The Integrated Perturbation Theory and its Applications

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@Workshop on Galaxy Bias: Non-linear, Non-local and Non-Guassian

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Integrated perturbation theory (iPT)

- Integration of Four “Non-’s”
  - nonlinear perturbation theory
  - nonlocal bias
  - nonlinear redshift-space distortions
  - non-Gaussianity of primordial density fields

Lagrangian perturbation theory with Lagrangian (nonlocal) bias

- The relation between Eulerian density fluctuations and Lagrangian variables

$$1+\delta_{\mathrm{X}}(\boldsymbol{x}) = \int d^3q \left[1+\delta^{\mathrm{L}}_{\mathrm{X}}(\boldsymbol{q})\right] \delta^3_{\mathrm{D}}[\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{\Psi}(\boldsymbol{q})]$$

Eulerian density field

Biased field in Lagrangian space

displacement (& redshift distortions)

- Perturbative expansion in Fourier space

Kernel of the Lagrangian bias

$$\delta^{\mathrm{L}}_{\mathrm{X}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta^3_{\mathrm{D}}(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k}) b^{\mathrm{L}}_n(\boldsymbol{k}_1,\dots,\boldsymbol{k}_n) \delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta_{\mathrm{L}}(\boldsymbol{k}_n)$$

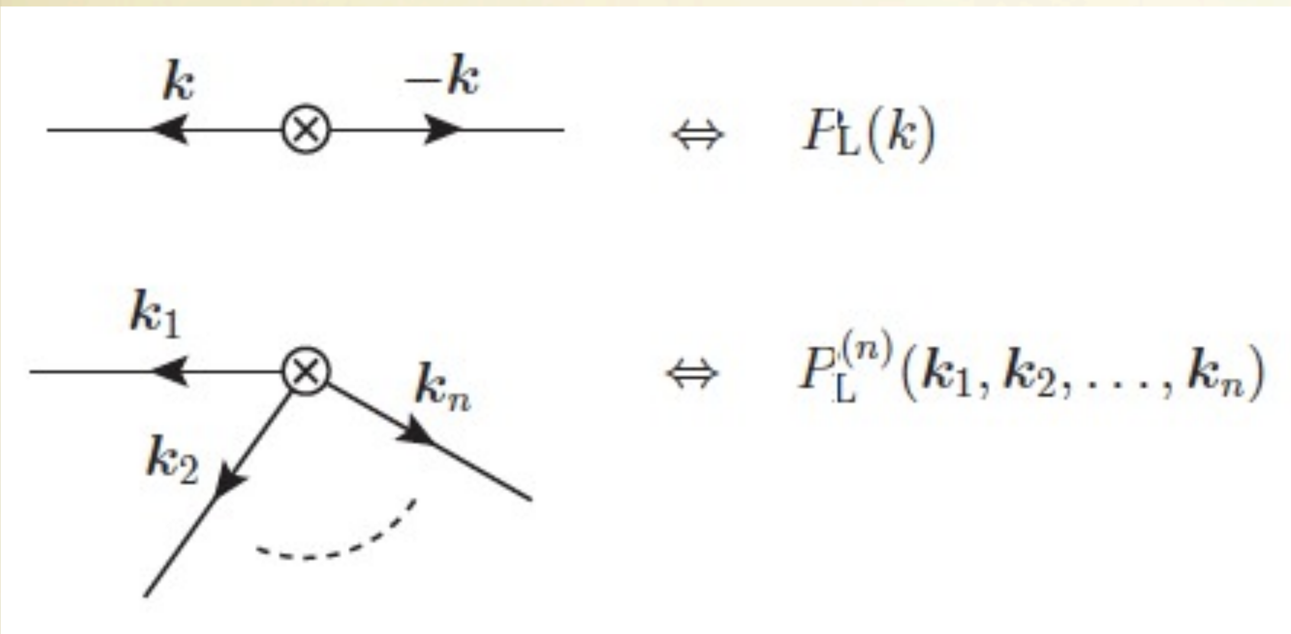
$$\tilde{\boldsymbol{\Psi}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \frac{i}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta^3_{\mathrm{D}}(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k}) \boldsymbol{L}_n(\boldsymbol{k}_1,\dots,\boldsymbol{k}_n) \delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta_{\mathrm{L}}(\boldsymbol{k}_n)$$

Kernel of the displacement field (& redshift distortions)

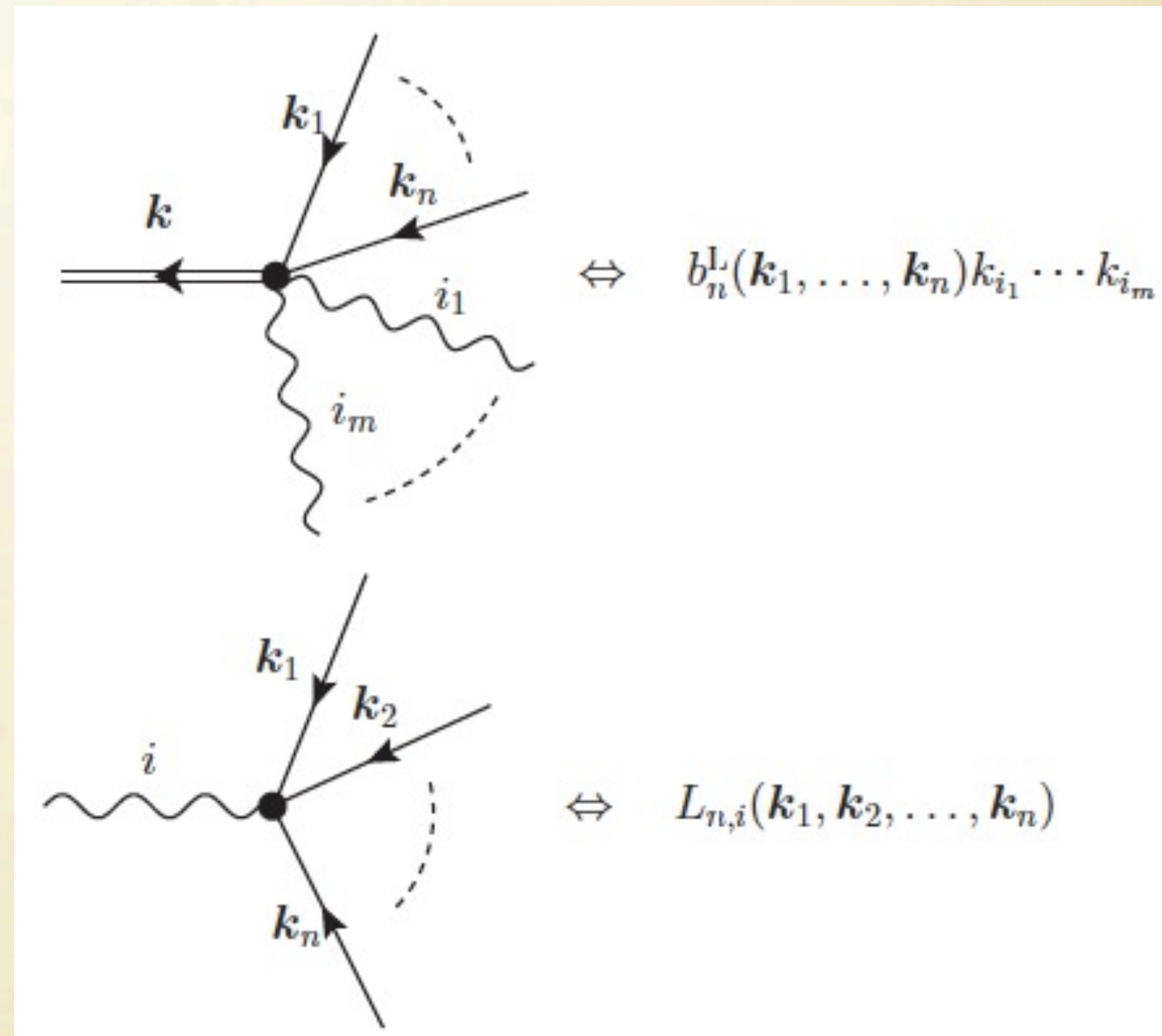
$$\boldsymbol{k}_{1\cdots n} \equiv \boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n$$

