Galaxy Clustering and the Peak-Background Split

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The bias problem

- What is our goal in “solving the bias problem” ?
- *Describe the connection between statistics of initial conditions and statistics of any observed large-scale structure tracer in terms of few, observationally determined numbers*
- How much (messy) astrophysics do we need to model ?
    - Trade-off between number of free parameters and restrictiveness of model

The bias problem

- The relation between tracer and matter perturbations in its most general form* is

$$n_h(\mathbf{x}) = F_h[\delta(\mathbf{y})]$$

- Clearly, we can't learn anything by having a general *functional* - need to condense it into a small set of numbers to be constrained observationally

- What are the most general constraints on biasing we can place without making specific model assumptions ?

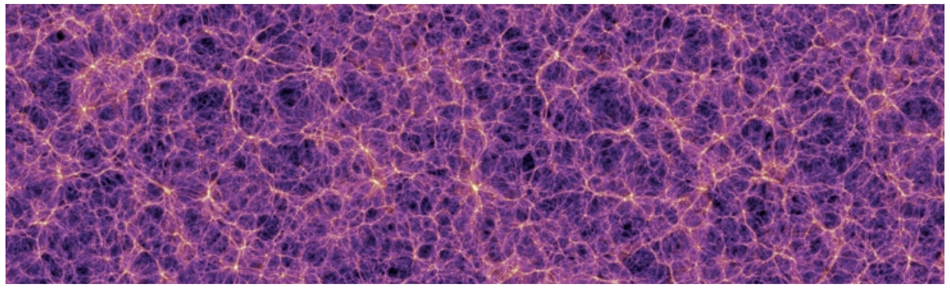
*On a constant-proper-time slice; also dependence on matter *velocities*.

I. "Model-independent" biasing approach

- Suppose the (expectation value of) local abundance of tracers at $\mathbf{x}$, at fixed proper time, is a function of the matter density field within a finite region of size R_*
- Physically, the largest possible such scale is the horizon: $R_* \leq H^{-1}(t)$
- But probably R_* is much smaller (few Mpc) for most tracers (since DM plays the dominant role)

Large-scale limit

- When looking at scales much larger than R_*, abundance of galaxies depends only on
  - (i) proper time (age of the Universe)
  - (ii) local matter density,
  - (iii) amplitude and shape of initial density fluctuations
- I.e. all the properties we need to initialize and run an N-body simulation



- For Gaussian initial conditions, (iii) is trivial: galaxy density is *a local function of the matter density*: local biasing

Coarse-graining

- Local biasing in a coarse-grained sense:

$$n_h(\mathbf{x}) = F_{h,L}[\delta_L(\mathbf{x});\ \mathbf{x}] \qquad \delta_L(\mathbf{x}) = \int d^3\mathbf{y}\, \delta(\mathbf{x}-\mathbf{y}) W_{R_L}(|\mathbf{y}|)$$

- The coarse-graining scale R_L is nothing physical:
  - a tool for our effective description
  - Arbitrarily chosen "UV cutoff"
- Expression for any *observable should be independent of* R_L

Tracer correlations

- Tracer correlation function is then, formally,

$$\xi_h(r) = \frac{\langle F_{h,L}(\delta_L(\mathbf{x}_1), \mathbf{x}_1) F_{h,L}(\delta_L(\mathbf{x}_2), \mathbf{x}_2)\rangle}{\langle F_{h,L}\rangle^2} - 1$$

Fry & Gaztanaga '93; Coles '93

- ξ_h is observable and should thus be R_L-independent. But, both $F_{h,L}$ and δ_L depend on R_L.

