The multiple faces of galaxy bias

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Overview

Local bias

Nonlocal bias

Exclusion bias

Peaks bias

Velocity bias

Summary: epicycles?

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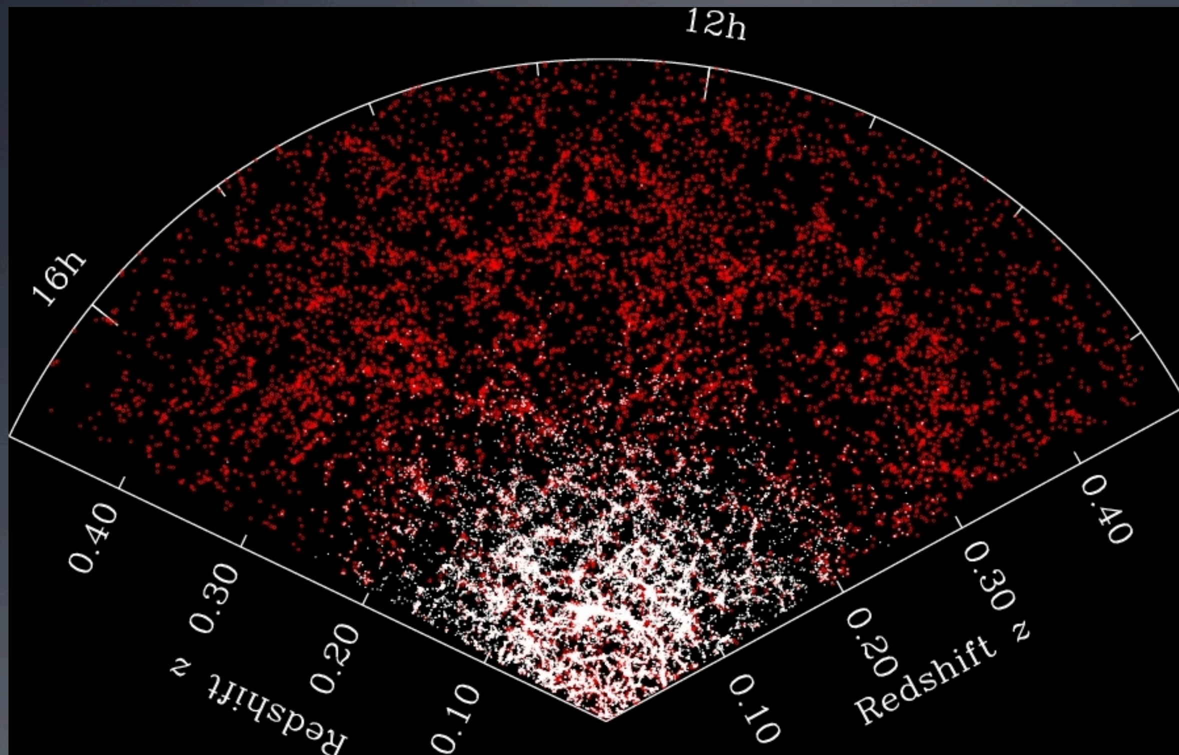
Galaxy clustering in redshift space



SDSS

1) Measures 3-d distribution, has many more modes than projected quantities like shear from weak lensing

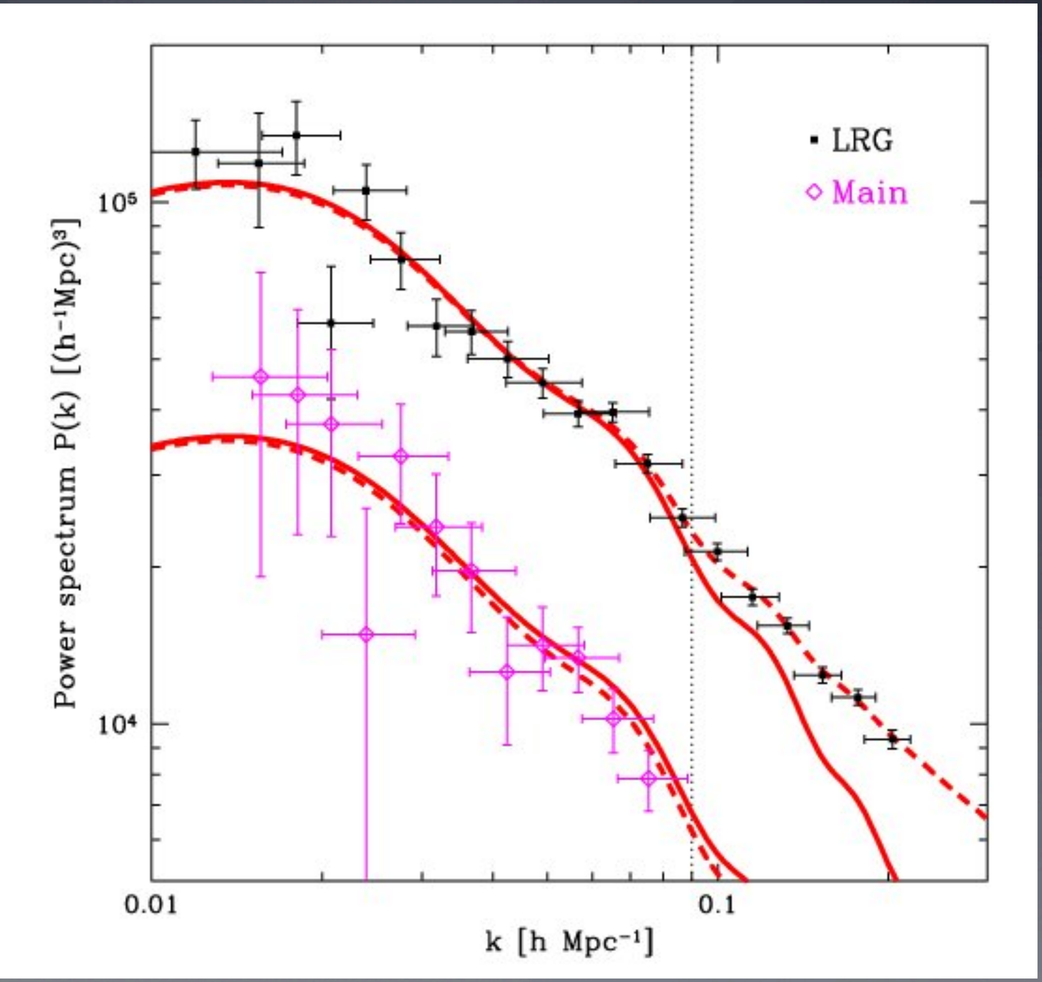
2) Easy to measure: effects of order unity, not 1%

Power Spectrum

- Galaxy clustering traces dark matter clustering: 3-d analysis contains a lot of statistical information
- Amplitude depends on galaxy type: galaxy bias b

