Nuclear matrix elements for $0\nu\beta^-\beta^-$ transitions (ICTP-2013)

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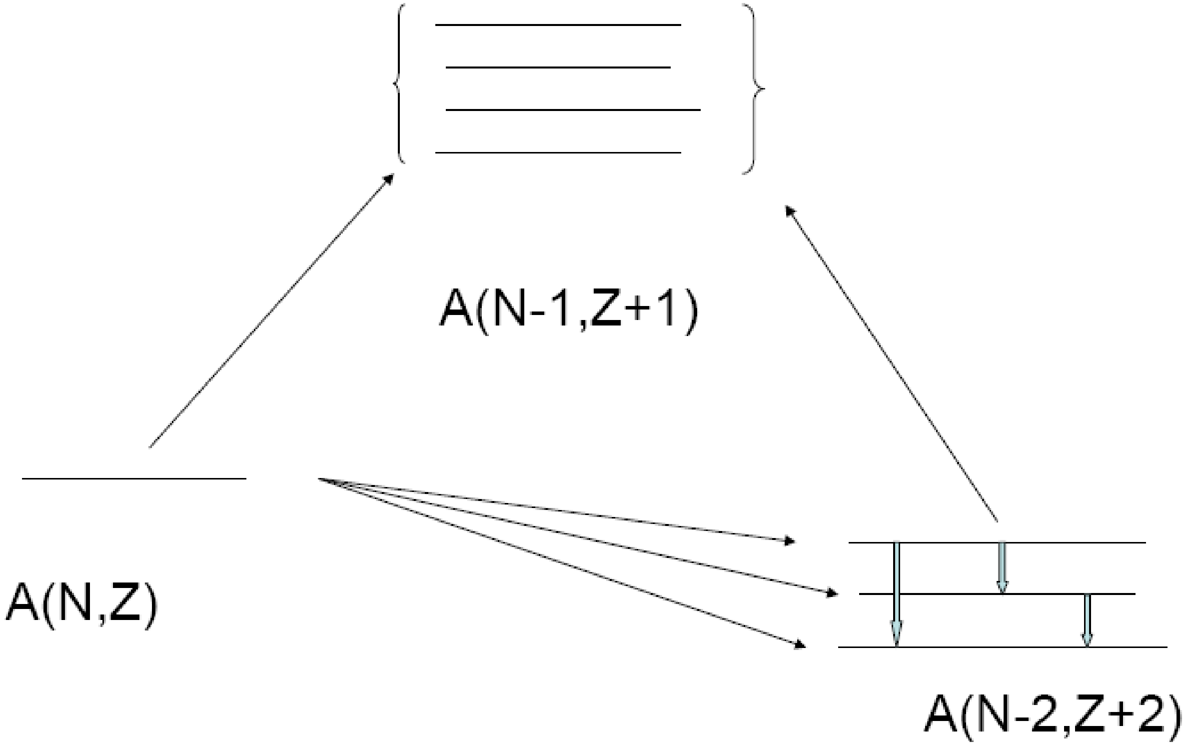
- Plan of the talk
  1. General properties of the nuclear matrix elements
  2. Summary of QRPA values (Jyväskylä-La Plata collaboration)
  3. Comparison with the available NME in other approximations
- Conclusions
- Main references: JPG 39 (2012)124009 JPG 39 (2012)085105

Schematic view of the decay path



Basic definitions ($2\nu\beta\beta - decays$)

$$\left[t_{1/2}^{(2\nu)}(0_i^+ \rightarrow J_f^+)\right]^{-1} = G^{(2\nu)}(J)\left|M^{(2\nu)}(J)\right|^2$$

$$M^{(2\nu)}(J) = \sum_{k_1 k_2} \frac{M_{\mathrm{F}}^J(1_{k_1}^+)\langle 1_{k_1}^+|1_{k_2}^+\rangle M_{\mathrm{I}}(1_{k_2}^+)}{(\frac{1}{2}\Delta + \frac{1}{2}[E(1_{k_1}^+) + \tilde{E}(1_{k_1}^+)] - M_i c^2)/m_{\mathrm{e}}c^2} \; .$$

$$\langle J_{k_1}^\pi | J_{k_2}^\pi \rangle = \sum_{pn} \left[X_{pn}^{J^\pi k_2} \bar{X}_{pn}^{J^\pi k_1} - Y_{pn}^{J^\pi k_2} \bar{Y}_{pn}^{J^\pi k_1}\right] \; .$$

Comments: it is a second order term, the intermediate states may be different from each side of the virtual transitions

Matrix elements and transition densities

$$M_{\rm I}(1^+_{k_2}) = (1^+_{k_2}\| \sum_n t^-_n \boldsymbol{\sigma}_n \|0^+_i) \ , \quad M^J_{\rm F}(1^+_{k_1}) = (J^+_f\| \sum_n t^-_n \boldsymbol{\sigma}_n \|1^+_{k_1})$$

$$\begin{aligned} M_{\rm I}(1^+_{k_2}) &= \frac{1}{\sqrt{3}} \sum_{pn} (p\|\boldsymbol{\sigma}\|n)(1^+_{k_2}\|[c^\dagger_p \tilde{c}_n]_1\|0^+_i) \ , \\ M^J_{\rm F}(1^+_{k_1}) &= \frac{1}{\sqrt{3}} \sum_{pn} (p\|\boldsymbol{\sigma}\|n)(J^+_f\|[c^\dagger_{p'} \tilde{c}_{n'}]_1\|1^+_{k_1}) \ . \end{aligned}$$

Comments: the use of the $t^{\pm}$ operators may not be correct for open shell systems, due to induced isospin violations

