



ROYAL INSTITUTE
OF TECHNOLOGY

Leptonic CP Violation in Neutrino Oscillations

Shun Zhou

KTH Royal Institute of Technology, Stockholm

Based on T. Ohlsson, H. Zhang and S.Z.,

[Phys. Rev. D 87 \(2013\) 013012;](#)

[Phys. Rev. D 87 \(2013\) 053006;](#)

[Phys. Rev. D 88 \(2013\) 013001.](#)

Outline

- Introduction
- RG running of Dirac CP-violating phase
- Leptonic CP violation in ν oscillations
- NSI Effects @ IceCube (DeepCore & PINGU)
- Summary

Open Questions in ν Physics

- **Are neutrinos Dirac or Majorana particles?**

Lepton number violation, neutrinoless double beta decays

- **What is the neutrino mass hierarchy?**

Normal ($m_1 < m_2 < m_3$) or inverted ($m_3 < m_1 < m_2$)?

- **What is the absolute neutrino mass scale?**

Is the lightest ν massless? Hierarchical or degenerate?

- **What is the origin of neutrino masses and flavor mixing?**

Seesaw mechanisms, flavor symmetries, ...

- **Is there CP violation in the lepton sector?**

What is the value of the Dirac CP-violating phase δ ?

Current Status

Precision measurements of neutrino parameters:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Global-fit analyses:

Forero *et al.*, Phys. Rev. D
86 (2012) 073012

Fogli *et al.*, Phys. Rev. D
86 (2012) 013012

Gonzalez-Garcia *et al.*,
JHEP 1212 (2012) 123

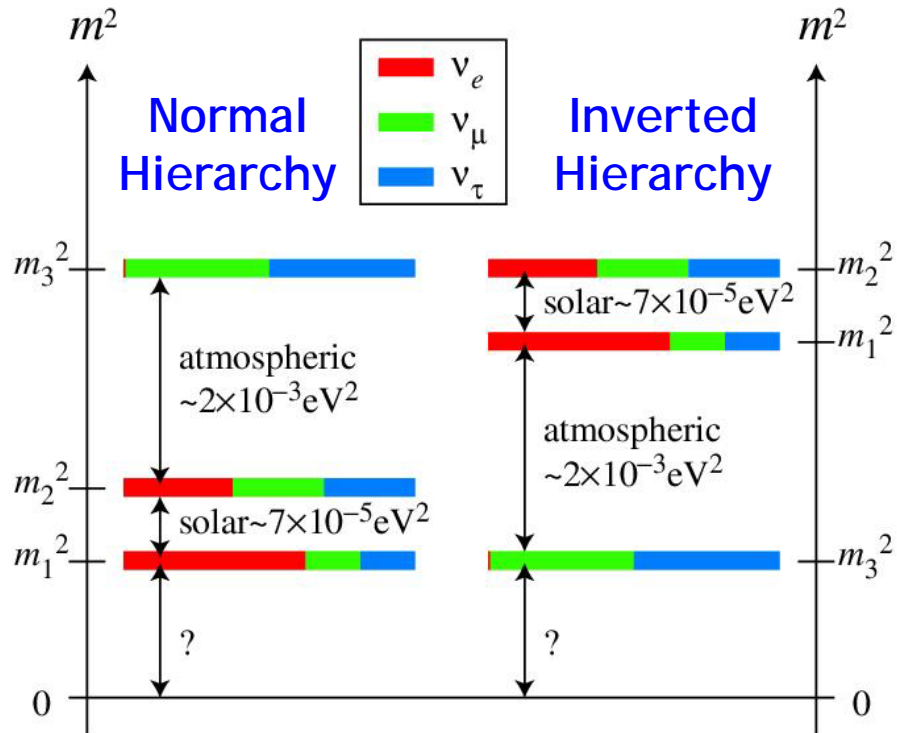
Atmospheric /
Accelerator

Reactor /
Accelerator

Solar / Reactor

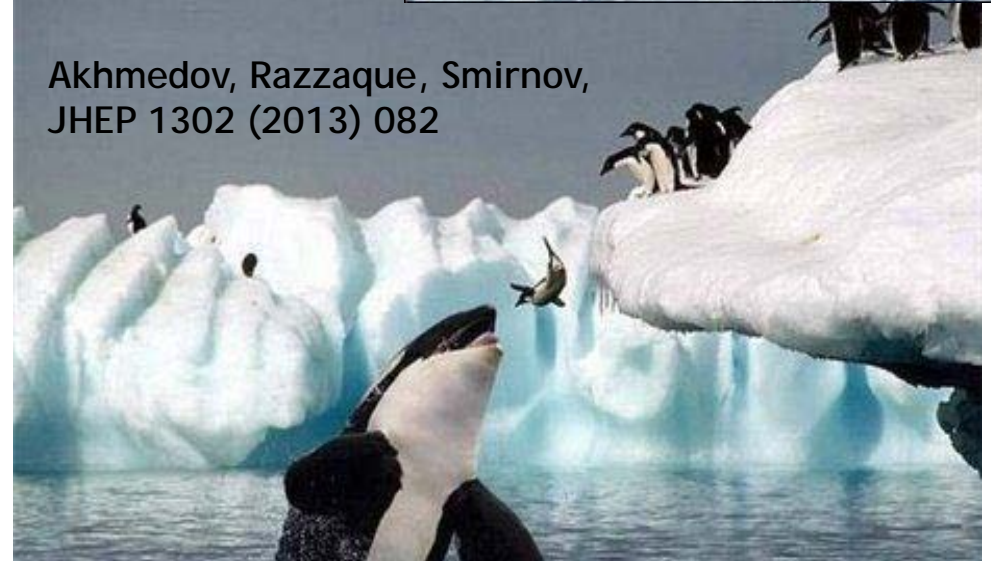
		bf μ $\pm 1\sigma$		3σ range
$\sin^2\theta_{12}$		0.30 ± 0.013		$0.27 \rightarrow 0.34$
$\theta_{12}/^\circ$		33.3 ± 0.8	Pascoli & Schwetz Adv. HEP, 2013	$31 \rightarrow 36$
$\sin^2\theta_{23}$		$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$		$0.34 \rightarrow 0.67$
$\theta_{23}/^\circ$		$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$		$36 \rightarrow 55$
$\sin^2\theta_{13}$		0.023 ± 0.0023		$0.016 \rightarrow 0.030$
$\theta_{13}/^\circ$		$8.6^{+0.44}_{-0.46}$		$7.2 \rightarrow 9.5$
$\delta/^\circ$	Leptonic CP violation?	300^{+66}_{-138}		$0 \rightarrow 360$
$\Delta m_{21}^2/10^{-5} \text{ eV}^2$		7.50 ± 0.185		$7.00 \rightarrow 8.09$
$\Delta m_{31}^2/10^{-3} \text{ eV}^2$ (NH)		$2.47^{+0.069}_{-0.067}$	Neutrino mass hierarchy?	$2.27 \rightarrow 2.69$
$\Delta m_{32}^2/10^{-3} \text{ eV}^2$ (IH)		$-2.43^{+0.042}_{-0.065}$		$-2.65 \rightarrow -2.24$

Neutrino Mass Hierarchy



AcreditSeQuiser.NET

Akhmedov, Razzaque, Smirnov,
JHEP 1302 (2013) 082



- Medium-baseline reactor experiments:
JUNO, RENO-50
- Long-baseline accelerator experiments:
T2K, NOvA, LBNO, LBNE
- Huge neutrino telescopes:
PINGU, ORCA

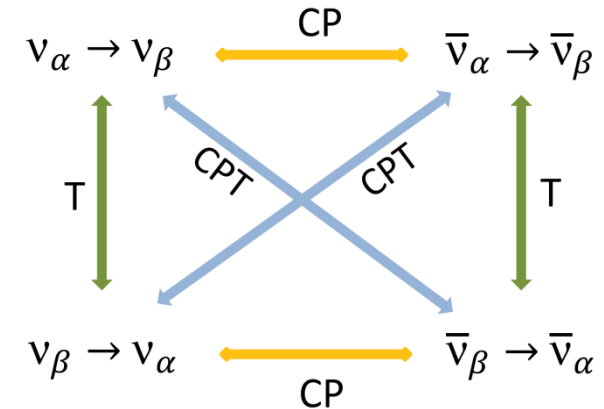
Leptonic CP Violation

Neutrino oscillations in vacuum:

$$A_{\alpha\beta}^{\text{CP}} \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$$

$$= 16 \underbrace{s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2}_{J : \text{Jarlskog Invariant}} \sin \delta \sin \frac{\Delta m_{21}^2 L}{4E} \sin \frac{\Delta m_{32}^2 L}{4E} \sin \frac{\Delta m_{31}^2 L}{4E}$$

Branco *et al.*, Rev. Mod. Phys. 84 (2012) 515



Parameter	Fogli <i>et al.</i>	Gonzalez-Garcia <i>et al.</i>	Forero <i>et al.</i>
$\sin^2 \theta_{12}$	0.307 0.291–0.325	0.300 0.287–0.313	0.320 0.303–0.336
$\sin^2 \theta_{13}$	0.0241 0.0216–0.0266	0.0230 0.0207–0.0253	0.0246 0.0218–0.0275
$\sin^2 \theta_{23}$	0.386 0.365–0.410	0.410 0.385–0.447	0.427 0.400–0.461
$\Delta m_{21}^2 / 10^{-5} \text{ eV}^2$	7.54 7.32–7.80	7.50 7.32–7.69	7.62 7.43–7.81
$\Delta m_{31}^2 / 10^{-3} \text{ eV}^2$	2.51 2.41–2.57	2.47 2.40–2.54	2.55 2.46–2.61
δ / π	1.08 0.77–1.36	1.67 0.90–2.03	0.8 0–2.0

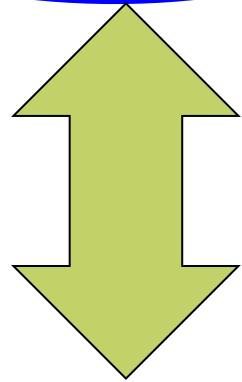
- First hint from global fits?
- Predictions from theories
- Compare between theory & exp. observation of δ :
Radiative corrections
- Optimize the exp. setup:
CP measures, ν -oscillogram
- E.g., NSI effects in the ice

RG Running of Neutrino Parameters

Why is RG running important?