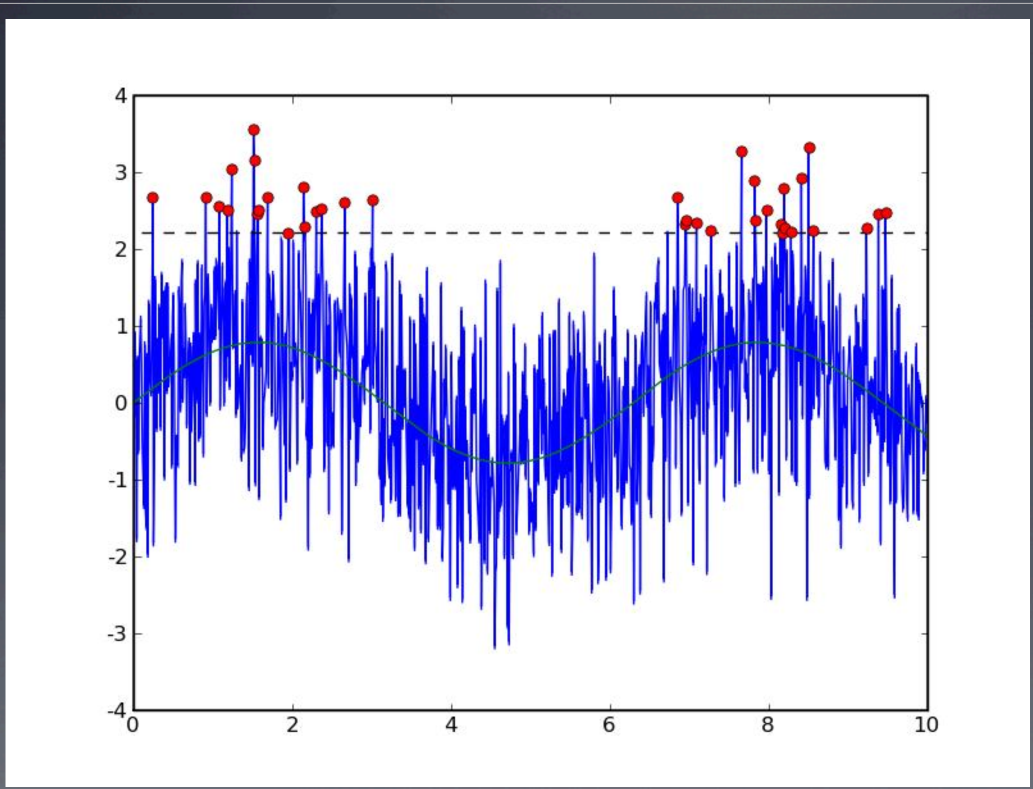
$$P_{gg}(k)=b^2(k)P_{mm}(k)$$

- To determine b(k) we need additional (external) information
- Galaxy bias can be scale dependent: b(k)
- Once we know bias we know how dark matter clustering grows in time



Tegmark et al. (2006)

Complication I: why are galaxies linearly biased?
Galaxies form at high density peaks of initial density:
rare peaks are more strongly clustered



The enhancement depends on the halo mass function slope

Entrenched concept by now: but it is a complication introduced to explain the difference between an observable (galaxy P(k)) and theory (dark matter P(k))

Complication II: local quadratic bias

Local bias model: $\delta_h = b_1 \delta_m + b_2 \delta_m^2 + \ldots$: Eulerian or Lagrangian?

Gravity develops nonlocal terms

$$^{(2)}\delta(\boldsymbol{x},\eta) = \frac{17}{21} {}^{(1)}\delta^2(\boldsymbol{x},\eta) - \boldsymbol{\Psi}(\boldsymbol{x},\eta)\cdot\boldsymbol{\nabla}\delta(\boldsymbol{x},\eta) + \frac{2}{7} s^2(\boldsymbol{x},\eta).$$

Tidal tensor

$$s_{ij}(\boldsymbol{x},\eta) = \partial_i \partial_j \Phi(\boldsymbol{x},\eta) - \frac{1}{3}\delta_{ij}^{(\mathrm{K})}\delta(\boldsymbol{x},\eta).$$

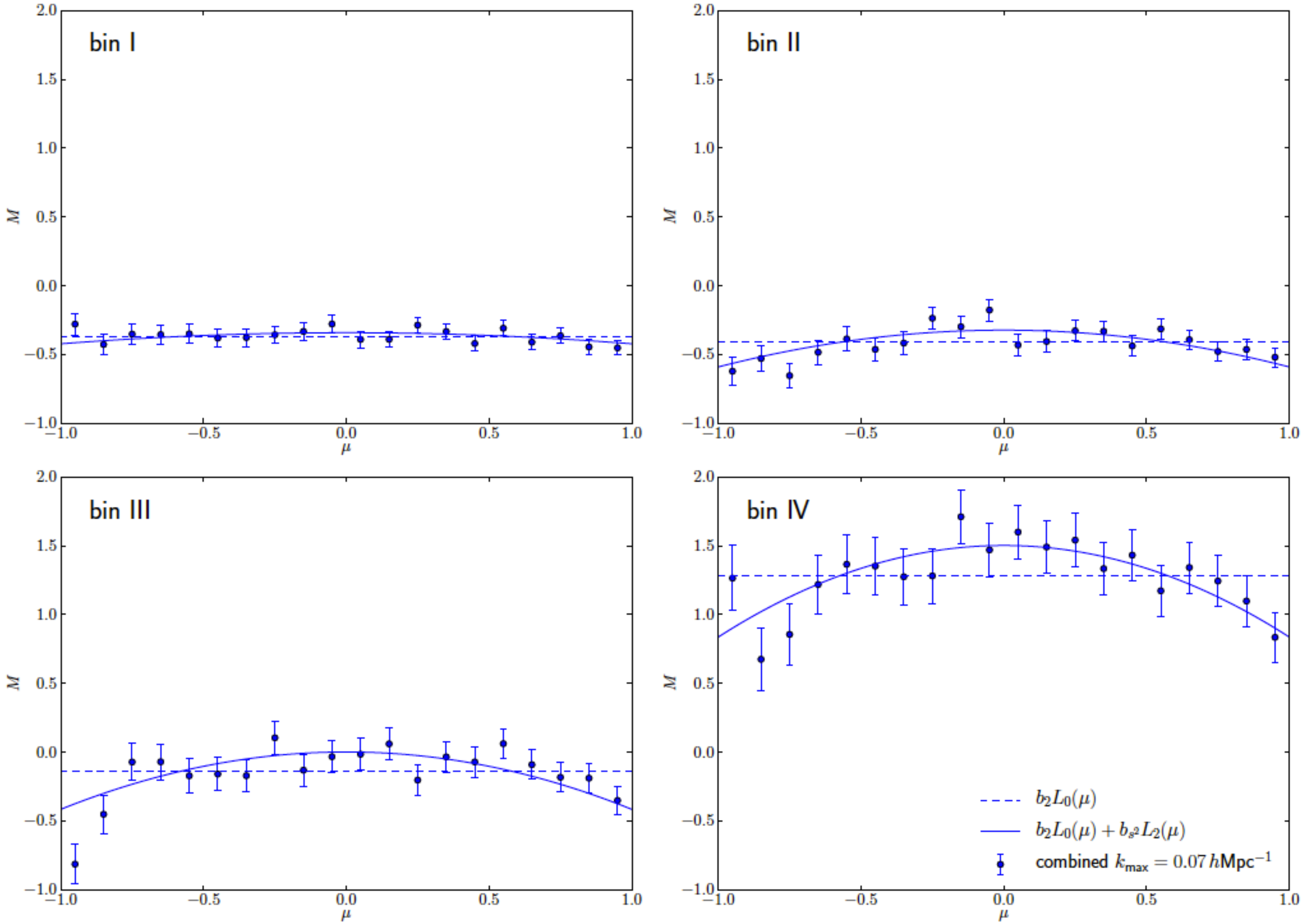
Baldauf etal, Kwan etal 2012

$$\delta_{\mathrm{h}}(\boldsymbol{x},\eta) = b_1 \delta(\boldsymbol{x},\eta) + b_2\left[\delta^2(\boldsymbol{x},\eta) - \langle\delta^2(\boldsymbol{x},\eta)\rangle\right] + b_{s^2}\left[s^2(\boldsymbol{x},\eta) - \langle s^2(\boldsymbol{x},\eta)\rangle\right]$$

