

1. Methodology / 2. Processing / 3. Examples

***Small Baseline Time Series
Method in InSAR***

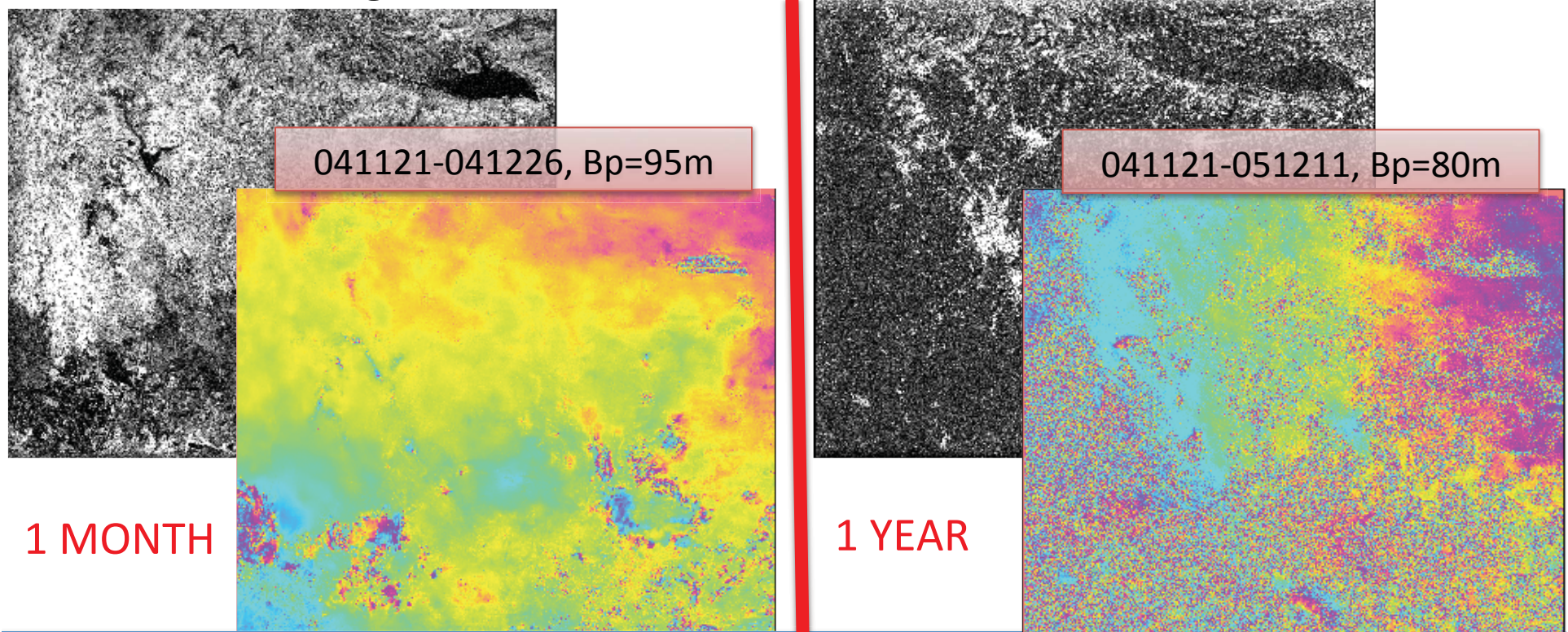
Hua Wang

Guangdong University of Technology



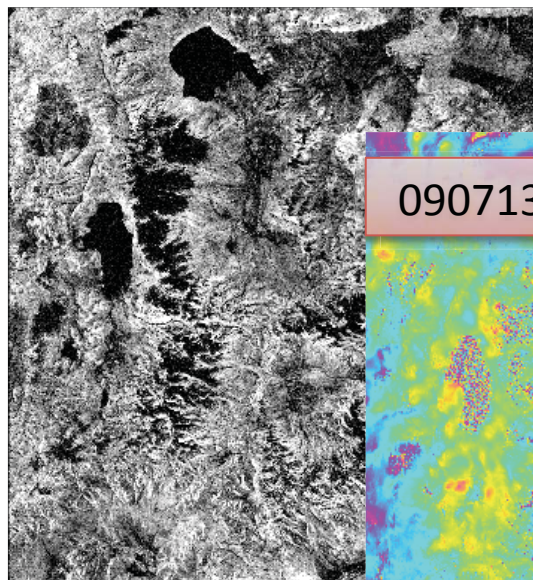
SBAS - Motivations

- **Coherence** is a key factor for InSAR
- Coherence mainly depends on temporal and perpendicular baselines given the same wavelength

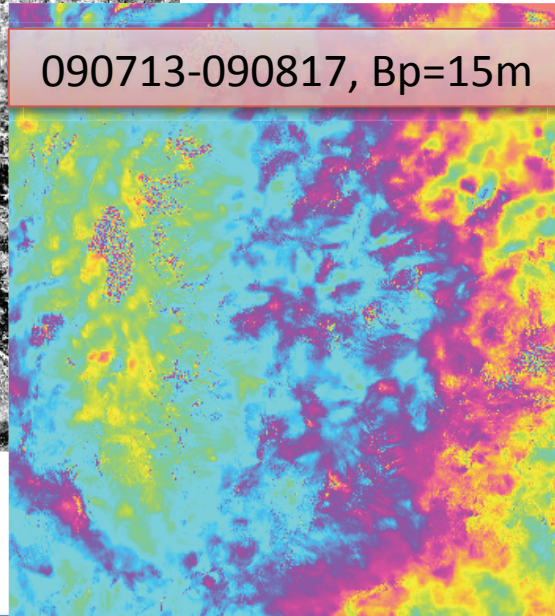


SBAS - Motivations

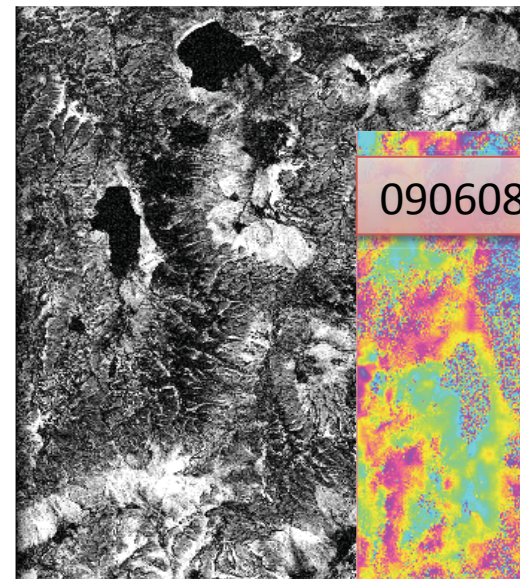
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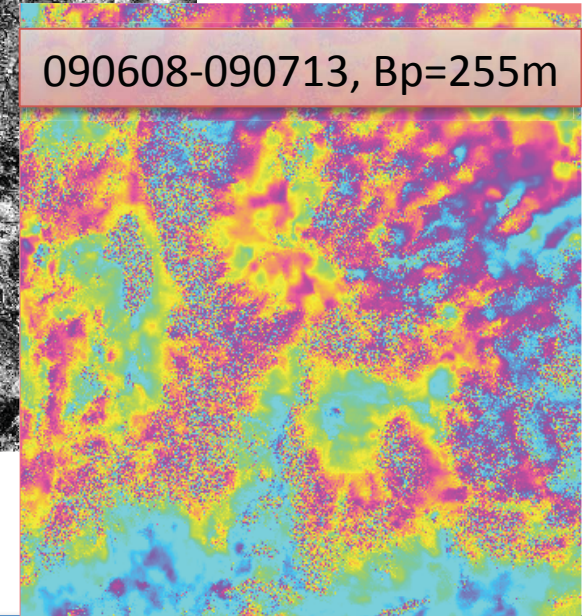
090713-090817, $B_p=15\text{m}$



15m



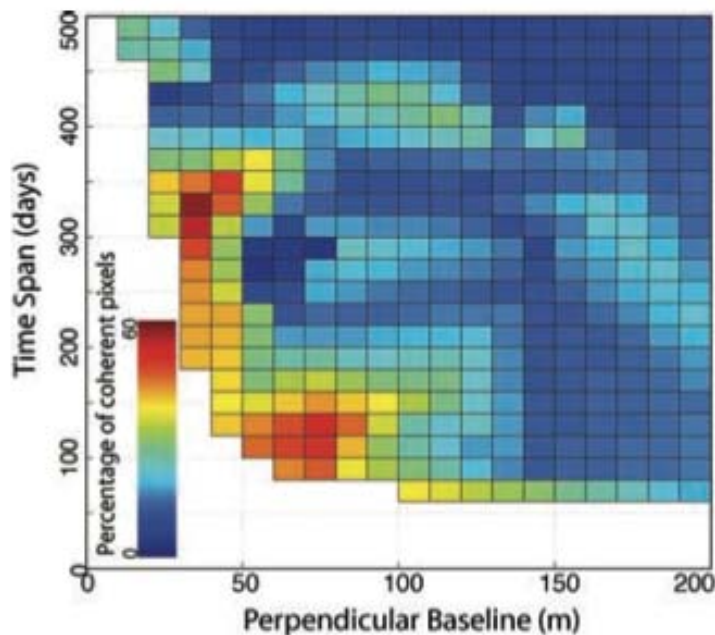
090608-090713, $B_p=255\text{m}$



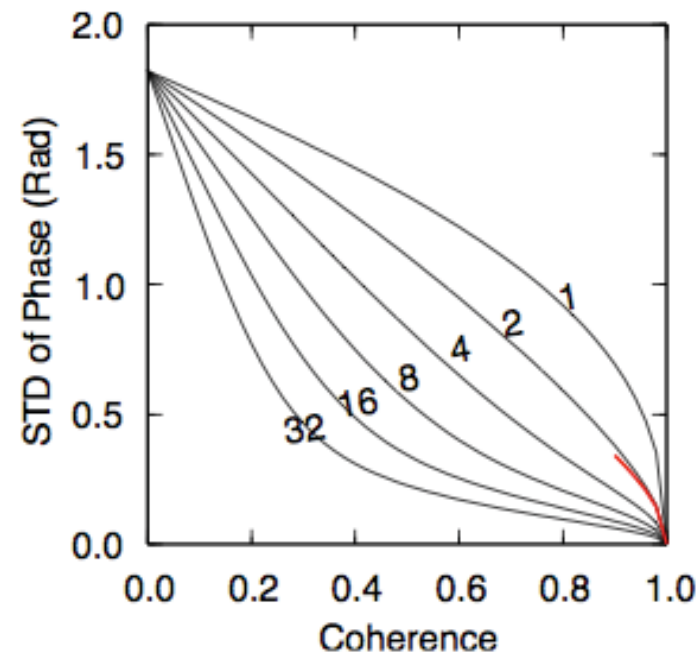
255m

SBAS - Motivations

- **Coherence** is a key factor for InSAR
- Coherence mainly depends on temporal and perpendicular baselines given the same wavelength
- **Smaller baseline gives higher coherence and more accurate phase**



Biggs et al., 2007



$$\sigma_{\phi}^2 = \frac{1 - \gamma^2}{2N\gamma^2}$$

Rodriguez
and Martin
(1992)



SBAS - Essentials

- Forming differential interferograms using small temporal and spatial baseline subsets
- Taking use of all coherent pixels (temporal vs persistent)
- Mitigating artifacts (e.g. atmosphere, orbit, DEM errors etc) by time series analysis (similar to PSInSAR)
- Usually starting from, but are not limited to, unwrapped interferograms



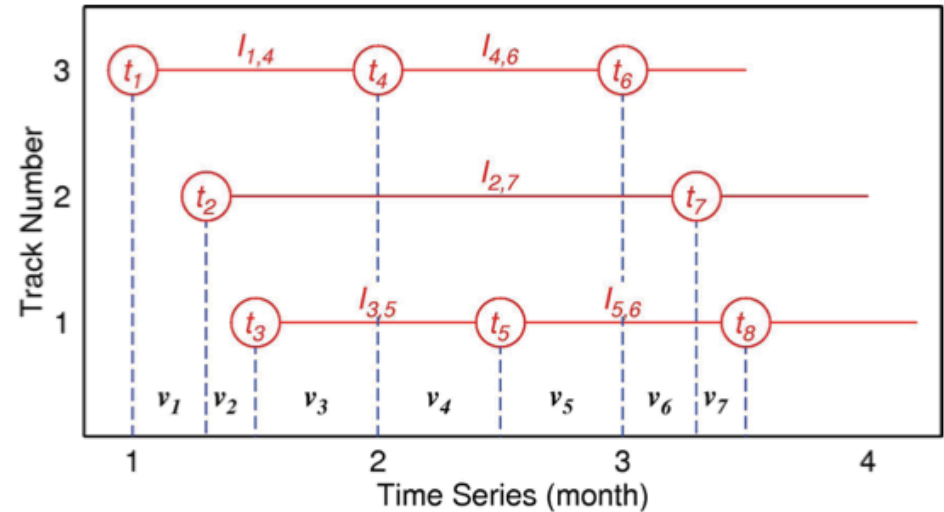
SBAS - Methodology

- System of equations

$$\mathbf{G}\mathbf{m} = \mathbf{d},$$

$$\mathbf{G}_{i,j} = \left[\underbrace{\mathbf{0}}_{i-1} \underbrace{\Delta t_i \cdots \Delta t_{j-1}}_{j-i} \underbrace{\mathbf{0}}_{n-j} \right],$$

$$\mathbf{m} = [v_1 \quad v_2 \quad \cdots \quad v_{n-1}]^T,$$



where $\Delta t_i = t_{i+1} - t_i$, t is the acquisition date, n is the total number of the acquisitions, $\mathbf{0}$ is a zero vector indicating acquisitions which are not covered by the interferogram $I_{i,j}$, v_i is the velocity of the i th time-span. Here, the acquisitions must be chronologically ordered