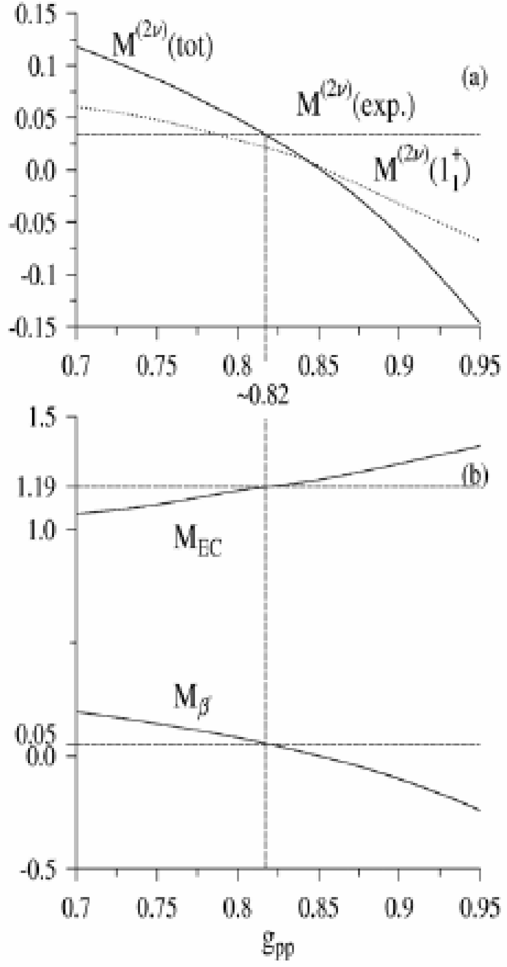
GT strengths

$$\mathsf{GT}_k^- = |(1_k^+ \| \sum_n t_n^- \boldsymbol{\sigma}_n \| 0_i^+)|^2$$

$$\mathsf{GT}_k^+ = |(1_k^+ \| \sum_n t_n^+ \boldsymbol{\sigma}_n \| 0_f^+)|^2 ,$$

Comments: other correlations, like IVSM modes, may add (substract) to the strength due to couplings with the GT correlations. GT strength distributions available from data.

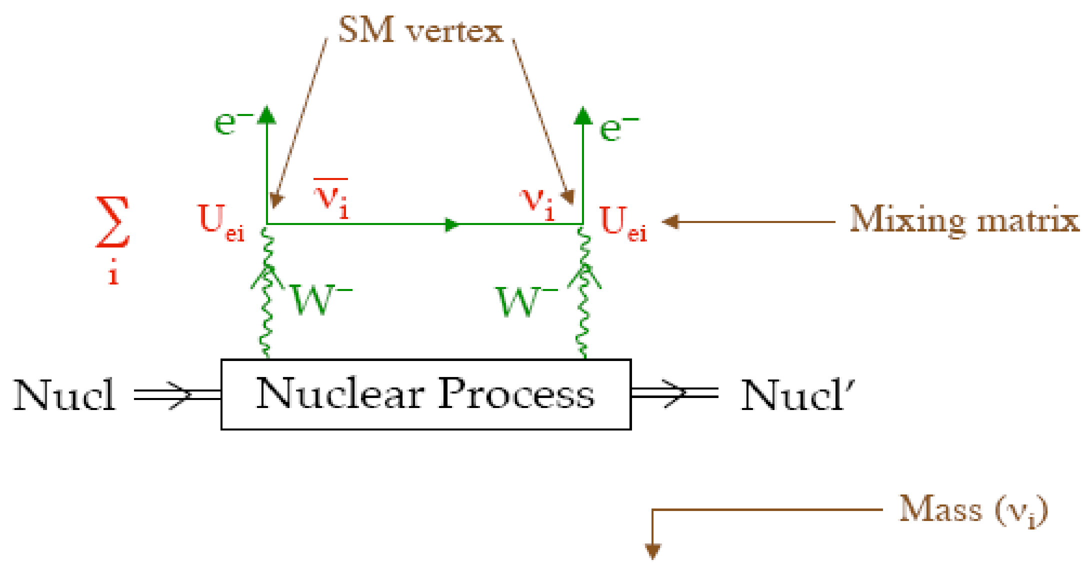
Adjusting the two neutrino mode



Mass	F_0 [yrs $^{-1}$]	$T_{1/2}^{(2\nu\beta\beta)}$ [yrs]
128	8.5 10^{-22}	2.5 $\pm$ 0.3 10^{24}
130	4.8 10^{-18}	0.9 $\pm$ 0.1 10^{21}
		7.6 $\pm$ 1.5 (stat.) $\pm$ 0.8(syst.) 10^{20}

Mass	Experimental value	Theory (this work)
128	0.025 $\pm$ 0.005	0.016
130	0.015 $\pm$ 0.001	0.012
	0.016 $\pm$ 0.002	

Basic notions on the decay



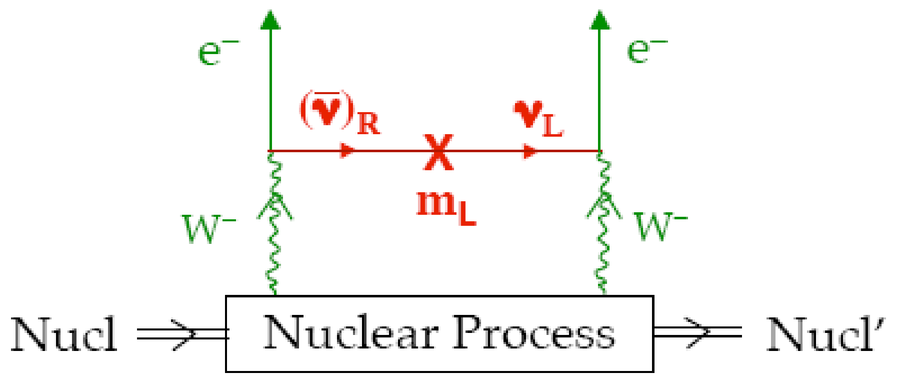
the $\overline{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] $\propto m_i$

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left| \sum_i m_i U_{ei}^2 \right| \equiv m_{\beta\beta}$$

Beyond the Standard Model

Assuming Standard Model vertices, 0νββ is —



The Majorana neutrino mass term plays two roles:

1) Violate L

2) Flip handedness

It will be needed for (1) even when not needed for (2).

