Co-seismic Deformation: Basic principles and modelling strategies

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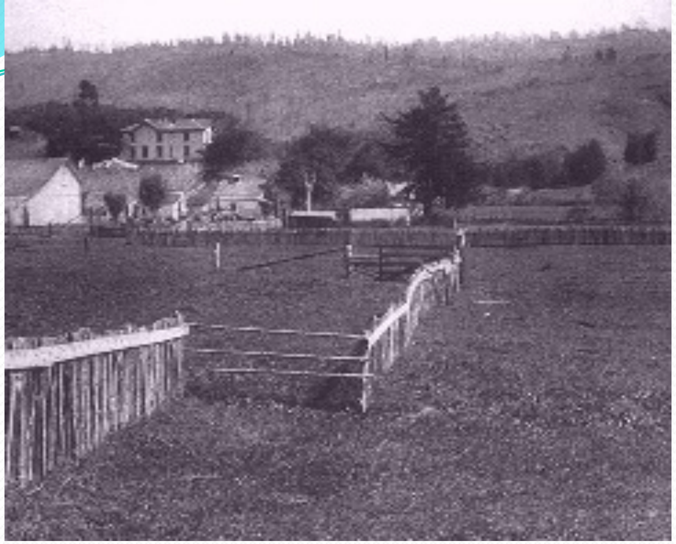
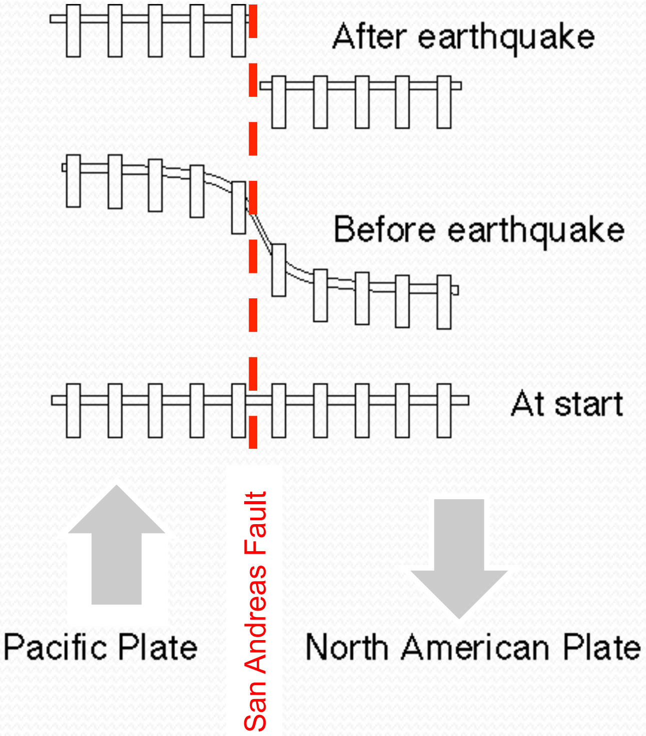
The beginning



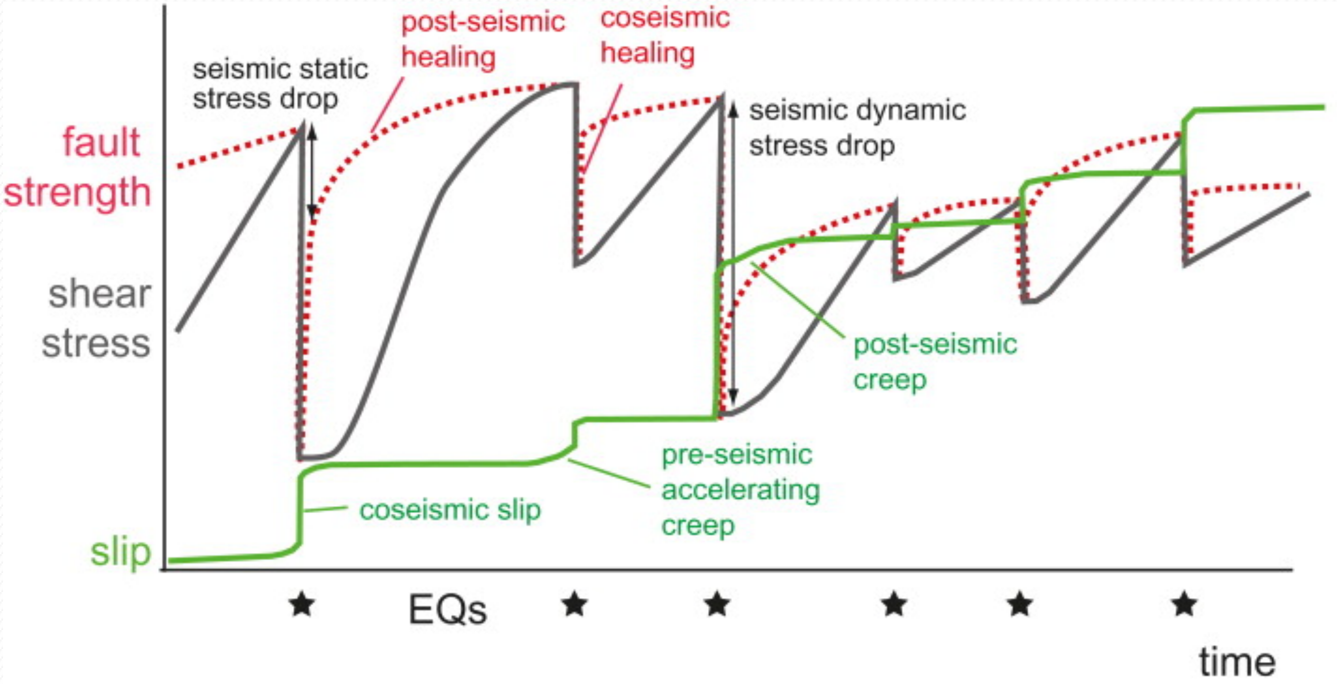


The elastic rebound theory formulated by Reid in 1910 posed the basis for the scientific developments which led to our ability to relate tectonic deformation to earthquakes.

Reid’s theory was based on a detailed study of the inter-seismic and co-seismic ground displacements associated to the 1906 San Francisco earthquake (the Lawson report).

He used field observations and triangulation surveys to measure the ground deformation.

The present



Di Toro et al., 2012

During the seismic cycle the fault strength and shear stress on a fault increase with time. The fault failure is the result of the complex feedback between the loading stress and the evolution of the fault strength with time. Both are controlled by several non-linear processes acting a very different scales.

The future

Our knowledge of all the processes involved in the fault rupture nucleation will probably never be sufficient to formulate deterministic predictions on fault failure.

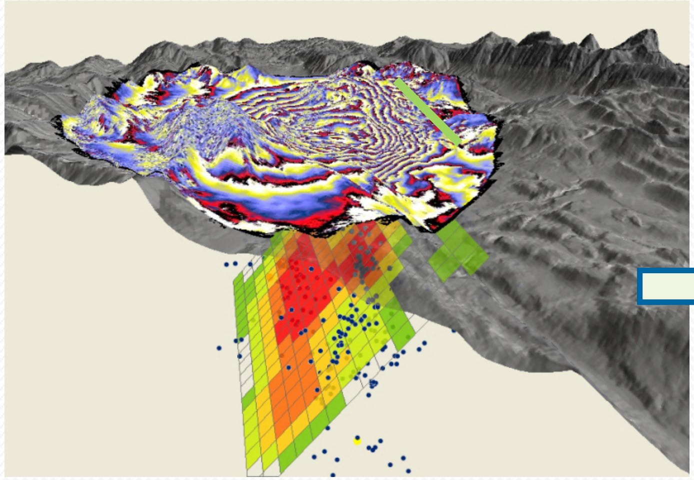
However, the study of past and present earthquake sources is crucial for the progress of the earthquake science and especially for the practical goal of improving the Seismic Hazard Assessment.

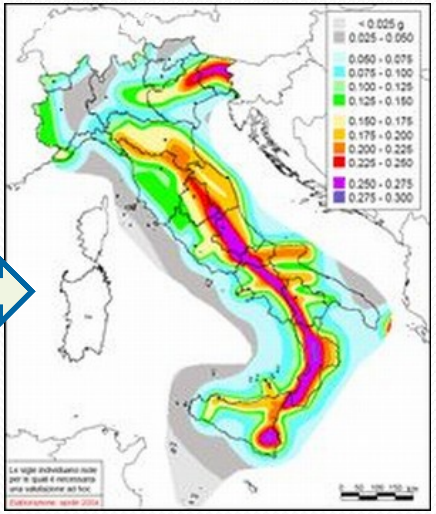
SH = the probability of exceeding a certain level of ground shaking (Peak Ground Acceleration) within a defined period of time (e.g. 50 yr).

The contribution of co-seismic deformation

The analysis of the co-seismic ground deformation is one of the most important tools to investigate the geometry and kinematics of present seismic sources.

Knowing the source and the deformation pattern we can use geological and geomorphological analyses to extend the knowledge of the co-seismic part of the seismic cycle back in time, effectively improving the estimation of the Seismic Hazard.