- For example, expanding $F_{h,L}$ in δ_L yields terms $\propto \sigma_L^{2n} = \langle \delta_L^2 \rangle^n$

- **Goal:** reorder ξ_h into an *R_L-independent expansion* in terms of matter correlators such as $\xi_L(r) = \langle \delta_L(1)\delta_L(2)\rangle$, i.e. no terms $\propto \sigma_L^{2n}$

Large-scale limit and local biasing

- On large scales, clustering of tracers should be determined by their *abundance in an unperturbed Universe with varying background density,* at fixed age, $\bar{n}_h(\bar{\rho}, t)$
- We can expand this function around fiducial $\bar{\rho}$ at fixed age by defining

$$b_N = \frac{\bar{\rho}^N}{\bar{n}_h(\bar{\rho}, t_0)} \frac{\partial^N \bar{n}_h(\bar{\rho}, t_0)}{\partial \bar{\rho}^N}$$

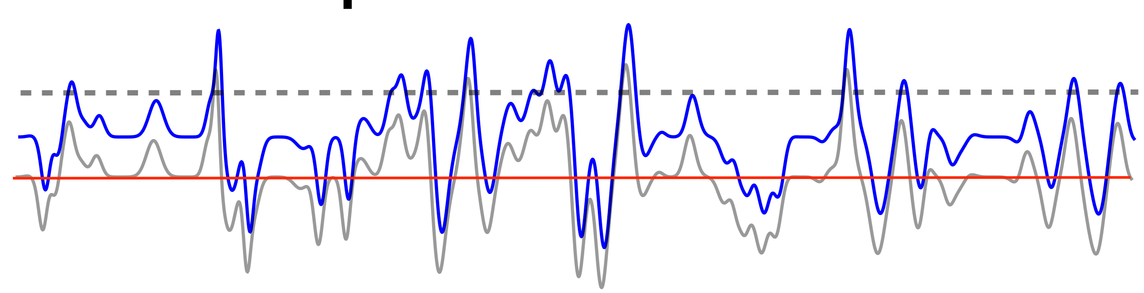
- b_N = *peak-background split (PBS) bias parameters* (following historic usage)
- The parameters b_N are *uniquely defined numbers* (at fixed t_0) for any given tracer - in particular, *independent of* R_L

PBS bias parameters

- To determine b_N *exactly*, run cosmological simulations with varying $\bar{\rho}$ and measure $\bar{n}_h$ at fixed age
- Simple *special case*: universal mass function prescription:

$$\bar{n}_h(M) \propto f(\nu), \quad \nu = \frac{\delta_c}{\sigma(M)}$$

- Adding a uniform matter density $D\bar{\rho}$ is equivalent to saying that the collapse threshold δ_c is reduced:



$$\delta_c \to \delta_c - D$$

Cole & Kaiser '89
Mo, Jing & White '97

- Thus*, $$b_N = \frac{(-1)^N}{\sigma(M)^N} \frac{1}{f(\nu)} \frac{d^N f(\nu)}{d\nu^N}$$

* There are small corrections from changes of the mass definition

PBS biases and correlations

- If residual dependence on x in $F_{h,L}(\delta_L, \mathbf{x})$ has *no large-scale correlations*, then, for a Gaussian density field:

$$\xi_h(r) = \sum_{N=1}^{\infty} \frac{b_N^2}{N!} \left[\xi_L(r)\right]^N$$

proofs in 1212.0868

- For a general density field:

$$\xi_h(r) = \sum_{N,M=1}^{\infty} \frac{b_N b_M}{N!M!} \left\langle \delta_L^N(1) \delta_L^M(2) \right\rangle_{\mathrm{nzl}}$$

← No zero-lag correlators

Analogous expression for tracer counts in cells

PBS biases and correlations

$$\xi_h(r) = \sum_{N=1}^{\infty} \frac{b_N^2}{N!} \left[\xi_L(r)\right]^N$$

- As desired, we have an expansion in terms of powers of $\xi(r)$ and PBS biases b_N
  - *Renormalized b_N* absorb all zero-lag terms (powers of σ_L)
  - Same b_N describe *both auto- and cross-correlations*
  - *Convergent whenever* $\xi(r) << 1$, independently of coarse-graining scale
- Local biasing as an effective description valid on large scales for any tracer, rather than a "microscopic" description valid at some fixed scale R_L
- Understood in this sense, *peak-background split is exact*

