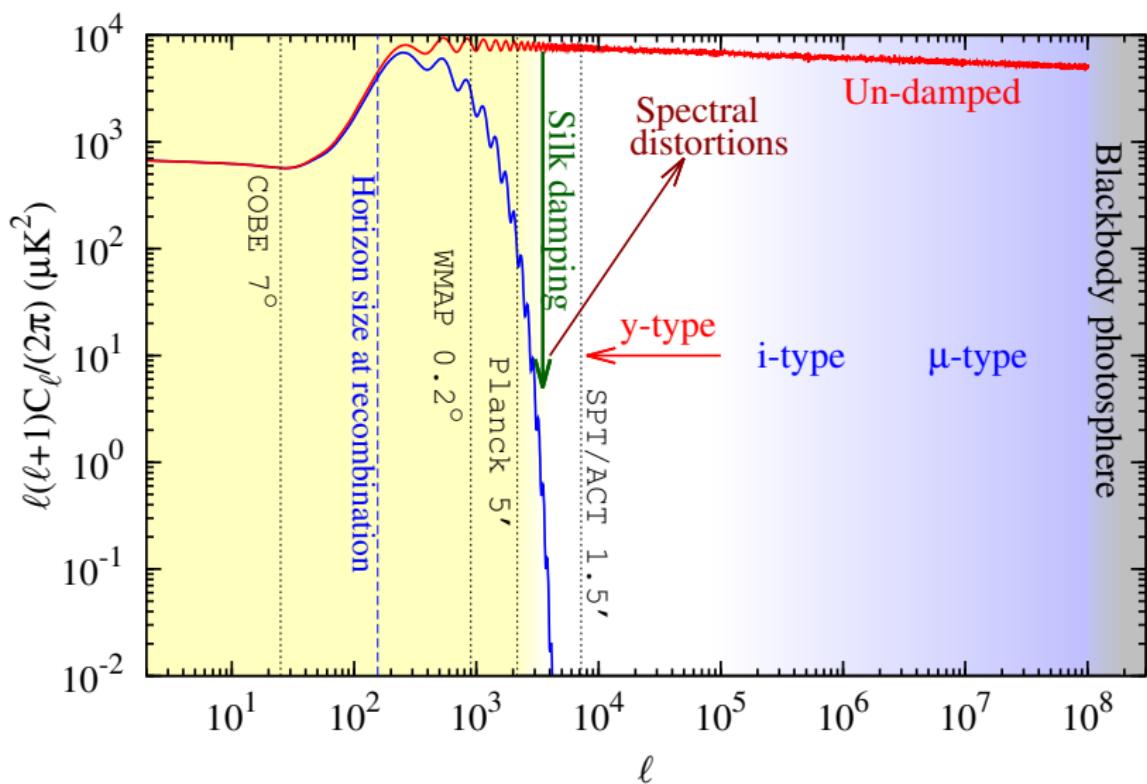
Silk damping: arXiv:1205.2871

Review: arXiv:1302.6553

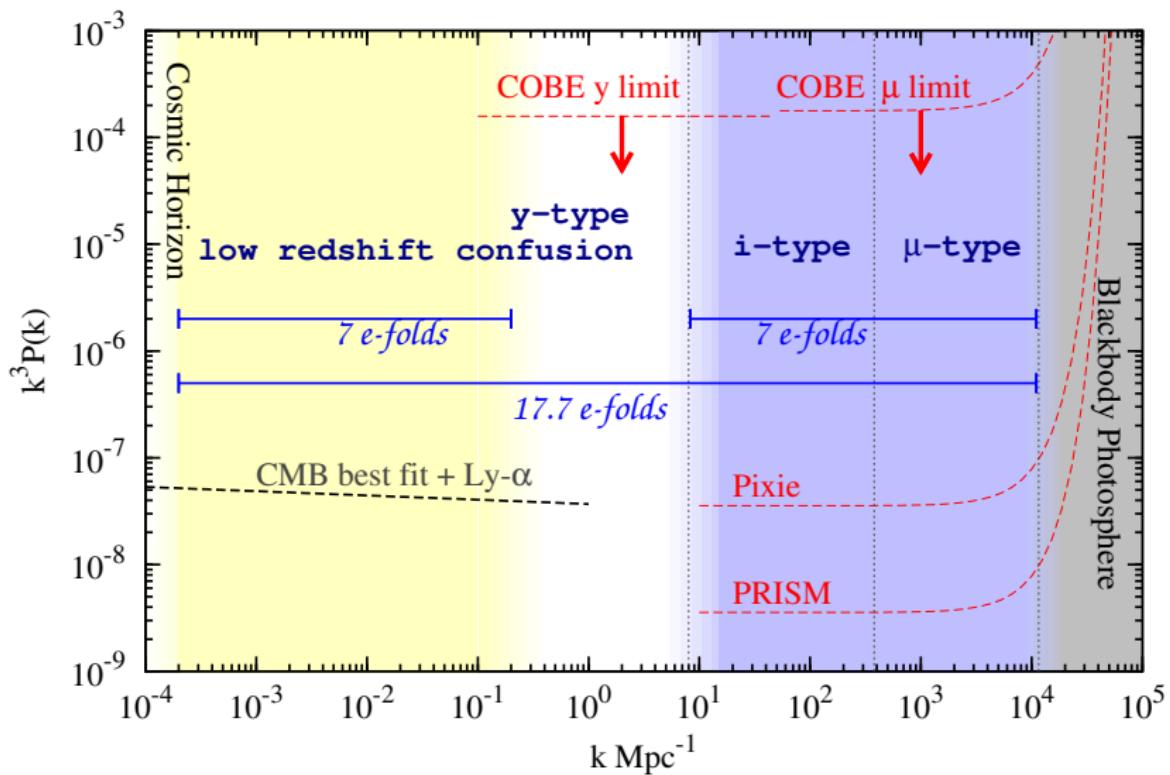
Forecasts: arXiv:1303.7212



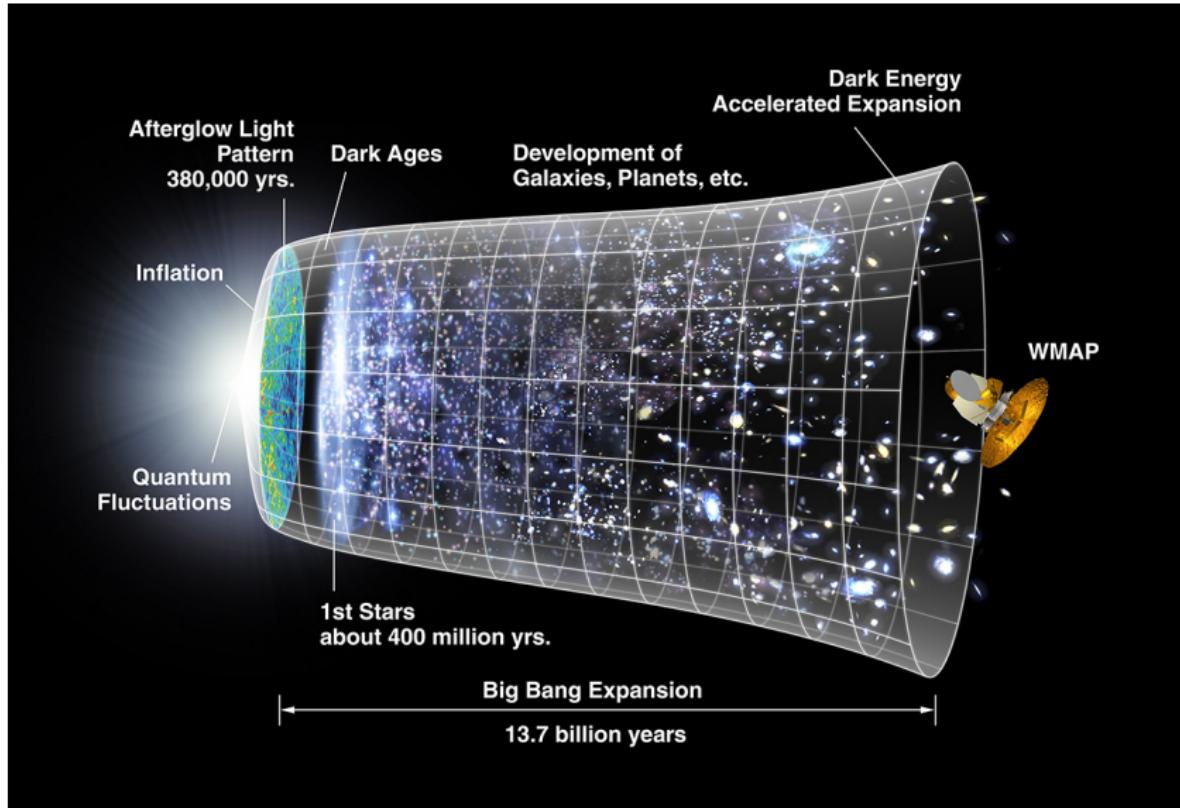
# We have reached the resolution limit for CMB anisotropies



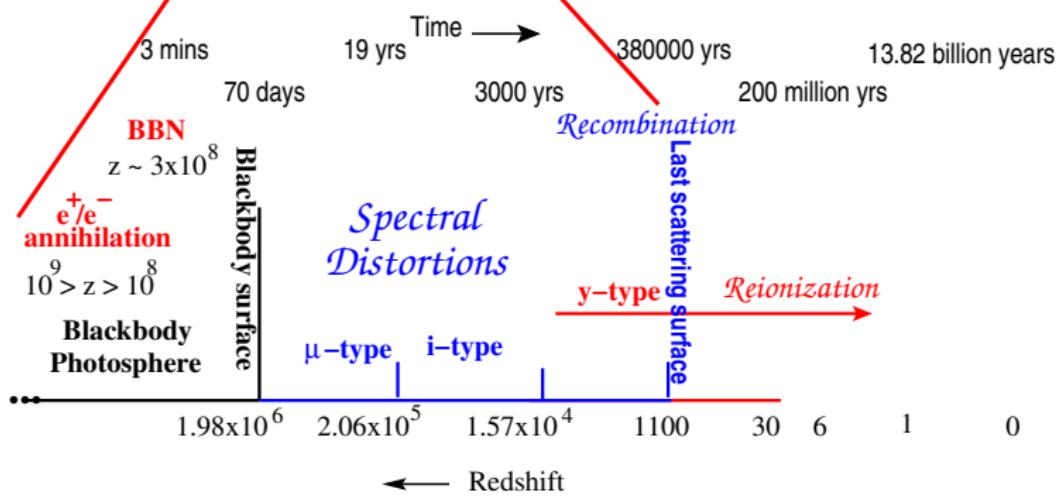
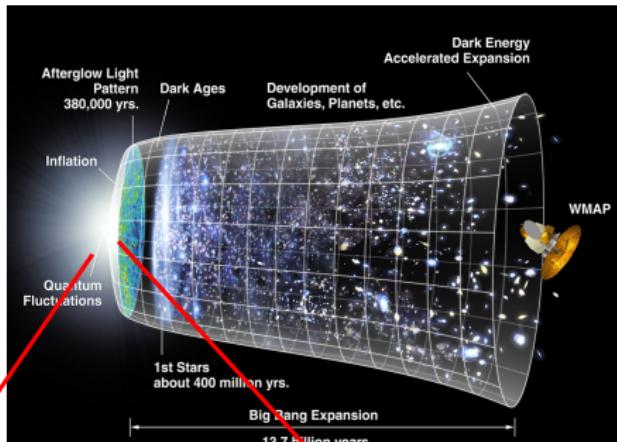
# Going from 7 to 17 e-folds of inflation



# Brief history of the Universe



# Brief history



## Planck spectrum

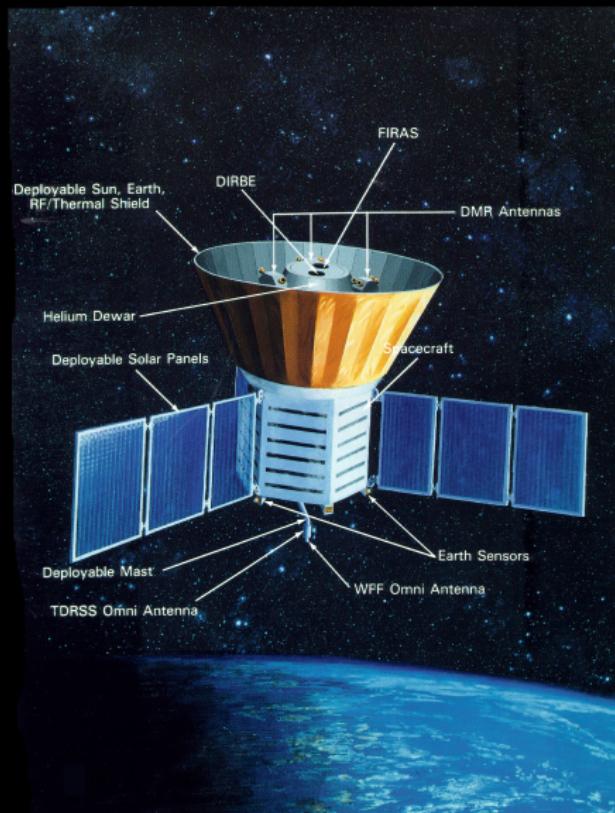
$$I_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_B T)} - 1}$$

