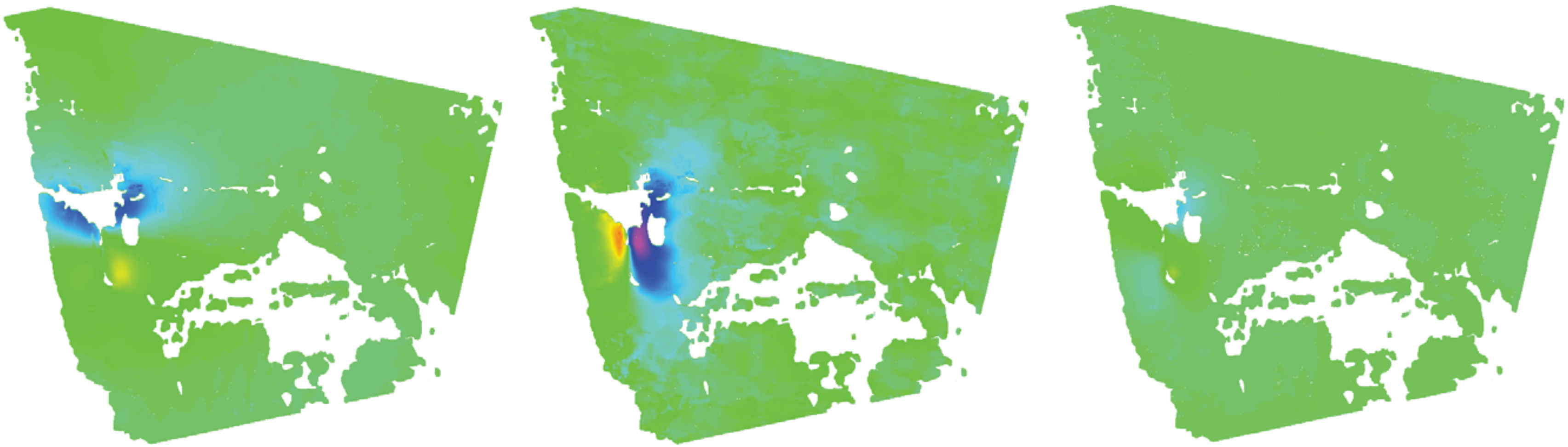
Determining 3D displacements with InSAR, azimuth offsets and multiple aperture interferometry

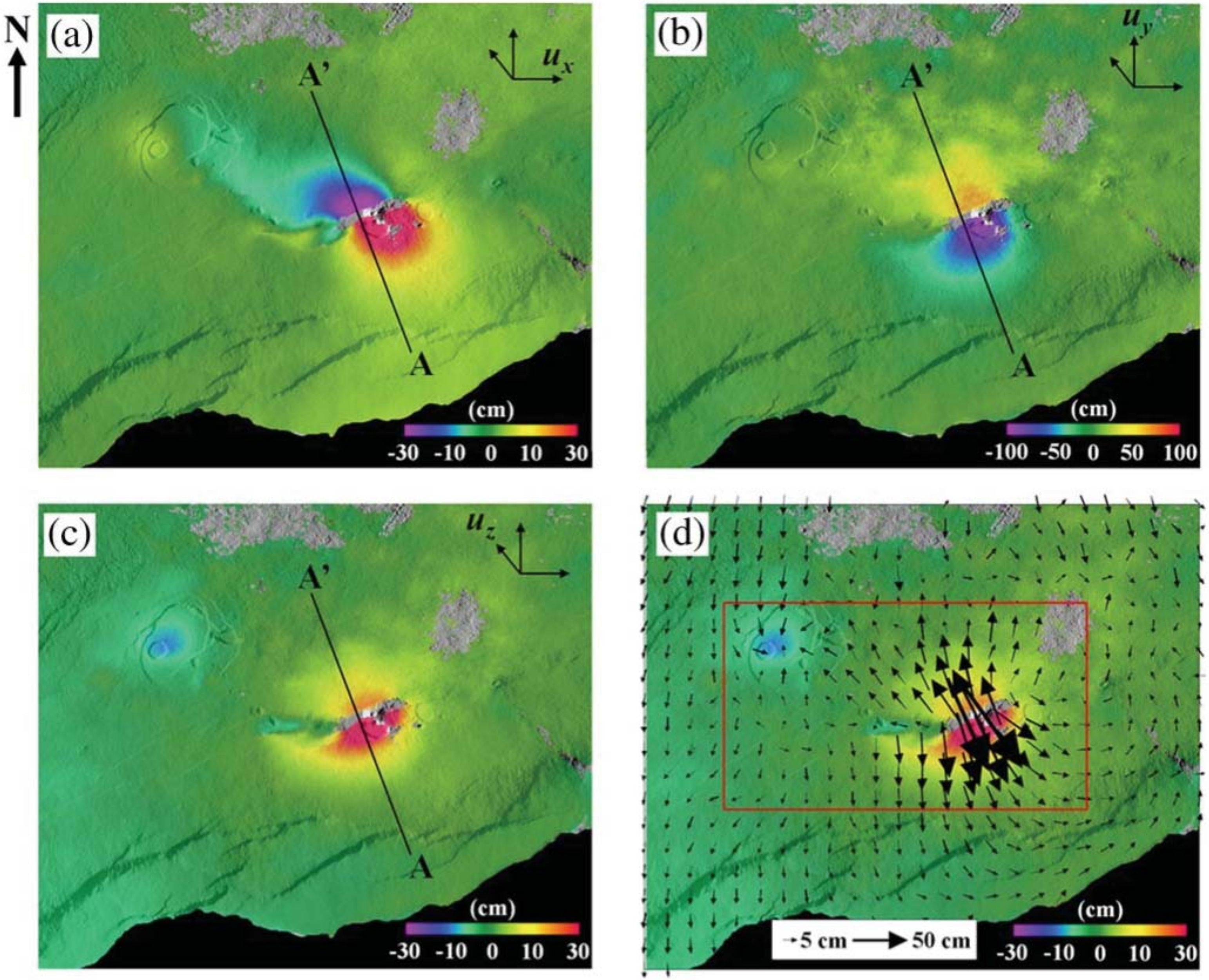


Gareth Funning

University of California, Riverside

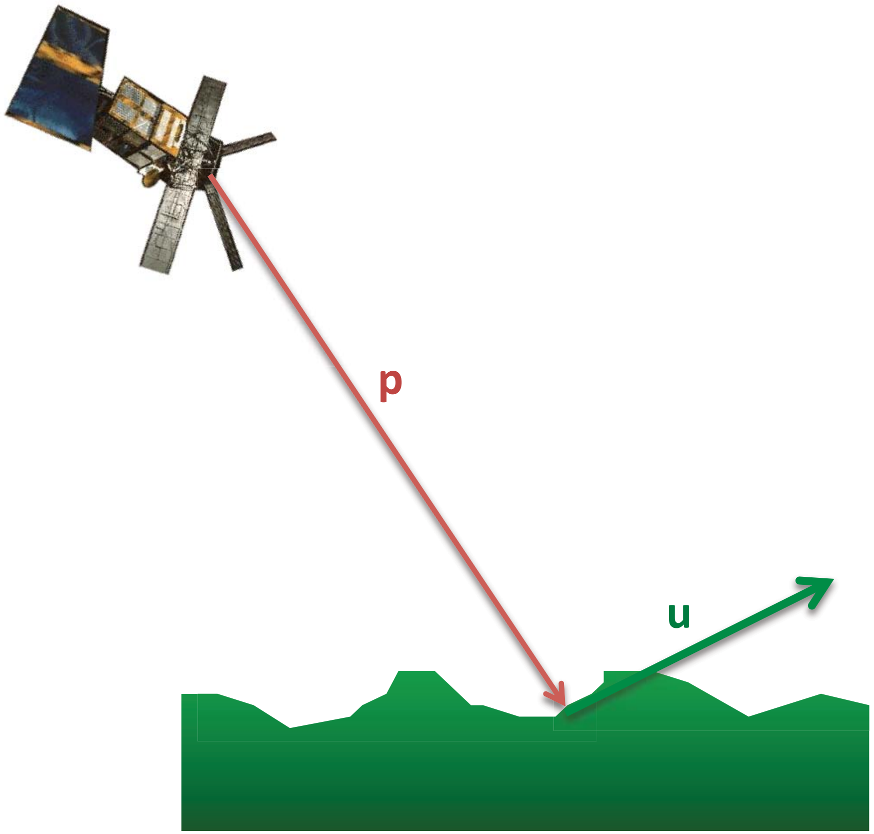
Outline

- 3D surface displacements: what we want
- How 3D displacements relate to range change
- Solving for 3D displacements
- How to constrain the N-S component
  - Azimuth offsets
  - MAI



Jung et al., 2011

Vector description of InSAR



u = ground displacement vector

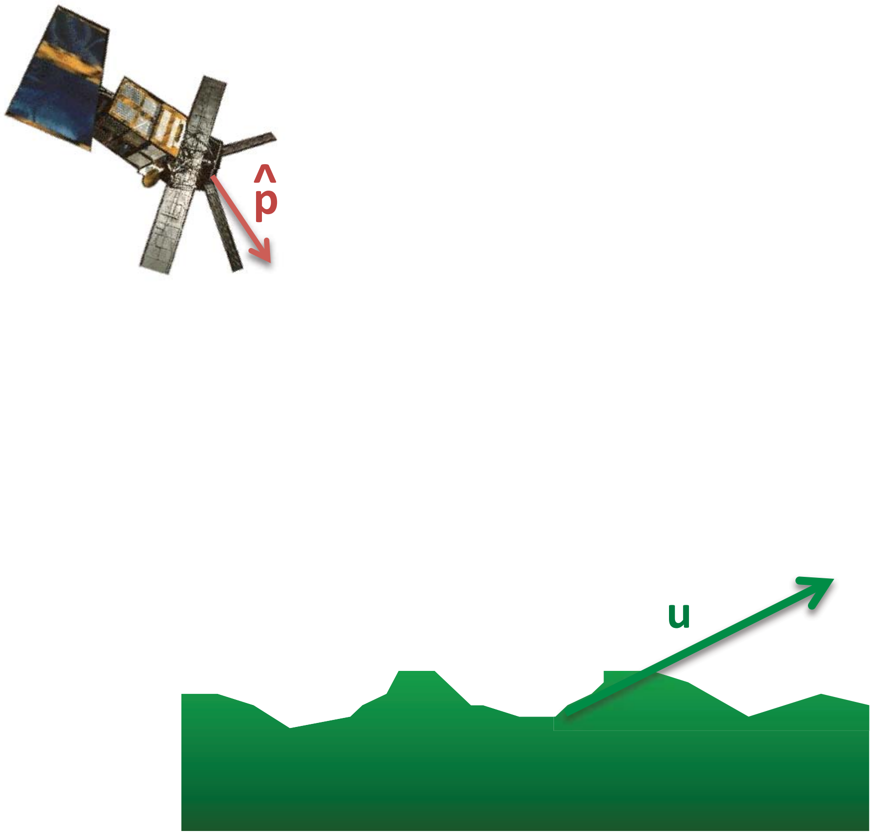
p = pointing vector (from satellite to ground target)

p is controlled by the satellite trajectory, beam mode (incidence angle) and position the pixel within the swath