Diagrams in iPT

Primordial spectra



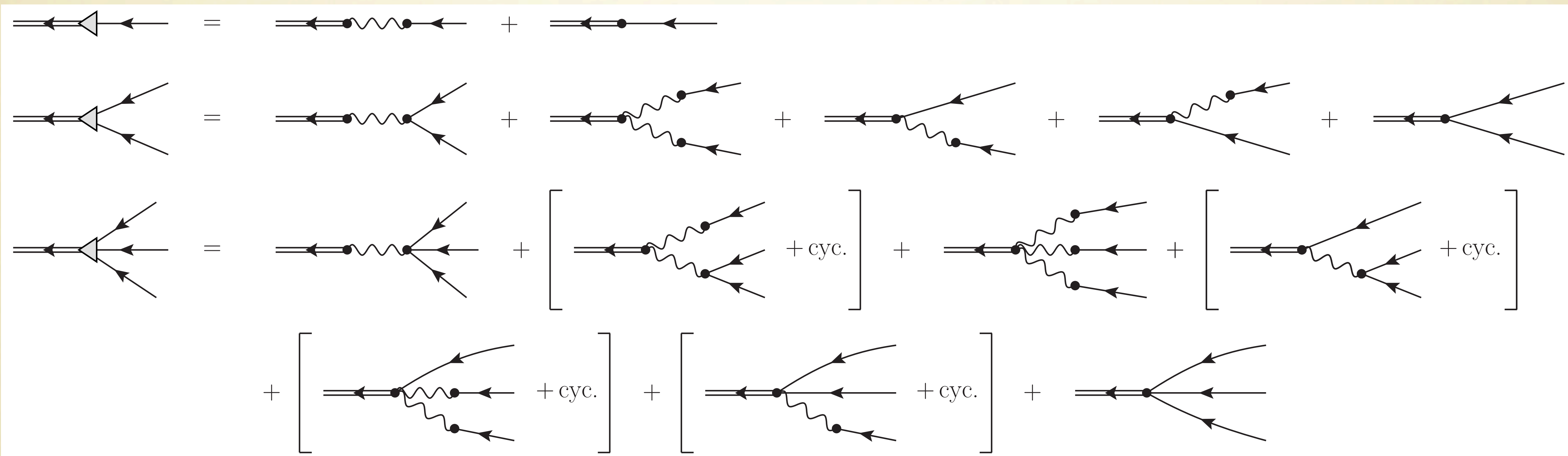
Vertices in Lagrangian PT



can naturally deal with RSD and nG

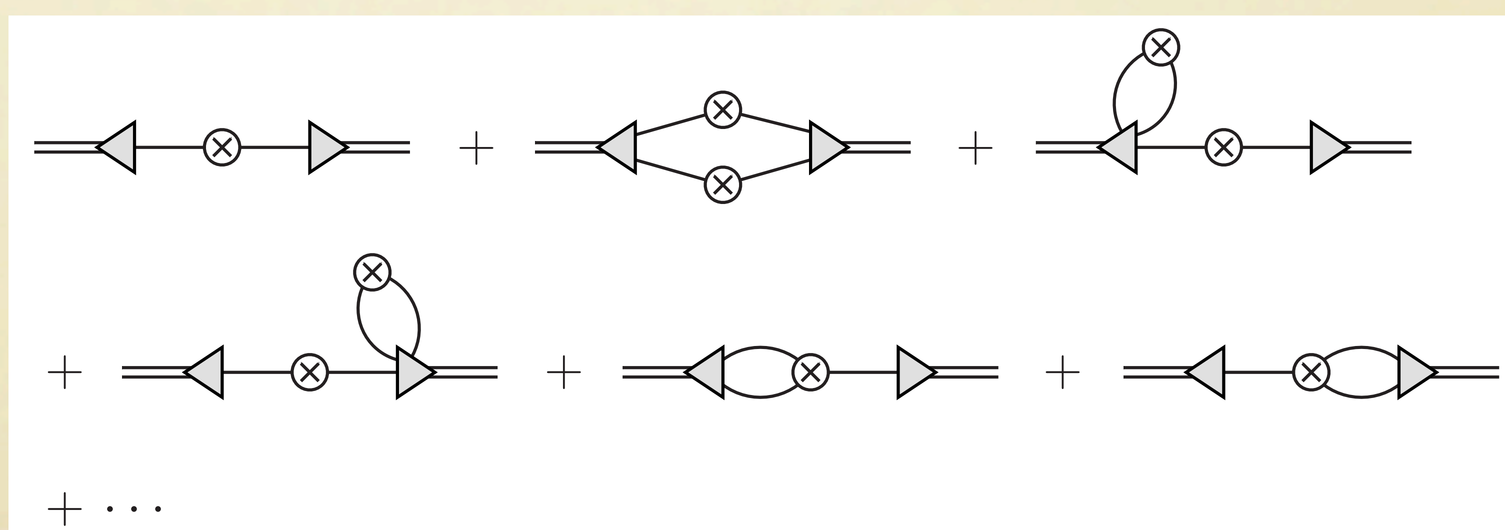
Vertex shrunk

- Shrunk vertices



- Ex.)