Local Lagrangian bias model predicts $b_{s2} = -2(b_1-1)/7$

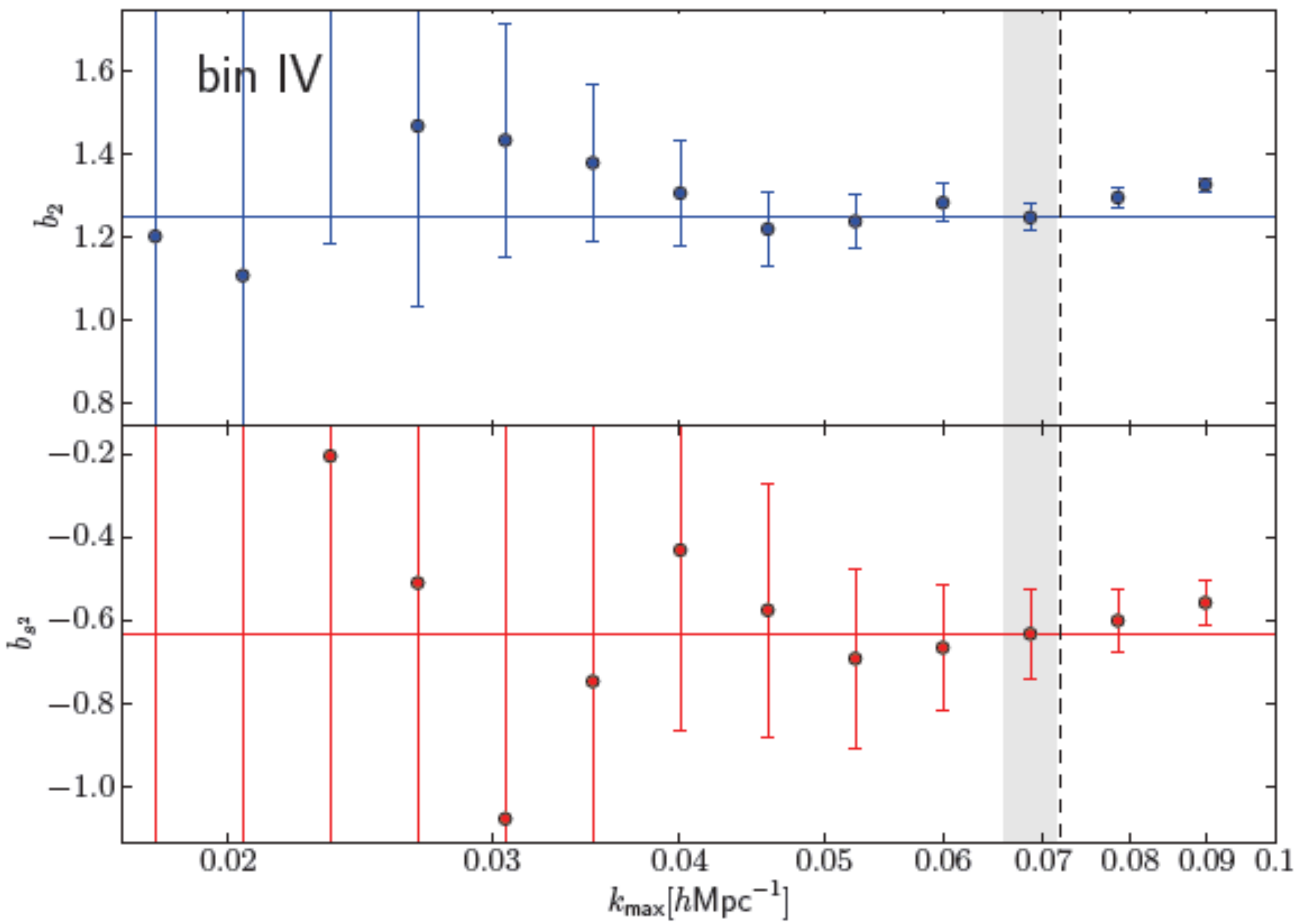
We can look for it in bispectrum B_{mmh}

$$B_{\mathrm{mmh}}^{(\mathrm{unsym})}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) - b_1 B_{\mathrm{mmm}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = 2P(k_1)P(k_2)\left[b_2 + b_{s^2}\left(\mu^2 - \frac{1}{3}\right)\right].$$

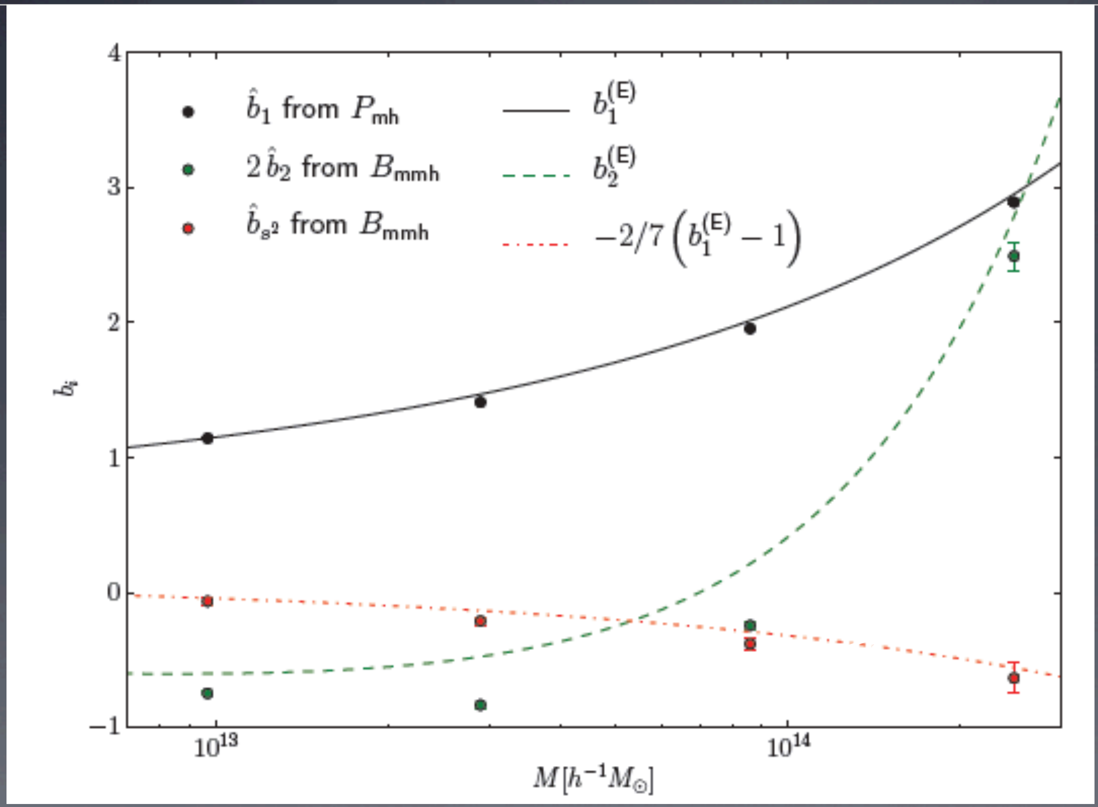


bin I
bin II
bin III
bin IV
M
μ
$b_2 L_0(\mu)$
$b_2 L_0(\mu) + b_{s^2} L_2(\mu)$
combined $k_{\rm max} = 0.07\,h$Mpc^{-1}

Imprint on the Bispectrum



Bispectrum fits to simulations vs peak-background split predictions



Lagrangian local bias is a better, but maybe not perfect description

Anything that is allowed by symmetry is also present

Do b_2 , b_{s2} help in modeling P(k)? No!

Saito et al, in prep

Complication III: 3^{rd} order bias

Let’s write all possible terms allowed by symmetry at 3^{rd} order (McDonald and Roy 2010)

2nd-order non-local bias (tidal bias)

$$B^{\mathrm{hmm}} \sim \langle \delta_{\mathrm{h}}^{(2)} \delta_{\mathrm{m}}^{(1)} \delta_{\mathrm{m}}^{(1)} \rangle$$

3rd-order non-local bias

$$T^{\mathrm{hmmm}} \sim \langle \delta_{\mathrm{h}}^{(3)} \delta_{\mathrm{m}}^{(1)} \delta_{\mathrm{m}}^{(1)} \delta_{\mathrm{m}}^{(1)} \rangle$$

$$P^{\mathrm{hm}} \sim \langle \delta_{\mathrm{h}}^{(1)} \delta_{\mathrm{m}}^{(1)} \rangle + \langle \delta_{\mathrm{h}}^{(1)} \delta_{\mathrm{m}}^{(3)} \rangle + \langle \delta_{\mathrm{h}}^{(2)} \delta_{\mathrm{m}}^{(2)} \rangle + \langle \delta_{\mathrm{h}}^{(3)} \delta_{\mathrm{m}}^{(1)} \rangle$$

linear bias x P^{NL}m

Combining P(k) with B(k) → 3rd-order nonlocal bias!