The unit pointing vector

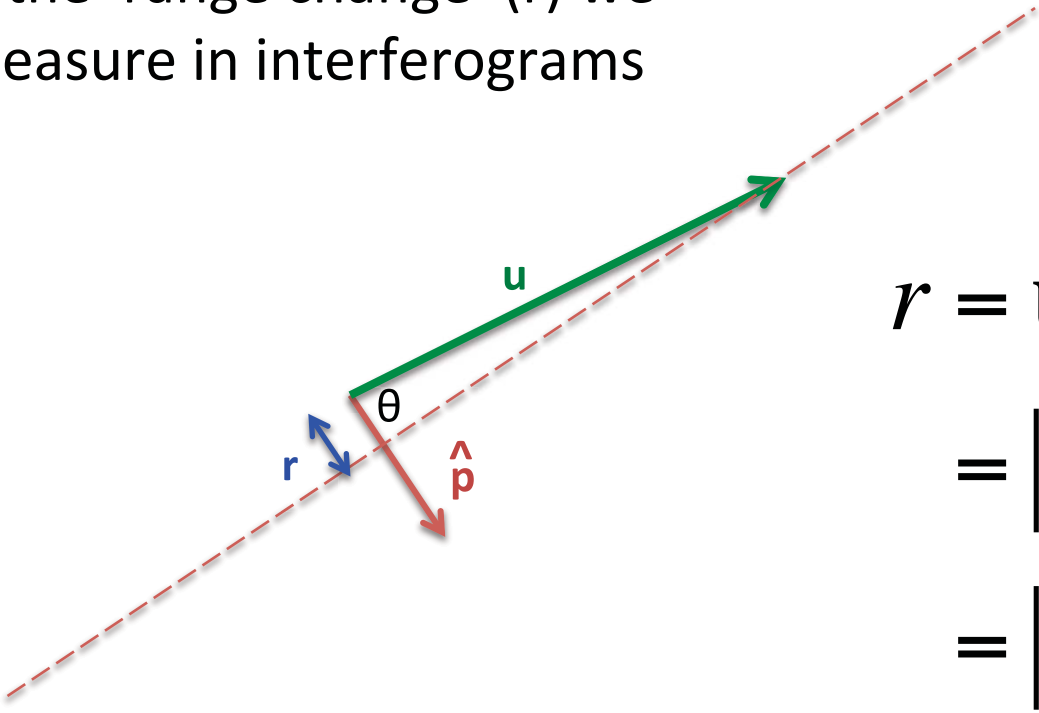
u = ground displacement vector

$\hat{\mathbf{p}}$ = <u>unit</u> pointing vector (from satellite to ground target)



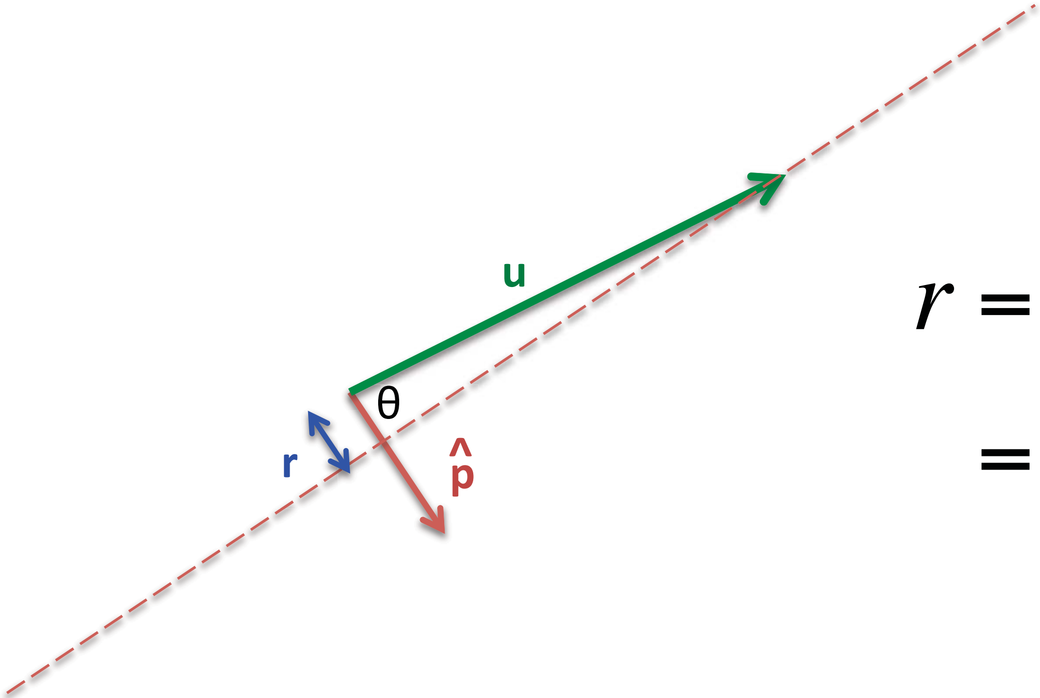
Range change

the scalar (dot) product of **u** and $\hat{\mathbf{p}}$ is the 'range change' (r) we measure in interferograms



$$r = \mathbf{u} \cdot \hat{\mathbf{p}}$$
$$= |\mathbf{u}||\hat{\mathbf{p}}|\cos\theta$$
$$= |\mathbf{u}|\cos\theta$$

These vectors are 3D!



$$r = \mathbf{u} \cdot \hat{\mathbf{p}}$$

$$= u_x \hat{p}_x + u_y \hat{p}_y + u_z \hat{p}_z$$

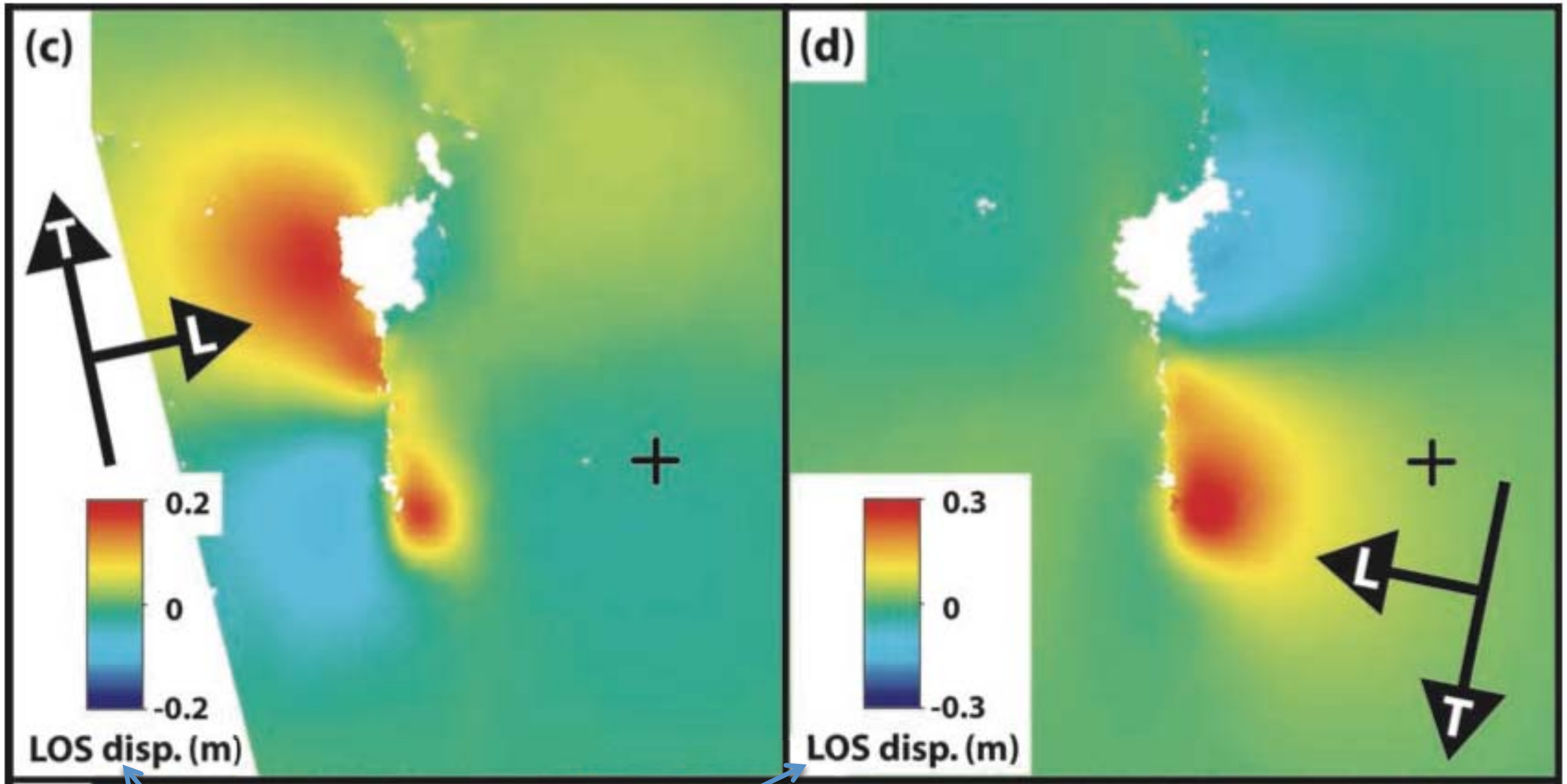
typically, we decompose **u** and $\hat{\mathbf{p}}$
into their Cartesian components

