Nonlocal peak bias factors
Theory vs Simulations

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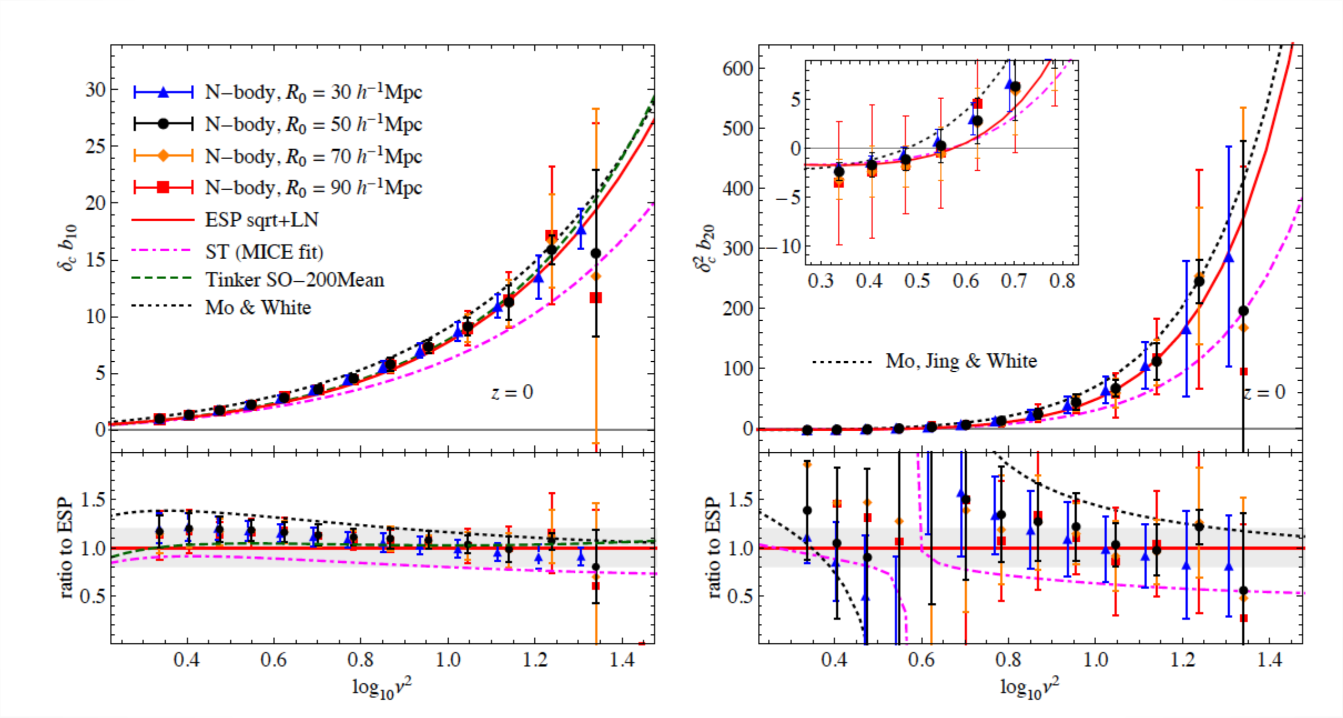
Based on: MB, Chan, Desjacques & Paranjape, ArXiv: 1310.1401

Motivation

Local bias factors can be measured in simulations using a **1-point cross-correlation technique** (Musso, Paranjape & Sheth (2012))



Paranjape, Sefusatti, Chan, Desjacques & Monaco (2013)

This technique can be extended to measure second-order **non-local Lagrangian bias factors**.

Modeling the clustering of dark matter haloes

Analytic and heuristic approaches:

- Peak model (BBKS 1986)
- Excursion set framework (Bond et al. 1991)
- Perturbation theory (Bernardeau et al. 2002 , review)
- Peak-background split (Kaiser 1984)
- Local bias (Fry & Gaztanaga 1993)

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Excursion set peaks

A brief summary on **excursion set peaks**:

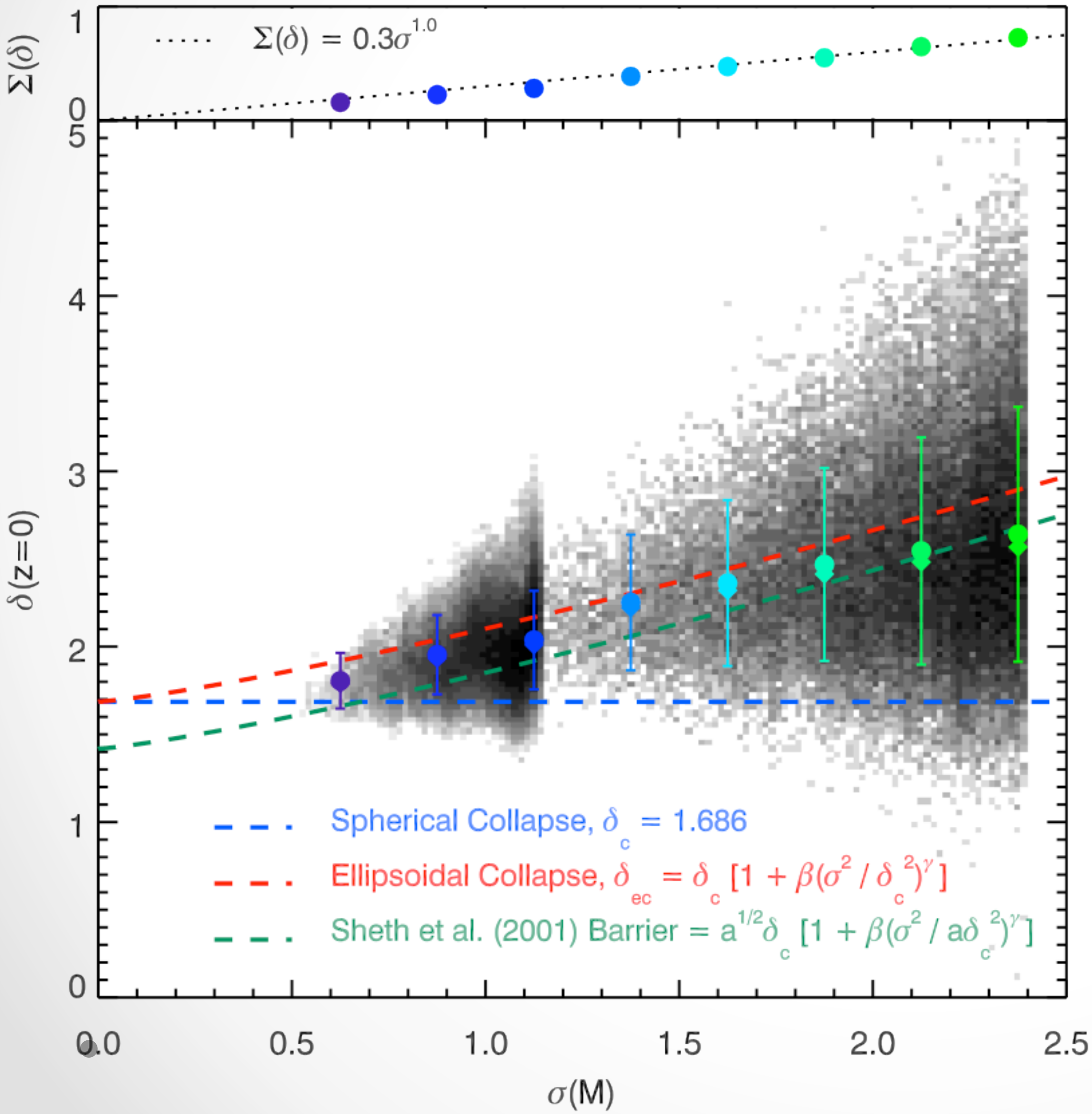
- The peak model is combined with excursion set theory by **imposing that peaks** on a given smoothing scale are counted only if they **satisfy a first crossing condition**;
- The barrier considered is $B = \delta_c + \beta\sigma_0$;
- The first crossing condition is then $B < \delta < B + (\not{B}' + \mu)\Delta R_s$;
- We assume that each halo "sees" a constant, flat barrier, whose height varies from halo to halo;

$$B' = 0 \Longrightarrow \mu = -\frac{d\delta_s}{dR_s} > 0$$

- This first crossing condition affects the number density of peaks through the variable μ .

$$n_{\mathrm{ESP}}(\mathbf{w}) = -\left(\frac{\mu}{\gamma_{\nu\mu}\nu_c}\right)\theta_H(\mu)\, n_{\mathrm{pk}}(\mathbf{y})$$

Excursion set peaks



Owing to the **triaxiality of collapse**, the critical density for collapse is not constant and equal to δ_c=1.68, but rather distributed around a value which **increases with decreasing of the halo mass.**

Robertson et al., APJ 696,636 (2009)

Excursion set peaks

With the assumptions taken above,

the peak multiplicity function is

$$f_{ESP}(\nu_c) = \left(\frac{V}{V_*}\right) \frac{1}{\gamma_{\nu\mu}\nu_c} \int_0^\infty d\beta\, p(\beta) \int_0^\infty d\mu\, \mu \int_0^\infty du\, f(u)\, \mathcal{N}(\nu_c, u, \mu)$$

Where we can apply Bayes' theorem and compute the integral over μ

$$\int_0^\infty d\mu\, \mu\, \mathcal{N}(\mu|\nu, u) = \bar{\mu}\left[\frac{1+\mathrm{erf}(\bar{\mu}/\sqrt{2}\Sigma)}{2} + \frac{\Sigma}{\sqrt{2\pi}\bar{\mu}} e^{-\bar{\mu}^2/2\Sigma^2}\right]$$