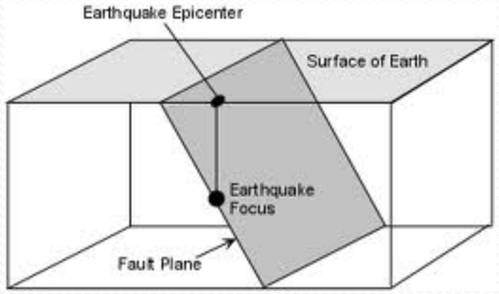
Faults as dislocations

When the shear stress on a fault overcomes the fault strength the rupture initiates. For nearly instantaneous ruptures the medium behaves elastically (e.g. Segall, 2010):

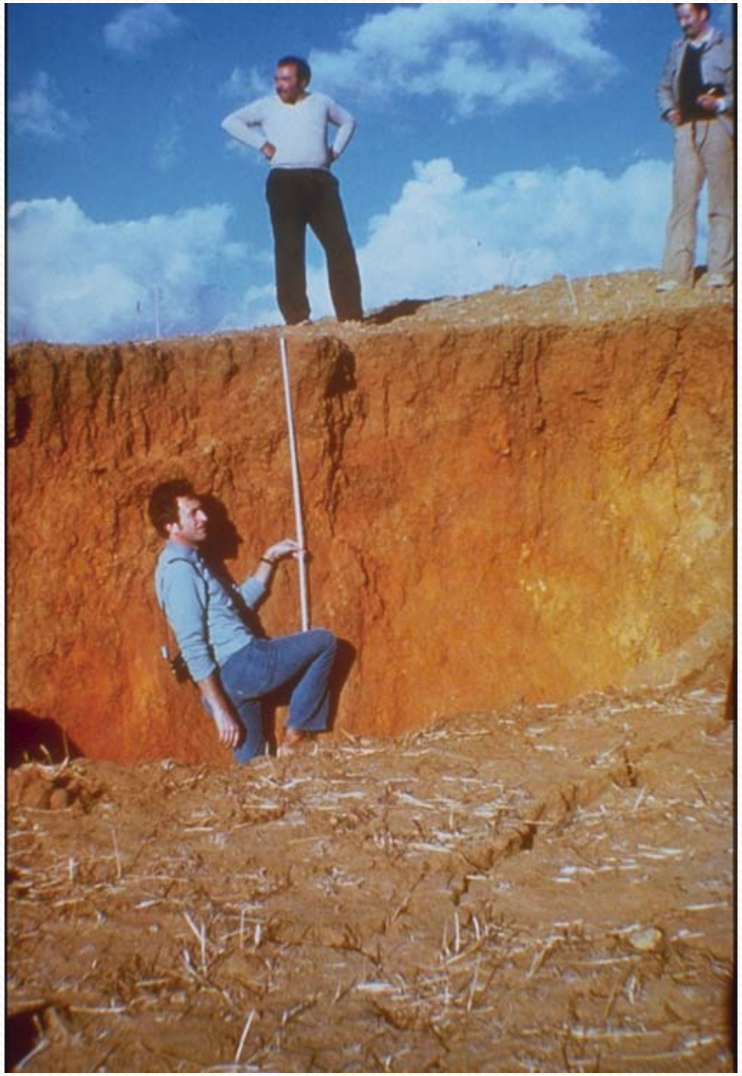
$$\sigma_{ij} = C_{ijkl}\epsilon_{kl}$$

Following the Elastic Dislocation theory, faults are considered as displacement discontinuities (or dislocations) in a continuous elastic medium (Steketee, 1958) .

It has been shown that the ED theory of can be effectively used to predict the static displacement field generated by seismic ruptures.



Fault dislocations at the surface



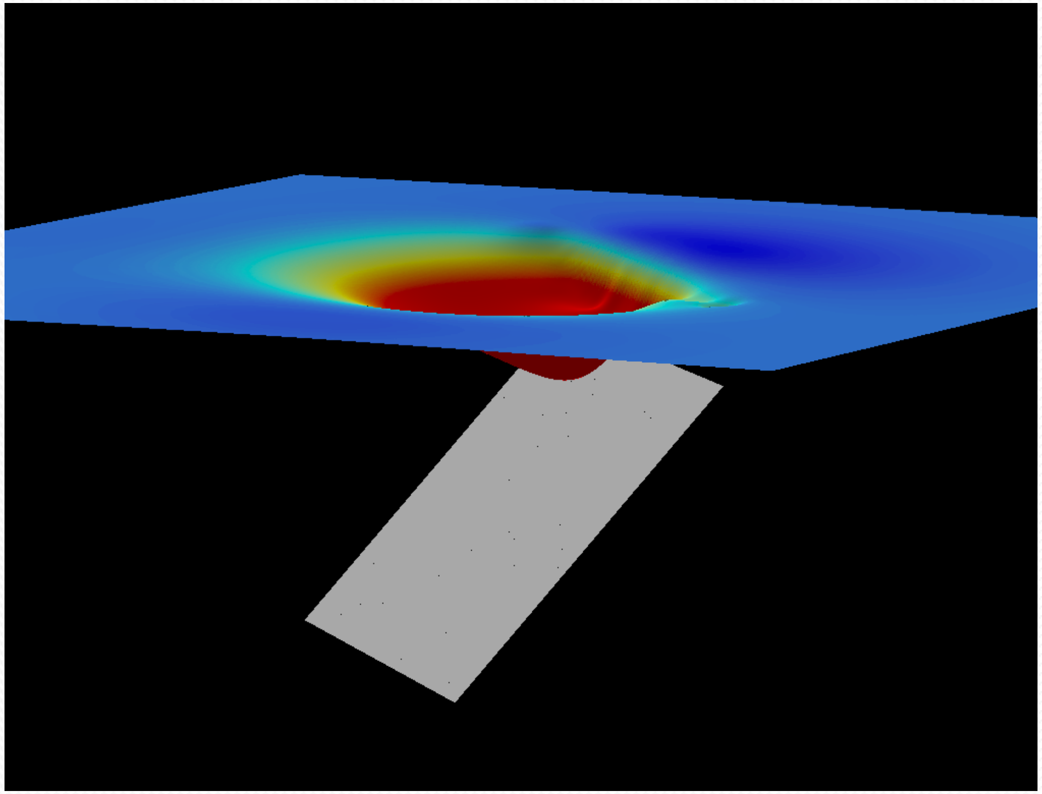
The magnitude and shape of the co-seismic 3D crustal displacements are a function of the fault slip direction and intensity, and of the crust properties .

Where the fault dislocation intersects the surface, the geometrical and kinematic parameters can often be measured directly.

However, local observations along fault scarps cannot be taken as representative of the entire dislocation.

Continuous displacement field

Away from the fault scarp, or when the rupture is "blind", the ground displacement needs to be measured by geodetic methods.