II. Beyond local bias

- Recall that there is non-locality in the tracer-matter relation on some scale R_*, i.e. at linear order (for simplicity)

$$\delta_h(\mathbf{x}) = \int f(|\mathbf{y}|)\delta(\mathbf{x}-\mathbf{y})d^3\mathbf{y} \qquad R_*^2 = \frac{1}{2}\int f(|\mathbf{y}|)y^2 d^3\mathbf{y}$$

- We can expand

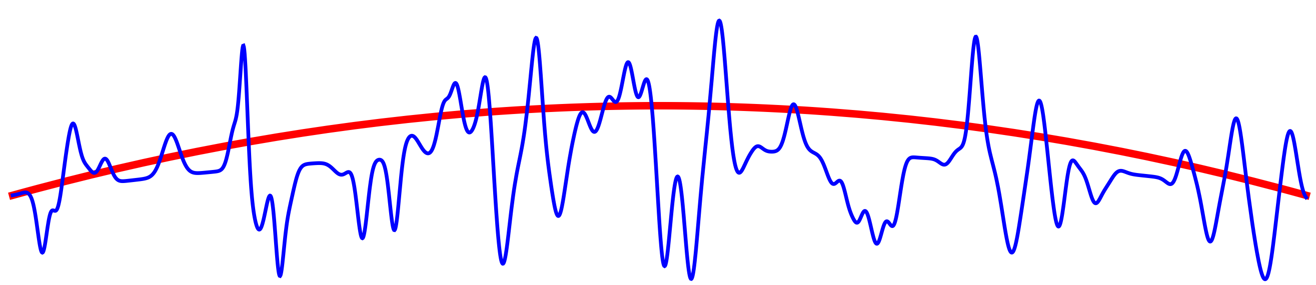
$$\delta_h(\mathbf{x}) = c_1\delta(\mathbf{x}) + R_*^2\nabla^2\delta(\mathbf{x}) + \cdots$$

- Thus, we expect a non-local bias w.r.t. $\nabla^2\delta$ to be necessary on sufficiently small scales

Curvature bias

- *Generalize local ansatz:* tracer density depends on local coarse-grained density as well as its curvature,

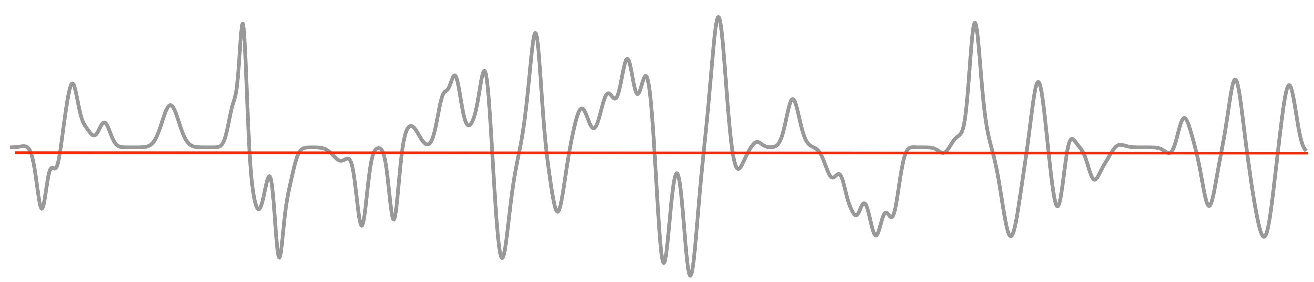
$$n_h(\mathbf{x}) = F_{h,L}\left[\delta_L(\mathbf{x}),\ \nabla^2\delta_L(\mathbf{x});\ \mathbf{x}\right]$$



- Leads to a bivariate expansion in $\delta_L,\ \nabla^2\delta_L$
  - coefficients again explicitly R_L-dependent
  - restrict to linear order here