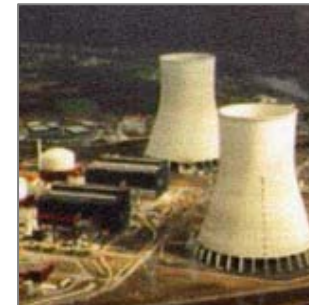
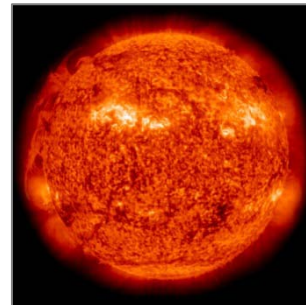
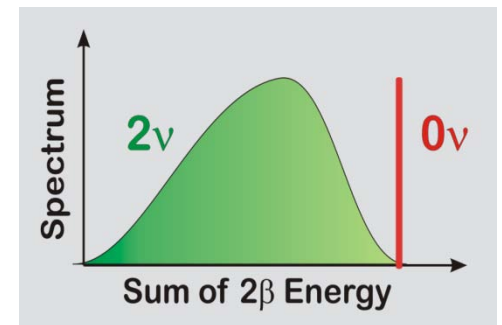
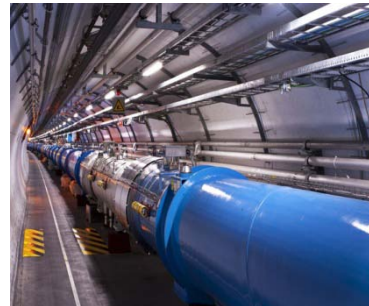
Predictions from
high-energy theories

RG running



Measurements in
low-energy experiments

- Grand Unified Theories: $SU(5)$, $SO(10)$, ...
- Extra-dimensional models: ADD, UED, ...
- Flavor symmetry models: A_4 , S_4 , ...



RG Running of Neutrino Parameters

The SM as an effective theory: **dimension-5 operator**

Weinberg, Phys. Rev. Lett.
43 (1979) 1566

$$\mathcal{L}_\nu = \frac{1}{2} (\bar{\ell} H) \cdot \underline{\kappa} \cdot (H^T \ell^C)$$

$[\Lambda]^{-1}$: high energy scale Λ

Majorana neutrino mass matrix:

$$M_\nu = \kappa v^2$$

Masses, mixing angles, and
leptonic CP-violating phases

Renormalization Group Equation:

$$16\pi^2 \frac{d\kappa}{dt} = \alpha_\kappa + C_\kappa [(Y_l Y_l^\dagger) \kappa + \kappa (Y_l Y_l^\dagger)^T]$$

$$t = \ln\left(\frac{\mu}{\Lambda_{EW}}\right) \quad C_\kappa = -\frac{3}{2}$$

$$\alpha_\kappa = -3g_2^2 + \lambda + 2\text{tr}\left[3(Y_u Y_u^+) + 3(Y_d Y_d^+) + (Y_l Y_l^+)\right]$$

Babu *et al.*, Phys. Lett. B
319 (1993) 191;
Chankowski & Pluciennik,
ibid., 316 (1993) 312

Antusch *et al.*, Phys. Lett.
B 519 (2001) 238

- Realized in various seesaw models
- Running of masses is dominated by the flavor-diagonal term (gauge, quark Yuk.)
- RG running of mixing angles and CP-violating phases is dominated by charged-lepton Yukawa couplings

MSSM and 5D-UED

Extensions of the SM: supersymmetry and extra dimensions

$$L_\nu = \frac{1}{2} LH_u \cdot \kappa \cdot LH_u$$

ν mass matrix:

$$M_\nu = \kappa (\nu \sin \beta)^2$$

RGE coefficients:

$$C_\kappa^{\text{MSSM}} = 1$$

Tau Yukawa coupling dominated:

$$y_\tau^2 = m_\tau^2 (1 + \tan^2 \beta) / \nu^2$$

$$L_\nu = \frac{1}{2} LH \cdot \hat{\kappa} \cdot LH$$

ν mass matrix:

$$M_\nu = \hat{\kappa} \nu^2 / (\pi R)$$

radius of the extra spatial dimension

RGE coefficients:

$$C_\kappa^{\text{UED}} = C_\kappa^{\text{SM}} (1 + s)$$

number of excited KK modes

Coefficient becomes larger at higher-energy scales:

$$s \equiv \left\lfloor \frac{\mu}{\mu_0} \right\rfloor$$

RG Running of Dirac CP-violating Phase

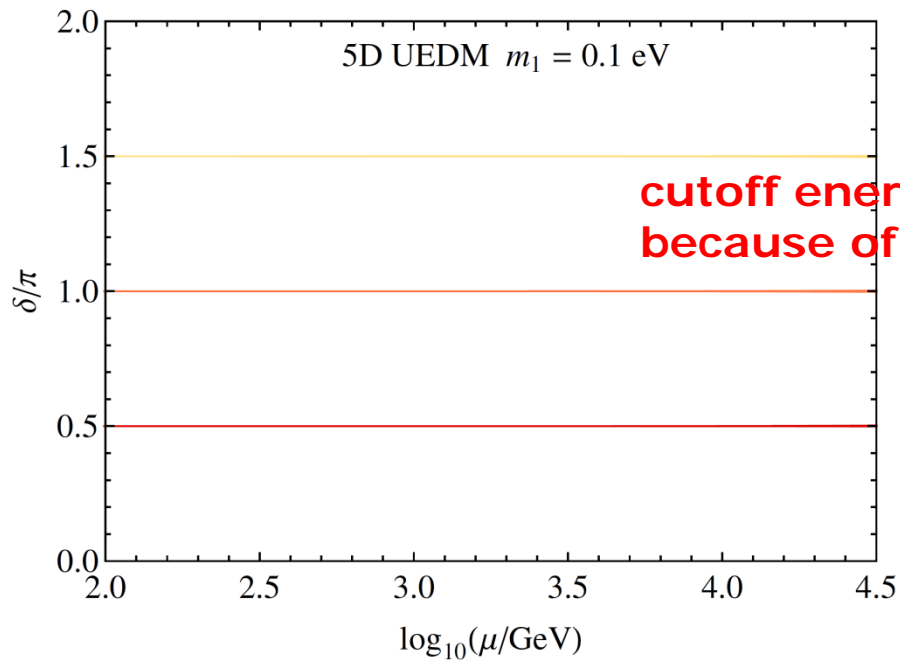
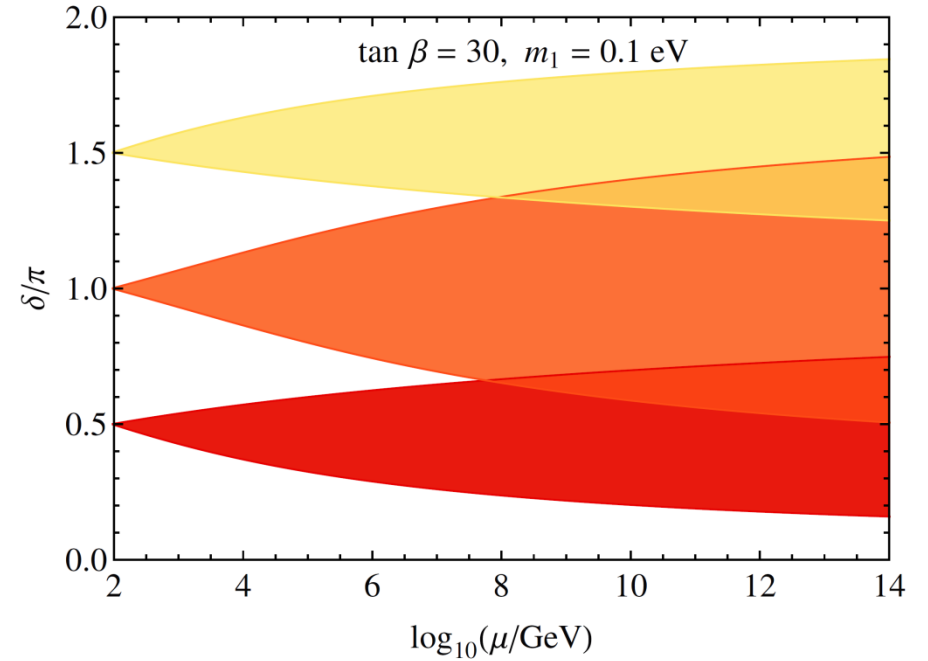
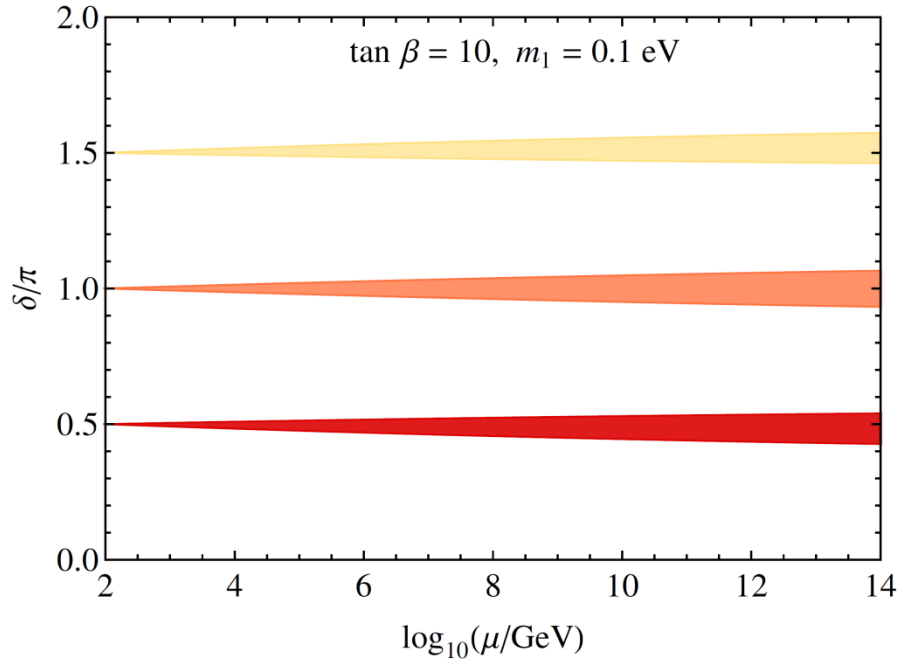
In the standard parametrization:

$$\delta \approx -\frac{C_\kappa y_\tau^2}{8\pi^2} \frac{m_1^2}{\Delta m_{21}^2} \left\{ s_{23}^2 s_{2(\rho-\sigma)} + \frac{2s_{23}c_{23}}{s_{12}c_{12}s_{13}} \left[s_{13}^2 c_{(\delta+\rho-\sigma)} + \frac{\Delta m_{21}^2}{\Delta m_{31}^2} s_{12}^2 c_{12}^2 c_{(\delta+\rho+\sigma)} s_{(\rho-\sigma)} \right] \right\}$$

