1. Methodology / 2. Algorithms / 3. Examples

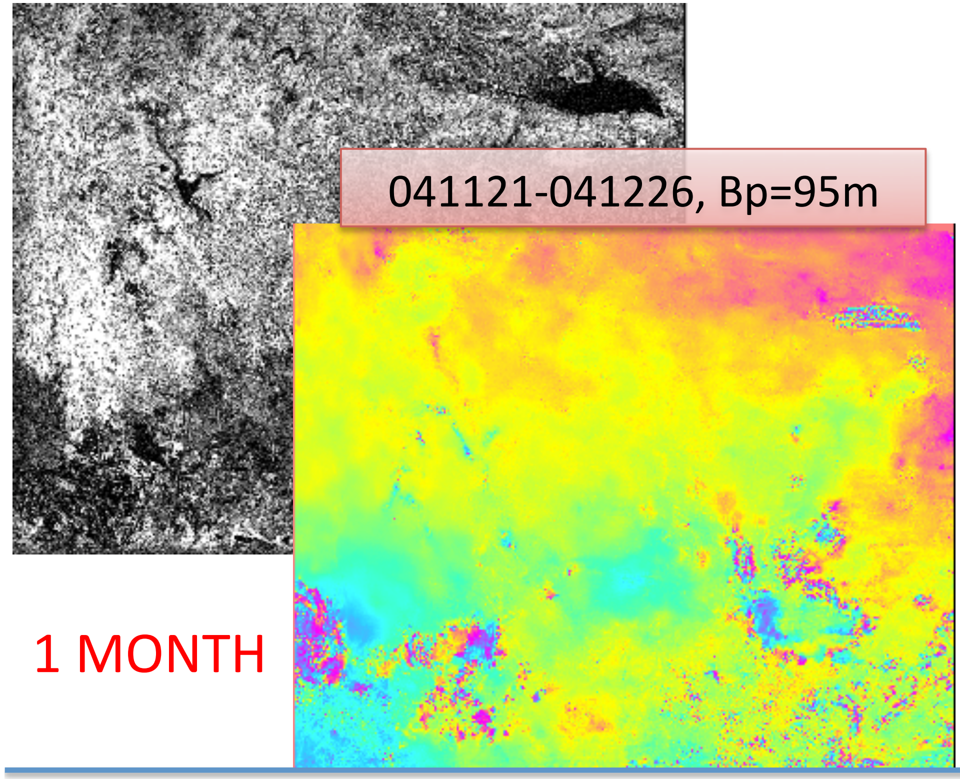
Small Baseline Time Series Methods in InSAR

Hua Wang

Guangdong University of Technology

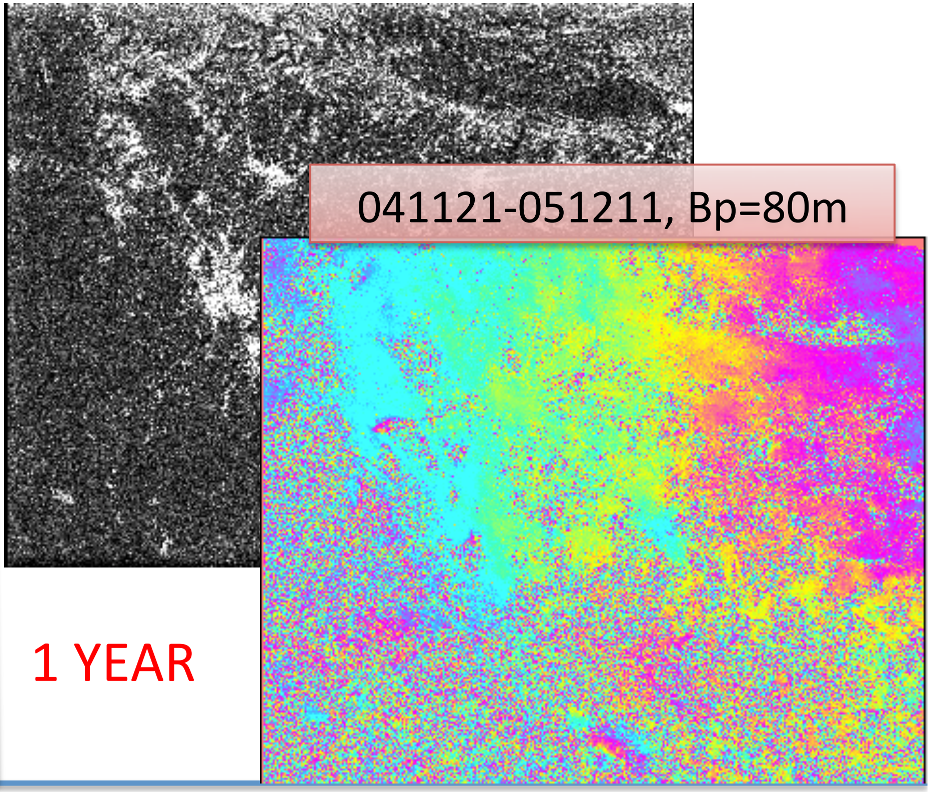
SBAS - Motivations

- Coherence is a key factor for InSAR
- Coherence is sensitive to temporal and perpendicular baselines given a specific wavelength



041121-041226, Bp=95m

1 MONTH

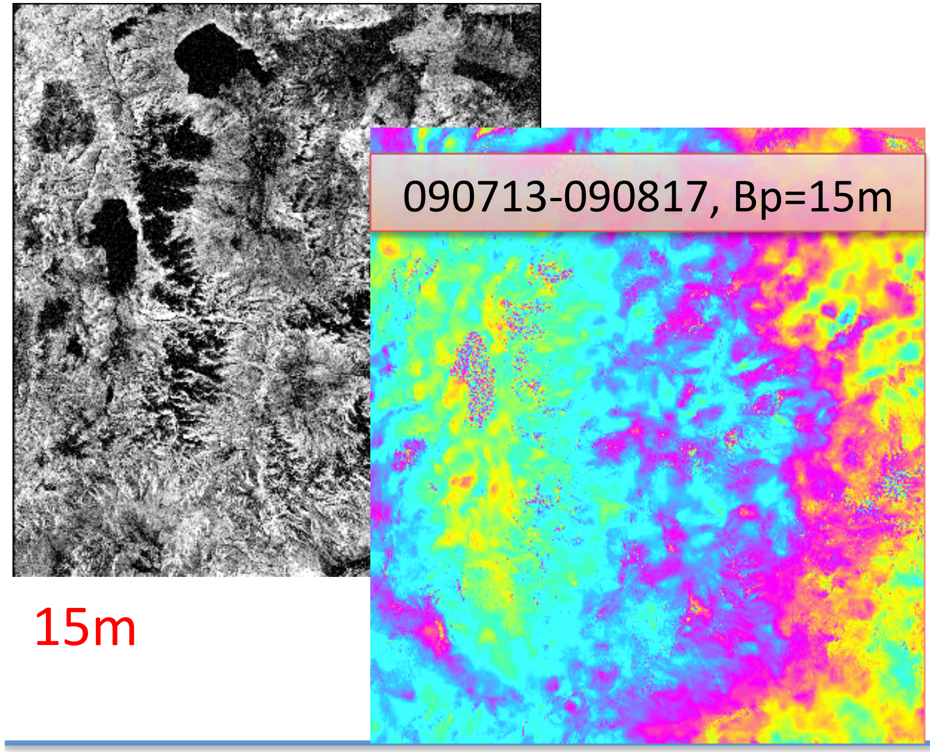


041121-051211, Bp=80m

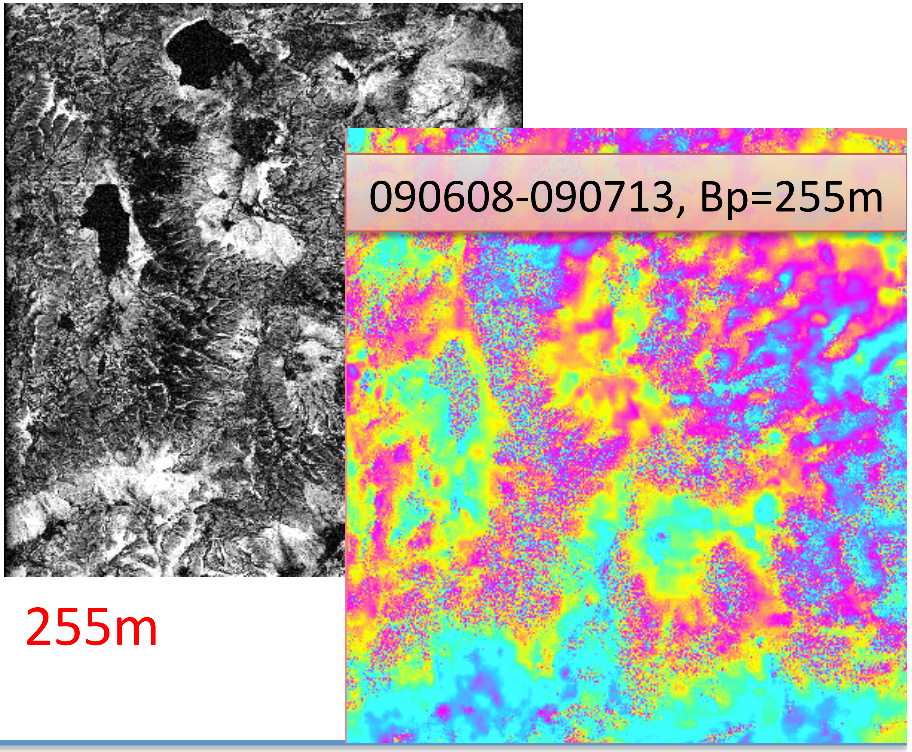
1 YEAR

SBAS - Motivations

- Coherence is a key factor for InSAR
- Coherence is sensitive to temporal and perpendicular baselines given a specific wavelength


090713-090817, Bp=15m

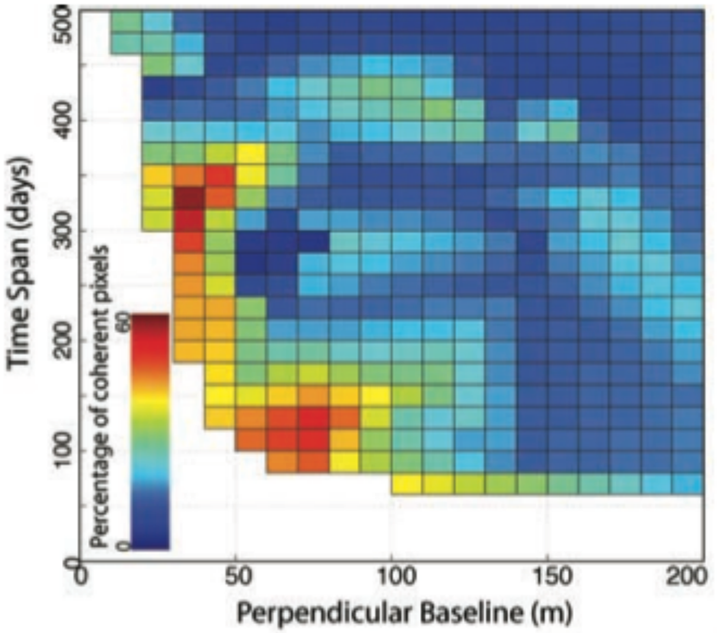
15m


090608-090713, Bp=255m

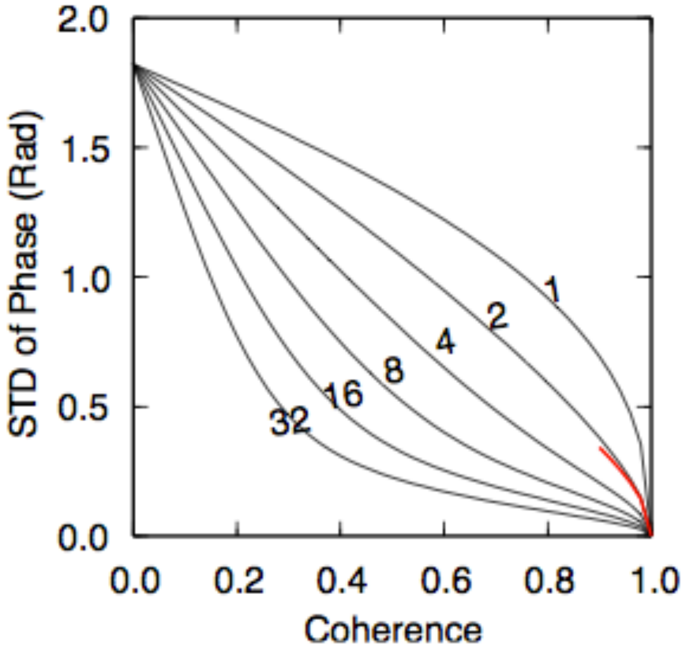
255m

SBAS - Motivations

- Coherence is a key factor for InSAR
- Coherence is sensitive to temporal and perpendicular baselines given a specific wavelength
- Smaller baseline gives higher coherence and more accurate phase



Biggs et al., 2007



$$\sigma_\phi^2 = \frac{1-\gamma^2}{2N\gamma^2}$$

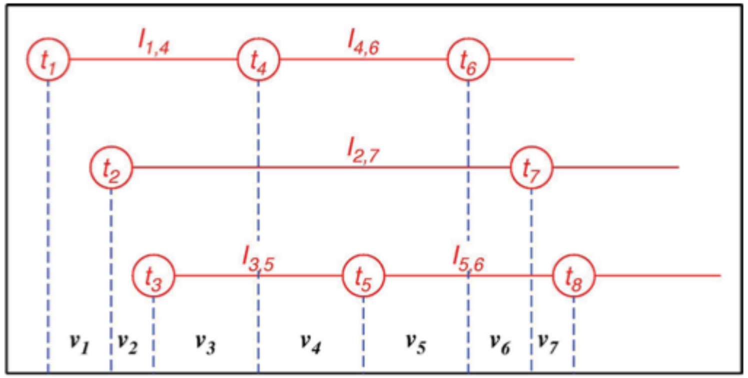
Rodriguez and Martin (1992)

SBAS - Essentials

- Forming differential interferograms using small temporal and spatial baseline subsets
- Taking use of all coherent pixels (temporal vs persistent)
- Mitigating artifacts (e.g. atmosphere, orbit, DEM errors etc) by time series analysis to derive high-precision deformation (similar to PSInSAR)
- Usually starting from, but is not limited to, unwrapped interferograms

SBAS – Methodology (TS)

- For an interferogram formed by image i, j, the displacement is



$$d_{ij} = d_i + d_{i+1} + \cdots + d_{j-1}$$

$$= \sum_{k=i}^{j-1} d_k$$

- If we replace incremental displacement by velocity,

$$d_{ij} = \Delta t_i v_i + \Delta t_{i+1} v_{i+1} + \cdots + \Delta t_{j-1} v_{j-1}$$