$$P_X(k) =$$

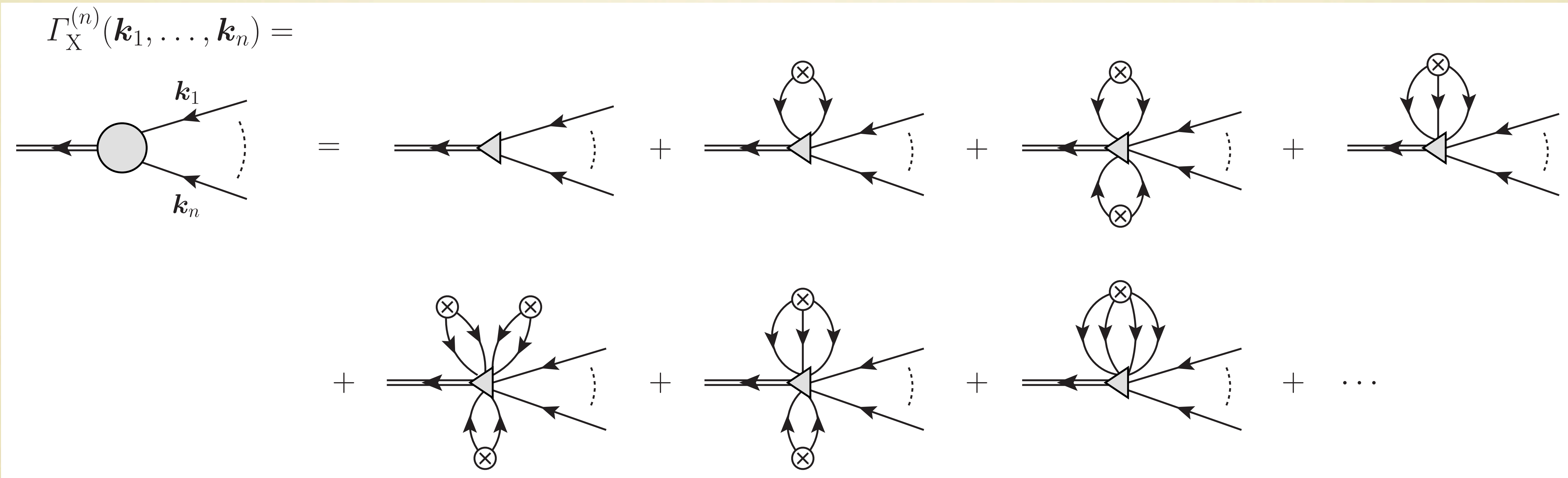


Multi-point propagator

TM (1995); Crocce & Scoccimarro (2006), Bernardeau et al. (2008)

- **Density sector of multi-point propagator with nonlocal bias and RSD**

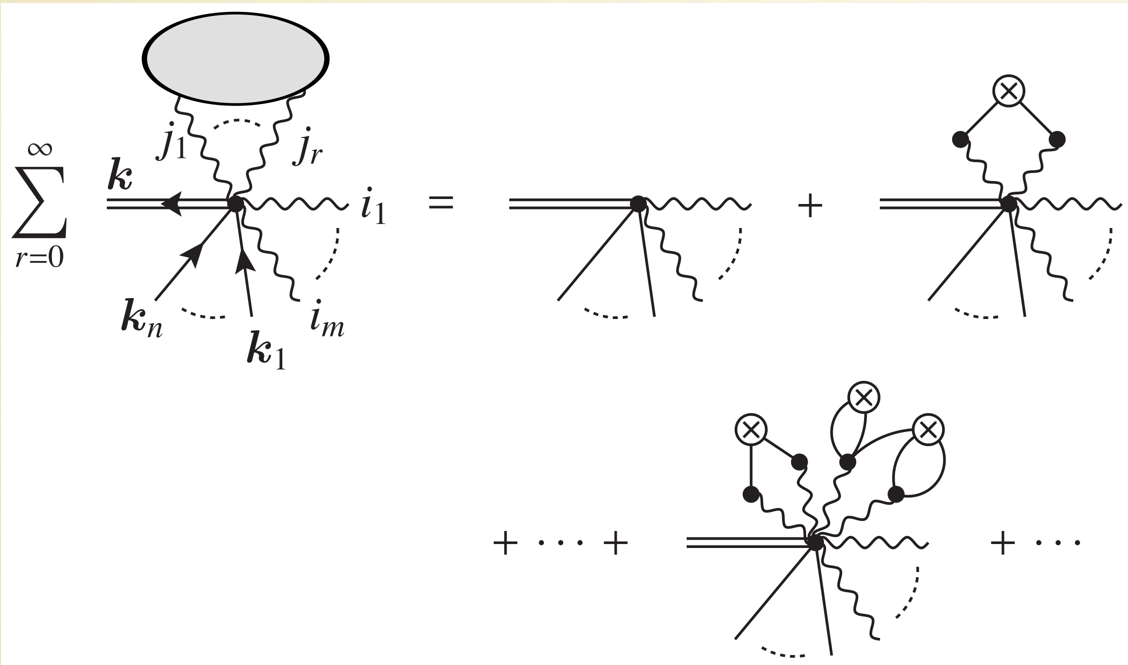
$$\left\langle \frac{\delta^n \delta_{\mathrm{X}}(\boldsymbol{k})}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta_{\mathrm{D}}^3(\boldsymbol{k}-\boldsymbol{k}_{1\cdots n})\Gamma_{\mathrm{X}}^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$