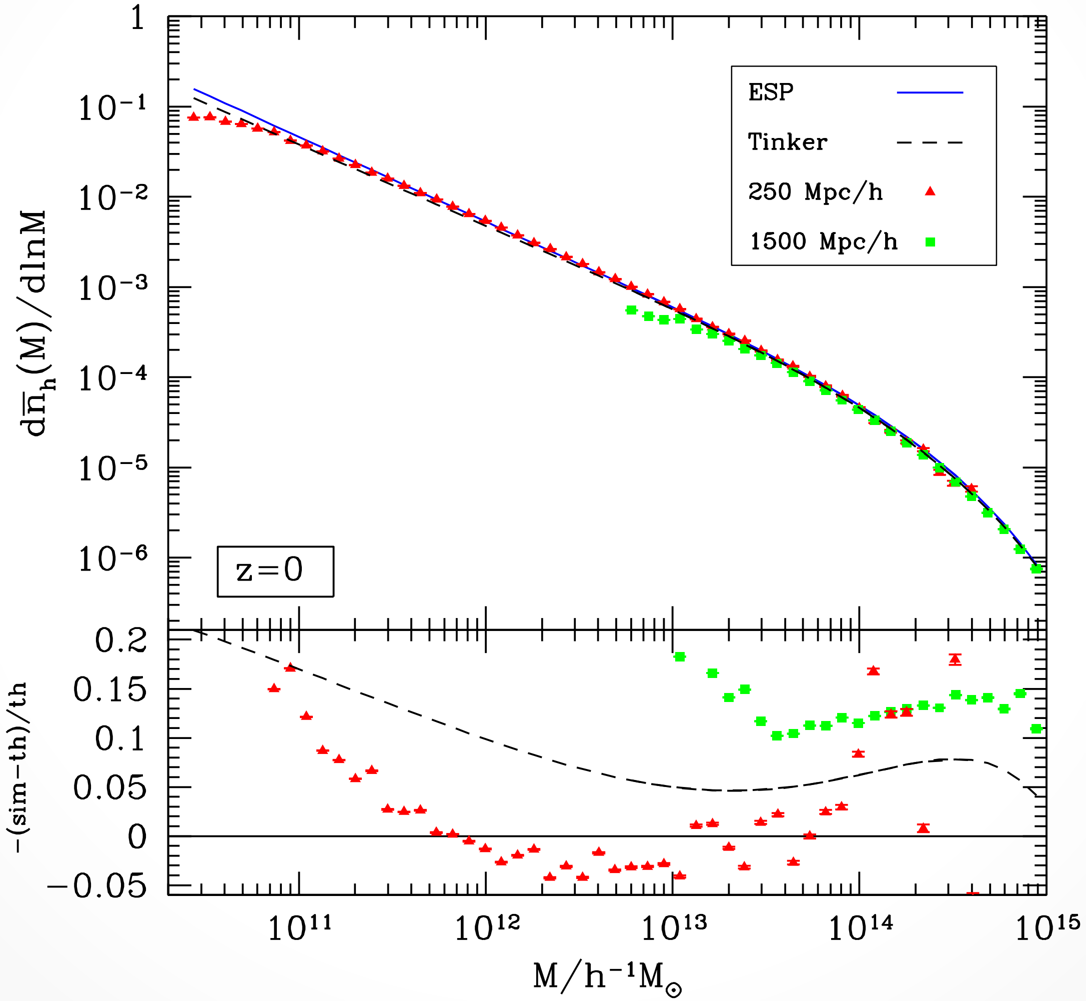
where

$$\bar{\mu} = u\left(\frac{\gamma_{u\mu} - \gamma_1\gamma_{\nu\mu}}{1-\gamma_1^2}\right) + (\nu+\beta)\left(\frac{\gamma_{\nu\mu} - \gamma_1\gamma_{u\mu}}{1-\gamma_1^2}\right)$$

$$\Sigma^2 = \Delta_0^2 - \frac{\gamma_{\nu\mu}^2 - 2\gamma_1\gamma_{\nu\mu}\gamma_{u\mu} + \gamma_{u\mu}^2}{1-\gamma_1^2}\,.$$

Excursion set peaks

Comparison with N-body simulations



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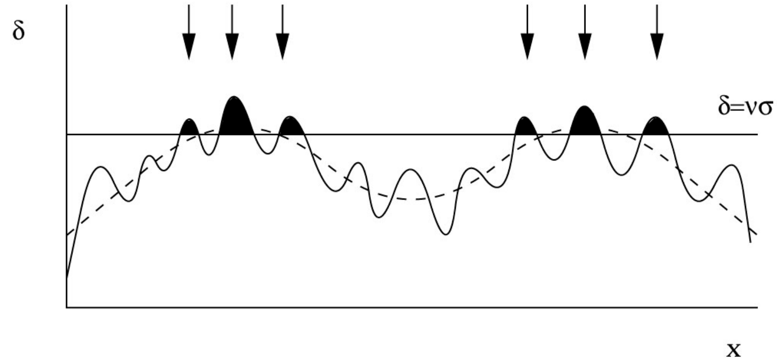
Bias factors from ESP

Local and nonlocal bias factors are computed using a **peak-background split argument**



$$\nu(x) = \frac{1}{\sigma_{0s}}\delta_s(x)$$

$$u(x) = -\frac{1}{\sigma_{2s}}\nabla^2\delta_s(x) = -Tr\left[\zeta\right]$$

$$\eta^2(x) = \frac{1}{\sigma_{1s}^2}\left(\nabla\delta_s\right)^2(x)$$

$$\zeta^2(x) = \frac{3}{2\sigma_{2s}^2}Tr\left[\left(\partial_i\partial_j\delta_s - \frac{1}{3}\delta_{ij}\nabla^2\delta_s\right)^2\right](x)$$

$$\mu(x) = -\frac{d\delta_s(x)}{dR_s}$$

The long-wavelength modes are uncorrelated with the short ones but modulate the mean of their distributions