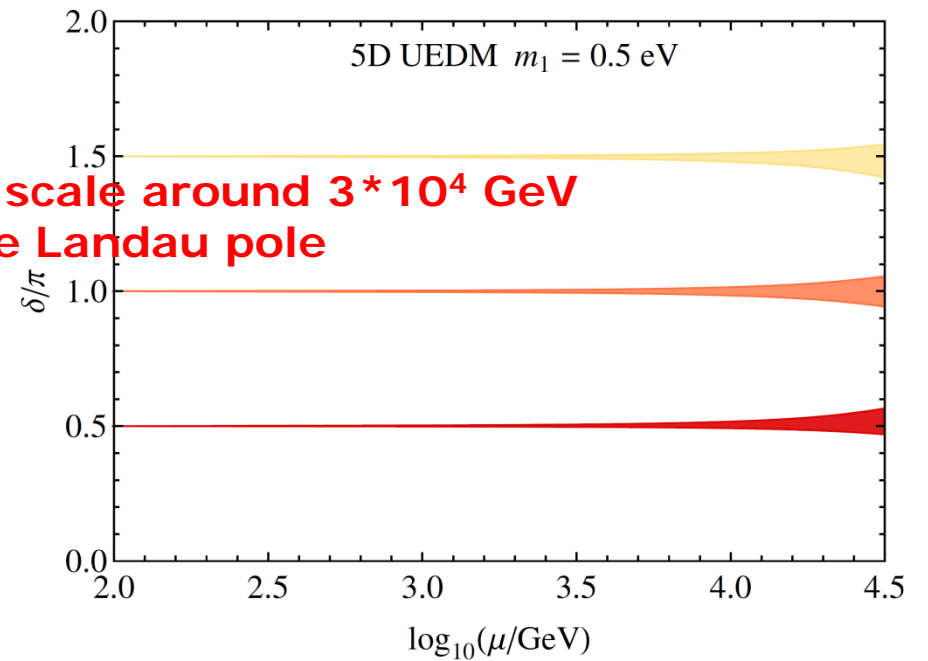
enhanced if neutrino masses are nearly degenerate

the same order of magnitude according to the latest global-fit results

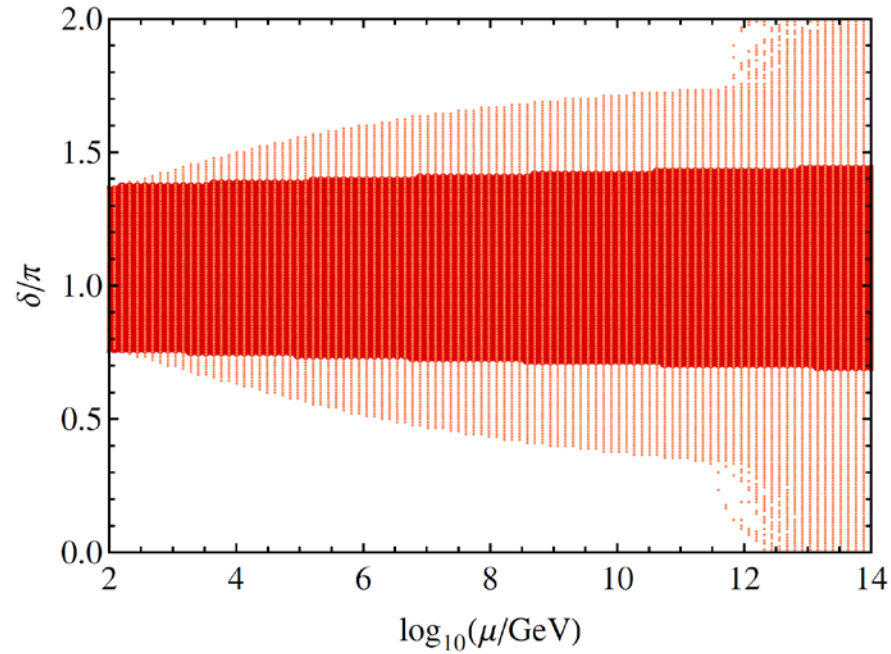
- Keep all contributions of the same order.
- RG running depends crucially on the Majorana CP-violating phases; insignificant running for $\rho=\sigma$.
- RG running is in the opposite direction for the MSSM, compared to the SM and 5D-UED.



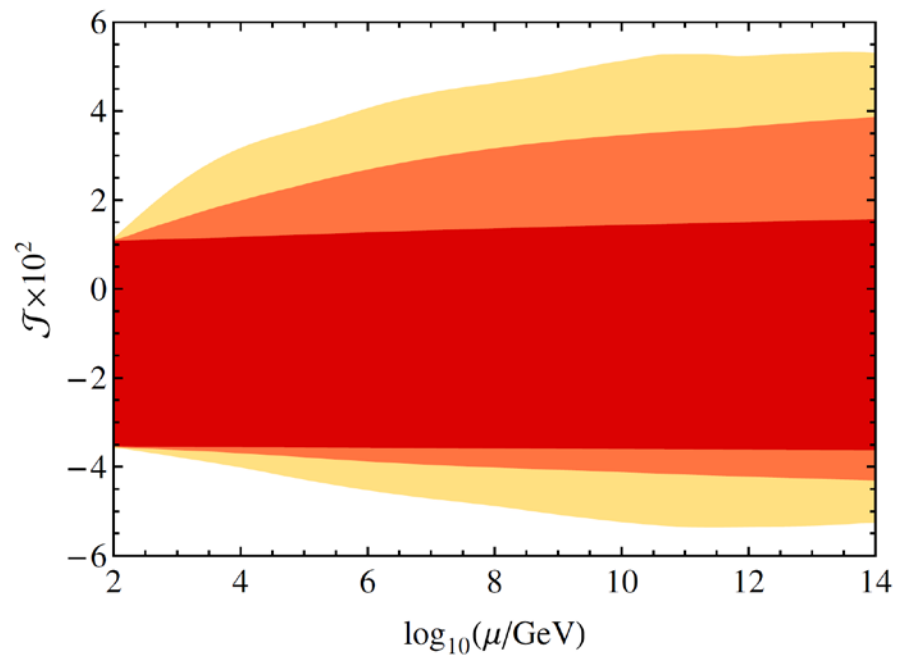
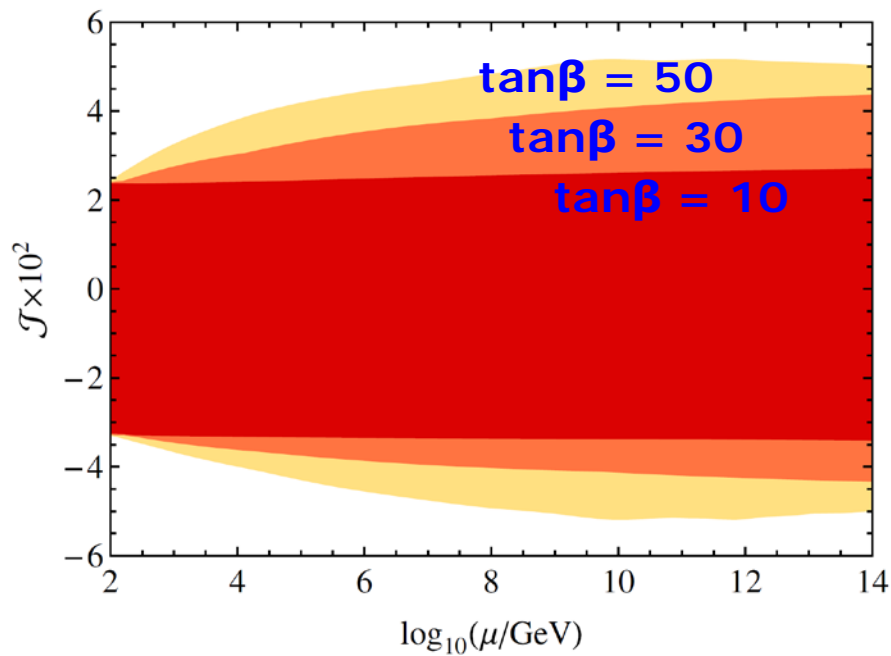
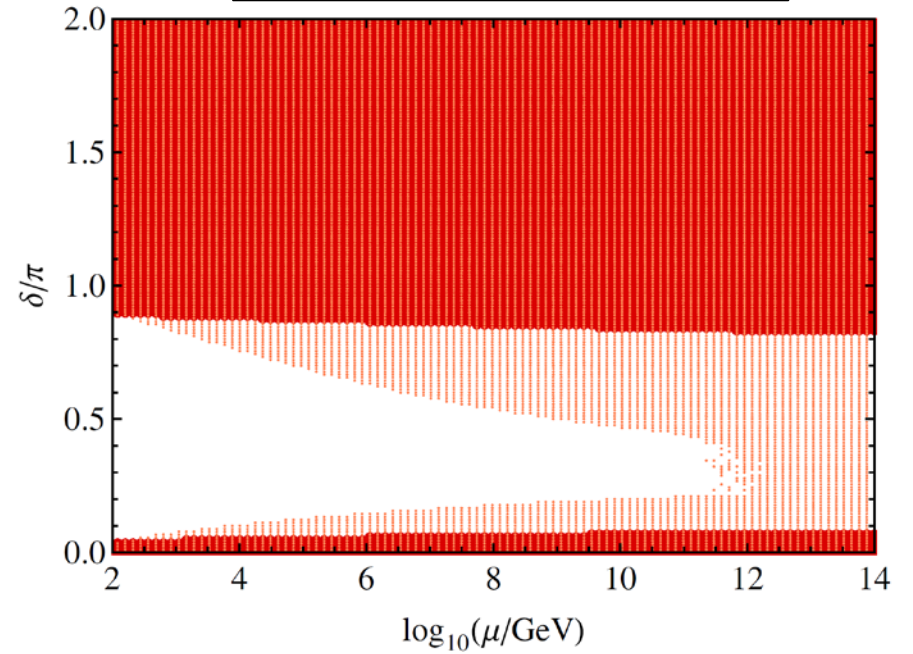
cutoff energy scale around $3 \cdot 10^4 \text{ GeV}$
because of the Landau pole



Fogli *et al.*



Gonzalez-Garcia *et al.*



Summary for RG Running of δ

- The RGE of the leptonic Dirac CP-violating phase is derived, and small contributions of the same order are included.
- Very tiny RG running effects in the SM, even for nearly-degenerate neutrino masses
- No significant RG running also in the MSSM and 5-UED, except for a very large $\tan\beta$ in the former case; however, note the dependence on the Majorana phases
- Non-zero δ can be radiatively generated at the low-energy scale even if $\delta = 0$ holds at a high-energy scale.
- For Dirac neutrinos, RG running is even smaller because of the absence of Majorana CP-violating phases.
- Constraints on δ from neutrino oscillation experiments can be directly applied to theory at a superhigh-energy scale.