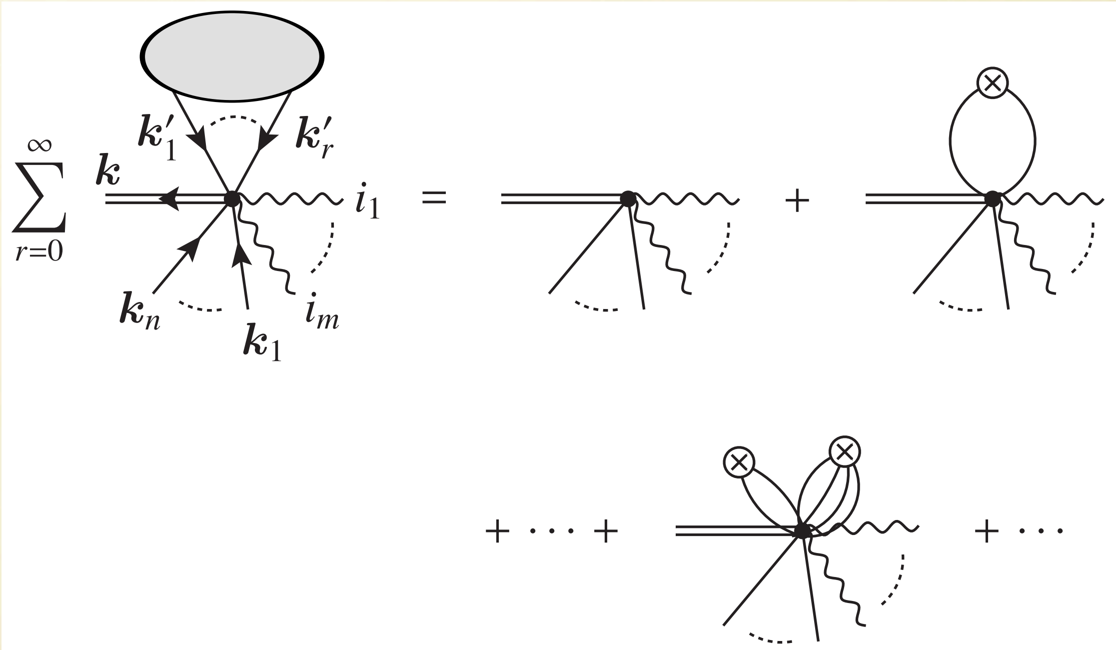
Multi-point propagator

- Full evaluations of MP propagator are difficult
- Partial resummations in the Lagrangian PT

Lagrangian vertex resummation



Lagrangian bias renormalization



$$\Pi(\boldsymbol{k}) = \left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Psi}} \right\rangle = \exp\left[\sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \langle (\boldsymbol{k}\cdot\boldsymbol{\Psi})^n \rangle_{\mathrm{c}} \right]$$

$$b_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k'}{(2\pi)^3} \left. \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right|_{\delta_{\mathrm{L}}=0}$$

$$\Rightarrow\ c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k'}{(2\pi)^3} \left\langle \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle$$

Renormalized bias functions

- Introduction of the "renormalized bias functions" is essential in iPT
  - Series of functions to characterize nonlocal bias

$$c_n^{\mathrm{L}}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k}{(2\pi)^3} \left\langle \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k})}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1) \cdots \delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle,$$

  - It can be viewed as a counterpart of multi-point propagator for Lagrangian biasing

$$\left\langle \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k})}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1) \cdots \delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n} \delta_{\mathrm{D}}^3(\boldsymbol{k}_{1\cdots n} - \boldsymbol{k}) c_n^{\mathrm{L}}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n).$$

A simple model of renormalized bias functions for halos

- Localization of PS model for the halo number density

$$n(M) = -\frac{2\bar{\rho}_0}{M}\frac{\partial}{\partial M}P(M,\delta_c), \quad \Rightarrow \quad n(\boldsymbol{x},M) = -\frac{2\bar{\rho}_0}{M}\frac{\partial}{\partial M}\Theta[\delta_M(\boldsymbol{x})-\delta_c],$$

- Applying the above model, we have

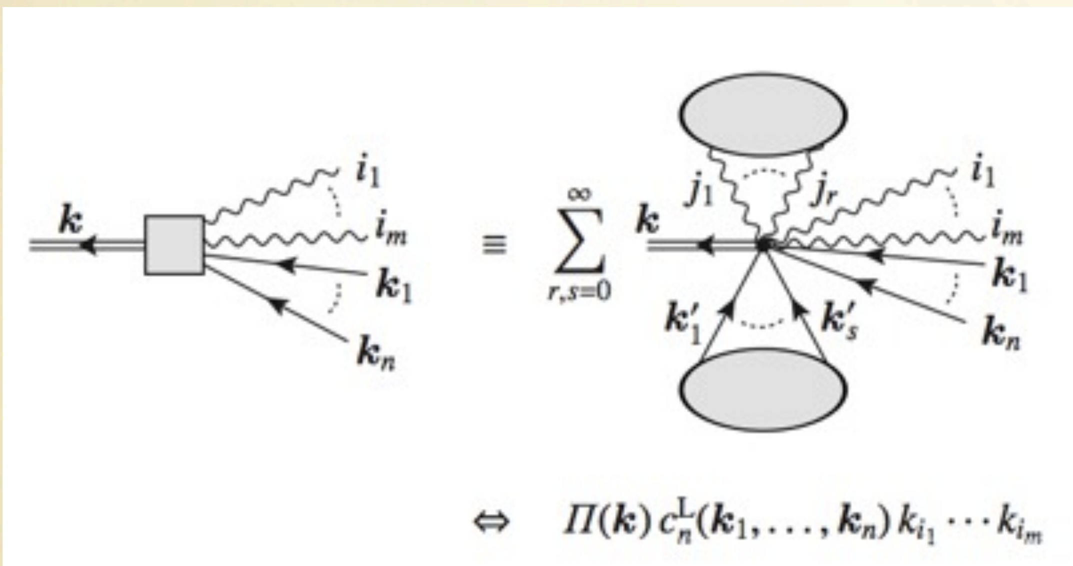
$$c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = \frac{A_n(M)}{{\delta_c}^n}W(k_1R)\cdots W(k_nR) + \frac{A_{n-1}(M)\,{\sigma_M}^n}{{\delta_c}^n}\frac{d}{d\ln\sigma_M}\left[\frac{W(k_1R)\cdots W(k_nR)}{{\sigma_M}^n}\right],$$