Relativistic invariant occupation number/phase space density

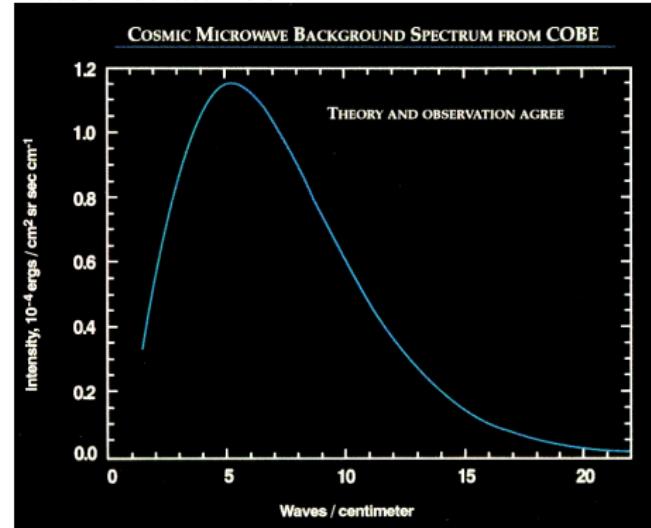
$$n(\nu) \equiv \frac{c^2}{2h\nu^3} I_\nu$$
$$n(x) = \frac{1}{e^x - 1} \quad , \quad x = \frac{h\nu}{k_B T}$$

# COBE-FIRAS confirmed blackbody spectrum of CMB at high precision

The COBE Satellite

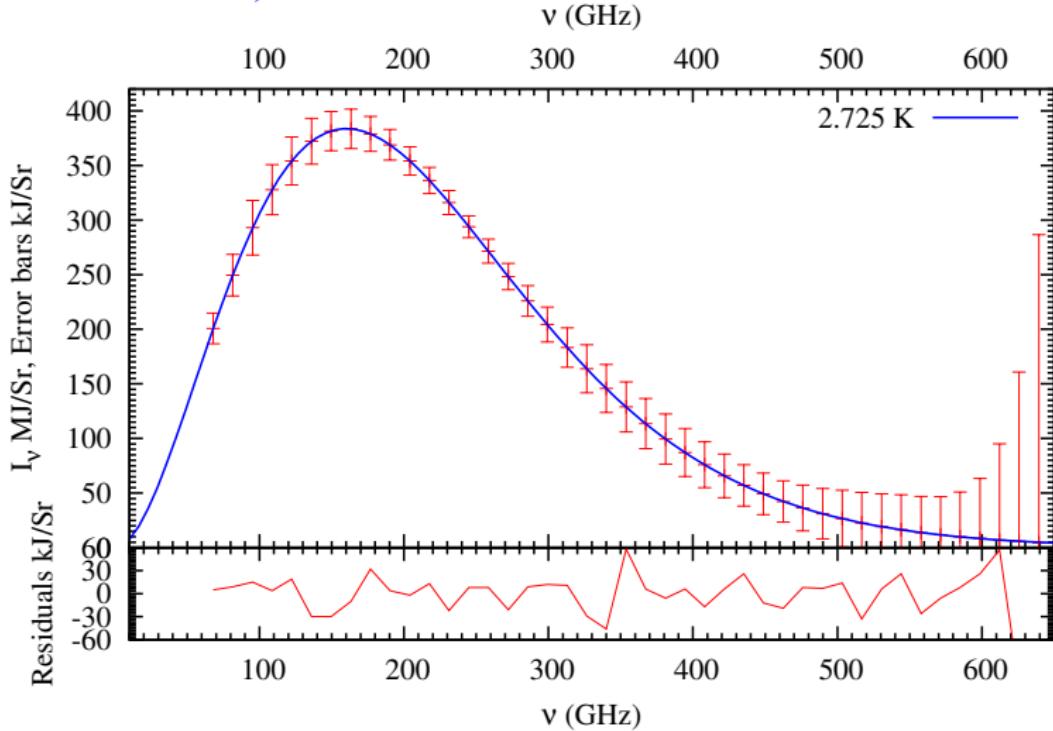


Nobel prize for Planck 1918  
Nobel Prize for Mather 2006  
*Fixsen et al. 1996*

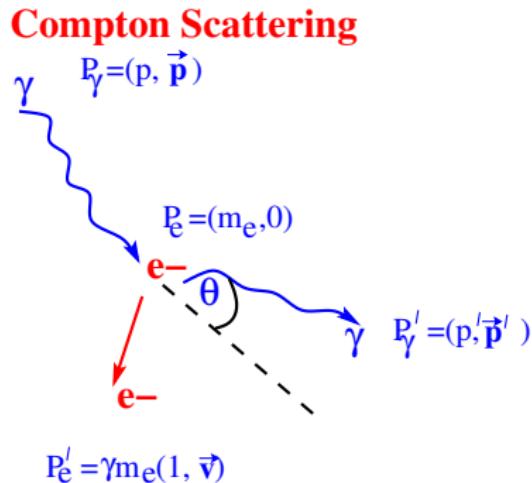


# The CMB spectrum from COBE agrees with the Planck spectrum

*Fixsen et al. 1996, Fixsen & Mather 2002*



# Compton scattering



$$\Delta p/p \approx -p/m_e(1 - \cos \theta)$$

# Efficiency of energy exchange between electrons and photons

Recoil:

$$y_\gamma = \int dt c \sigma_T n_e \frac{k_B T_\gamma}{m_e c^2}, \quad T_\gamma = 2.725(1+z)$$

Doppler effect:

$$y_e = \int dt c \sigma_T n_e \frac{k_B T_e}{m_e c^2}$$

In early Universe  $y_\gamma \approx y_e$

$y$ : Amplitude of distortion

$$y = \int dt c \sigma_T n_e \frac{k_B (T_e - T_\gamma)}{m_e c^2}$$

# Efficiency of energy exchange between electrons and photons

Recoil:

$$y_\gamma = \int dt c \sigma_T n_e \frac{k_B T_\gamma}{m_e c^2}, \quad T_\gamma = 2.725(1+z)$$

No. of scatterings

Doppler effect:

$$y_e = \int dt c \sigma_T n_e \frac{k_B T_e}{m_e c^2}$$

In early Universe  $y_\gamma \approx y_e$

$y$ : Amplitude of distortion

$$y = \int dt c \sigma_T n_e \frac{k_B (T_e - T_\gamma)}{m_e c^2}$$

# Efficiency of energy exchange between electrons and photons

Recoil:

$$y_\gamma = \int dt c \sigma_T n_e \frac{k_B T_\gamma}{m_e c^2}, \quad T_\gamma = 2.725(1+z)$$

No. of scatterings

Energy transfer per scattering

Doppler effect:

$$y_e = \int dt c \sigma_T n_e \frac{k_B T_e}{m_e c^2}$$

In early Universe  $y_\gamma \approx y_e$

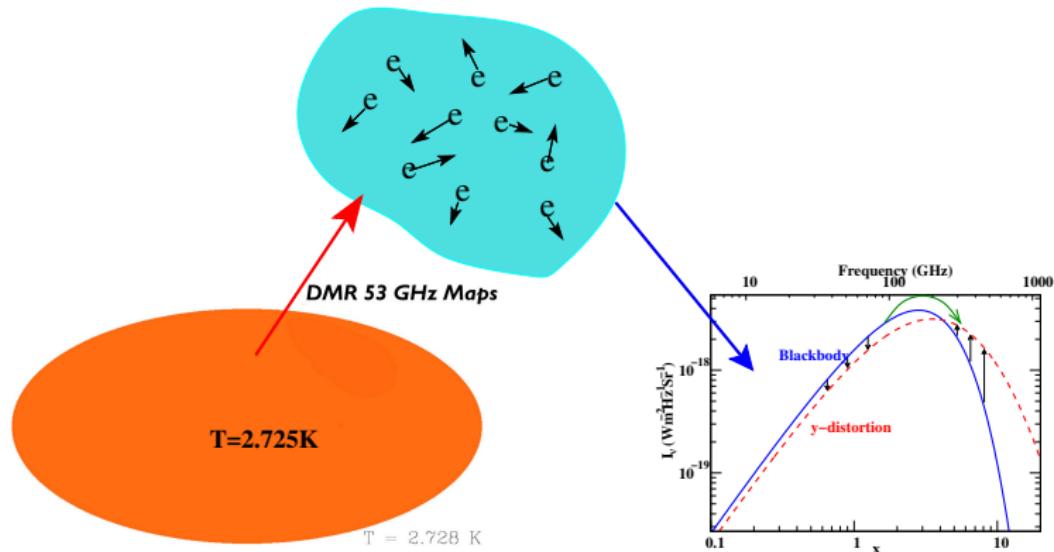
$y$ : Amplitude of distortion

$$y = \int dt c \sigma_T n_e \frac{k_B (T_e - T_\gamma)}{m_e c^2}$$

# $y$ -type (Sunyaev-Zeldovich effect) from clusters/reionization

$$y_\gamma \ll 1, T_e \sim 10^4$$

$$y = (\tau_{\text{reionization}}) \frac{k_B T_e}{m_e c^2} \sim (0.1)(1.6 \times 10^{-6}) \sim 10^{-7}$$



## $y$ -type (Sunyaev-Zeldovich effect) from clusters/reionization

$$n_{SZ} = y \ T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{Pl}}{\partial T}$$

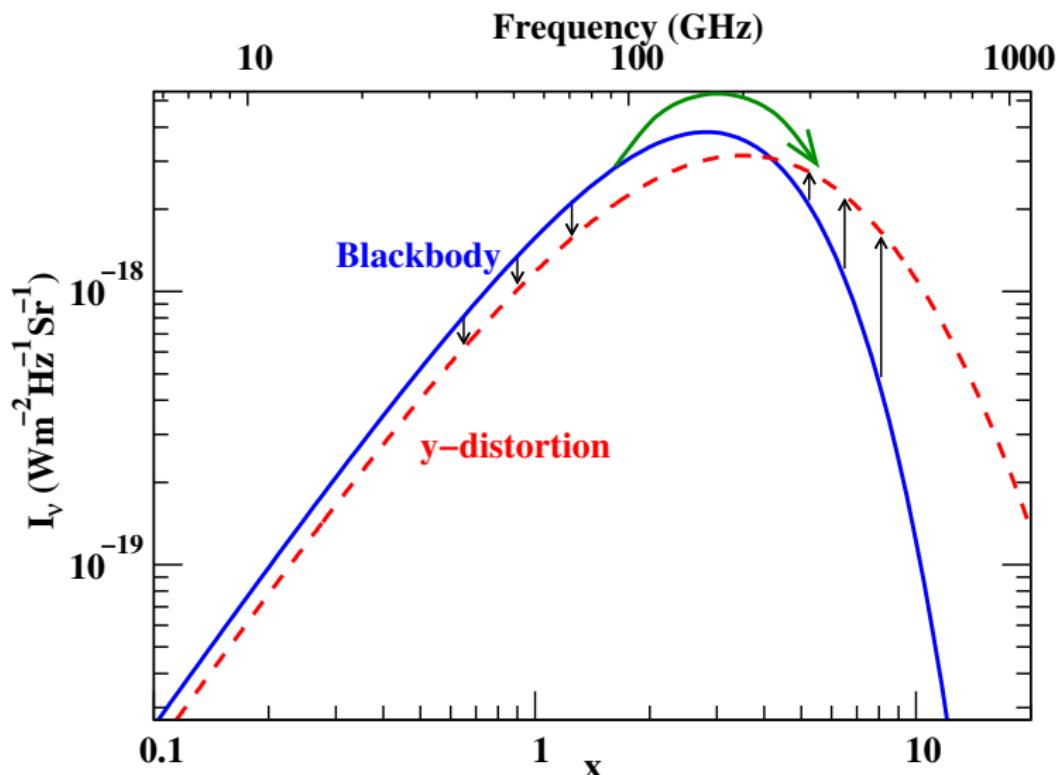
$$= y \ \frac{x e^x}{(e^x - 1)^2} \left( x \frac{e^x + 1}{e^x - 1} - 4 \right)$$

$$\Delta I_{SZ} = I_{SZ} - I_{planck} = \frac{2h\nu^3}{c^2} n_{SZ}$$

# Average $y$ -distortion (Sunyaev-Zeldovich effect) limits

(Zeldovich and Sunyaev 1969)

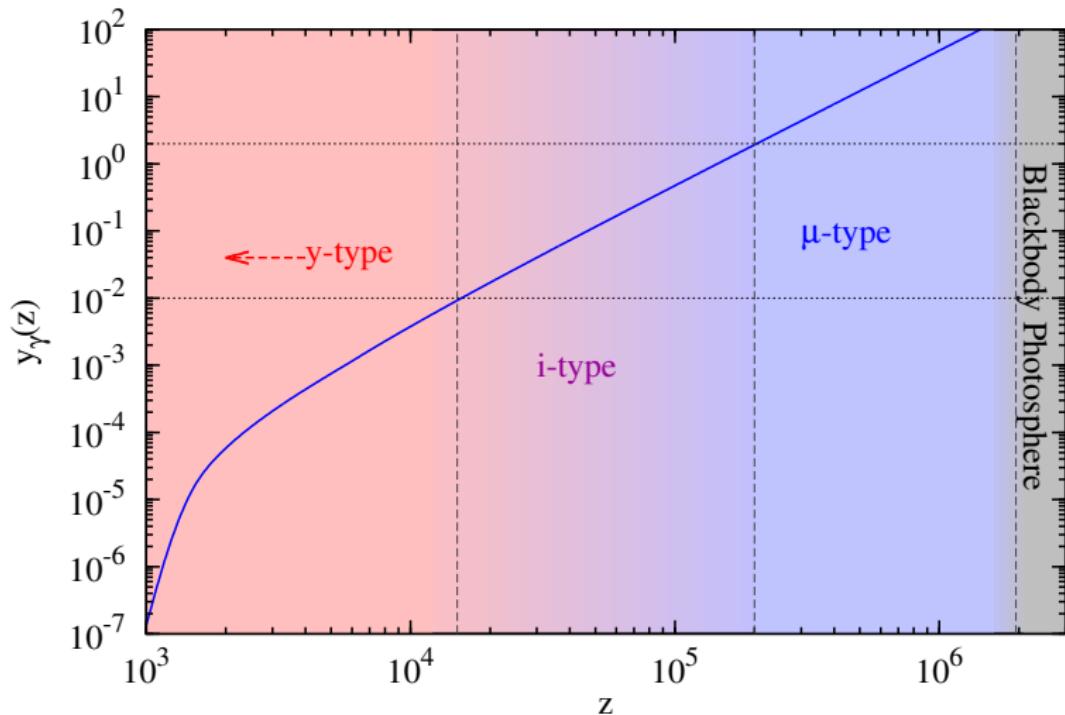
COBE-FIRAS limit (95%):  $y \lesssim 1.5 \times 10^{-5}$  (Fixsen et al. 1996)



For  $y_\gamma \gg 1$  equilibrium is established.