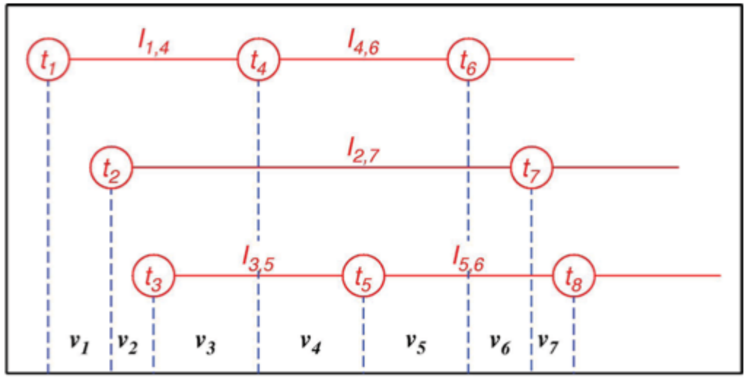
$$= \sum_{k=i}^{j-1} \Delta t_k v_k$$

SBAS – Methodology (TS)

- System of equations

$$\mathbf{Gm} = \mathbf{d},$$

$$\mathbf{G}_{i,j} = [\underbrace{\mathbf{0}}_{i-1}\ \underbrace{\Delta t_i \cdots \Delta t_{j-1}}_{j-i}\ \underbrace{\mathbf{0}}_{n-j}],$$



$$\mathbf{m} = [v_1 \quad v_2 \quad \cdots \quad v_{n-1}]^T,$$

where $\Delta t_i = t_{i+1} - t_i$, t is the acquisition date, n is the total number of the acquisitions, $\mathbf{0}$ is a zero vector indicating acquisitions which are not covered by the interferogram $I_{i,j}$, v_i is the velocity of the ith time-span. Here, the acquisitions must be chronologically ordered

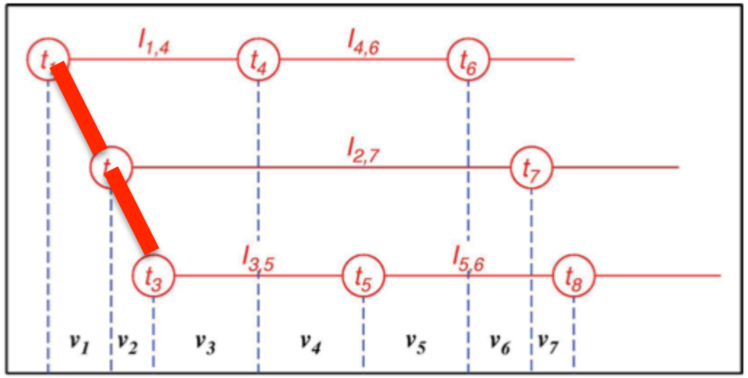
SBAS – Methodology (TS)

- System of equations

$$\mathbf{G}\mathbf{m} = \mathbf{d},$$

$$\mathbf{G}_{i,j} = [\underbrace{\mathbf{0}}_{i-1}\ \underbrace{\Delta t_i \cdots \Delta t_{j-1}}_{j-i}\ \underbrace{\mathbf{0}}_{n-j}],$$

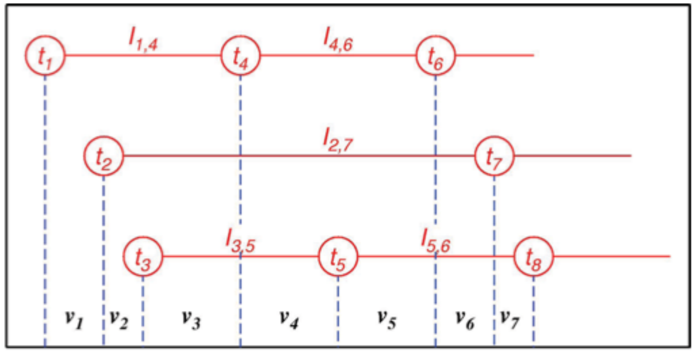
$$\mathbf{m} = [v_1 \quad v_2 \quad \cdots \quad v_{n-1}]^T,$$



- Some isolated subsets exist, G is rank deficit
- All epochs are connected in a network, G is full rank
- Solution is not stable due to noise in d

SBAS – Methodology (TS)

- Unknown parameters
  - ○ π-RATE (described above): velocity of each interval
  - ○ Berardino et al. (2002): velocity of each epoch (rank deficit)
  - ○ Schmidt and Burgmann (2003): incremental displacement (irregular interval)



- Solution
  - – SVD (Berardino et al. 2002)

$$\mathbf{m} = \mathbf{G}^{+}\mathbf{d}$$

  - – Laplacian smoothing (Schmidt and Burgman, 2003)

$$\begin{bmatrix} \mathbf{G} \\ \kappa^2 \nabla^2 \end{bmatrix} \mathbf{m} = \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix}$$

SBAS – Methodology (TS)

- System of equations

$$\mathbf{G}\mathbf{m} = \mathbf{d},$$

- Considering DEM errors

$$\begin{bmatrix} \mathbf{G} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ \Delta h \end{bmatrix} = \mathbf{d}$$

$$\mathbf{B} = \begin{bmatrix} -\frac{B_{\perp}^{1}}{\rho \sin \vartheta} & & \\ & \ddots & \\ & & -\frac{B_{\perp}^{m}}{\rho \sin \vartheta} \end{bmatrix}$$

Assuming DEM errors are invariant in time

SBAS – Methodology (TS)

- System of equations

$$\mathbf{G}\mathbf{m} = \mathbf{d},$$

- Considering DEM errors

Assuming DEM errors are invariant in time

$$\begin{bmatrix} \mathbf{G} & \mathbf{B} \\ \kappa^2\nabla^2 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ \Delta h \end{bmatrix} = \begin{bmatrix} \mathbf{d} \\ 0 \end{bmatrix}$$

$$\mathbf{m}' = \left(\mathbf{G}'^T \mathbf{C}_d^{-1} \mathbf{G}' + \mathbf{K}^T \mathbf{K}\right)^{-1} \mathbf{G}'^T \mathbf{C}_d^{-1} \mathbf{d},$$

$$\mathbf{C}_{\mathbf{m}'} = \left(\mathbf{G}'^T \mathbf{C}_d^{-1} \mathbf{G}' + \mathbf{K}^T \mathbf{K}\right)^{-1},$$

where $\mathbf{K} = \kappa^2\nabla^2$.

SBAS – Methodology (mean velocity)

- System of equations

$$\mathbf{G}\mathbf{m} = \mathbf{d},$$

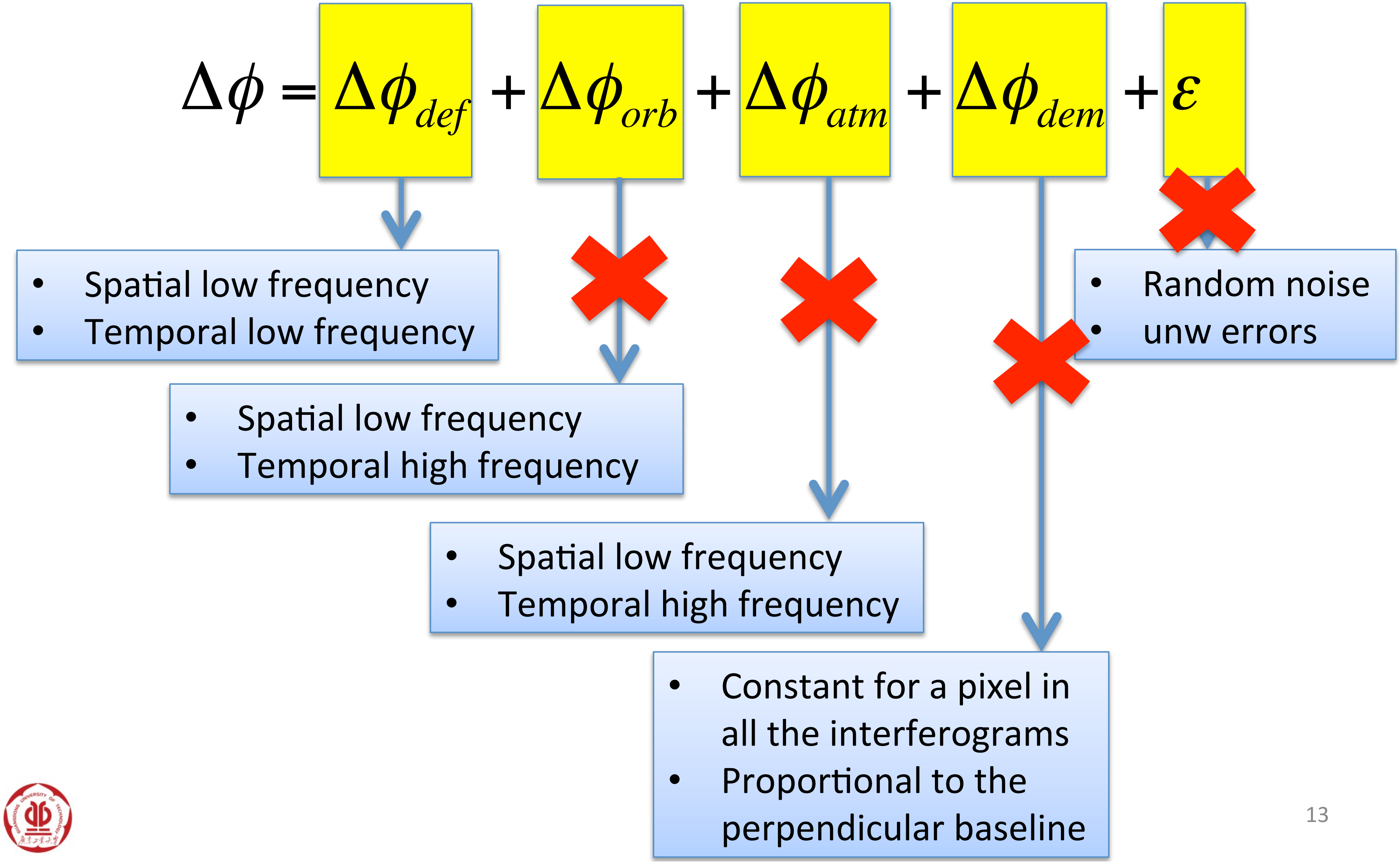
- Considering DEM errors

$$\begin{bmatrix}\mathbf{G} & \mathbf{B}\end{bmatrix}\begin{bmatrix}\mathbf{m} \\ \Delta h\end{bmatrix} = \mathbf{d}$$

Assuming DEM errors are invariant in time

- Here, G is the time span and B is the coefficient for DEM correction for each interferogram.
- The design matrix is full rank once more than 1 observations are obtained.

Components of interferometric phase



$$\Delta\phi = \Delta\phi_{def} + \Delta\phi_{orb} + \Delta\phi_{atm} + \Delta\phi_{dem} + \varepsilon$$

- Spatial low frequency
- Temporal low frequency

- Spatial low frequency
- Temporal high frequency

- Spatial low frequency
- Temporal high frequency

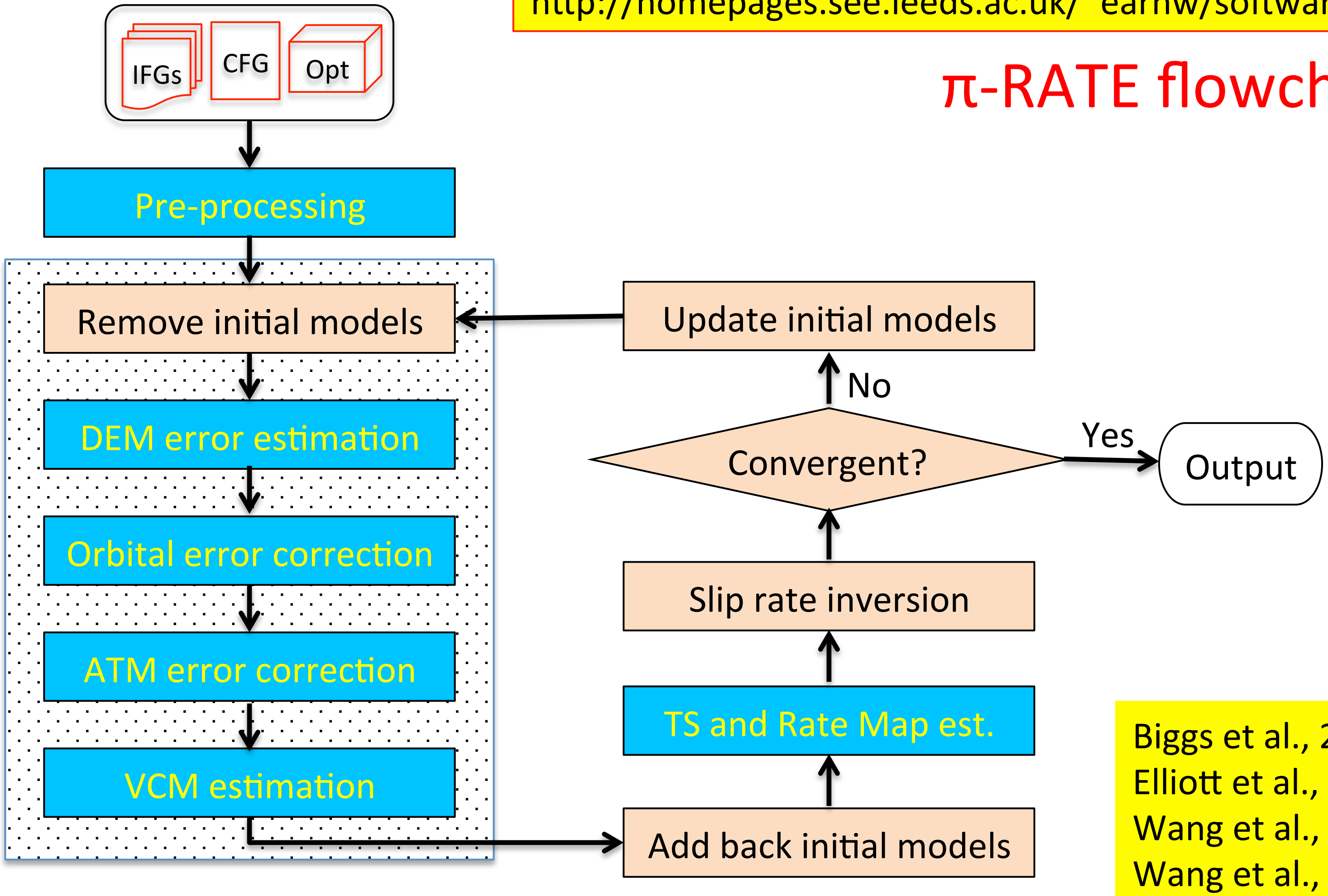
- Constant for a pixel in all the interferograms
- Proportional to the perpendicular baseline