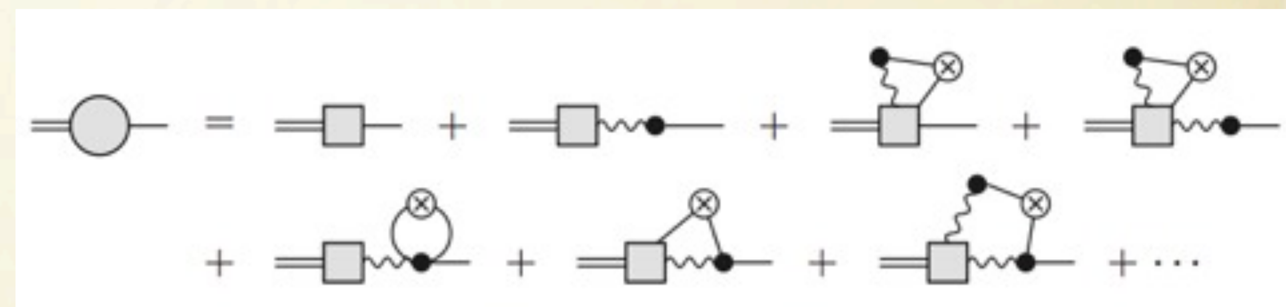
$$A_n(M) \equiv \sum_{j=0}^{n}\frac{n!}{j!}{\delta_c}^j\,b_j^{\mathrm{L}}(M).$$

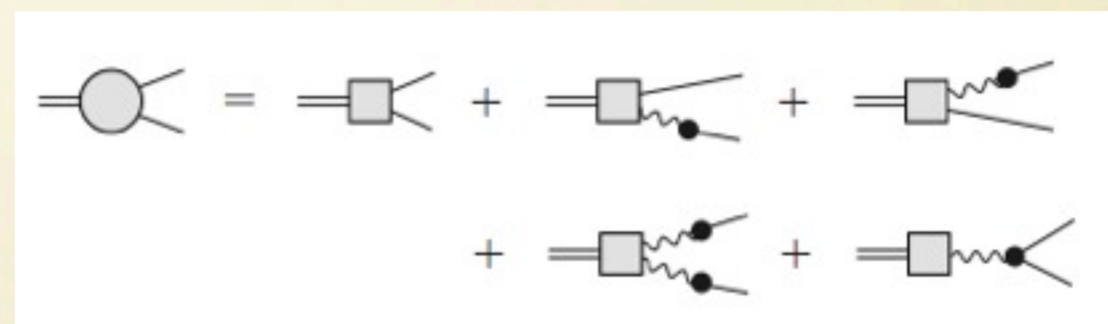
$$A_0(M) = 1,$$
$$A_1(M) = 1 + \delta_c b_1^{\mathrm{L}}(M),$$
$$A_2(M) = 2 + 2\delta_c b_1^{\mathrm{L}}(M) + {\delta_c}^2 b_2^{\mathrm{L}}(M).$$

Vertex summations

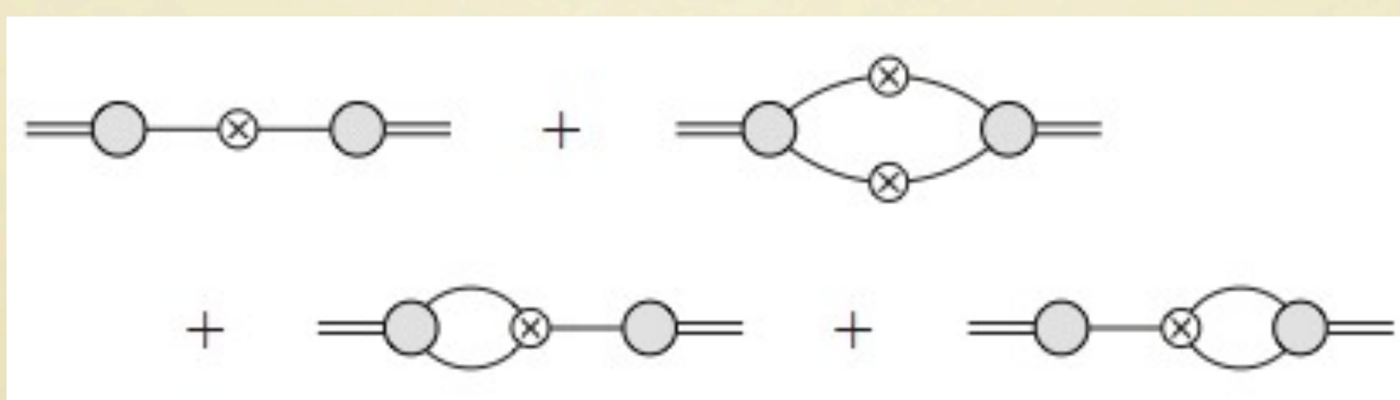
- Partially renormalized vertex, propagators