Optical leveling

EDM

VLBI

Tiltmeters

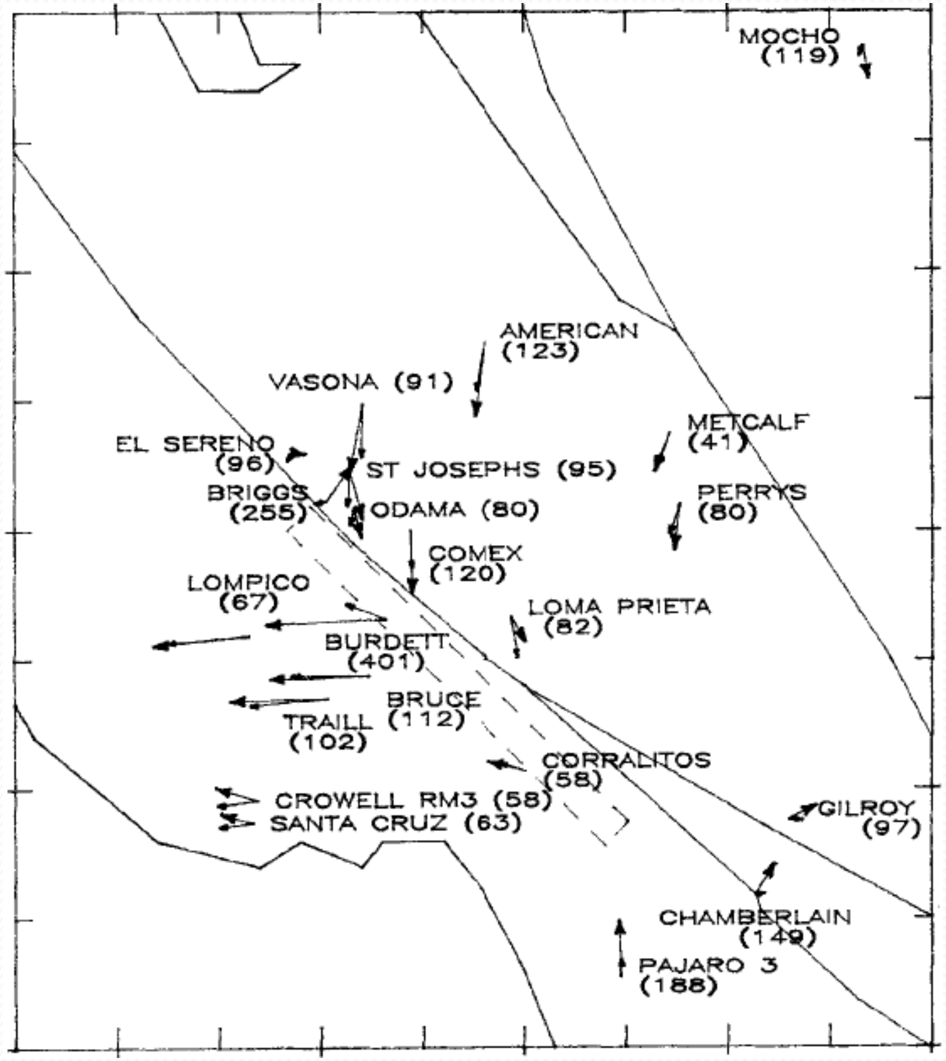
Tide gauges

GPS

LIDAR

SAR methods

Co-seismic displacement observations for the Loma Prieta earthquake - 1989

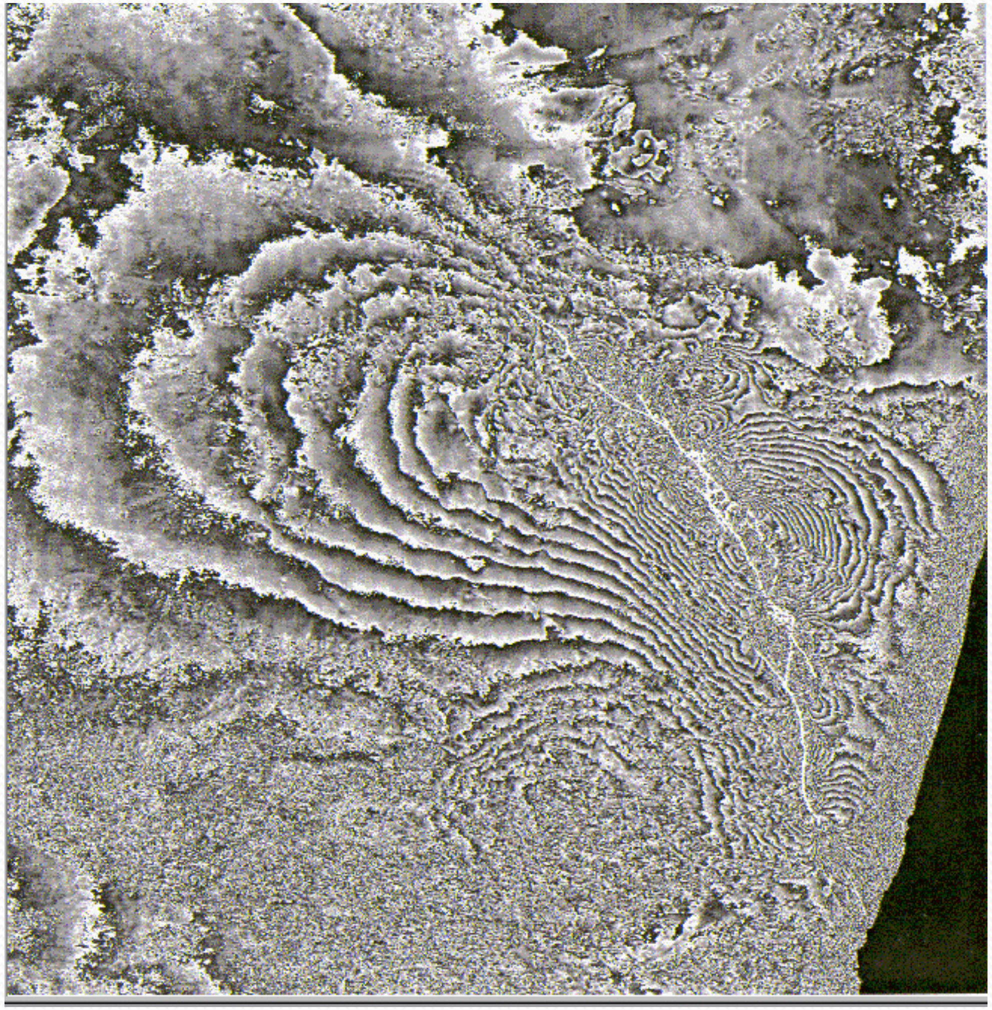


Available observations of co-seismic displacement:

- 84 EDM line changes,
- 4 GPS displacement vectors,
- 2 VLBI displacement vectors,
- 21 displacement vectors obtained by re-measuring a pre-event triangulation survey with GPS

(Snay et al., 1991).

Co-seismic displacement observations for the Landers earthquake - 1992



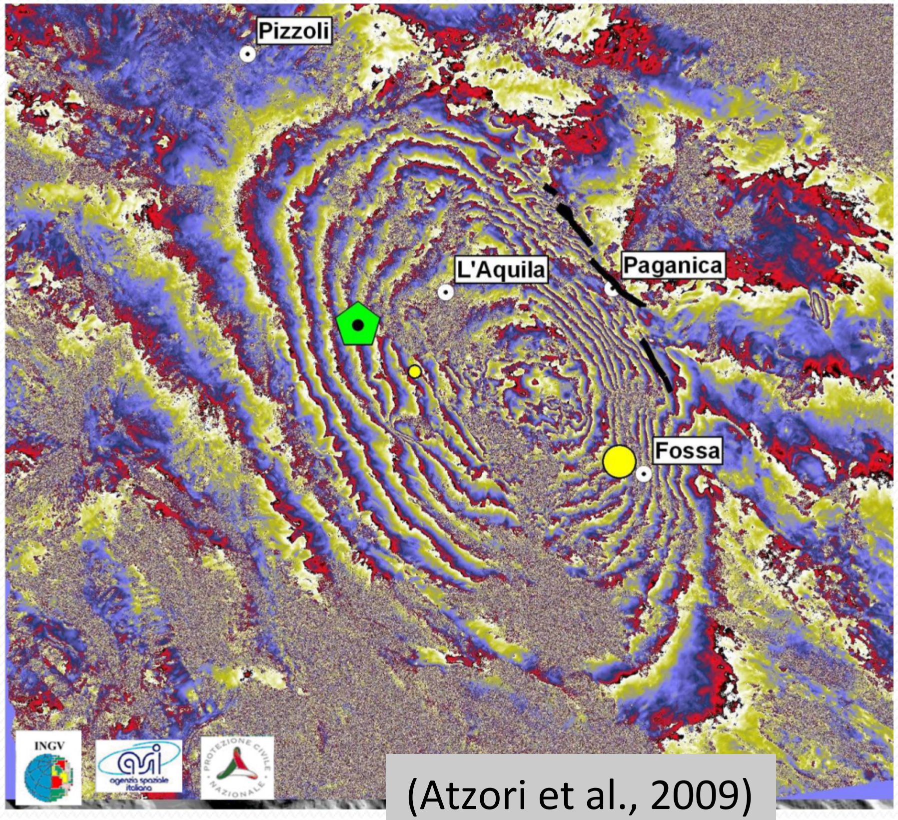
Available observations of co-seismic displacement:

- 10^5 Line of Sight displacement values from an ERS-1 differentialSAR interferogram

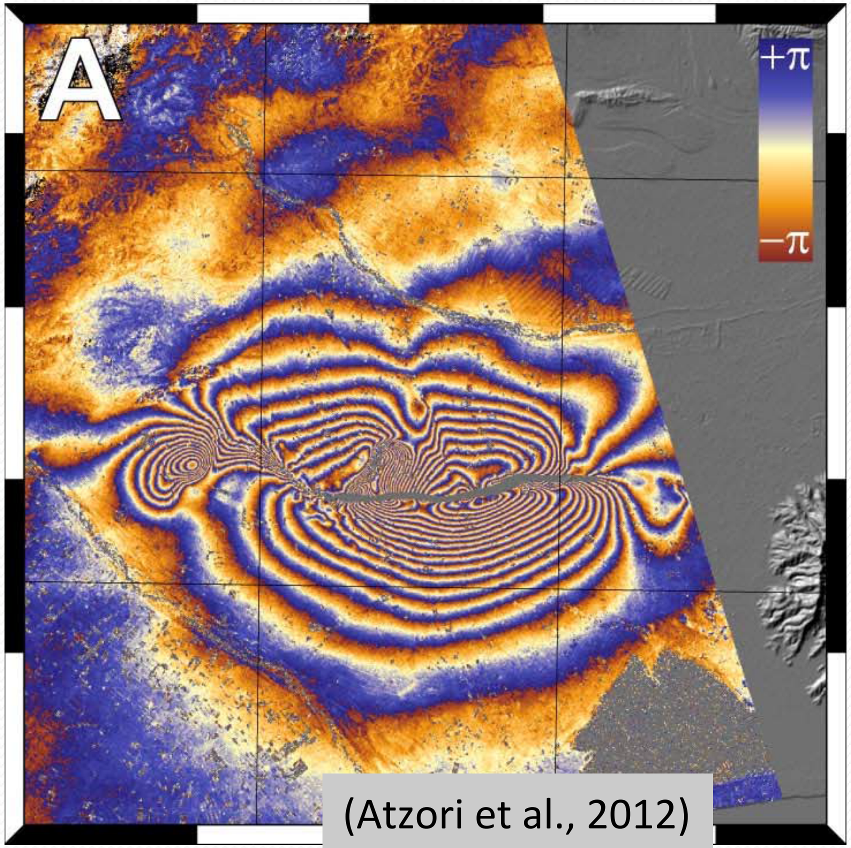
(Massonnet et al., 1993).

The InSAR era: imaging the deformation

SAR images allow to obtain continuous maps of co-seismic ground deformation, with the same resolution and effort at different scales, as opposed to discontinuous point sampling provided by GPS.