Bias factors from ESP

Local and nonlocal bias factors are computed using a **peak-background split argument**

$$\sigma_{0T}^{i}\sigma_{2G}^{j}b_{ijk} = \frac{1}{\bar{n}_{ESP}}\int d^{11}\mathbf{w}\, n_{\rm ESP}(\mathbf{w})H_{ijk}(\nu,u,\mu)P_1(\mathbf{w})$$

$$\sigma_{1G}^{2k}\chi_{k0} = \frac{(-1)^k}{\bar{n}_{ESP}}\int d^{11}\mathbf{w}\, n_{\rm ESP}(\mathbf{w})L_k^{(1/2)}\left(\frac{3\eta^2}{2}\right)P_1(\mathbf{w})$$

$$\sigma_{2G}^{2k}\chi_{0k} = \frac{(-1)^k}{\bar{n}_{ESP}}\int d^{11}\mathbf{w}\, n_{\rm ESP}(\mathbf{w})L_k^{(3/2)}\left(\frac{5\zeta^2}{2}\right)P_1(\mathbf{w})$$

where

$$P_1(\mathbf{w})d^{11}\mathbf{w} = \mathcal{N}(\nu,u,\mu)d\nu du d\mu \times \chi_3^2(3\eta^2)d(3\eta^2) \times \chi_5^2(5\zeta^2)d(5\zeta^2) \times P(\text{angles})$$

Effective bias expansion

We can write a **effective bias expansion** using rotational invariants

$$\begin{aligned}\delta_{\rm pk}(\mathbf{x}) = &\ \sigma_{0T} b_{100} \nu(\mathbf{x}) + \sigma_{2G} b_{010} u(\mathbf{x}) + b_{001} \mu(\mathbf{x}) \\ &+ \frac{1}{2}\sigma_{0T}^2 b_{200} \nu^2(\mathbf{x}) + \sigma_{0T}\sigma_{2G} b_{110} \nu(\mathbf{x}) u(\mathbf{x}) \\ &+ \frac{1}{2}\sigma_{2G}^2 b_{020} u^2(\mathbf{x}) + \frac{1}{2} b_{002} \mu^2(\mathbf{x}) \\ &+ \sigma_{0T} b_{101} \nu(\mathbf{x}) \mu(\mathbf{x}) + \sigma_{2G} b_{011} u(\mathbf{x}) \mu(\mathbf{x}) \\ &+ \sigma_{1G}^2 \chi_{10} \eta^2(\mathbf{x}) + \sigma_{2G}^2 \chi_{01} \zeta^2(\mathbf{x}) + \cdots\end{aligned}$$

This expansion can be used to **calculate the N-point correlation function** in the excursion set peak framework

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Bias from cross-correlation

Bias factors of discrete tracers can be computed from **1-point measurements** Musso, Paranjape & Sheth (2012)

The recipe is

- Find haloes at z=0;
- Track back the particles at initial conditions. There we have our proto halo;
- Smooth the density field on a "large" scale ($R=10,15,20\ h^{-1}\mathrm{Mpc}$)
- The quantity $H_n(\nu_l)$ is computed at the location of the proto-halo. For the ESP considered, the ensemble average over all proto-haloes reads

$$\frac{1}{N}\sum_{i=1}^{N} H_n(\nu_l) = \frac{1}{\bar{n}_{\mathrm{ESP}}}\int_{-\infty}^{+\infty} d\nu_l\, \mathcal{N}(\nu_l)\langle n_{\mathrm{ESP}}|\nu_l\rangle H_n(\nu_l)$$

Bias from cross-correlation

We want to measure nonlocal bias factors χ_{10} and χ_{01} related to the rotational invariants

$$\eta^2(\mathbf{x}) = \frac{1}{\sigma_{1G}^2}(\nabla\delta)^2(\mathbf{x})$$

$$\zeta^2(\mathbf{x}) = \frac{3}{2\sigma_{2G}^2}\mathrm{tr}\left[\left(\partial_i\partial_j\delta - \frac{1}{3}\delta_{ij}\nabla^2\delta\right)^2\right](\mathbf{x})$$

In this case we have to deal with χ^2 **distributions with 3- and 5- degrees of freedom** and consequently with their orthogonal, **Laguerre polynomials.**

Bias from cross-correlation

In analogy with the derivation of the bias factors associated to Hermite polynomials, a **first way** to get a measurement of χ_{10} is

$$\int_0^\infty d(3\eta_l^2)\,\chi_3^2(3\eta_l^2)\langle n_{ESP}|3\eta_l^2\rangle L_n^{(\alpha)}\left(\frac{3\eta_l^2}{2}\right) = \left\langle L_n^{(\alpha)}\left(\frac{3\eta_l^2}{2}\right)\Big|\mathrm{peak}\right\rangle$$