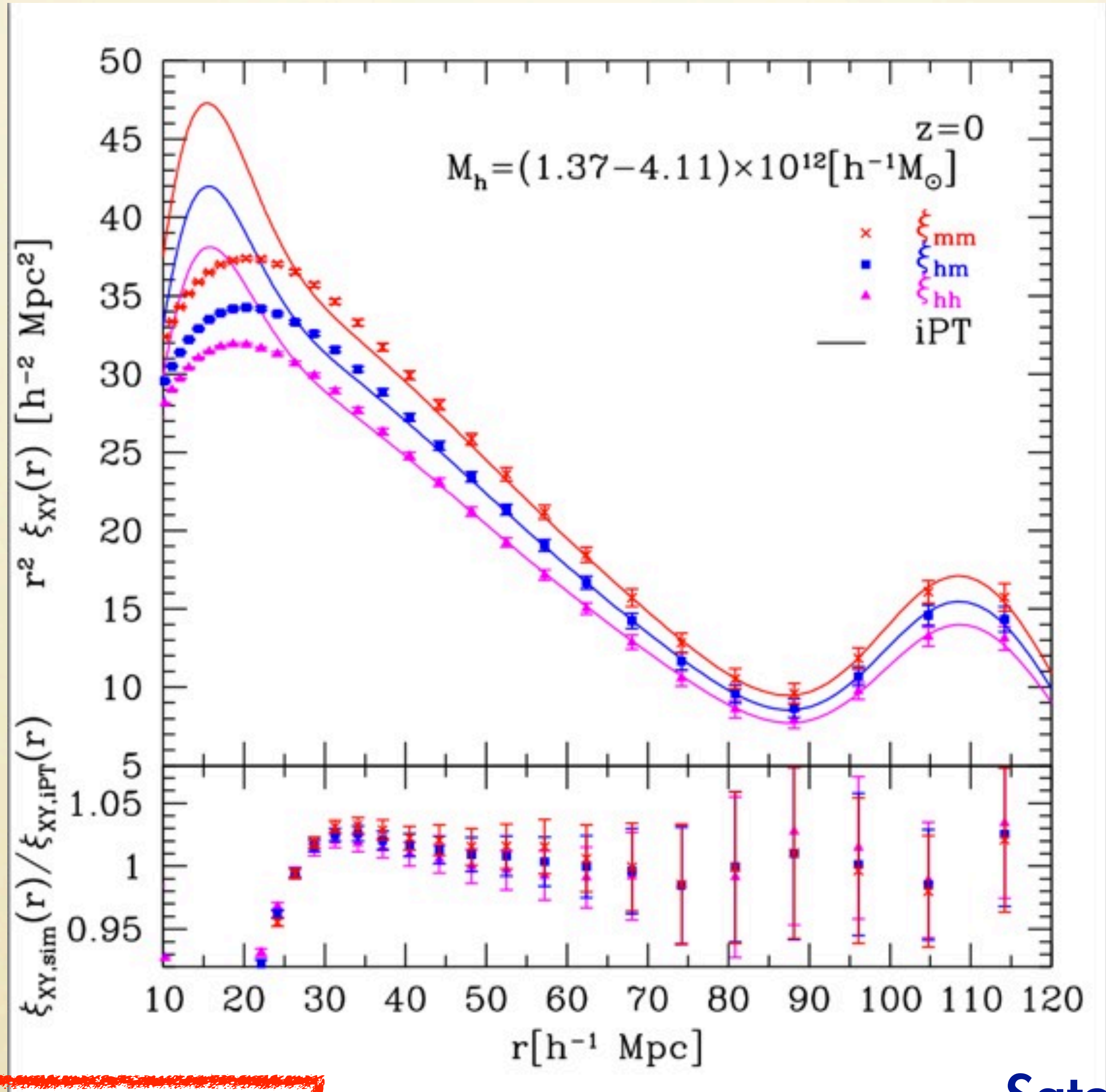


- Power spectrum (up to 1-loop, including nG)



