Nonlocal Halo Bias

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Outline

- Argue specific form of nonlocal bias inevitably induced by large scale gravitational evolution. Present evidence from simulations.
- Discuss the possibility that bias is not local at the Lagrangian space.

References

- KCC, R Scoccimarro and R K Sheth, Gravity and nonlocal bias, Phys. Rev. D 85, 083509 (2012).
- R K Sheth, KCC and R Scoccimarro, Nonlocal Lagrangian bias, Phys. Rev. D 87, 083002 (2013).

Review of halo bias

- Nonlinear deterministic local bias (Fry and Gaztanaga 1993)

$$\delta_{\rm h} = b_0 + b_1\delta + \frac{b_2}{2}\delta^2 + \frac{b_3}{3!}\delta^3 + \ldots$$

- The bias parameters b_i, often taken as free parameters.
- Local: δ_h depends on the local dark matter δ, small δ, can be described by a simple polynomial in δ.
- Used to interpret observational data.
- Works reasonably well, some problems when compared to simulations, e.g. bias from different measurements don't agree.
- For precision cosmology, further corrections may be required: nonlocality and stochasticity.

Nonlocal bias from large scale gravitational evolution: Halo conservation model

- Local at the formation time, subsequent gravitational evolution induces nonlocality.
- Nonlocal: extra other degrees of freedom other than d and the stochastic noise. (Many forms of nonlocal bias, e.g. different Peak bias in Vincent and Matteo's talk)
- Assume halo number density conserves after formation.
- Velocity bias. Velocity field of the halos can be different from that of the dark matter.
- Use the fluid approximation for dark matter and halos.

Halo conservation model

$$
\begin{aligned}
\frac{\partial\delta}{\partial\tau} + \theta &= -\int \mathrm{d}^3k_1\,\mathrm{d}^3k_2\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{12})\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\theta(\mathbf{k}_1)\,\delta(\mathbf{k}_2),\\
\frac{\partial\theta}{\partial\tau} + \mathcal{H}\,\theta + \frac{3}{2}\mathcal{H}^2\,\Omega_{\mathrm{m}}\,\delta &= -\int \mathrm{d}^3k_1\,\mathrm{d}^3k_2\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{12})\,\beta(\mathbf{k}_1,\mathbf{k}_2)\,\theta(\mathbf{k}_1)\,\theta(\mathbf{k}_2),\\
\frac{\partial\delta_{\mathrm{g}}}{\partial\tau} + \theta_{\mathrm{g}} &= -\int \mathrm{d}^3k_1\,\mathrm{d}^3k_2\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{12})\,\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\theta_{\mathrm{g}}(\mathbf{k}_1)\,\delta_{\mathrm{g}}(\mathbf{k}_2),\\
\frac{\partial\theta_{\mathrm{g}}}{\partial\tau} + \mathcal{H}\,\theta_{\mathrm{g}} + \frac{3}{2}\mathcal{H}^2\,\Omega_{\mathrm{m}}\,\delta &= -\int \mathrm{d}^3k_1\,\mathrm{d}^3k_2\,\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{12})\,\beta(\mathbf{k}_1,\mathbf{k}_2)\,\theta_{\mathrm{g}}(\mathbf{k}_1)\,\theta_{\mathrm{g}}(\mathbf{k}_2).
\end{aligned}
$$

The mode-coupling kernels α and β are defined as

$$
\alpha(\mathbf{k}_1,\mathbf{k}_2) = \frac{\mathbf{k}_{12}\cdot\mathbf{k}_1}{k_1^2}, \qquad \beta(\mathbf{k}_1,\mathbf{k}_2) = \frac{k_{12}^2(\mathbf{k}_1\cdot\mathbf{k}_2)}{2k_1^2k_2^2}.
$$