$T_e$  and  $T_\gamma$  converge to common value

The photon spectrum relaxes to equilibrium Bose-Einstein distribution



## Bose-Einstein spectrum- Chemical potential ( $\mu$ )

$$n(x) = \frac{1}{e^{x+\mu} - 1}$$

## Bose-Einstein spectrum- Chemical potential ( $\mu$ )

$$n(x) = \frac{1}{e^{x+\mu} - 1}$$

Given two constraints, energy density ( $E$ ) and number density ( $N$ ) of photons,  $T, \mu$  uniquely determined.

## Bose-Einstein spectrum- Chemical potential ( $\mu$ )

$$n(x) = \frac{1}{e^{x+\mu} - 1}$$

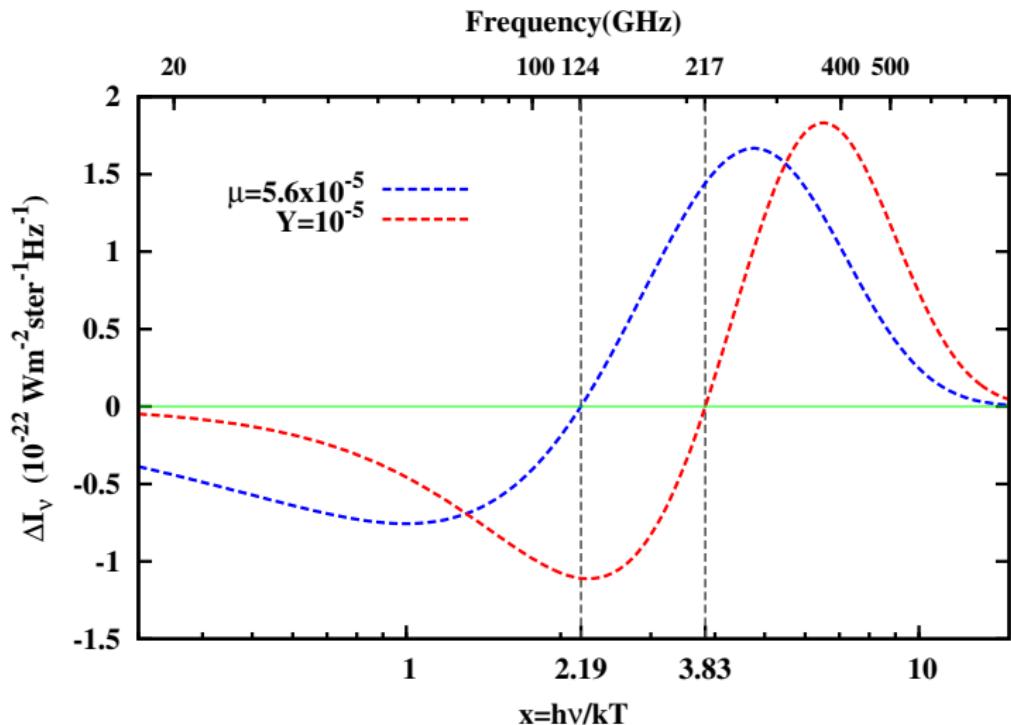
Given two constraints, energy density ( $E$ ) and number density ( $N$ ) of photons,  $T, \mu$  uniquely determined.

Idea behind analytic solutions:

If we know rate of production of photons and energy injection rate, we can calculate the evolution/production of  $\mu$  (and T)

# $\mu$ -distortion: Bose-Einstein spectrum, $y_\gamma \gg 1$

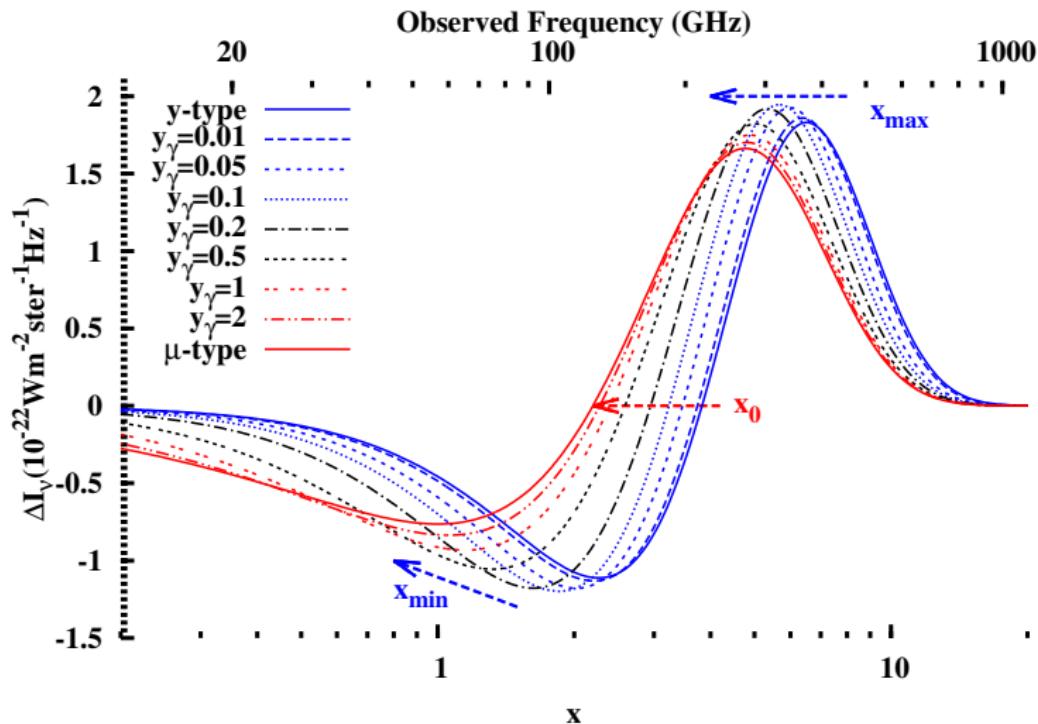
COBE-FIRAS limit (95%):  $\mu \lesssim 9 \times 10^{-5}$  (Fixsen et al. 1996)



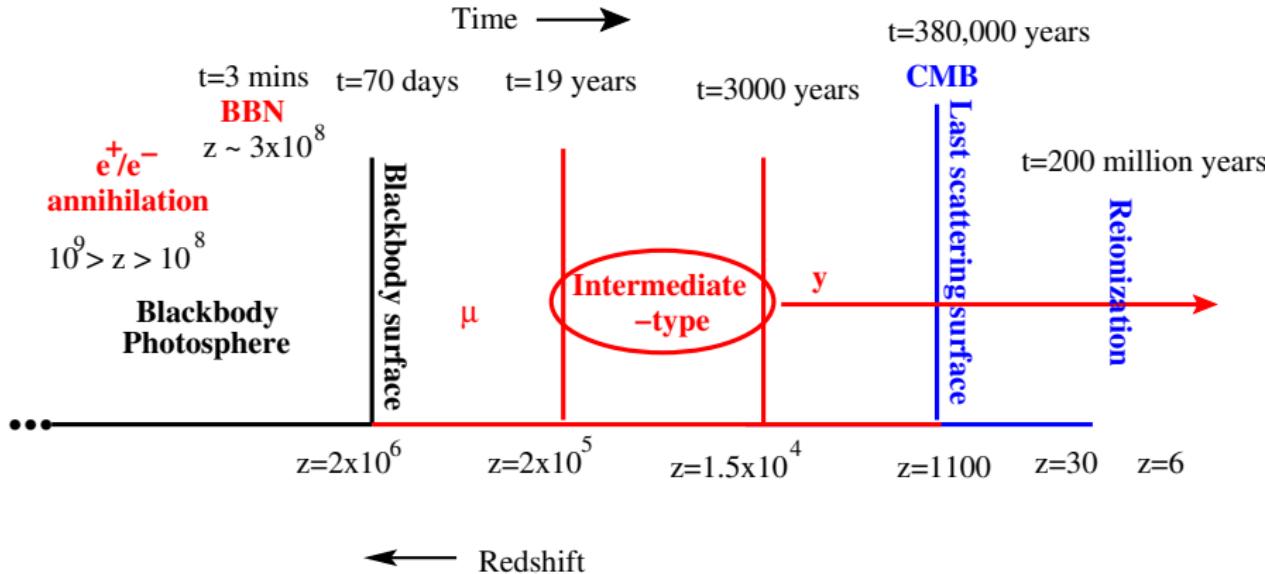
## Intermediate-type distortions (Khatri and Sunyaev 2012b)

Solve Kompaneets equation with initial condition of  $y$ -type solution.

$$\frac{\partial n}{\partial y_\gamma} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \left( n + n^2 + \frac{T_e}{T} \frac{\partial n}{\partial x} \right), \quad \frac{T_e}{T} = \frac{\int (n+n^2)x^4 dx}{4 \int nx^3 dx}$$

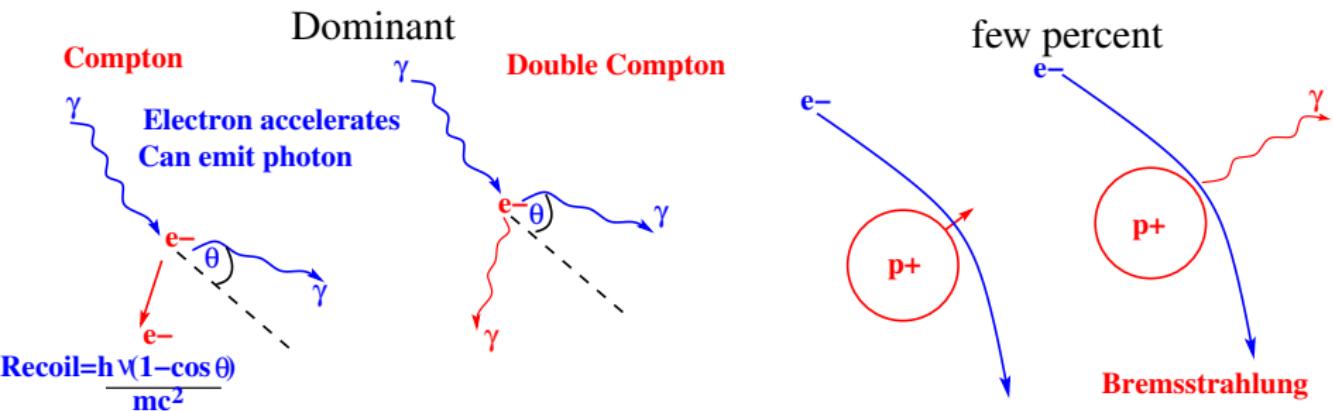


# Intermediate-type distortions



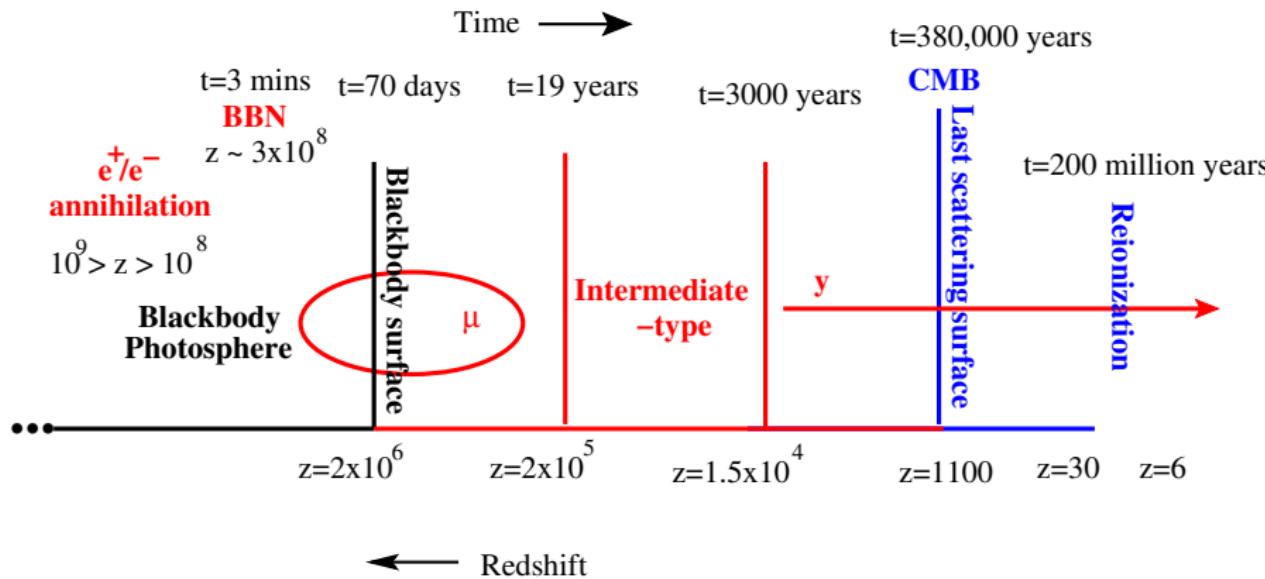
intermediate-type distortions: Numerically solve Kompaneets equation

# Processes responsible for creation of CMB spectrum



- ▶ Double Compton and bremsstrahlung create/absorb photons ( $\propto 1/x^2$ )
- ▶ Compton scattering distributes them over the whole spectrum