(Atzori et al., 2009)



(Atzori et al., 2012)

Operational deformation mapping by InSAR

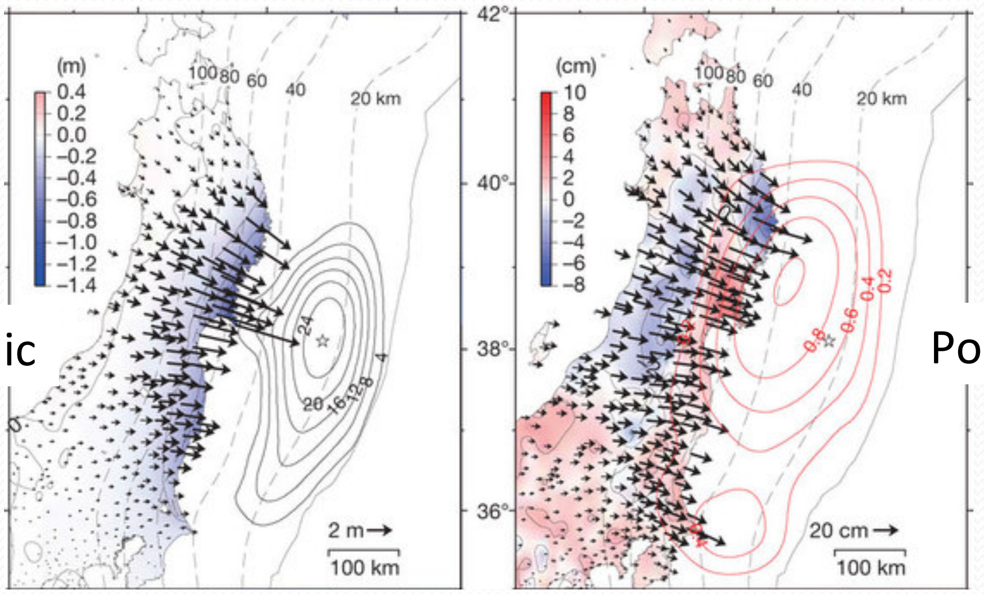
SAR data are cheap (not all of them!); can be available in NRT; can have a pre-defined acquisition schedule and a sampling rate (up to 1 image/day) suitable for static co-seismic displacement mapping.

⬇

Satellite SAR Interferometry has now become a routine technique for co-seismic ground displacement mapping, and can be applied on an operational basis at a global scale, provided a constant flow of data is assured (as expected with Sentinel-1).

Important aspects-1

The fixed revisit times of SAR satellites may cause the co-seismic displacement field to be contaminated by some post-seismic signal.



Co-seismic

Post-seismic

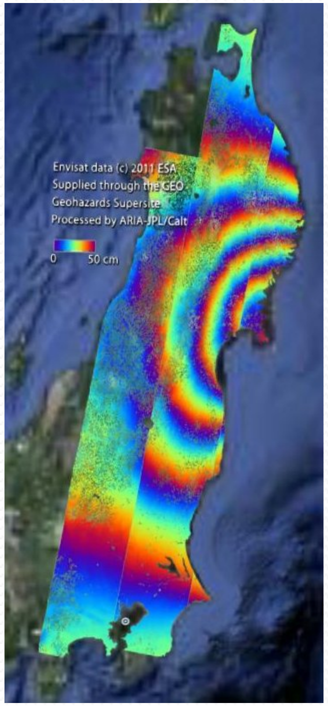
(Tohoku eq,
Ozawa et al., 2011)

It has been shown for the 2004 Parkfield earthquake, that a one-day contamination by post-seismic signal generated a factor of 2 difference in the geodetic moment (Langbein et al., 2005).

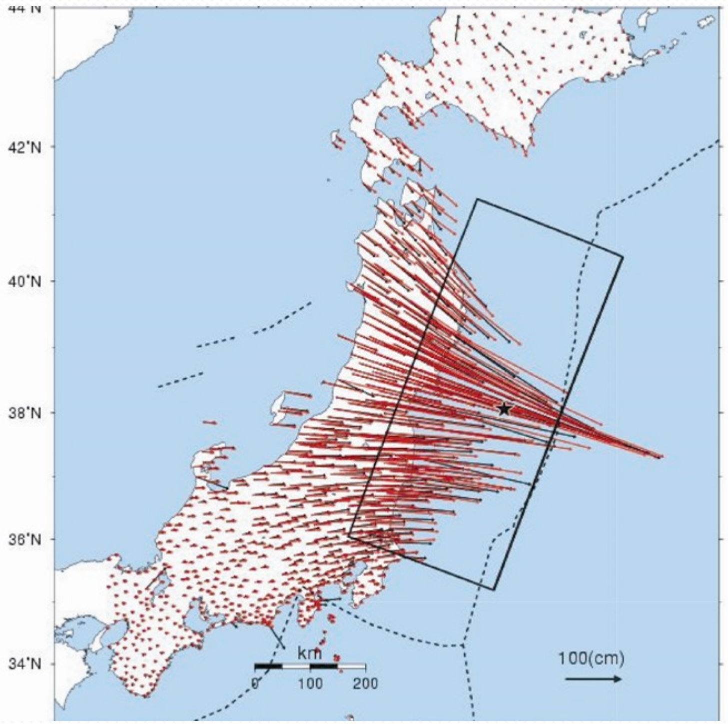
Important aspects-2

Using InSAR with other techniques (Offset Tracking, MAI) can allow to estimate 3D surface displacements.

However, with a dense CGPS network, and for a very large earthquake, the GPS displacement vectors are the best data to describe the most important details of the co-seismic deformation field.



(Agram et al., 2011)

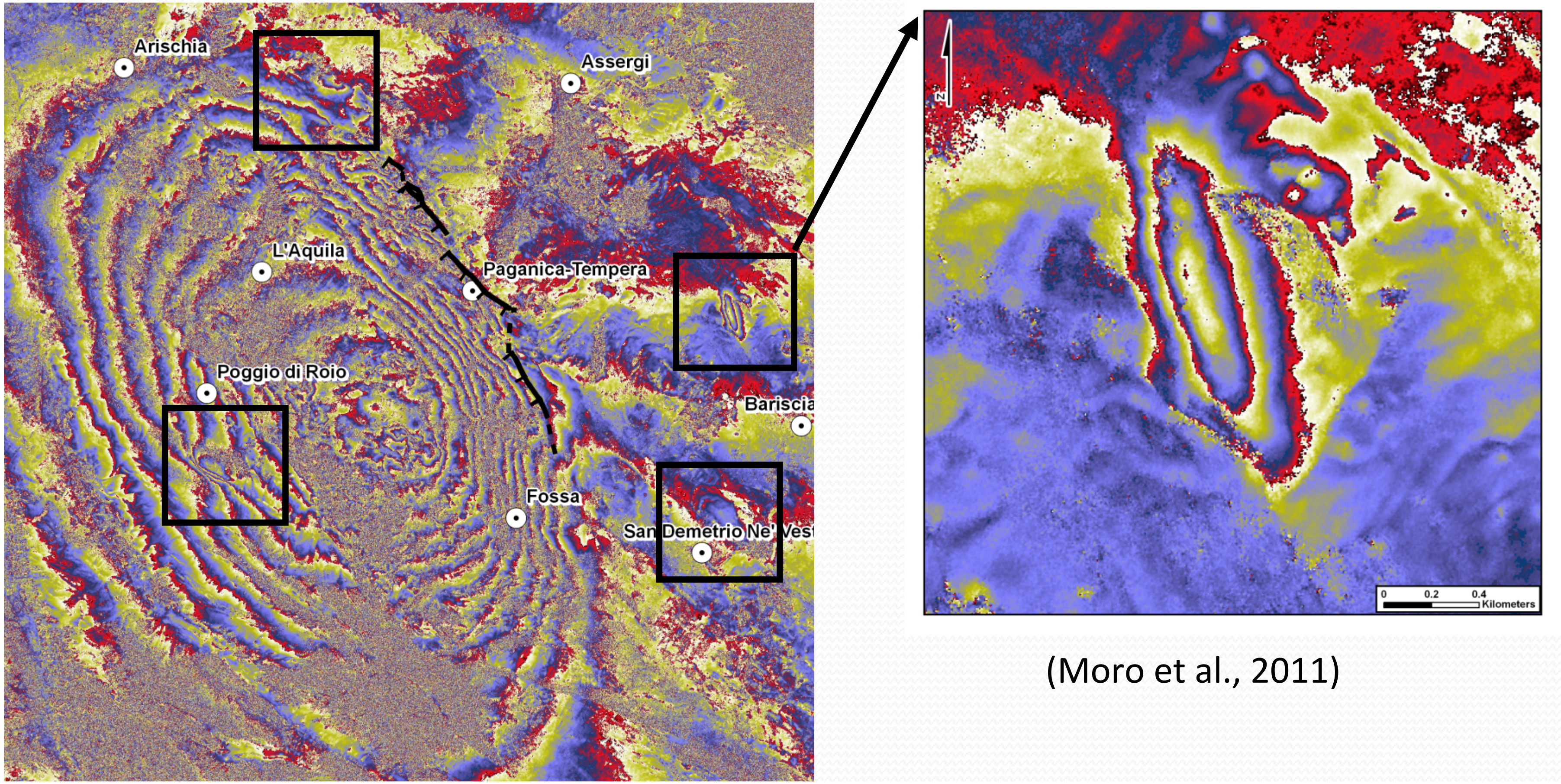


(Caltech-JPL ARIA group, 2011)

Important aspects-3

The co-seismic displacement field may be contaminated also by non-tectonic signals (gravitational deformation, shallow deformation due to acquifer pressurization).