- Random noise
- unw errors

SBAS - Implementation

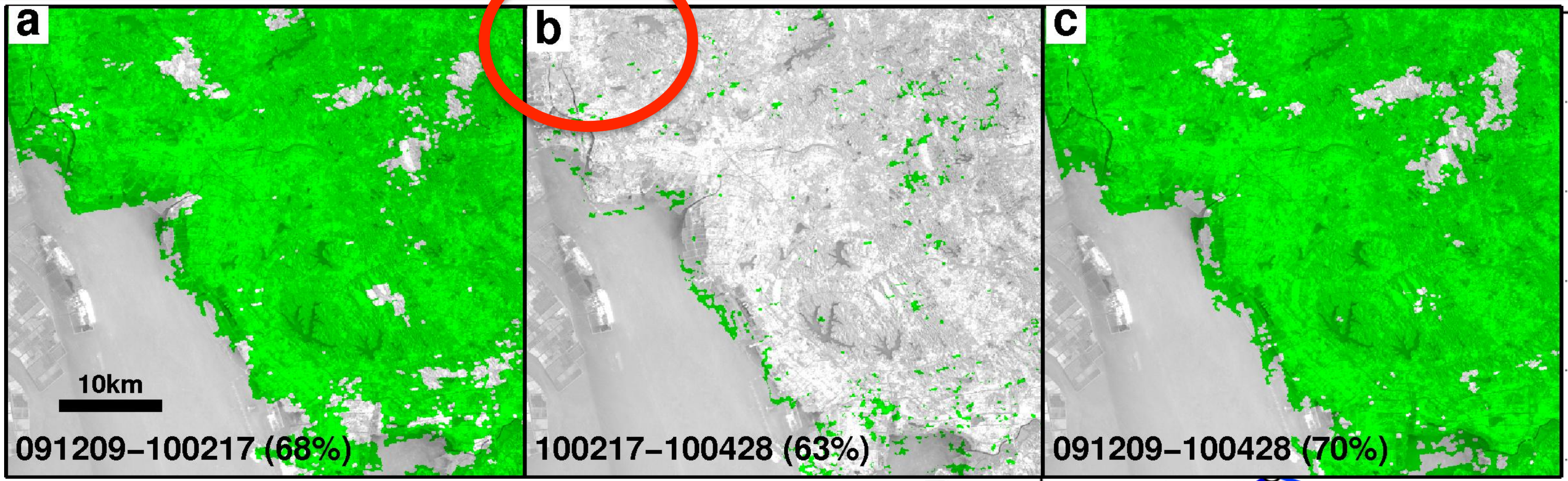
http://homepages.see.leeds.ac.uk/~earhw/software/pi-rate

π-RATE flowchart

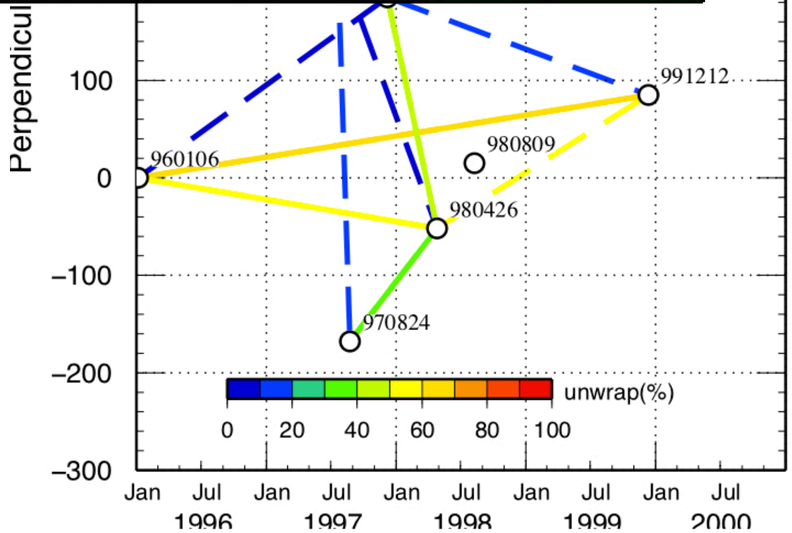


Biggs et al., 2007, GJI
Elliott et al., 2008, GRL
Wang et al., 2009, GRL
Wang et al., 2012, GJI

(1) Interferogram Selection

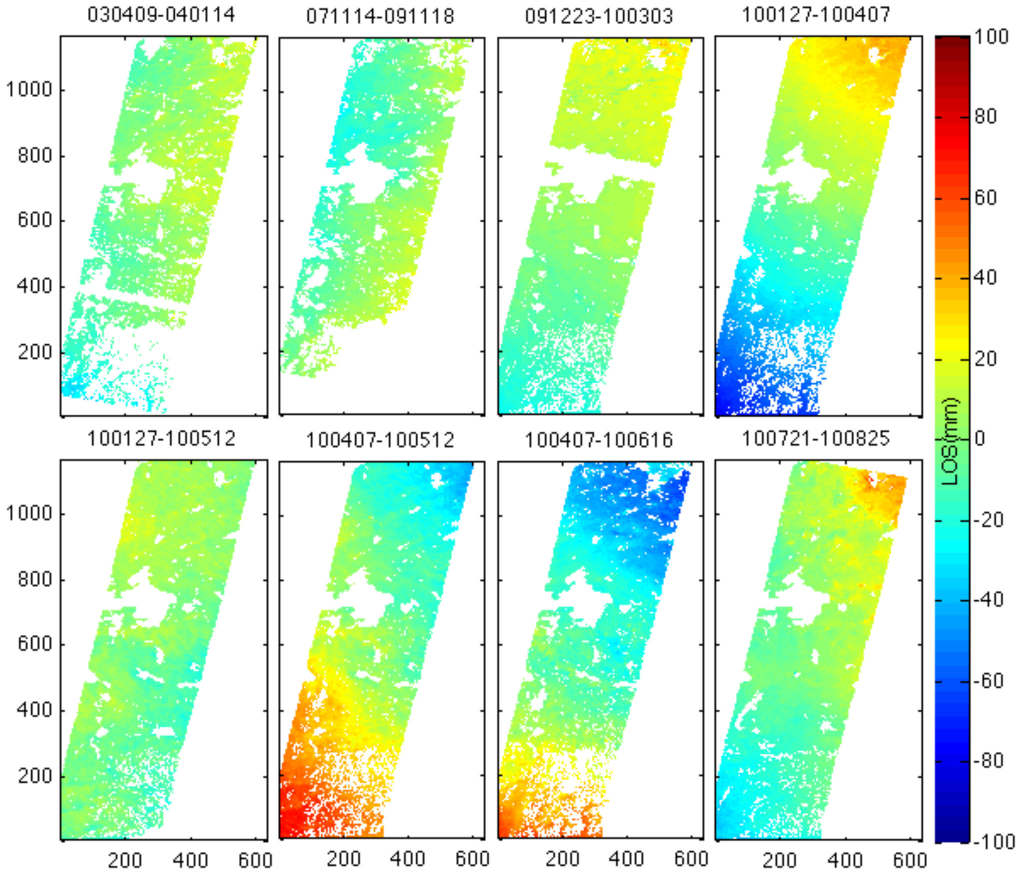


- MST (select a tree for each pixel from a network)
    - Independent observations
    - Different subset of interferograms for each pixel



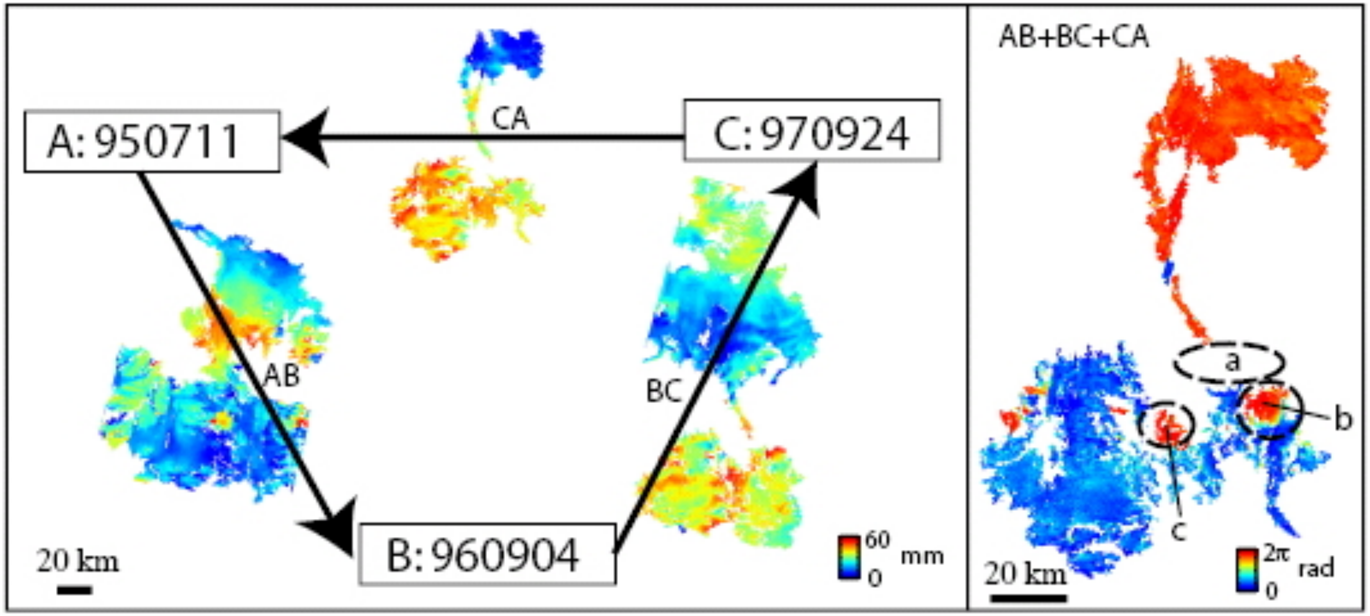
(2) Coregistration

- Coregistered to a single mater image
- Coregistered to a single DEM, and crop/fill to the same size



(3) Phase unwrapping errors

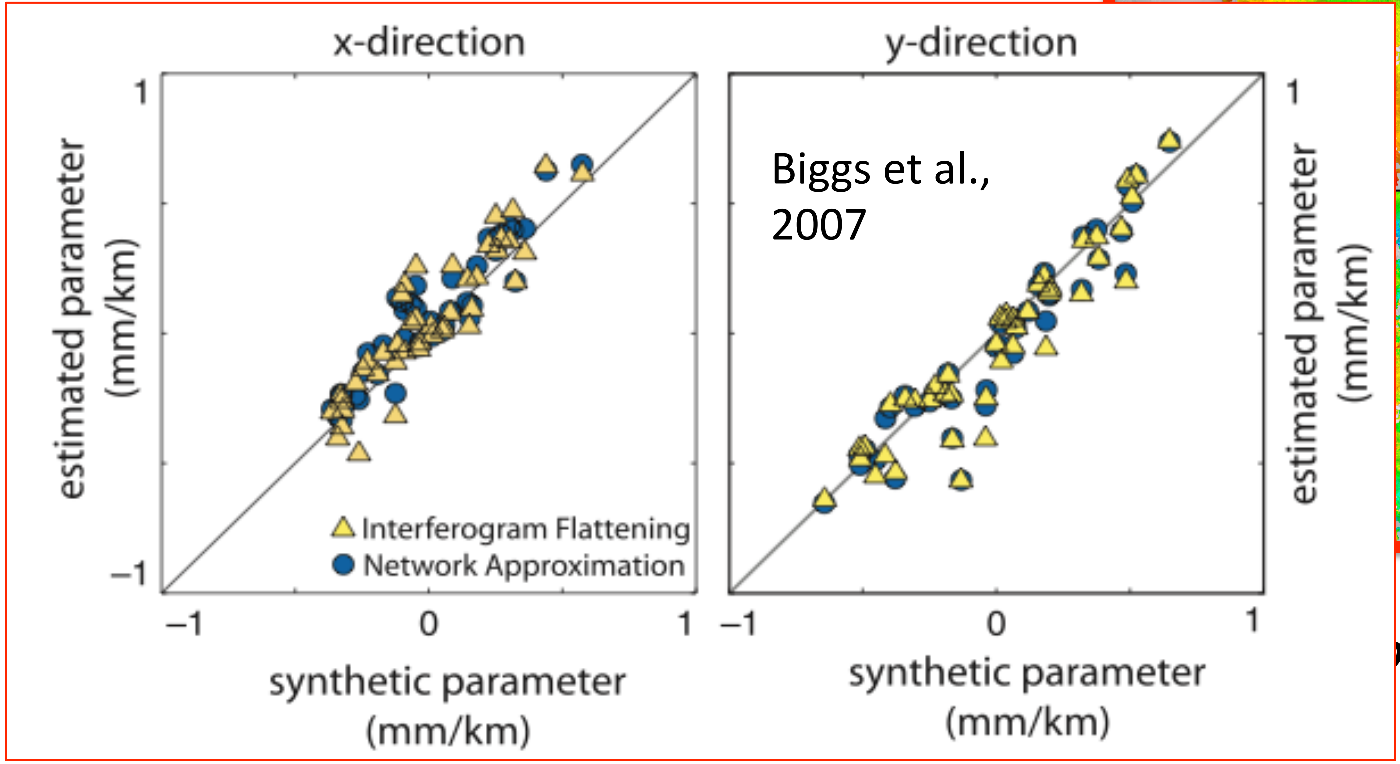
- In theory, the sum of phase in a closure is zero.
- Jump exists once phase unwrapping is wrong.
- Mask or correct phase unwrapping errors after detection