Curvature bias

- What is the physical (renormalized) curvature bias ?
- Consider adding a component with uniform curvature to the density: $\delta(\mathbf{x}) \to \delta(\mathbf{x}) + \frac{\alpha}{6\ell^2}\mathbf{x}^2$



- Then, define $b_{\nabla^2\delta} = \frac{\ell^2}{\bar{n}_h}\frac{\partial \bar{n}_h}{\partial \alpha}\Big|_{\alpha=0}$

Note: dimension [length]^2

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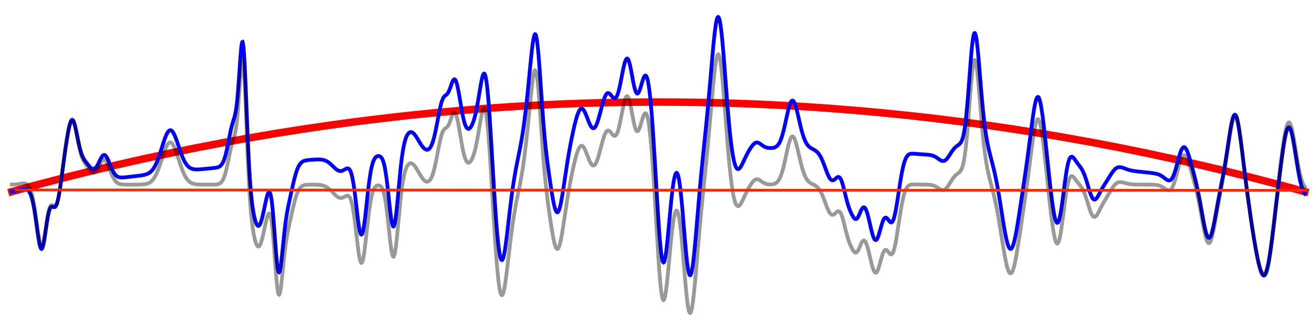

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Curvature bias

- The tracer correlation then becomes
$$\xi_h(r) = b_1^2 \xi(r) + 2 b_1 b_{\nabla^2 \delta} \nabla^2 \xi(r) + \cdots$$
- Introducing curvature bias has thus removed the R_L-dependence in $\xi_L(r)$!

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- Introducing curvature bias has thus removed the R_L-dependence in $\xi_L(r)$!
- In fact, we can generalize to include higher derivatives of the density field $\nabla^{2n}\delta$ - can show that we then remove the effects of smoothing exactly - *the scale R_L disappears completely !*
- If $b_{\nabla^{2n}\delta} \sim R_*^{2n}$, scale R_* is physical "cut-off" of the effective description: we cannot hope to describe the tracer statistics on scales R_* and smaller

Curvature bias = scale-dependent bias

- Tracer correlation in Fourier space becomes
  $P_h(k) = b_1^2 P(k) + 2 b_1 b_{\nabla^2\delta}\, k^2\, P(k) + \cdots$

- $b_{\nabla^2\delta}$ is a scale-dependent bias which can boost or suppress BAO feature in $\xi_h(r)$

See Desjacques et al, 2010 for peaks of a Gaussian density field

- Our general renormalization approach predicts that a *scale-dependent bias needs to be introduced whenever* $(R_* k)^2$ *becomes relevant*

III. Non-Gaussianity

- Previous results formally apply to non-Gaussian density fields as well
- The lowest-order non-Gaussian contribution to halo clustering is the 3-pt function:

$$\xi_h(r) = b_1^2 \xi_L(r) + b_1 b_2 \langle \delta_L(1) \delta_L^2(2) \rangle + \cdots$$

Verde & Matarrese, '08

- With local NG, the second term is*

$$\left\langle \delta_L(1) \delta_L^2(2) \right\rangle = 4 f_{\rm NL} \left\langle \delta_L(1) \phi(2) \right\rangle \sigma_L^2$$

- Despite having used the *renormalized* b_N here, we get an unwanted R_L-dependence. Something is missing !