Using the variable Ψ

$$
\Psi = \begin{pmatrix} \delta \\ -\theta/f\mathcal{H} \\ \delta_{\mathrm{g}} \\ -\theta_{\mathrm{g}}/f\mathcal{H} \end{pmatrix},
$$

we can write the equations compactly as

$$
\partial_y\Psi_a(\mathbf{k}) + \Omega_{ab}\Psi_b(\mathbf{k}) = \gamma_{abc}\,\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2),
$$

where the entries of γ_{abc} are zero except for

$$
\begin{aligned}
\gamma_{121} &= \gamma_{343} = \delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{12})\,\alpha(\mathbf{k}_1,\mathbf{k}_2),\\
\gamma_{222} &= \gamma_{444} = \delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_{12})\,\beta(\mathbf{k}_1,\mathbf{k}_2).
\end{aligned}
$$

Halo conservation model

Assume the initial condition can be written as

$$\phi_a(\mathbf{k}) = \sum_n \int \mathrm{d}^3q_1 \ldots \mathrm{d}^3q_n \, \delta_\mathrm{D}(\mathbf{k} - \mathbf{q}_{12\ldots n}) \, \mathcal{I}_a^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n) \, \delta_0(\mathbf{q}_1) \ldots \delta_0(\mathbf{q}_n),$$

and the vector $\Psi_a(\mathbf{k}, y)$ can be similarly expanded as

$$\Psi_a(\mathbf{k}, y) = \sum_n \int \mathrm{d}^3q_1 \ldots \mathrm{d}^3q_n \, \delta_\mathrm{D}(\mathbf{k} - \mathbf{q}_{12\ldots n}) \, \mathcal{K}_a^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n, y) \, \delta_0(\mathbf{q}_1) \ldots \delta_0(\mathbf{q}_n).$$

Then the $\mathcal{K}_a^{(n)}$ kernels is given by

$$\begin{aligned} \mathcal{K}_a^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n, y) &= g_{ab}(y) \, \mathcal{I}_b^{(n)}(\mathbf{q}_1, \ldots, \mathbf{q}_n) + \sum_{j=1}^{n-1} \int_0^y \mathrm{d}y' g_{ab}(y - y') \, \gamma_{bcd}(\mathbf{k}_1 = \mathbf{q}_{1\ldots j}, \mathbf{k}_2 = \mathbf{q}_{j+1\ldots n}) \\ &\times \; \mathcal{K}_c^{(j)}(\mathbf{q}_1, \ldots, \mathbf{q}_j, y') \, \mathcal{K}_d^{(n-j)}(\mathbf{q}_{j+1}, \ldots, \mathbf{q}_n, y'), \end{aligned}$$

where g_{ab} is

$$g_{ab} = \begin{pmatrix} \frac{2}{5}\mathrm{e}^{-3y/2} + \frac{3}{5}\mathrm{e}^{y} & -\frac{2}{5}\mathrm{e}^{-3y/2} + \frac{2}{5}\mathrm{e}^{y} & 0 & 0 \\ -\frac{3}{5}\mathrm{e}^{-3y/2} + \frac{3}{5}\mathrm{e}^{y} & \frac{3}{5}\mathrm{e}^{-3y/2} + \frac{2}{5}\mathrm{e}^{y} & 0 & 0 \\ -1 + \frac{2}{5}\mathrm{e}^{-3y/2} + \frac{3\mathrm{e}^{y}}{5} & \left(-2 - \frac{2}{5}\mathrm{e}^{-3y/2} + 2\mathrm{e}^{-y/2} + \frac{2\mathrm{e}^{y}}{5}\right) & 1 & 2(1 - \mathrm{e}^{-y/2}) \\ -\frac{3}{5}\mathrm{e}^{-3y/2} + \frac{3\mathrm{e}^{y}}{5} & \frac{3}{5}\mathrm{e}^{-3y/2} - \mathrm{e}^{-y/2} + \frac{2\mathrm{e}^{y}}{5} & 0 & \mathrm{e}^{-y/2} \end{pmatrix}.$$

e^y denotes the linear growth factor.