$0\nu\beta\beta$

$$
\begin{aligned}
t_{1/2}^{(0\nu)} &= g^{(0\nu)} \left| M^{(0\nu)\prime} \right|^{-2} (|\langle m_\nu \rangle|[\mathrm{eV}])^{-2} \\
\langle m_\nu \rangle &= \sum_j \lambda_j^{\mathrm{CP}} m_j |U_{\mathrm{e}j}|^2 \,. \\
M^{(0\nu)\prime} &= \left(\frac{g_\mathrm{A}}{g_\mathrm{A}^\mathrm{b}} \right)^2 \left[M_\mathrm{GT}^{(0\nu)} - \left(\frac{g_\mathrm{V}}{g_\mathrm{A}} \right)^2 M_\mathrm{F}^{(0\nu)} + M_\mathrm{T}^{(0\nu)} \right] \\
M_\mathrm{F}^{(0\nu)} &= \sum_k (0_f^+ || \sum_{mn} h_\mathrm{F}(r_{mn}, E_k) || 0_i^+) \,, \quad r_{mn} = |\mathbf{r}_m - \mathbf{r}_n| \,, \\
M_\mathrm{GT}^{(0\nu)} &= \sum_k (0_f^+ || \sum_{mn} h_\mathrm{GT}(r_{mn}, E_k)(\boldsymbol{\sigma}_m \cdot \boldsymbol{\sigma}_n) || 0_i^+) \,,
\end{aligned}
$$

$$h_K(r_{mn}, E_k) = \frac{2}{\pi} R_{\rm A} \int dq \frac{q h_K(q^2)}{q + E_k - (E_{\rm i} + E_{\rm f})/2} j_0(q r_{mn})$$

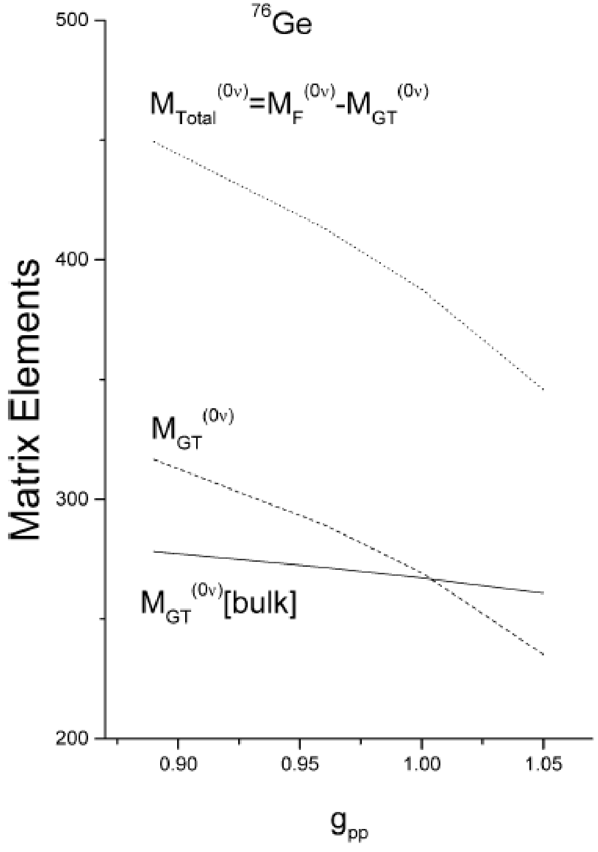
$$M_K^{(0\nu)} = \sum_{J^\pi, k_1, k_2, J'} \sum_{pp'nn'} (-1)^{j_n + j_{p'} + J + J'} \sqrt{2J'+1} \times \left\{ \begin{array}{ccc} j_p & j_n & J \\ j_{n'} & j_{p'} & J' \end{array} \right\}$$

$$(pp' : J' || \mathcal{O}_K || nn' : J') \times (0_f^+ || \left[c_{p'}^\dagger \tilde{c}_{n'} \right]_J || J_{k_1}^\pi) \langle J_{k_1}^\pi | J_{k_2}^\pi \rangle (J_{k_2}^\pi || \left[c_p^\dagger \tilde{c}_n \right]_J || 0_i^+)$$

$$\mathcal{O}_{\rm F} = h_{\rm F}(r, E_k) \ , \quad \mathcal{O}_{\rm GT} = h_{\rm GT}(r, E_k) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \ , \quad r = \left| \mathbf{r}_1 - \mathbf{r}_2 \right| \ ,$$

Comments: transitions between structureless nucleons, momentum cut-off dependent, renormalization effects upon g_A

Suppression effects



Comparison of NME

- QRPA, IBA-2,ISM,PHFB, EDF
- The g_{pp} question
- Dependence on single particle states and occupancies
- Short range correlations (Jastrow, UCOM)
- Shell effects
- Decay to excited states
- Overall values of the NME

Calculated ground-state-to-ground-state NMEs for $g_{\mathrm{A}} = 1.25$ using the Jastrow short-range correlations. The last line summarizes the overall magnitude and the associated dispersion of the NMEs of the cited nuclear model (without ^{48}Ca included).