These can be detected through their peculiar spatial frequencies.



(Moro et al., 2011)

Modelling the fault dislocation

Analytical equations for the displacement and strain fields generated by a dislocation in a homogeneous elastic half space have long been formulated (Chinnery, 1961; Maruyama, 1964; Press, 1965; Okada, 1985, 1992).

However the full capacity of elastic dislocation models to yield realistic simulations of the crustal deformation generated during a seismic dislocation remained largely unexplored until the start of the InSAR era.

Since then ~60 earthquakes have been studied using SAR data, and a recent review shows that source parameters estimated with geodetic and seismological data are in good agreement, with few exceptions (Weston et al., 2011).

Practical uses of a source model

- Support the deployment of temporary seismic/geodetic networks.
- Support the development of an event scenario for the crisis management.
- Allow an independent validation of seismological source parameters.
- Support, through stress-transfer analysis, the estimation of time-dependent seismic hazard.
- Improve the knowledge of the active tectonics and the SHA of the area.

Elastic dislocation models

Medium	**Source type**	**Type of simulation**	**Model**
Homogeneous, isotropic, elastic half space	Infinite	2D, static	Analytical: 2D solution, Savage e Burford, 1973
Homogeneous, isotropic, elastic half space	Finite (rectangular)	3D, static	Analytical: Okada, 1985
Homogeneous, isotropic, elastic half space	Finite and complex (triangular)	3D, static	Analytical: Meade, 2007
Homogeneous, elastic layered space	Finite (rectangular)	3D, static	Numerical/analytical: Wang et al., 2003; Fukahata and Matsu'ura, 2005.
Heterogeneous medium with variable rheologies	Finite and complex	3D, static and dynamic	Numerical: FEM (e.g. Masterlark, 2003; Zhao et al., 2004;Trasatti et al., 2011; Wang et al., 2013)

The Okada model

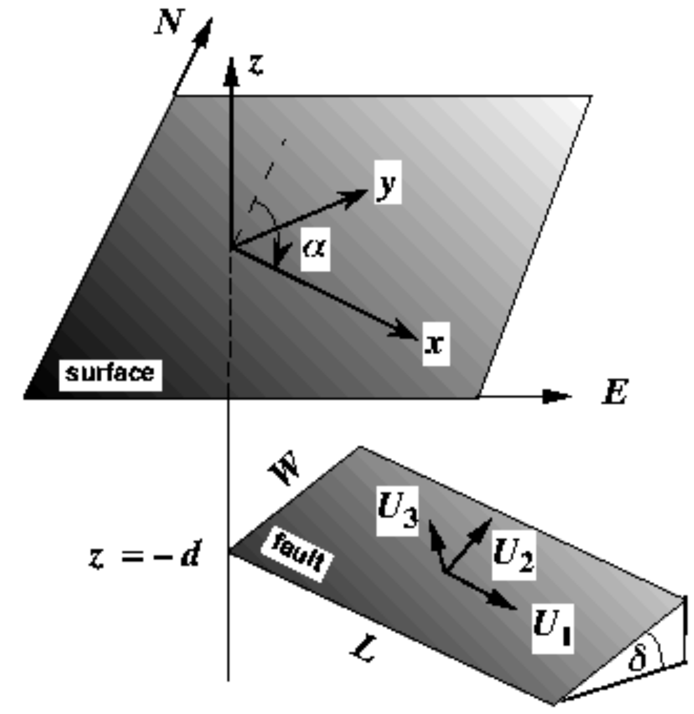
In geodesy, the most common equations used for fault dislocation modelling are those provided by Okada (1985) for a rectangular source.

The Okada analytical formulations are relative to a crustal medium idealised as a homogeneous, isotropic half space.

They provide an effective way to calculate surface displacements (also strains and tilts) due to shear and tensile dislocations occurring over rectangular faults, and are thus well suited to simulate images of co-seismic deformation as those provided by SAR data.

The Okada model parameters

The Okada model for a rectangular source is defined by 10 parameters, referred to a fault-centered cartesian plane:



Position of fault bottom edge,

z (z = -depth)

Length, L

Width, W

Dip, δ

Slip along strike, u1

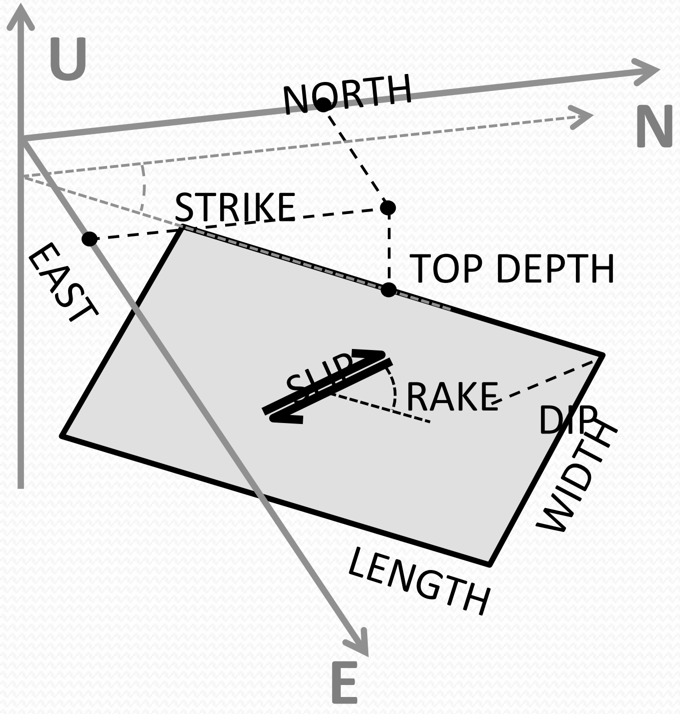
Slip along dip, u2

Tensile opening , u3

Surface displacements, dx, dy, dz

For shear dislocation modeling

By neglecting the tensile slip u3, and transforming the parameters as to be more manageable for geophysicists, we obtain the following list of 9 input parameters for the model:

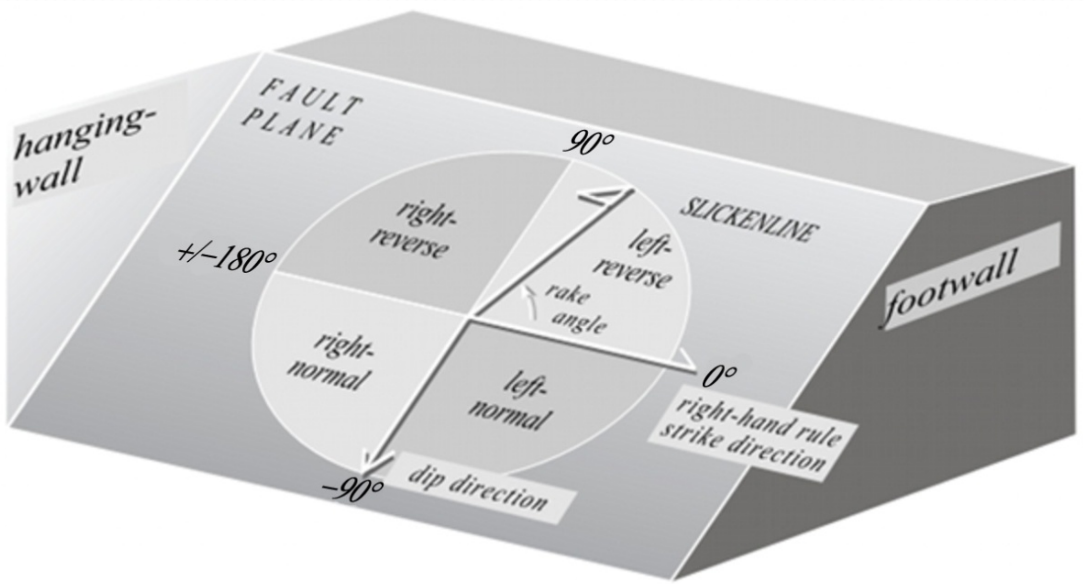


Length
Width
Top depth
Strike
Dip
Rake
North coordinate of top center
East coordinate of top center
Slip

Strike and rake conventions (from Aki and Richards, 1980)

The strike angle is determined clockwise from North if you look along strike and the fault is dipping to your right.

The rake angle (orientation of the slip vector of the hanging-wall with respect to the footwall) is measured on the fault plane from the right horizontal direction . Reverse movements have positive rake values, while normal movements have negative rakes. E.g. pure left lateral slip -> rake = 0°; pure normal slip -> rake = -90°.



The medium

The medium in the Okada equations is considered a homogeneous, isotropic half space, characterised by its Lamé constants λ and μ.