Evolution of linear density bias and velocity bias

- Using the initial condition

$$\mathcal{I}_a^{(1)} = \begin{pmatrix} 1 \\ 1 \\ b_1^* \\ b_\mathrm{v}^* \end{pmatrix}$$

- The linear density bias is given by

$$b_1 \equiv \frac{\mathcal{K}_3^{(1)}(y)}{\mathcal{K}_1^{(1)}(y)} = 1 + (b_1^* - 1)\mathrm{e}^{-y} + 2(b_\mathrm{v}^* - 1)\mathrm{e}^{-y}(1 - \mathrm{e}^{-y/2})$$

- The velocity bias is

$$b_\mathrm{v} \equiv \frac{\mathcal{K}_4^{(1)}(y)}{\mathcal{K}_2^{(1)}(y)} = 1 + (b_\mathrm{v}^* - 1)\,\mathrm{e}^{-3y/2}$$

- Bias is local at linear order.

Evolution of bias to second order

- We are particularly interested in the nonlocal terms induced by evolution, we consider $\chi^{(2)} = \delta_{\rm h}^{(2)} - b_1 \delta^{(2)}$
- The kernel for $\chi^{(2)}$ in Fourier space is quadratic in the angular variable, so further decompose them in Legendre polynomials
- When there is no initial velocity bias, i.e. $b_v = 1$, the results reduce to

$$\chi_0^{(2)} = \frac{b_2^*}{2} + \frac{4}{21}\epsilon_\delta, \quad \chi_1^{(2)} = 0, \quad \chi_2^{(2)} = -\frac{4}{21}\epsilon_\delta,$$

with $\epsilon_\delta \equiv (b_1 - 1)\, {\rm e}^y \left({\rm e}^y - 1\right)$.

- No dipole term is induced as there is no velocity difference between dark matter and halo.
- The induced quadrupole and monopole are of the same magnitude but opposite sign.
- The presence of quadrupole indicates that the bias is nonlocal.

Bias evolution with galaxy formation/merger source

- Previously we assume halo number density conserves. But real galaxies form and merge.
- We introduce an effective source term to model galaxy formation and merger.
- But assume galaxies and dark matter follow the same velocity field.

The number density n_g obeys

$$\frac{\partial n_g^{(c)}}{\partial \ln a} + \frac{1}{\mathcal{H}} \nabla \cdot (n_g^{(c)} \mathbf{u}) = A^{(c)} j(\rho).$$

Expanded to second order, the source function is given by

$$A^{(c)} j(\rho) \simeq A^{(c)} j(\bar{\rho}) \Big[1 + \frac{j'(\bar{\rho})\bar{\rho}}{j(\bar{\rho})} \delta + \frac{1}{2} \frac{j''(\bar{\rho})\bar{\rho}^2}{j(\bar{\rho})} \delta^2 \Big],$$

$$b_1^*(t) \equiv \frac{j'(\bar{\rho})\bar{\rho}}{j(\bar{\rho})}, \quad b_2^*(t) \equiv \frac{j''(\bar{\rho})\bar{\rho}^2}{j(\bar{\rho})}.$$

- Solving the equation order by order $n_{\rm g}^{\rm (c)} = \bar{n}_{\rm g}^{\rm (c)}(1 + \delta_{\rm g}^{(1)} + \delta_{\rm g}^{(2)})$
- Background solution

$$\bar{n}_{\rm g}^{\rm (c)} = \int_{\ln a_{\rm ini}}^{\ln a} {\rm d}(\ln a)\, A^{\rm (c)} j(\bar{\rho})$$

- First order solution

$$D_{\rm g}(t) = D(t) + \frac{1}{\bar{n}_{\rm g}^{\rm (c)}} \int_0^{\bar{n}_{\rm g}^{\rm (c)}} {\rm d}n_* \left(b_1^* - 1\right) D_*,$$

Linear bias $b_1 = D_{\rm g}/D$,

$$b_1(t) = 1 + \frac{1}{\bar{n}_{\rm g}^{\rm (c)} D} \int_0^{\bar{n}_{\rm g}^{\rm (c)}} {\rm d}n_* \left(b_1^* - 1\right) D_*.$$

- Second order solution

$$\chi_2^{(2)} = -\frac{4}{21\bar{n}_{\rm g}^{(c)}} \int_0^{\bar{n}_{\rm g}^{\rm (c)}} {\rm d}n_* D_*(D - D_*)(b_1^* - 1),$$

$$\chi_1^{(2)} = 0,$$

$$\chi_0^{(2)} = -\chi_2^{(2)} + \frac{1}{\bar{n}_{\rm g}^{\rm (c)}} \int_0^{\bar{n}_{\rm g}^{\rm (c)}} {\rm d}n_* D_*^2 \left(\frac{b_2^*}{2}\right).$$

- The structure of the second order kernel is the same as that in the conservation model. The bias is still nonlocal.