* In the large-scale limit

LSS with non-Gaussianity

- In the Gaussian case, the dependence of the tracer abundance on the small-scale fluctuations $\delta_s(\mathbf{x}) = \delta(\mathbf{x}) - \delta_L(\mathbf{x})$ is irrelevant for clustering
- The R_L-dependence in NG case tells us that we can no longer ignore this dependence
  - Not surprising since for local f_{NL},

$$\delta_s(\mathbf{x}) \rightarrow [1 + 2f_{\rm NL}\phi(\mathbf{x})]\delta_s(\mathbf{x})$$

Small-scale fluctuations

- Thus, we explicitly introduce the dependence of the tracer density on the amplitude of δ_s:

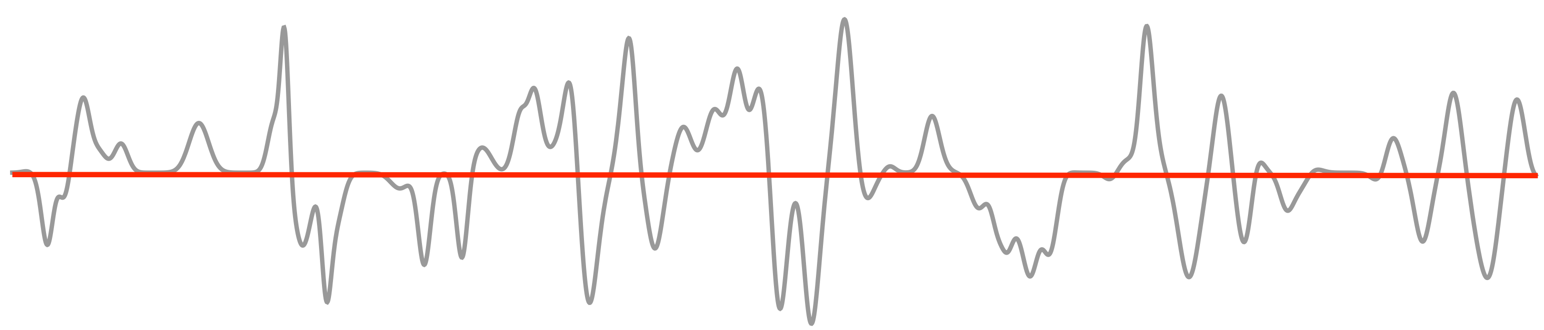
$$n_h(\mathbf{x}) = F_{h,L}\left[\delta_L(\mathbf{x}),\ y_*(\mathbf{x});\ \mathbf{x}\right]$$

$$y_*(\mathbf{x}) \equiv \frac{1}{2}\left(\frac{\delta_s^2(\mathbf{x})}{\sigma_s^2} - 1\right)$$

- As in the case of $\nabla^2\delta_L$, we now have a bivariate bias expansion.

Bivariate halo bias

- Bivariate bias parameters encode the dependence of $\bar{n}_h$ on the mean matter density as well as the amplitude of initial density fluctuations



- Thus, e.g. $b_{01} = \dfrac{1}{\bar{n}_h} \dfrac{\partial \bar{n}_h}{\partial \ln \sigma_8}$