Usually the Lamé constants are set to have the same value, which means that we use a Poisson ratio $\nu = 0.25$.

$$\nu = \frac{\lambda}{2(\lambda + \mu)}$$

(A more complicated medium can be conceived using other models, see later…)

Forward and inverse modeling

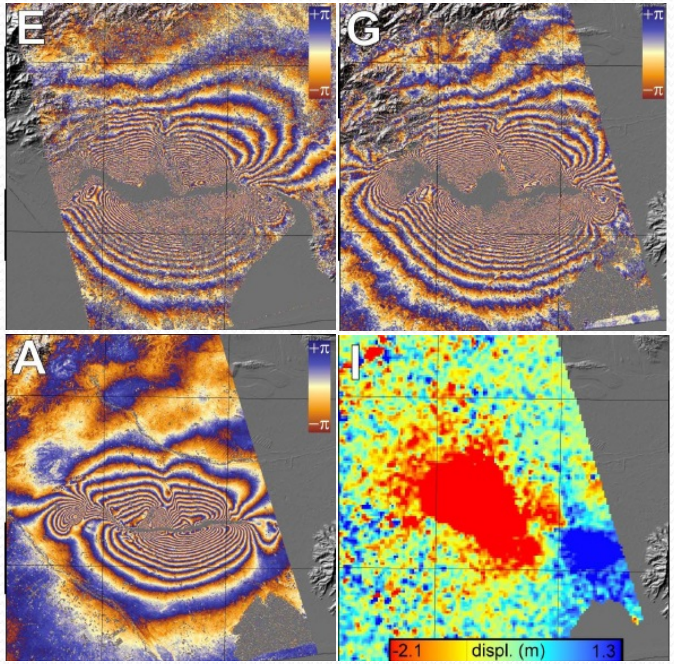
If we had knowledge of the geometrical and kinematic fault parameters we could then calculate the displacement at the surface for any given slip value. This is done in the **forward modeling** approach.

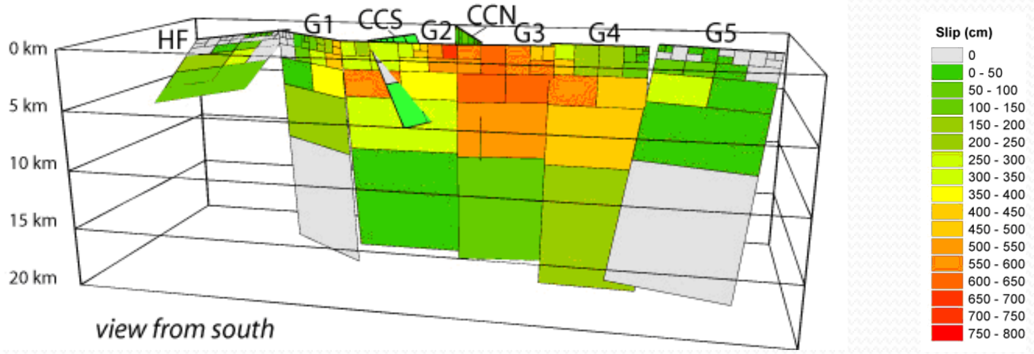
If instead we want to estimate the model parameters from the surface displacement observations, we have to use an **inversion modeling** approach.

The inversion is the (mathematical) process of estimating the values (and uncertainties) of a set of parameters of a given model, based on a set of observations related to the model.

Typical inversion modeling

We take one or (better) more co-seismic displacement data sets, with their uncertainties, we run our inversion code for a specific dislocation model, and we obtain best fit estimates of the parameters of our model fault, together with their uncertainties and trade offs.





						Fixed patches	
Fault name	**Strike** (°)	**Dip** (°)	**Rake** (°)	**Lon** (°)	**Lat** (°)	**M_0** (N·m)	**#**
Greendale East	90	90	-175	172.408	-43.575	$1.13 \cdot 10^{19}$	252
Greendale Center	86	90	175	172.211	-43.592	$2.52 \cdot 10^{19}$	280
Greendale West	305	80	-170	172.046	-43.584	$0.80 \cdot 10^{19}$	140
Charing Cross North	165	70	2	172.200	-43.496	$0.25 \cdot 10^{19}$	72
Charing Cross South	27	60	117	172.122	-43.567	$0.68 \cdot 10^{19}$	112
Hororata	240	60	90	171.921	-43.577	$0.12 \cdot 10^{19}$	100
					TOTAL	$5.50 \cdot 10^{19}$	952

Linear and non linear inversion

We can distinguish two different inversion problems.

In the linear problems the data and the model parameters are linearly related. In a general form:

$$\mathbf{d}=\mathbf{G}\mathbf{m}$$

d is an N-dimensional data vector,

m is an M-dimensional model parameter vector,

G is an N × M matrix containing only constant coefficients which depend on the selected model (the Green function matrix) .

Non linear inversion deals with more complex, non linear relationships between data and model parameters. In a general form:

$$\mathbf{d}=\mathbf{G}(\mathbf{m})$$

Inversion of Okada model parameters

In the case of the Okada model, all model parameters other than slip are non-linearly related to the surface deformation.

This is intuitively clear if we think that doubling the strike (or rake, or etc.) of a model dislocation certainly we do not double the surface displacement.

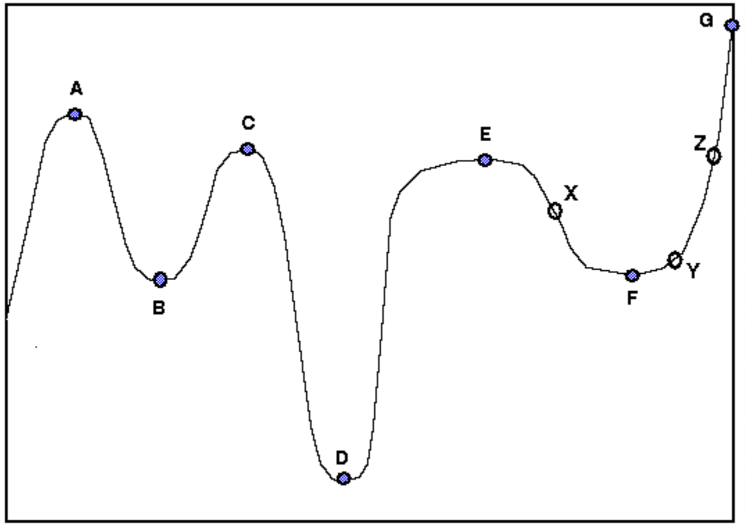
Thus to estimate the geometrical and kinematic fault model parameters (strike, dip, rake, etc.) we need to use **non linear inversion**.

Surface displacement is instead proportional to Slip, and a **linear inversion** approach can be used to estimate, for a given dislocation, the slip distribution able to predict the observed surface deformation.

Non linear inversion of co-seismic data

Most mathematical techniques to carry out non linear inversion seek to minimise a Cost Function, defined as:

CF = f(observed - modeled) e.g. $CF = \frac{1}{N}\sum_{i}^{N}\frac{\left(d_{i,\text{obs}} - d_{i,\text{mod}}\right)^2}{\sigma_i}$



The difficult task in NL inversion is to find a the actual global minimum of a CF which contains also local minima.

Various techniques exist to search for the global minimum in the fastest and most efficient way (see e.g. Menke, 1984; Mueller &Siltanen, 2012).

Linear inversion of co-seismic data

Since the slip is the only parameter linearly related to the surface displacement (doubling the slip the co-seismic displacement is doubled), we can use a linear inversion approach to estimate the variable distribution of slip on the fault.

To do this we need to know the other 8 parameters of the Okada dislocation model, typically obtained from a previous (non linear) inversion of the same co-seismic data.

This task is simpler than the non linear inversion, and essentially consists of inverting the Green matrix in the least square sense, by calculating its generalised inverse (with InSAR data the problem is always overdetermined).

Beyond half space models

It has been shown that, accounting for a layered medium with heterogeneous elastic properties, can have a strong influence on the dislocation models, for instance on moment and centroid depth estimates (Hearn & Bürgmann,2005; Masterlark, 2003).

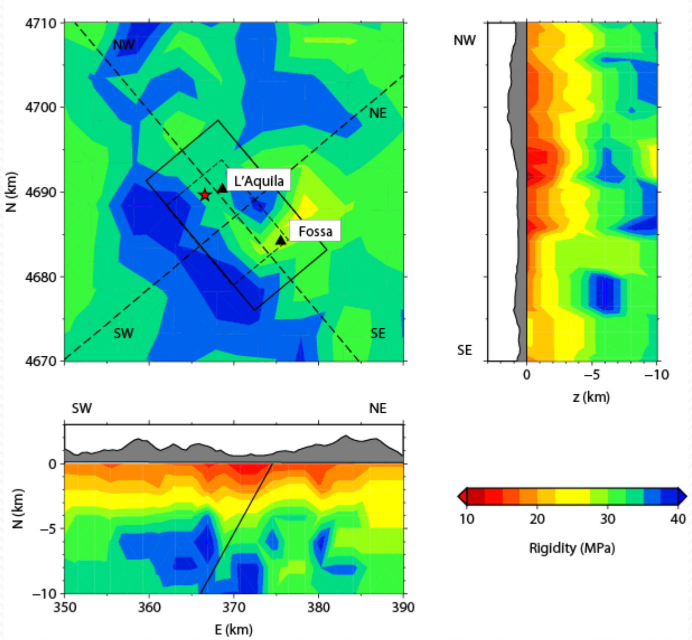
This in particular for deep dislocations in complex tectonic settings, as subduction zones (Moreno et al., 2010).

For smaller, shallow crustal earthquakes, the effect of considering the elastic layering seems less pronounced, with differences in slip distributions of 10-20% (Trasatti et al., 2011; Zhao et al., 2004).