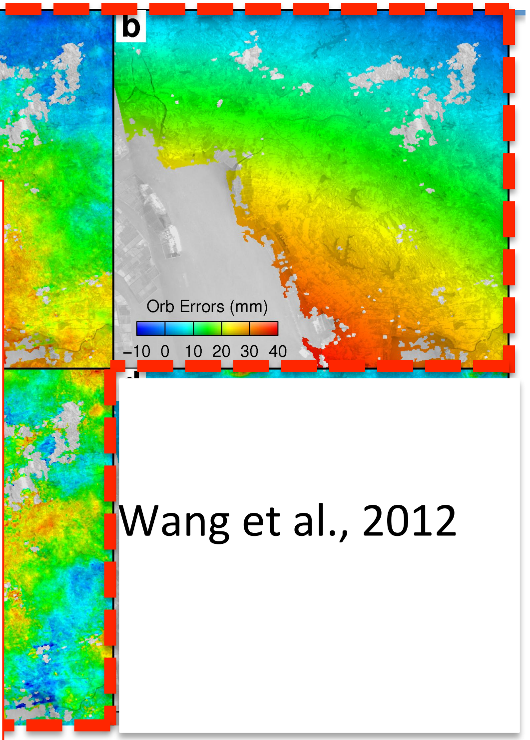
Biggs et al., 2007

(4) Orbital errors

- Polynomial fitting



Biggs et al., 2007

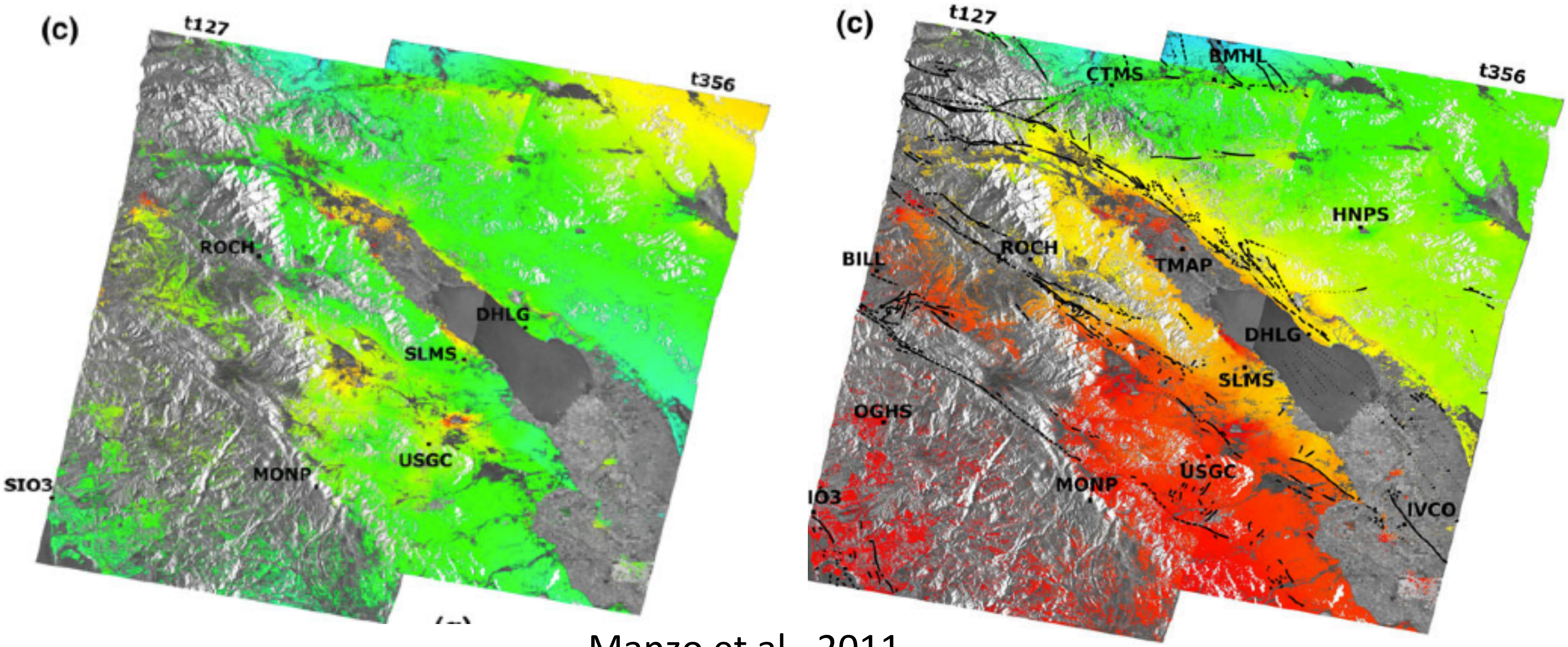


Wang et al., 2012

$$_j(y^p - y_0) + c_{i,j}$$

(4) Orbital errors

- Polynomial fitting
- GPS time series calibration



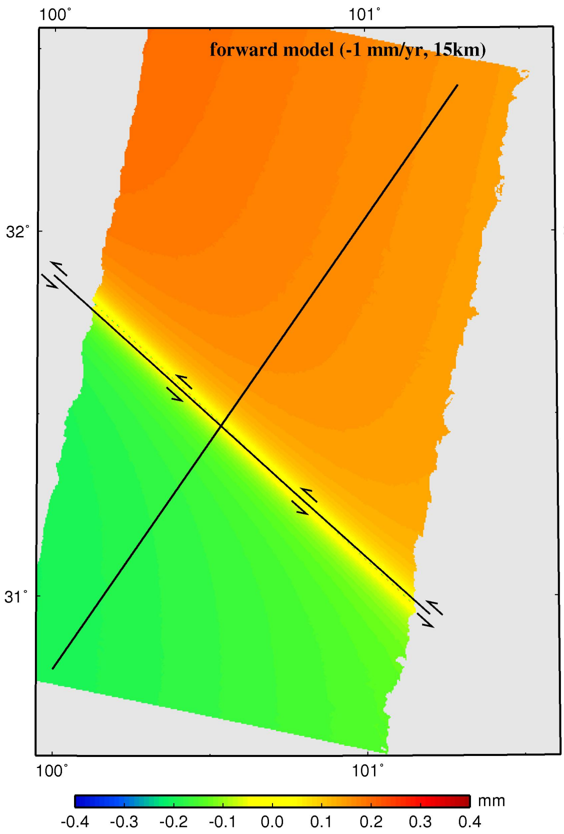
Manzo et al., 2011

(5) Initial models

- Why do we use initial model?

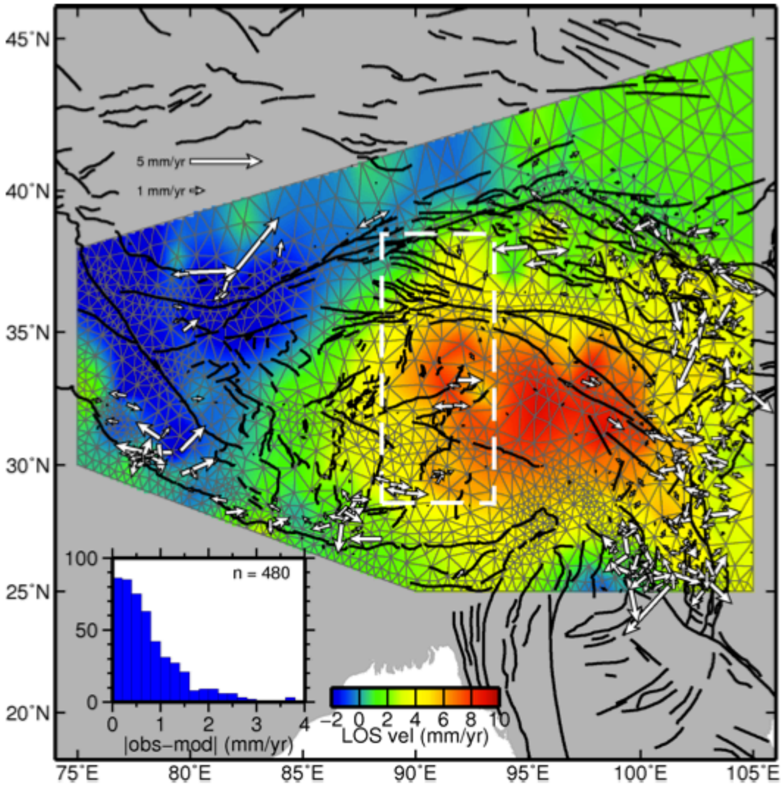
  spatial low frequency: deformation, atmosphere, orbit

Geophysical models



Wang, Wright, Biggs, 2009

Velocity field



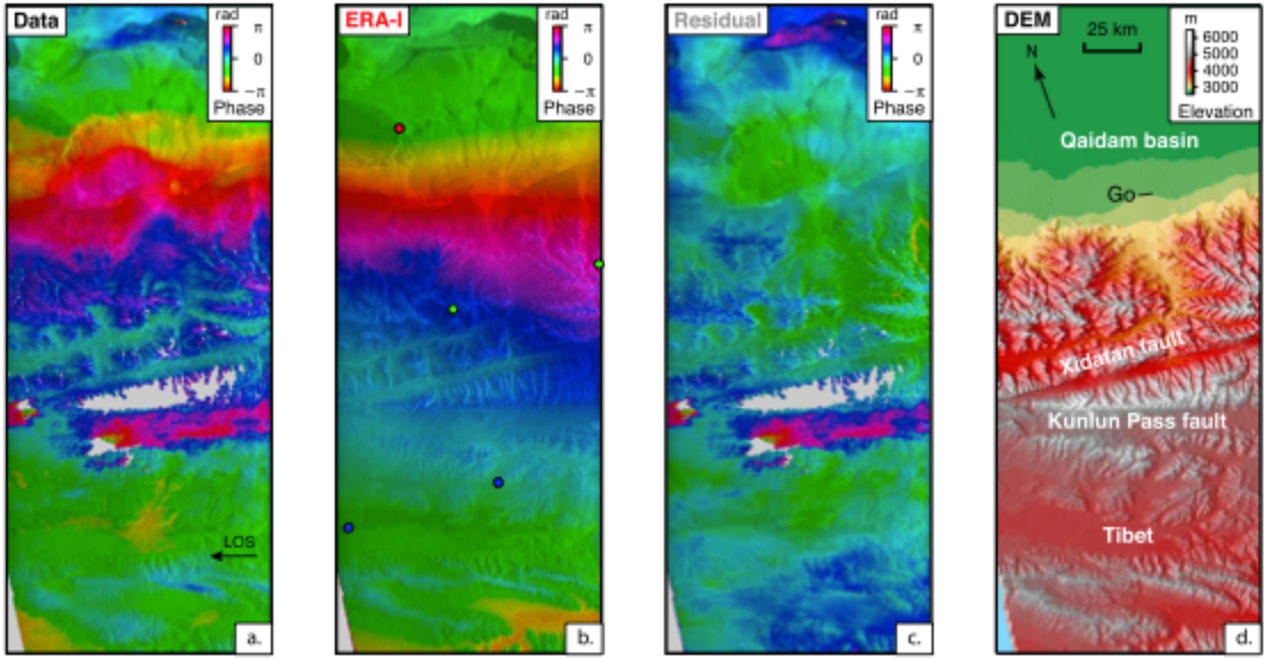
Garthwaite, Wang, Wright, 2013

(6) Atmospheric delay errors

- External calibration (GPS, MODIS, MERIS, Metrological data)

☐ Advantage: independent of InSAR

☐ Disadvantage: spatial and temporal resolution discrepancies, availability of GPS data



Jolivet et al., 2011

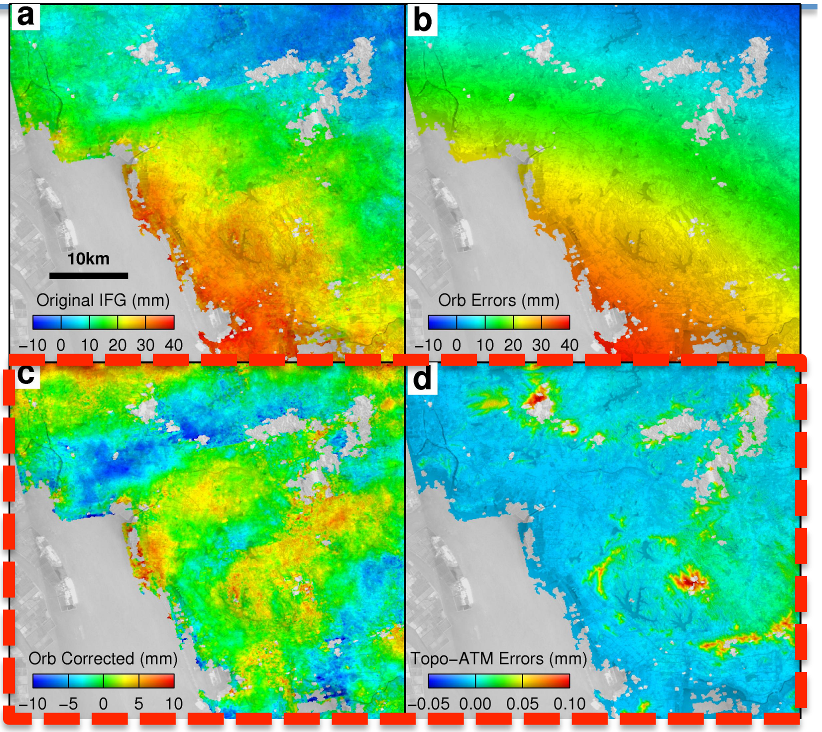
(6) Atmospheric delay errors

- External calibration
- Empirical Estimation
  - Topo-correlated (stratified)
    - Interferogram by interferogram

$$\Delta\phi_{i,j}^{p} = a_{i,j} \cdot (H^{p} - H_{0}) + b_{i,j}$$

    - Network approach (Elliott et al., 2008)

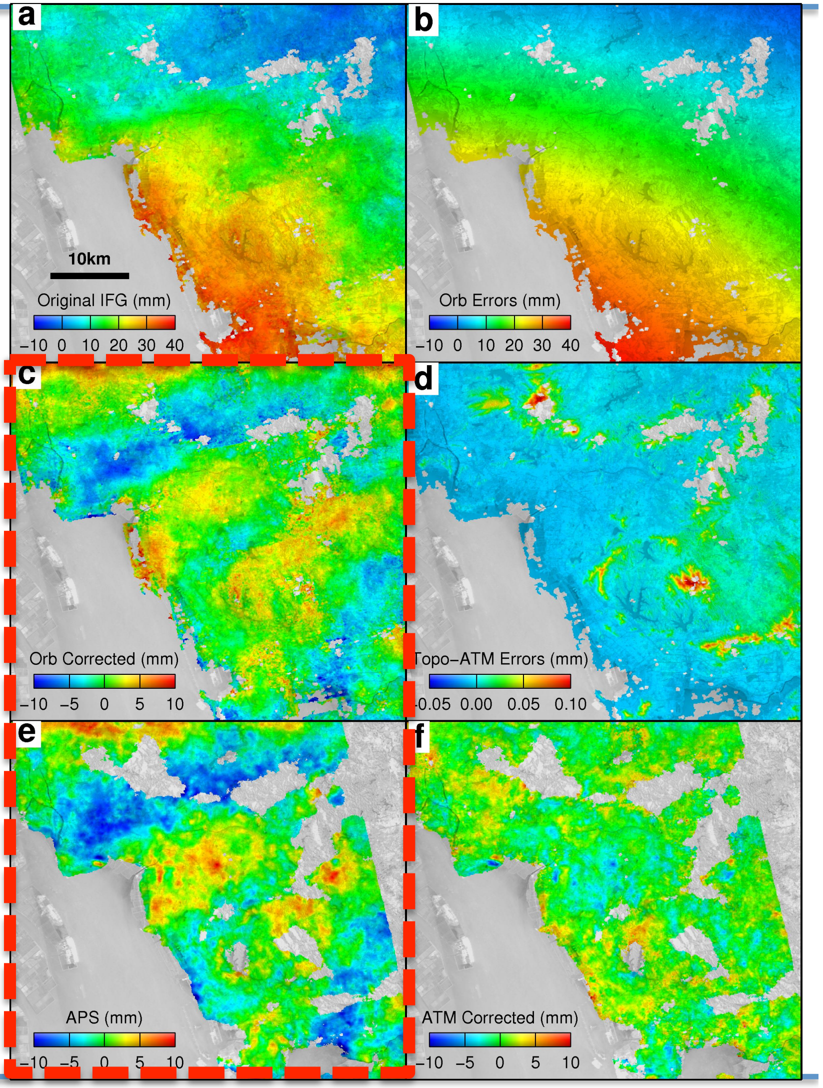
$$\Delta\phi_{i,j}^{p} = -a_{i} \cdot (H^{p} - H_{0}) + a_{j} \cdot (H^{p} - H_{0}) + b_{i,j}$$



Wang et al., 2012

(6) Atmospheric delay errors

- External calibration
- Empirical Estimation
  - Topo-correlated (stratified)
  - APS estimation (turbulent)
    - Raw time series inversion
    - Sudden deformation removal
    - Temporal low-pass filter
    - Spatial high-pass filter