# $\mu$ -type distortions



Compton + double Compton + bremsstrahlung  
Analytic solution:  $\mu = 1.4 \int \frac{dQ}{dz} e^{-\mathcal{T}(z)} dz$   
(Sunyaev and Zeldovich 1970)

# Solutions for $\mathcal{T}(Z)$

Old solutions

(Sunyaev and Zeldovich 1970, Danese and de Zotti 1982)

Extension of old solutions to include both double Compton and bremsstrahlung

$$\mathcal{T}(z) \approx \left[ \left( \frac{1+z}{1+z_{dC}} \right)^5 + \left( \frac{1+z}{1+z_{br}} \right)^{5/2} \right]^{1/2} + \varepsilon \ln \left[ \left( \frac{1+z}{1+z_\varepsilon} \right)^{5/4} + \sqrt{1 + \left( \frac{1+z}{1+z_\varepsilon} \right)^{5/2}} \right]$$

This solution has accuracy of  $\sim 10\%$ ,  $z_{dC} \approx 1.96 \times 10^6$

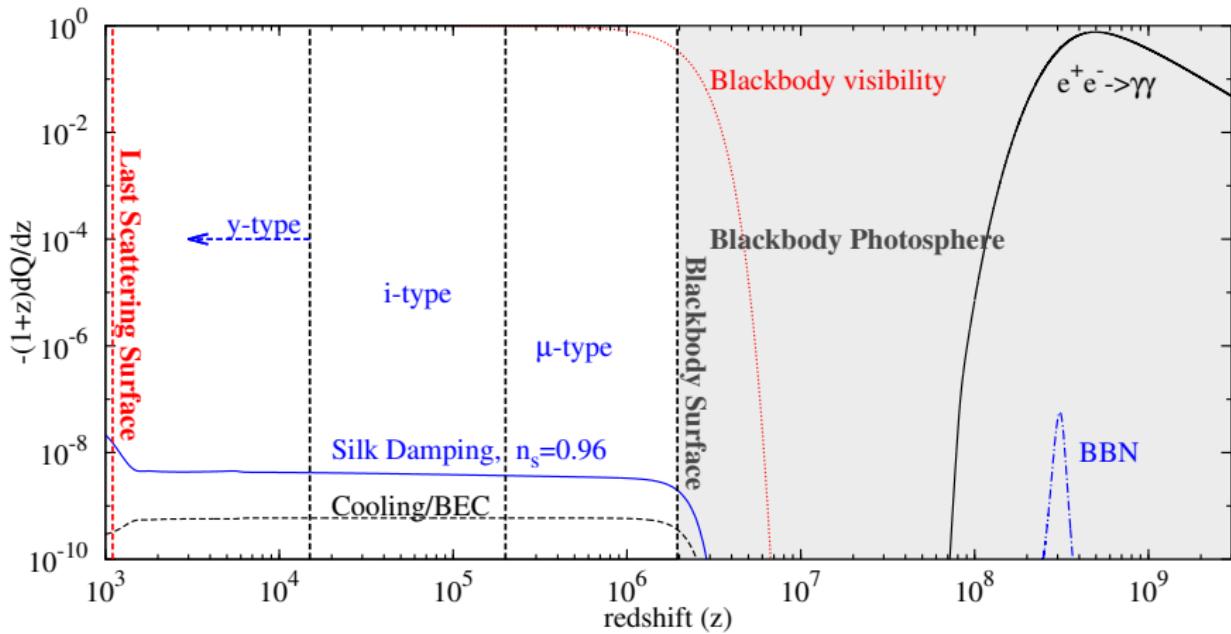
Numerical studies: Illarionov and Sunyaev 1975, Burigana, Danese, de Zotti 1991, Hu and Silk 1993, Chluba and Sunyaev 2012

New solution, accuracy  $\sim 1\%$

(Khatri and Sunyaev 2012a)

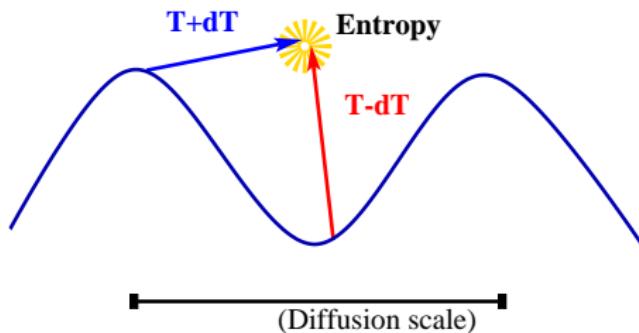
$$\begin{aligned} \mathcal{T}(z) \approx & 1.007 \left[ \left( \frac{1+z}{1+z_{dC}} \right)^5 + \left( \frac{1+z}{1+z_{br}} \right)^{5/2} \right]^{1/2} + 1.007 \varepsilon \ln \left[ \left( \frac{1+z}{1+z_\varepsilon} \right)^{5/4} + \sqrt{1 + \left( \frac{1+z}{1+z_\varepsilon} \right)^{5/2}} \right] \\ & + \left[ \left( \frac{1+z}{1+z_{dC'}} \right)^3 + \left( \frac{1+z}{1+z_{br'}} \right)^{1/2} \right], \end{aligned}$$

# The general picture



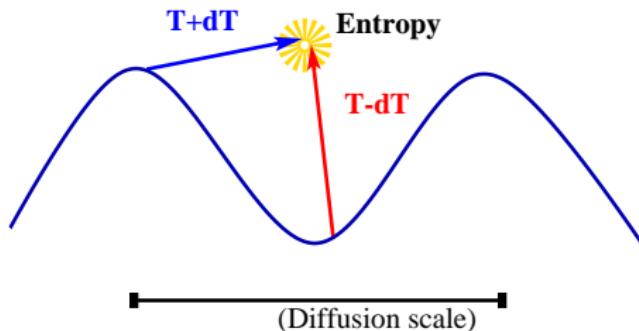
# Silk damping

Photon diffusion  $\longrightarrow$  mixing of blackbodies



# Silk damping

Photon diffusion  $\longrightarrow$  mixing of blackbodies

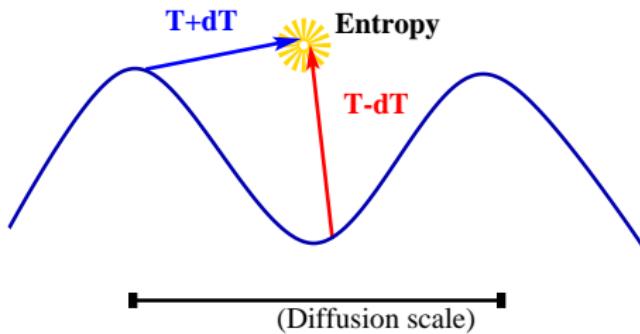


Mixing of blackbodies gives  $\gamma$ -type distortion

*Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004*

# Silk damping

Photon diffusion → mixing of blackbodies



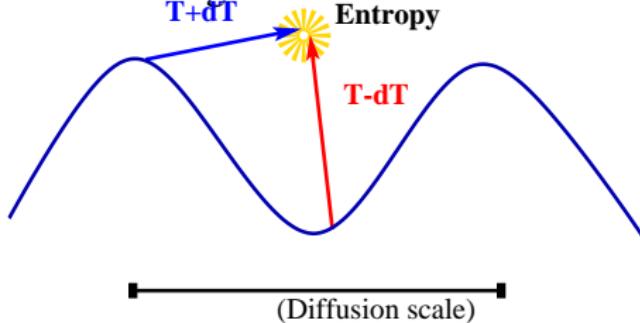
Mixing of blackbodies gives  $\gamma$ -type distortion

Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004

$$\langle n_{\text{Planck}} \left( T + \frac{\delta T}{T} \right) \rangle = \langle \frac{1}{e^{\frac{h\nu}{k(T+\delta T/T)}} - 1} \rangle$$

# Silk damping

Photon diffusion → mixing of blackbodies



Mixing of blackbodies gives  $\gamma$ -type distortion

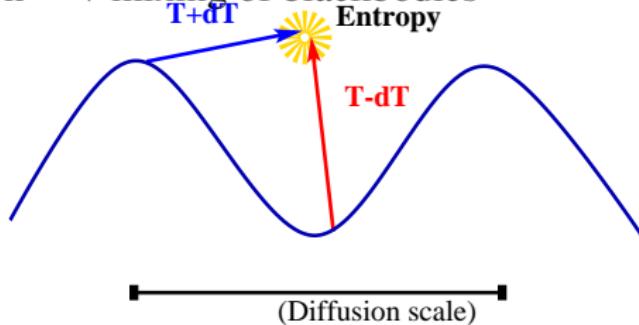
Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004

$$\langle n_{\text{Planck}} \rangle = \frac{1}{e^{\frac{hv}{kT}} - 1} + \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle T \frac{\partial n_{\text{Pl}}}{\partial T} + \frac{1}{2} \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{\text{Pl}}}{\partial T}$$

A red oval highlights the term  $T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{\text{Pl}}}{\partial T}$ .

## Silk damping

Photon diffusion → mixing of blackbodies



Mixing of blackbodies gives y-type distortion

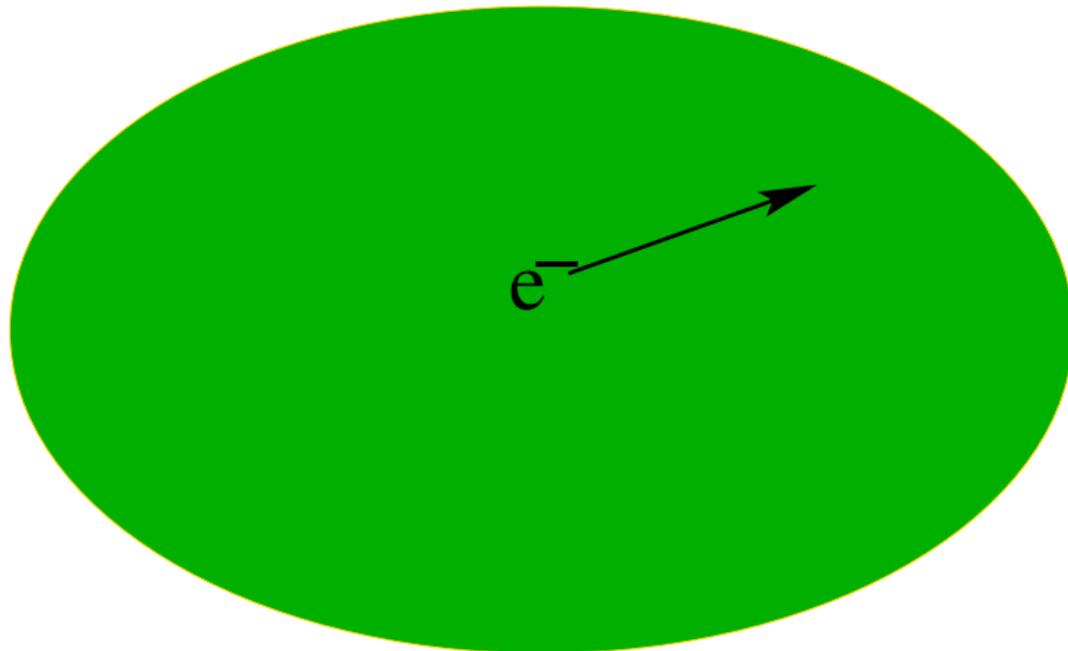
Zeldovich, Illarionov & Sunyaev 1972, Chluba & Sunyaev 2004

$$\begin{aligned}
< n_{\text{Planck}} > &= \frac{1}{e^{\frac{h\nu}{kT}} - 1} + \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle T \frac{\partial n_{\text{Pl}}}{\partial T} + \frac{1}{2} \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle T^4 \frac{\partial}{\partial T} \frac{1}{T^2} \frac{\partial n_{\text{Pl}}}{\partial T} \\
&= n_{\text{Planck}} \left( T + \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle \right) + \frac{1}{2} \left\langle \left( \frac{\delta T}{T} \right)^2 \right\rangle n_{\text{SZ}}
\end{aligned}$$

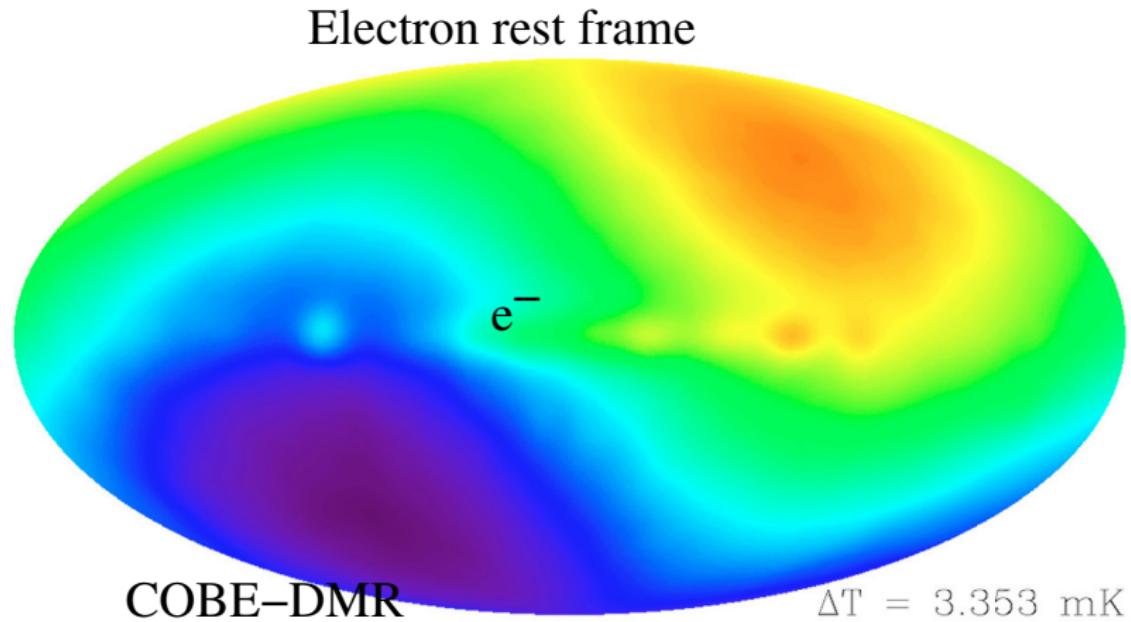
2/3 ↑ Black body      1/3 ↑ Kompaneets operator/SZ