local bias

non-local bias

$$\delta_{\rm h}(\boldsymbol{x}) = c_\delta \delta_{\rm m}(\boldsymbol{x}) + \frac{1}{2}c_{\delta^2}\delta_{\rm m}(\boldsymbol{x})^2 + \frac{1}{2}c_{s^2}s(\boldsymbol{x})^2 + \frac{1}{3!}c_{\delta^3}\delta_{\rm m}(\boldsymbol{x})^3 + \frac{1}{2}c_{\delta s^2}\delta_{\rm m}(\boldsymbol{x})s(\boldsymbol{x})^2 + c_\psi\psi(\boldsymbol{x}) + c_{st}s(\boldsymbol{x})t(\boldsymbol{x}) + \frac{1}{3!}c_{s^3}s(\boldsymbol{x})^3 + c_\epsilon\epsilon + \ldots,$$

linear: can be measured via $P^{\rm hm}(k)$ at large scales

2nd-order: measured via $B^{\rm hmm}(k)$ at large scales s

3rd-order

where

$$s_{ij}(\boldsymbol{x}) \equiv \partial_i\partial_j\phi(\boldsymbol{x}) - \frac{1}{3}\delta^{\rm K}_{ij}\delta_{\rm m}(\boldsymbol{x}) = \left[\partial_i\partial_j\partial^{-2} - \frac{1}{3}\delta^{\rm K}_{ij}\right]\delta_{\rm m}(\boldsymbol{x}),$$ tidal field

$$t_{ij}(\boldsymbol{x}) \equiv \partial_i v_j - \frac{1}{3}\delta^{\rm K}_{ij}\theta_{\rm m}(\boldsymbol{x}) - s_{ij}(\boldsymbol{x}) = \left[\partial_i\partial_j\partial^{-2} - \frac{1}{3}\delta^{\rm K}_{ij}\right][\theta(\boldsymbol{x}) - \delta_{\rm m}(\boldsymbol{x})],$$

$$\psi(\boldsymbol{x}) \equiv [\theta(\boldsymbol{x}) - \delta_{\rm m}(\boldsymbol{x})] - \frac{2}{7}s(\boldsymbol{x})^2 + \frac{4}{21}\delta_{\rm m}(\boldsymbol{x})^2.$$

(halo density)-(matter density) McDonald & Roy (2010)

$$\begin{aligned} P^{\rm hm}_{00}(k) = & \left(c_\delta + \frac{34}{21}c_{\delta^2}\sigma^2 + \frac{1}{2}c_{\delta^3}\sigma^2 + \frac{1}{3}c_{\delta s^2}\sigma^2 + \frac{1}{2}c_{\delta\epsilon^2}\sigma^2_\epsilon + \frac{68}{63}c_{s^2}\sigma^2 - \frac{16}{63}c_{st}\sigma^2\right)P^{\rm NL}_{\delta\delta}(k) \\ & + c_{\delta^2}\int\frac{d^3q}{(2\pi)^3}P(q)P(|\boldsymbol{k}-\boldsymbol{q}|)F^{(2)}_{\rm S}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}) \\ & + c_{s^2}\int\frac{d^3q}{(2\pi)^3}P(q)P(|\boldsymbol{k}-\boldsymbol{q}|)F^{(2)}_{\rm S}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})S^{(2)}(\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}) \\ & + \left(-\frac{16}{21}c_{s^2} + \frac{32}{105}c_{st} + \frac{512}{2205}c_\psi\right)\sigma^2_3(k)P(k) \\ = & \; b_1 P^{\rm NL}_{\delta\delta}(k) + b_2 P_{b2,\delta}(k) + b_{s^2}P_{bs2,\delta}(k) + b_{3\rm nl}\,\sigma^2_3(k)P(k), \end{aligned}$$

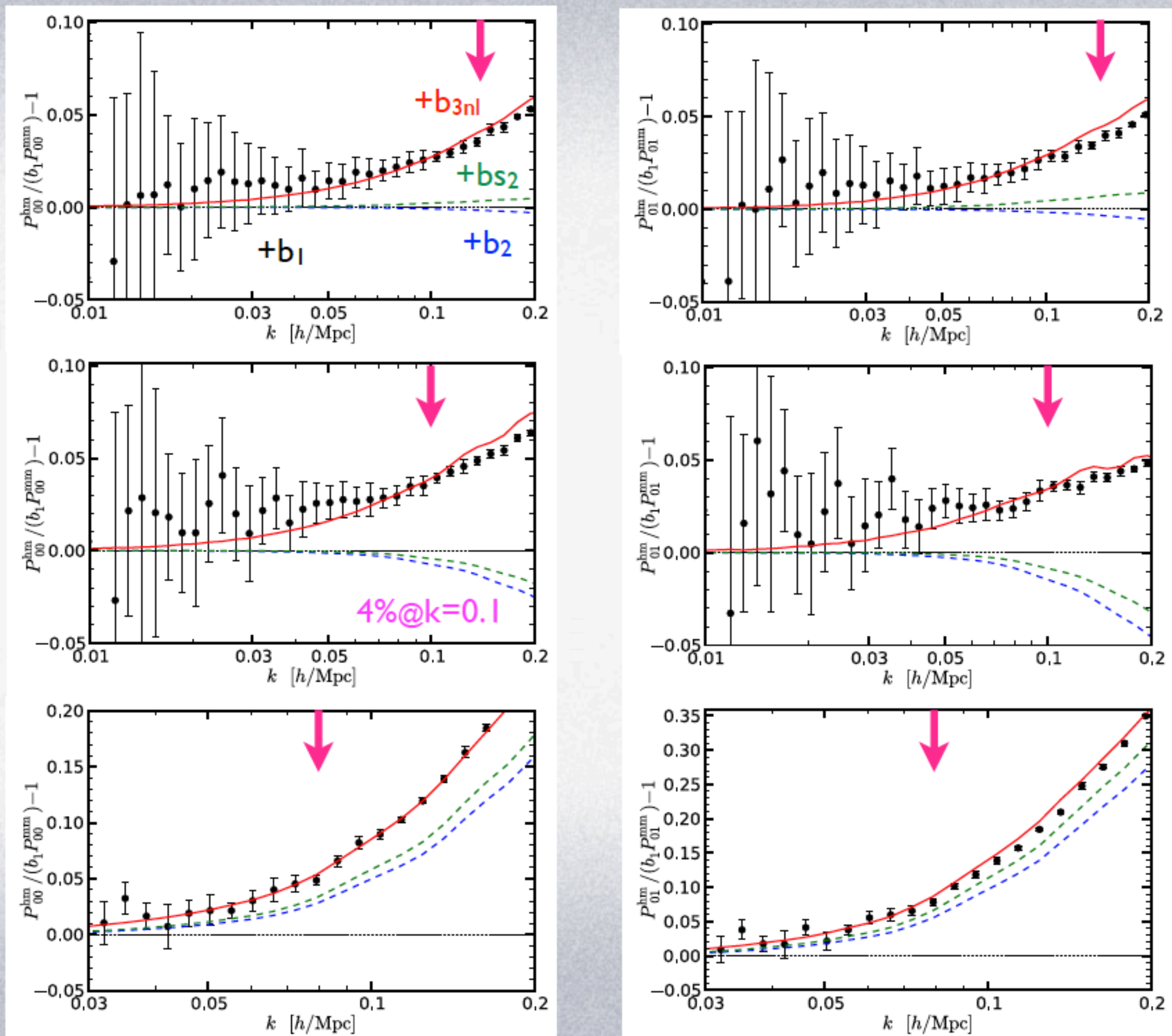
origin: (1)x(1) or (1)x(3)→linear bias

(halo density)-(matter density) | (halo density)-(matter momentum)