Nonlocal bias to third order

- Assuming DM and halos follows the same velocity field.
- Bias is local at the formation time.

$$\chi^{(2)} = \frac{b_2}{2}\,[\delta^{(1)}]^2 + \gamma_2 \mathcal{G}_2^{(2)}(\Phi_{\rm v})$$

$$\begin{aligned}\chi^{(3)} &= b_2\,\delta^{(1)}\delta^{(2)} + \frac{b_3}{6}\,[\delta^{(1)}]^3 \\ &+ \gamma_2\,\mathcal{G}_2^{(3)}\,(\Phi_{\rm v}) + \gamma_2\beta\,\delta^{(1)}\mathcal{G}_2^{(2)}\,(\Phi_{\rm v}) \\ &+ \gamma_3\left(\mathcal{G}_3(\Phi_{\rm v}) + \frac{6}{7}\,\mathcal{G}_2(\Phi_{\rm v}^{(1)}, \Phi_{\rm 2LPT})\right)\end{aligned}$$

$$\begin{aligned}\mathcal{G}_2(\Phi_1, \Phi_2) &= \nabla_{ij}\Phi_1\nabla_{ij}\Phi_2 - \nabla^2\Phi_1\nabla^2\Phi_2 \\ \nabla^2\Phi_{\rm 2LPT} &= -\mathcal{G}_2(\Phi_{\rm v}^{(1)}) \\ \mathcal{G}_3(\Phi_{\rm v}) = (\nabla^2\Phi_{\rm v})^3 &+ 2\nabla_{ij}\Phi_{\rm v}\nabla_{jk}\Phi_{\rm v}\nabla_{ki}\Phi_{\rm v} - 3(\nabla_{ij}\Phi_{\rm v})^2\nabla^2\Phi_{\rm v}\end{aligned}$$

$$\gamma_2 = -\frac{2}{7}(b_1 - 1)(1 - {\rm e}^{-y}), \qquad \beta = \frac{b_2}{(b_1 - 1)},$$

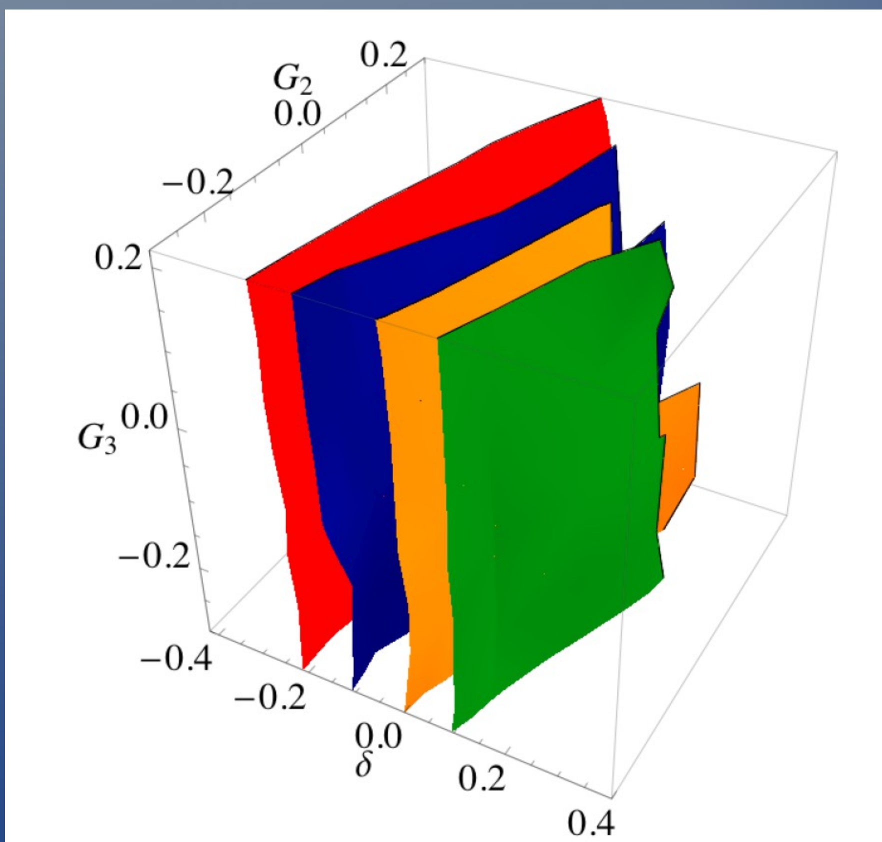
$$\gamma_3 = \frac{1}{63}(b_1 - 1)(1 - {\rm e}^{-y})(11 - 7{\rm e}^{-y}).$$