CP Violation and Matter Effects

Neutrino oscillations in matter:

$$H_{\text{eff}} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger + 2\sqrt{2}G_F N_e E \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} \right]$$

Oscillation probabilities:

Kimura *et al.*, Phys. Lett.
B 537 (2012) 86

$$\left. \begin{aligned} P_{\mu e}(\delta) &\equiv P(\nu_\mu \rightarrow \nu_e) = a \cos \delta + b \sin \delta + c \\ \bar{P}_{\mu e}(\delta) &\equiv P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \bar{a} \cos \delta + \bar{b} \sin \delta + \bar{c} \end{aligned} \right\} \Rightarrow A_{\mu e}^{\text{CP}}(\delta) = \Delta a \cos \delta + \Delta b \sin \delta + \Delta c$$

- Fake CP violation is induced by matter effects:
 - obscuring the intrinsic CP-violating effects by δ
- How to describe leptonic CP violation?
 - working observables based on oscillation probabilities

Working Observables

Series expansions of the probabilities in terms of $\alpha = \Delta m^2_{21} / \Delta m^2_{31}$ and s_{13} :

$$a \approx +8\alpha J_r \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{A-1} \cos \Delta$$

$$b \approx -8\alpha J_r \frac{\sin A\Delta}{A} \frac{\sin(A-1)\Delta}{A-1} \sin \Delta$$

$$c \approx 4s_{13}^2 s_{23}^2 \frac{\sin^2(A-1)\Delta}{(A-1)^2}$$

$$J_r \equiv s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2$$

$$\Delta a \approx +8\alpha J_r \Theta_- \frac{\sin A\Delta}{A} \cos \Delta$$

$$\Delta b \approx -8\alpha J_r \Theta_+ \frac{\sin A\Delta}{A} \sin \Delta$$

$$\Delta c \approx 4s_{13}^2 s_{23}^2 \Theta_+ \Theta_-$$

$$\Theta_{\pm} \equiv \frac{\sin(A-1)\Delta}{A-1} \pm \frac{\sin(A+1)\Delta}{A+1}$$

Here $\Delta = \Delta m^2_{31} L/4E$ and $A = VL/2\Delta$, with V being the matter potential, Δ the oscillation phase, and L the distance between the source and the detector.

$$\underline{\Delta P_{\mu e}^{\text{CP}}(\delta) \ \& \ \Delta P_{\mu e}^{\text{m}}}$$

Definitions:

$$\Delta P_{\mu e}^{\text{CP}}(\delta) \equiv P_{\mu e}(\delta) - P_{\mu e}(\delta = 0)$$

$$\Delta P_{\mu e}^{\text{m}} \equiv \max[P_{\mu e}(\delta)] - \min[P_{\mu e}(\delta)]$$

$$\underline{\Delta A_{\mu e}^{\text{CP}}(\delta) \ \& \ \Delta A_{\mu e}^{\text{m}}}$$

Definitions:

$$\Delta A_{\mu e}^{\text{CP}}(\delta) \equiv A_{\mu e}^{\text{CP}}(\delta) - A_{\mu e}^{\text{CP}}(\delta = 0)$$

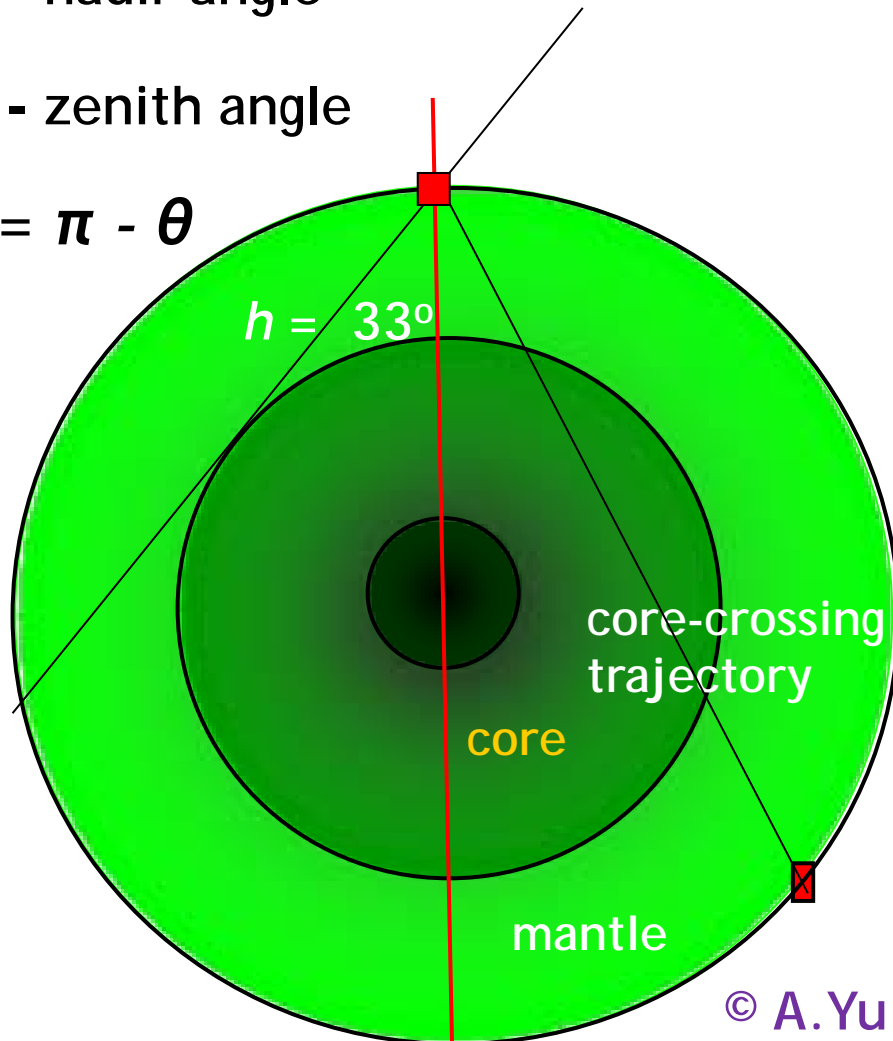
$$\Delta A_{\mu e}^{\text{m}} \equiv \max[A_{\mu e}^{\text{CP}}(\delta)] - \min[A_{\mu e}^{\text{CP}}(\delta)]$$

Neutrino Oscillograms of the Earth

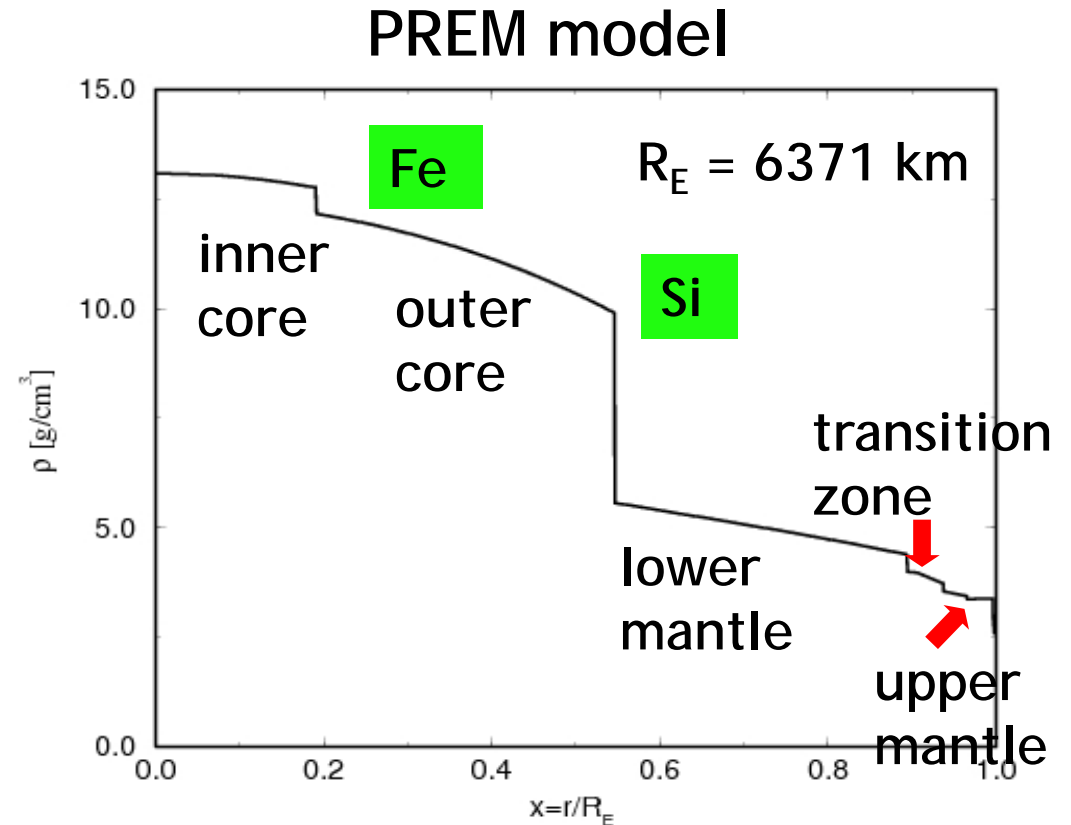
h - nadir angle

θ - zenith angle

$$h = \pi - \theta$$



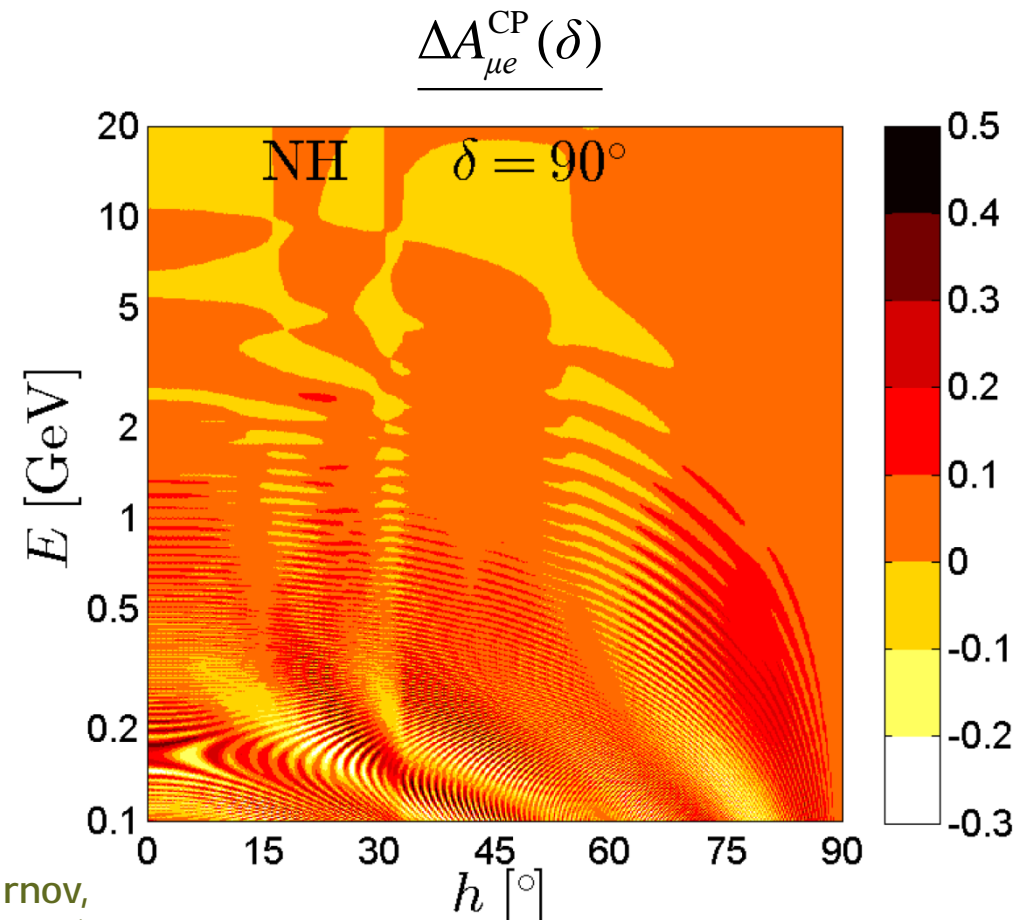
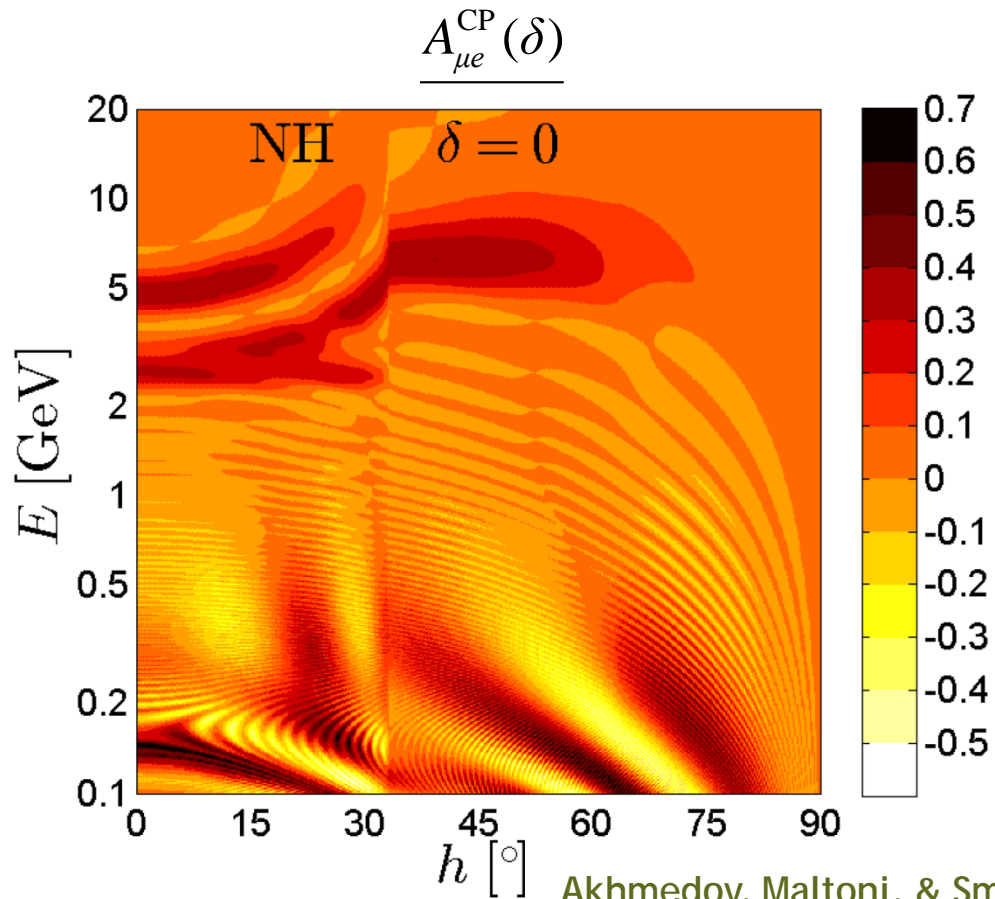
© A.Yu. Smirnov