## SZ effect in CMB rest frame: Doppler boost

CMB rest frame

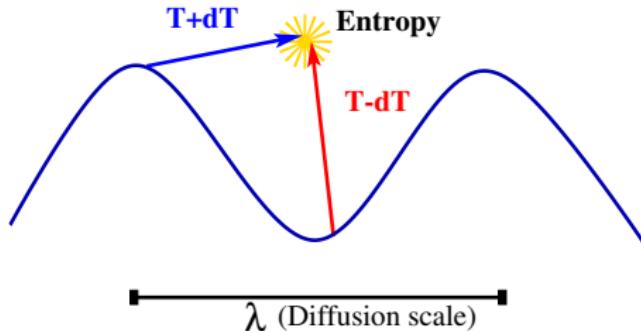


## SZ effect in electron rest frame: Mixing of blackbodies in the dipole seen by the electron



# Silk damping

Photon diffusion → mixing of blackbodies



Apply mixing of blackbodies result to CMB

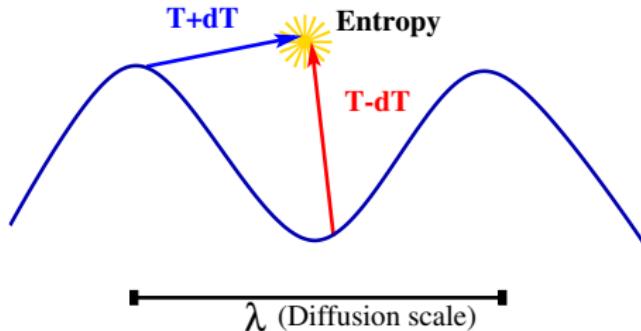
*Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012*

$$\left. \frac{d}{dt} \frac{\Delta E}{E_\gamma} \right|_{\text{distortion}} \approx - \frac{d}{dt} 2 \int \frac{k^2 dk}{2\pi^2} P_i(k) [\Theta_0^2 + 3\Theta_1^2 + (\ell > 1 \text{ terms})]$$

$$\frac{\Delta T}{T} = \sum_\ell (-i)^\ell (2\ell + 1) P_\ell \Theta_\ell$$

# Silk damping

Photon diffusion  $\longrightarrow$  mixing of blackbodies



Apply mixing of blackbodies result to CMB

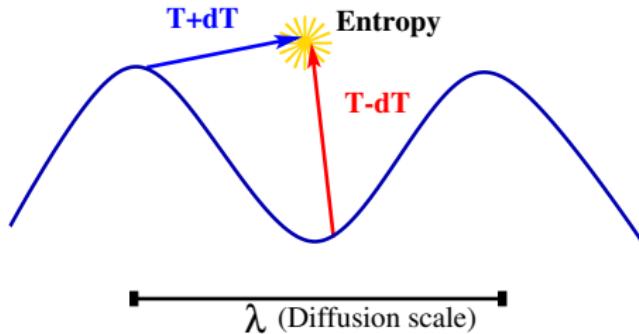
*Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012*

$$\left. \frac{d}{dt} \frac{\Delta E}{E_\gamma} \right|_{\text{distortion}} \approx - \frac{d}{dt} 2 \int \frac{k^2 dk}{2\pi^2} P_i(k) [\Theta_0^2 + 3\Theta_1^2 + (\ell > 1 \text{ terms})]$$

$$1.5 \times 10^4 \lesssim z \lesssim 2 \times 10^6 \implies 8 \lesssim k_D \lesssim 10^4 \text{ Mpc}^{-1}$$

# Silk damping

Photon diffusion → mixing of blackbodies



Apply mixing of blackbodies result to CMB

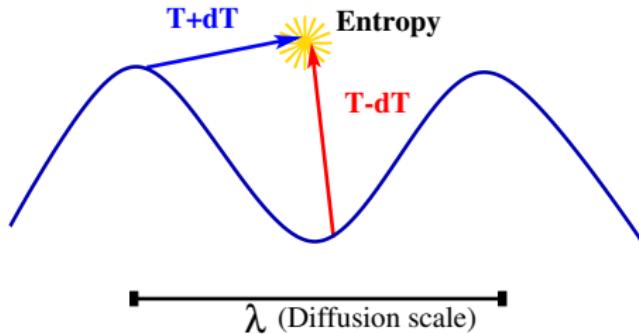
*Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012*

$$\left. \frac{d}{dt} \frac{\Delta E}{E_\gamma} \right|_{\text{distortion}} \approx - \frac{d}{dt} 2 \int \frac{k^2 dk}{2\pi^2} P_i(k) [\Theta_0^2 + 3\Theta_1^2 + (\ell > 1 \text{ terms})]$$

density  $\Theta_0 \propto \cos(kr_s)e^{-k^2/k_D^2}$ , velocity  $\Theta_1 \propto \sin(kr_s)e^{-k^2/k_D^2}$

# Silk damping

Photon diffusion → mixing of blackbodies



Apply mixing of blackbodies result to CMB

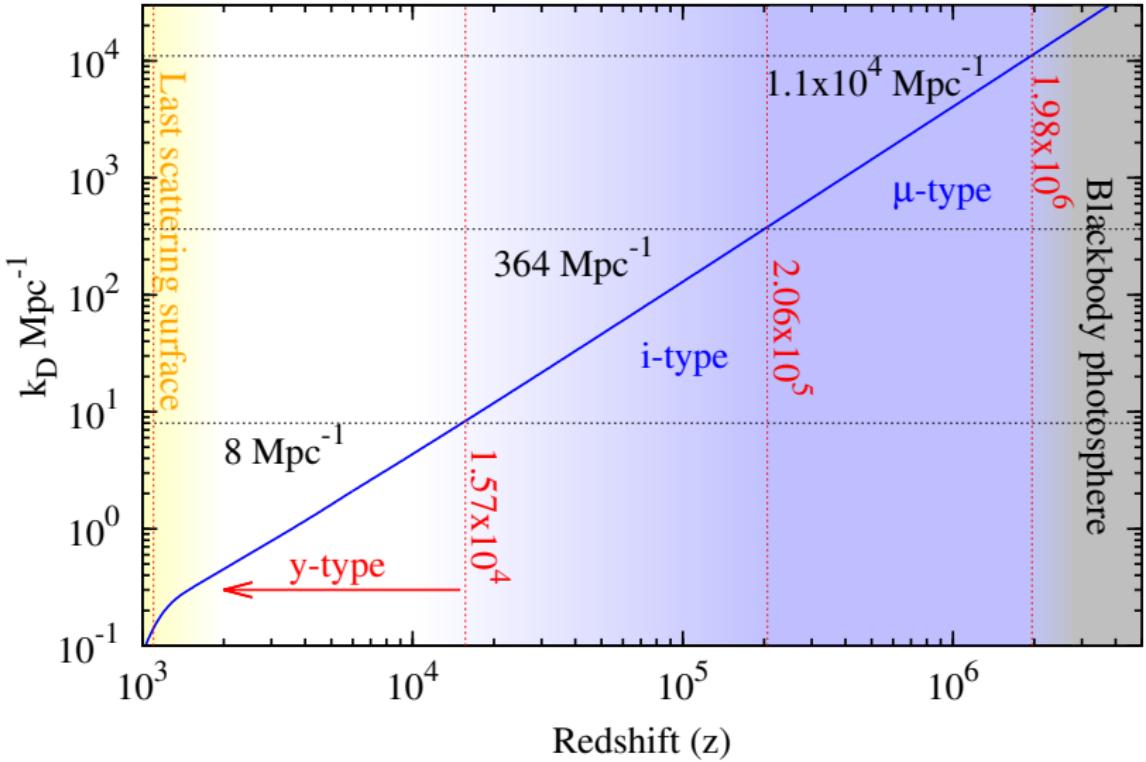
*Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012*

$$\left. \frac{d}{dt} \frac{\Delta E}{E_\gamma} \right|_{\text{distortion}} \approx - \frac{d}{dt} 2 \int \frac{k^2 dk}{2\pi^2} P_i(k) [\Theta_0^2 + 3\Theta_1^2 + (\ell > 1 \text{ terms})]$$

density  $\Theta_0 \propto \cos(kr_s)e^{-k^2/k_D^2}$ , velocity  $\Theta_1 \propto \sin(kr_s)e^{-k^2/k_D^2}$

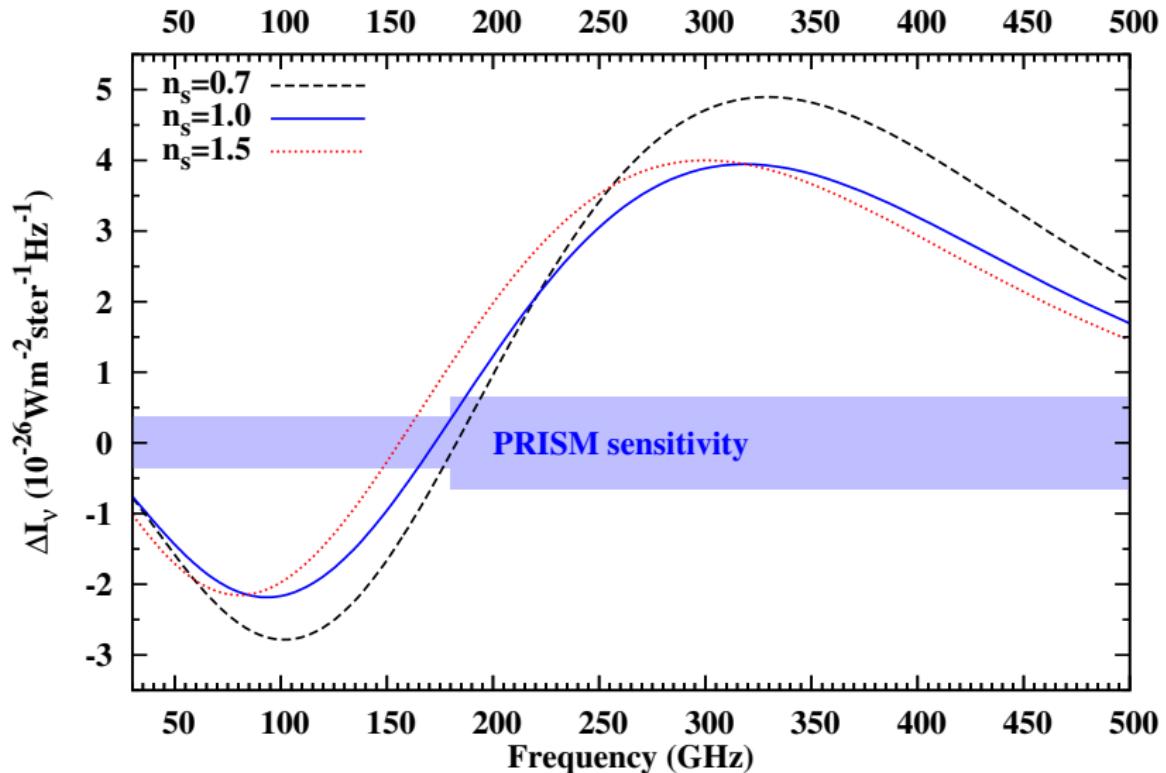
Total energy in the standing wave is independent of time

# The Silk damping scale



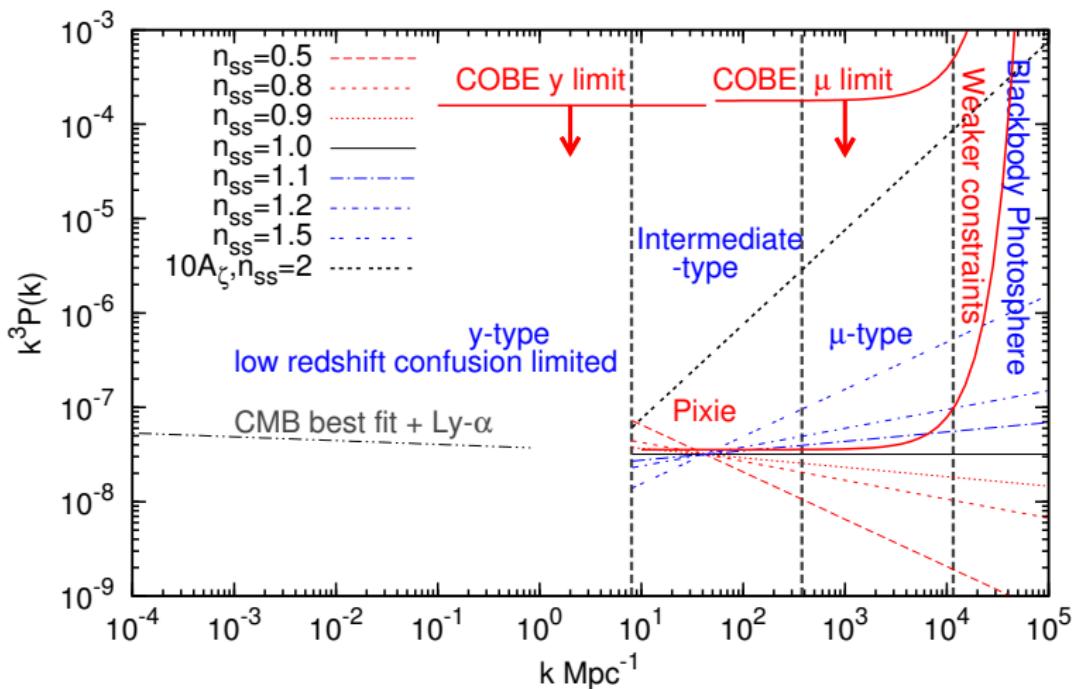
## Silk damping (*Khatri and Sunyaev 2012b*)