z=1 lightest

z=0.5 CMASS

z=0 massive bin

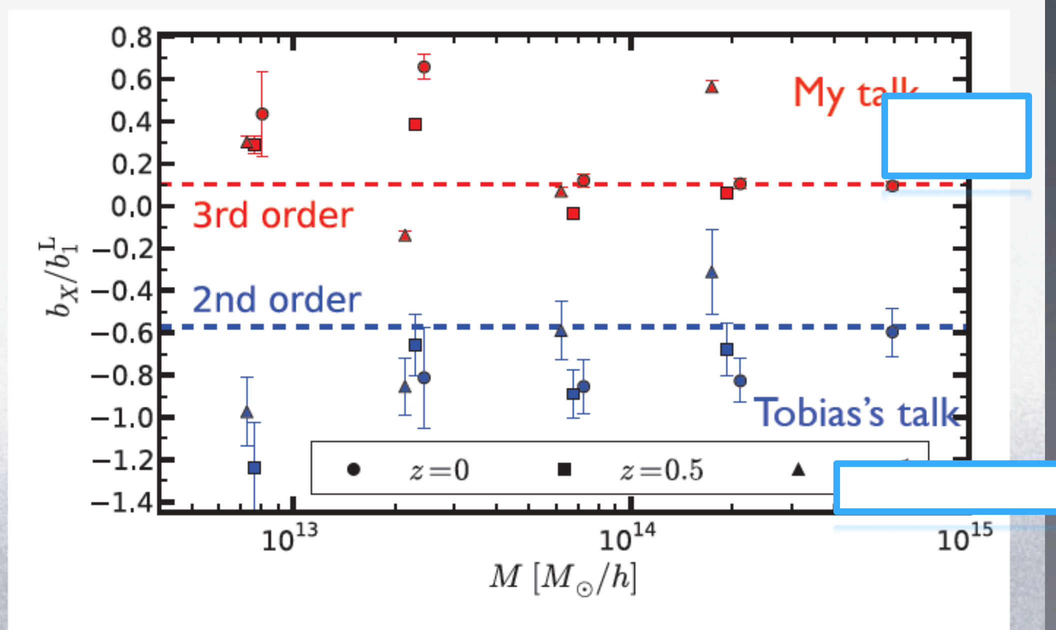


Prediction from local bias in Lagrangian space

The simple co-evolution picture predicts non-local bias as

2nd order $$b_{s^2} = -\frac{4}{7} b_1^{\mathrm{L}} = -\frac{4}{7}(b_1^{\mathrm{E}} - 1)$$

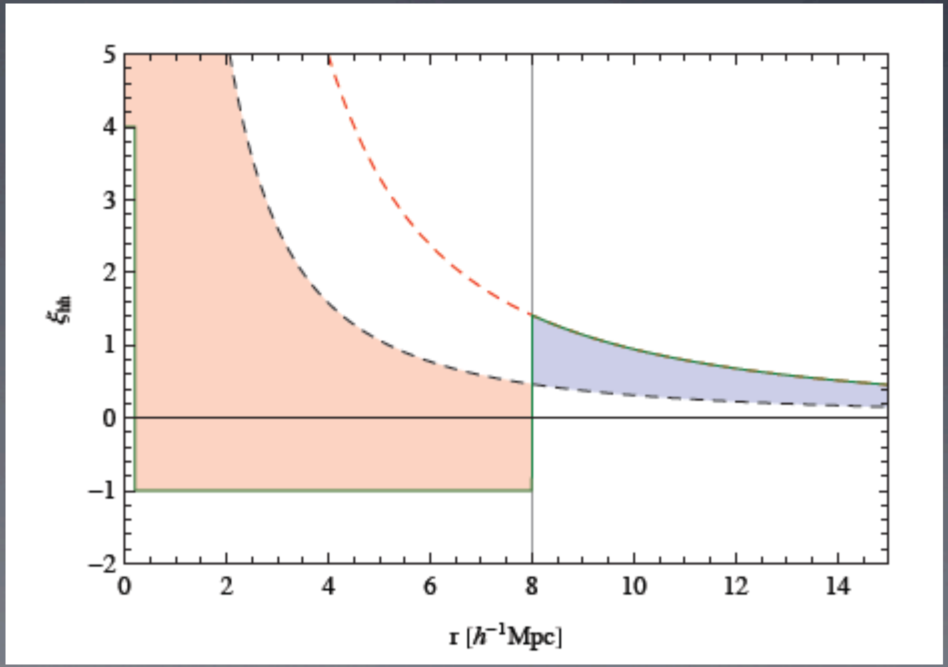
3rd order $$b_{3\mathrm{nl}} = \frac{32}{315} b_1^{\mathrm{L}} = \frac{32}{315}(b_1^{\mathrm{E}} - 1)$$



We now have introduced 4 free parameters to explain P_{hm} , B_{hmm}

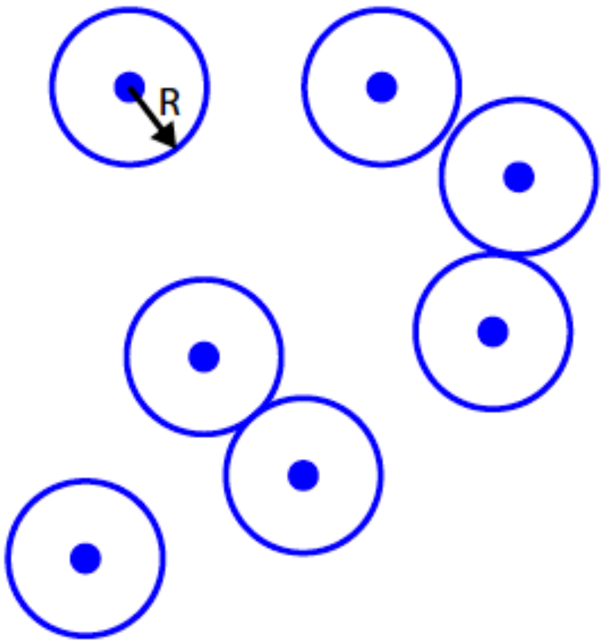
Complication IV: stochasticity from exclusion and nonlinearity

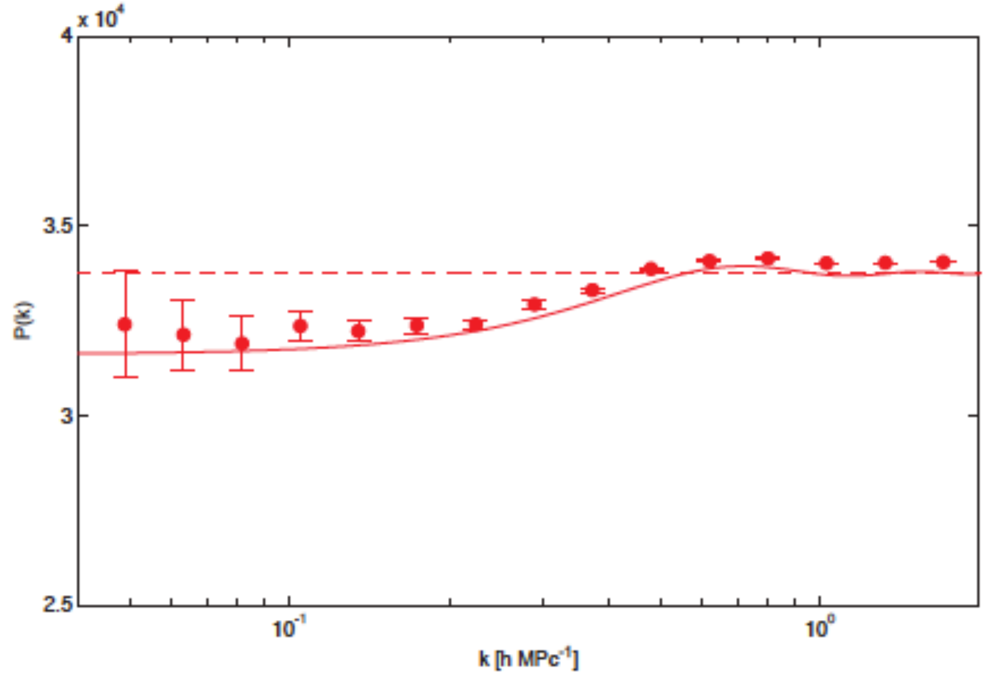
- We managed to explain P_{hm}, what about P_{hh}?
- Define stochasticity as $\sigma^2=\langle(\delta_h-b\delta_m)^2\rangle=P_{hh}-2bP_{hm}+b^2P_{mm}$
- If we can model $\sigma^2(k)$ we can model P_{hh}
- Standard model: $\sigma^2=1/n$
- 2 corrections: exclusion, nonlinear clustering