Wang et al., 2012

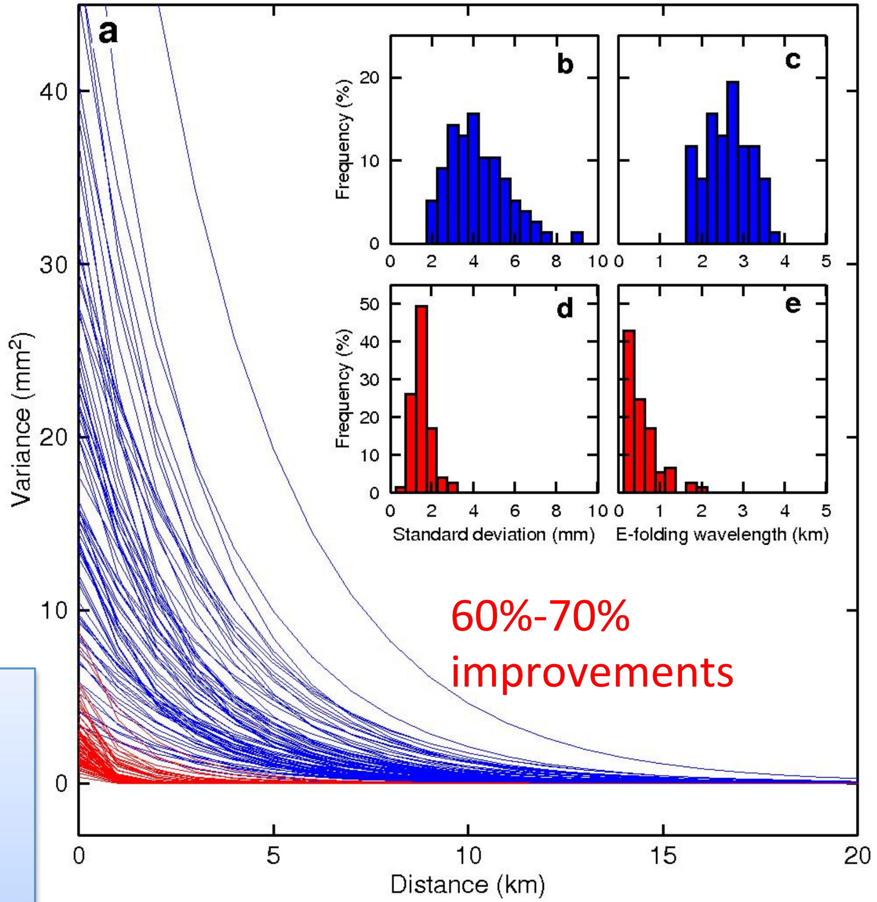
(6) Atmospheric delay errors

- External calibration
- Empirical Estimation
  - Topo-correlated (stratified)
  - APS estimation (turbulent)
    - Raw time series inversion
    - Sudden deformation removal
    - Temporal low-pass filter
    - Spatial high-pass filter

☐ Advantage: only depends on InSAR data

☐ Disadvantages: (1) non-linear relationship exists between topography and delay; (2) how to determine smoothing windows for APS estimation

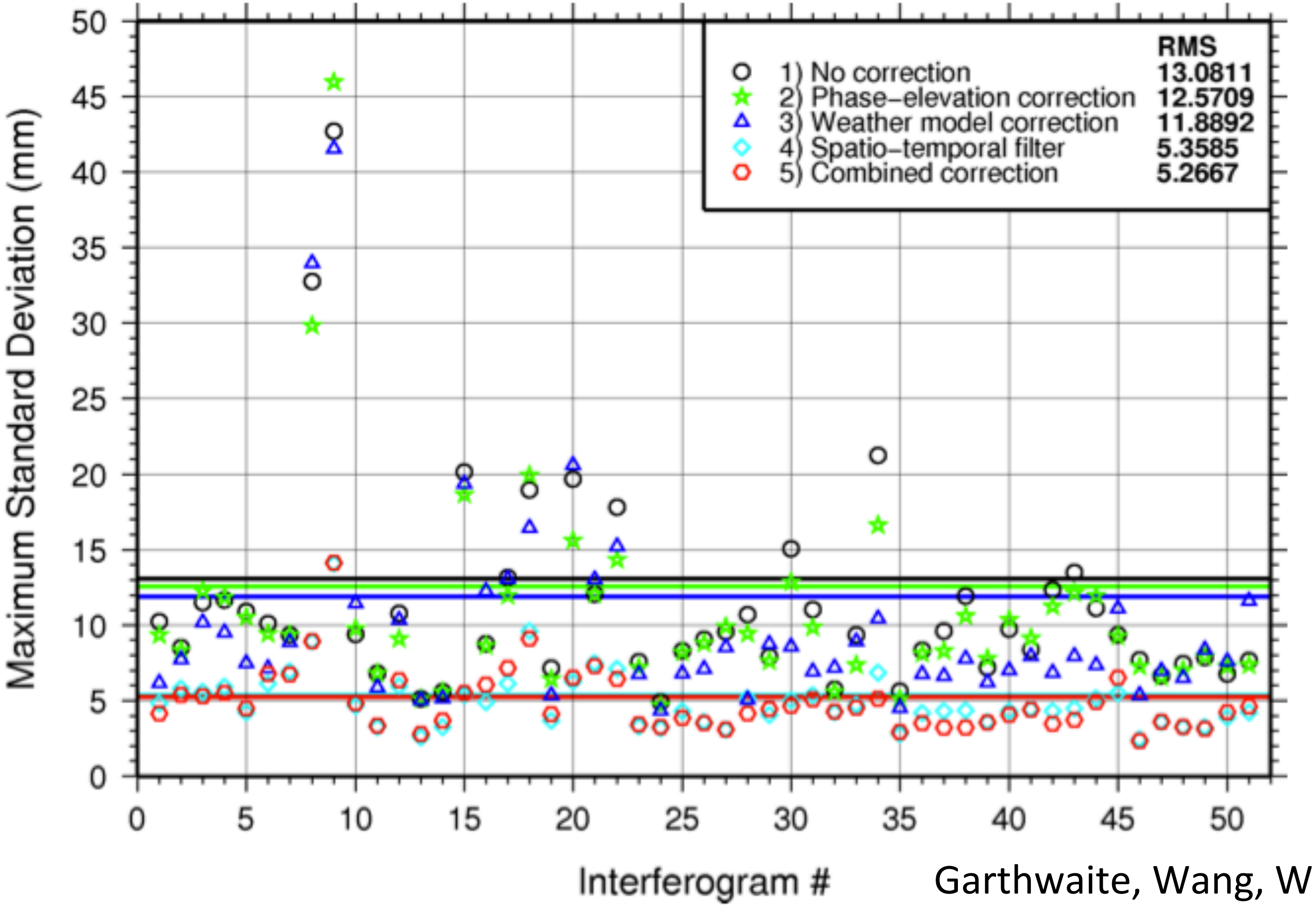
$$c_{jk} = \sigma^2 e^{-d_{jk}/\alpha}$$



60%-70% improvements

Wang et al., 2012

(6) Atmospheric delay errors

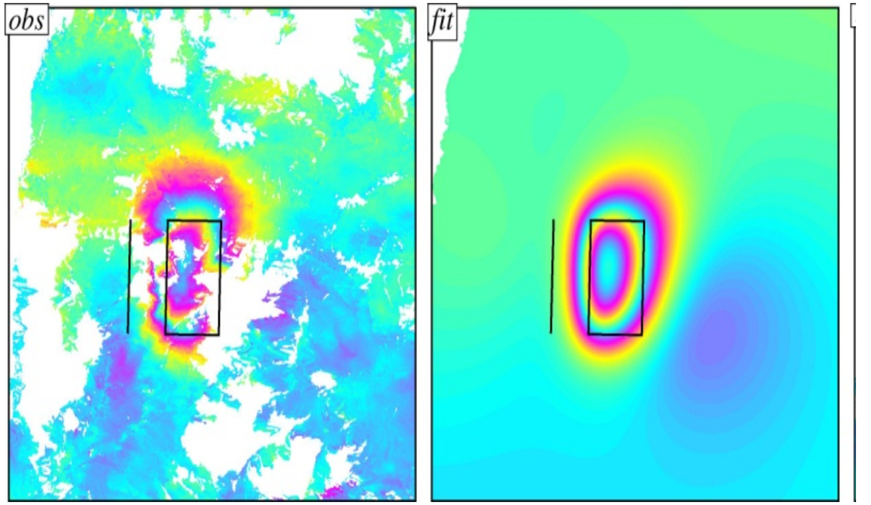


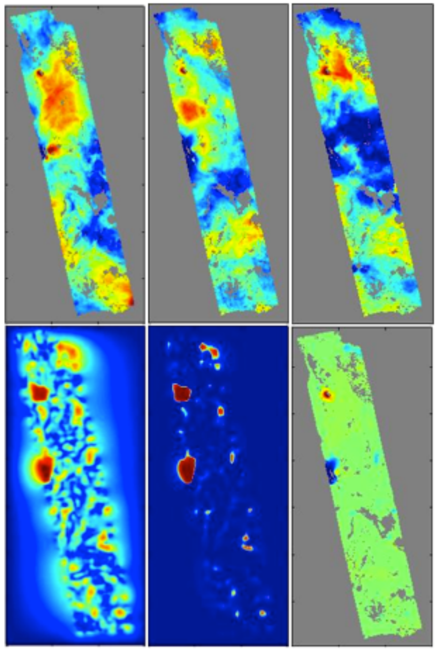
Garthwaite, Wang, Wright, 2013

(6) Atmospheric delay errors

- External calibration
- Empirical Estimation
  - Topo-correlated (stratified)
  - APS estimation (turbulent)
    - Raw time series inversion
    - Sudden deformation removal
    - Temporal low-pass filter
    - Spatial high-pass filter

Models

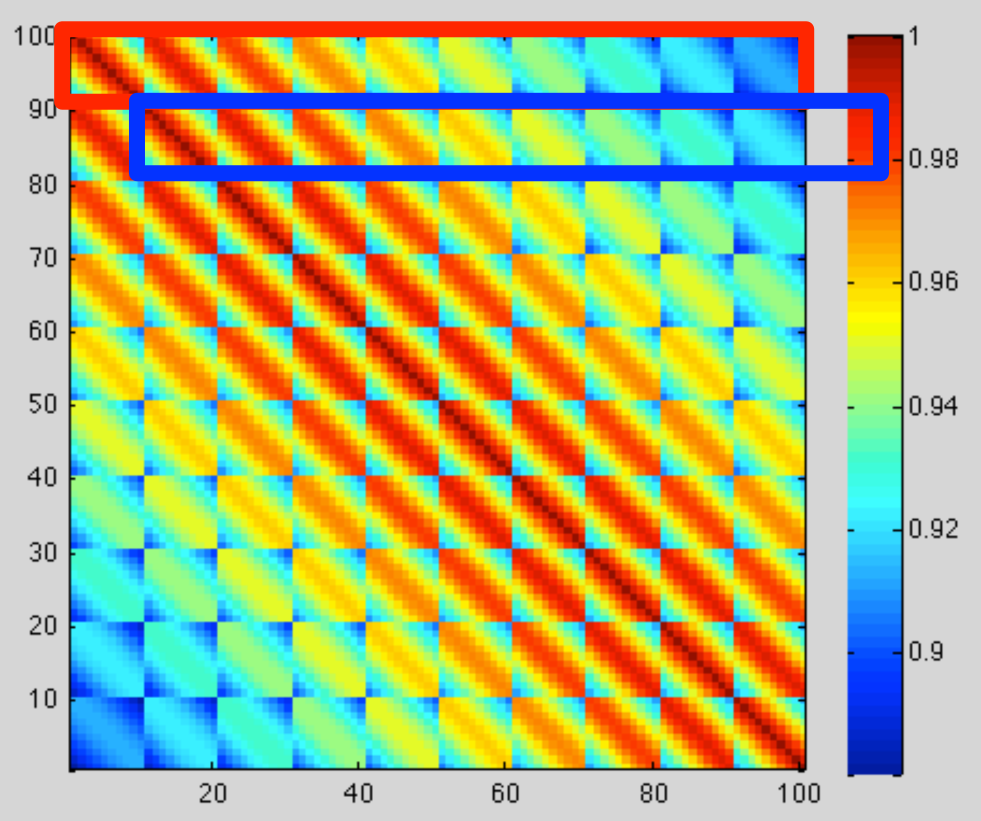




(7) VCM estimation

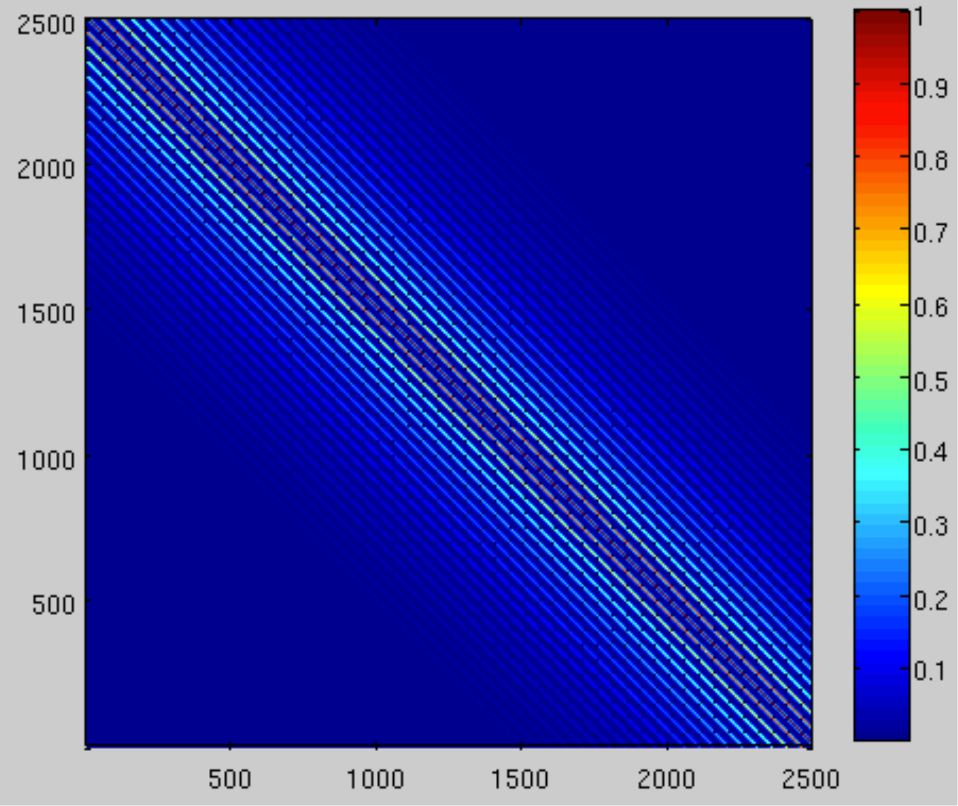
- VCM in space (to refine the initial model)