WARNING

Historically, people did not all use the same sign conventions in InSAR (including me…)

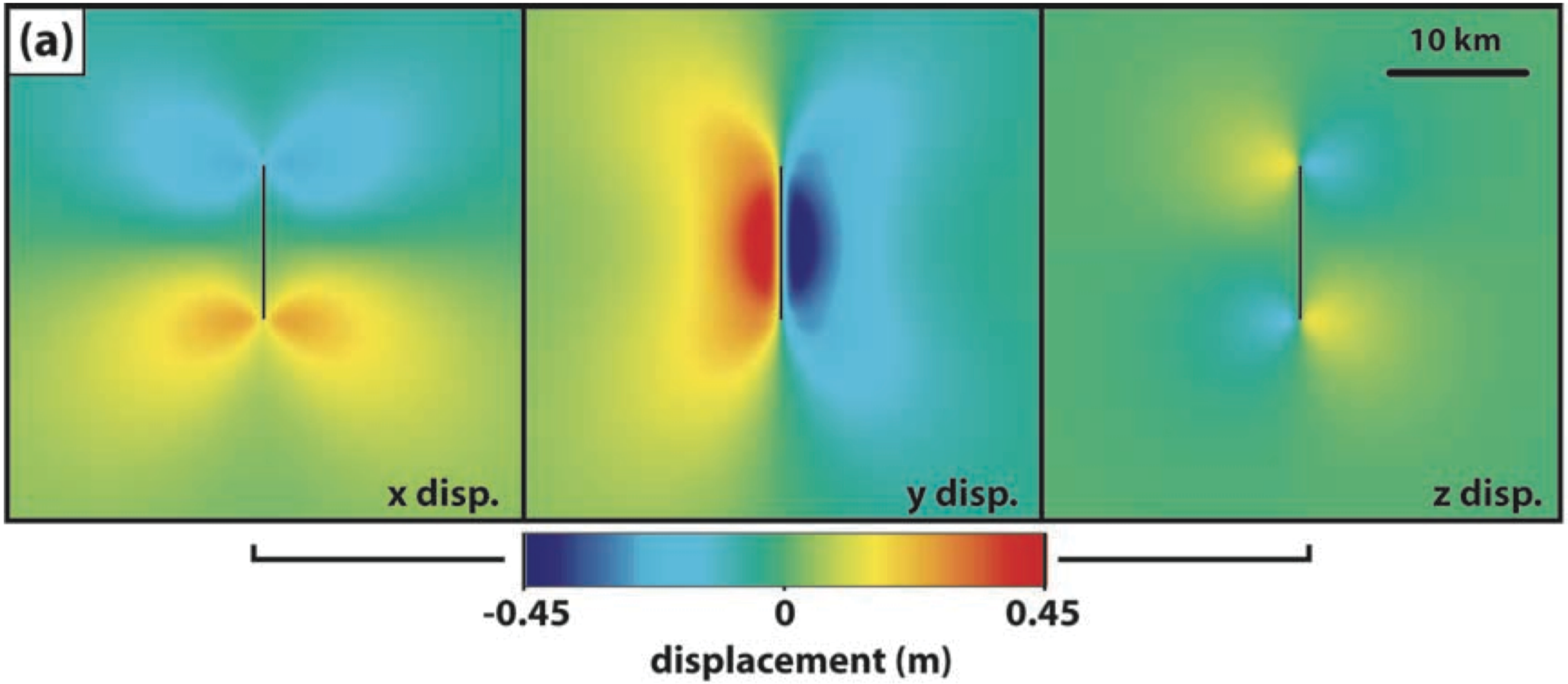
- Check whether your interferograms are 'range change' or 'ground LOS displacement'
- Check if your pointing vectors are consistent with your interferograms (pointing from satellite to ground, or ground to satellite?)

Example: 2003 Bam, Iran



ground LOS displacement(!)

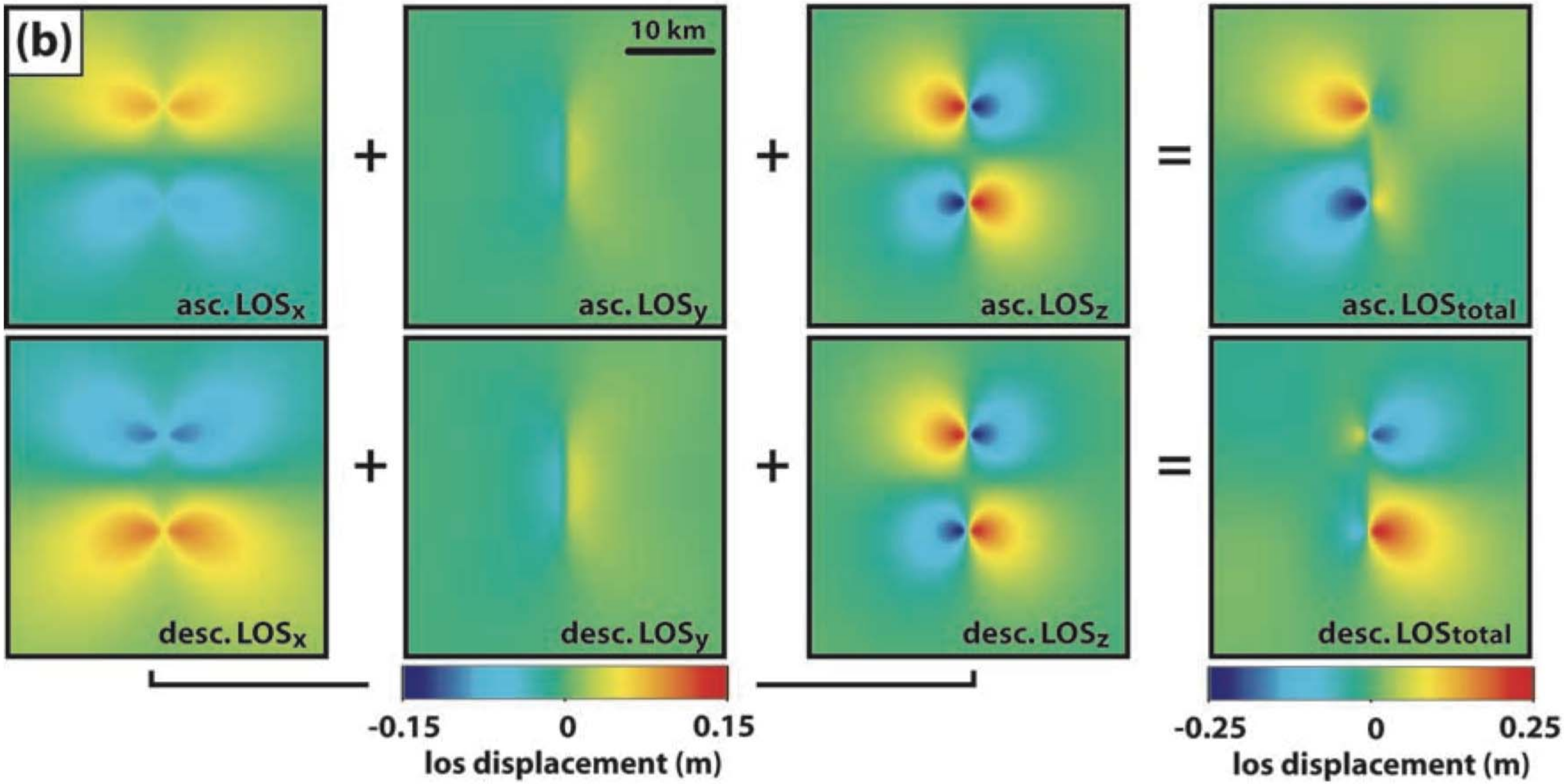
Funning et al., 2005



Forward model

- pure right-lateral strike-slip
- vertical dip
- N–S strike
- 1.8 m slip
- top 0.6 km depth
- bottom 13 km depth

Funning et al., 2005



When scaled by their pointing vector coefficients

– the north component contributes little to LOS

– the east and vertical components add on one side of the fault and cancel on the other

Funning et al., 2005

With 3 unknowns, we need 3 equations

(u_x, u_y, u_z)

3 different range-change observations...

$$r_1 = \mathbf{u} \cdot \hat{\mathbf{p}}_1$$

$$r_2 = \mathbf{u} \cdot \hat{\mathbf{p}}_2$$

$$r_3 = \mathbf{u} \cdot \hat{\mathbf{p}}_3$$

...from 3 different imaging geometries

[This is for a single pixel, assuming all data sets are detrended, referenced to a pixel in the far-field and sampled onto the same grid.]

expanding…

$$r_1 = \mathbf{u} \cdot \widehat{\mathbf{p}}_1 = u_x(\hat{p}_x)_1 + u_y(\hat{p}_y)_1 + u_z(\hat{p}_z)_1$$

$$r_2 = \mathbf{u} \cdot \widehat{\mathbf{p}}_2 = u_x(\hat{p}_x)_2 + u_y(\hat{p}_y)_2 + u_z(\hat{p}_z)_2$$

$$r_3 = \mathbf{u} \cdot \widehat{\mathbf{p}}_3 = u_x(\hat{p}_x)_3 + u_y(\hat{p}_y)_3 + u_z(\hat{p}_z)_3$$

in matrix form, this is

$$\begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = \begin{pmatrix} (\hat{p}_x)_1 & (\hat{p}_y)_1 & (\hat{p}_z)_1 \\ (\hat{p}_x)_2 & (\hat{p}_y)_2 & (\hat{p}_z)_2 \\ (\hat{p}_x)_3 & (\hat{p}_y)_3 & (\hat{p}_z)_3 \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

$$\mathbf{r} = \mathbf{P} \qquad \mathbf{u}$$

this can be solved by standard least squares methods:

$$\mathbf{r} = \mathbf{P}\,\mathbf{u}$$

$$\mathbf{P}^T \mathbf{r} = \mathbf{P}^T \mathbf{P}\,\mathbf{u}$$

$$(\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \mathbf{r} = (\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \mathbf{P}\,\mathbf{u}$$

$$\boxed{(\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \mathbf{r} = \mathbf{u}}$$

