Change of variables as a universal approach to local and global regimes for β matrix models

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We consider the β matrix model with an arbitrary admissible potential V. Assuming that V is smooth enough and the corresponding equilibrium density ρ has a one-interval support $\sigma=[-2,2]$, we consider the change of variables $\lambda_i\to\zeta(\lambda_i)$ with $\zeta(\lambda)$ chosen from the equation

$\zeta'(\lambda)\rho(\zeta(\lambda))=\rho_{sc}(\lambda)$,

where $\rho_{sc}(\lambda)$ is the standard semicircle density. This gives us the "deformed" β matrix model which has an additional interaction term. Evidently the partition function and all marginal densities of the initial β ensemble can be expressed in terms of the new ensemble.
Then after some transformations we obtain that the "deformed" β matrix model may be reduced to the standard Gaussian β matrix model with a small perturbation $n^{-1}h(\lambda)$.
This reduces most of the problems of local and global regimes for β matrix models to the corresponding problems for the Gaussian model with a small perturbation.