Add spectra for different  $k_D(y_\gamma)$  with weights  $\propto P_i(k_D)$

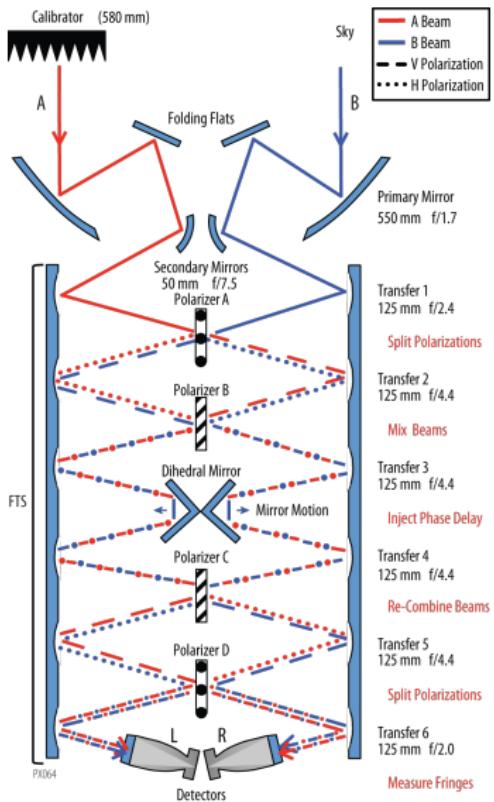


# Pivot point $k_0 = 42 \text{ Mpc}^{-1}$

$$P_\zeta = (A_\zeta 2\pi^2/k^3)(k/k_0)^{n_s - 1 + \frac{1}{2}dn_s/d\ln k(\ln k/k_0)}$$

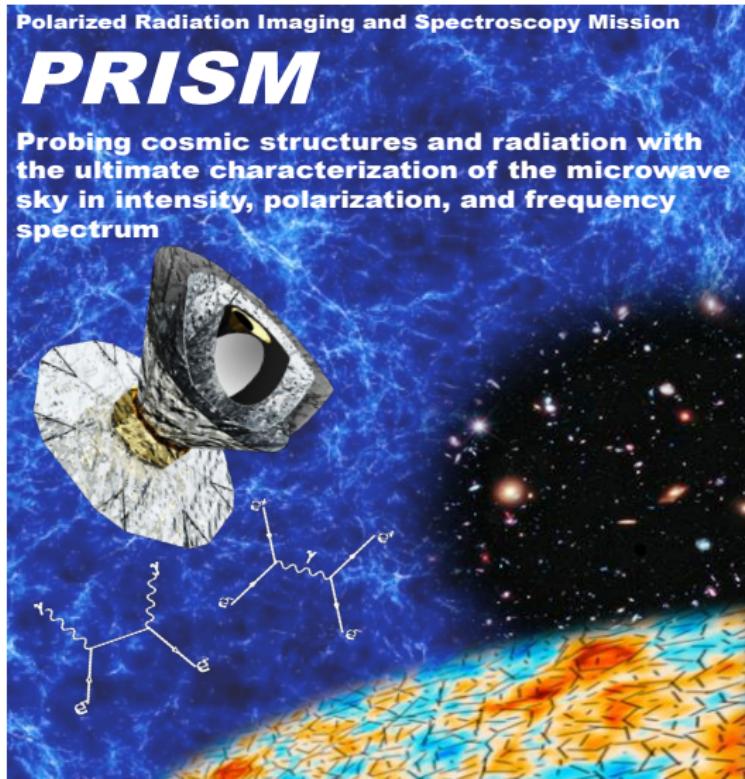


# Spectrum: Pixie will improve over the COBE precision by at least 3 orders of magnitude (*Kogut et al. 2011*)



# New proposal to ESA for L class mission

10 times more sensitive than Pixie



# Please Sign Up to show your support for PRISM

<http://www.prism-mission.org/>

The screenshot shows a web browser window with the following details:

- Menu Bar:** File, Edit, View, History, Bookmarks, Tools, Help.
- Title Bar:** PRISM Mission.
- Address Bar:** www.prism-mission.org.
- Toolbar:** Back, Forward, Stop, Refresh, Home, Google search bar, Download, and other icons.
- Navigation Bar:** Ent, astro, mail, LEO Deutsch-Englis..., PDG Particle Data Group, NASA ADS, NASA ADS.
- Content Area:**
  - Image:** A graphic of vertical dashed lines in various colors (green, red, blue) on a black background.
  - Title:** PRISM  
Polarized Radiation Imaging  
and Spectroscopy Mission
  - Text:** A white paper in response to the European Space Agency Call for white papers for the definition of the L2 and L3 missions in the ESA Science Programme
  - Text:** Probing cosmic structures and radiation with the ultimate polarimetric spectro-imaging of the microwave and far-infrared sky
  - Image:** An illustration of a large, multi-layered dish antenna, likely the PRISM instrument itself.
- Left Sidebar:** A vertical menu with buttons for Home, SIGN UP!, Executive summary, Clusters survey, Infrared background, Gravity waves, Spectral distortions, Legacy archive, Strawman mission, and PRISM White Paper.

## Fisher matrix forecasts

Model:

$$\Delta I_V = t I_V^t + y I_V^y + I_V^{\text{damping}}(n_s, A_\zeta, dn_s/d\ln k).$$

Marginalize over temperature ( $t$ ) and SZ effect ( $y$ )

$I_V^{\text{damping}}$  contains  $i$ -type and  $\mu$ -type distortions

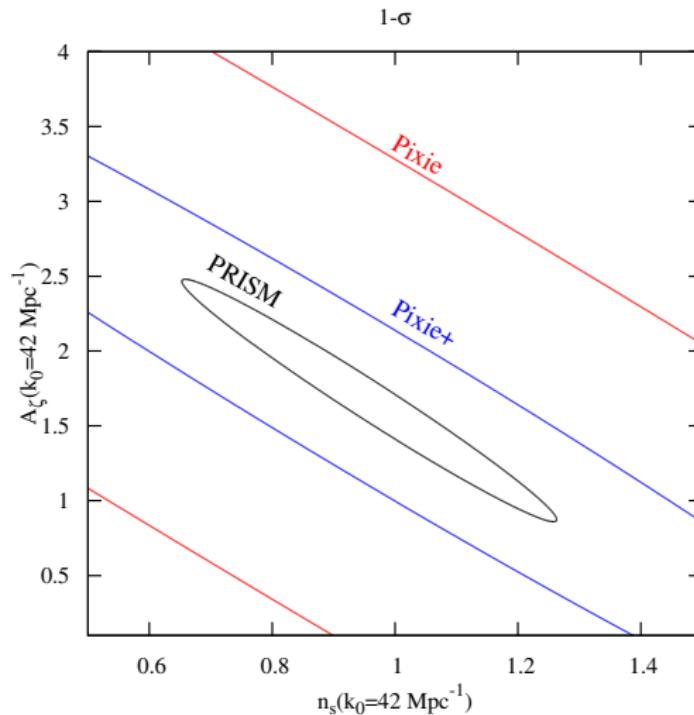
# Fisher matrix forecasts

(Khatri and Sunyaev 2013)

Pixie-like experiments:

$(x,y) \equiv (\text{Resolution GHz}, \delta I(v) = 10^{-26} \text{Wm}^{-2}\text{Sr}^{-1}\text{Hz}^{-1})$

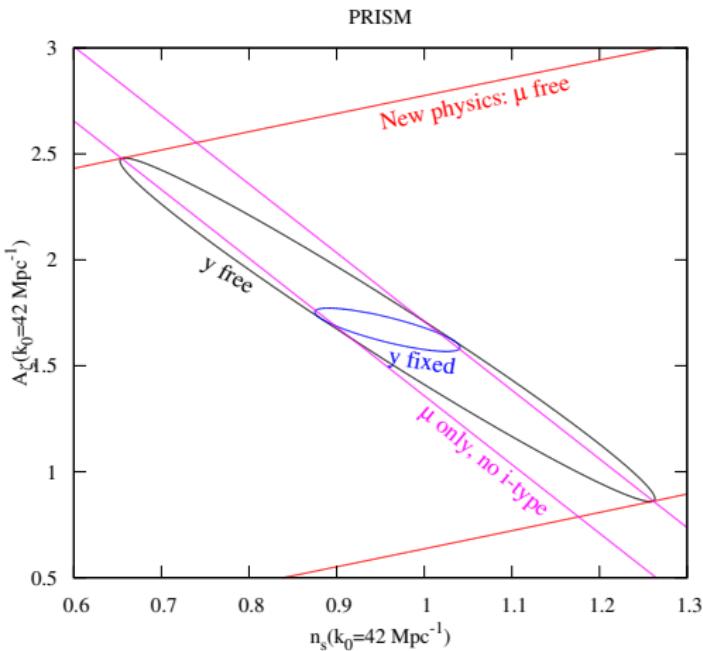
Pixie=(15,5)



# Importance of $i$ -type distortions, degeneracies

(Khatri and Sunyaev 2013)

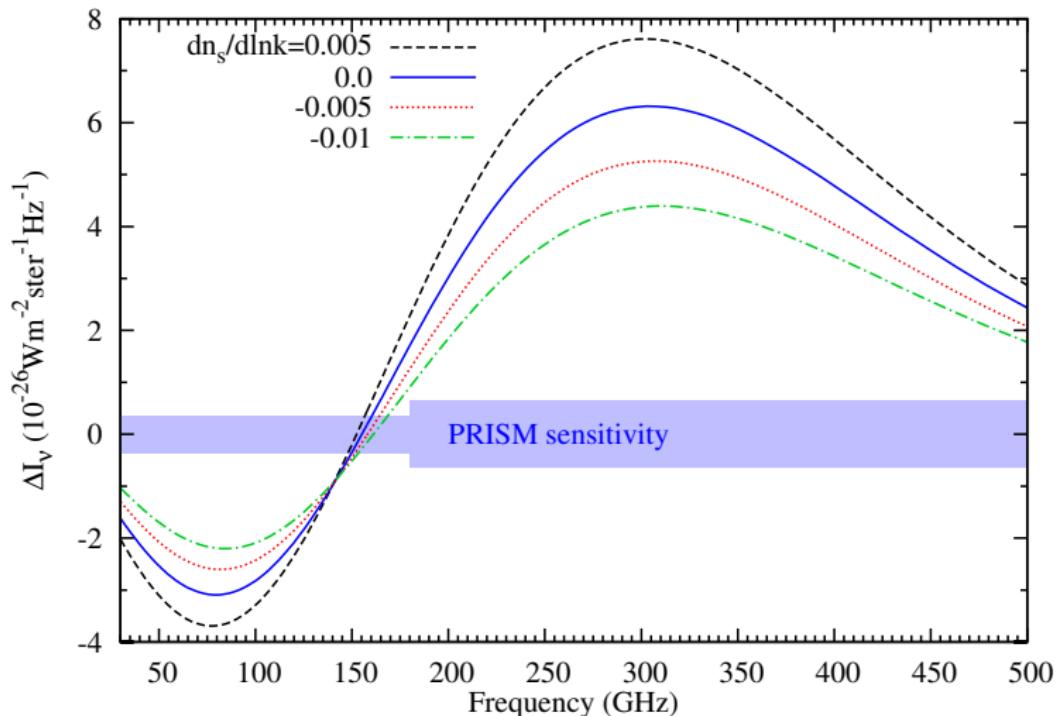
Information in the shape of  $i$ -type distortions breaks the  $A_\zeta - n_s$  degeneracy



# Running spectral index

Fix the pivot point at  $k = 0.05 \text{ Mpc}^{-1}$

Long lever arm: Main effect in the amplitude of distortion



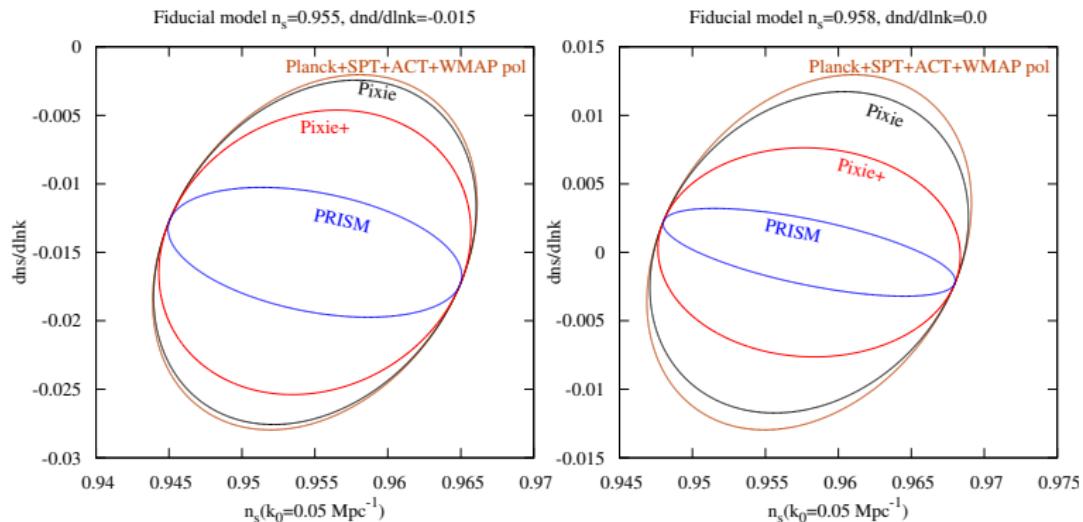
# Fisher matrix forecasts with Planck+SPT+ACT+WMAP-pol

(Khatri and Sunyaev 2013)