$$\mathbf{u} = (\mathbf{P}^T \mathbf{P})^{-1} \mathbf{P}^T \mathbf{r}$$

if you have estimates of the uncertainties in range change, you can use them to weight the inversion…

uncertainty in r_1 → σ_1^2

$$\mathbf{E} = \begin{pmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{pmatrix}$$

including weighting, we get:

$$\mathbf{u} = (\mathbf{P}^T \mathbf{E}^{-1} \mathbf{P})^{-1} \mathbf{P}^T \mathbf{E}^{-1} \mathbf{r}$$

with covariances in the estimate of **u** of:

$$\mathbf{C} = (\mathbf{P}^T \mathbf{E}^{-1} \mathbf{P})^{-1}$$

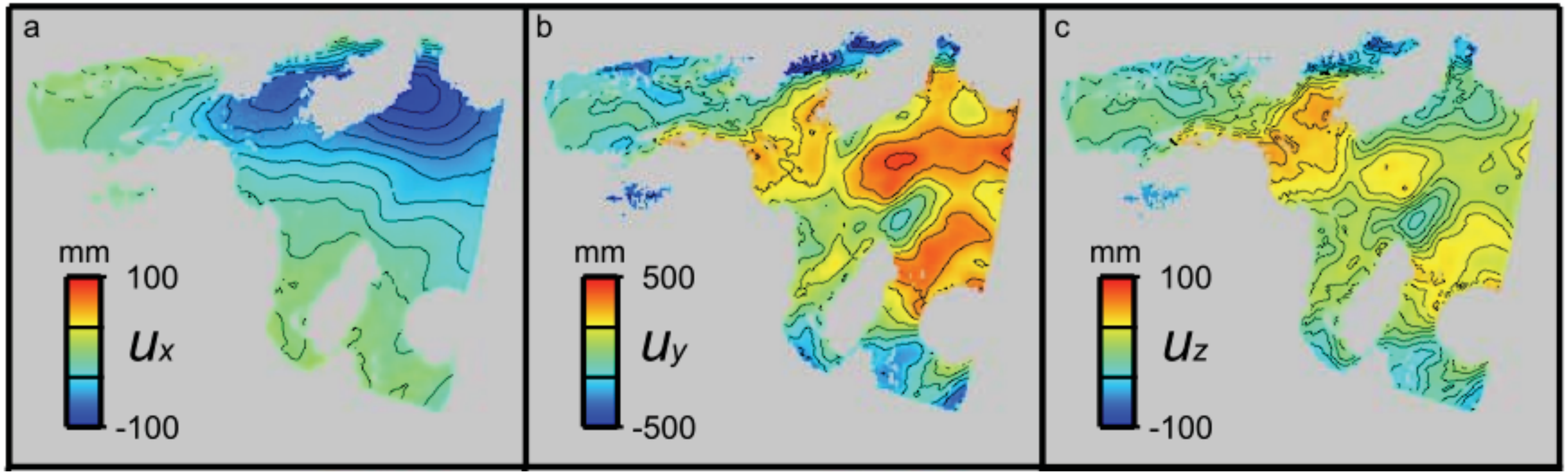
With 3 unknowns, we need at least 3 equations

Ascending + descending + ?

- Interferogram with a different incidence angle?
- Left- and right-looking interferograms?
- Some other measure of displacement?

2002 Nenana Mountain, AK

σ_x = 6 mm σ_y = 286 mm σ_z = 41 mm



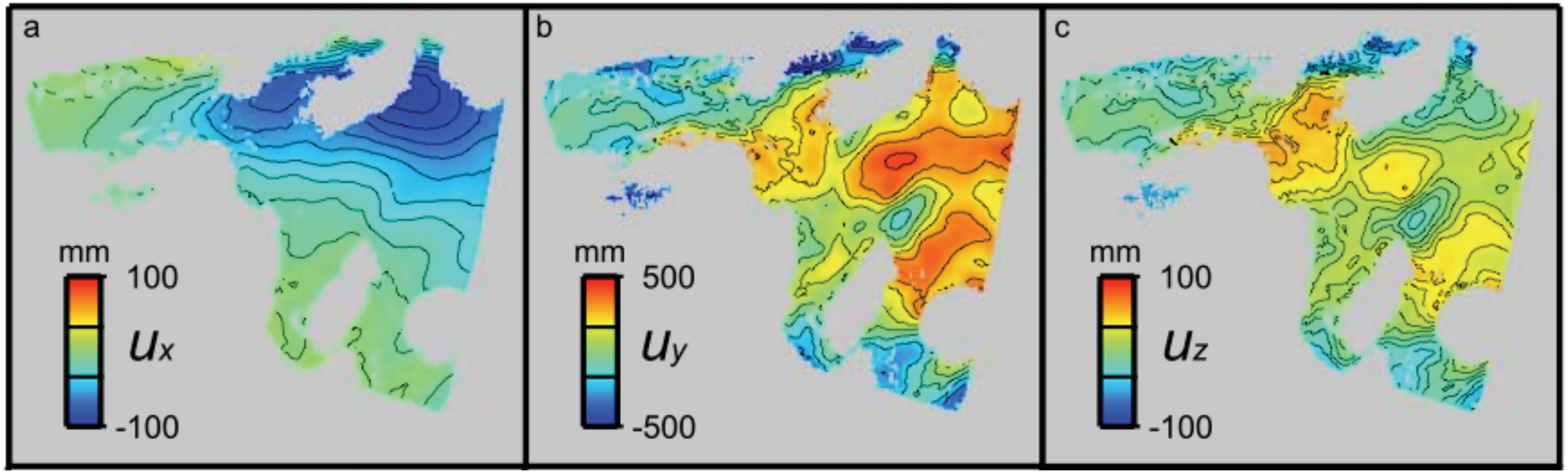
4 input interferograms:

Ascending and descending RADARSAT data with 24° incidence

Ascending and descending RADARSAT data with 45° incidence

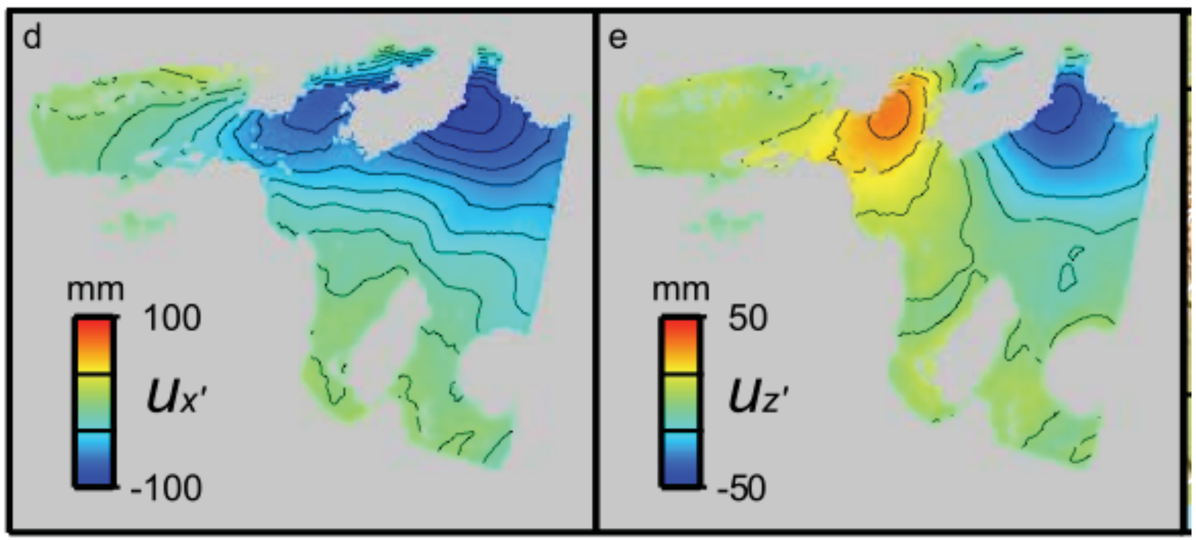
Wright et al., 2003

$\sigma_x = 6$ mm $\sigma_y = 286$ mm $\sigma_z = 41$ mm



Set u_y to zero, and you get:

$\sigma_{x'} = 6$ mm $\sigma_{z'} = 4$ mm



Wright et al., 2003

With 3 unknowns, we need at least 3 equations

Ascending + descending + ?

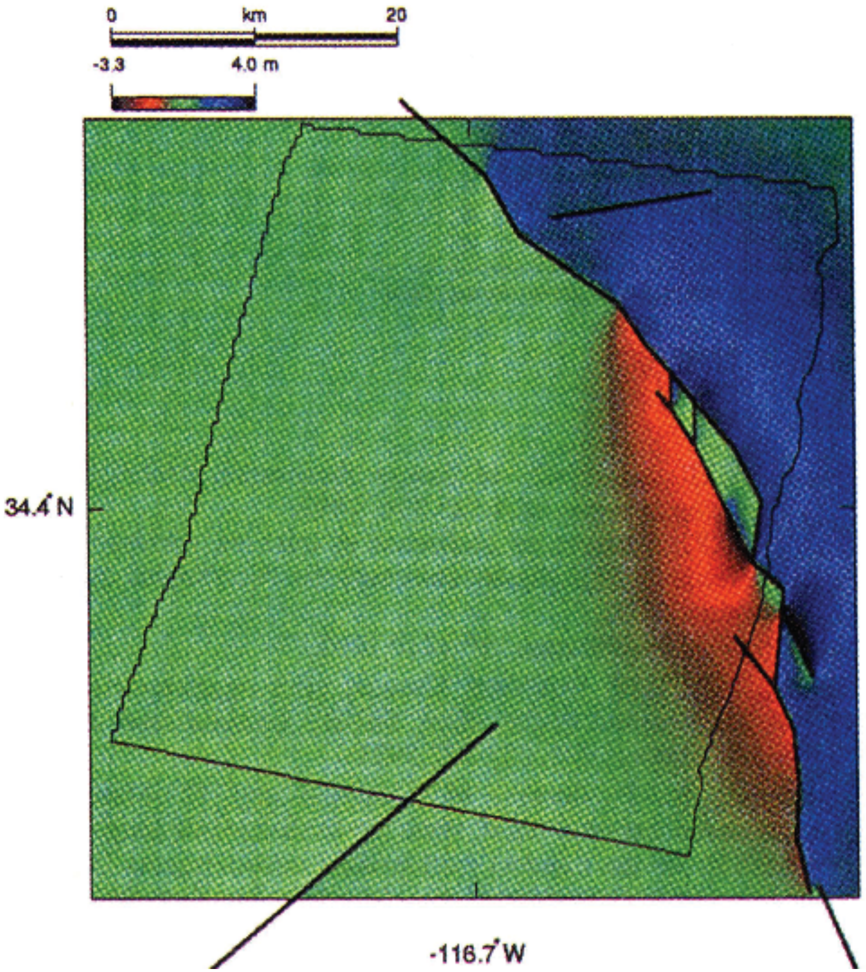
- Interferogram with a different incidence angle?
  doesn't work – not enough constraint on u_y
- Left- and right-looking interferograms?
  should work, but we have no satellites that can do it
- Some other measure of displacement?
  along-track component of deformation from azimuth offsets or MAI