Halo clustering: Comparison with N-body simulations

One-loop

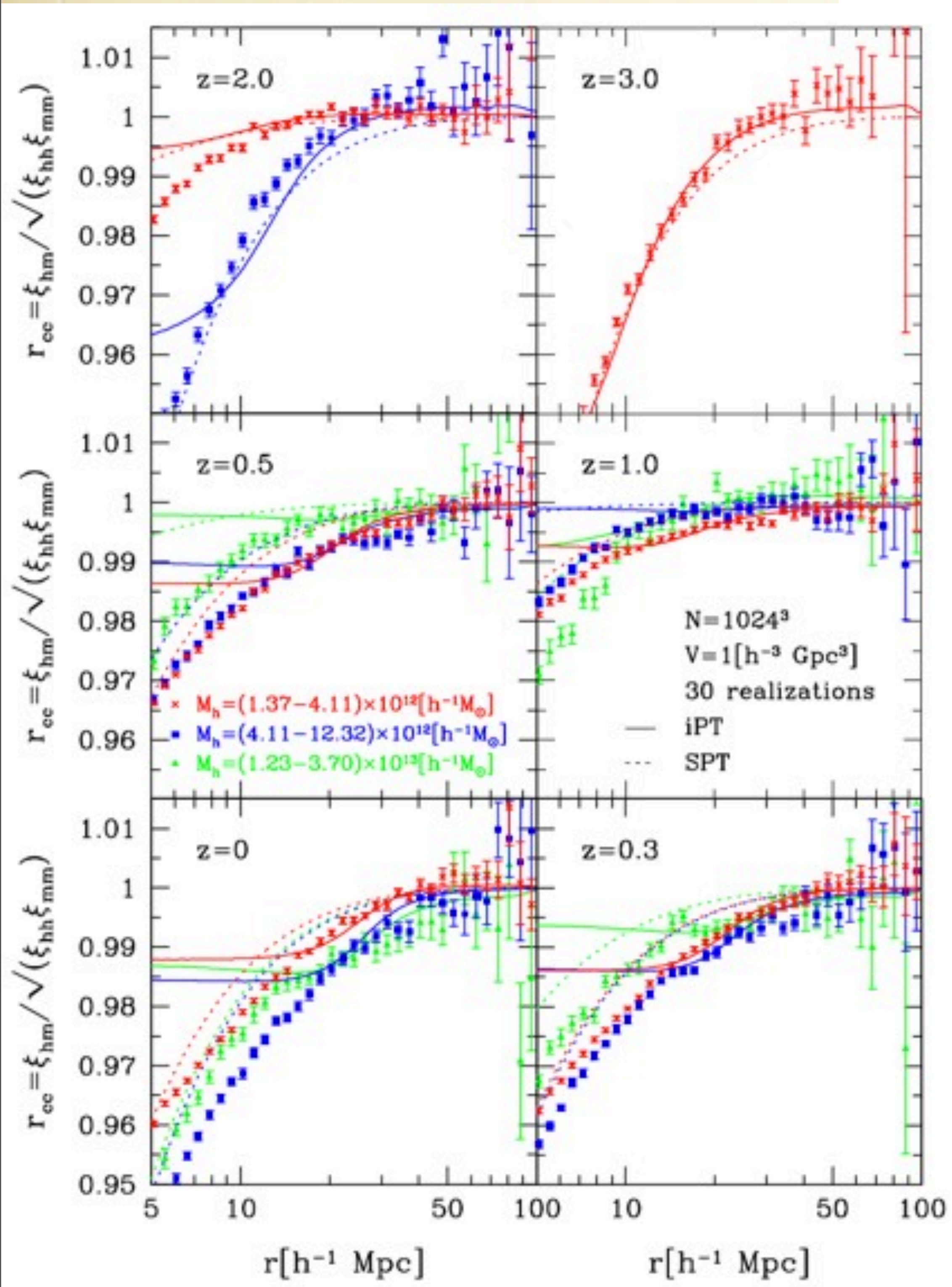


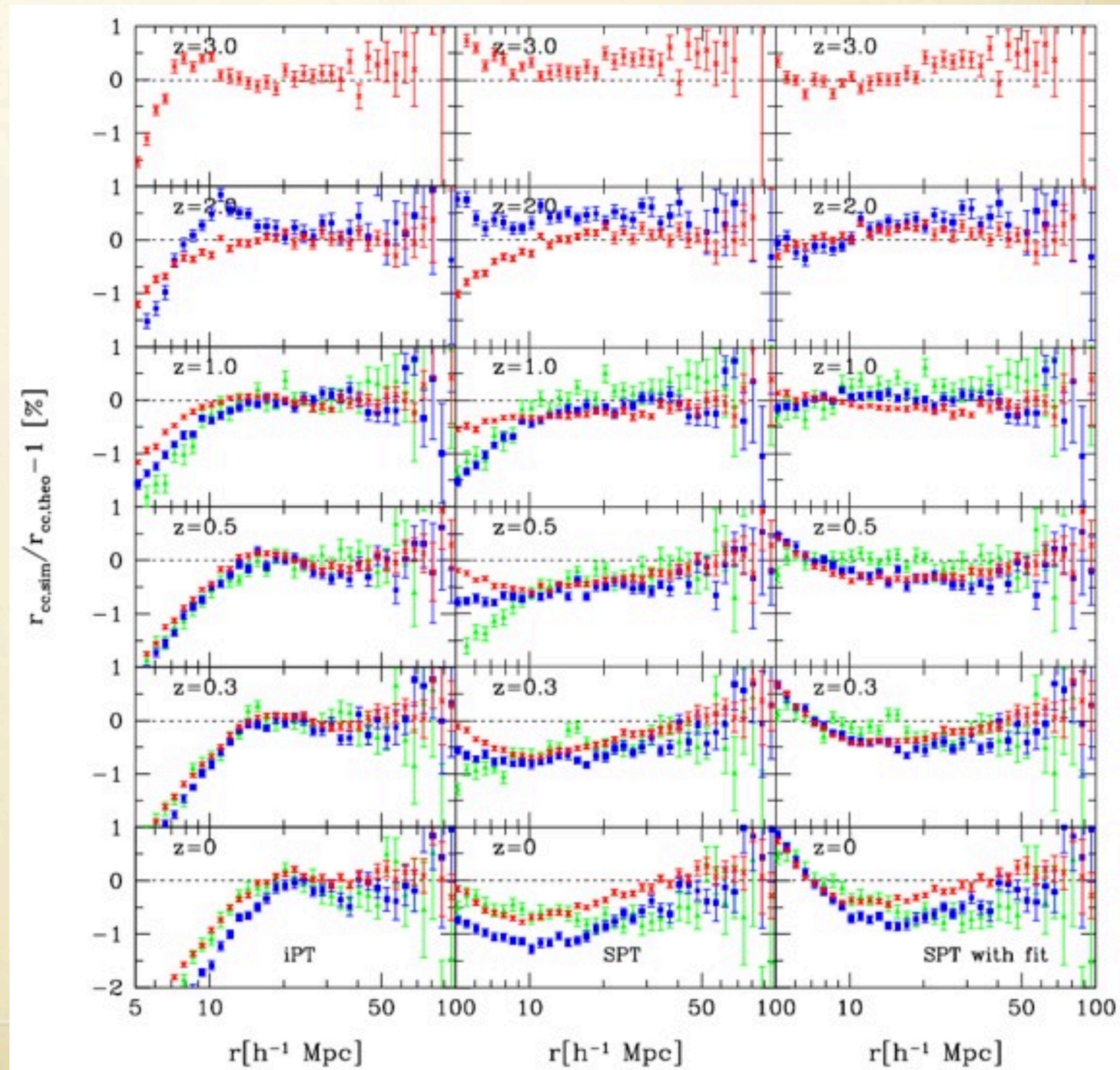
Sato & TM (2013)

No fitting parameter

Bias stochasticity and iPT

$$r_{cc} = \xi_{hm} / \sqrt{(\xi_{hh}\xi_{mm})}$$

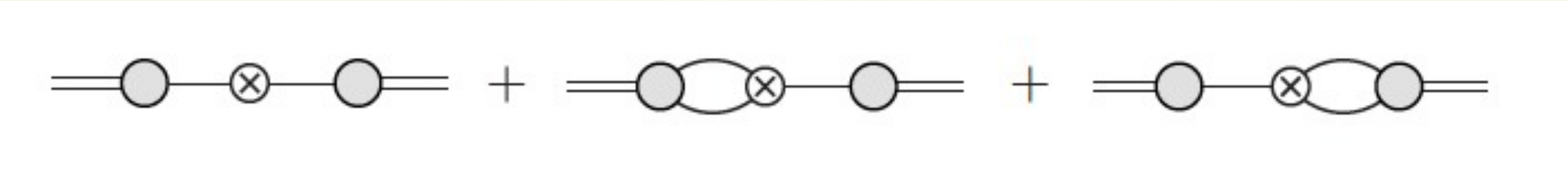




Scale-dependent bias and iPT

- iPT predicts scale-dependent bias in the presence of primordial non-Gaussianity

$$P_X(k) = [\Gamma_X^{(1)}(\boldsymbol{k})]^2 P_L(k) + \Gamma_X^{(1)}(\boldsymbol{k}) \int \frac{d^3k'}{(2\pi)^3} \Gamma_X^{(2)}(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}') B_L(k, k', |\boldsymbol{k}-\boldsymbol{k}'|) + \cdots , \quad (2)$$



General results

- dominant term on large scales:

$$\Delta b(k) \approx \frac{1}{2P_{\rm L}(k)} \int \frac{d^3k'}{(2\pi)^3} c_2^{\rm L}(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}') B_{\rm L}(k, k', \boldsymbol{k}-\boldsymbol{k}')$$

- scalings do not depend on details of bias

$$\Delta b^{\rm loc.} \propto k^{-2},$$
$$\Delta b^{\rm eql.} \propto k^{0},$$
$$\Delta b^{\rm fol.} \propto k^{-1},$$
$$\Delta b^{\rm ort.} \propto k^{-1}$$