Planck parameters, running spectrum, Pivot point  $k_0 = 0.05$

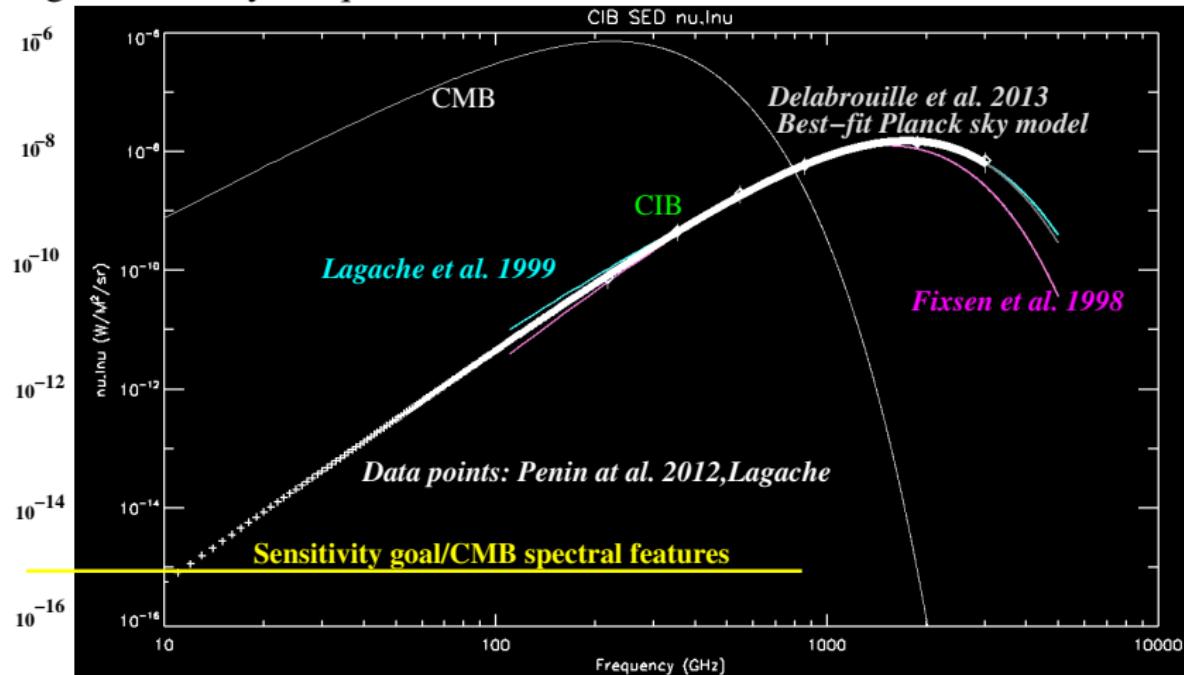
(x,y)  $\equiv$  (Resolution GHz ,  $\delta I(v) = 10^{-26} \text{Wm}^{-2}\text{Sr}^{-1}\text{Hz}^{-1}$  )

Pixie=(15,5)



# Foregrounds

Figure courtesy Jacques Delabrouille



## Summary

- The shape of the  $\mu$  and intermediate type distortions is rich in information

## Summary

- ▶ The shape of the  $\mu$  and intermediate type distortions is rich in information
- ▶ With spectral distortions we can extend our 'view' of inflation from 6-7 e-folds at present to 17 e-folds

## Summary

- ▶ The shape of the  $\mu$  and intermediate type distortions is rich in information
- ▶ With spectral distortions we can extend our 'view' of inflation from 6-7 e-folds at present to 17 e-folds
- ▶ Spectral distortions take us a little nearer to the end of inflation

## Summary

- ▶ The shape of the  $\mu$  and intermediate type distortions is rich in information
- ▶ With spectral distortions we can extend our 'view' of inflation from 6-7 e-folds at present to 17 e-folds
- ▶ Spectral distortions take us a little nearer to the end of inflation
- ▶  $\mu$ -type and intermediate type distortions can be calculated very fast using analytic and pre-calculated cosmology-independent high precision numerical solutions (Green's functions). This allows us to explore the rich multidimensional parameter space

# Summary

- ▶ The shape of the  $\mu$  and intermediate type distortions is rich in information
- ▶ With spectral distortions we can extend our 'view' of inflation from 6-7 e-folds at present to 17 e-folds
- ▶ Spectral distortions take us a little nearer to the end of inflation
- ▶  $\mu$ -type and intermediate type distortions can be calculated very fast using analytic and pre-calculated cosmology-independent high precision numerical solutions (Green's functions). This allows us to explore the rich multidimensional parameter space
- ▶  $i$ -type distortions are quite powerful in removing degeneracies between power spectrum parameters. The extra information comes from the shape of the  $i$ -type distortion

## Summary continued

With intermediate-type distortions we can distinguish between different mechanisms of energy injection which have different redshift dependence

## Summary continued

With intermediate-type distortions we can distinguish between different mechanisms of energy injection which have different redshift dependence

There is more....

- ▶ Cosmological recombination spectrum gives measurement of primordial helium

*Kurt,Zeldovich,Sunyaev,Peebles,Dubrovich,Chluba,Rubino-Martin*

## Summary continued

With intermediate-type distortions we can distinguish between different mechanisms of energy injection which have different redshift dependence

There is more....

- ▶ Cosmological recombination spectrum gives measurement of primordial helium

*Kurt,Zeldovich,Sunyaev,Peebles,Dubrovich,Chluba,Rubino-Martin*

- ▶ Resonant scattering on C,N,O and other ions during and after **reionization** makes the optical depth to the last scattering surface frequency dependent

*Basu, Hernandez-Monteagudo, and Sunyaev 2004*

## Summary continued

With intermediate-type distortions we can distinguish between different mechanisms of energy injection which have different redshift dependence

There is more....

- ▶ Cosmological recombination spectrum gives measurement of primordial helium  
*Kurt,Zeldovich,Sunyaev,Peebles,Dubrovich,Chluba,Rubino-Martin*
- ▶ Resonant scattering on C,N,O and other ions during and after **reionization** makes the optical depth to the last scattering surface frequency dependent  
*Basu, Hernandez-Monteagudo, and Sunyaev 2004*
- ▶ Sunyaev-Zeldovich effect from hot electrons during reionization/WHIM can give a measurement of average electron temperature, **find missing baryons**  
*Zeldovich & Sunyaev 1969, Hu, Scott, Silk 1994, Cen and Ostriker 1999,2006,*

## Summary continued

With intermediate-type distortions we can distinguish between different mechanisms of energy injection which have different redshift dependence

There is more....

- ▶ Cosmological recombination spectrum gives measurement of primordial helium  
*Kurt,Zeldovich,Sunyaev,Peebles,Dubrovich,Chluba,Rubino-Martin*
- ▶ Resonant scattering on C,N,O and other ions during and after **reionization** makes the optical depth to the last scattering surface frequency dependent  
*Basu, Hernandez-Monteagudo, and Sunyaev 2004*
- ▶ Sunyaev-Zeldovich effect from hot electrons during reionization/WHIM can give a measurement of average electron temperature, **find missing baryons**  
*Zeldovich & Sunyaev 1969, Hu, Scott, Silk 1994, Cen and Ostriker 1999,2006,*
- ▶ Primordial non-gaussianity on extremely small scales  
*Pajer and Zaldarriaga 2012, Ganc and Komatsu 2012*

## Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$

*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*

## Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$   
*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*
- ▶ Adiabatic cooling: Opposite sign to Silk damping with  $n_s = 1$   
*(Chluba and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012b)*

## Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$   
*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*
- ▶ Adiabatic cooling: Opposite sign to Silk damping with  $n_s = 1$   
*(Chluba and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012b)*
- ▶ Particle decay:  $\frac{dQ}{dz} \propto \frac{e^{-\left(\frac{1+z_{\text{decay}}}{1+z}\right)^2}}{(1+z)^4}$   
*(Hu and Silk 1993, Chluba and Sunyaev 2012, Khatri and Sunyaev 2012a, 2012b)*

## Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$   
*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*
- ▶ Adiabatic cooling: Opposite sign to Silk damping with  $n_s = 1$   
*(Chluba and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012b)*
- ▶ Particle decay:  $\frac{dQ}{dz} \propto \frac{e^{-\left(\frac{1+z_{\text{decay}}}{1+z}\right)^2}}{(1+z)^4}$   
*(Hu and Silk 1993, Chluba and Sunyaev 2012, Khatri and Sunyaev 2012a, 2012b)*
- ▶ Cosmic strings:  $\frac{dQ}{dz} \propto \text{constant}$   
*Tashiro, Sabancilar, Vachaspati 2012*

## Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$   
*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*
- ▶ Adiabatic cooling: Opposite sign to Silk damping with  $n_s = 1$   
*(Chluba and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012b)*
- ▶ Particle decay:  $\frac{dQ}{dz} \propto \frac{e^{-\left(\frac{1+z_{\text{decay}}}{1+z}\right)^2}}{(1+z)^4}$   
*(Hu and Silk 1993, Chluba and Sunyaev 2012, Khatri and Sunyaev 2012a, 2012b)*
- ▶ Cosmic strings:  $\frac{dQ}{dz} \propto \text{constant}$   
*Tashiro, Sabancilar, Vachaspati 2012*
- ▶ Primordial magnetic fields :  $\propto (1+z)^{(3n+7)/2}$ ,  $n$  is the spectral index of magnetic field power spectrum  
*(Jedamzik, Katalinic, and Olinto 2000)*

## Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$   
*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*
- ▶ Adiabatic cooling: Opposite sign to Silk damping with  $n_s = 1$   
*(Chluba and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012b)*
- ▶ Particle decay:  $\frac{dQ}{dz} \propto \frac{e^{-\left(\frac{1+z_{\text{decay}}}{1+z}\right)^2}}{(1+z)^4}$   
*(Hu and Silk 1993, Chluba and Sunyaev 2012, Khatri and Sunyaev 2012a, 2012b)*
- ▶ Cosmic strings:  $\frac{dQ}{dz} \propto \text{constant}$   
*Tashiro, Sabancilar, Vachaspati 2012*
- ▶ Primordial magnetic fields :  $\propto (1+z)^{(3n+7)/2}$ ,  $n$  is the spectral index of magnetic field power spectrum  
*(Jedamzik, Katalinic, and Olinto 2000)*
- ▶ Black holes: Depends on the mass function  
*Tashiro and Sugiyama 2008, Carr et al. 2010*

# Summary continued

- ▶ Silk damping:  $\frac{dQ}{dz} \propto (1+z)^{(3n_s-5)/2}$   
*(Chluba, Khatri and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012)*
- ▶ Adiabatic cooling: Opposite sign to Silk damping with  $n_s = 1$   
*(Chluba and Sunyaev 2012, Khatri, Sunyaev and Chluba 2012b)*
- ▶ Particle decay:  $\frac{dQ}{dz} \propto \frac{e^{-\left(\frac{1+z_{\text{decay}}}{1+z}\right)^2}}{(1+z)^4}$   
*(Hu and Silk 1993, Chluba and Sunyaev 2012, Khatri and Sunyaev 2012a, 2012b)*
- ▶ Cosmic strings:  $\frac{dQ}{dz} \propto \text{constant}$   
*Tashiro, Sabancilar, Vachaspati 2012*
- ▶ Primordial magnetic fields :  $\propto (1+z)^{(3n+7)/2}$ ,  $n$  is the spectral index of magnetic field power spectrum  
*(Jedamzik, Katalinic, and Olinto 2000)*
- ▶ Black holes: Depends on the mass function  
*Tashiro and Sugiyama 2008, Carr et al. 2010*
- ▶ Quantum wave function collapse:  $\frac{dQ}{dz} \propto (1+z)^{-4}$   
*Lochan, Das and Bassi 2012*

## **There is still a long road ahead for CMB cosmology**

CMB spectrum is very rich in information about the early Universe,  
late time Universe and fundamental physics

# There is still a long road ahead for CMB cosmology

CMB spectrum is very rich in information about the early Universe,  
late time Universe and fundamental physics

This information is accessible and within reach of experiments in not  
too far future: Pixie, PRISM

## Public code/pre-calculated numerical solutions

Example Mathematica code + high precision pre-calculated numerical solutions for i-type distortions available at  
<http://www.mpa-garching.mpg.de/~khatri/idistort.html>  
Fortran version soon.

## Algorithm for fast solution, $\sim 1\%$ level accuracy

(Khatri and Sunyaev 2012b, arXiv:1207.6654)

- ▶ Calculate  $\mu$  type distortion using the analytic solution, integrating up to the redshift when  $y_\gamma = 2$ .

$$n_{\mu-type} = 1.4 n_\mu \int_{\infty}^{z(y_\gamma=2)} \frac{dQ}{dz} e^{-\mathcal{T}} \quad (1)$$

# Algorithm for fast solution, $\sim 1\%$ level accuracy

(Khatri and Sunyaev 2012b, arXiv:1207.6654)

- ▶ Calculate  $\mu$  type distortion using the analytic solution, integrating up to the redshift when  $y_\gamma = 2$ .

$$n_{\mu-type} = 1.4 n_\mu \int_{\infty}^{z(y_\gamma=2)} \frac{dQ}{dz} e^{-\mathcal{T}} \quad (1)$$

- ▶ Calculate intermediate type distortions by adding up pre-calculated numerical solutions (Green's functions) in  $\delta y_\gamma$  bins.

$$n_{i-type} = \frac{1}{Q_{num}} \sum_i \frac{dQ}{dy_\gamma} (y_\gamma^i) \delta y_\gamma^i n(y_\gamma^i) \quad (2)$$

<http://www.mpa-garching.mpg.de/khatri/idistort.html>

# Algorithm for fast solution, $\sim 1\%$ level accuracy

(Khatri and Sunyaev 2012b, arXiv:1207.6654)

- ▶ Calculate  $\mu$  type distortion using the analytic solution, integrating up to the redshift when  $y_\gamma = 2$ .

$$n_{\mu-type} = 1.4 n_\mu \int_{\infty}^{z(y_\gamma=2)} \frac{dQ}{dz} e^{-\mathcal{T}} \quad (1)$$

- ▶ Calculate intermediate type distortions by adding up pre-calculated numerical solutions (Green's functions) in  $\delta y_\gamma$  bins.