1) Azimuth offsets

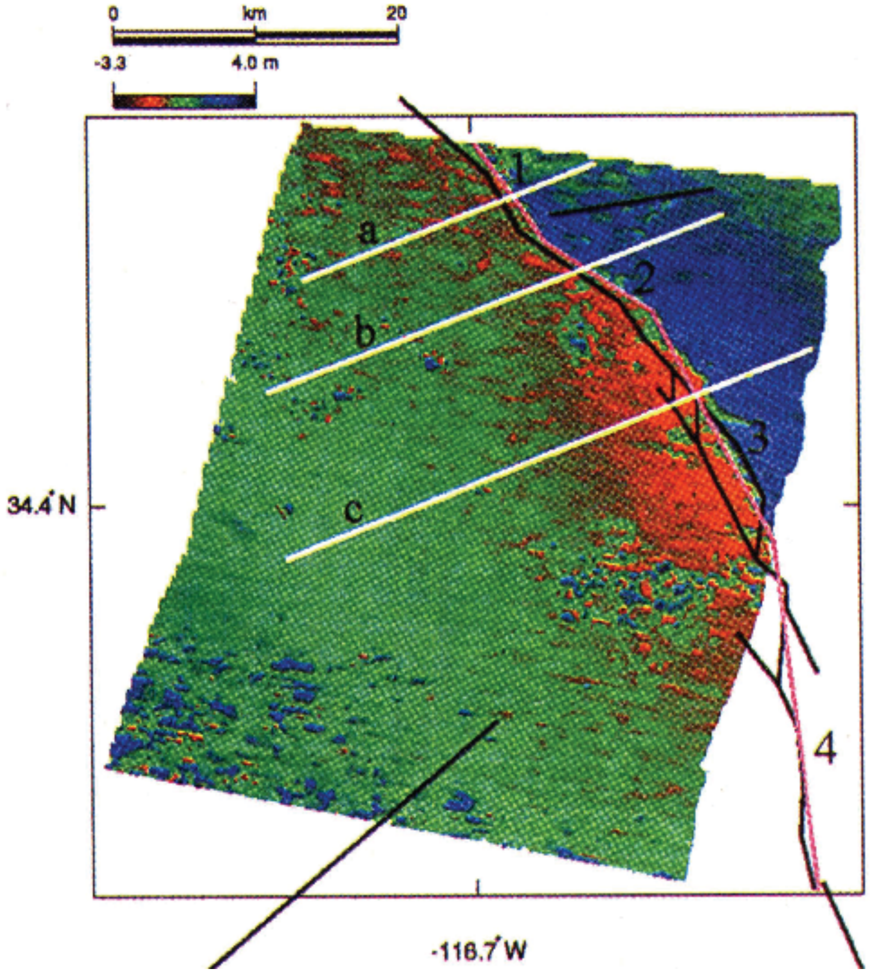
- Distortion to post-earthquake SAR image caused by displacement in along-track direction
- Obtained by sub-pixel matching of the SLC images
- Same process as used to coregister SLCs for InSAR, but at much greater density (e.g. 4 range looks), with resulting increase in time taken
- Low precision compared with InSAR (10s of cm)

1992 Landers, CA earthquake

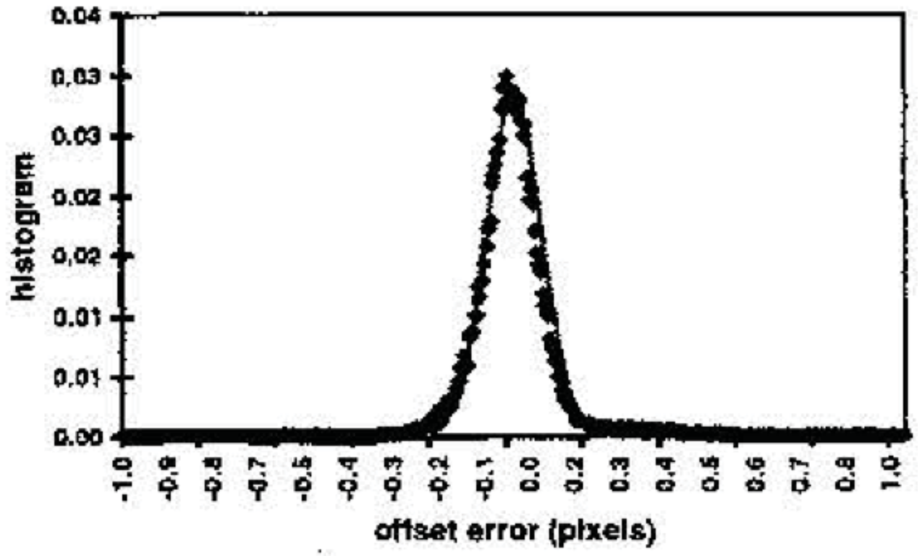
model (from Hudnut et al., 1994)



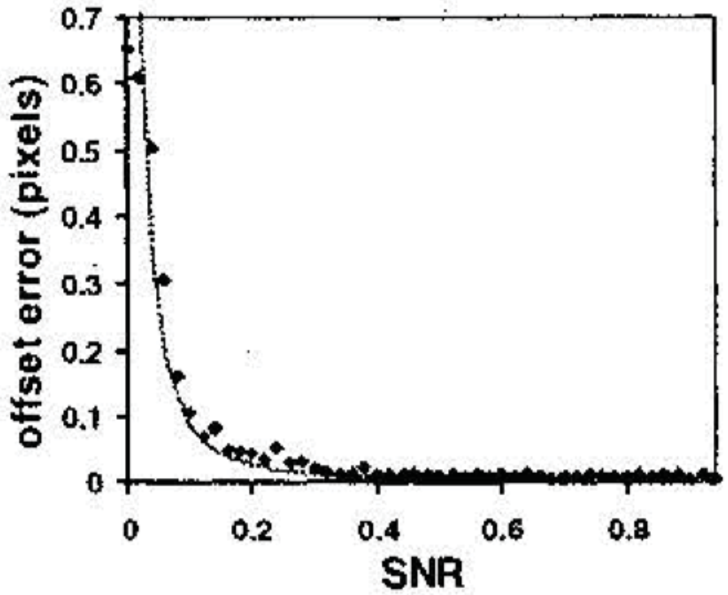
azimuth offsets



Michel et al., 1999



Precision estimated from matching images with no deformation – σ ≈ 0.1 pixel



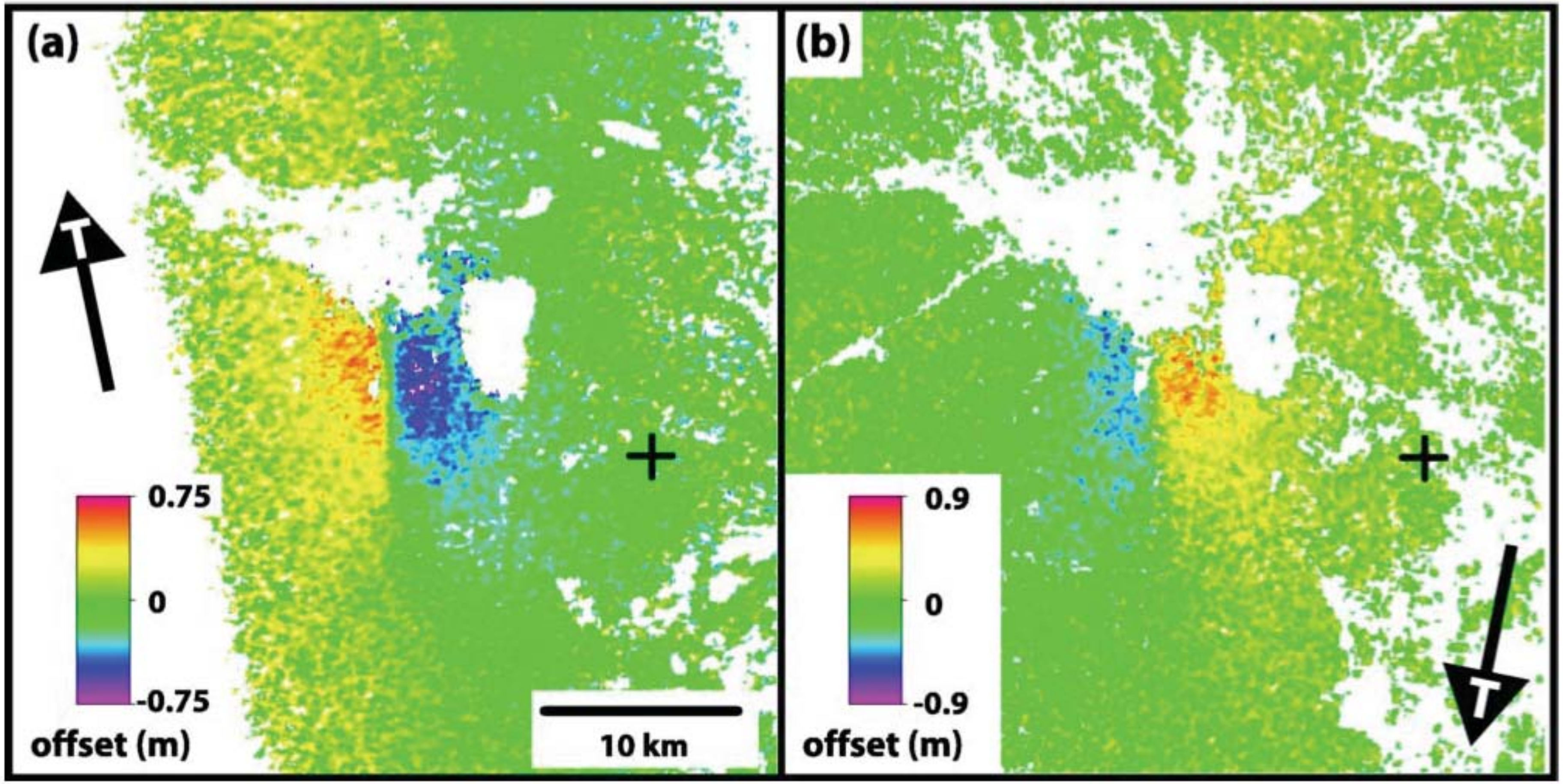
Relationship between signal-noise ratio and offset errors (mismatches)