Oscillograms for CP Violation

Conventional CP asymmetry

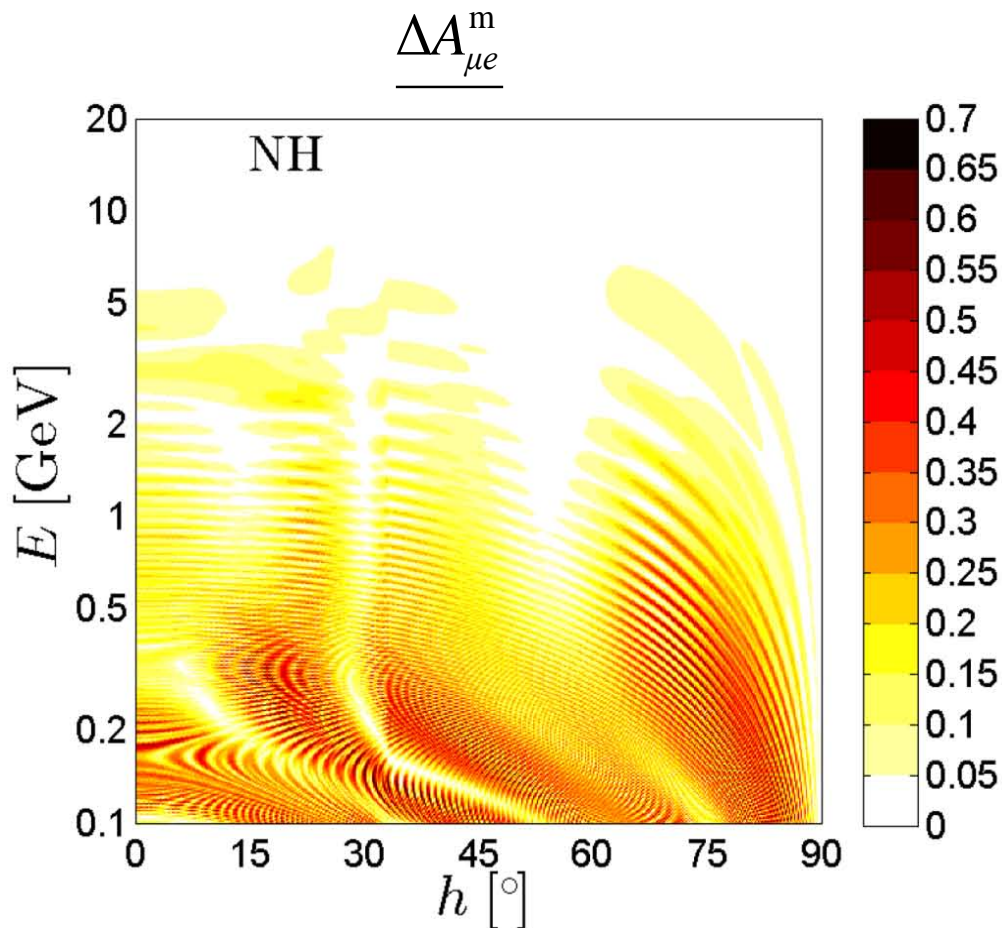
Working observable



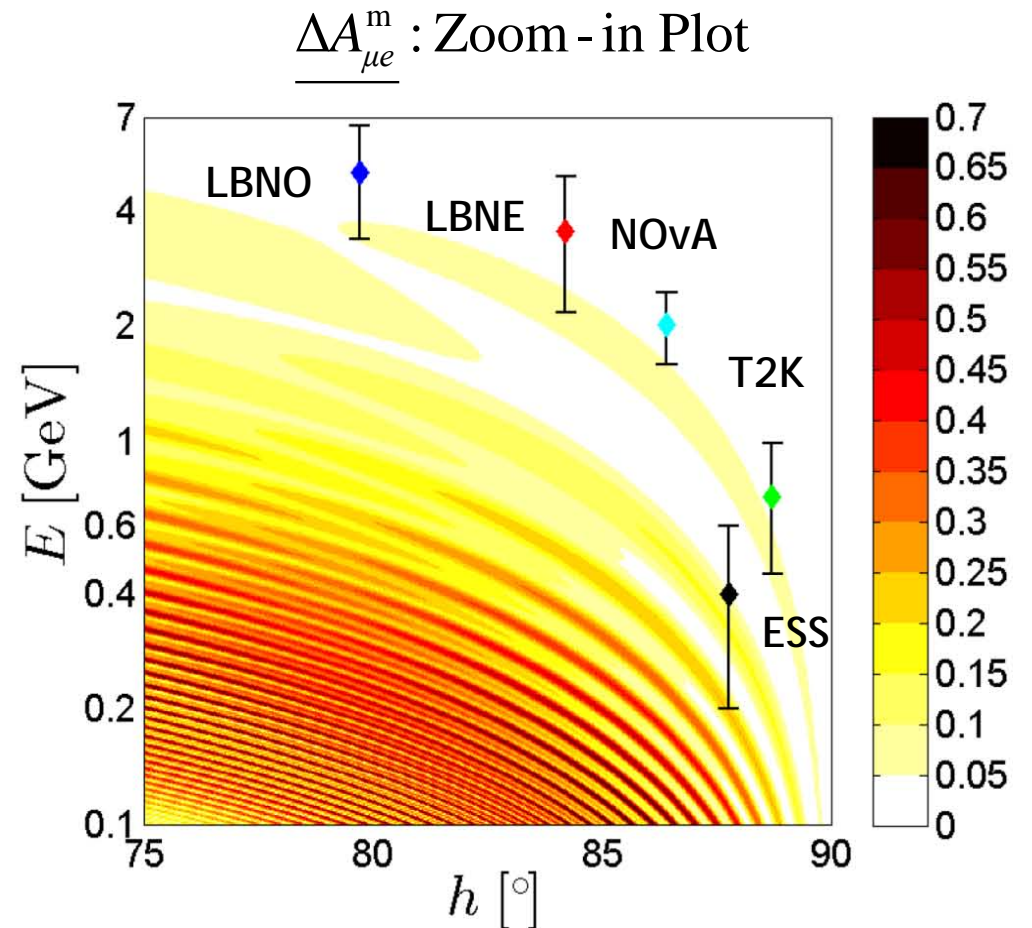
Akhmedov, Maltoni, & Smirnov,
JHEP 05 (2007) 077; 06 (2008) 072

Oscillograms for CP Violation

Working observable



Locating future experiments



Non-Standard Neutrino Interactions

Neutrino oscillations in matter with NSIs:

For a review, Ohlsson, Rept. Prog. Phys.
76 (2013) 044201

$$H_{\text{eff}} = \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^+ + 2\sqrt{2}G_F N_e E \left[\begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} + \begin{pmatrix} \mathcal{E}_{ee} & \mathcal{E}_{e\mu} & \mathcal{E}_{e\tau} \\ \mathcal{E}_{e\mu}^* & \mathcal{E}_{\mu\mu} & \mathcal{E}_{\mu\tau} \\ \mathcal{E}_{e\tau}^* & \mathcal{E}_{\mu\tau}^* & \mathcal{E}_{\tau\tau} \end{pmatrix} \right] \right\}$$

Oscillation probability with NSIs:

$$P_{\mu\mu}^{\text{NSI}} \simeq P_{\mu\mu}^{\text{SD}} - |\mathcal{E}_{\mu\tau}| c_{\phi_{\mu\tau}} \left(s_{2\times 23}^3 A \Delta \sin \Delta + 4s_{2\times 23} c_{2\times 23}^2 A \sin^2 \frac{\Delta}{2} \right) \\ + (|\mathcal{E}_{\mu\mu}| - |\mathcal{E}_{\tau\tau}|) s_{2\times 23}^2 c_{2\times 23} \left(\frac{A\Delta}{2} \sin \Delta - 2A \sin^2 \frac{\Delta}{2} \right)$$

- Deviation from the std. probability; only mu-tau flavors are involved
- Not suppressed by mixing angle θ_{13} or the mass ratio $\Delta m_{21}^2 / \Delta m_{31}^2$