Measuring the nonlocal terms by scatter plot

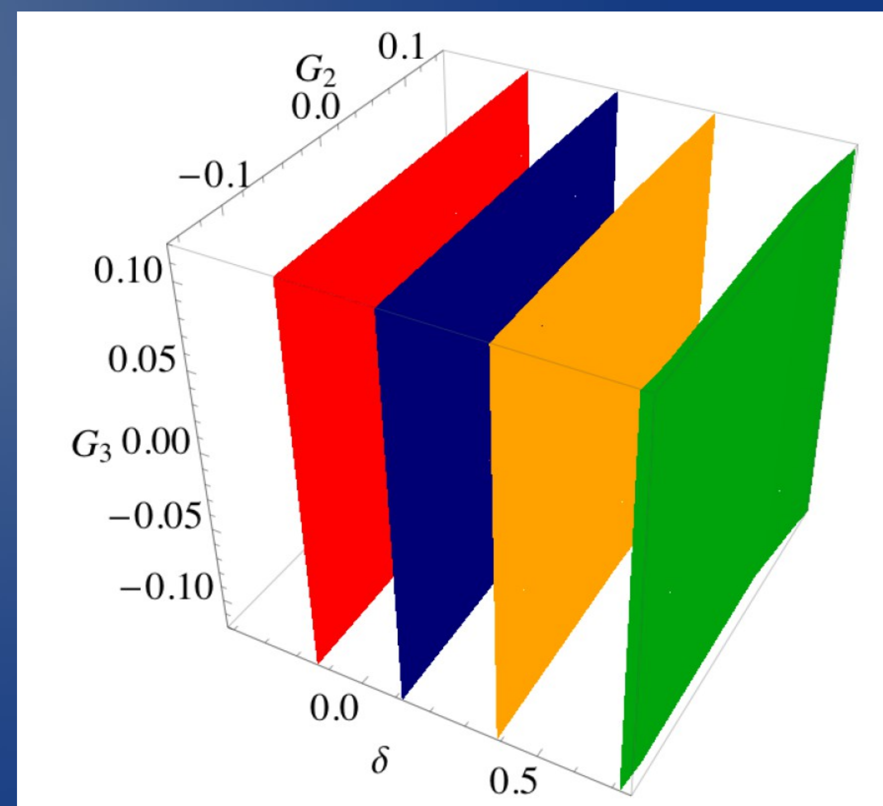
- At each point in the simulation box, compute δ, and other variables G_2 (and G_3). Plotting δ_h against these variables.
- The first time nonlocal term detected in simulations.

$$\begin{aligned}
\mathcal{G}_2(\Phi_v) &= (\nabla_{ij}\Phi_v)^2 - (\nabla^2\Phi_v)^2, \\
\mathcal{G}_3(\Phi_v) &= (\nabla^2\Phi_v)^3 + 2\nabla_{ij}\Phi_v\nabla_{jk}\Phi_v\nabla_{ki}\Phi_v \\
&\quad -3(\nabla_{ij}\Phi_v)^2\nabla^2\Phi_v.
\end{aligned}$$