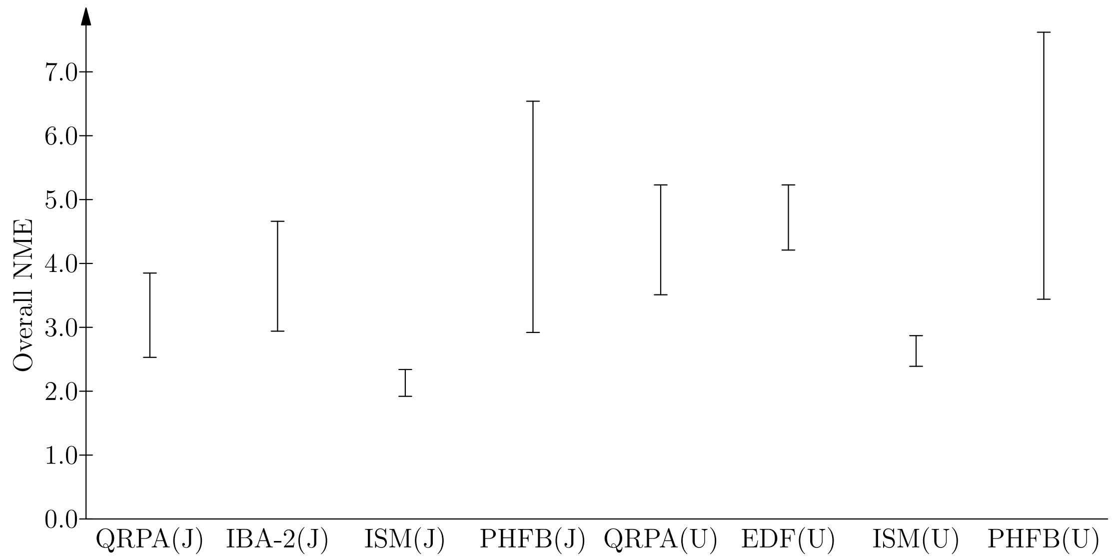
Transition	pnQRPA(J)	IBA-2(J)	ISM(J)	PHFB(J)
^{48}Ca $\longrightarrow$ ^{48}Ti	1.67 ± 0.09	2.00	0.61	-
^{76}Ge $\longrightarrow$ ^{76}Se	3.83 ± 0.53	5.46	2.30	-
^{82}Se $\longrightarrow$ ^{82}Kr	3.15 ± 0.30	4.41	2.18	-
^{96}Zr $\longrightarrow$ ^{96}Mo	2.07	2.53	-	2.80 ± 0.10
^{100}Mo $\longrightarrow$ ^{100}Ru	2.74	3.73	-	6.19 ± 0.46
^{110}Pd $\longrightarrow$ ^{110}Cd	4.15 ± 0.41	3.62	-	7.07 ± 0.58
^{116}Cd $\longrightarrow$ ^{116}Sn	3.03	2.78	-	-
^{124}Sn $\longrightarrow$ ^{124}Te	3.30 ± 0.92	3.53	2.10	-
^{128}Te $\longrightarrow$ ^{128}Xe	3.80 ± 0.37	4.52	2.34	3.59 ± 0.28
^{130}Te $\longrightarrow$ ^{130}Xe	3.47 ± 0.37	4.06	2.12	4.01 ± 0.45
^{136}Xe $\longrightarrow$ ^{136}Ba	2.36 ± 0.22	3.35	1.76	-
Overall NME	3.19 ± 0.66	3.80 ± 0.86	2.13 ± 0.21	4.73 ± 1.81

Calculated ground-state-to-ground-state NMEs for $g_{\mathrm{A}} = 1.25$ using the UCOM short-range correlations. The last line summarizes the overall magnitude and the associated dispersion of the NMEs of the cited nuclear model (without ^{48}Ca included).

Transition	pnQRPA(U)	EDF(U)	ISM(U)	PHFB(U)
^{48}Ca $\longrightarrow$ ^{48}Ti	-	2.37	0.85	-
^{76}Ge $\longrightarrow$ ^{76}Se	5.18 ± 0.54	4.60	2.81	-
^{82}Se $\longrightarrow$ ^{82}Kr	4.20 ± 0.35	4.22	2.64	-
^{96}Zr $\longrightarrow$ ^{96}Mo	3.12	5.65	-	3.32 ± 0.12
^{100}Mo $\longrightarrow$ ^{100}Ru	3.93	5.08	-	7.22 ± 0.50
^{110}Pd $\longrightarrow$ ^{110}Cd	5.63 ± 0.49	-	-	8.23 ± 0.62
^{116}Cd $\longrightarrow$ ^{116}Sn	3.93	4.72	-	-
^{124}Sn $\longrightarrow$ ^{124}Te	4.57 ± 1.33	4.81	2.62	-
^{128}Te $\longrightarrow$ ^{128}Xe	5.26 ± 0.40	4.11	2.88	4.22 ± 0.31
^{130}Te $\longrightarrow$ ^{130}Xe	4.76 ± 0.41	5.13	2.65	4.66 ± 0.43
^{136}Xe $\longrightarrow$ ^{136}Ba	3.16 ± 0.25	4.20	2.19	-
Overall NME	4.37 ± 0.86	4.72 ± 0.51	2.63 ± 0.24	5.53 ± 2.09

Overall $0\nu\beta\beta$ NME gs-gs transitions

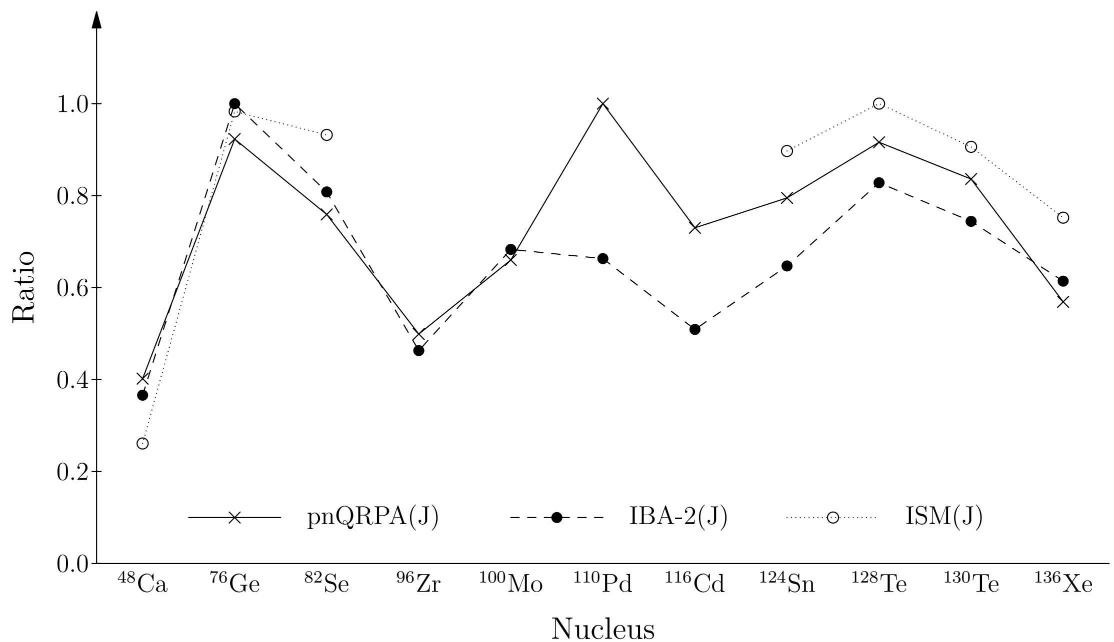
Ranges of values of the overall nuclear matrix elements for $0\nu\beta\beta$ ground state to ground state transitions