Neutrino Parameter Mappings

Leptonic mixing matrix elements in matter:

$$U_{e3}^m = \sin \hat{\theta}_{13} + \frac{\cos \hat{\theta}_{13}}{2\hat{C}} \left\{ \sin 2\hat{\theta}_{13} [\alpha s_{12}^2 + A(\varepsilon_{ee} - \tilde{\varepsilon}_{\tau\tau})] + 2A [\cos 2\hat{\theta}_{13} \text{Re}(\tilde{\varepsilon}_{e\tau}) + i\text{Im}(\tilde{\varepsilon}_{e\tau})] \right\}$$

$$U_{e2}^m = -\frac{c_{13}}{2A} \alpha \sin 2\theta_{12} - \tilde{\varepsilon}_{e\mu} + \frac{\tan \theta_{13}}{A} \tilde{\varepsilon}_{\mu\tau}^*$$

$$U_{\mu 3}^m = s_{23} \cos \hat{\theta}_{13} e^{i\delta} \left\{ 1 - \frac{A \tan \hat{\theta}_{13}}{\hat{C}} [\cos 2\hat{\theta}_{13} \text{Re}(\tilde{\varepsilon}_{e\tau}) + i\text{Im}(\tilde{\varepsilon}_{e\tau})] \right\} + \frac{\alpha c_{23} \sin 2\theta_{12} \sin \hat{\theta}_{13}}{1 + A + \hat{C}},$$

Modified NSI parameters:

$$\tilde{\varepsilon}_{e\mu} = \varepsilon_{e\mu} c_{23} - \varepsilon_{e\tau} s_{23},$$

$$\tilde{\varepsilon}_{e\tau} = (\varepsilon_{e\mu} s_{23} + \varepsilon_{e\tau} c_{23}) e^{i\delta},$$

$$\tilde{\varepsilon}_{\mu\mu} = (\varepsilon_{\mu\mu} c_{23}^2 + \varepsilon_{\tau\tau} s_{23}^2) - 2s_{23} c_{23} \text{Re}[\varepsilon_{\mu\tau}],$$

$$\tilde{\varepsilon}_{\tau\tau} = (\varepsilon_{\mu\mu} s_{23}^2 + \varepsilon_{\tau\tau} c_{23}^2) + 2s_{23} c_{23} \text{Re}[\varepsilon_{\mu\tau}],$$

$$\tilde{\varepsilon}_{\mu\tau} = [(\varepsilon_{\mu\tau} c_{23}^2 - \varepsilon_{\mu\tau}^* s_{23}^2) + (\varepsilon_{\mu\mu} - \varepsilon_{\tau\tau}) s_{23} c_{23}] e^{i\delta}$$

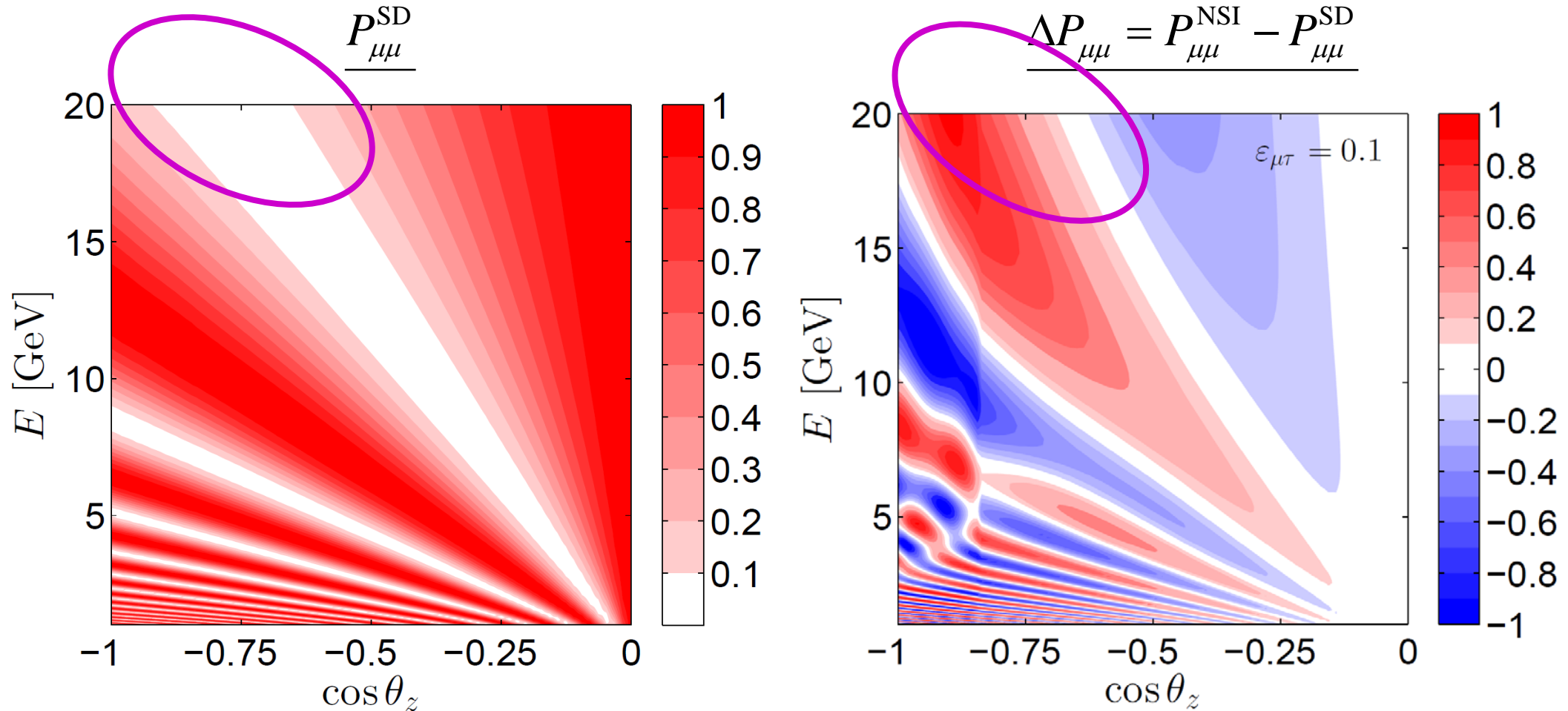
Modified mixing angle:

$$\sin^2 \hat{\theta}_{13} = \frac{\hat{C} - \cos 2\theta_{13} + A}{2\hat{C}}$$

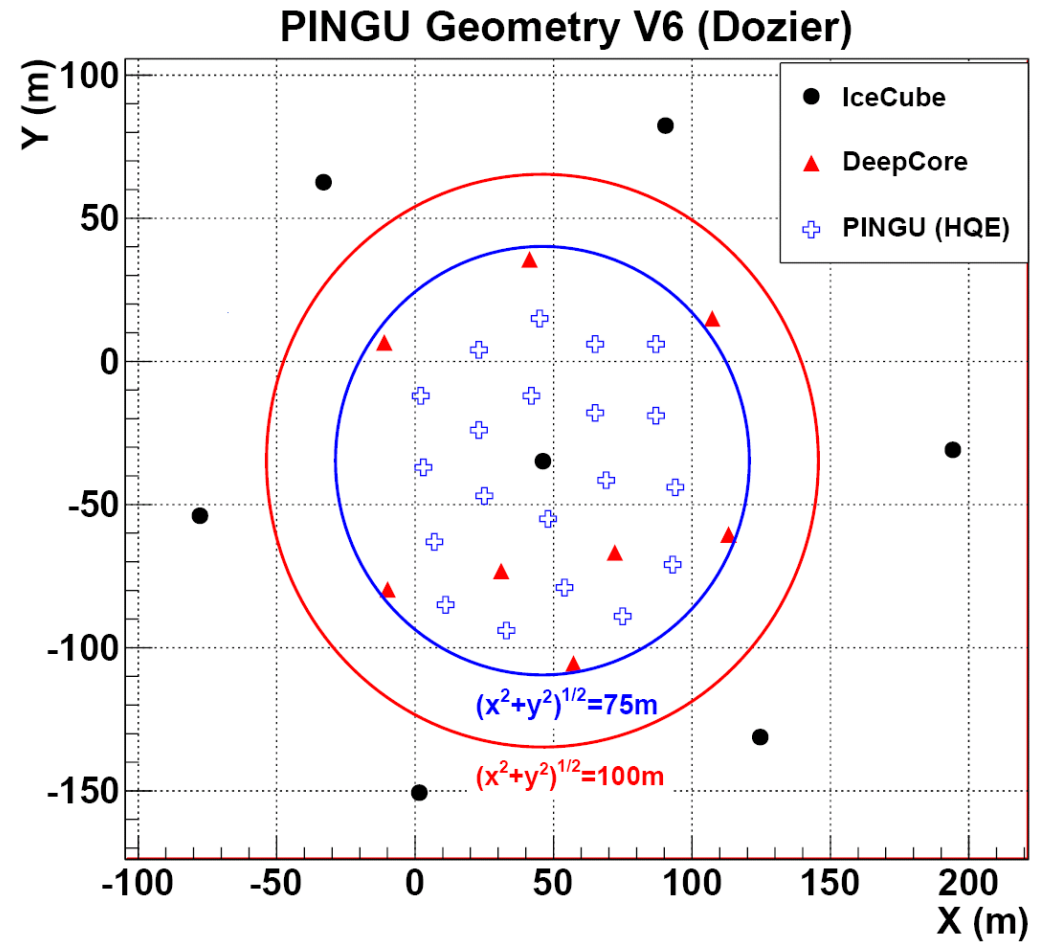
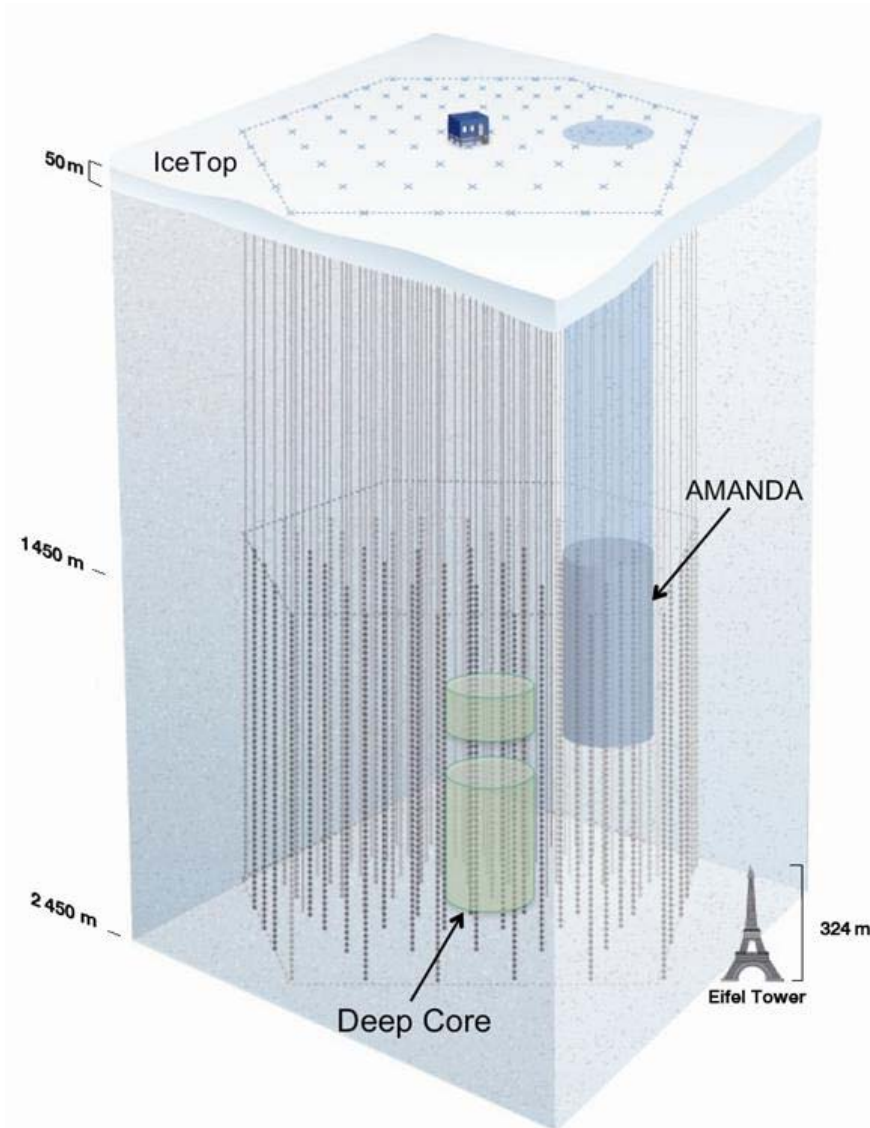
$$\hat{C} = \sqrt{(\cos 2\theta_{13} - A)^2 + \sin^2 2\theta_{13}}$$