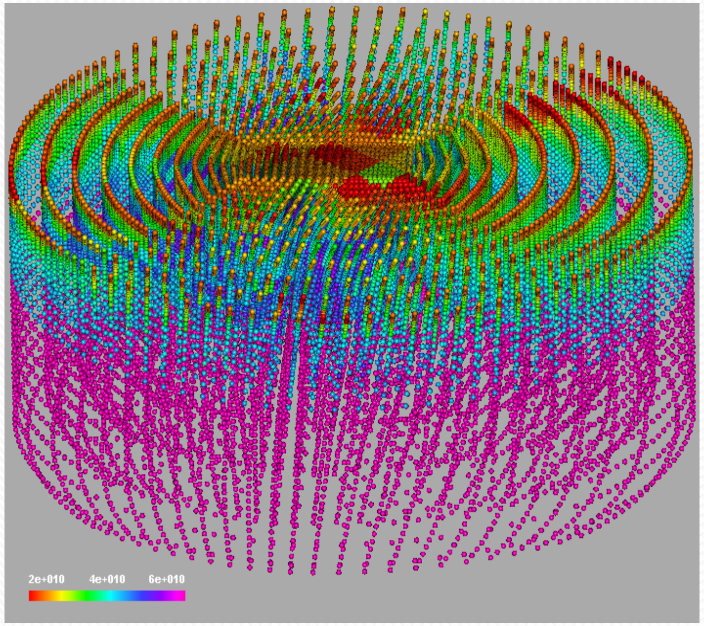
FEM modelling

The Finite Element Method is a numerical approximation technique by which the partial differential equations governing the (elastic) deformation are solved by numerical approximation over a grid describing a discrete medium.

The medium heterogeneities are defined by independent data.



Seismic tomography of the L'Aquila region

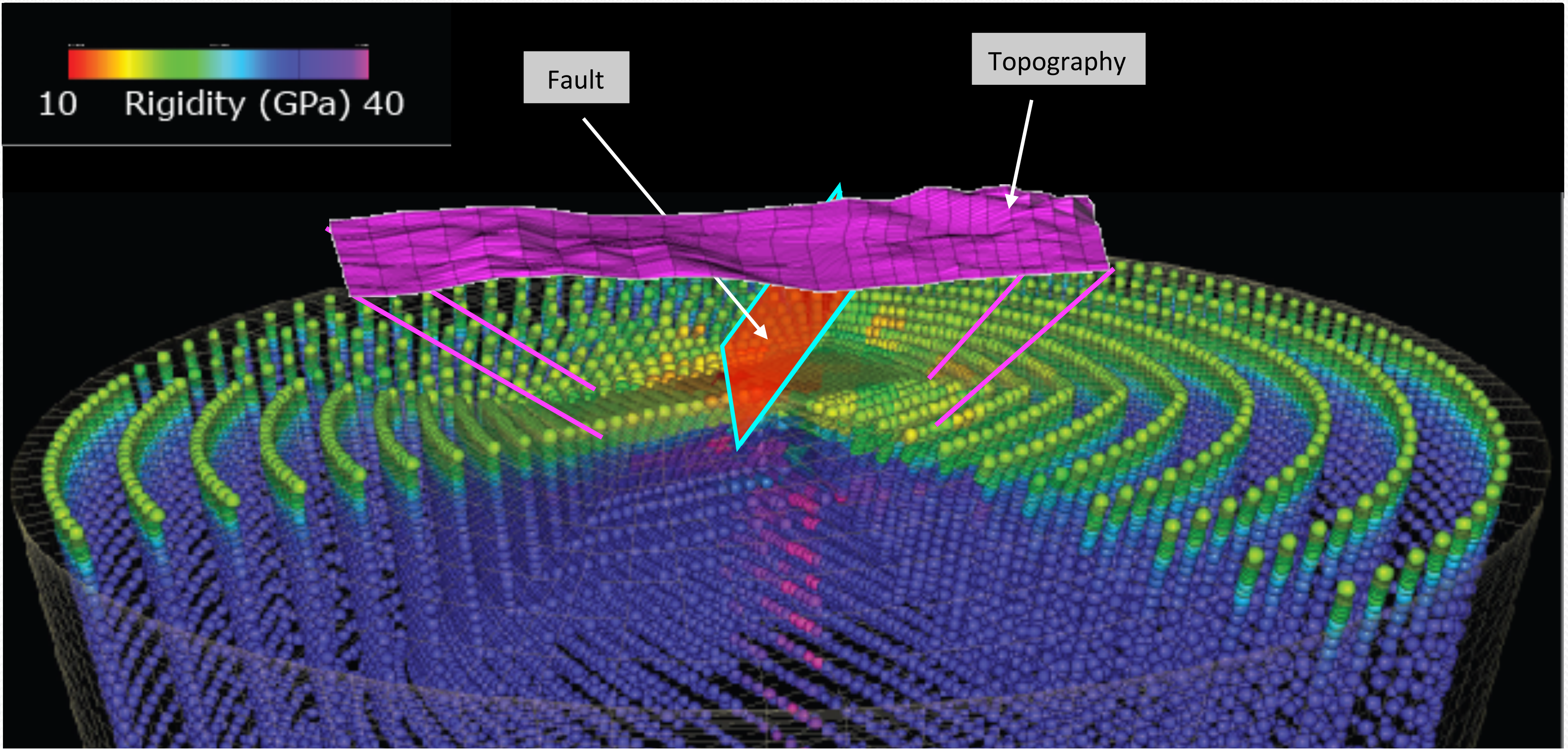


Heterogeneous medium

FEM modelling

FEM-derived Green functions for unitary slip on fault patches can be used for the linear inversion of the co-seismic displacements.

The advantage is that we can have a more realistic medium, more complicate fault surfaces, and account for topographic variations.



Issues in inversion modelling-1

Best data

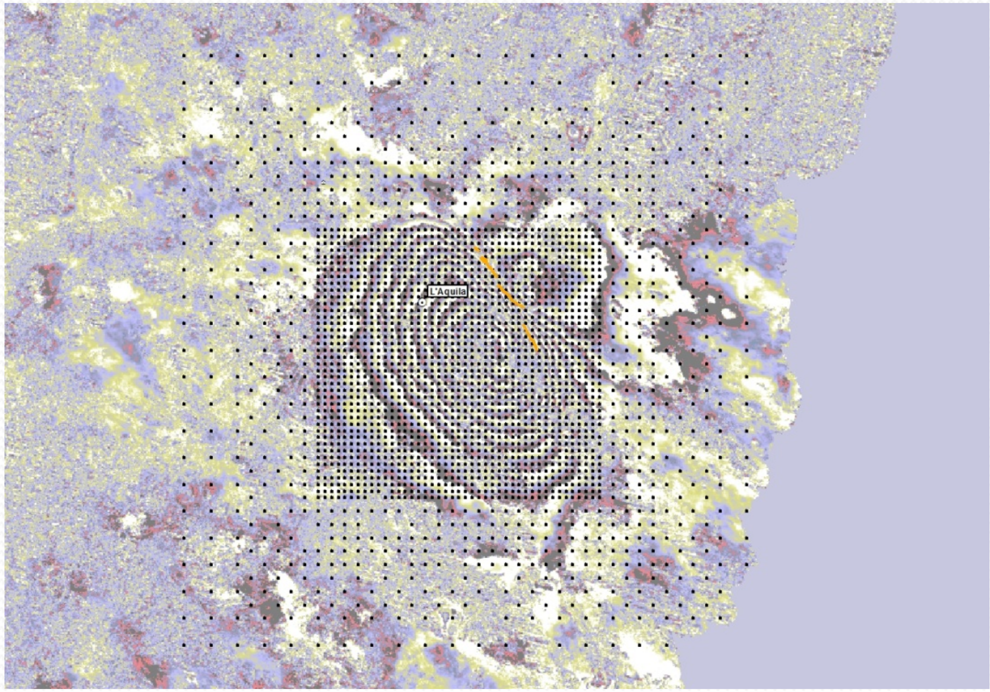
While the best situation is certainly when the data set allows a 3-D measurement of displacements (e.g. with InSAR ascending and descending LoS displacements, and azimuth displacements by offset tracking), it has been shown with synthetic data that, at least for large displacements, two data sets with opposite LoS give reasonably good results (Wright et al., 2004).

With only one Line of Sight, it is advisable to use other constraints, for instance from seismological data…

Issues in inversion modelling-2

Data sub-sampling

For a manageable inversion, the number of observations should be limited to a few thousands. Data sub-sampling is usually carried out, using dense sampling grids near the fault and more spaced grids away from it (Lohman and Simons, 2005; Jónsson et al., 2002)

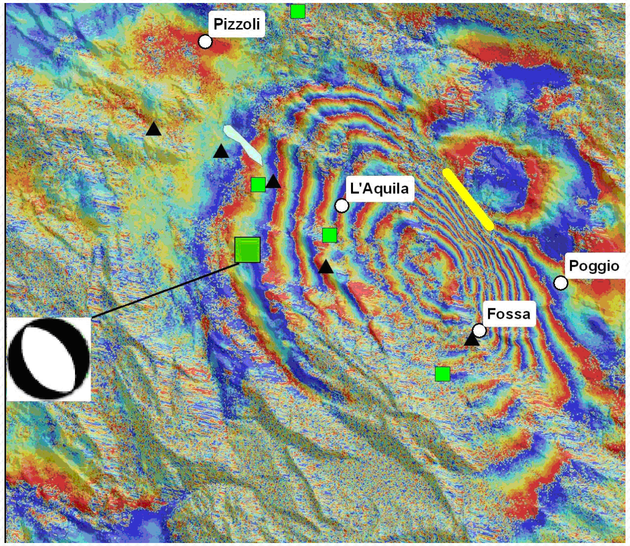


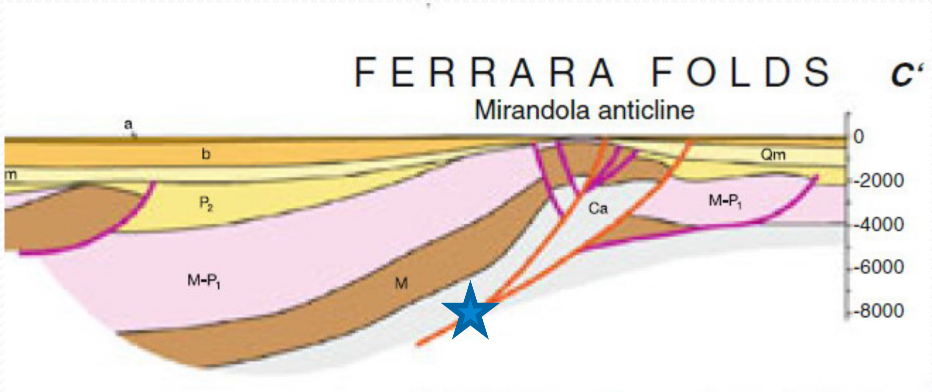
Issues in inversion modelling-3

Further constraints

To reduce the effects of non-uniqueness of inversion results (different sets of parameters can generate very similar displacements), we should use as much as possible other constraints (*a priori* information).