Michel et al., 1999

2003 Bam, Iran earthquake

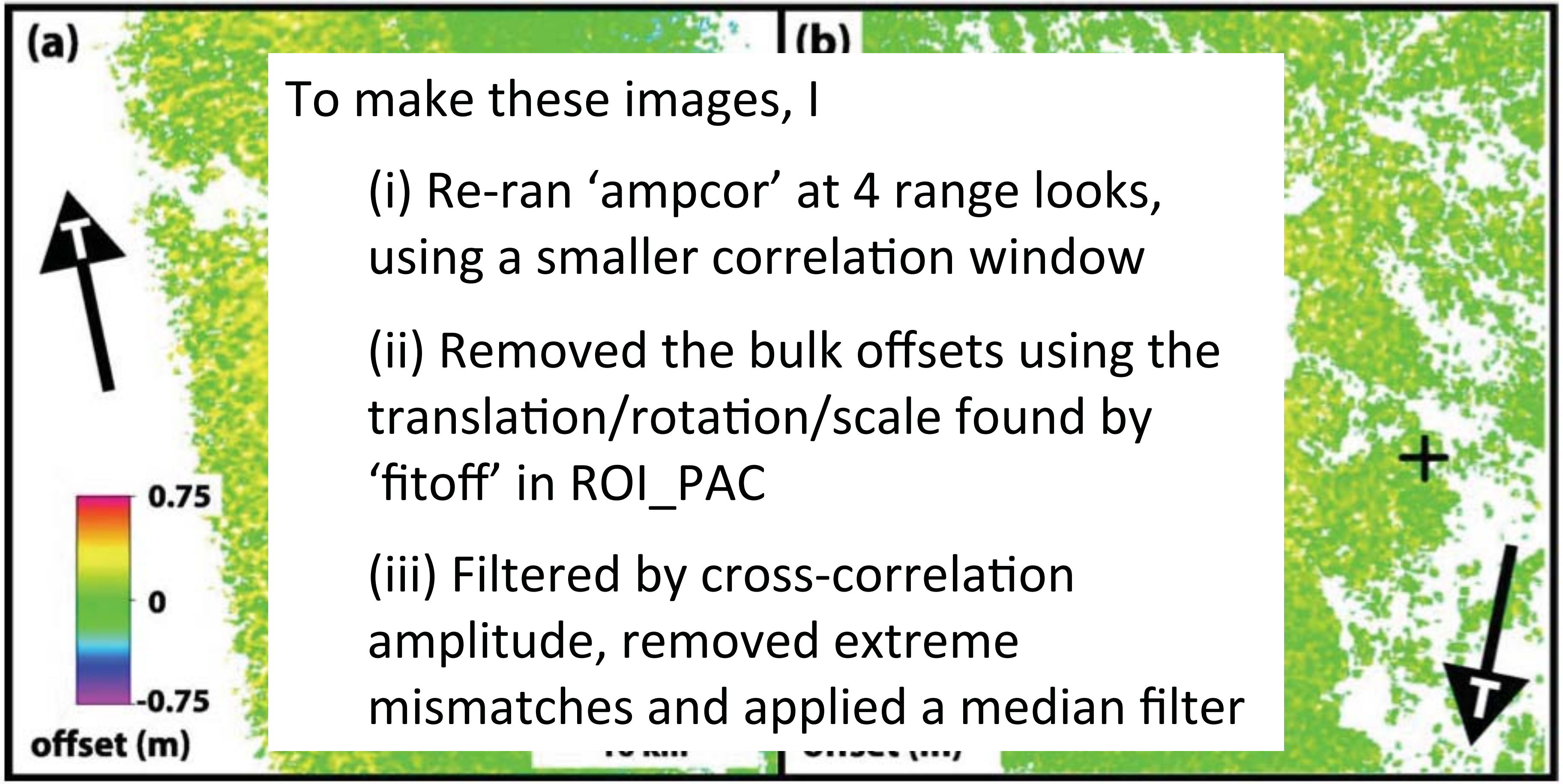
σ = 114 mm

σ = 117 mm



Funning et al., 2005

2003 Bam, Iran earthquake



To make these images, I

(i) Re-ran ‘ampcor’ at 4 range looks, using a smaller correlation window

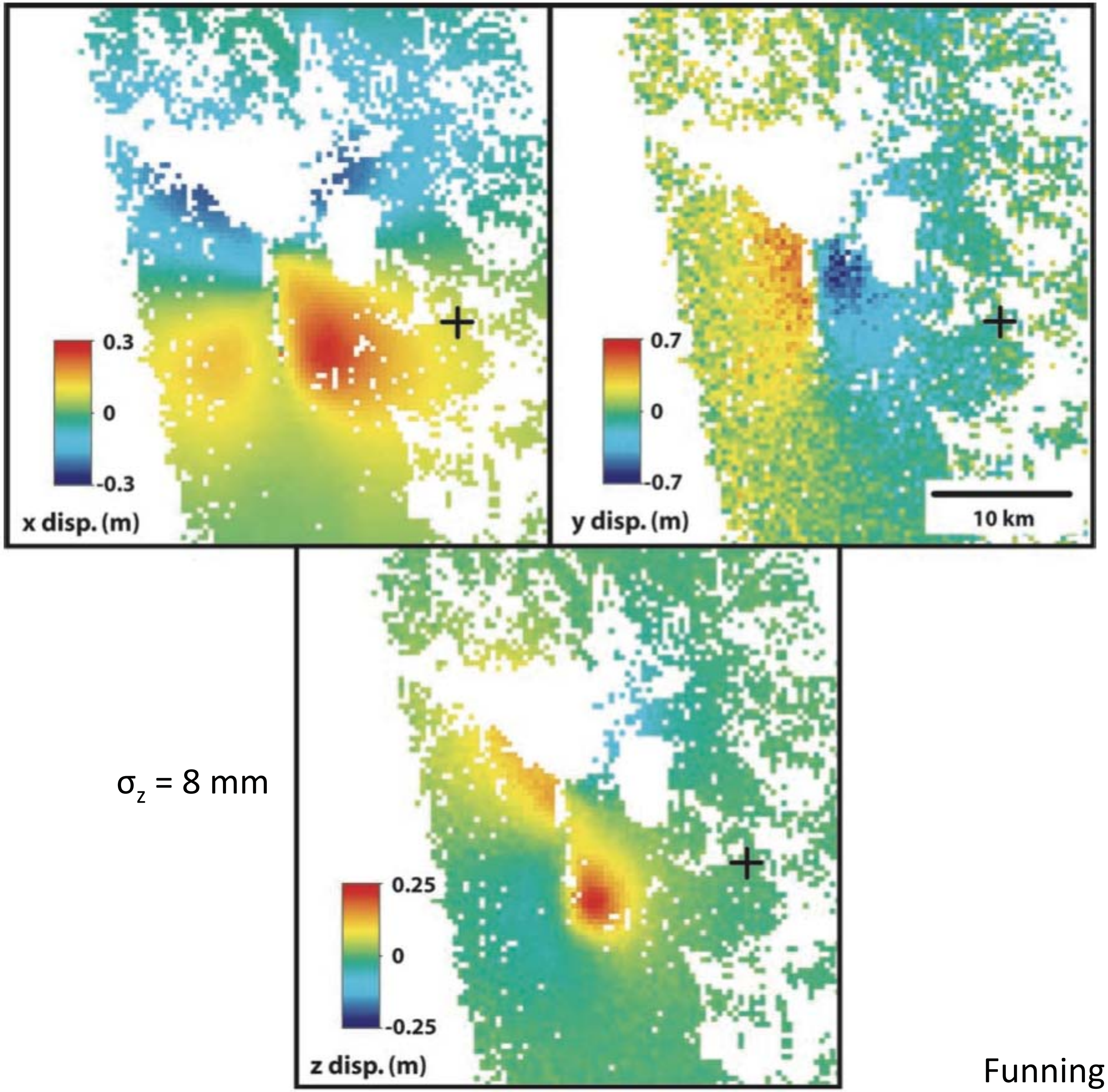
(ii) Removed the bulk offsets using the translation/rotation/scale found by ‘fitoff’ in ROI_PAC

(iii) Filtered by cross-correlation amplitude, removed extreme mismatches and applied a median filter