Oscillation Probabilities with NSIs

Significant deviation in the high-energy range:



IceCube (DeepCore and PINGU)

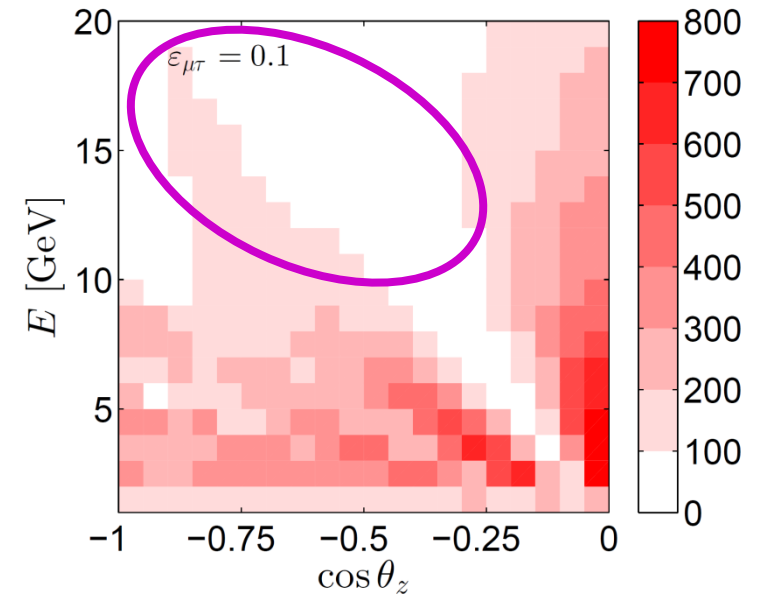
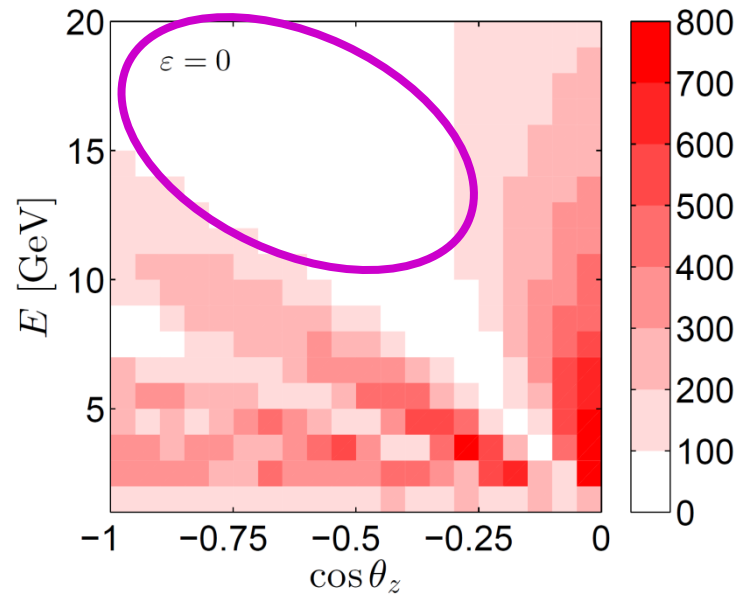


(PINGU, 12/2012)

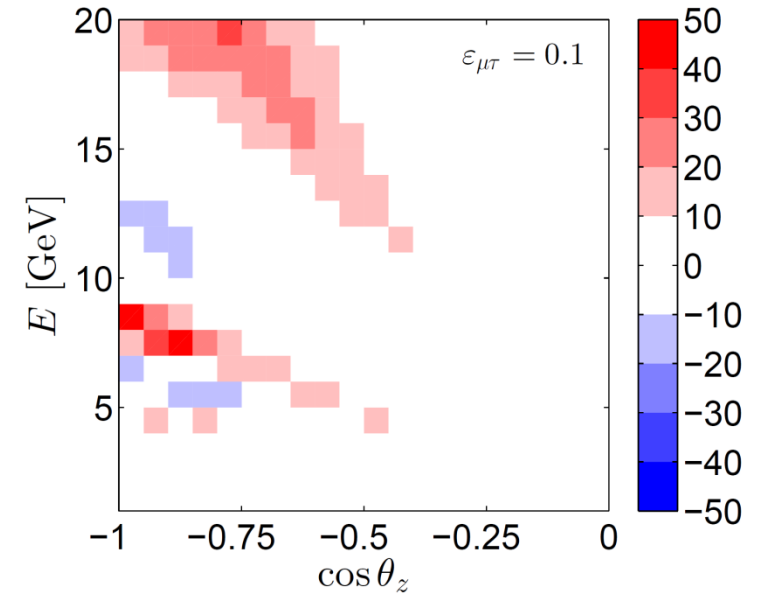
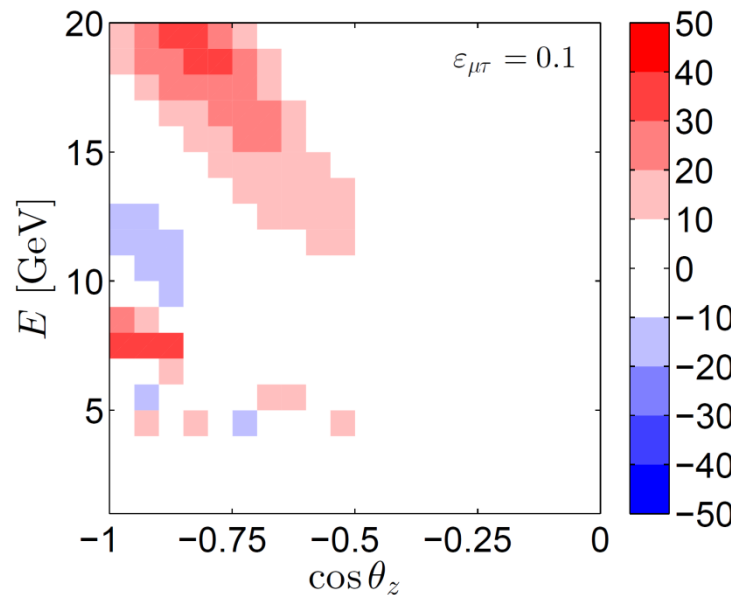
Number of Events @ PINGU

Compare between standard and NSIs

NH



$$A \equiv \frac{N_{\mu}^{\text{SD}} - N_{\mu}^{\text{NSI}}}{\sqrt{N_{\mu}^{\text{SD}}}}$$