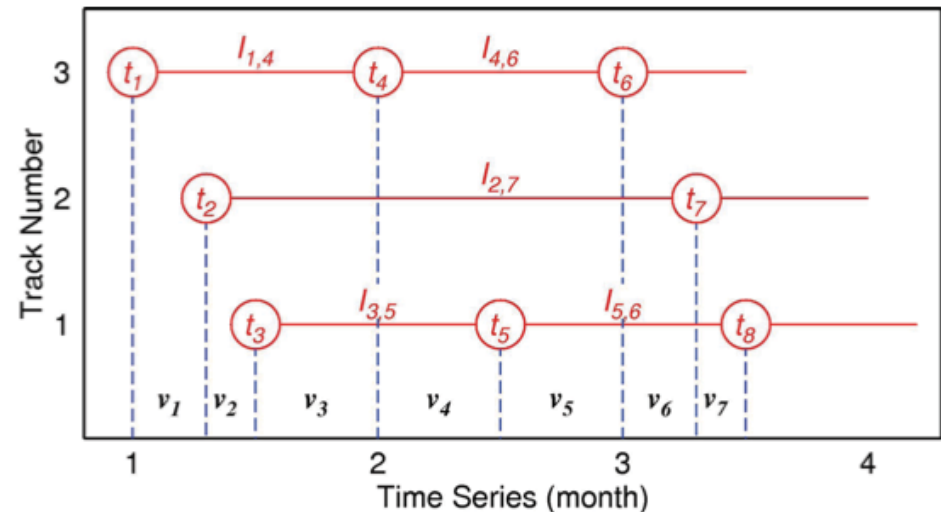
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- Unknown parameters
 - Berardino et al. (2002): velocity of each epoch
 - Schmidt and Burgmann (2003): incremental displacement
 - Pi-RATE (above): velocity of each interval

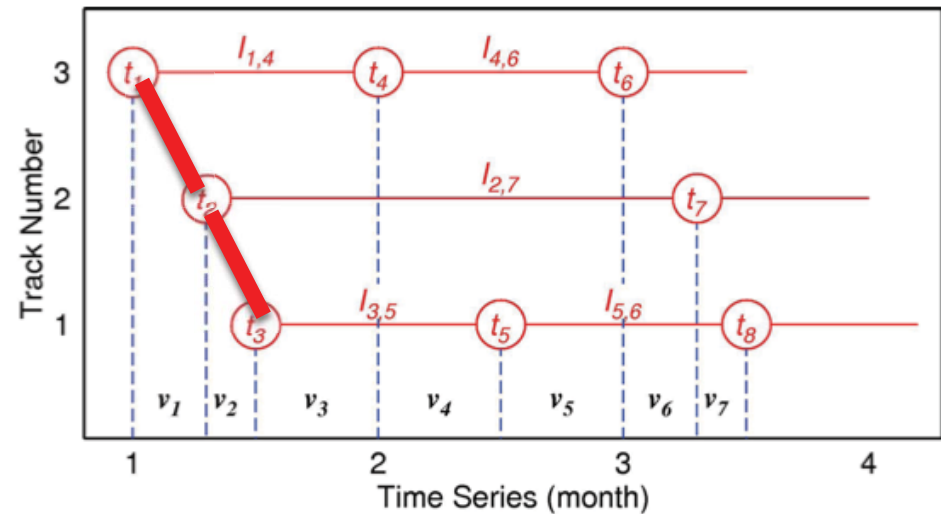
SBAS - Methodology

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$$\mathbf{m} = [v_1 \quad v_2 \quad \cdots \quad v_{n-1}]^T,$$



- Some isolated subsets exist, G is rank deficit
- All epochs are connected in a network, G is full rank
- Solution is not stable due to noise in d

SBAS - Methodology

- System of equations

$$\mathbf{G}\mathbf{m} = \mathbf{d},$$

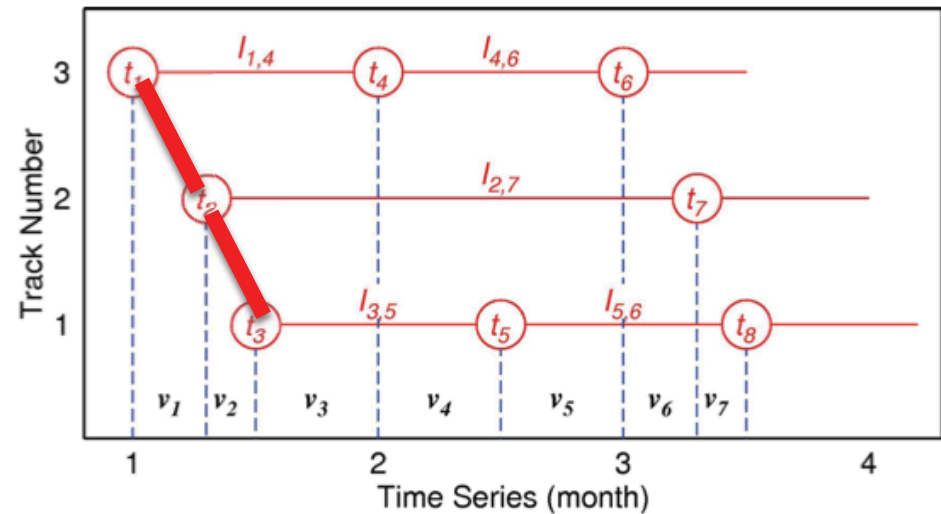
- Solution

– SVD

$$\mathbf{m} = \mathbf{G}^+ \mathbf{d}$$

– Laplacian smoothing

$$\begin{bmatrix} \mathbf{G} \\ k\nabla^2 \end{bmatrix} \mathbf{m} = \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix}$$



SBAS - Methodology

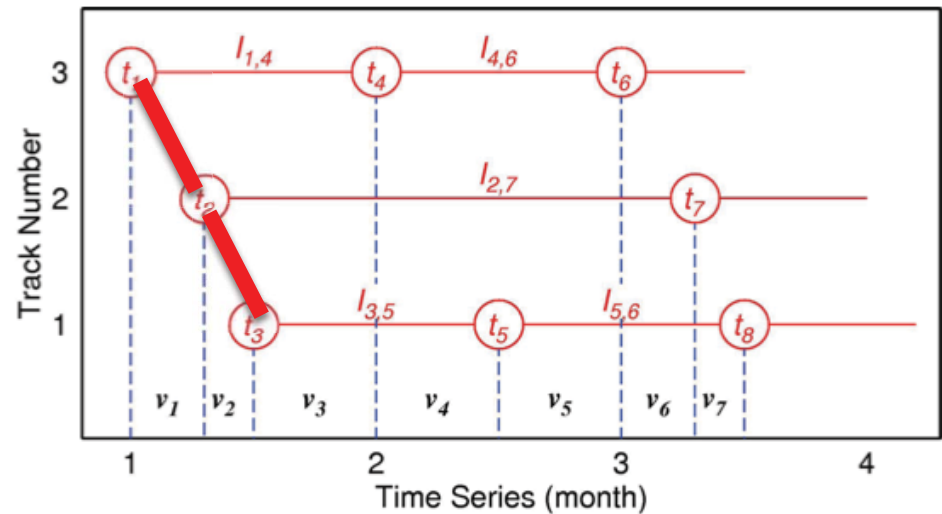
- System of equations

$$\mathbf{Gm} = \mathbf{d},$$

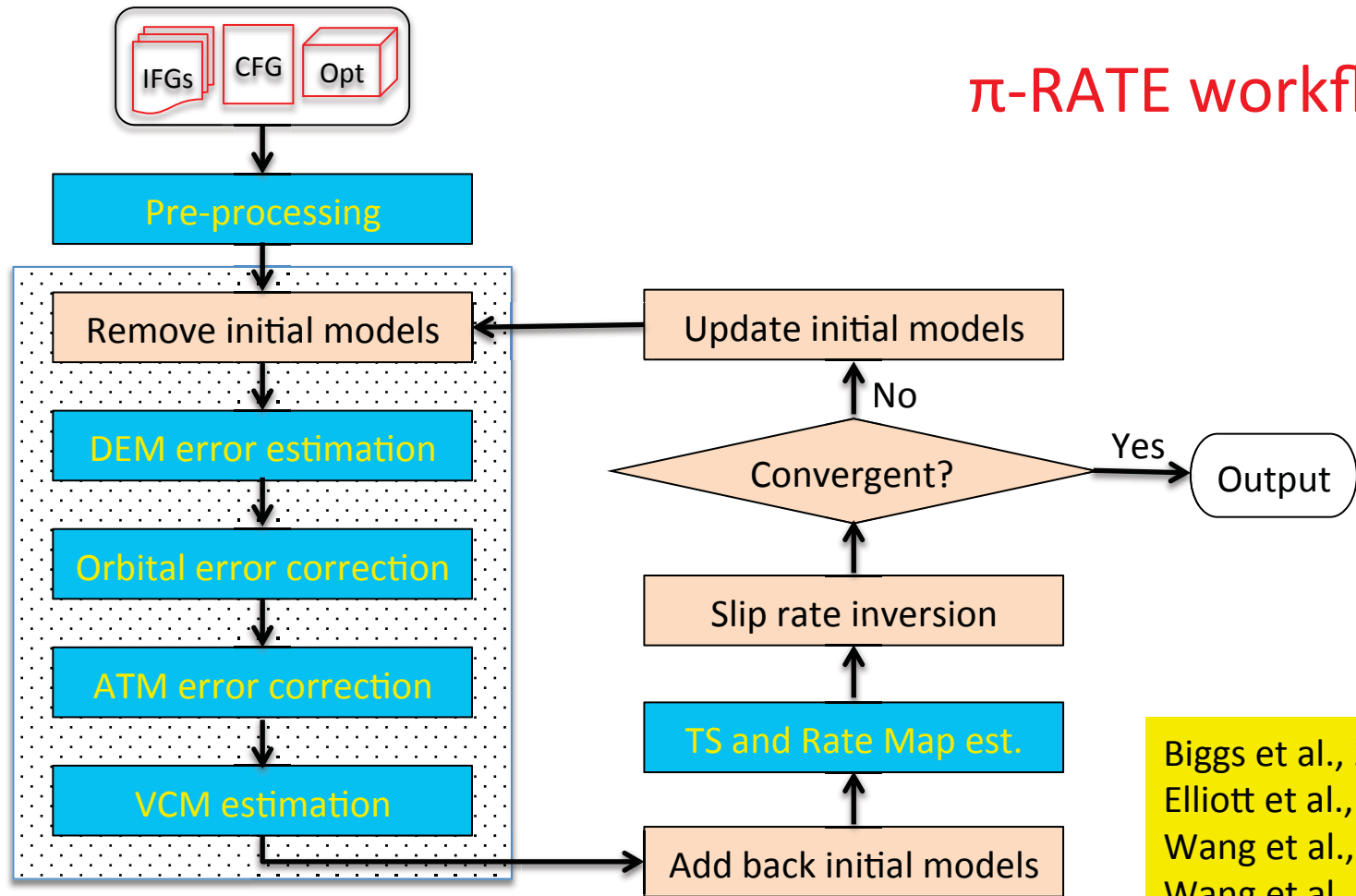
- Considering DEM errors

$$\begin{bmatrix} \mathbf{G} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{m} \\ \Delta h \end{bmatrix} = \mathbf{d}$$

$$\mathbf{B} = \begin{bmatrix} -\frac{B_{\perp}^1}{\rho \sin \vartheta} & & & \\ & \ddots & & \\ & & & -\frac{B_{\perp}^m}{\rho \sin \vartheta} \end{bmatrix}$$



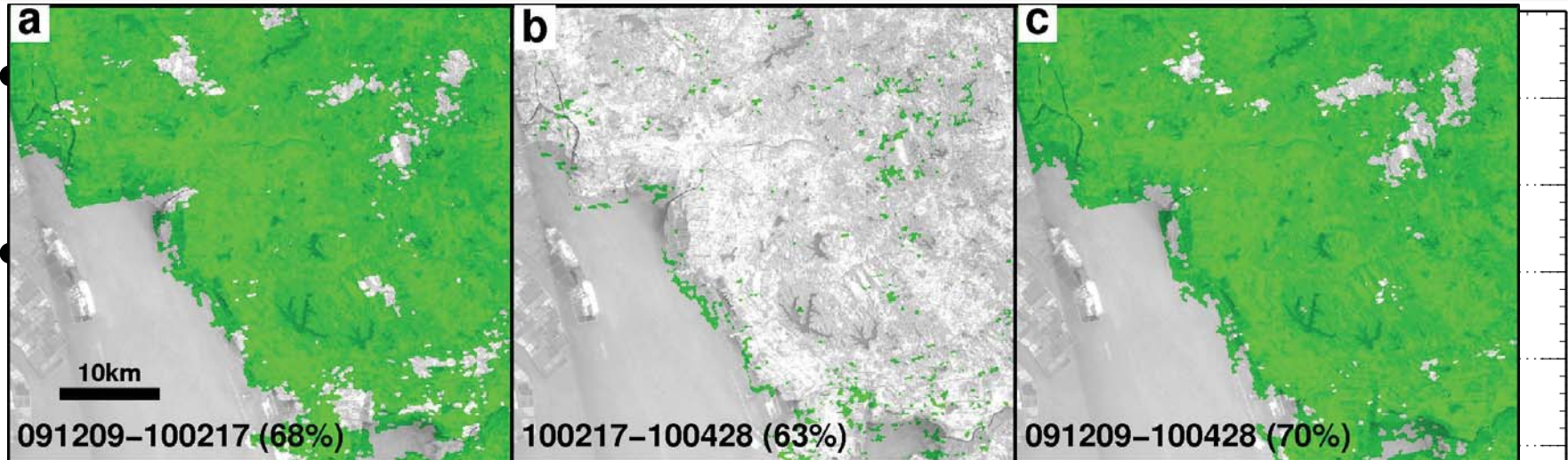
SBAS - Processing



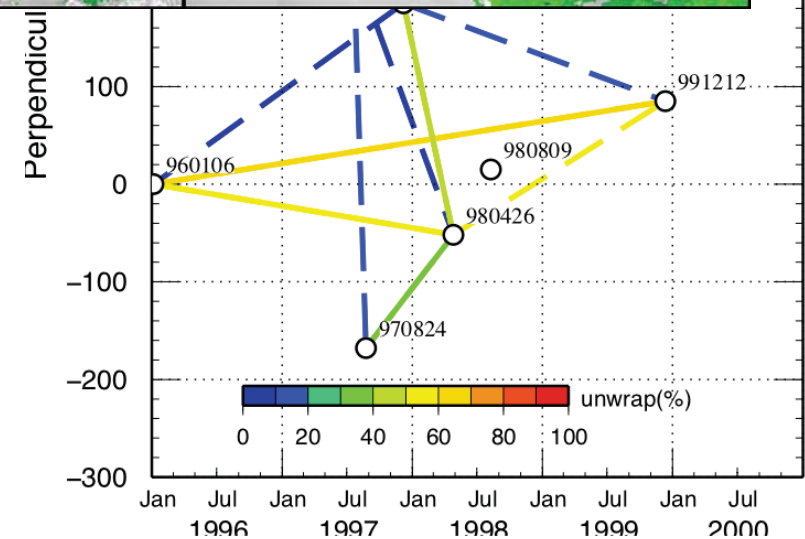
Biggs et al., 2007, GJI
Elliott et al., 2008, GRL
Wang et al., 2009, GRL
Wang et al., 2012, GJI