R=40 Mpc/*h*



z=1, δ_h = -0.3, 0.1, 0.5 and 0.9, $M > 5.7 \times 10^{13}\ M_o/h$



z=0, $M = (4\text{-}7) \times 10^{13}\ M_o/h$

Measuring the nonlocal terms by bispectrum

- The biasing prescription is

$$\delta_{\rm h} = b_1\delta + \frac{b_2}{2}\delta^2 + \gamma_2\mathcal{G}_2,$$
$$\mathcal{G}_2(\Phi_{\rm v}) = (\nabla_{ij}\Phi_{\rm v})^2 - (\nabla^2\Phi_{\rm v})^2$$

- Consider the cross bispectrum

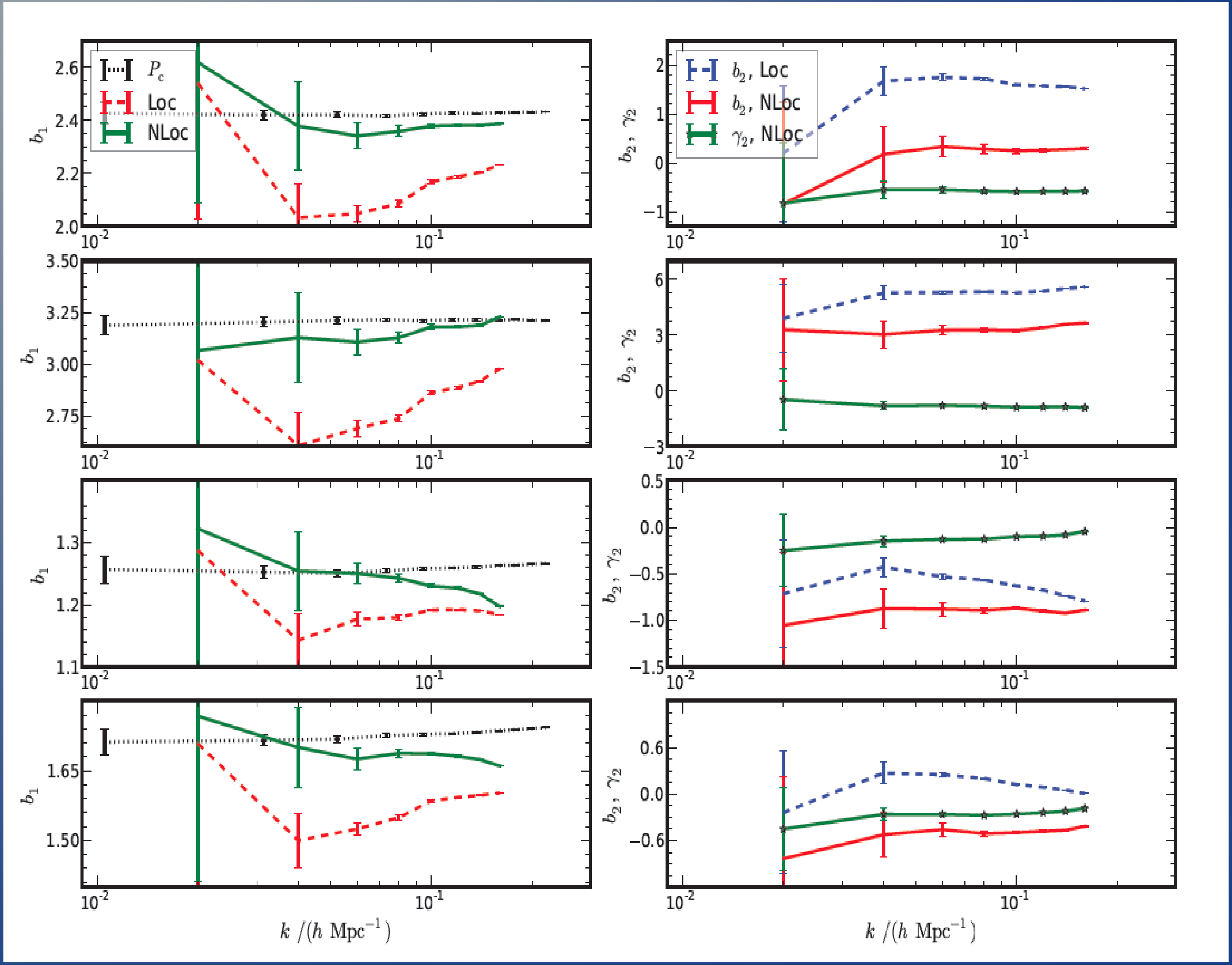
$$B_{\rm c}\delta_{\rm D}(\mathbf{k}_{123}) = \langle\delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta_{\rm h}(\mathbf{k}_3)\rangle$$