Baldauf et al 2013

Random Sample with Exclusion

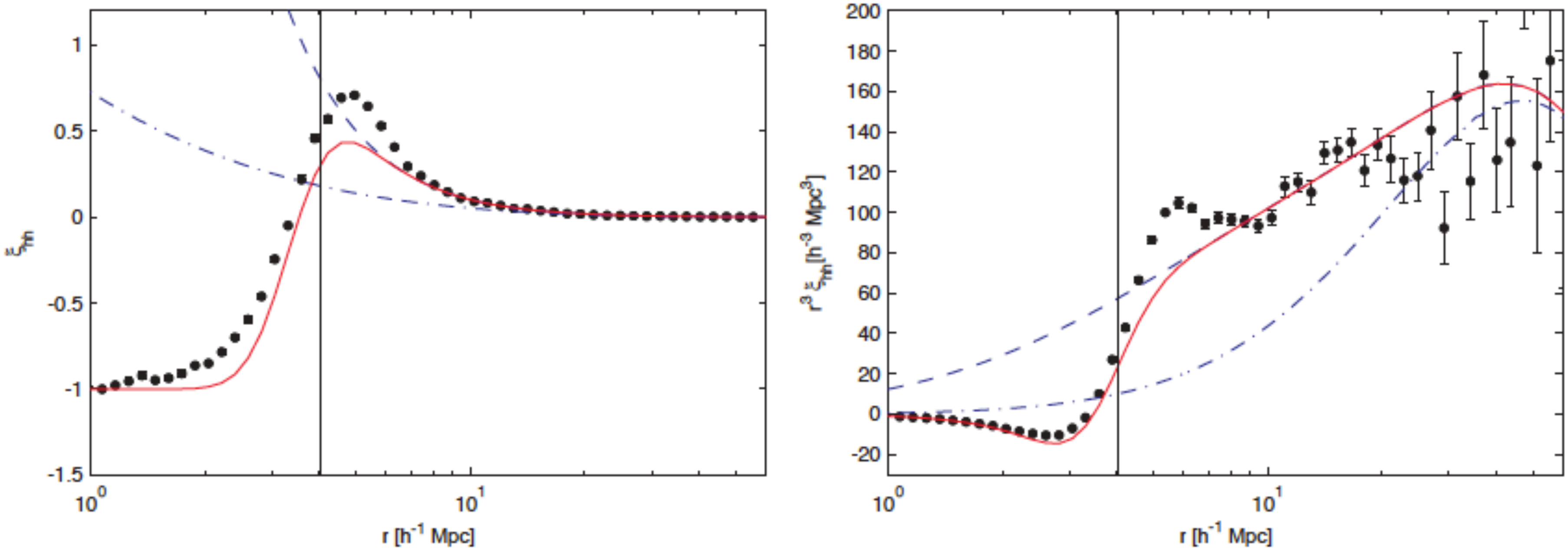




Perturbation Theory + Exclusion

$$\sigma_{ij}(k) = \frac{1}{\bar{n}} - [P_{\text{disc}} * W](k) + \frac{1}{2} b_{2,i} b_{2,j} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} P(q) P(\mathbf{k} - \mathbf{q})$$

Proto-halos in initial conditions



$\sigma^2(k=0)$ is given by integral of $r^3(\xi_{hh}-b^2\xi_{lin})$

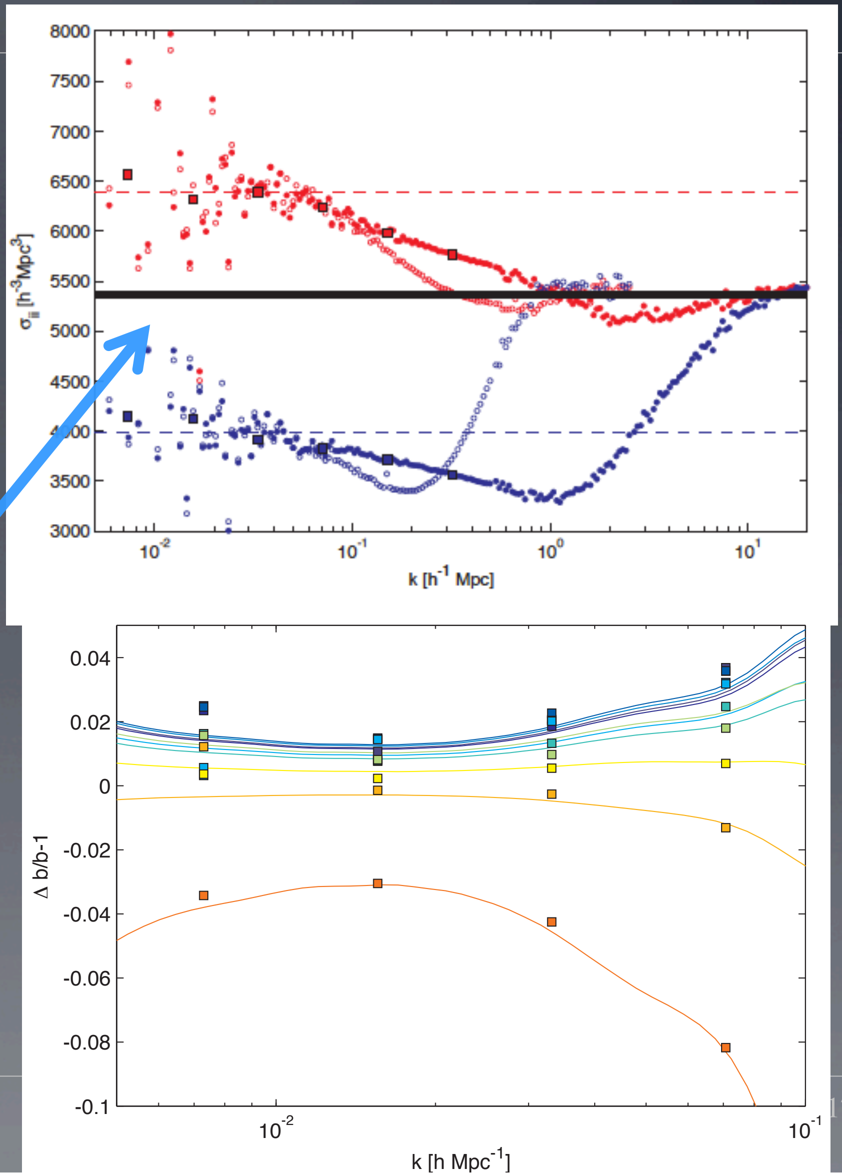
This value is preserved to z=0

Dependence on halo mass

Same value of $\sigma^2(k=0)$ between $z=z_{in}$ and $z=0$: gravity cannot modify it

Positive (nonlinear effects dominate) for low mass, negative (exclusion dominates) for high mass