Ratios of NME (Jastrow)

$$R = M^{(0\nu)}/M_{\rm max}^{(0\nu)} ,$$

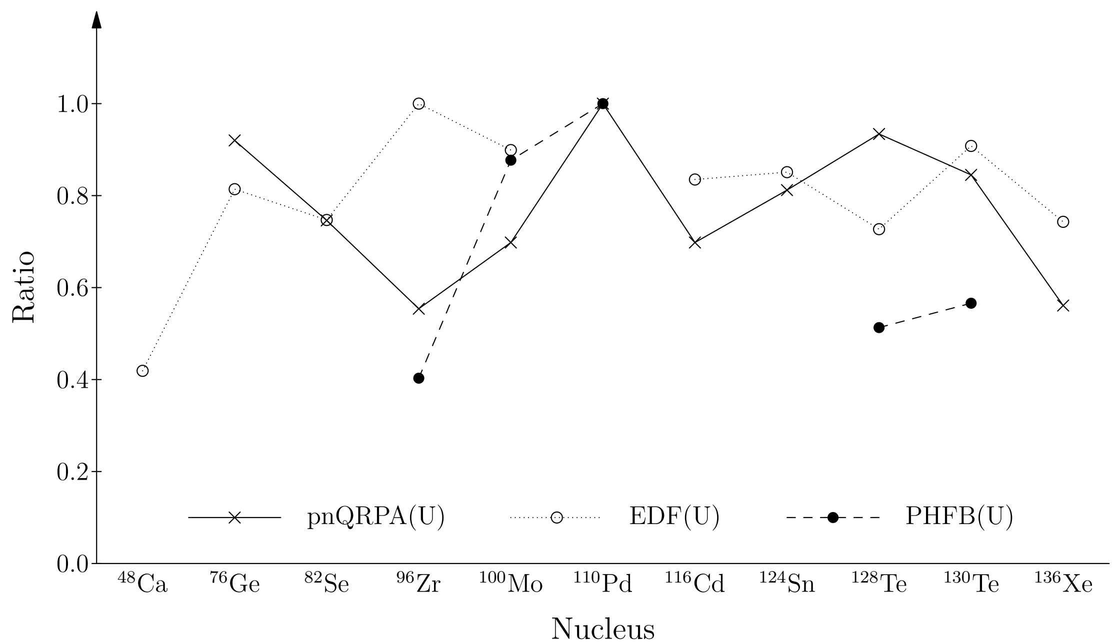
Ratios of the NME (gs-gs)(average values) with the largest value (for a given model(QRPA,IBA-2,ISM))



Ratios of NME (UCOM)

$$R = M^{(0\nu)}/M^{(0\nu)}_{\rm max} ,$$

Ratios of the NME (gs-gs)(average values) with the largest value (for a given model(QRPA,IBA-2,ISM))



Shell effects upon NME

$$M_{\mathrm{GS}}^{(0\nu)'} = \kappa\sqrt{(N_\pi+1)N_\nu(16-N_\pi)(16-N_\nu+1)}$$
$$R = M^{(0\nu)'}/M_{\mathrm{GS}}^{(0\nu)'} ,$$

Ratios for the indicated nuclei in the five different model frameworks (first column). The second column gives the value of the fitting parameter κ (fitted to the decay of 128Te).

Model	κ	$A=124$	$A=128$	$A=130$	$A=136$
IBA-2	0.114	0.912	1.00	1.00	1.13
ISM	0.059	1.05	1.00	1.01	1.15
pnQRPA	0.096	1.01	1.00	1.02	0.95
EDF	0.104	1.36	1.00	1.39	1.56
PHFB	0.091	-	1.00	1.24	-

Comments: the analysis of the ratio of NME for 128Te/130Te gives ≈ 1.10 for all models except EDF (0.80) and PHFB (0.90)