Funning et al., 2005

σ_x = 9 mm

σ_y = 89 mm



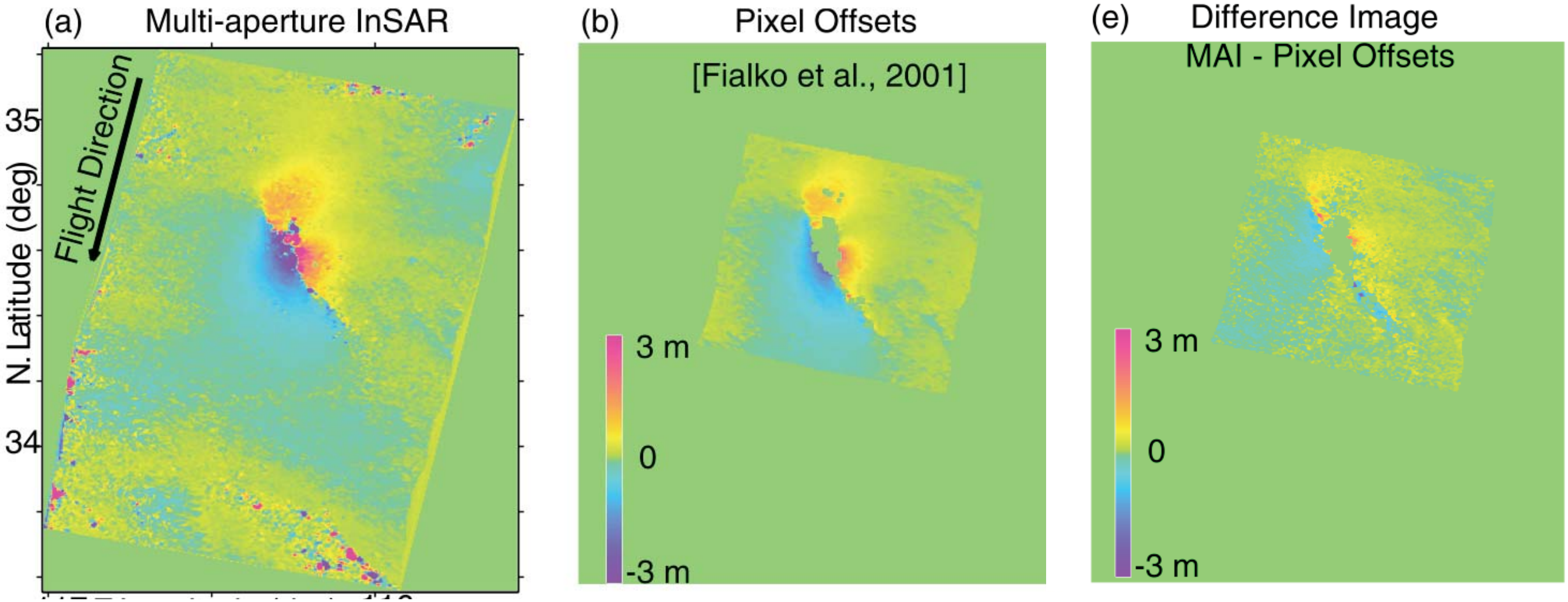
σ_z = 8 mm

Funning et al., 2005

2) Multi-Aperture Interferometry

- Split the synthetic aperture for both SAR images into forward- and backward-looking apertures
- Form forward- and backward-looking SLCs, and then interferograms
- The difference of those two interferograms is a measure of the along-track displacement
- Much faster to compute than azimuth offsets
- Lower signal-to-noise and precision than regular InSAR

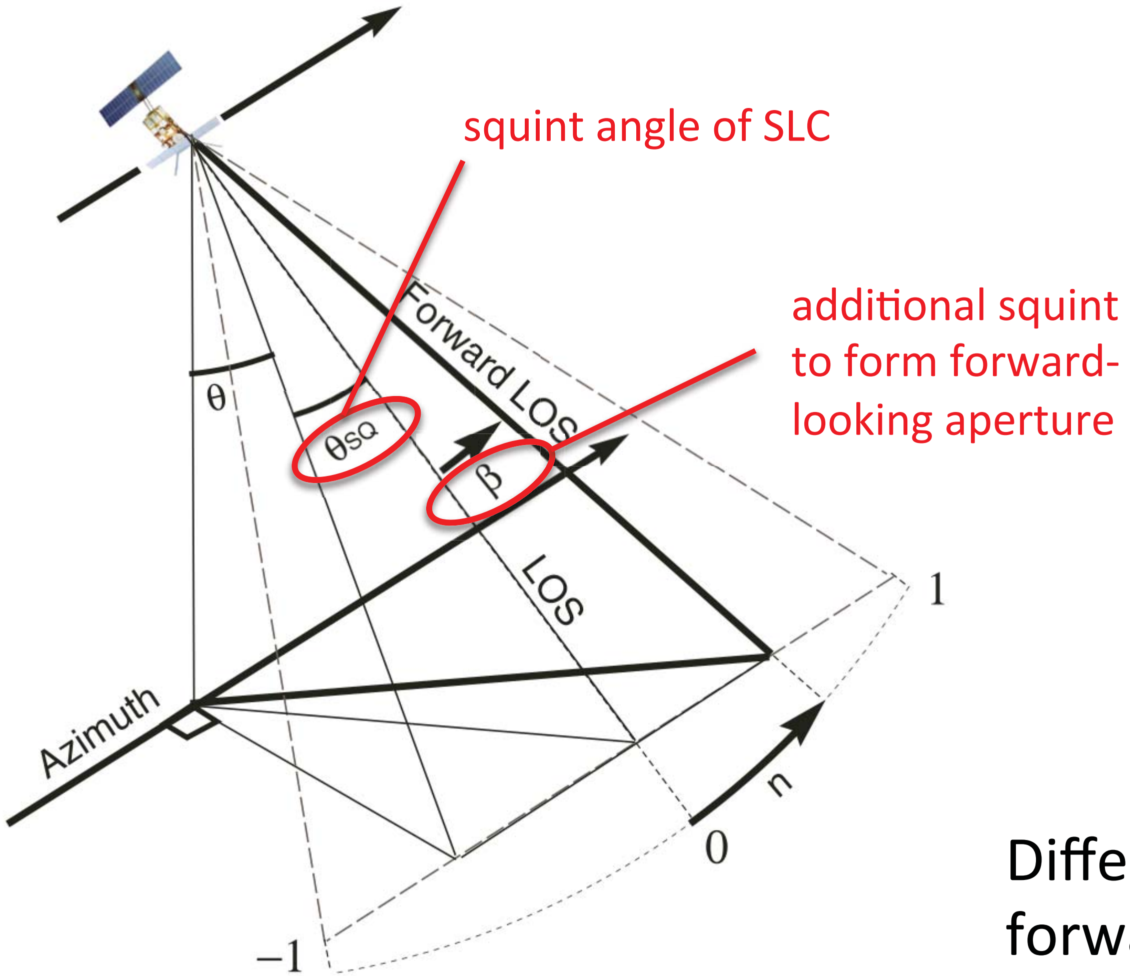
1999 Hector Mine, CA earthquake



σ ≈ 60 mm

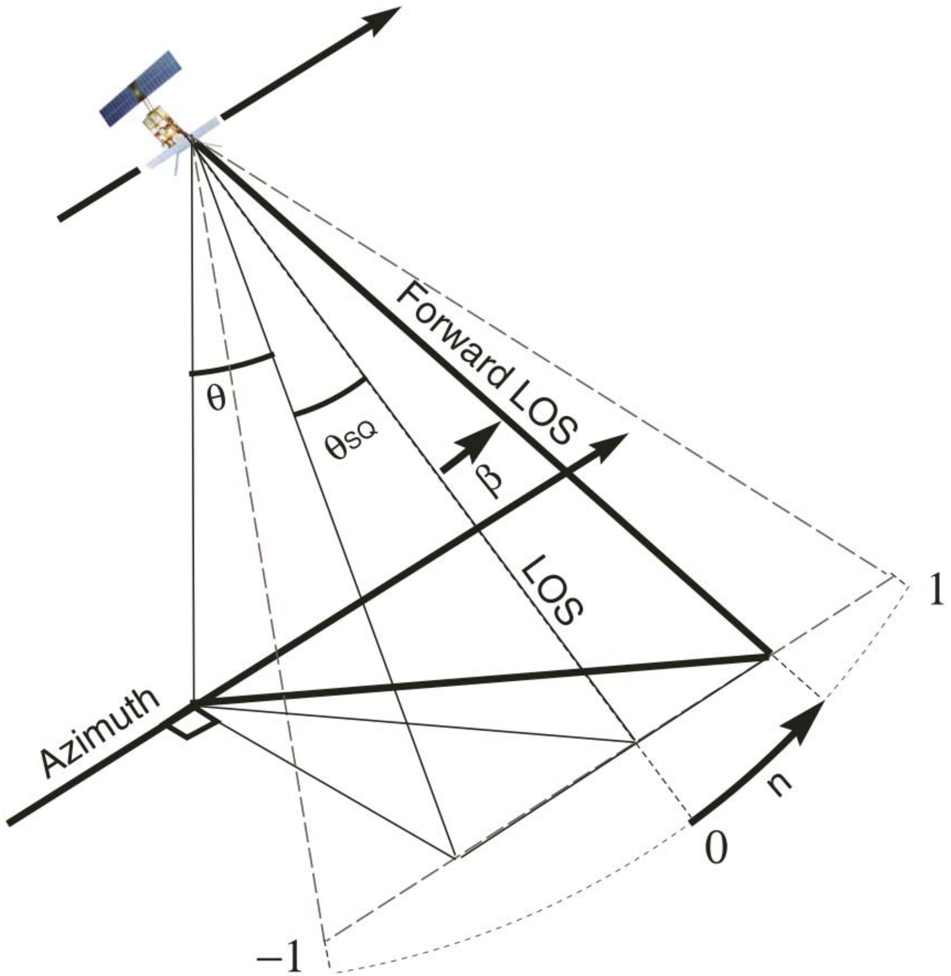
σ ≈ 105 mm

Bechor and Zebker, 2006



Difference between forward-looking and backward-looking range change is a function of along-track displacement

Bechor and Zebker, 2006



Interferogram phases for an along-track displacement x:

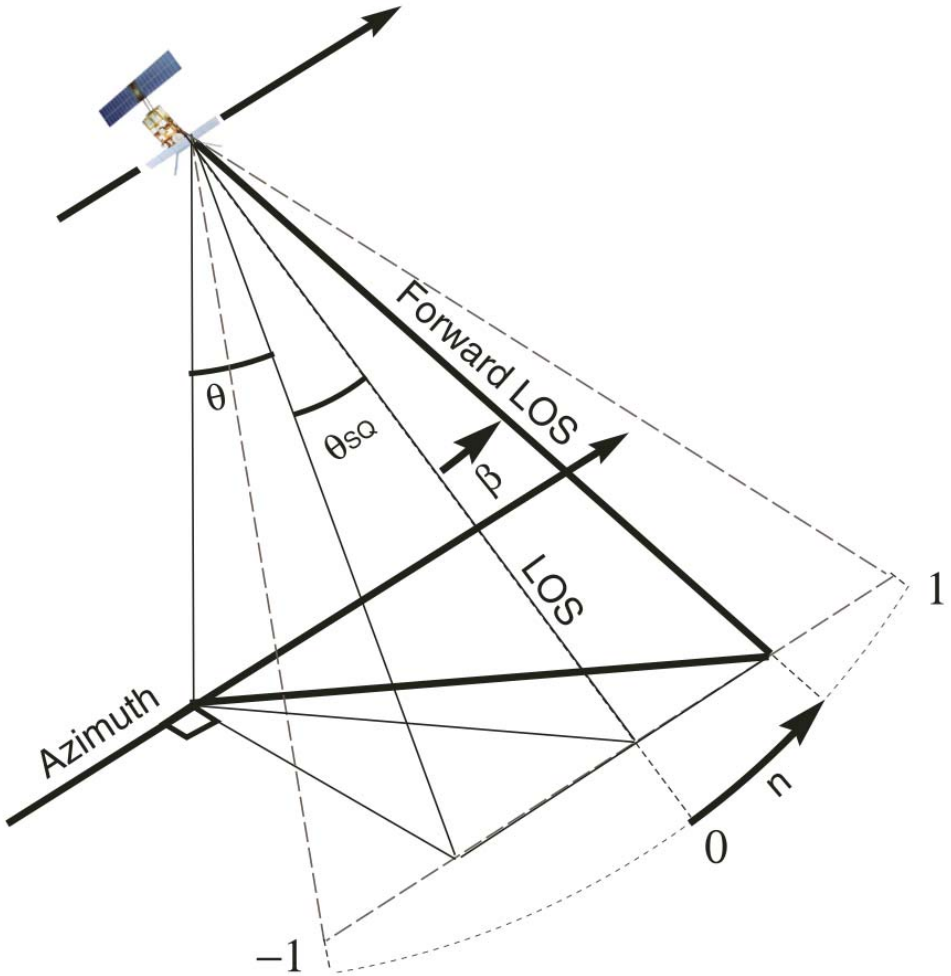
$$\phi_{forward} = -\frac{4\pi x}{\lambda}\sin\left(\theta_{SQ} + \beta\right)$$

$$\phi_{backward} = -\frac{4\pi x}{\lambda}\sin\left(\theta_{SQ} - \beta\right)$$

The MAI phase

$$\phi_{MAI} = \phi_{forward} - \phi_{backward}$$

$$= -\frac{4\pi x}{\lambda}\left(2\sin\beta\cos\theta_{SQ}\right)$$



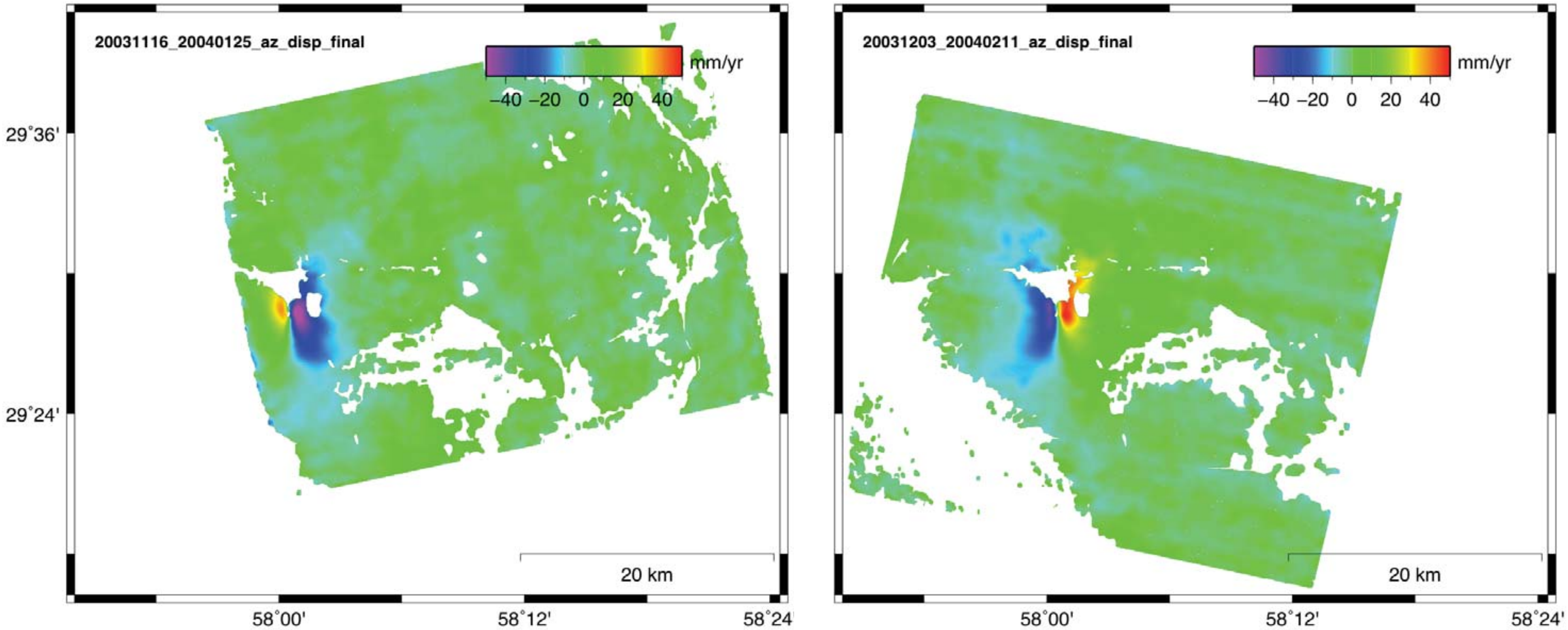
The MAI phase:

$$\phi_{MAI} = \phi_{forward} - \phi_{backward}$$
$$= -\frac{4\pi x}{\lambda}\left(2\sin\beta\cos\theta_{SQ}\right)$$

In practice, if we choose β to be one quarter of the antenna beam width, α, and if all angles are small…

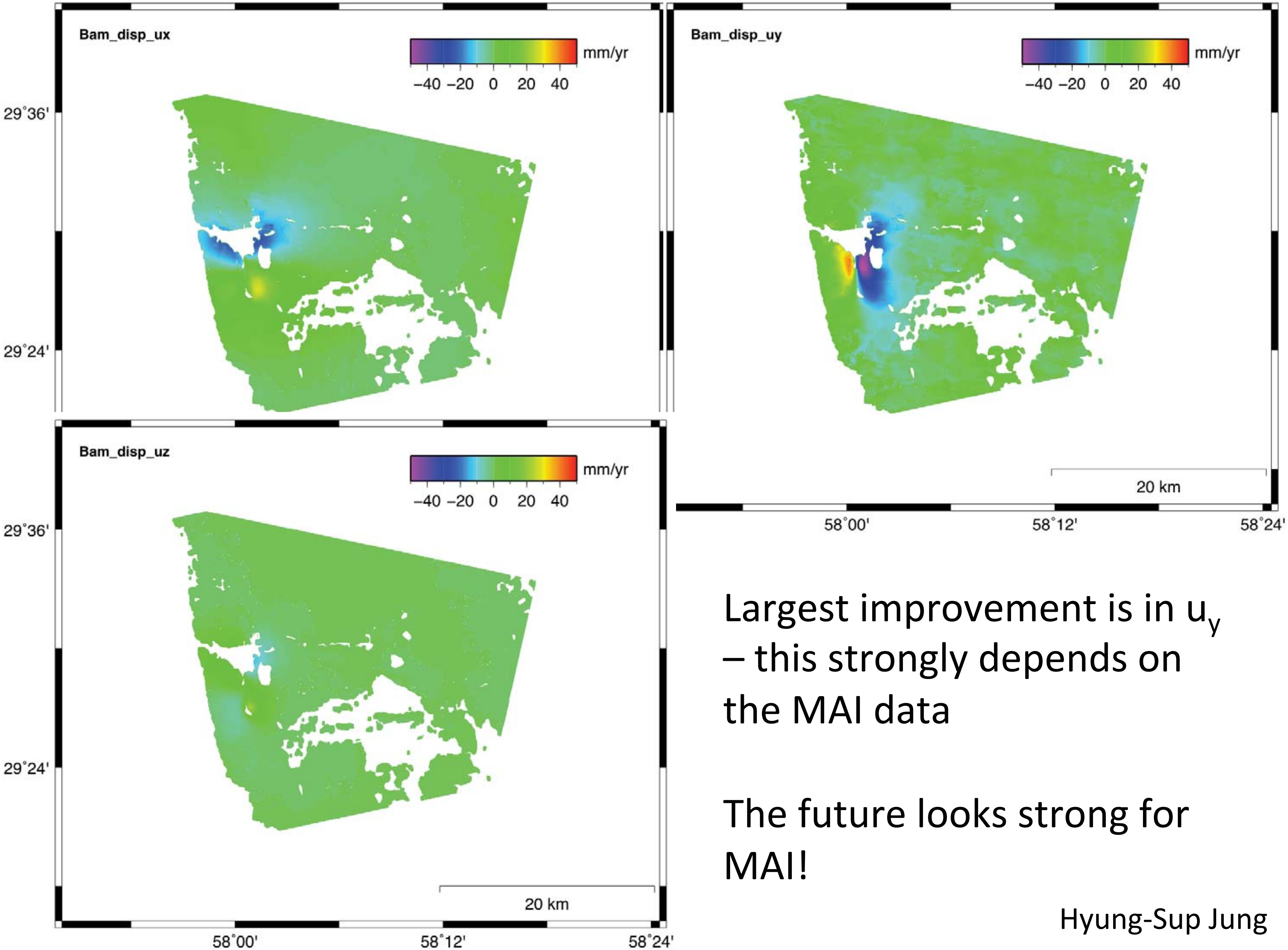
$$\phi_{MAI} = \frac{2\pi x}{\lambda}\alpha$$
$$= \frac{2\pi x}{l}$$

[assuming antenna length l ≈ λ / α]



MAI result is much 'cleaner' than the azimuth offsets

Hyung-Sup Jung



Largest improvement is in u_y – this strongly depends on the MAI data

The future looks strong for MAI!

Hyung-Sup Jung

Final thoughts

- Making a 3D displacement field is straightforward…
- …if you have along-track displacements
- Azimuth offsets are noisy and slow to compute…
- …but require no special software
- MAI measurements have approximately half the uncertainties of azimuth offsets…
- …but are not widely available at present (hopefully soon, as part of ISCE)