Interferogram Selection



- MST (select a chain for each pixel from a network)
 - Independent observations
 - Different subset of interferograms for each pixel



Components of interferometric phase

$$\Delta\phi = \Delta\phi_{def} + \Delta\phi_{orb} + \Delta\phi_{atm} + \Delta\phi_{dem} + \varepsilon$$

- Spatial low frequency
- Temporal low frequency

- Spatial low frequency
- Temporal high frequency

- Spatial low frequency
- Temporal high frequency

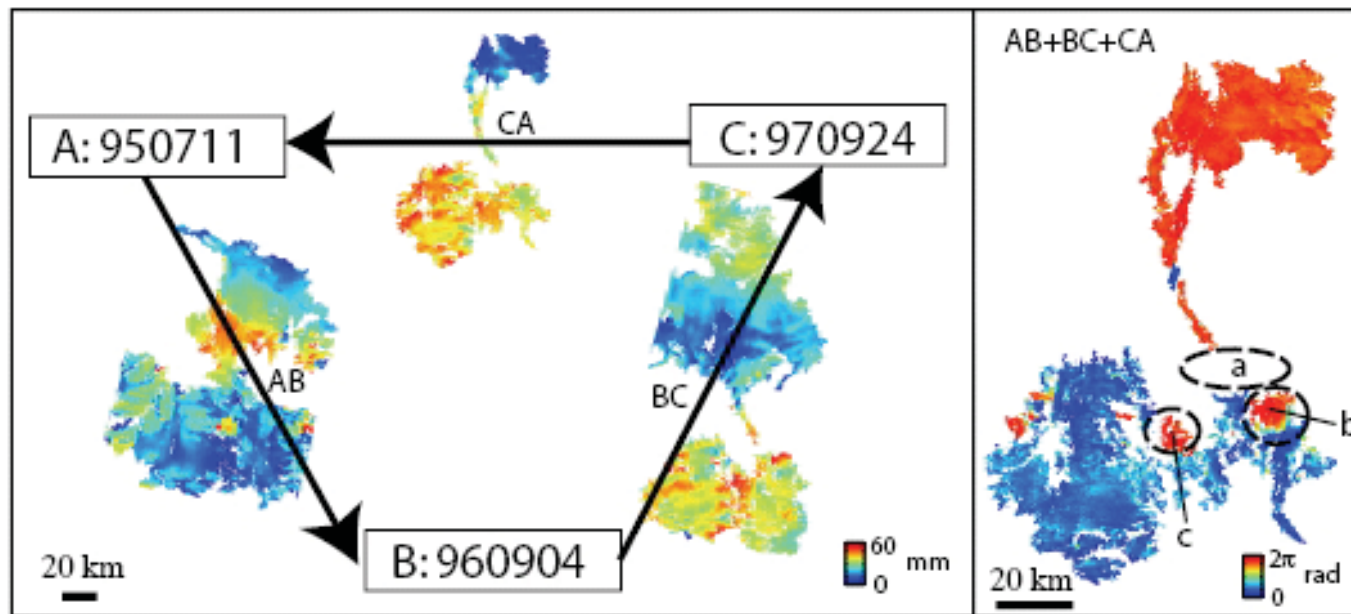
- Constant for a pixel in all the interferograms
- Proportional to perpendicular baseline

- Random noise
- unw errors



Phase unwrapping errors

- In theory, the sum of phase in a closure is zero.
- Jump exists once phase unwrapping is wrong.
- Mask or correct phase unwrapping errors after detection

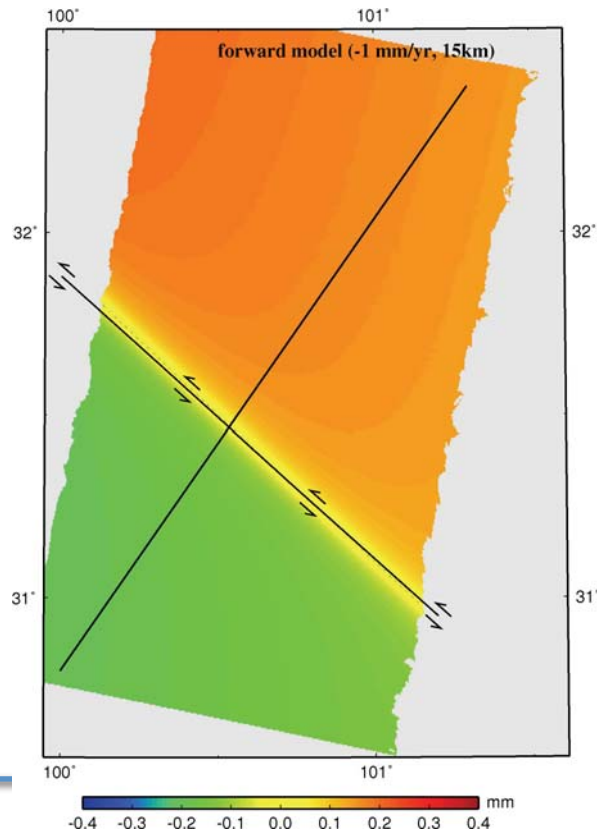


Biggs et al., 2007

Initial models

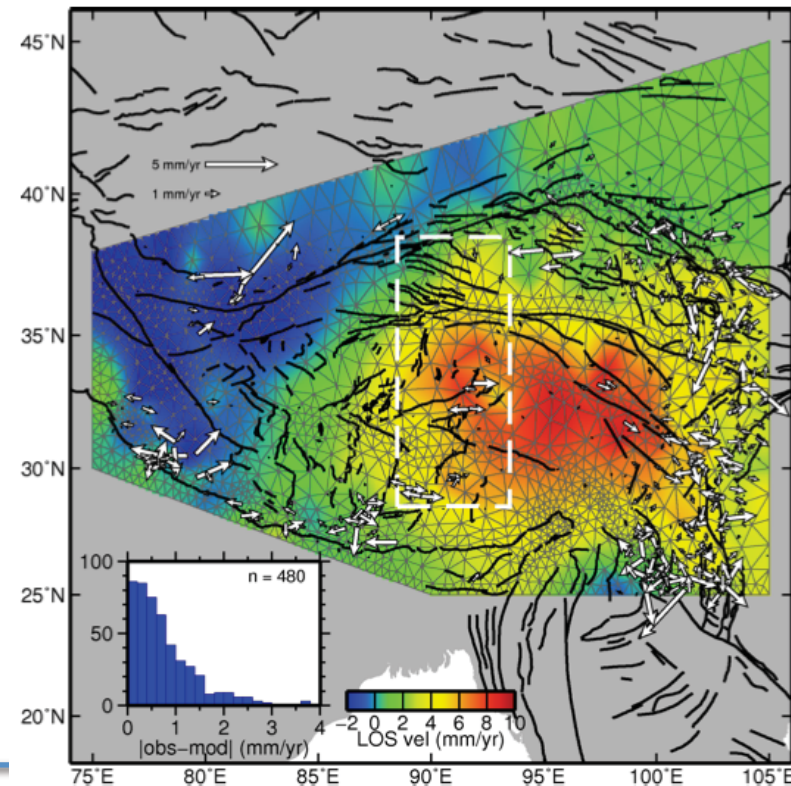
- Why do we use initial model?
spatial low frequency: deformation, atmosphere, orbit

Geophysical models



Wang, Wright, Biggs, 2009

Velocity field

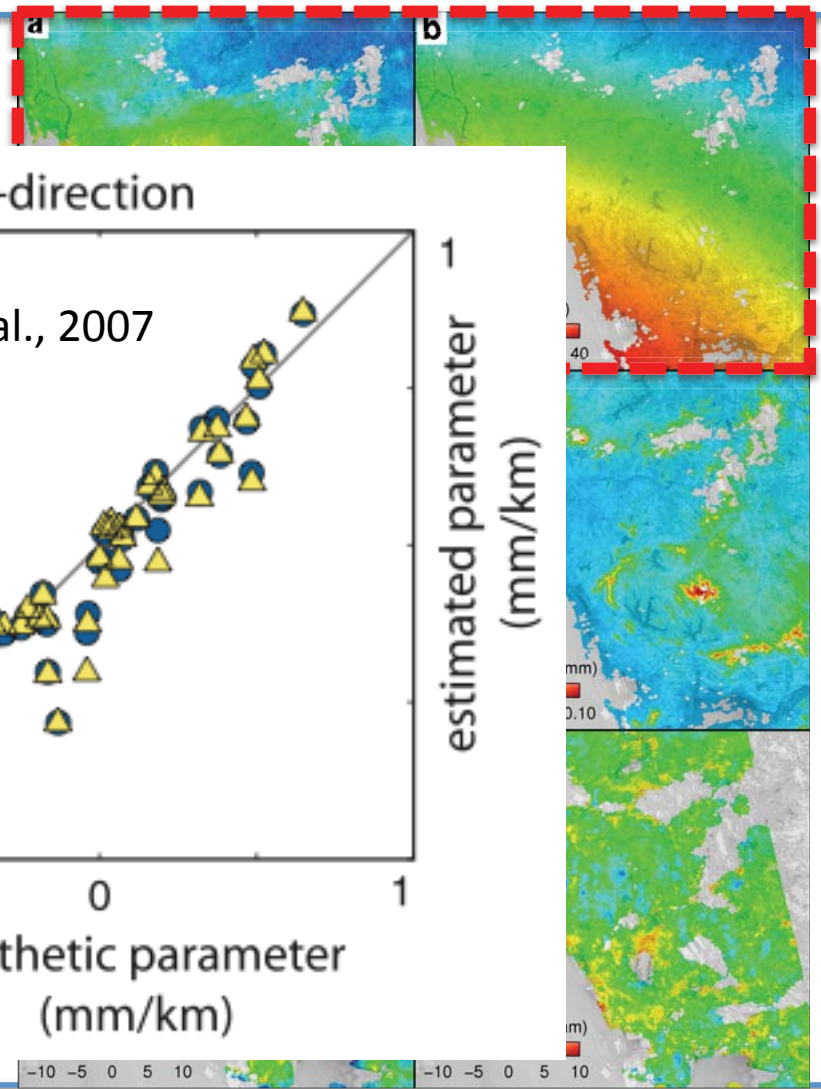
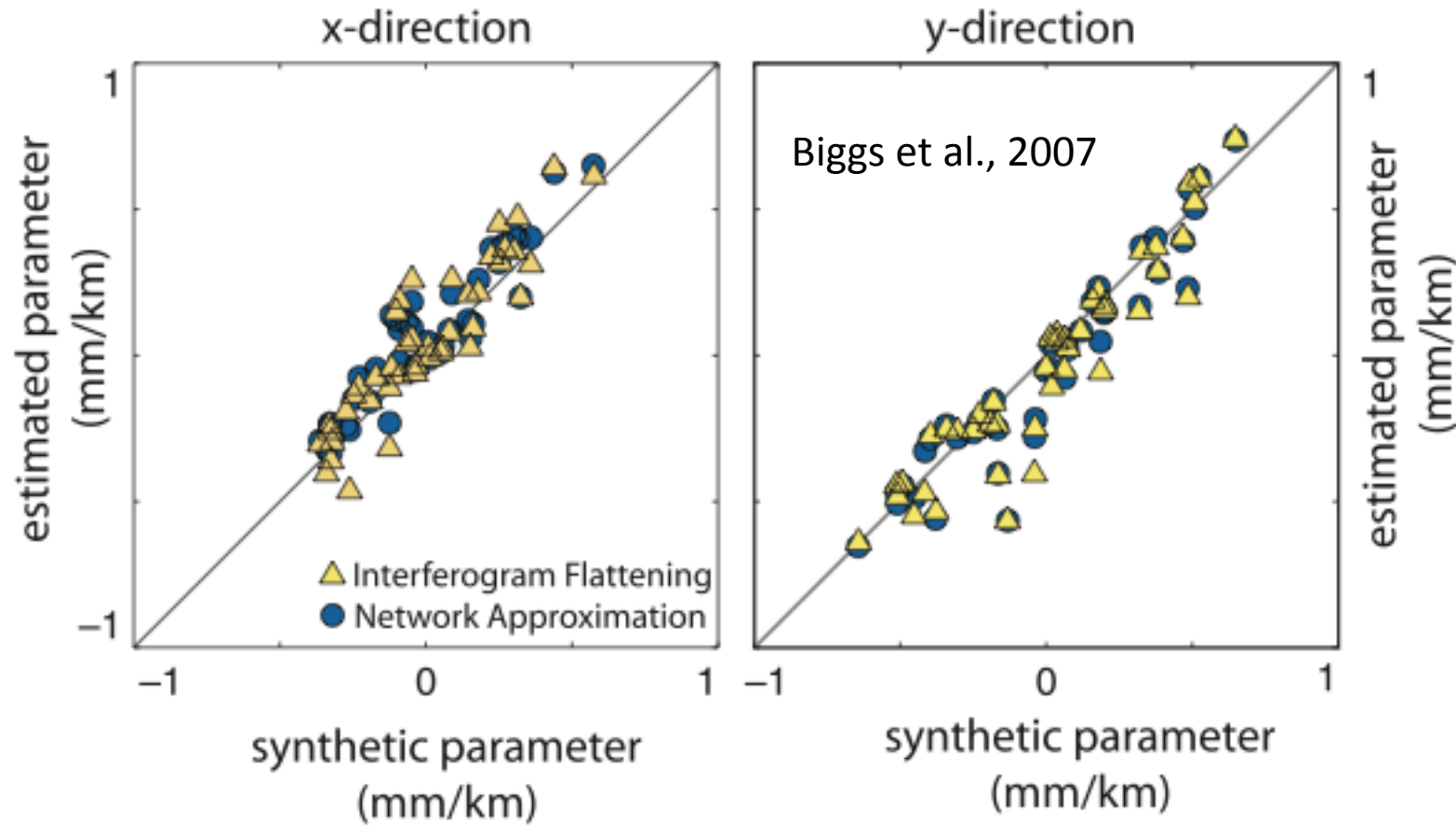