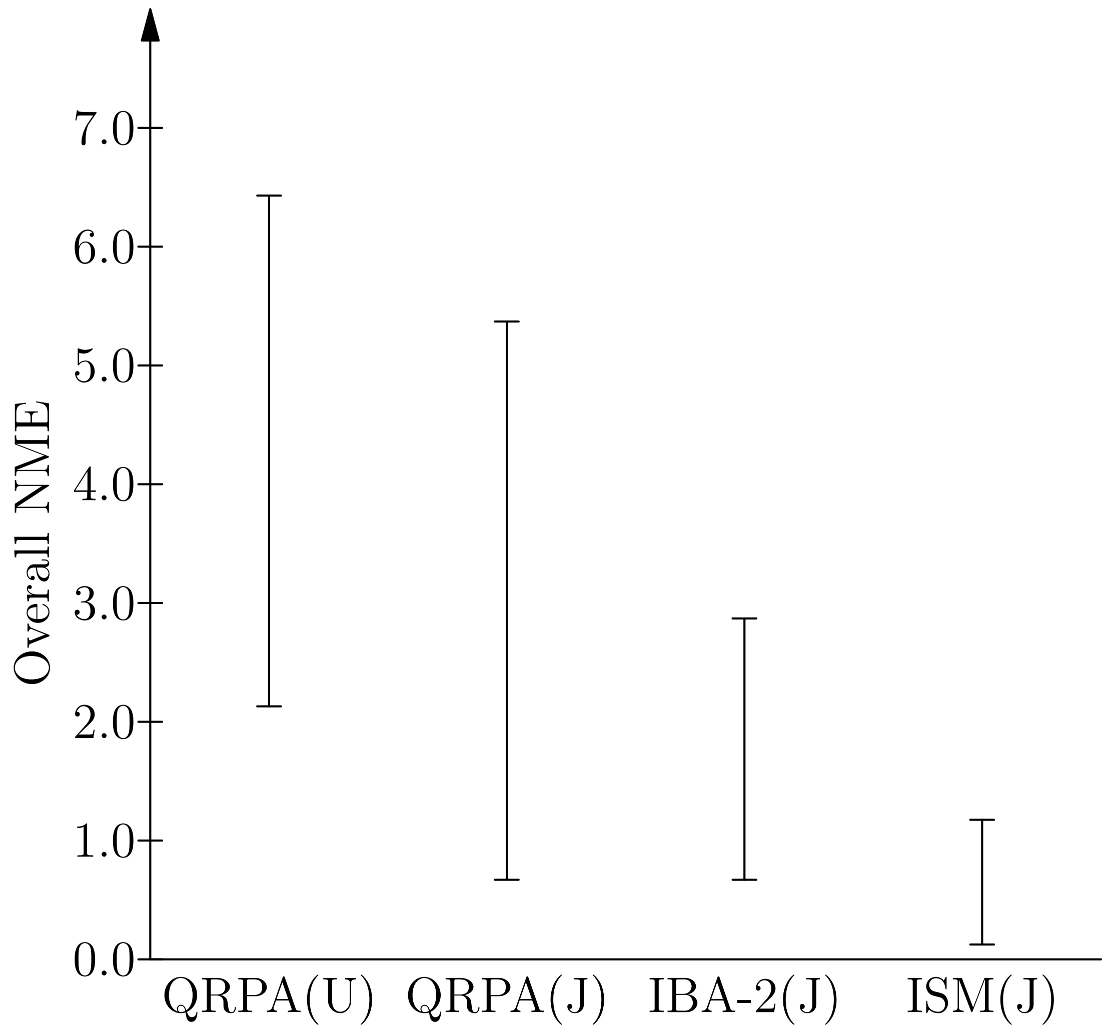
Calculated ground-state-to-excited-state NMEs for $g_{\mathrm{A}} = 1.25$ using the Jastrow (J) and UCOM (U) short-range correlations. The last line summarizes the overall magnitude and the associated dispersion of the NMEs of the cited nuclear model (^{48}Ca excluded).

Transition	pnQRPA (U)	pnQRPA (J)	IBA-2 (J)	ISM (J)
^{48}Ca $\longrightarrow$ ^{48}Ti	-	-	5.90	0.68
^{76}Ge $\longrightarrow$ ^{76}Se	4.87 ± 0.73	4.67 ± 0.70	2.48	1.49
^{82}Se $\longrightarrow$ ^{82}Kr	1.53 ± 0.31	1.46 ± 0.29	1.25	0.28
^{96}Zr $\longrightarrow$ ^{96}Mo	-	1.96	0.04	-
^{100}Mo $\longrightarrow$ ^{100}Ru	-	0.31	0.42	-
^{110}Pd $\longrightarrow$ ^{110}Cd	1.63 ± 0.21	1.59 ± 0.21	1.60	-
^{116}Cd $\longrightarrow$ ^{116}Sn	-	0.25	1.05	-
^{124}Sn $\longrightarrow$ ^{124}Te	6.00 ± 0.18	5.74 ± 0.19	2.72	0.80
^{128}Te $\longrightarrow$ ^{128}Xe	-	-	3.24 (Q-fbdden)	-
^{130}Te $\longrightarrow$ ^{130}Xe	$6.31 \pm 0.58^{*}$	$6.06 \pm 0.57^{*}$	3.09	0.19
^{136}Xe $\longrightarrow$ ^{136}Ba	5.34 ± 0.94	5.11 ± 0.92	1.84	0.49
Overall NME	4.28 ± 2.15	3.02 ± 2.35	1.77 ± 1.10	0.65 ± 0.52

* This work.

NME gs-exc.states transitions

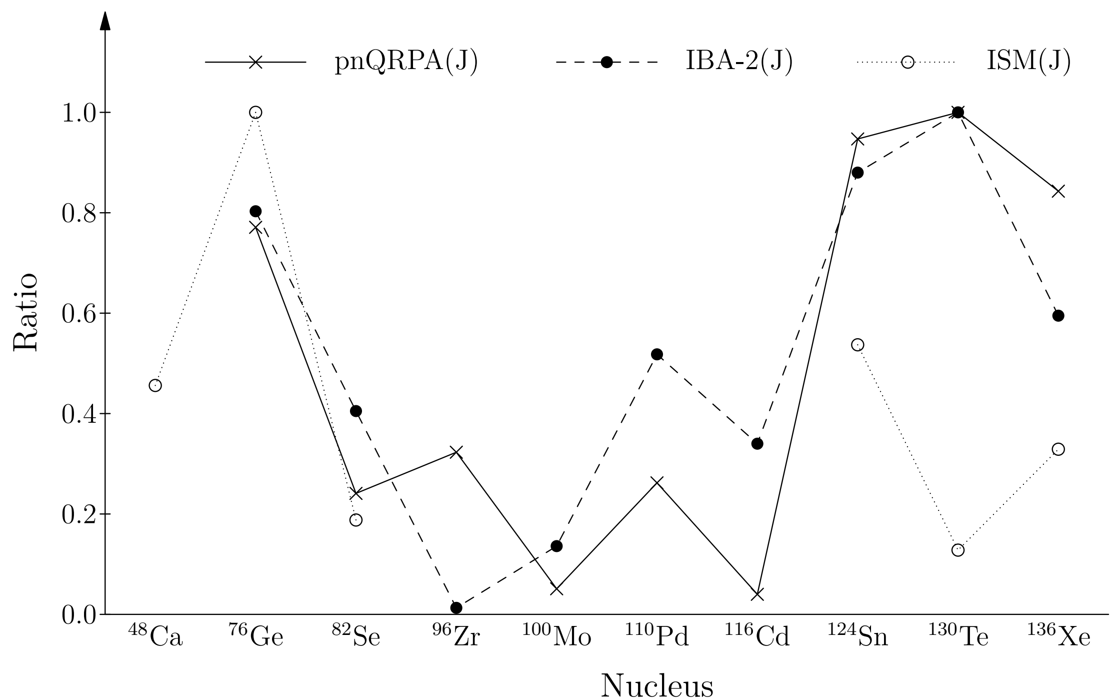
Ranges of values of the overall nuclear matrix elements for ground state to excited state transitions