$\sigma_8 \propto \mathcal{A}_s^{1/2}$

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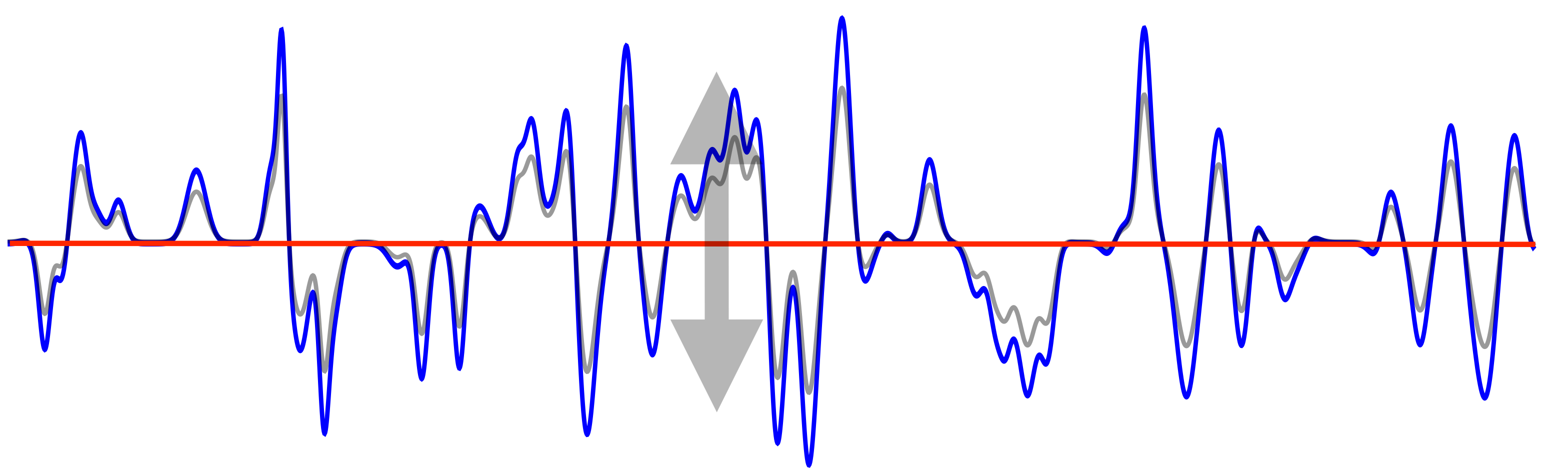

- Thus, e.g. $b_{01} = \frac{1}{\bar{n}_h} \frac{\partial \bar{n}_h}{\partial \ln \sigma_8}$ $\qquad \sigma_8 \propto \mathcal{A}_s^{1/2}$

Prediction for LSS statistics

- We then obtain

see 1212.0868

$$\xi_h(r) = b_{10}^2\,\xi_L(r) + \frac{b_{20}^2}{2}\,\xi_L^2(r) + 2b_{10}b_{01}\,\langle\delta_L(1)y_*(2)\rangle$$

- Again, we have obtained an R_L-independent result
  - The term $\propto b_2\sigma_L^2$ has been *absorbed into* b_{01}
  - Scale-dependent bias is encoded by

$$\langle\delta_L(1)y_*(2)\rangle = 2f_{\rm NL}\,\langle\delta_L(1)\phi(2)\rangle$$

Generalizations

(for NG aficionados)

- Argument generalizes to *non-local and higher order non-Gaussianity*
  - Key property is the *squeezed limit scaling* of the primordial bispectrum (or N-pt function)
  - The value of scale-dependent bias b_{01} depends on this scaling: tracers respond differently to different shapes of non-Gaussianity
- For a universal mass function, recover the previously found $b_{01} = \delta_c b_{10}$ for local NG
  - Again, special case of the general *exact definition*

Generalizations

(for NG aficionados)

- We can also generalize beyond the squeezed limit (see *arXiv:1304.1817*)
- Now, long-wavelength modes modify the shape as well as the amplitude of small-scale fluctuations
  - Leads to trivariate bias expansion...
- Clarifies the differences beyond squeezed limit between various scale-dependent bias predictions

Conclusions

- Gravity and formation of tracers are local processes
- We use this as starting point of a general, model-independent bias expansion
  - Fictitious coarse-graining scale R_L indicates the regime of validity of the bias expansion
  - PBS biases (when understood in this way) are *exact*
  - Source of scale-dependent bias from primordial non-Gaussianity clarified (no term $\propto b_2\sigma_L^2$)
- *Framework* in which specific approaches (peaks, excursion set, ...) can be embedded to provide relations between bias parameters (cf. scale-dep bias from f_{NL})