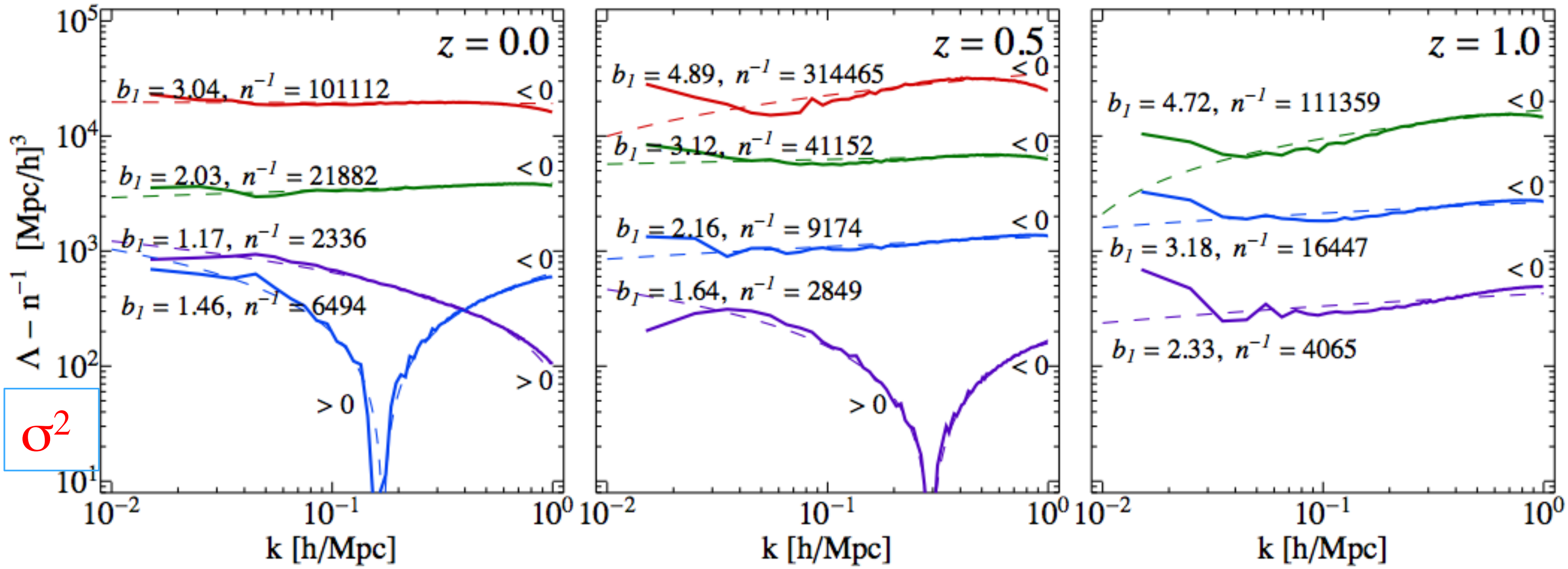
At k ➞ 0 correction to 1/n



Transition scale shrinks at z=0 relative to $z=z_{in}$

Effects of order a few %

Can we predict it? Sort of, not really… (Tobias's, Zvonimir's talk)



Vlah et al 2013

Scale dependent, redshift dependent, halo mass dependent

We can explain the full covariance matrix of halos of different mass

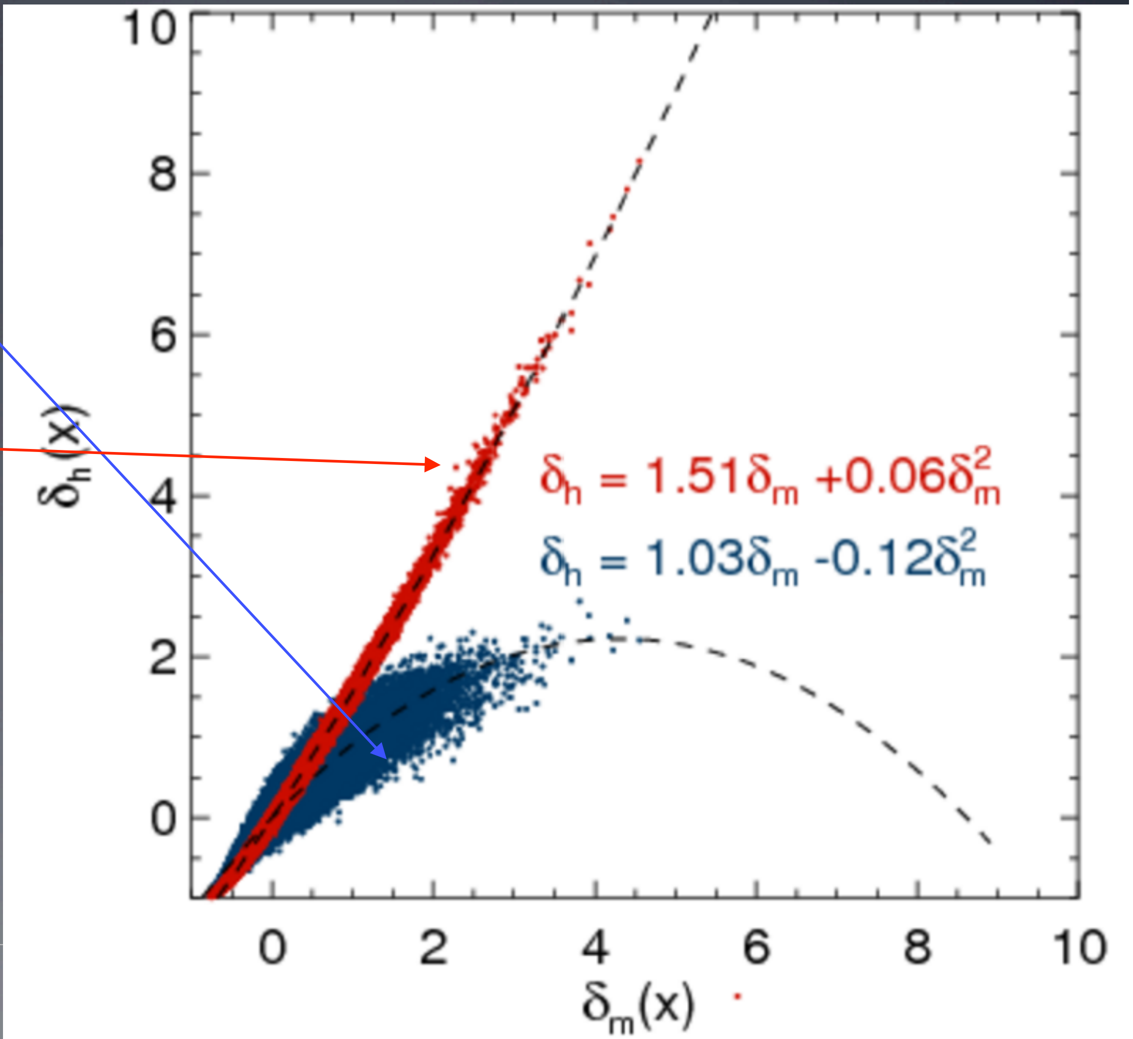
Diagonalization reveals one low eigenvalue: low stochasticity

stochasticity reduction

- Uniform weighting has large stochasticity
- Weighting galaxies by halo mass reduces scatter

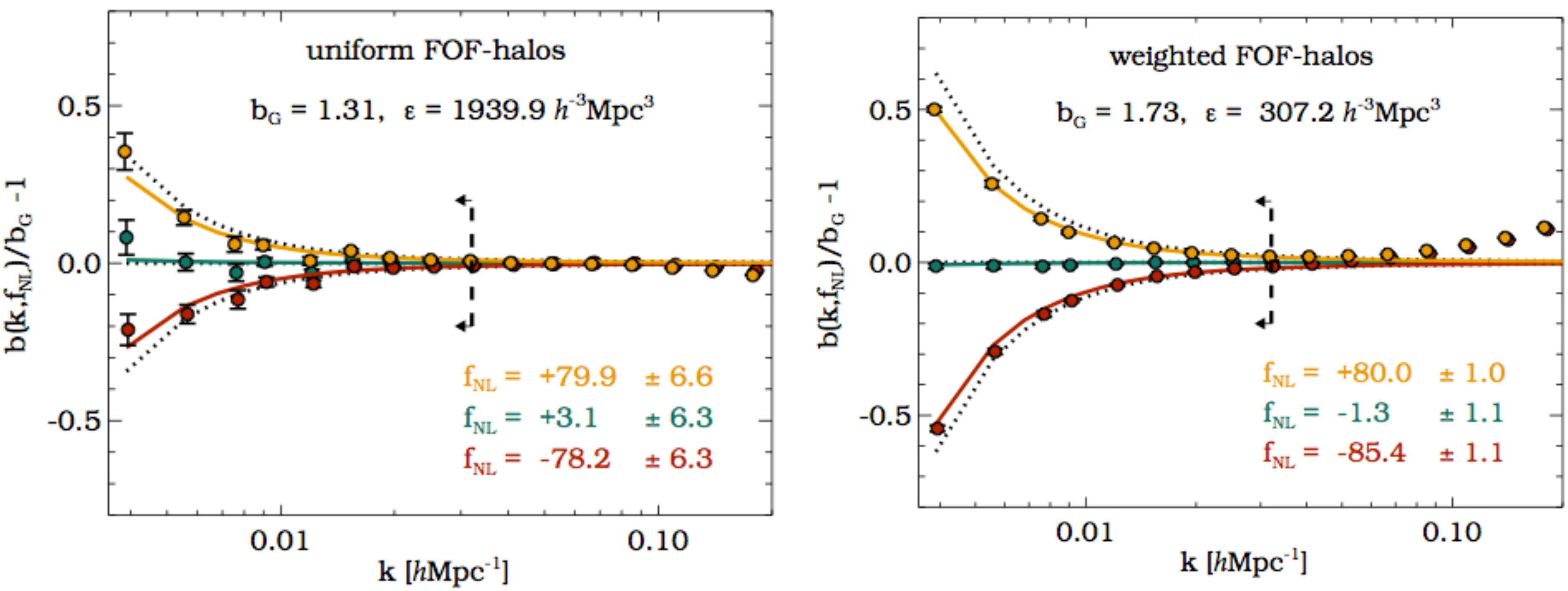
(Hamaus, US, Desjacques 2010)