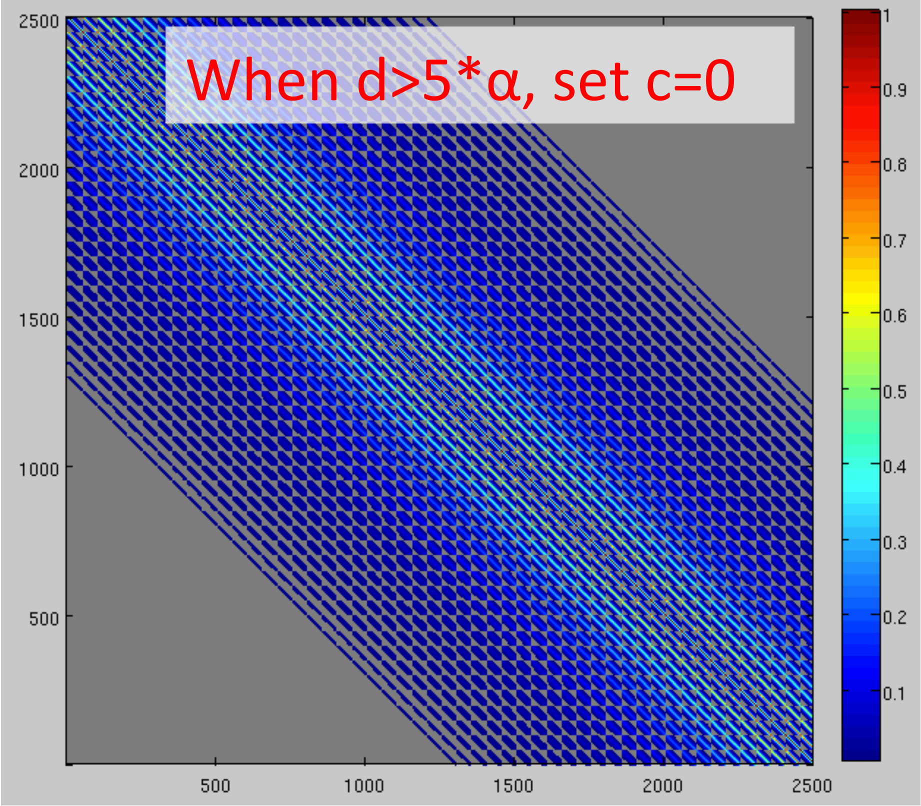
$$c_{jk} = \sigma^2 e^{-d_{jk}/\alpha}$$



(7) VCM estimation

- VCM in space (to refine the initial model)



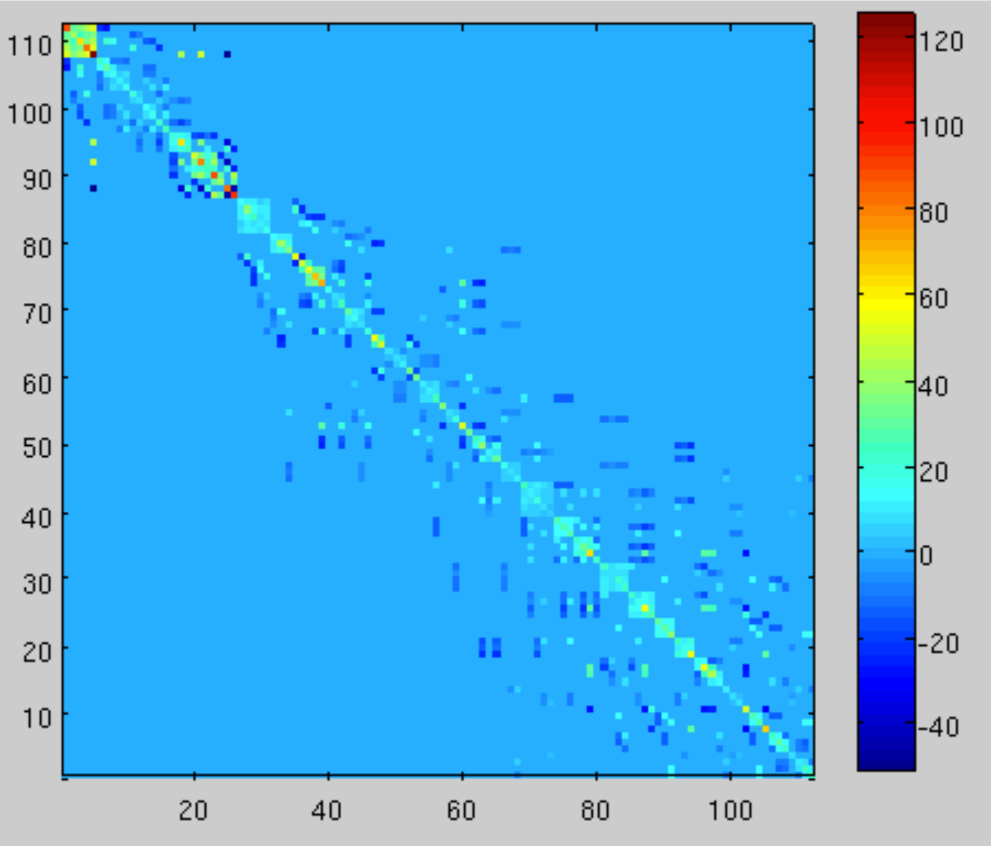

When d>5*α, set c=0

(7) VCM estimation

- VCM in time (for time series and rate map inversion)

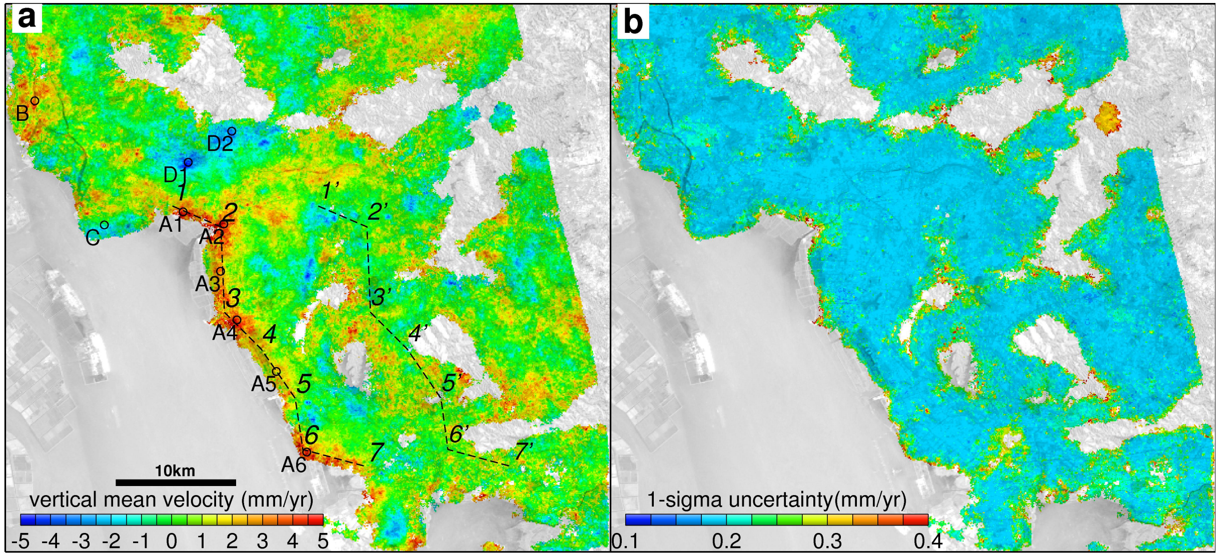
$$C_{lm,nq} = \begin{cases} 1 & (l = n, m = q) \\ -0.5 & (l = q \text{ or } m = n) \\ 0.5 & (l = n \text{ or } m = q) \\ 0 & (\text{otherwise}) \end{cases}$$

Biggs et al., 2007



(8) Final products estimation

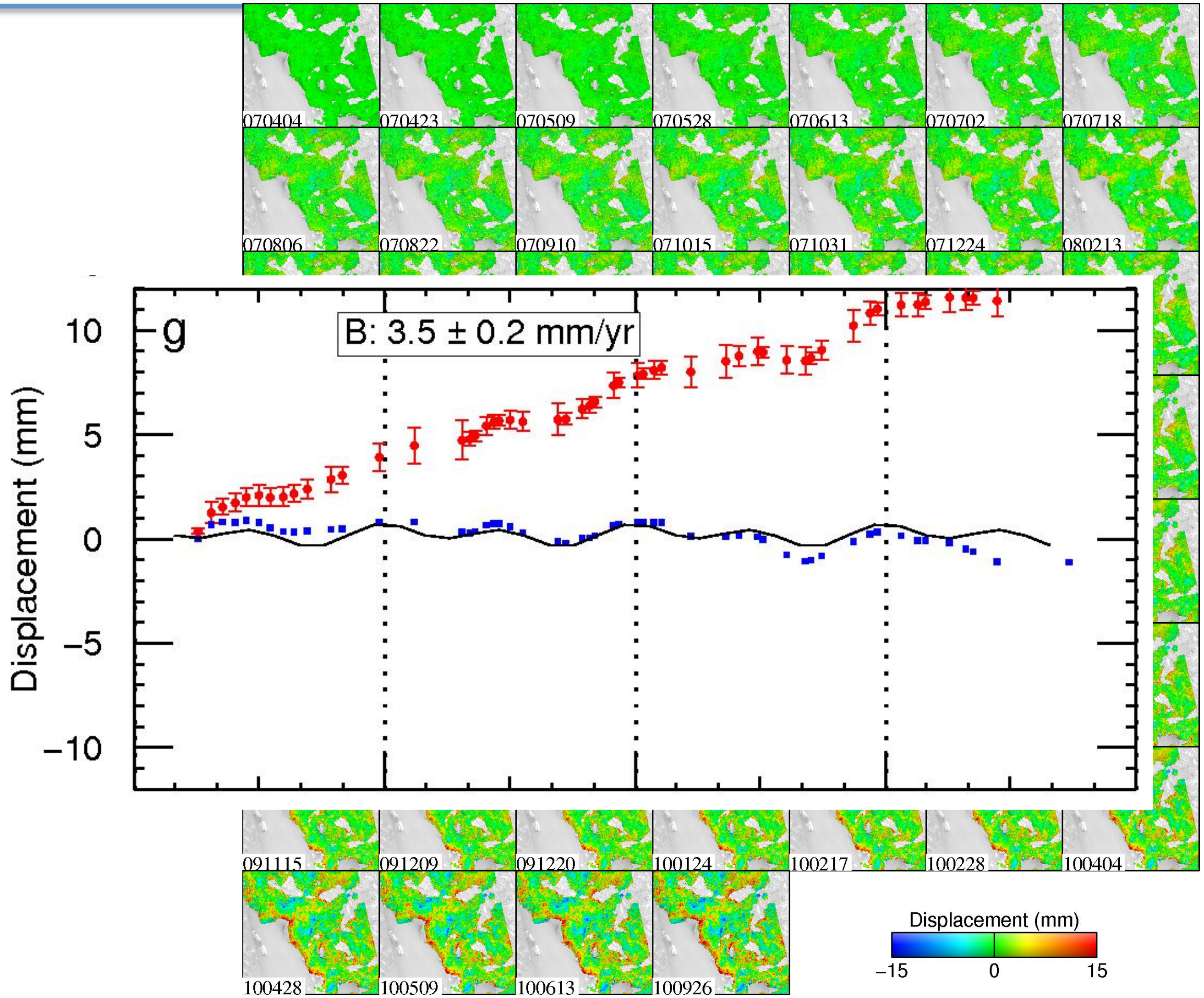
- Rate map
- Error map
- DEM errors
- Time series



Wang et al., 2012

(8) Final products estimation

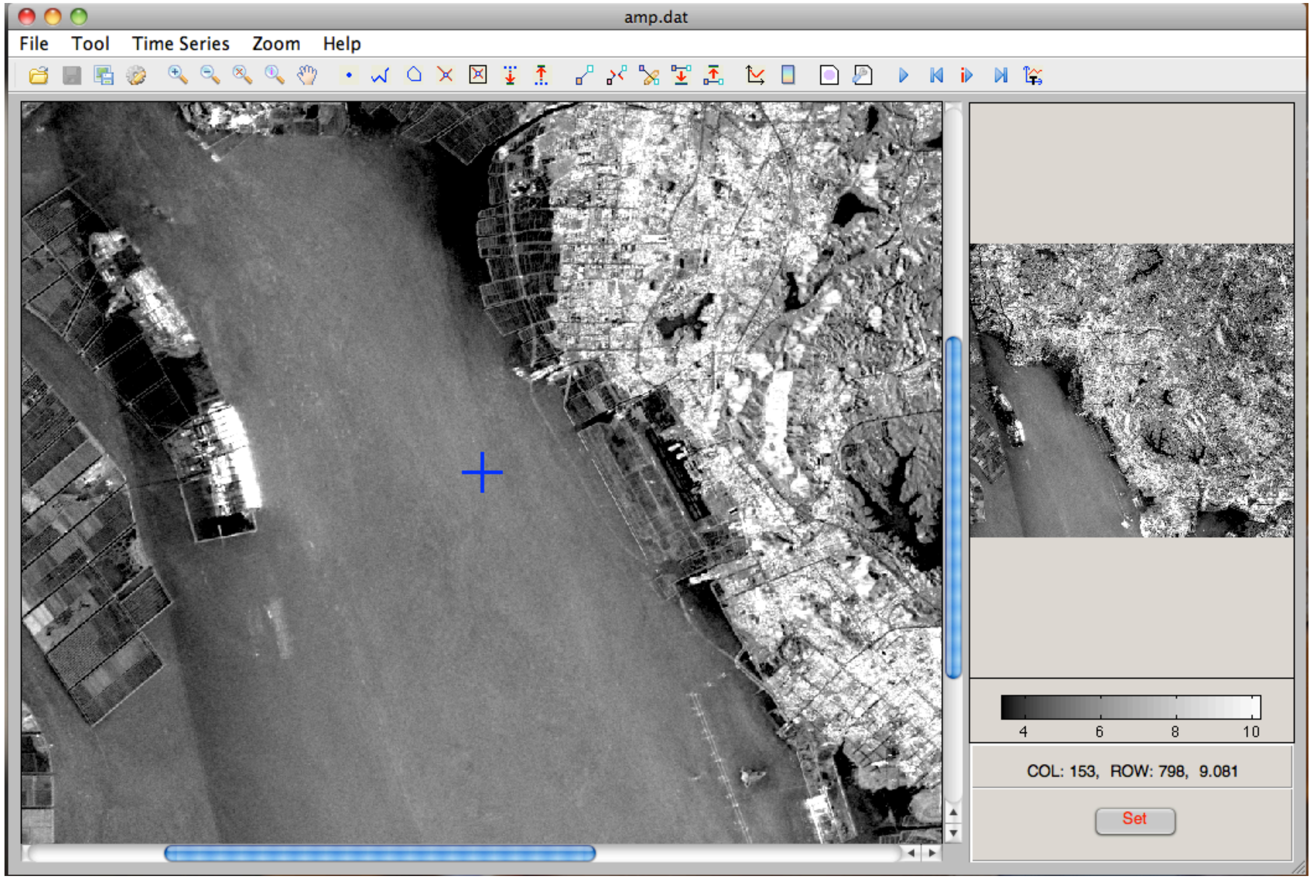
- Rate map
- Error map
- DEM errors
- Time series