Garthwaite, Wang, Wright, 2013



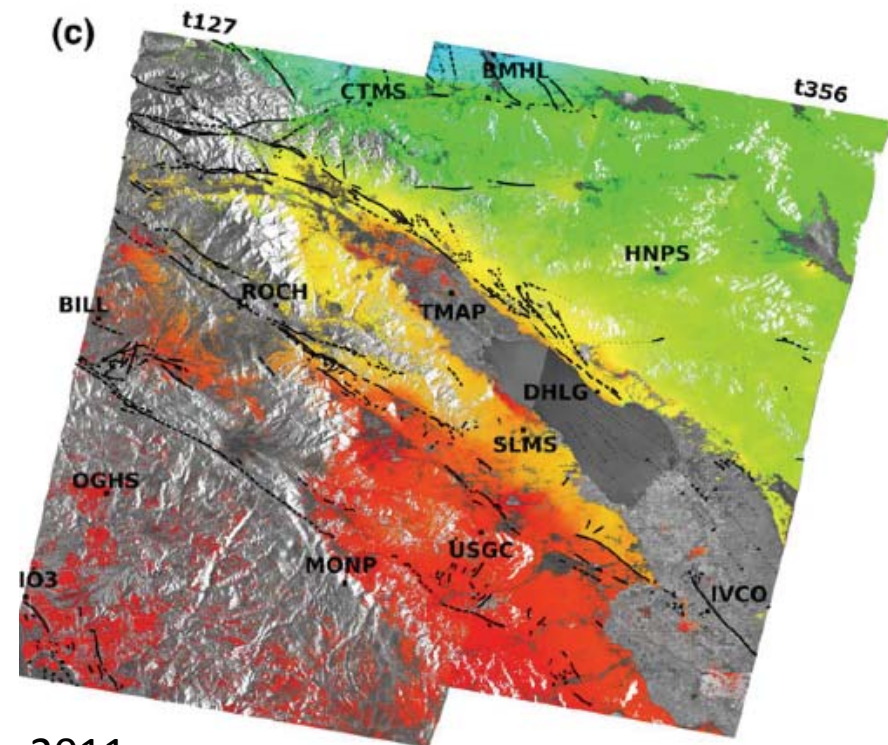
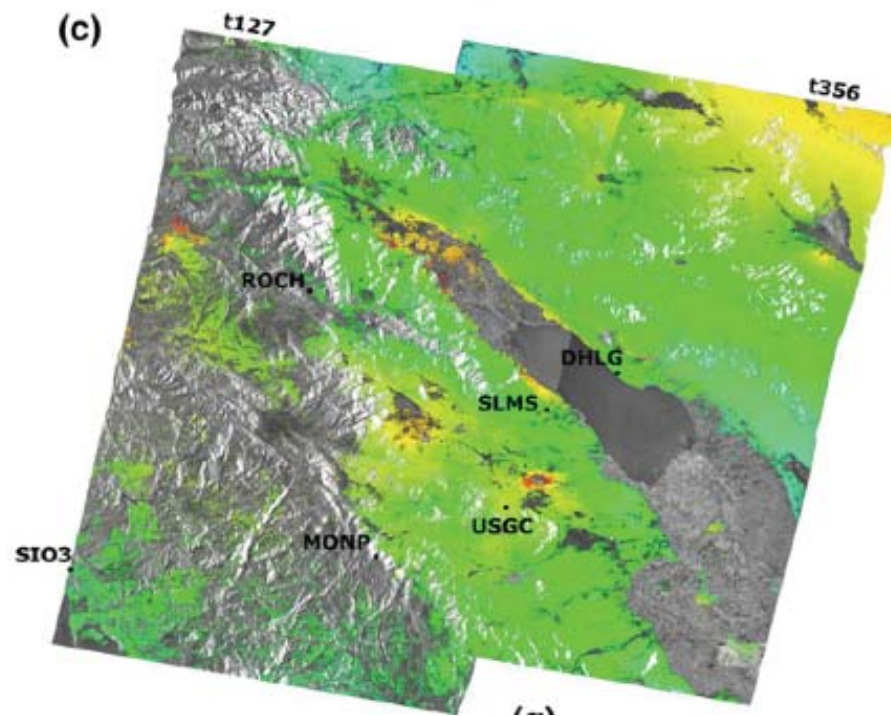
Orbital error correction

- Polynomial fitting



Orbital error correction

- Polynomial fitting
- **GPS time series calibration**

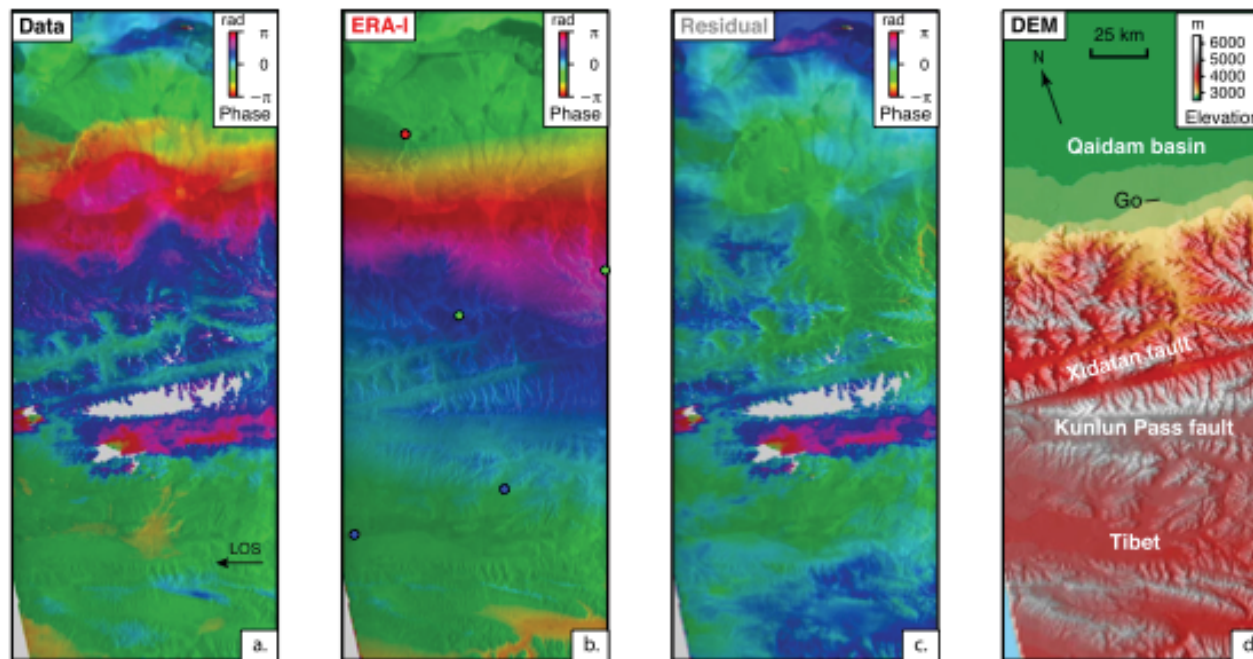


Manzo et al., 2011

Atmospheric delay errors

- External calibration (GPS, MODIS, MERIS, Metrological data)

- ❑ **Advantage:** independent of InSAR
- ❑ **Disadvantage:** spatial and temporal resolution discrepancies, GPS data availability



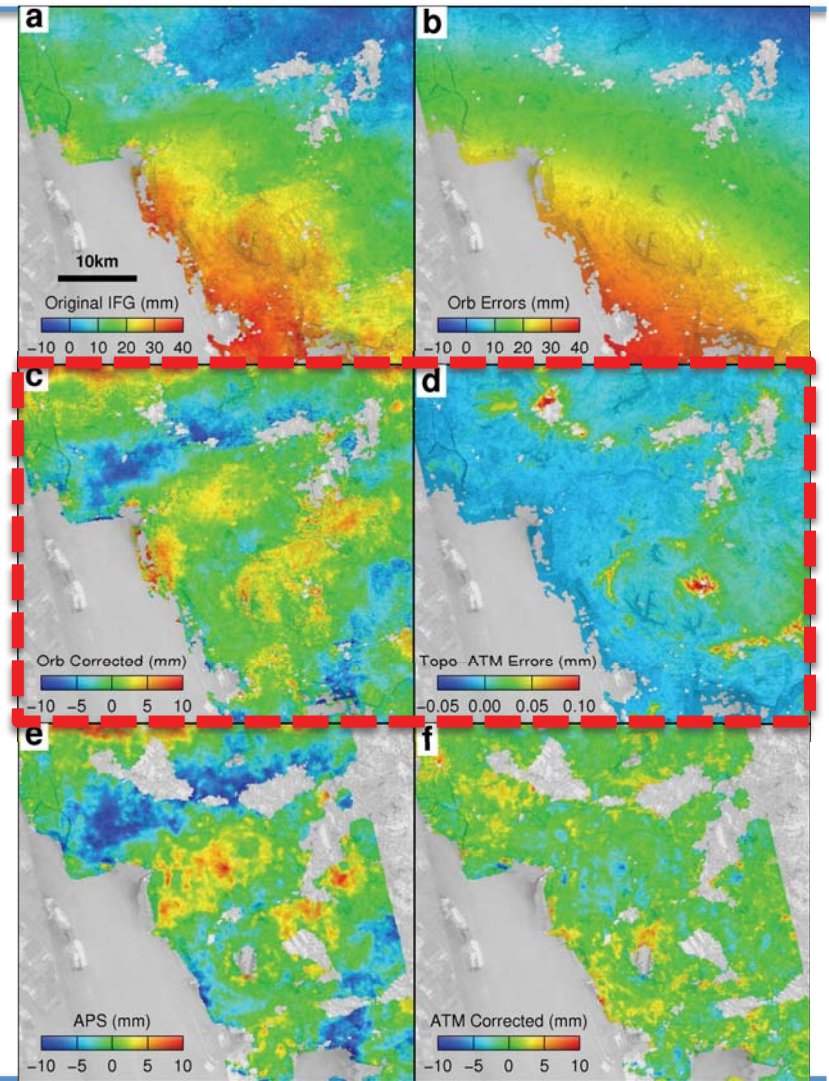
Jolivet et al., 2011

Atmospheric delay errors

- External calibration
- Empirical Estimation
 - Topo-correlated (stratified)

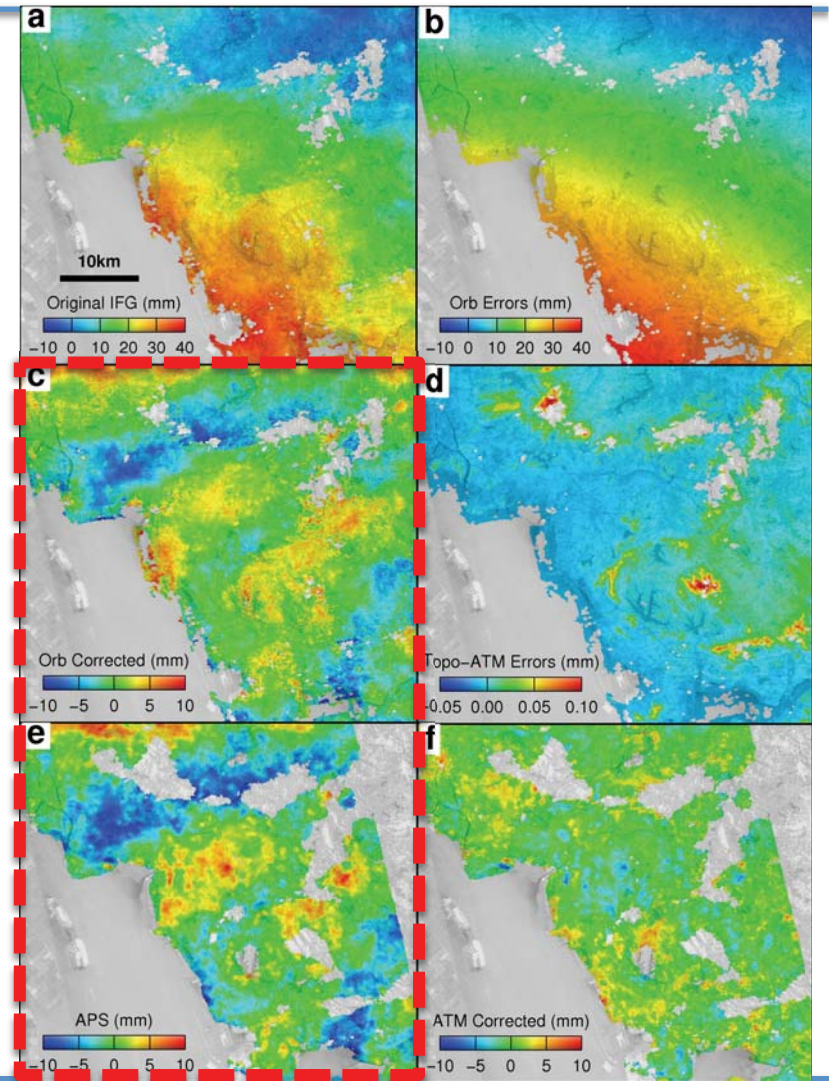
$$\Delta\phi = a \cdot (H - H_0) + b$$

- APS estimation (turbulent)



Atmospheric delay errors

- External calibration
- Empirical Estimation
 - Topo-correlated (stratified)
 - APS estimation (turbulent)
 - Raw time series inversion
 - Sudden deformation removal
 - Temporal low-pass filter
 - Spatial high-pass filter