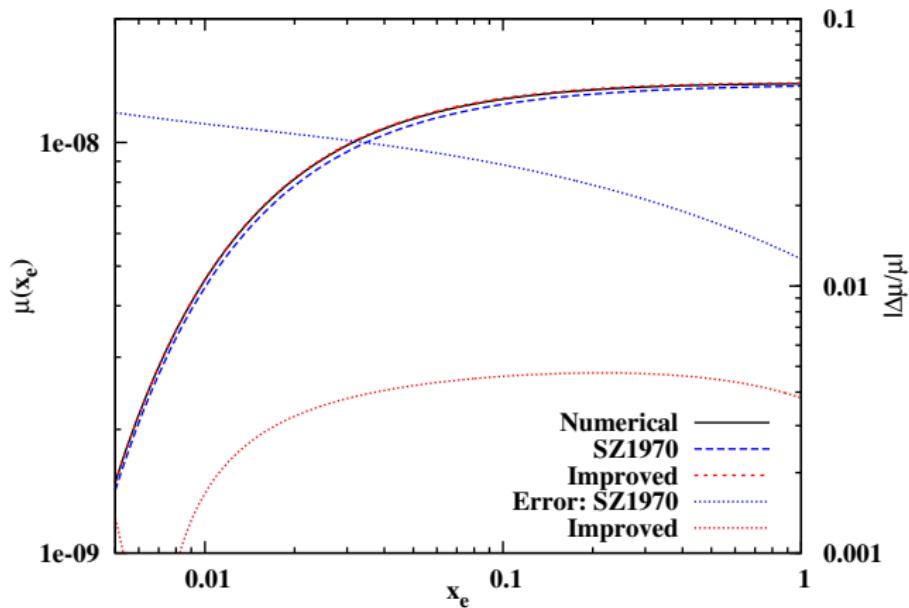
$$n_{i-type} = \frac{1}{Q_{num}} \sum_i \frac{dQ}{dy_\gamma} (y_\gamma^i) \delta y_\gamma^i n(y_\gamma^i) \quad (2)$$

<http://www.mpa-garching.mpg.de/khatri/idistort.html>

- ▶ Add rest of the energy to  $y$ -type distortions.

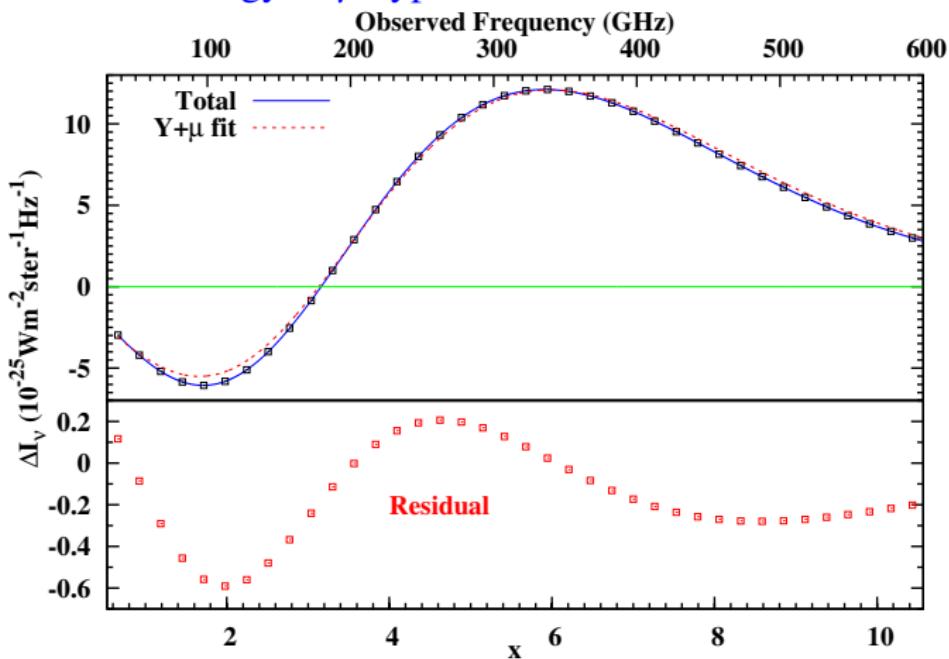
$$n_{y-type} = 0.25 n_y \int_{z(y_\gamma=0.01)}^{z=0} \frac{dQ}{dz} \quad (3)$$

# Accuracy of new solutions is better than 1%

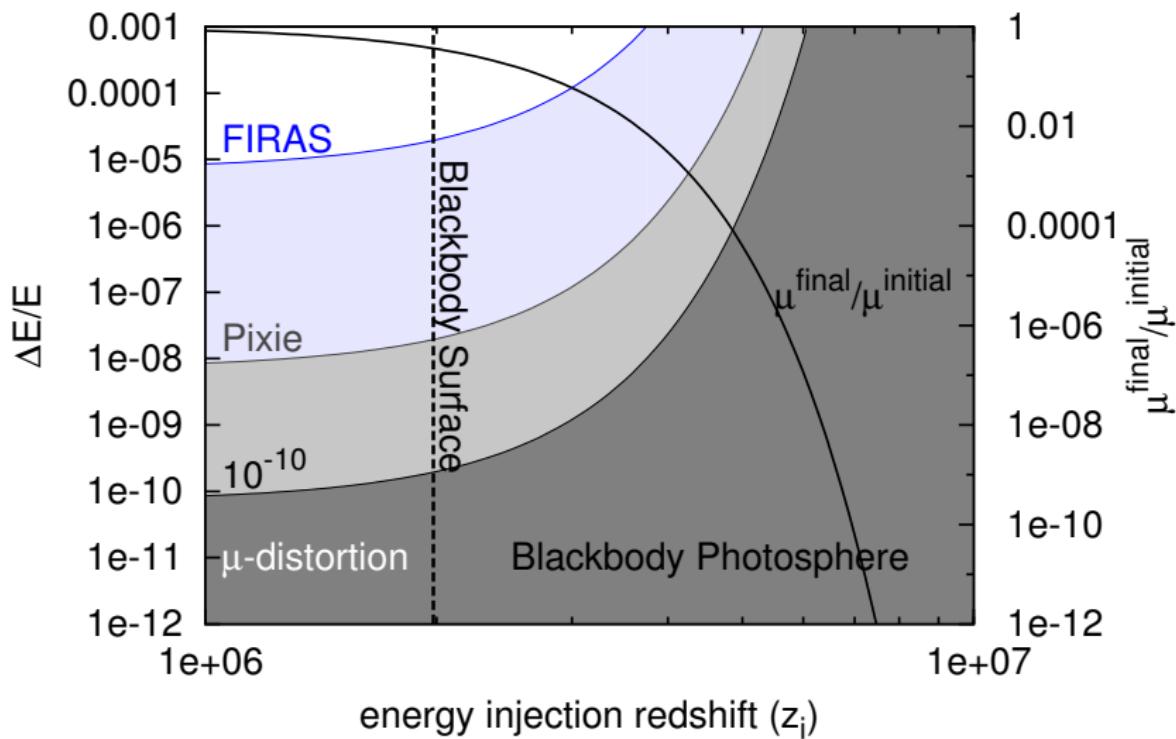


## $y+\mu$ cannot fully mimic $i$ -type distortion

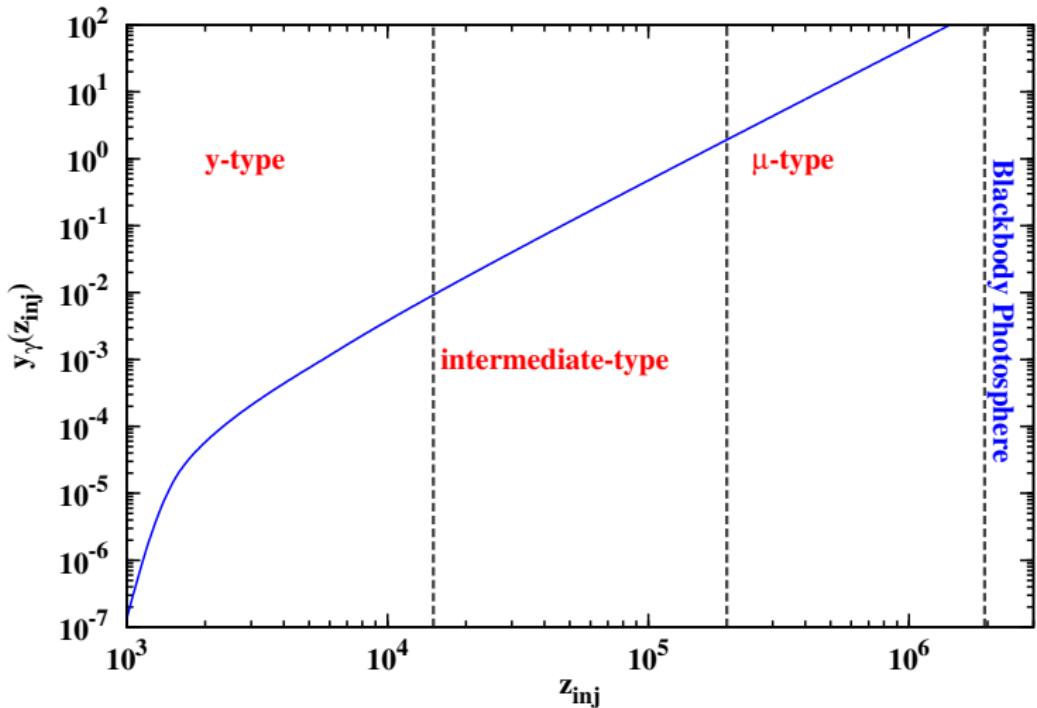
$\mu$  type and intermediate-type distortions are not independent. For Silk damping, intermediate-type distortions must contain about the same amount of energy as  $\mu$ -type distortions.



# Blackbody photosphere



$$y_\gamma = \int_{z_{\text{inj}}}^0 dt \frac{k_B \sigma_T n_e}{m_e c} T_\gamma \quad |y_\gamma \gg 1 \implies \mu, \quad y_\gamma \ll 1 \implies y$$



## Intermediate-type distortions (*Khatri and Sunyaev 2012b*)

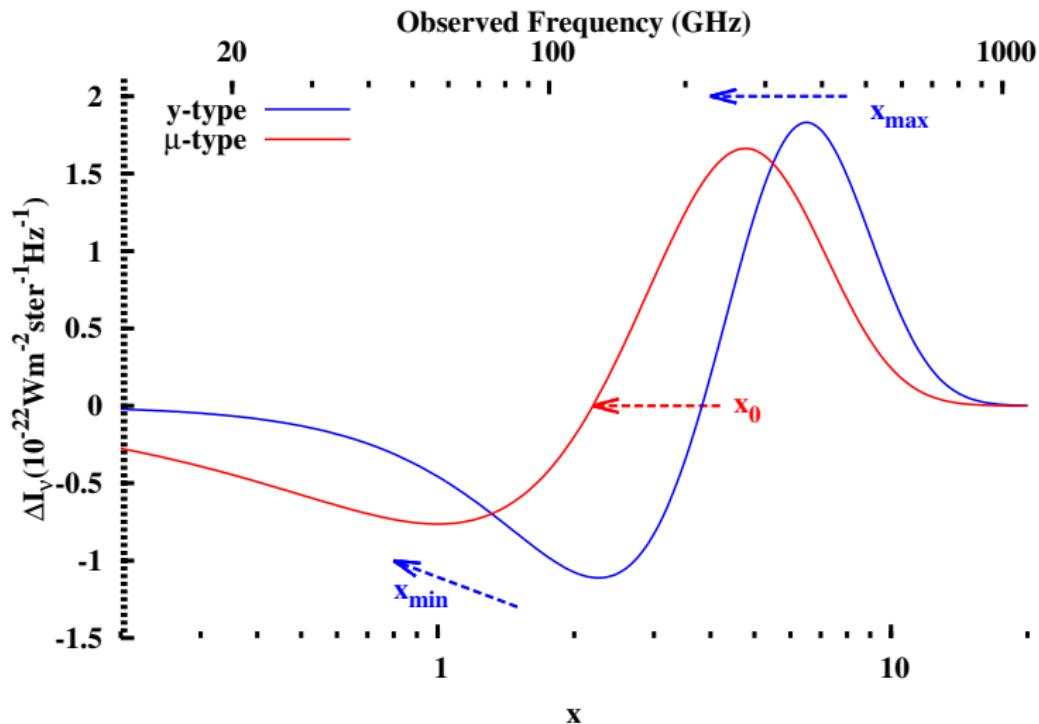
Solve Kompaneets equation with initial condition of y-type solution.

$$\frac{\partial n}{\partial y_\gamma} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \left( n + n^2 + \frac{T_e}{T} \frac{\partial n}{\partial x} \right), \quad \frac{T_e}{T} = \frac{\int (n + n^2) x^4 dx}{4 \int n x^3 dx}$$

## Intermediate-type distortions (Khatri and Sunyaev 2012b)

Solve Kompaneets equation with initial condition of y-type solution.

$$\frac{\partial n}{\partial y_\gamma} = \frac{1}{x^2} \frac{\partial}{\partial x} x^4 \left( n + n^2 + \frac{T_e}{T} \frac{\partial n}{\partial x} \right), \quad \frac{T_e}{T} = \frac{\int (n+n^2)x^4 dx}{4 \int nx^3 dx}$$



## Sensitivity of Pixie-like Experiment, resolution 15 GHz

$\frac{-2}{W \text{ m Hz Sr}}$   $\frac{-1}{\text{Sr}}$   $\frac{-1}{}$