- The bispectrum from the local model and nonlocal model are

$$\begin{aligned} B_{\rm c}^{\rm Loc}(k_1,k_2,k_3) &= b_1B(k_1,k_2,k_3) + b_2P(k_1)P(k_2) \\ B_{\rm c}^{\rm NLoc}(k_1,k_2,k_3) &= b_1B(k_1,k_2,k_3) + b_2P(k_1)P(k_2) \\ &+ 2\gamma_2(\mu^2-1)P(k_1)P(k_2) \end{aligned}$$

P and *B* are nonlinear power spectrum and bispectrum measured from simulations

Bias parameters from bispectrum



With the nonlocal term, b_1 from P_c agrees with b_1 from B_c much better.

See also Baldaulf et al 2012

Nonlocal term in Lagrangian space

- Suppose the collapse barrier *B* depends on the local shear field in addition to δ.
- Take a simple model

$$B(q) = \delta_c(1+\sqrt{q^2/q_c^2}) \rightarrow \delta_c(1+\sqrt{\sigma^2/q_c^2}),$$

$$q_c^2 = 8\delta_c^2, \qquad q^2 \equiv \frac{3}{2}\left[(\nabla_{ij}\phi)^2 - \frac{1}{3}(\nabla^2\phi)^2\right]$$

- *B* decreases as mass increases.
- Using this barrier

uncorrelated walks

$$\begin{aligned}
\delta_c\, b_1^{\rm L} &= \nu^2 - 1 + \frac{\nu}{q_c/\delta_c} + \frac{1}{1+4\nu q_c/\delta_c},\\
\delta_c^2\, b_2^{\rm L} &= \nu\, H_3(\nu) + \frac{\nu^3}{q_c/\delta_c} + \frac{2\nu^2}{1+4\nu q_c/\delta_c} - \delta_c^2\, c_2^{\rm L},\\
\delta_c^2\, c_2^{\rm L} &= -\frac{\nu^2}{(q_c/\delta_c)^2}\left[1+\nu(q_c/\delta_c) - \frac{9(q_c/\delta_c)^2}{1+(128/35)\nu q_c/\delta_c}\right].
\end{aligned}$$