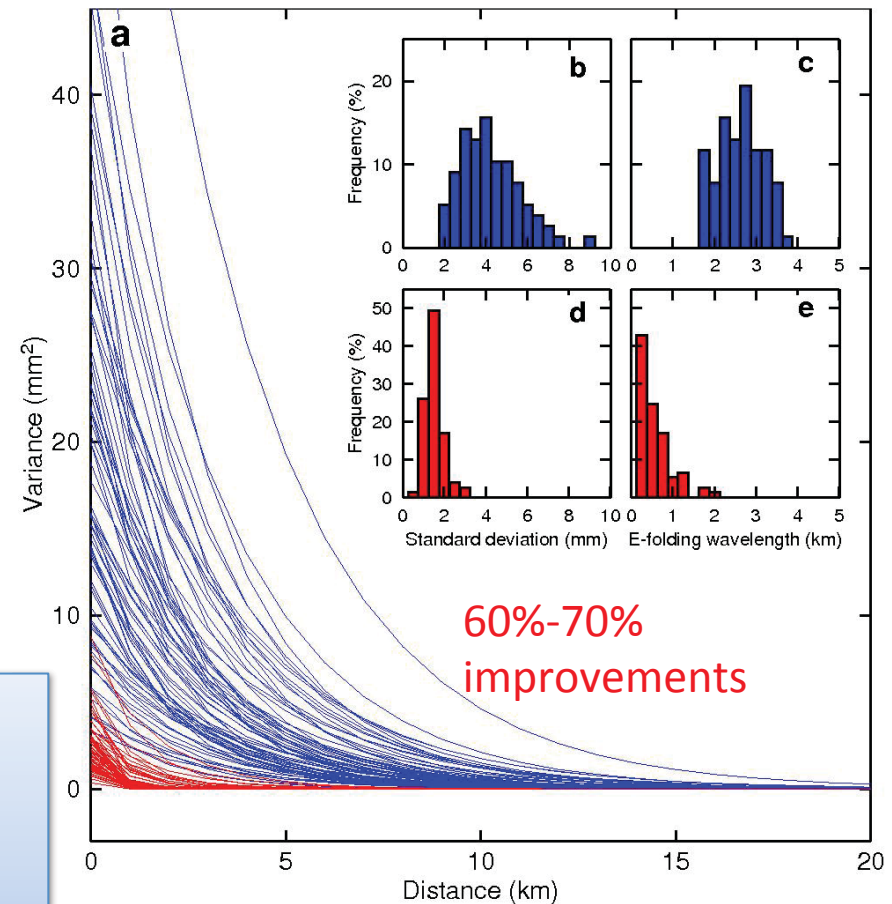


Atmospheric delay errors

- External calibration
- Empirical Estimation
 - Topo-correlated (stratified)
 - **APS estimation (turbulent)**
 - Raw time series inversion
 - Sudden deformation removal
 - Temporal low-pass filter
 - Spatial high-pass filter

- Advantage:** only depends on InSAR data
- Disadvantages:** (1) non-linear relationship exists between topography and delay; (2) how to determine smoothing windows for APS estimation

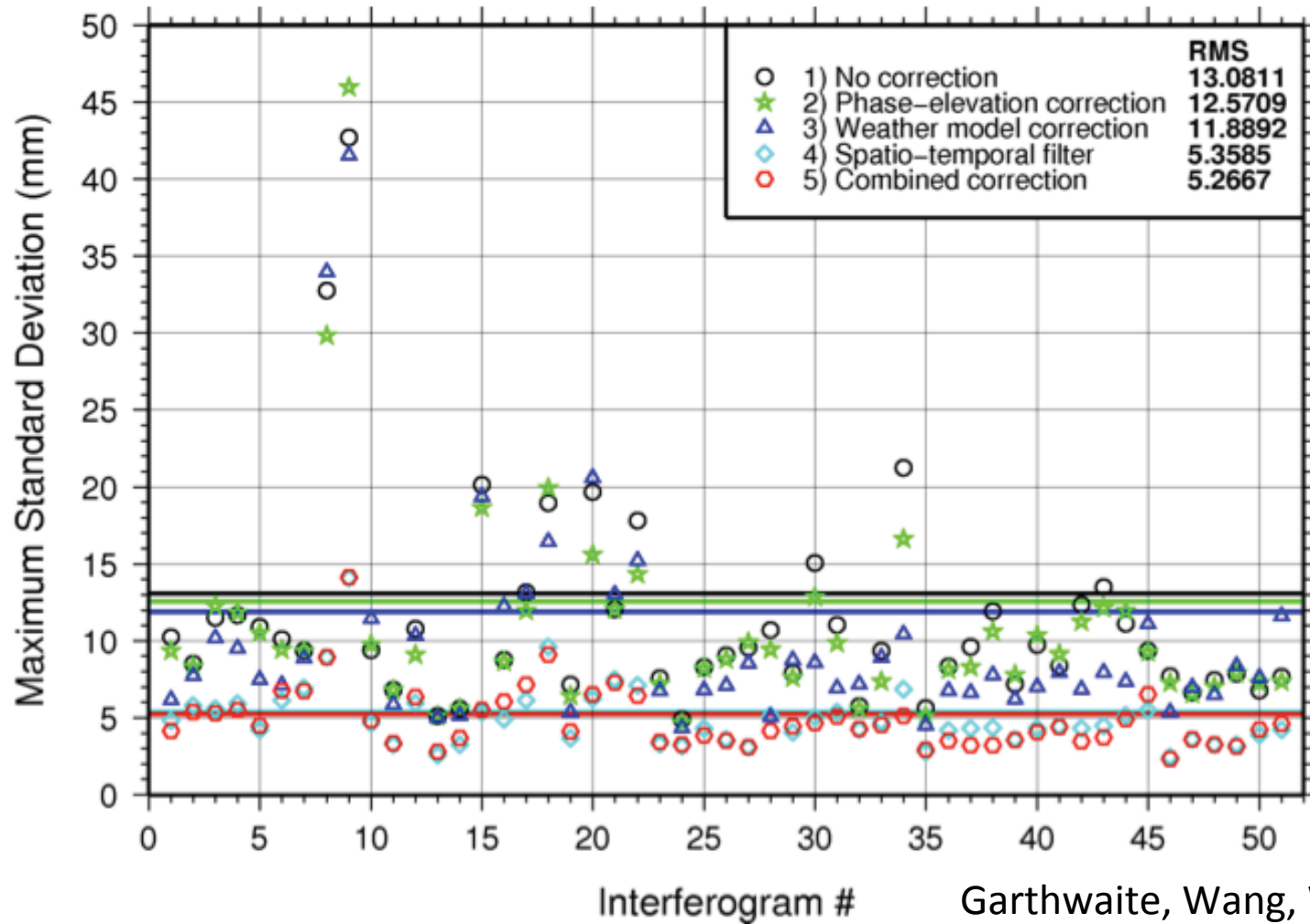
$$C_{jk} = \sigma^2 e^{-d_{jk}/\alpha}$$



Wang et al., 2012



Atmospheric delay errors

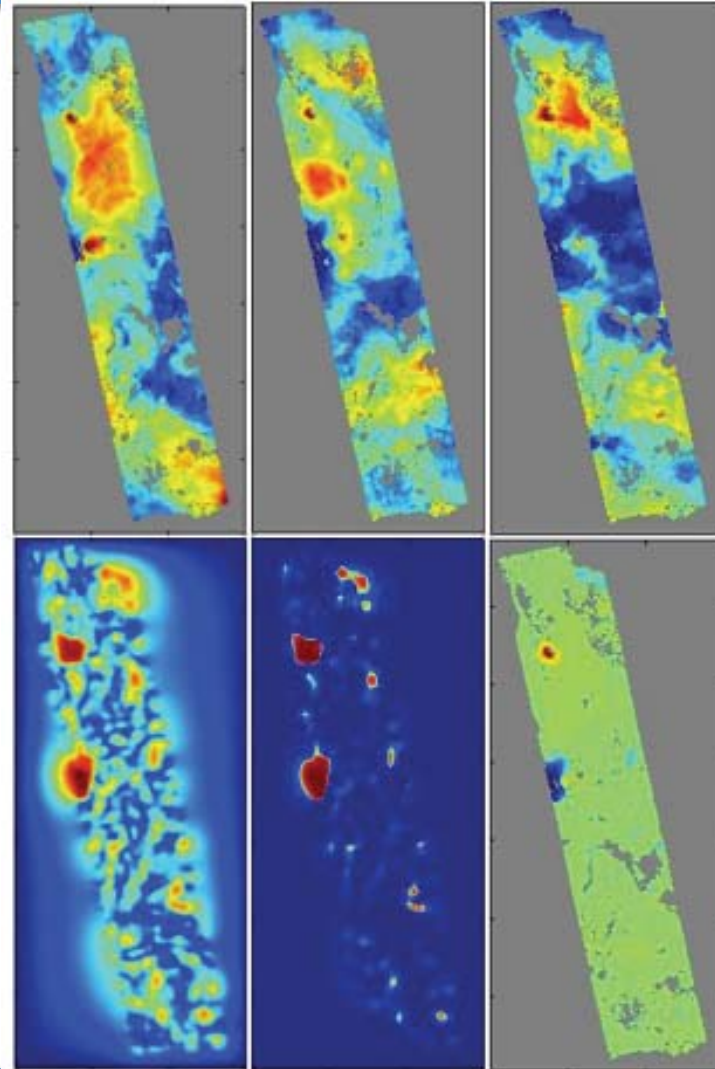


Garthwaite, Wang, Wright, 2013



Atmospheric delay errors

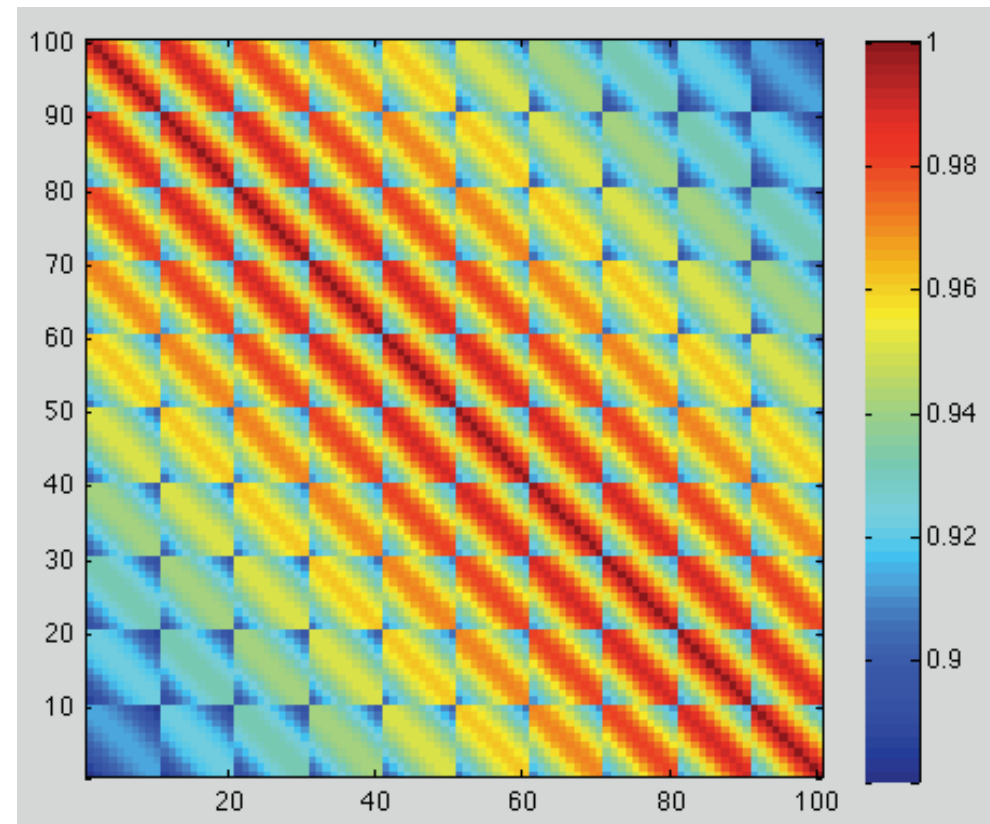
- External calibration
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VCM estimation

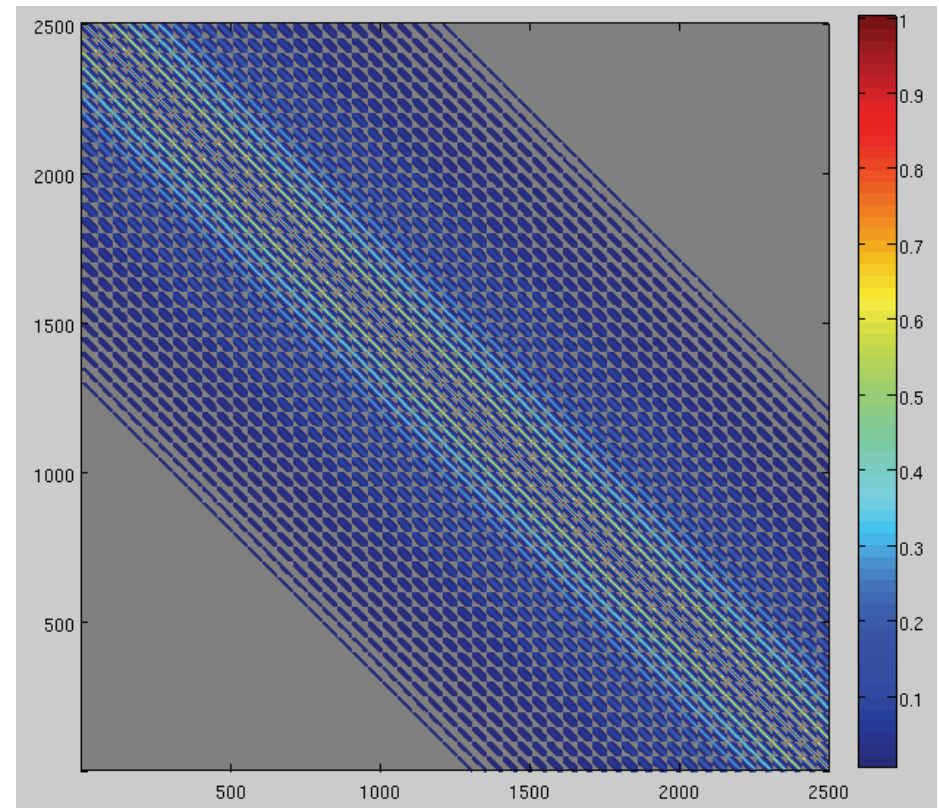
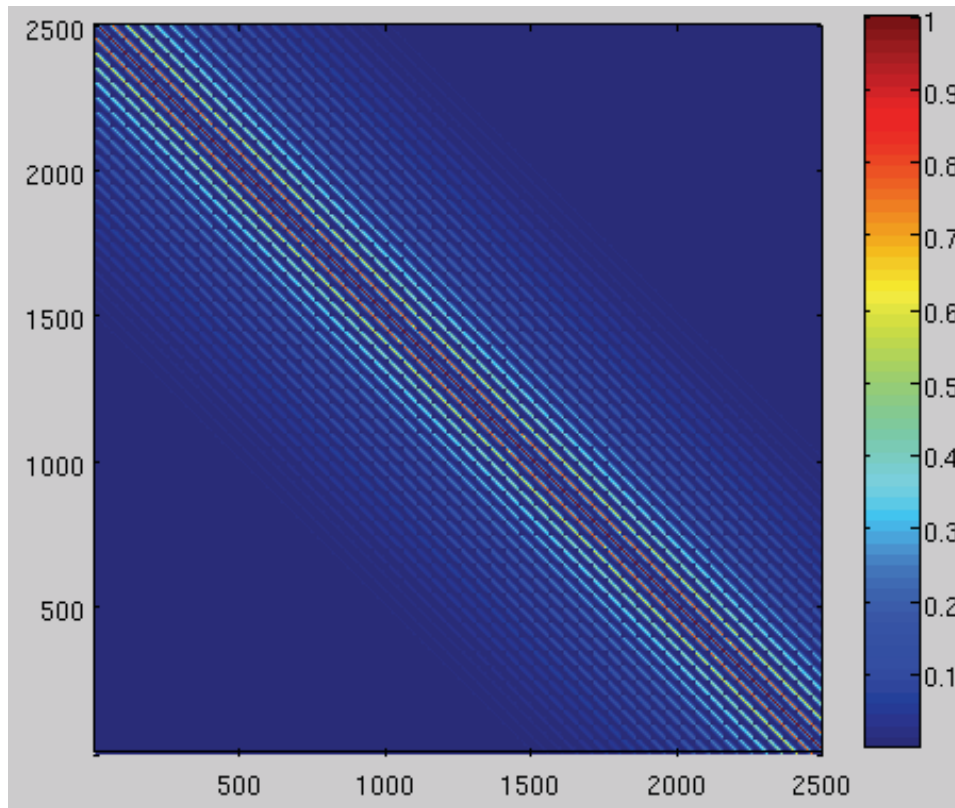
- VCM in space (for initial model inversion)

$$C_{jk} = \sigma^2 e^{-d_{jk}/\alpha}$$



VCM estimation

- VCM in space (for initial model inversion)