And one can show that

$$L_1^{(1/2)}(3\eta_l^2/2) = \epsilon^2 L_1^{(1/2)}(3\eta^2/2)\chi_3^2(3\eta^2)$$

Which brings to

$$\left\langle L_1^{(1/2)}\left(\frac{3\eta_l^2}{2}\right)\Big|\mathrm{peak}\right\rangle = -\epsilon^2\sigma_1^2\chi_{10}$$

where $$\epsilon^2 = \left(\frac{\sigma_{1\times}^2}{\sigma_{1s}\sigma_{1l}}\right)^2$$

Bias from cross-correlation

A **second (equivalent) way** is to start from the original formula

$$\sigma_{1G}^2 \chi_{10} = -\frac{1}{\bar{n}_{\rm ESP}} \int d^{11}\mathbf{w}\, n_{\rm ESP}(\mathbf{w}) L_1^{(1/2)}\left(\frac{3\eta^2}{2}\right) P_1(\mathbf{w})$$

$$= -\left\langle L_1^{(1/2)}\left(\frac{3\eta^2}{2}\right) \Big| \text{peak} \right\rangle$$

$$= -\frac{3}{2} + \frac{1}{2}\langle 3\eta^2 | \text{peak}\rangle$$

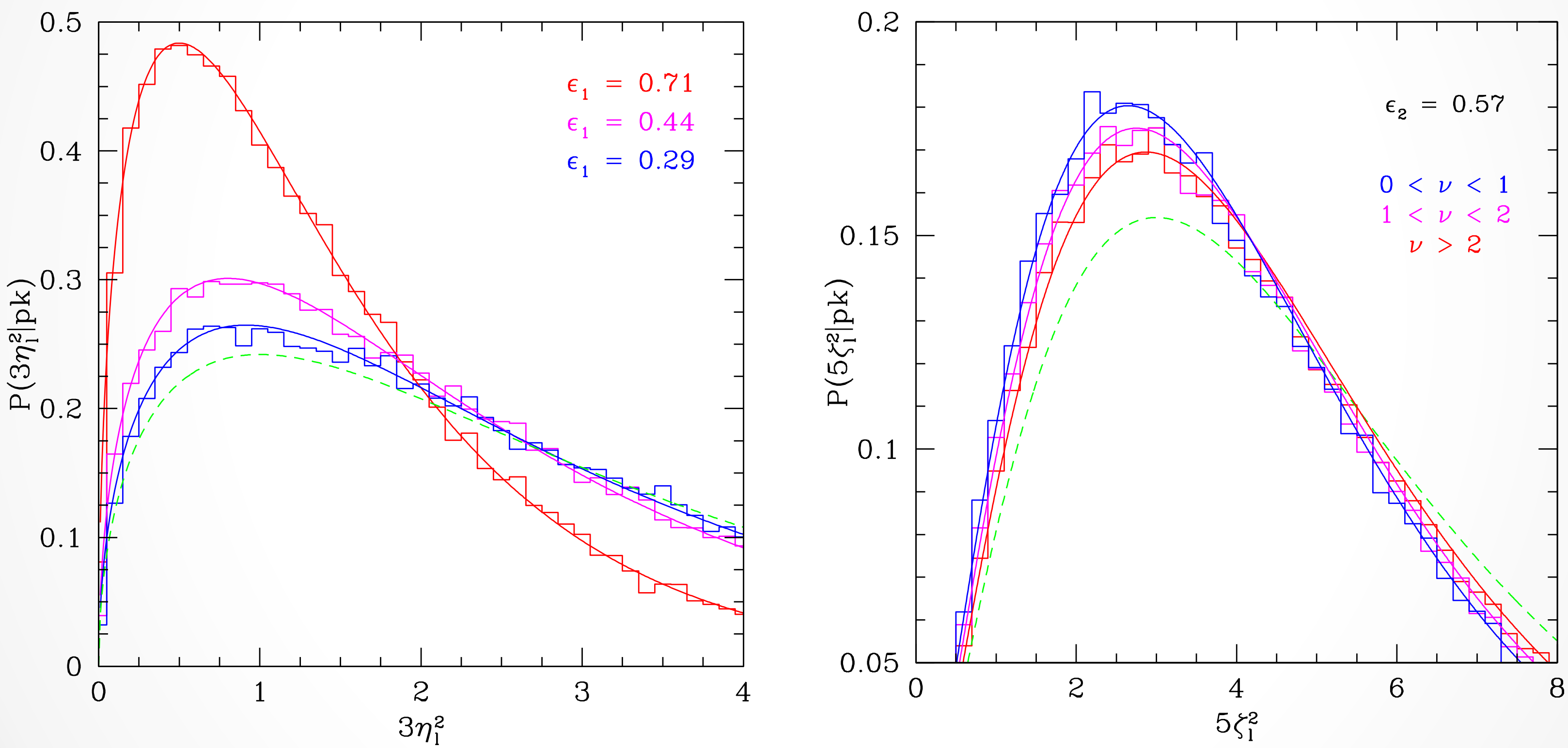
And consider the conditional chi-squared distribution $\chi_k^2(y|x;\epsilon)$

Where $k = 3$ $y = 3\eta_l^2$ $x = 3\eta^2$ and ε is the correlation between y and x.