Wang et al., 2012

(9) By-products

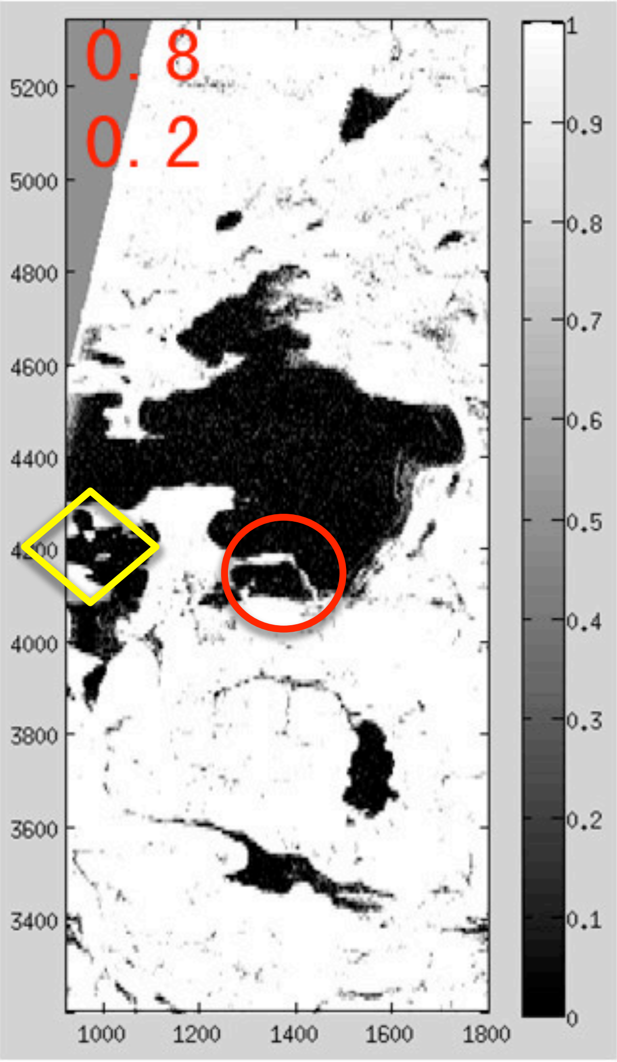
- Amplitude
- Coherence
- …

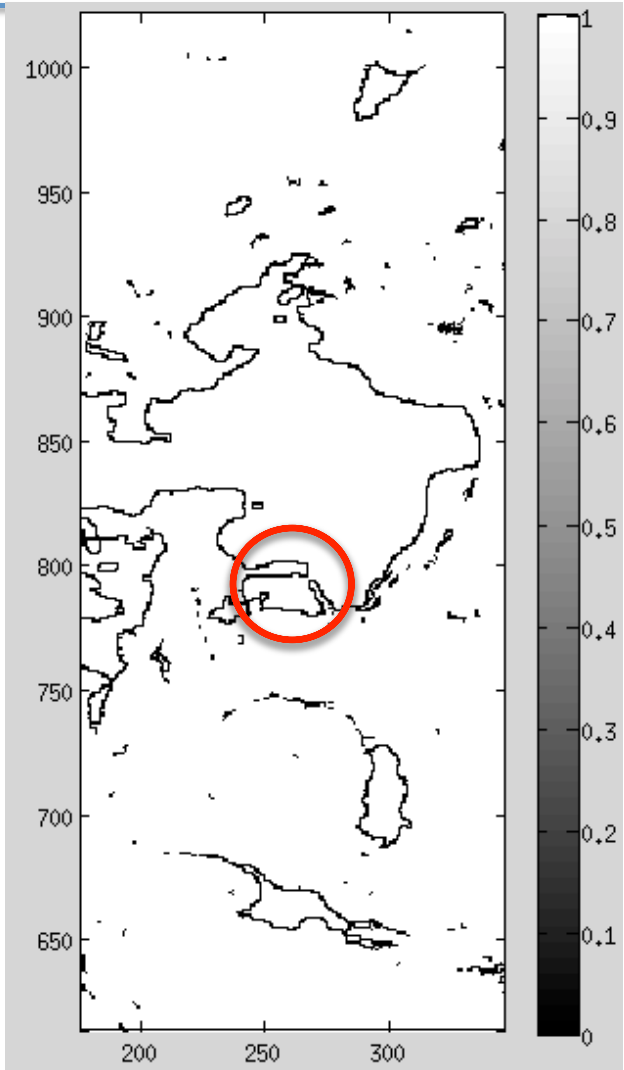


COL: 153, ROW: 798, 9.081

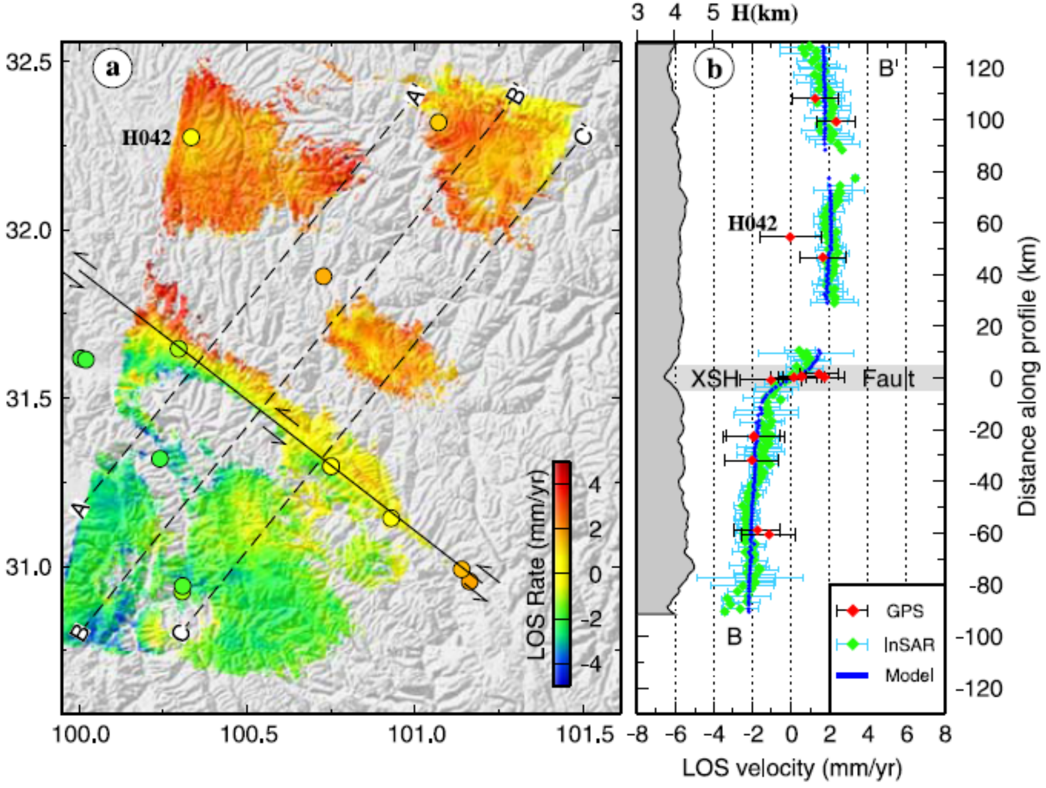
(9) By-products

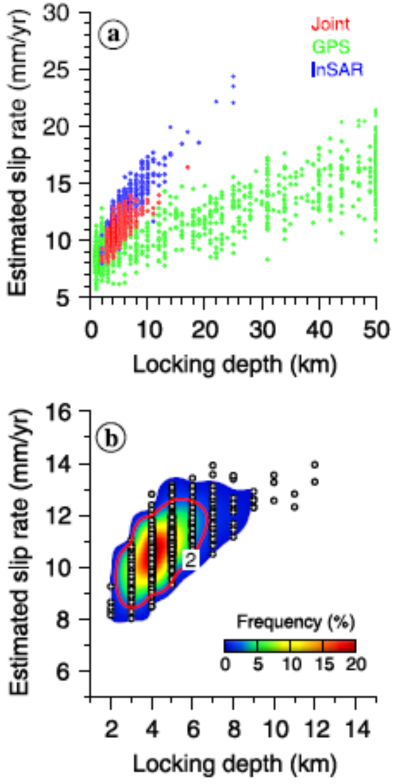
- Amplitude
- Coherence
- …





Examples: Eastern Tibet (XSH)

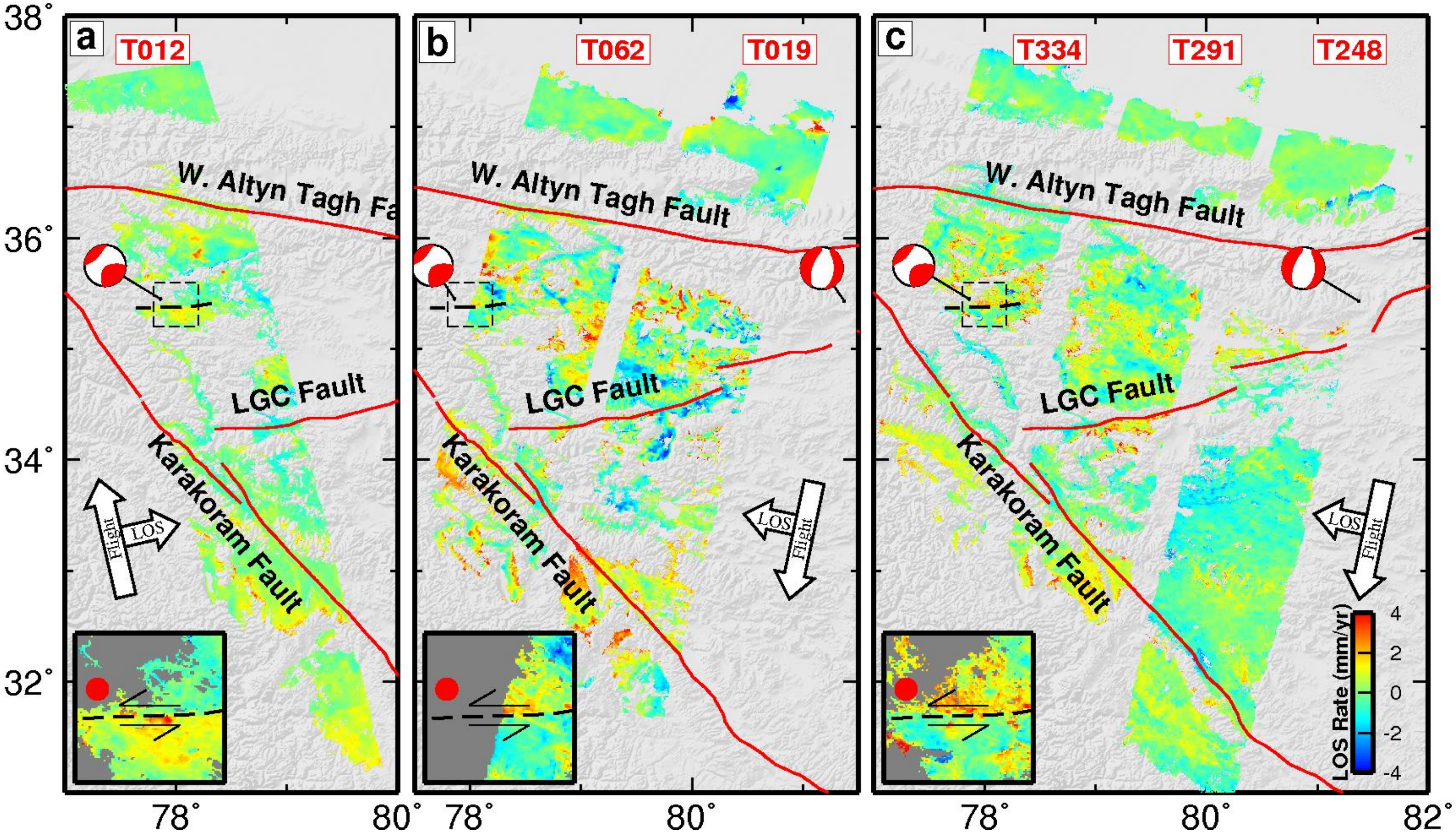




- Consistent interseismic deformation measured by InSAR and GPS
- Improvement on the constraint of locking depth using InSAR and GPS
- Slip rate: 9-12 mm/yr; locking depth: 3-6 km.