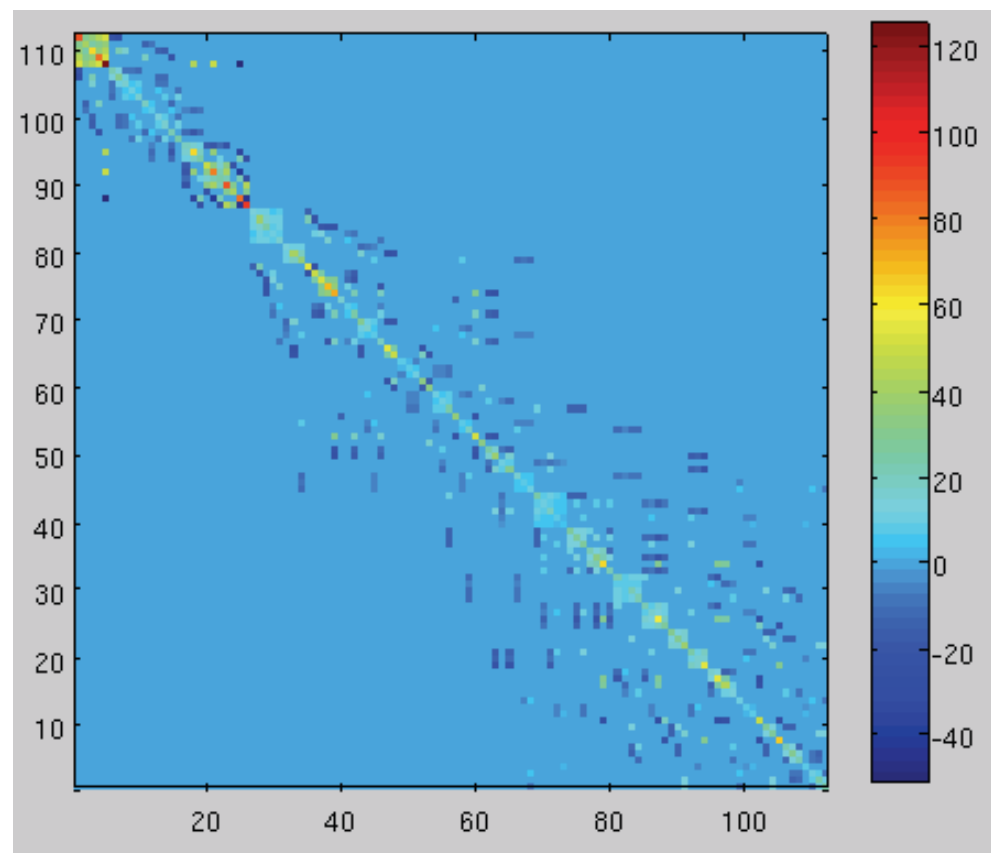


VCM estimation

- **VCM in time** (for time series and rate map inversion)

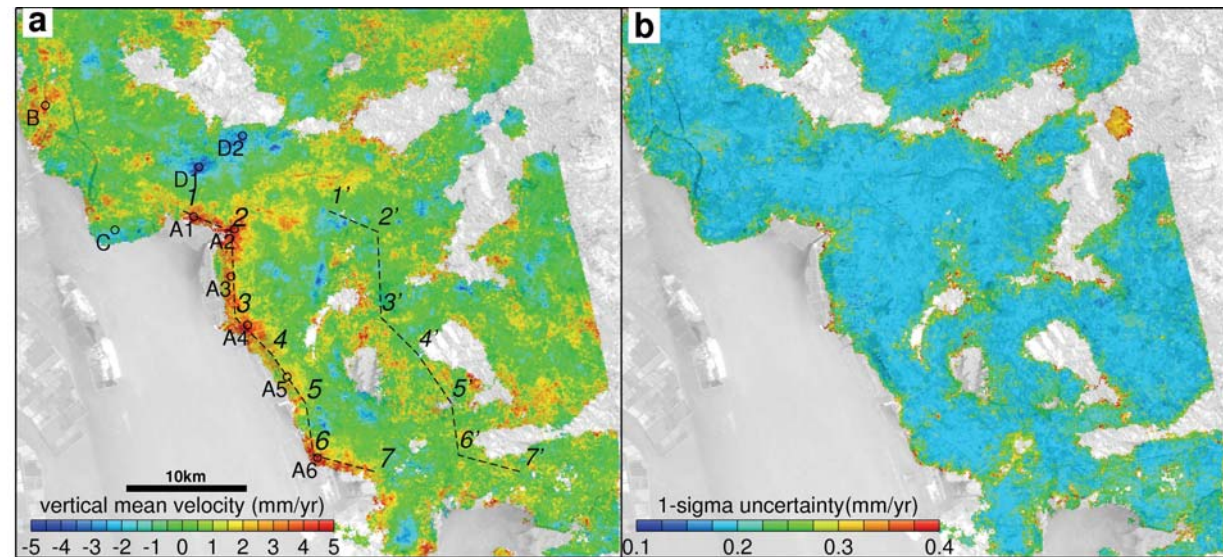
$$C_{lm,nq} = \begin{cases} 1 & (l = n, m = q) \\ -0.5 & (l = q \text{ or } m = n) \\ 0.5 & (l = n \text{ or } m = q) \\ 0 & (\text{otherwise}) \end{cases}$$

Biggs et al., 2007



Final products estimation

- Rate map
- Error map
- DEM errors
- Time series



Wang et al., 2012

$$\mathbf{Gm} = \mathbf{d}$$

$$\mathbf{m} = (\mathbf{G}^T \mathbf{C}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{C}^{-1} \mathbf{d}$$

$$\mathbf{C}_m = (\mathbf{G}^T \mathbf{C}^{-1} \mathbf{G})^{-1}$$

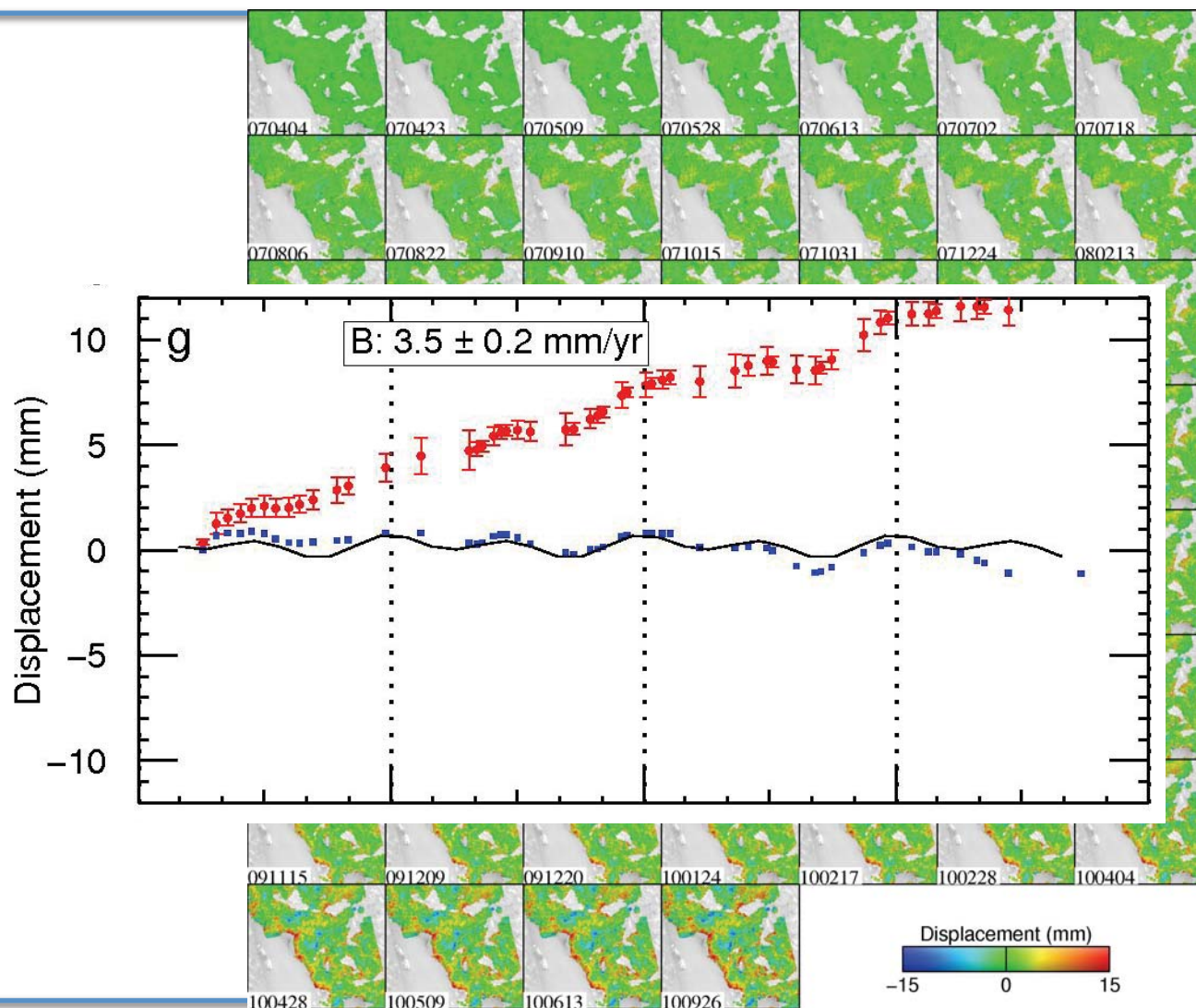
Final products estimation

- Rate map
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$$\mathbf{Gm} = \mathbf{d}$$

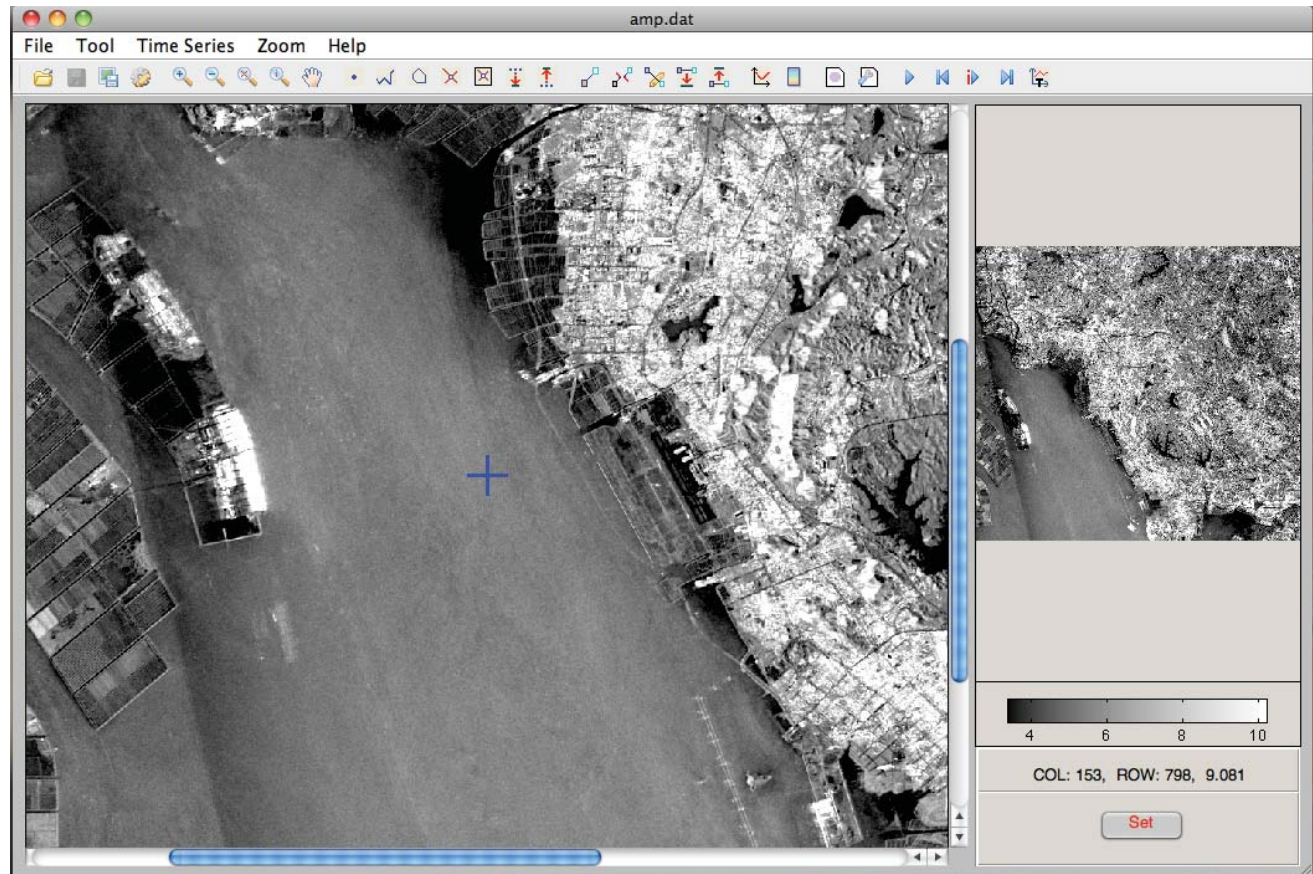
$$\mathbf{m} = (\mathbf{G}^T \mathbf{C}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{C}^{-1} \mathbf{d}$$