Measuring the bias of the Lagrangian halos

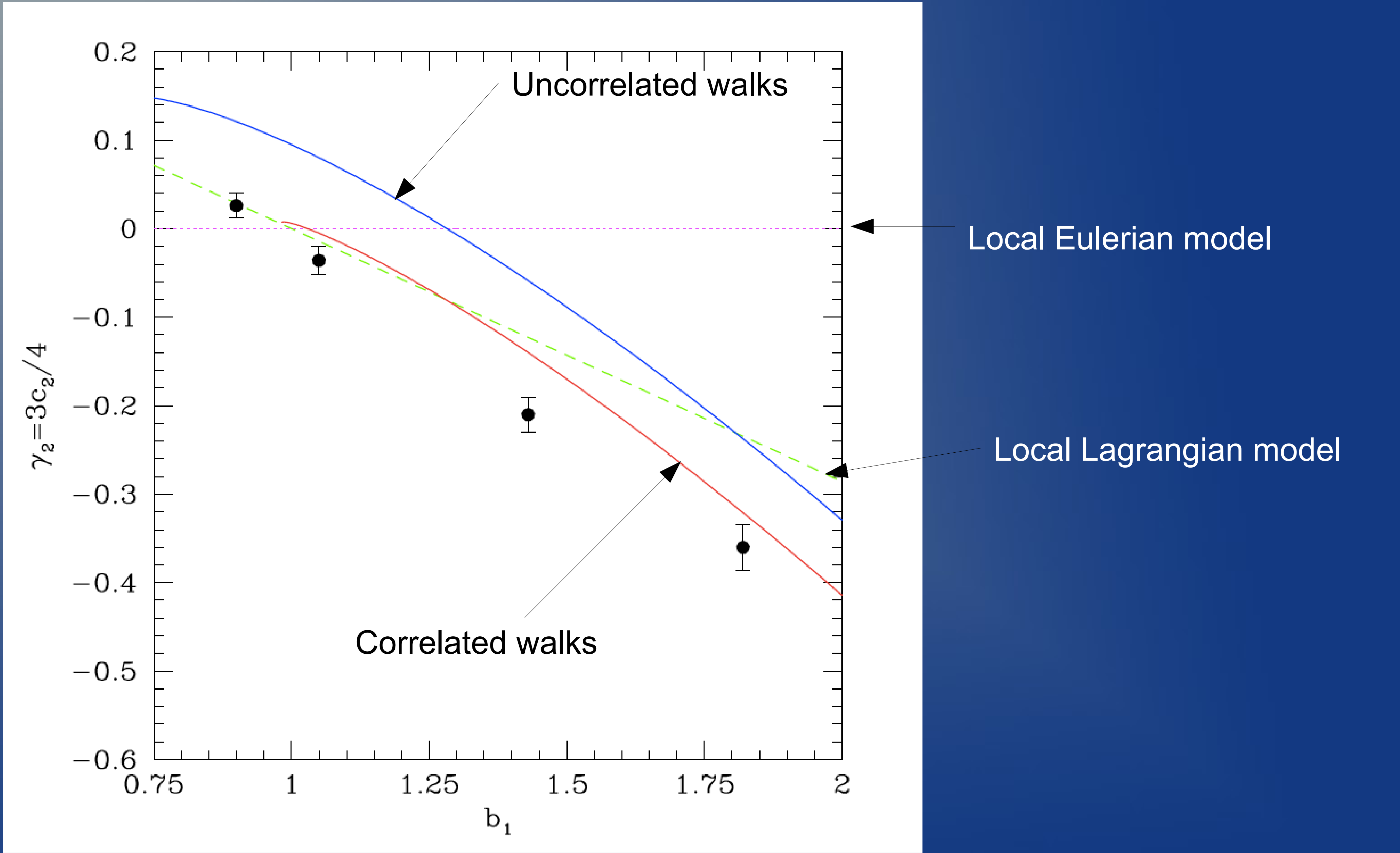
- Tracing the dark matter particles in the Eulerian halos back to the initial conditions. Using the Lagrangian DM particles to construct the Lagrangian halos.
- Measure the Lagrangian bias parameters using the model

$$B_{\mathrm{c}}^{\mathrm{L}} = b_1^{\mathrm{L}} B^{\mathrm{L}} + b_2^{\mathrm{L}} P^{\mathrm{L}}(k_1) P^{\mathrm{L}}(k_2) + 2\gamma_2^{\mathrm{L}}(\mu^2 - 1) P^{\mathrm{L}}(k_1) P^{\mathrm{L}}(k_2)$$

- Relationship between Lagrangian bias parameters and Eulerian ones

$$\begin{aligned} b_1 &= 1 + \frac{b_1^{\mathrm{L}} - 1}{D_*} \\ \gamma_2 &= \frac{\gamma_2^{\mathrm{L}}}{D_*^2} - \frac{2}{7}(b_1 - 1)(1 - \frac{1}{D_*}) \end{aligned}$$

Measuring the bias of the Lagrangian halos



Conclusions

- Using the halo conservation model and continuous galaxy formation model, we show that large scale gravitational evolution naturally induces nonlocal term in the halo biasing prescription even if the Lagrangian bias is local.
- We present evidences of the nonlocal term from simulations, using scatter plot and bispectrum. In particular, linear bias from bispectrum agrees with that from the cross power spectrum much better if the nonlocal term is included.
- The Lagrangian bias is nonlocal if the collapse threshold also depends on the local shear field. Measurement of the Lagrangian halo bias suggests that the Lagrangian bias could be already nonlocal.