Wang, Wright, Biggs, 2009, GRL

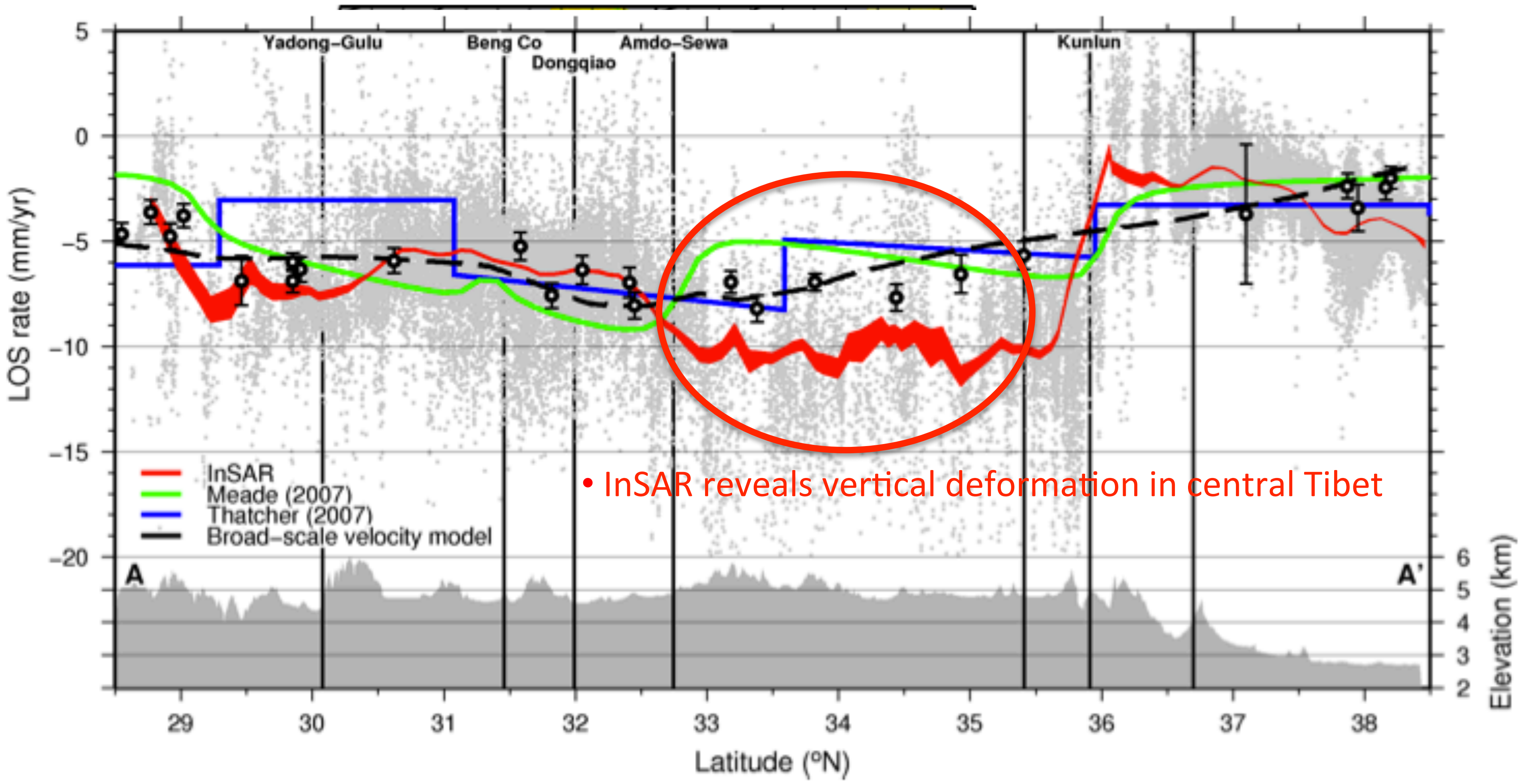
Examples: Western Tibet



- InSAR reveals internal deformation in western Tibet

Wang and Wright, 2012, GRL

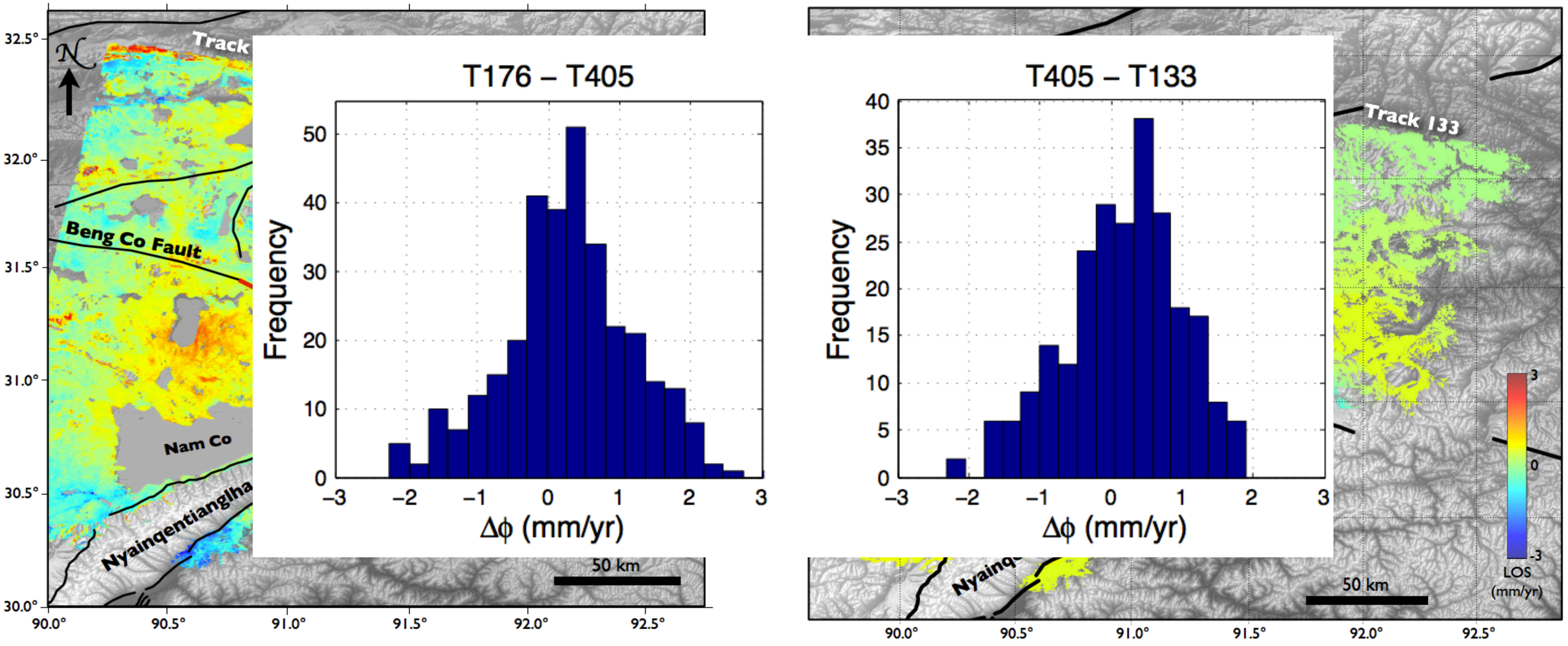
Examples: Central Tibet



- InSAR reveals vertical deformation in central Tibet

Garthwaite, Wang and Wright, 2013, JGR

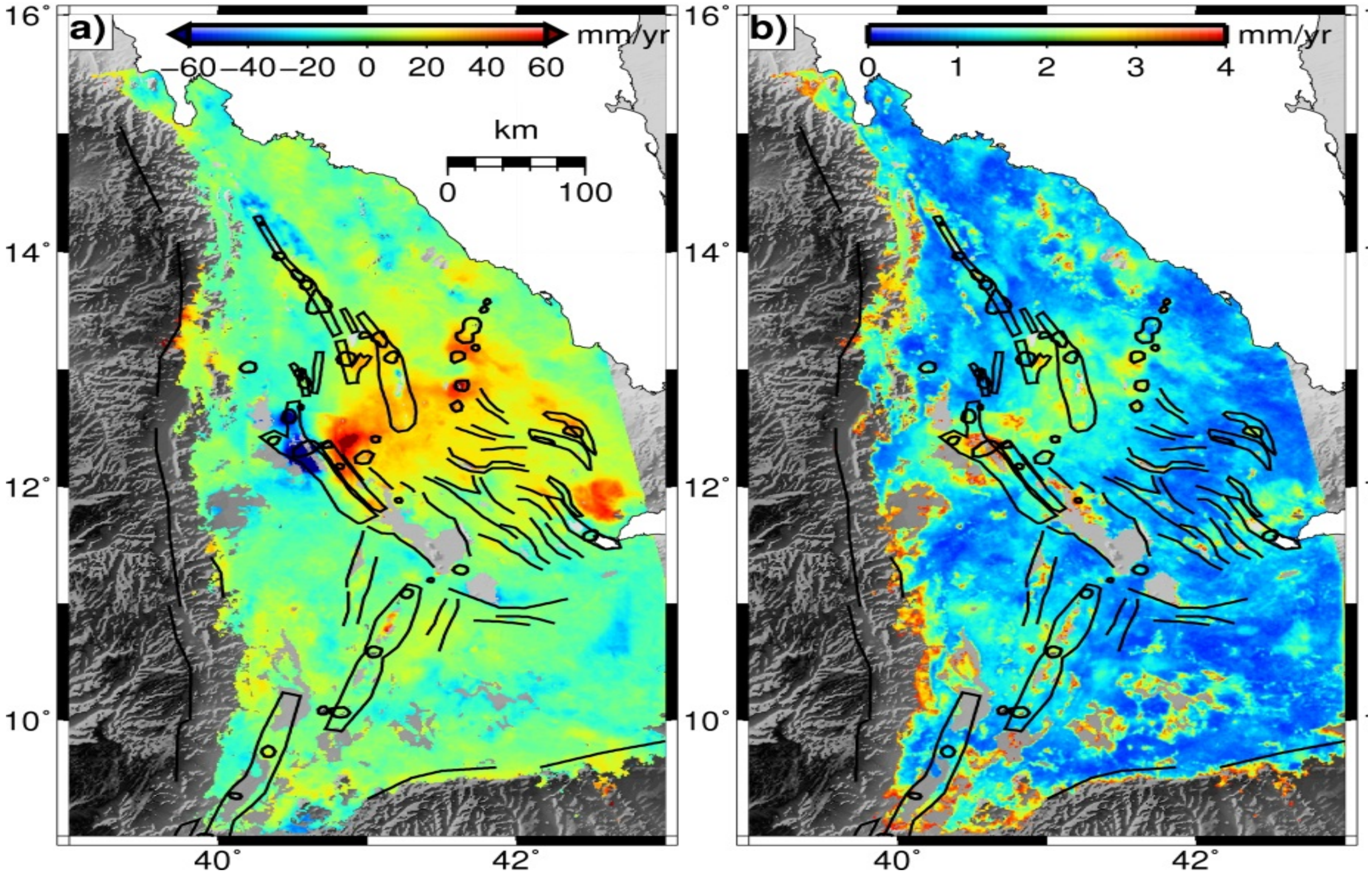
Examples: Beng Co and Yadong-Gulu Rift



- Postseismic deformation after 60 years
- Viscoelastic stress relaxation in the lower crust (viscosity = 3e19)

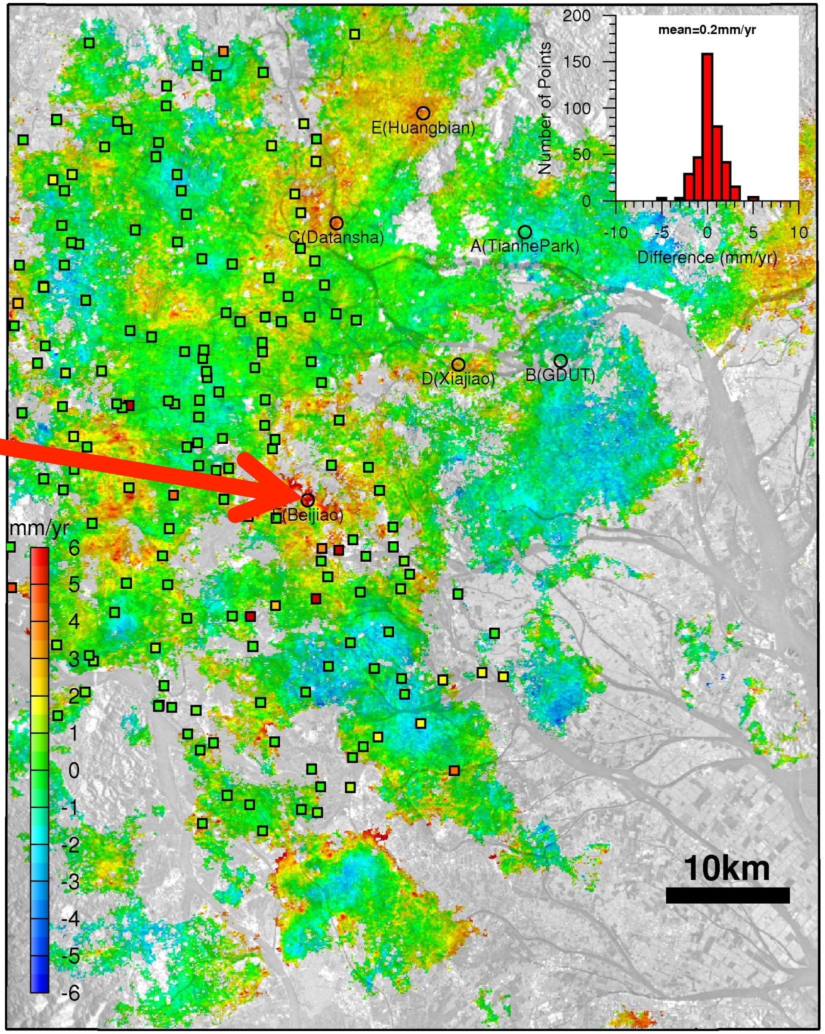
Ryder et al., in prep

Examples: Afar-wide swath rate map



Pagli et al., in prep

Examples: PRD subsidence

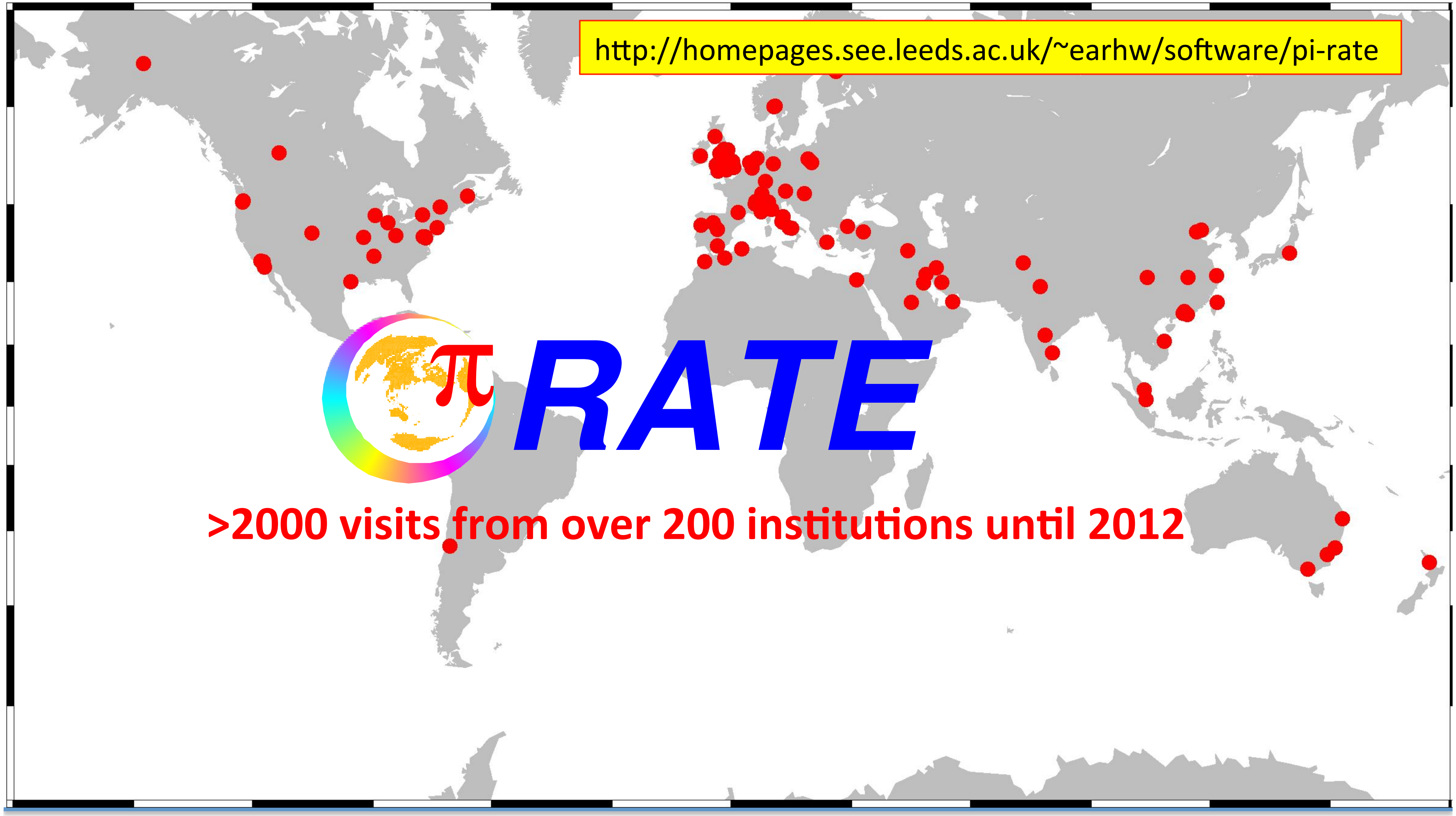


InSAR - Leveling: 0.2 mm/yr

Conclusions and future work

- SBAS method has been widely used for measuring deformation.
- No generic method can reliably correct all atmospheric delay errors.
- Phase unwrapping is challenging and time consuming before SBAS.
- Full-resolution phase unwrapping is required to improve spatial resolution of SBAS products.
- It's important to distinguish different components in InSAR time series, e.g., stable, transient, periodic, sudden offset etc.
- New satellites with shorter revisit time can increase coherence, thus can hopefully eliminate the prejudice between SBAS and PSInSAR.

Thank You!



http://homepages.see.leeds.ac.uk/~earhw/software/pi-rate

>2000 visits from over 200 institutions until 2012