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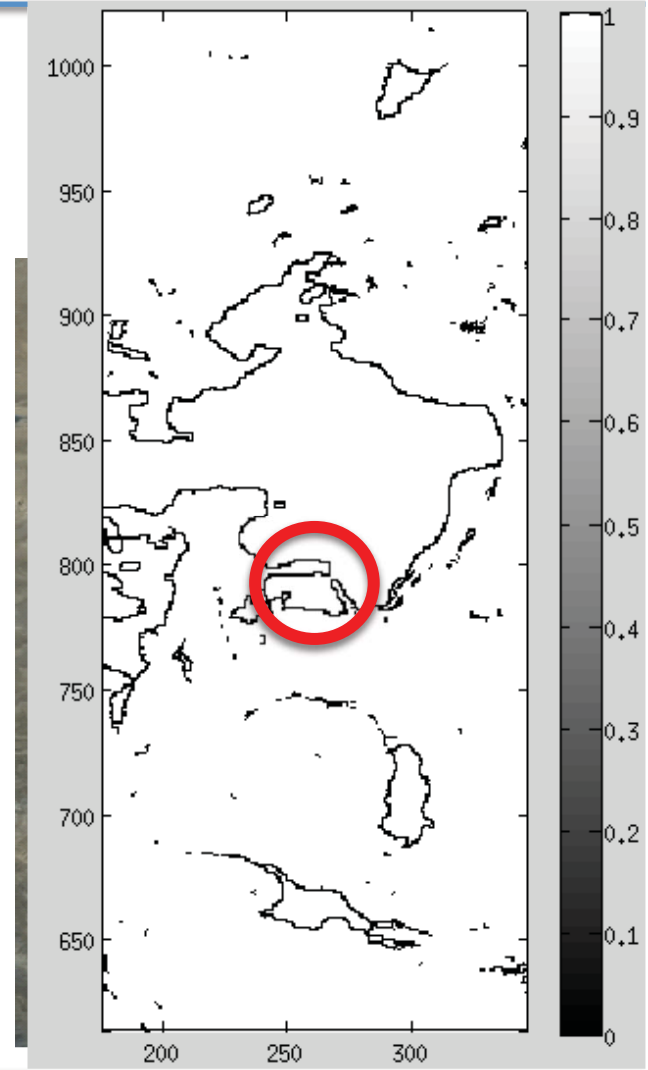
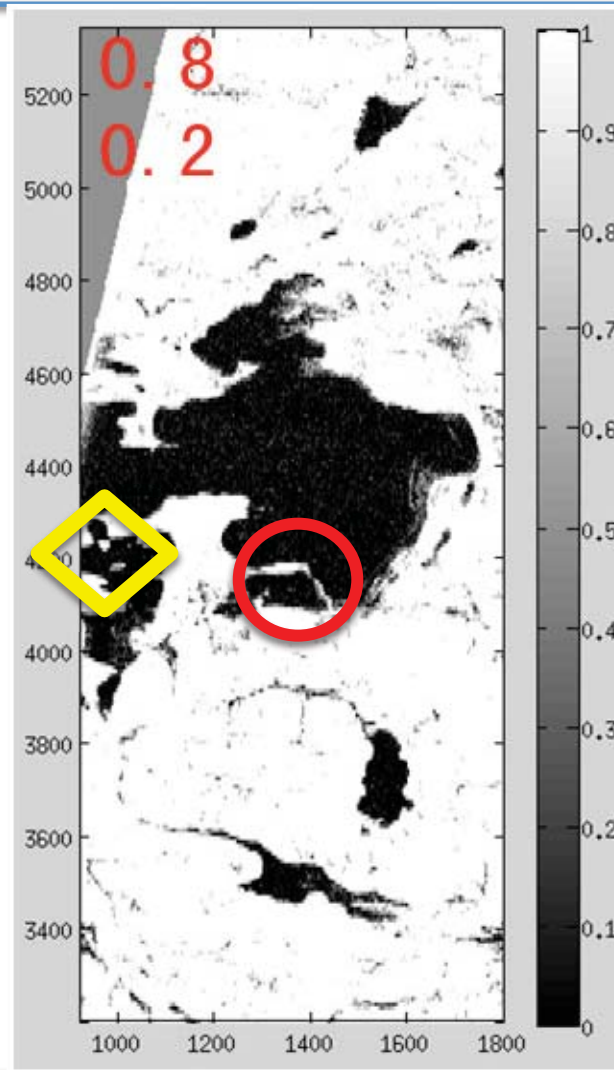
By-products

- Amplitude
- Coherence
- ...

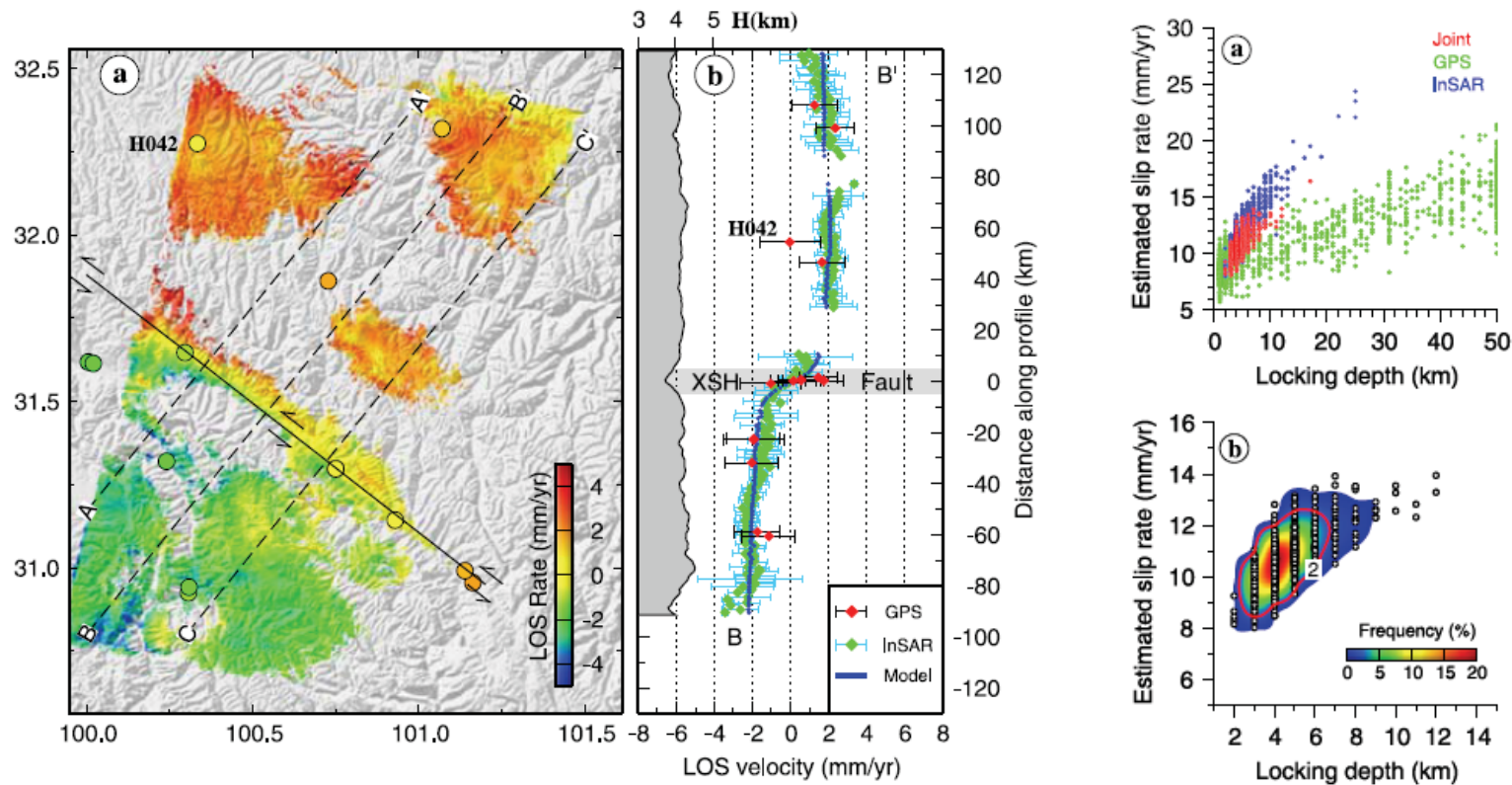


By-products

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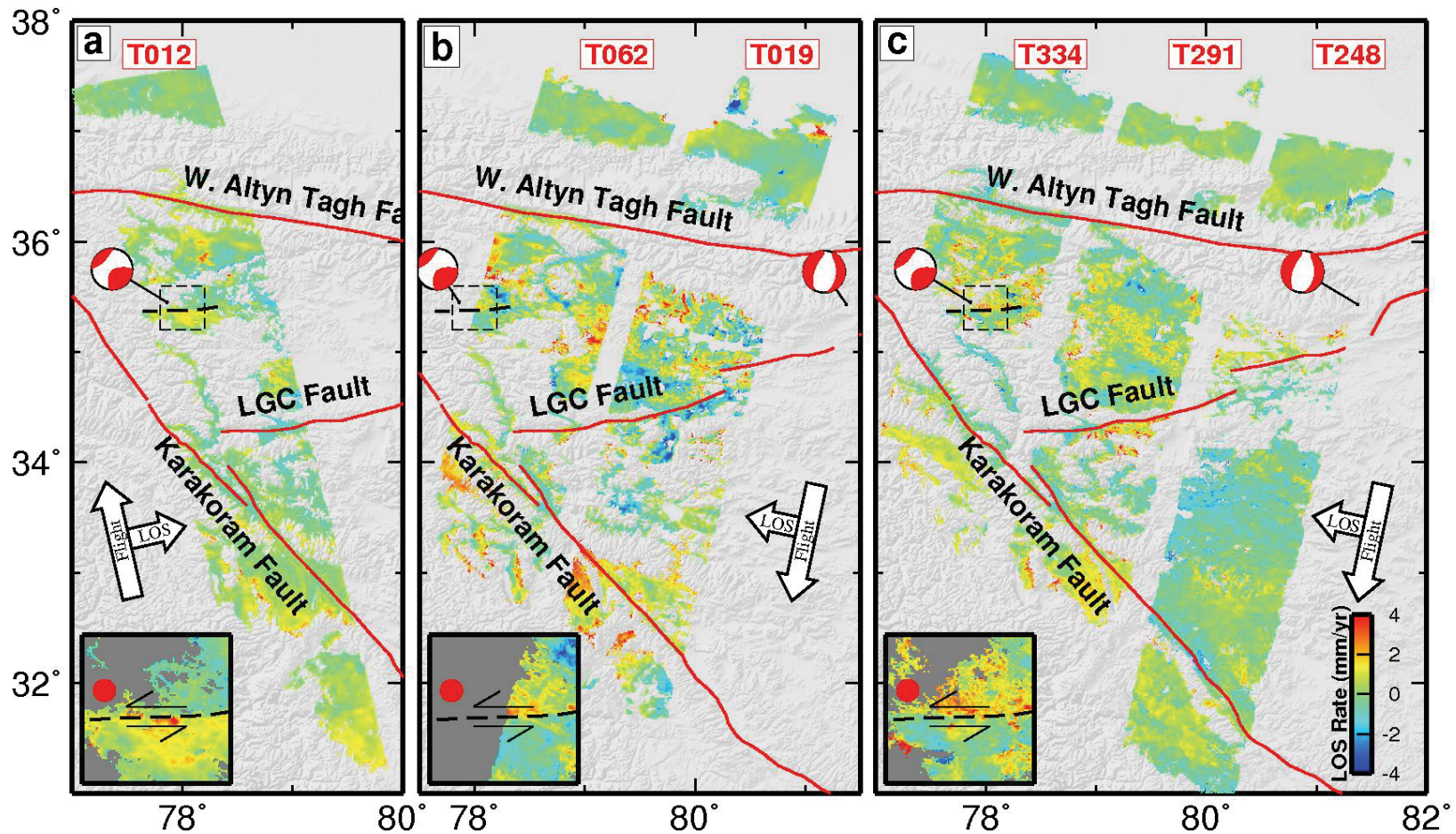
Examples: Eastern Tibet (XSH)



- Consistent interseismic deformation measured by InSAR and GPS
- Improvement on the constraint of locking depth using InSAR and GPS
- Slip rate: 9-12 mm/yr; locking depth: 3-6 km.