Ratios of NME gs-exc.states (Jastrow)

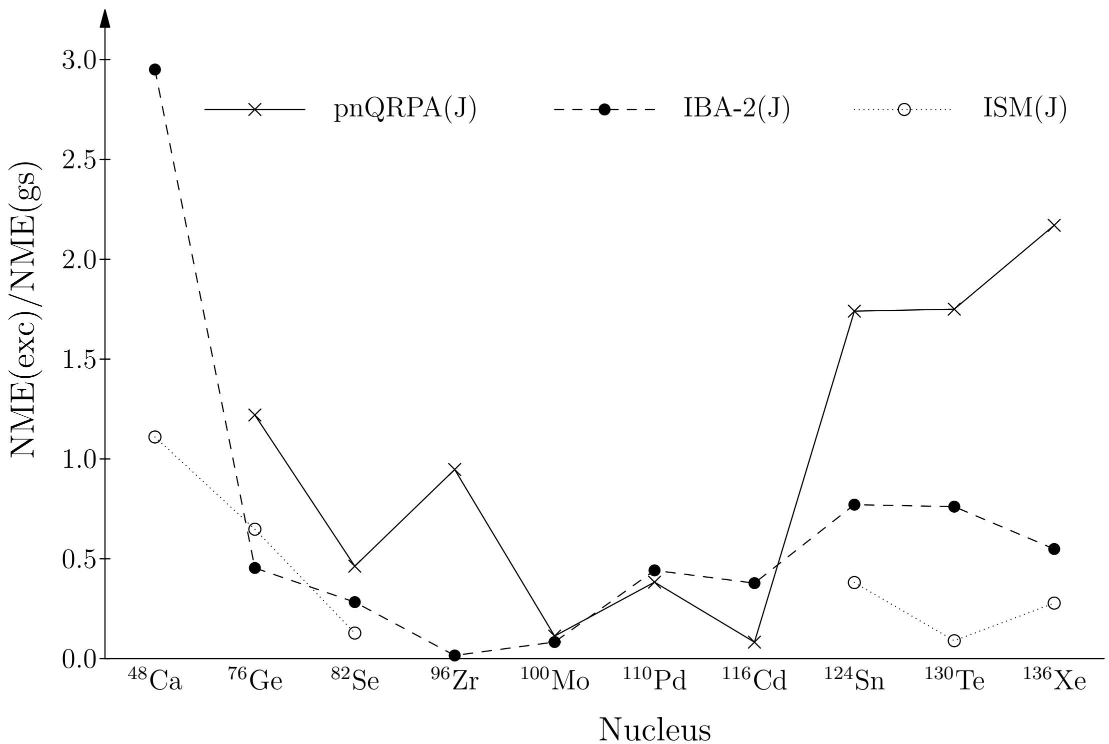
$$R = M^{(0\nu)}/M^{(0\nu)}_{\rm max} \; ,$$

Ratios of the NME (gs-first 0^+ states)(average values) with the largest value (for a given model(QRPA,IBA-2,ISM))