And we can measure the conditional chi-squared distribution at the position of the maxima and fit it to the best value of x.

Testing for Gaussian random fields…

We first test the technique with peaks of Gaussian random fields (with the same power spectrum used in our N-body simulations)



Conditional probability distributions for the variables $3\,\eta_l^{\,2}$ and $5\,\xi_l^{\,2}$ measured at the position of the maxima of the linear density field where the theoretical prediction gives $x = \langle 3\eta^2|\mathrm{peak}\rangle = 0$

… and measuring χ_{10} and χ_{01}

In principle, we could go directly to the **first way** and measure

$$\sigma_{1s}^2 \hat{\chi}_{10} = -\frac{1}{N\epsilon_1^2} \sum_{i=1}^{N} L_1^{(1/2)} \left(\frac{3\eta_l^2}{2} \right)$$

The problem is that the cross-correlation coefficient ε_1 is too small unless we take R_l very close to the halo R_G.

So we have to use the **second way**

$$\sigma_{1s}^2 \hat{\chi}_{10} = \frac{1}{2} \left(\langle 3\eta^2 | \mathrm{halo} \rangle - 3 \right)$$

By fitting the probability distribution $P(3\eta_l^2 | halo)$ with the conditional chi-squared distribution and getting $x = \langle 3\eta^2 | \mathrm{halo} \rangle$

Measuring χ_{10} and χ_{01}

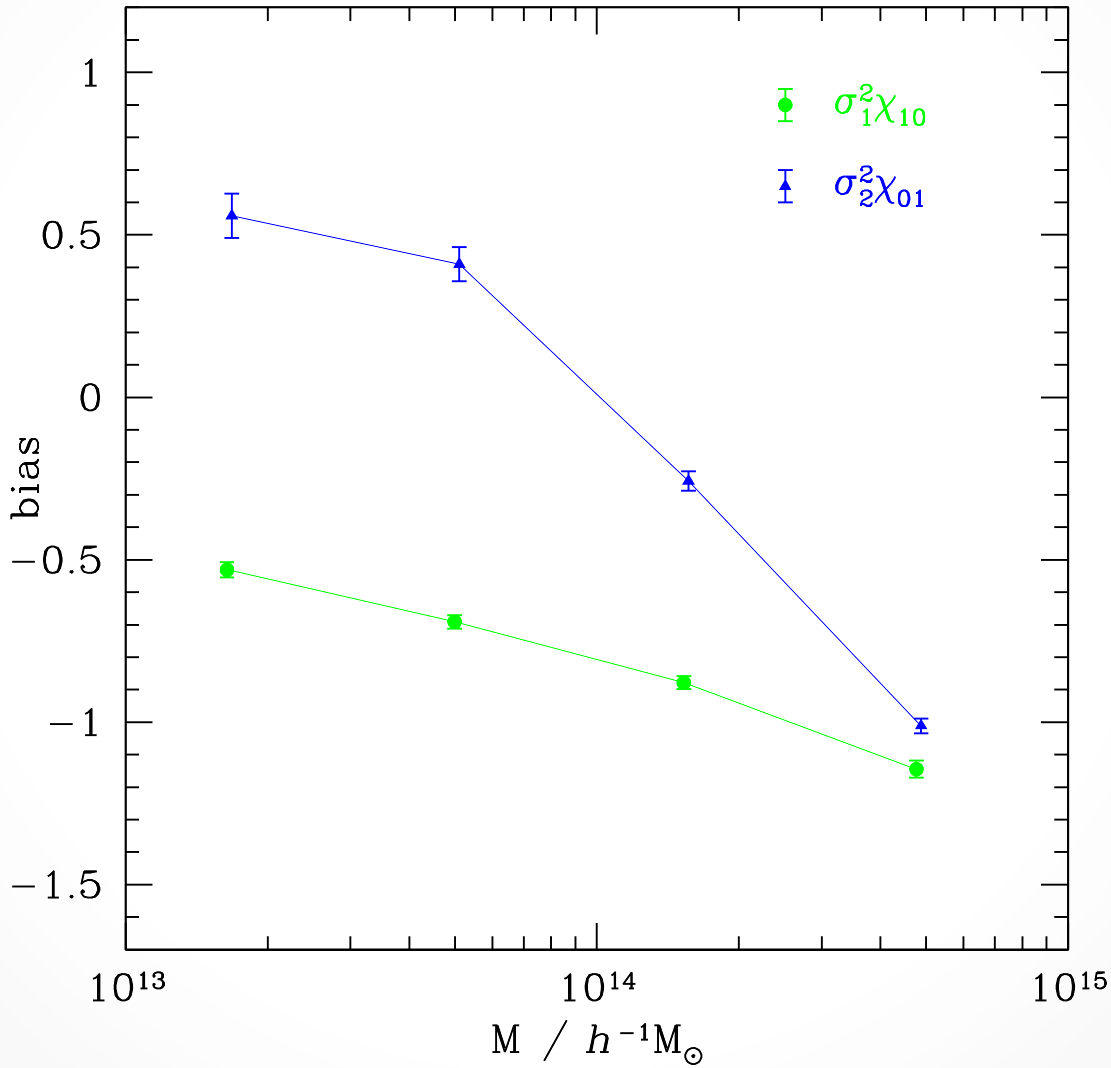
We choose R_l=10 h^{-1}Mpc and we get the halo R_G as a function of R_T requiring that

$$\langle \delta_G | \delta_T \rangle = \delta_T$$

The R_G gives us a value for the cross-correlation coefficients, but it **leads to unphysical negative values of x**.
What we do is then to **use the following recipe**:

- Estimate both ε_1 and x by fitting the model $\chi^2_k(y|x;\epsilon)$ to the measured $P(3\eta_l^2|halo)$;
- Compute ε_2 assuming that the same R_G enters the spectral moments;
- Estimate $x = \langle 5\zeta^2|\text{halo}\rangle$ by fitting the theoretical model $\chi^2_5(y|x;\epsilon_2)$ to the measured $P(5\zeta_l^2|\text{halo})$.

Measuring χ_{10} and χ_{01}



MB, Chan, Desjacques & Paranjape, ArXiv 1310.1401

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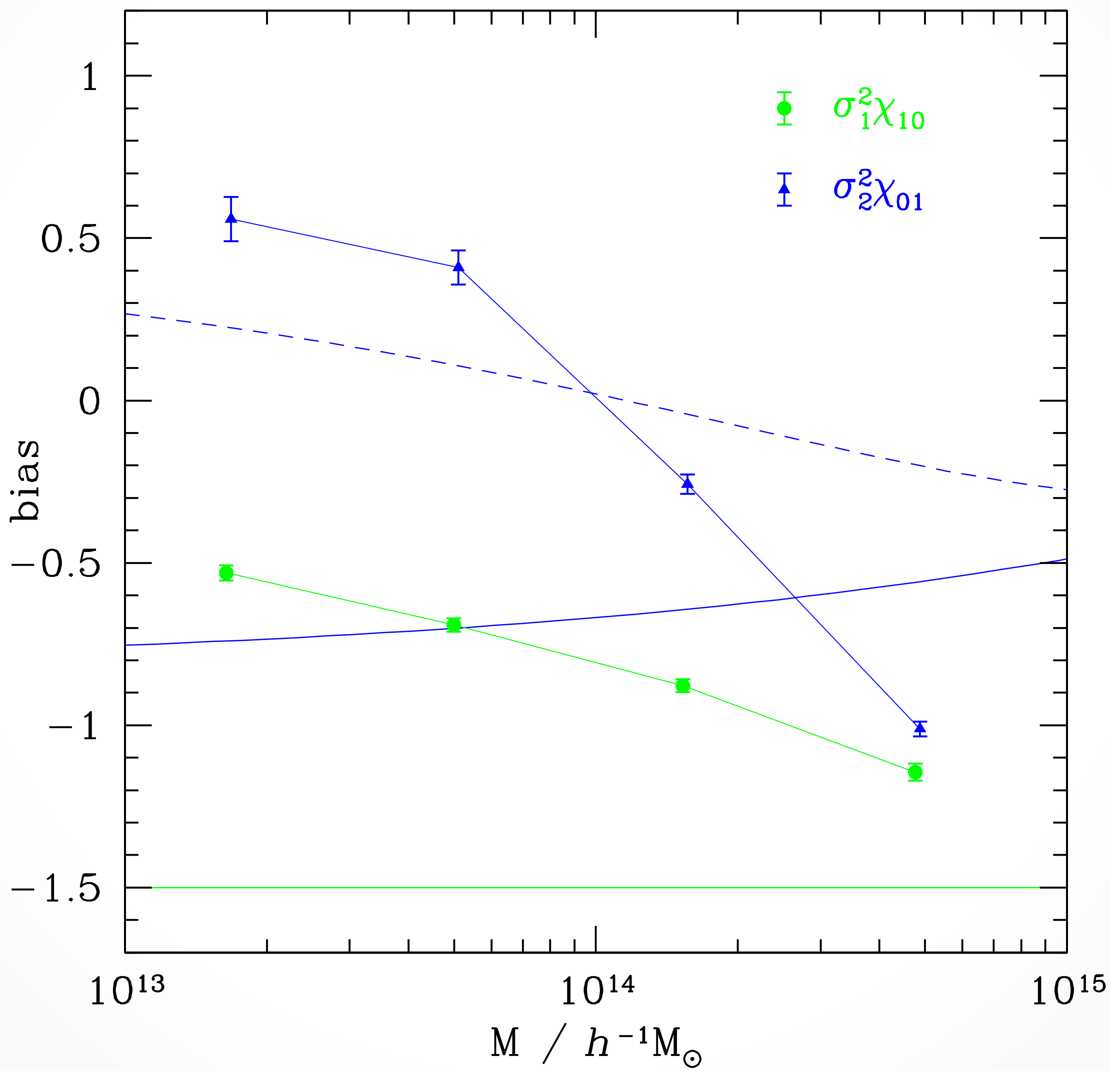


MB, Chan, Desjacques & Paranjape, ArXiv 1310.1401

Measuring χ_{10} and χ_{01}

Interpretation of the measurement:

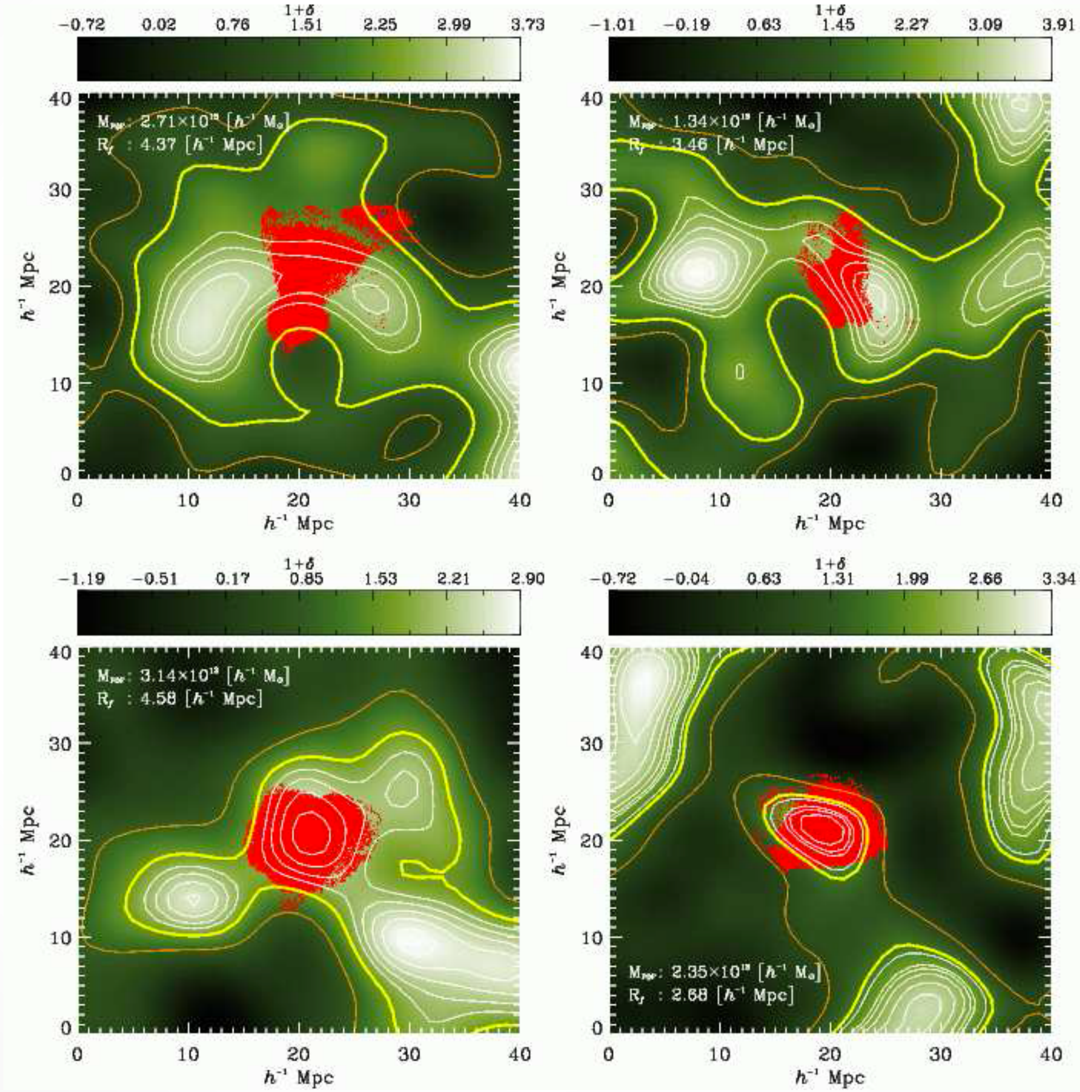
- **If haloes were forming out of randomly distributed patches** in the initial conditions then **we would measure something consistent with 0** since for random points: $\langle 3\eta^2 \rangle = 3$ and $\langle 5\zeta^2 \rangle = 5$
- There could be an **offset between the proto-halo center of mass and the position of the linear density peak;**
- We assumed that proto-haloes always form around a density peak. However, **N-body simulations suggest that a fraction of the proto-haloes collapse along the ridges or filaments** connecting two density maxima (especially significant for low halo masses);
- We note that if the Lagrangian clustering of haloes also depends on

$$s_2(x) = s_{ij}(x)s^{ij}(x), \quad s_{ij}(x) = \partial_i\partial_j\phi(x) - \frac{1}{3}\delta_{ij}\delta(x)$$

then **we are not measuring χ_{01}, but some weighted and scale-dependent combination of both χ_{01} and the Lagrangian bias γ_2** associated with s_2.

Chan, Scoccimarro & Sheth, PRD 85,083509 (2012)

Measuring χ_{10} and χ_{01}



Ludlow & Porciani, MNRAS 413,1961 (2011)

Take home message

- We **combine excursion set theory with the peak model**, exploiting advantages from both approaches;
- Theoretical prediction for bias parameters is made using a simple **peak-background split**;
- We get bias from N-body simulation with a **1-point measurement** and no higher order correlation functions;
- We can use **correlation between wavelengths modes** in N-body simulations to measure bias parameters.