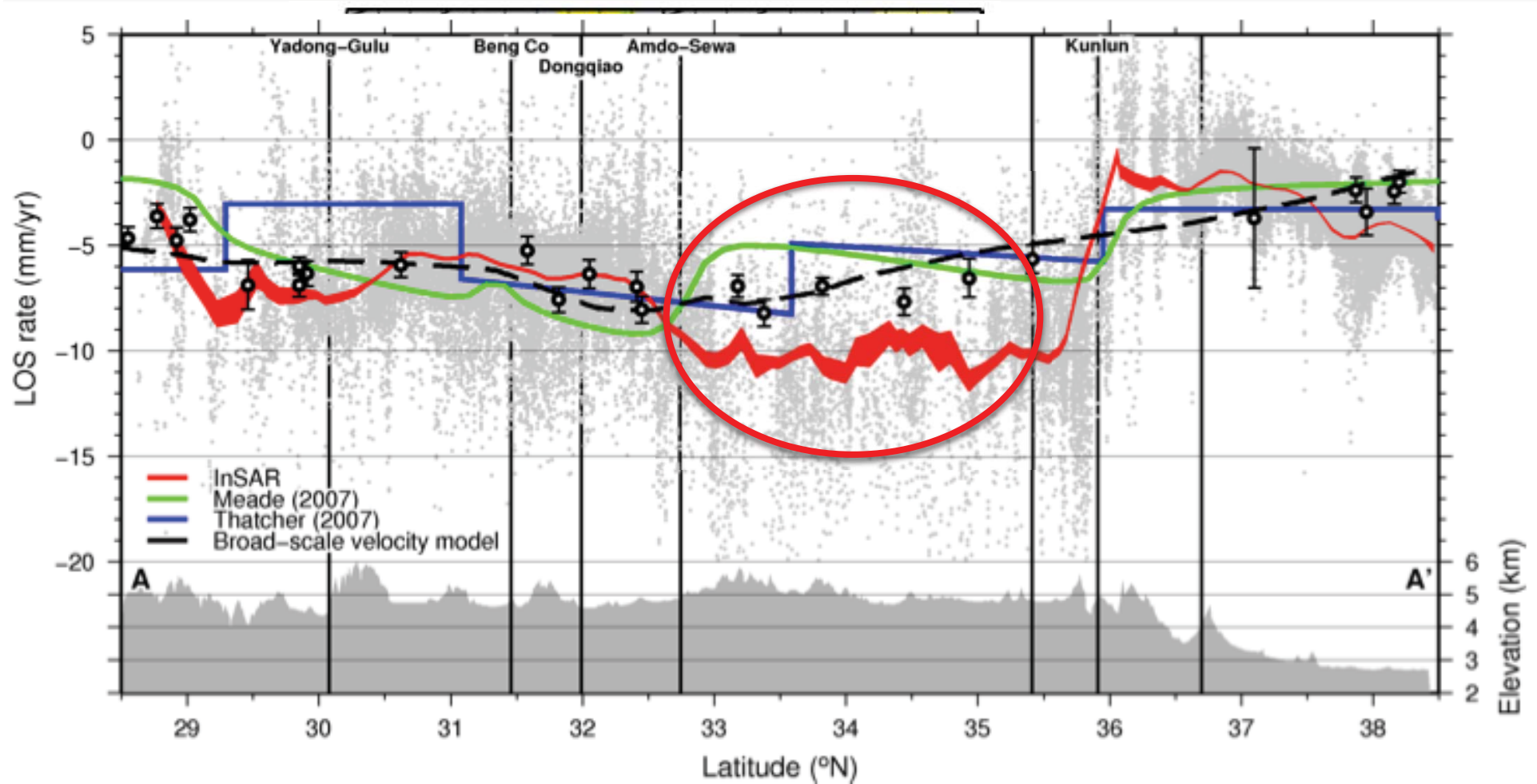
Examples: Western Tibet



- InSAR reveals internal deformation in western Tibet



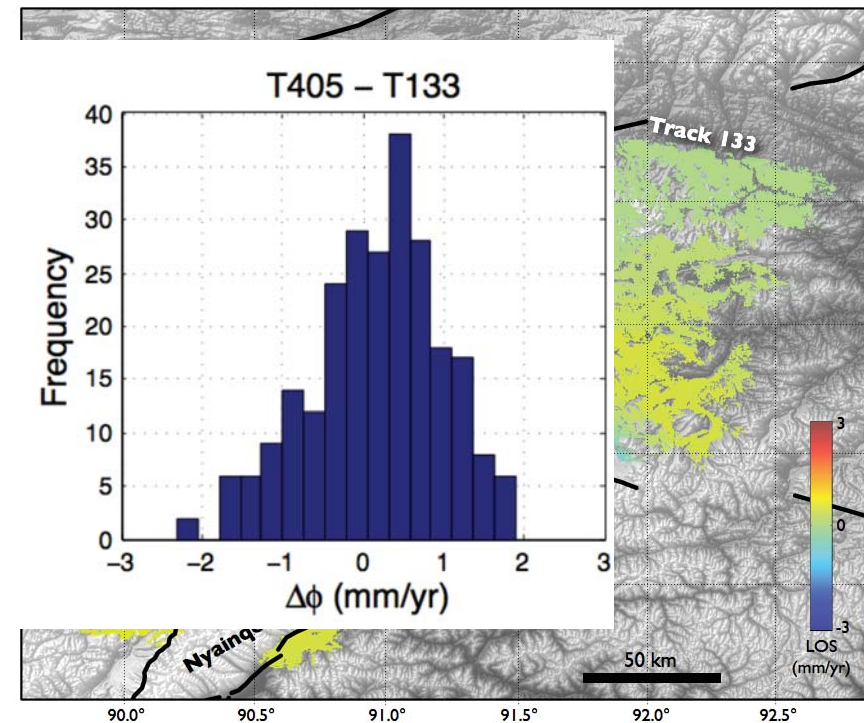
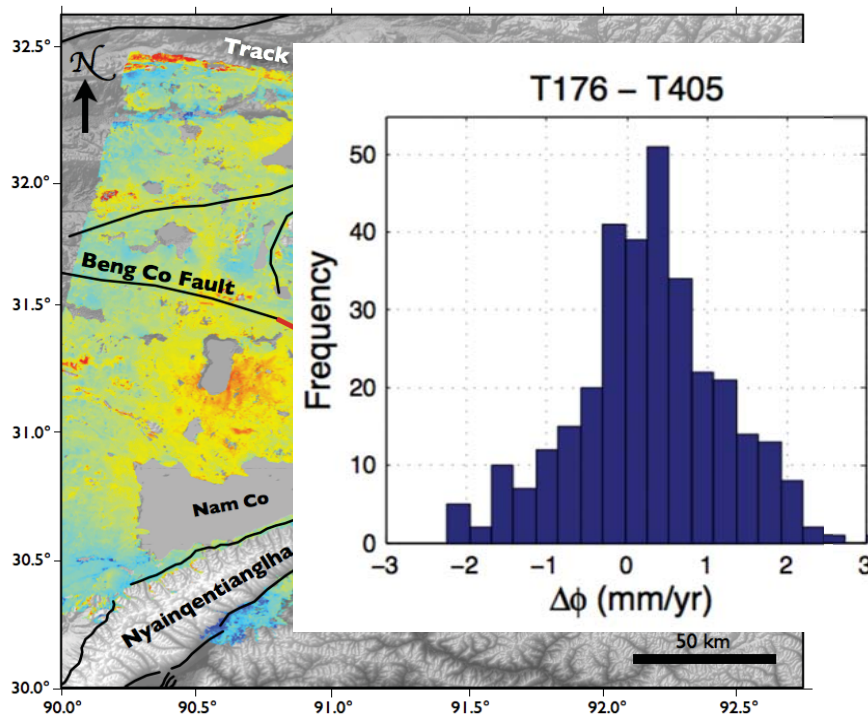
Examples: Central Tibet



- InSAR reveals vertical deformation in central Tibet



Beng Co and Yadong-Gulu Rift

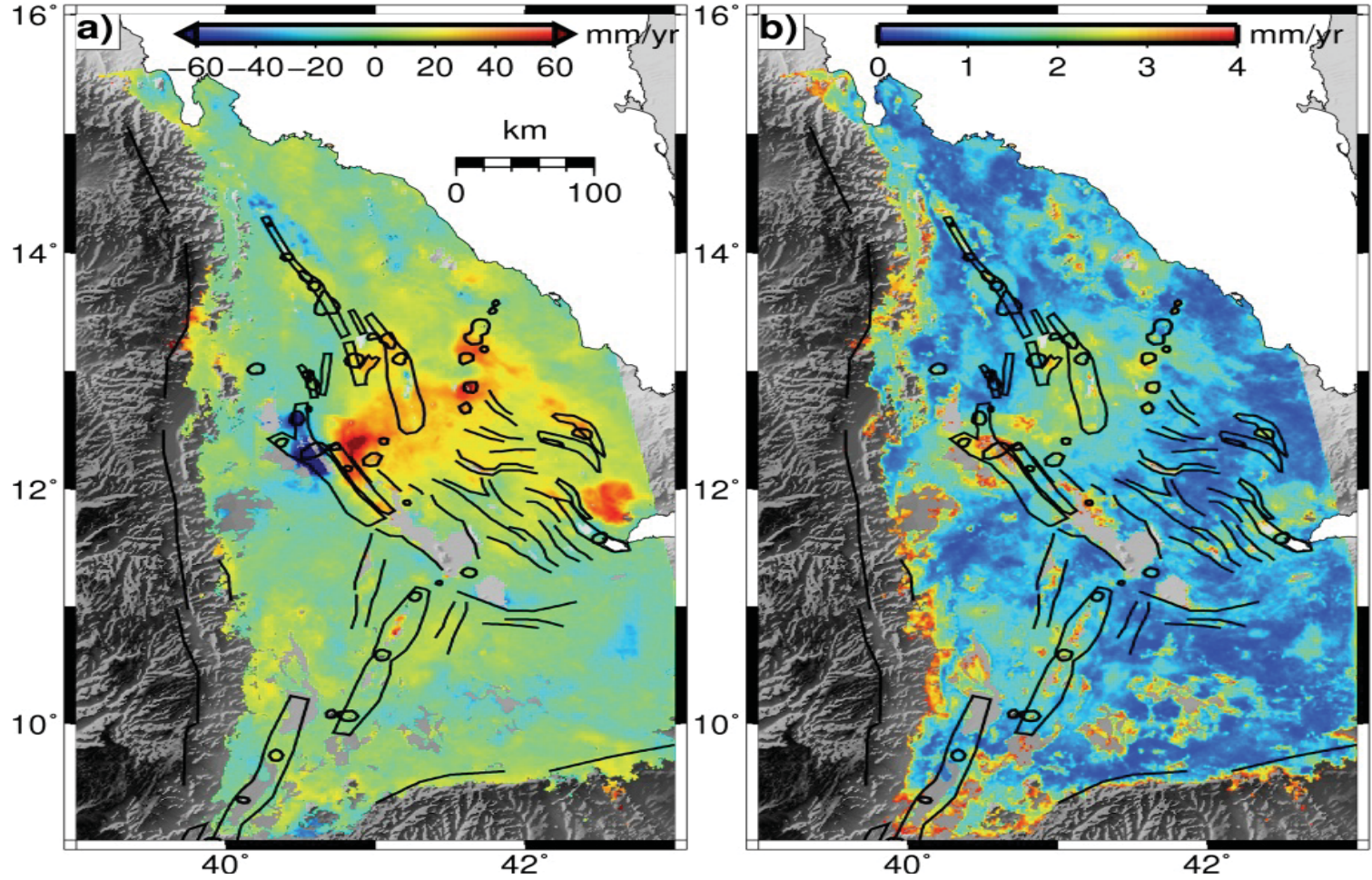


- Postseismic deformation following the 1951/1952 earthquakes
- Viscoelastic stress relaxation in the lower crust (viscosity = $3e19$)

Ryder et al.,
in prep



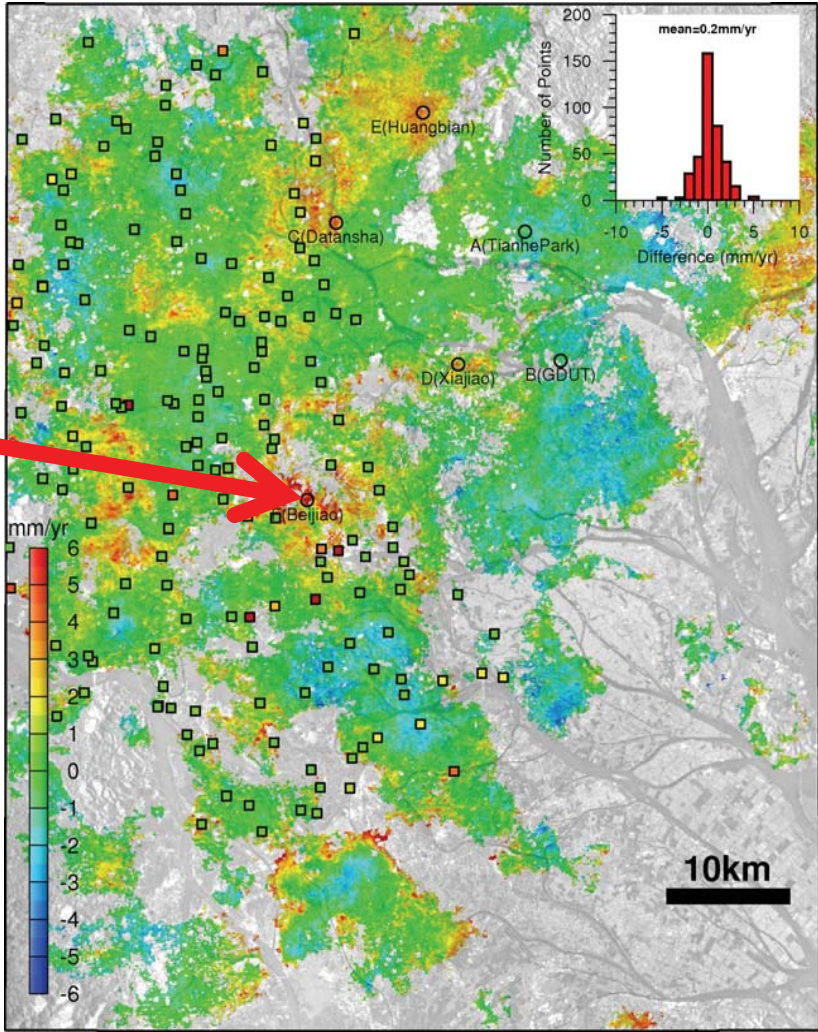
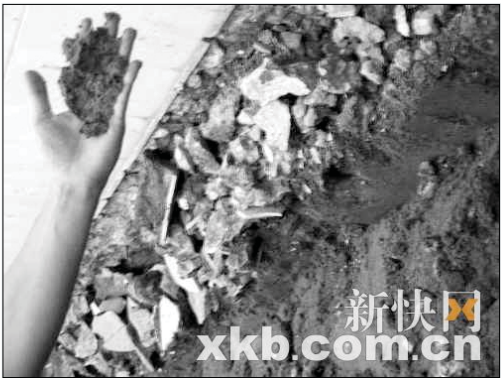
Examples: Afar-wide swath rate map



Pagli et al., in prep



Examples: PRD subsidence



InSAR - Leveling: 0.2 mm/yr



Conclusions and future work

- SBAS method has been widely used for measuring deformation due to its easy realization.
- No general method can reliably correct atmospheric delay errors.
- Phase unwrapping is challenging and time consuming in SBAS.
- Spatial resolution is usually limited due to multi-look processing for phase unwrapping.
- Extraction of different components in InSAR time series.
- New satellites with shorter revisit time will increase coherence, thus make phase unwrapping much easier.



Thank You!

<http://homepages.see.leeds.ac.uk/~earhw/software/pi-rate>



>2000 visits from over 200 institutions until 2012