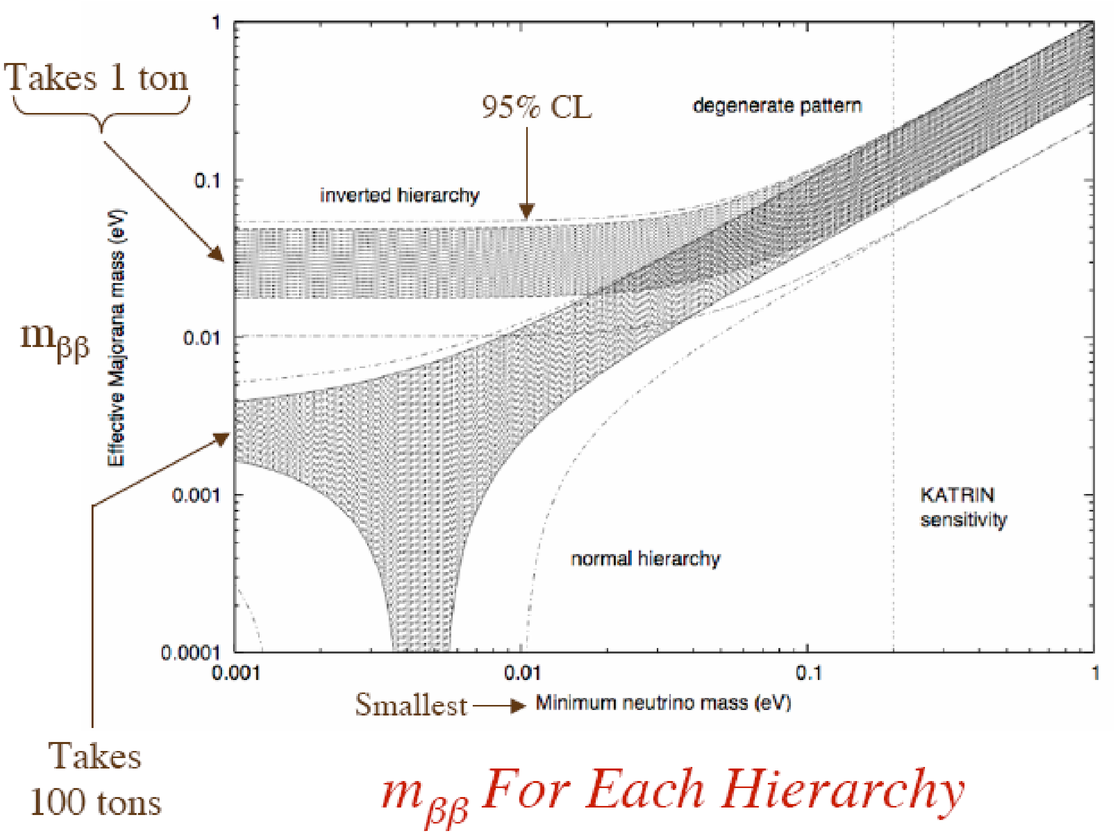


NME(exc)/NME(gs)

Ratios of the corresponding excited-state and ground-state NME



Extracted average neutrino mass



$m_{\beta\beta}$ For Each Hierarchy

Adopted n.m.e

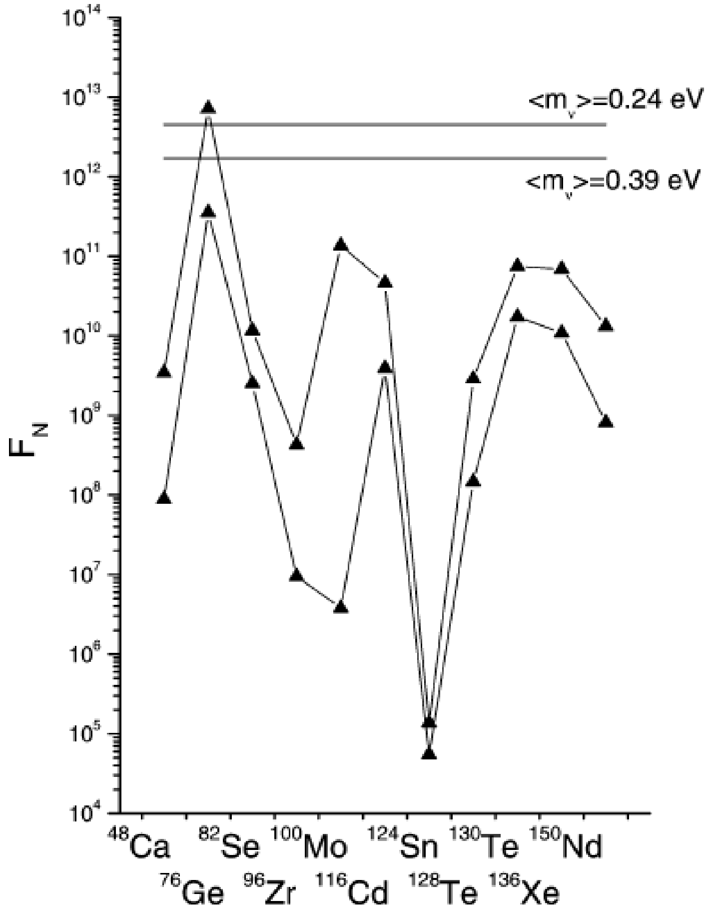
$$[t_{1/2}^{(0\nu)}]^{-1} = G^{(0\nu)}\left(M^{(0\nu)\prime}\right)^2\left(\langle m_\nu\rangle\ [\mathrm{eV}]\right)^2,$$

$$t_{1/2}^{(0\nu)} = \frac{C^{(0\nu)}}{(\langle m_\nu\rangle\ [\mathrm{eV}])^2} \times 10^{24}\ \mathrm{yr}.$$

Quantity	NME ($g_A = 1.00$)	NME ($g_A = 1.25$)	$G^{(0\nu)}$ (yr^{-1})	$C^{(0\nu)}$
^{76}Ge	3.23	5.52	2.42×10^{-26}	1.36–3.96
^{82}Se	2.77	4.57	1.05×10^{-25}	0.46–1.24
^{128}Te	3.74	5.62	6.36×10^{-27}	4.98–11.2
^{130}Te	3.48	5.12	1.59×10^{-25}	0.24–0.52
^{136}Xe	2.38	3.35	1.67×10^{-25}	0.53–1.06

Calculated light-neutrino mass

$$F_N = t_{1/2}^{(0\nu)} C_{mm}^{(0\nu)} = \left(\frac{\langle m_\nu \rangle}{m_e} \right)^{-2}$$



Present limits (electron-neutrino mass)

Nuclei	Half-life [10^(25)years] (experimental lower limit)	mass [eV] (upper limit) (with average n.m.e.)
76Ge	1.6	0.6
82Se	0.04	1.8
100Mo	0.11	1.0
116Cd	0.02	2.2
130Te	0.28	0.8
136Xe	0.05	2.3

Summary

- Thus far only the pnQRPA manages to avoid the use of the closure approximation in the NME calculations.
- The IBA-2 and the ISM use only one closed major shell as their model space whereas the pnQRPA and the mean-field based models allow larger valence spaces.
- Although the IBA-2 maps to the seniority scheme and is thus closely related to the ISM, the magnitudes of the NMEs of the two models deviate notably from each other.
- These models also suffer of the lack of explicit inclusion of the spin-orbit partner orbitals in the calculations.
- Summarizing the above features one could (provocatively) argue that the pnQRPA is still the scheme that is most complete and best suited for reliable calculation of the values of the double-beta-decay nuclear matrix elements.