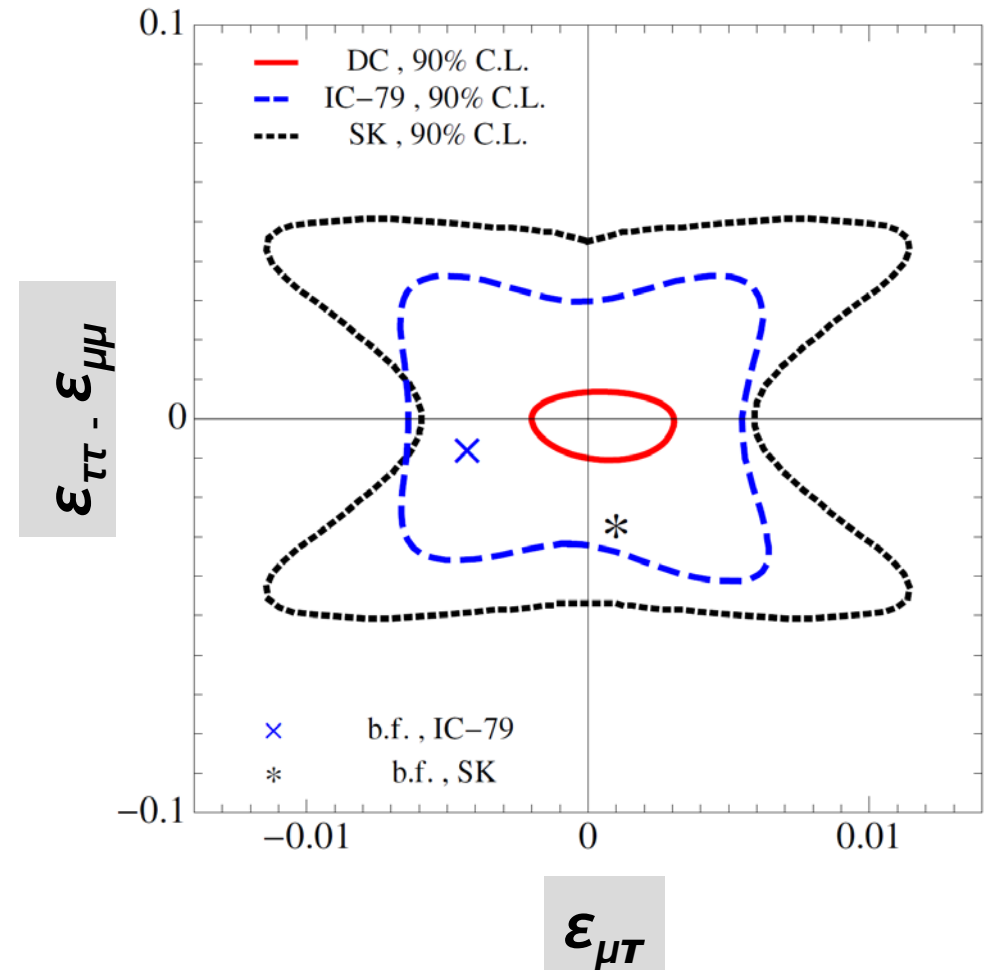
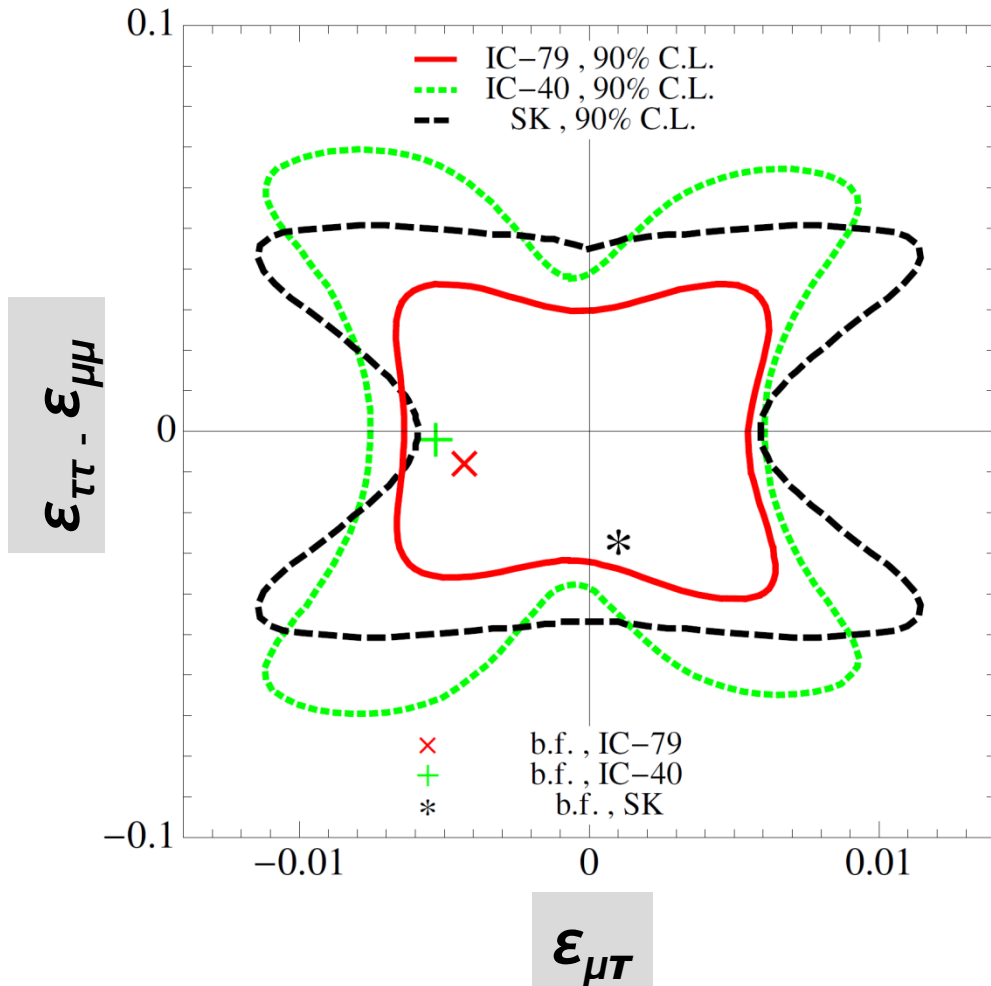


NH (left) IH (right)

Sensitivity @ IceCube (DeepCore)

Higher resonance energy for smaller $\epsilon_{\alpha\beta}$

Esmaili & Smirnov, arXiv:1304.1042

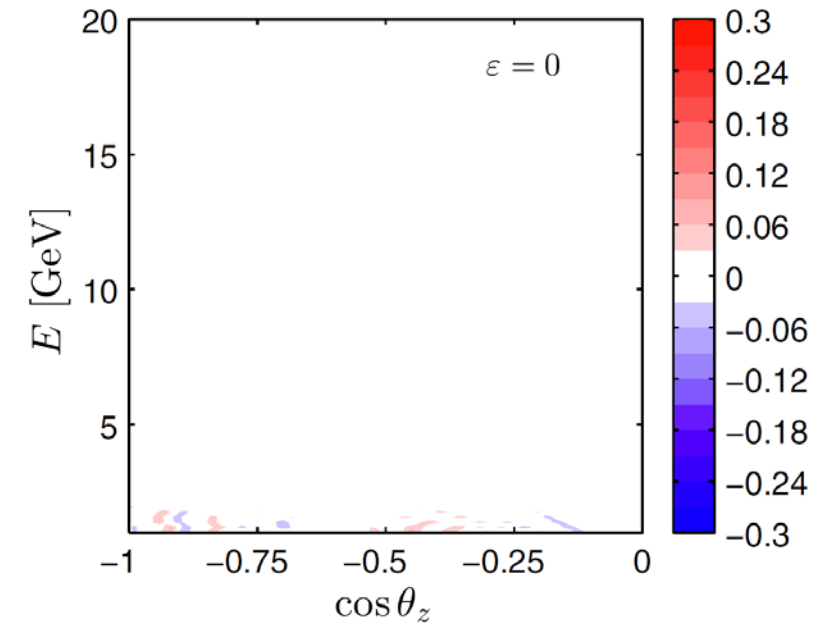
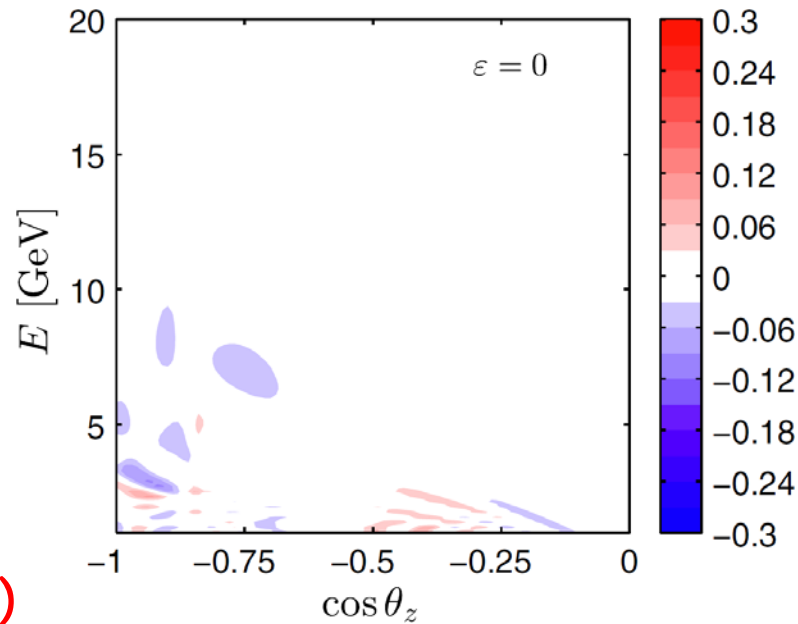


CP Violation with NSIs

$$P_{e\mu}^{\text{SD}}(\delta) - P_{e\mu}^{\text{SD}}(0)$$

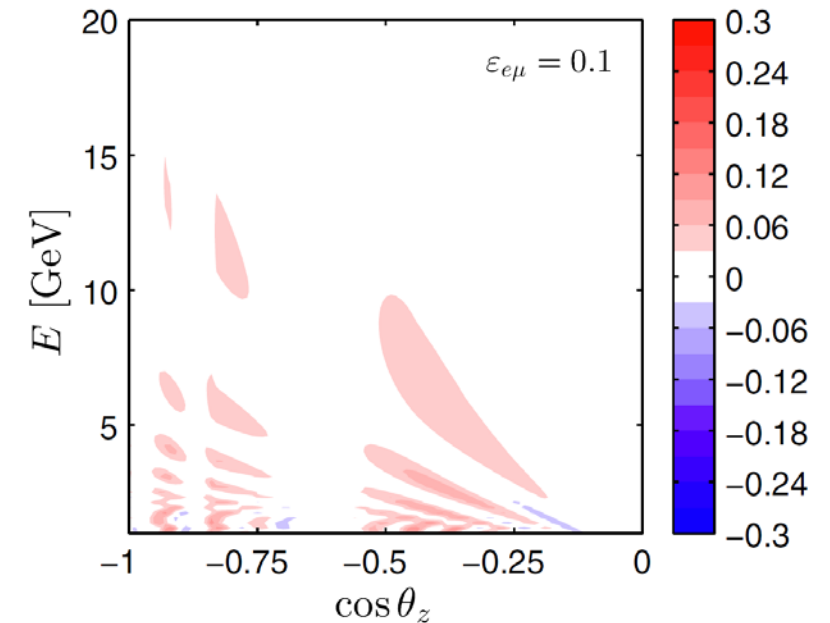
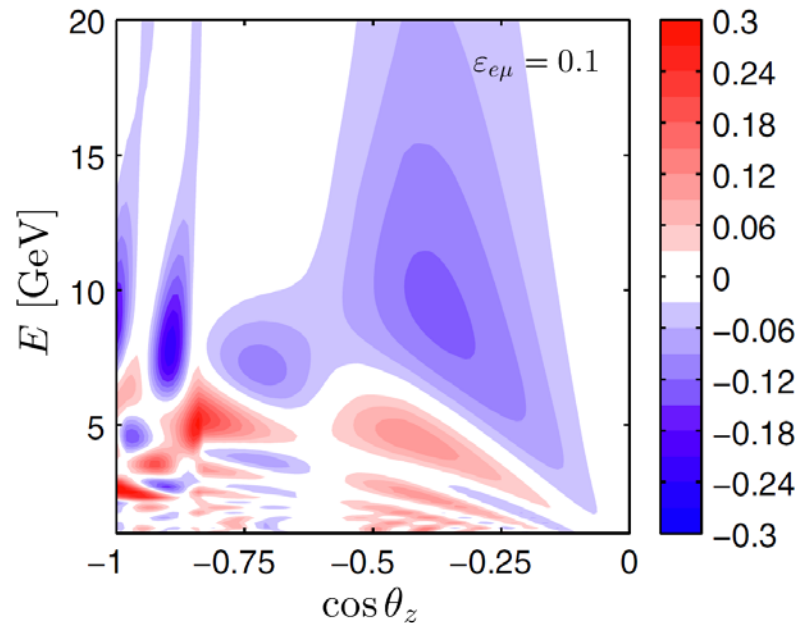
$$\delta = \frac{\pi}{2}$$

NH (left) IH (right)



$$P_{e\mu}^{\text{NSI}}(\delta) - P_{e\mu}^{\text{NSI}}(0)$$

$$\delta = \frac{\pi}{2}$$



Summary

- Future neutrino oscillation experiments aim to determine the neutrino mass ordering and measure the Dirac CP-violating phase.
- Usually theoretical prediction for the CP phase is given at a super-high energy scale, which should be compared with low-energy measurements. So, RG running has to be taken into account.
- Two new working observables have been introduced to describe intrinsic leptonic CP violation, disentangle the fake CP-violating effects, and optimize the experimental setup.
- The possibility to constrain NSIs has been investigated at IceCube (DeepCore and PINGU).



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**Thanks a lot
for your attention!**