Useful for reconstruction, f_{nl}…



f_{nl}: sampling variance canceling

Hamaus, US, Desjacques 2011



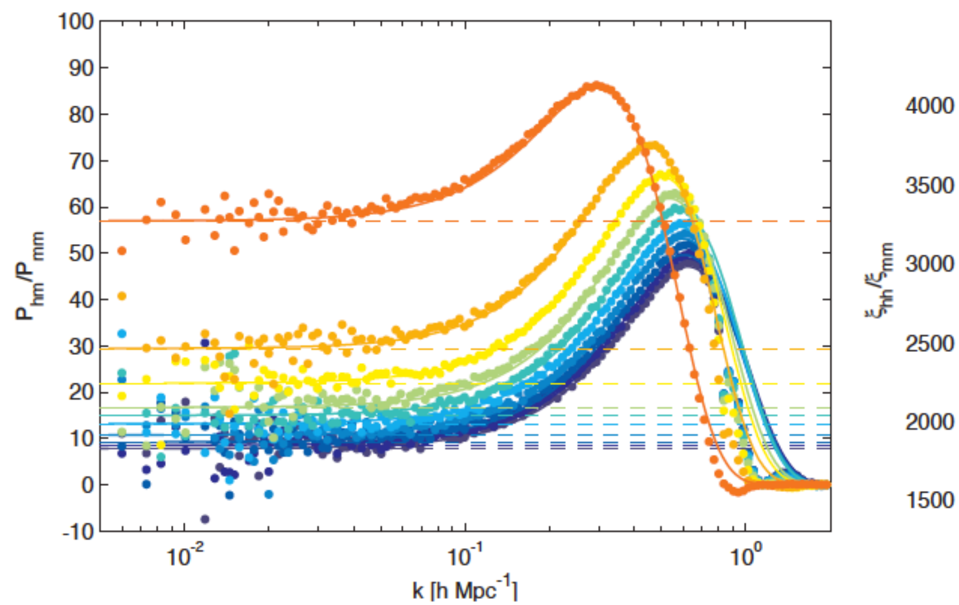
Future surveys (MS-DESI, Euclid) could reach f_{nl} around 1

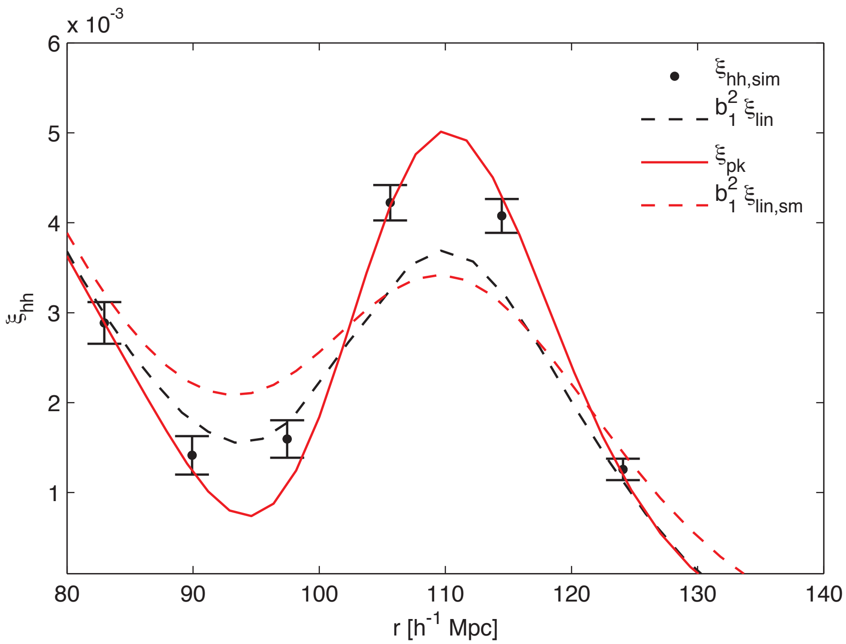
Complication V: linear peak biasing for proto-halos

- Peak constraints depend also on 2nd derivatives of density field, in low k limit one can expand to find (Desjacques 2008)

$$P_{\rm hm}(k) = \left(b_\nu + b_\zeta k^2\right) W_{{\rm G},R_{\rm pk}}(k) P(k)$$

$$P_{\rm hh}(k) \approx \left(b_\nu + b_\zeta k^2\right)^2 W^2_{{\rm G},R_{\rm pk}}(k) P(k)$$





k² effects likely survive to z=0, but are pushed to smaller scales. Similar effects in velocity or momentum (Tobias's talk)

State of the field

- For each new statistic we introduced new parameters to explain them, along with a theoretical explanation why it is natural to expect them and why we have no choice not to introduce them. Epicycles?
- So far we have more parameters than statistics, making explicit demonstration of effects like k^2 difficult
- Can we predict all nonlinear biasing terms as a function of z and b_1 and do we have a consistent description for all N-point functions in real and redshift space?
- To what k can we model galaxy power spectrum in real and redshift space? k=0.1 or 0.2h/Mpc? Other NL effects like FoG require even more poorly constrained parameters

Alternative: graceful transition to ignorance (McDonald 2012)

observable → $\delta_o(\mathbf{x}) = f[\delta_t(\mathbf{x}')]$ ← theory $\quad |\mathbf{x}-\mathbf{x}'| \lesssim R$

$$\delta_o(\mathbf{x}) = f[\delta_t(\mathbf{x}')] = f[0] + \int d\mathbf{x}'\ K(|\mathbf{x}-\mathbf{x}'|)\,\delta_t(\mathbf{x}') + \ldots$$

$$\delta_o(\mathbf{x}) = b\left[\delta_t(\mathbf{x}) + \frac{\tilde{b}_{k^2}}{2}R^2\nabla^2\delta_t(\mathbf{x}) + \ldots\right] + \ldots$$

$$\delta_o(\mathbf{k}) = b\left[1 - \frac{\tilde{b}_{k^2}}{2}R^2k^2 + \frac{\tilde{b}_{k^4}}{8}R^4k^4 + \ldots\right]\delta_t(\mathbf{k}) + \ldots$$

$$P_o(k) = b^2\left[1 - \tilde{b}_{k^2}\ R^2k^2 + \ldots\right]P_t(k) + \ldots$$

- We can parametrize our ignorance as a series in powers of k^2

Graceful transition to ignorance

- What should δ_t be? Options for dark matter: linear theory, SPT, fully NL matter. Can also include some biasing effects (eg local bias etc). SPT may be a good choice.
- b_{k2} etc are different for each statistic, but can include all complications mentioned before
- Stochasticity: $\sigma^2(k)=b_{k0}+b_{k2}k^2+\ldots$
- In redshift space $b_{k2}(\mu^0)$, $b_{k2}(\mu^2)\ldots$
- If δ_t linear then no PT needed. We are simply imposing locality ($kR<<1$) and nothing else
- All these coefficients can still be fully deterministic, eg just a function of b_1 (perhaps unlikely in any realistic galaxy case), or come with strong priors

Summary

- Biasing is hard: more parameters than statistics to determine them
- Redshift space disortions are even harder (Zvonimir's talk)
- Perhaps we should just parametrize our ignorance and let the data determine it (possibly with some priors from simulations)