E.g.: we may reduce the number of model parameters using CMT focal solutions (or geological data) to constrain the source geometrical parameters within a range of values.





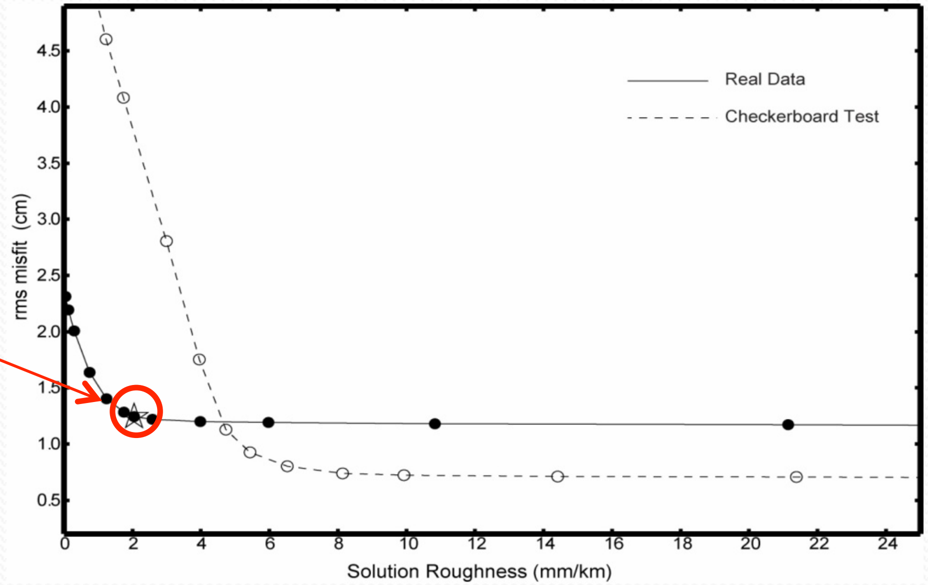
Issues in inversion modelling-4

Further constraints

In the linear inversion , unrealistic variations of slip values across adjacent patches are avoided by applying a smoothing constraint (Freymuller et al., 1994; Jónsson et al., 2002;).

A positivity constraint is also usually applied to avoid retrograde fault movement.

These are added as further rows to the Green function matrix.



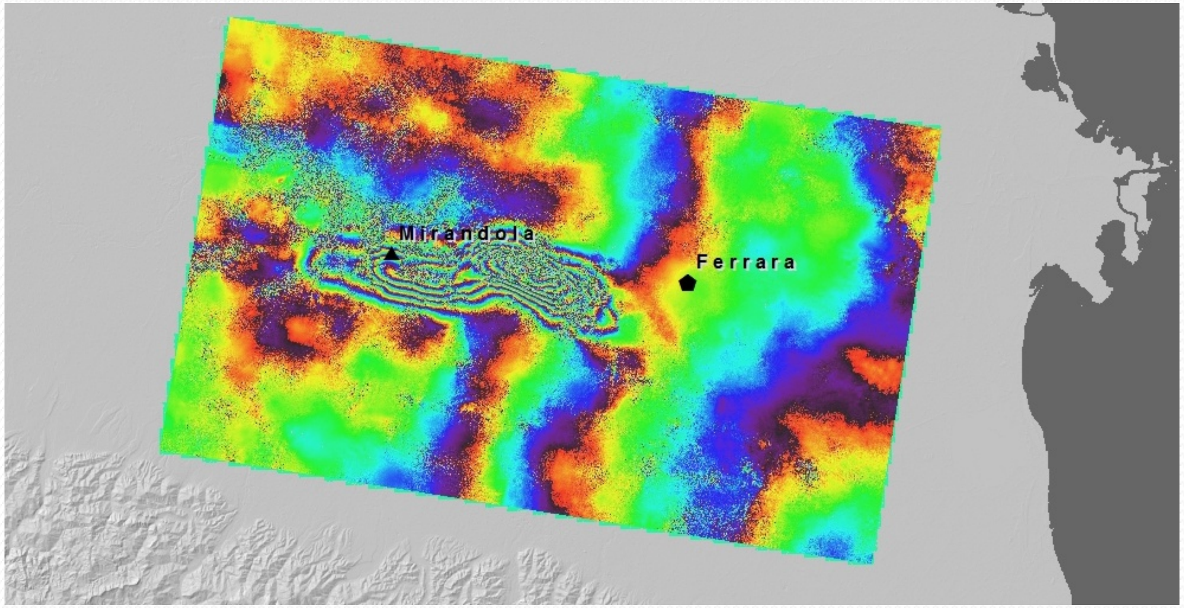
Best smoothing factor

(Wright et al., 2003).

Issues in inversion modeling-5

Other model parameters

It is also common practice to include in the model parameter vector the coefficients of an orbital ramp (usually linear or quadratic), and an offset to account for the ambiguity of the zero level in the LoS displacements.

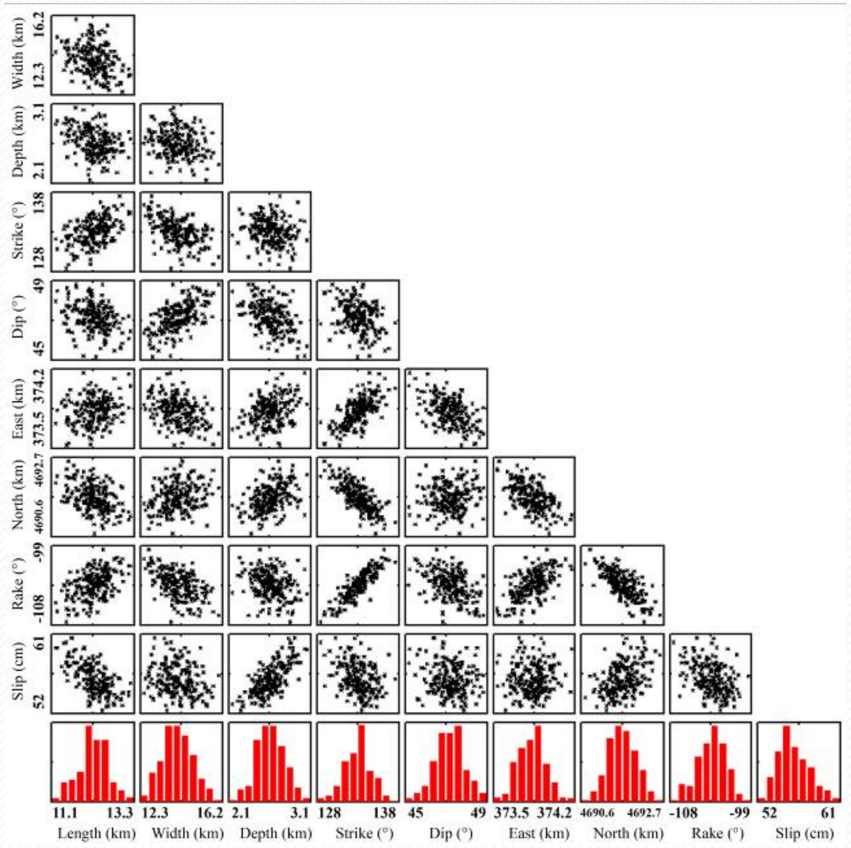


Issues in inversion modeling-6

Uncertainties and trade offs for model parameters

The noise structure of the ground displacement data (essentially due to atmospheric water vapour variations) is estimated in areas of low deformation.

Then parameter uncertainties and trade offs can be estimated through the inversions of many synthetic datasets to which a similar noise has been added (Lohman and Simons, 2005; Parsons et al., 2006).



Issues in inversion modeling-7a

Patch size and model resolution

The density of our observations affects the model resolution, that is the minimum slip patch dimension that can be resolved. For instance a partial coverage of a displacement field will degrade the ability of the model to estimate the details of fault slip in that part of the fault.

Loss of detail in the slip distribution is also increasing with the fault depth, and at a depth of 15 km the minimum resolvable patch may be ten times larger that the shallower ones.

Issues in inversion modeling-7b

Patch size and model resolution

The choice of the patch size will influence the maximum depth of slip. The patch dimensions giving the optimal resolution can be automatically calculated (e.g. Atzori & Antonioli, 2011).