- amplitudes depend on details of bias

Cancellation of highest-order bias parameters in PS mass function

- For the Press-Schechter mass function,

$$b_n^{\mathrm{L}}(M) = \frac{\nu^{n-1} H_{n+1}(\nu)}{{\delta_{\mathrm{c}}}^n}, \quad \Rightarrow \quad A_n(M) = \nu^n H_n(\nu) = \frac{{\delta_{\mathrm{c}}}^{n+1}}{{\sigma_M}^2} b_{n-1}^{\mathrm{L}}(M),$$

$$c_n^{\mathrm{L}} \sim \frac{\delta_{\mathrm{c}}}{\sigma_M^2} b_{n-1}^{\mathrm{L}} + (\text{nonlocal corrections})$$

  - This explains why scale-dependent bias Δb is proportional to b^{L}_{1} instead of b^{L}_{2}

- Reproduces PBS results in this model with iPT

New general formula

- When the mass function is arbitrary, we have a new formula

$$\Delta b(k) \approx \frac{{\sigma_M}^2}{2{\delta_c}^2}\left[\left(2 + 2\delta_c b_1^L + {\delta_c}^2 b_2^L\right)\mathcal{I}(k) + \left(1 + \delta_c b_1^L\right)\frac{d\mathcal{I}(k)}{d\ln\sigma_M}\right].$$

  - E.g., Sheth-Tormen mass function:

$$\Delta b(k) \approx \left[\frac{q\delta_c b_1^L}{2} + \frac{1}{\nu^2}\frac{p(q\nu^2 + 2p + 1)}{1 + (q\nu^2)^p}\right]\mathcal{I}(k) + \left[\frac{q}{2} + \frac{1}{\nu^2}\frac{p}{1 + (q\nu^2)^p}\right]\frac{d\mathcal{I}(k)}{d\ln\sigma_M}.$$

Scale-dependent bias in redshift space

- **Redshift-space distortions are straightforwardly included in iPT**

$$\Delta p_0(k) \approx \left(\frac{f}{3} + b_1\right) Q_2(k), \quad \Delta p_2(k) \approx \frac{2f}{3} Q_2(k),$$

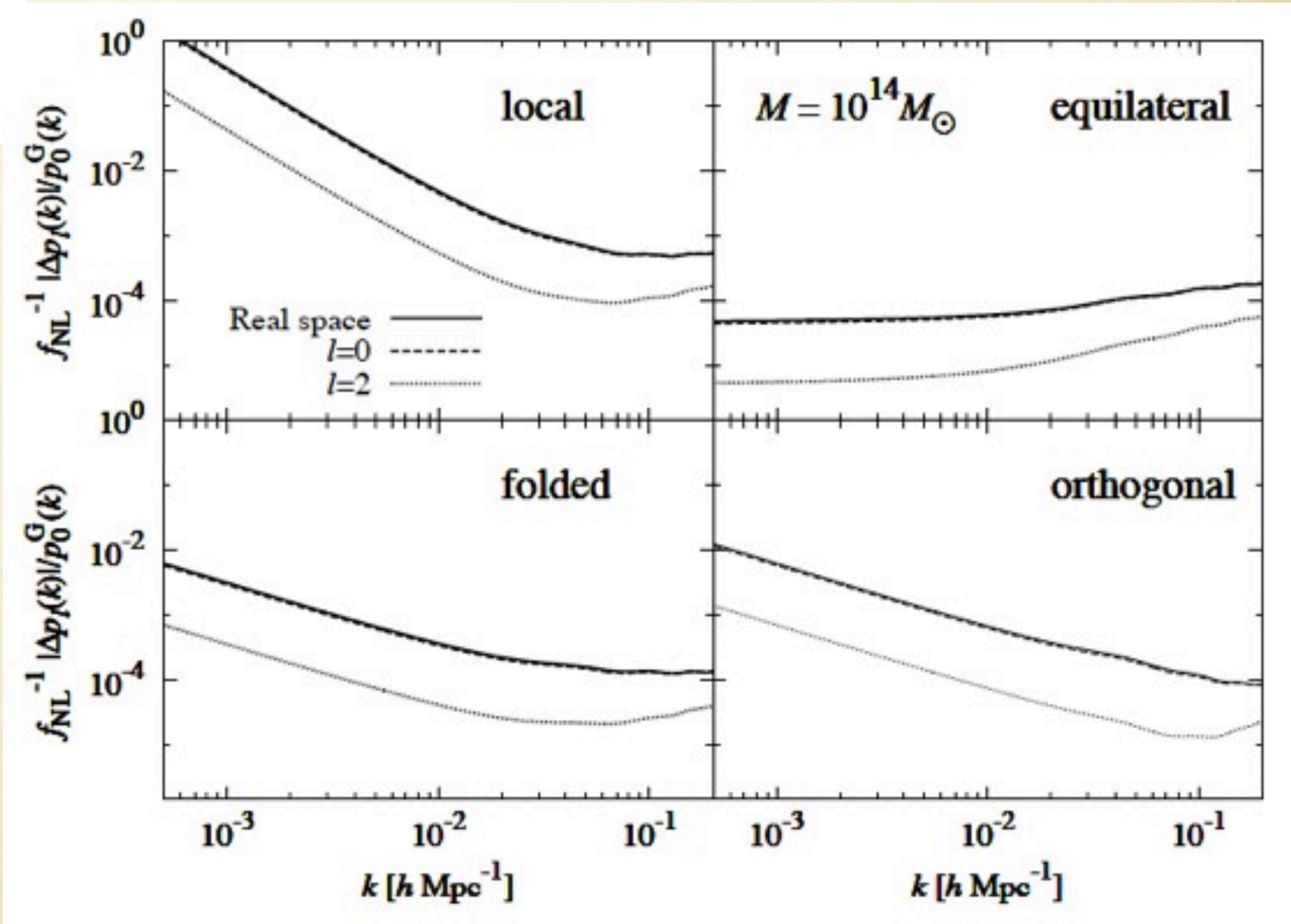
$$\Delta p_4(k), \Delta p_6(k) \ll \Delta p_0(k), \Delta p_2(k),$$

$$\begin{aligned}\Delta p_0(k) = {} & 2\left[\frac{f}{3} + \frac{2f^2}{5} + \frac{f^3}{7} + \left(1 + \frac{2f}{3} + \frac{f^2}{5}\right) b_1(k)\right] R_1(k) \\ & - 2\left[\frac{2f}{21} + \frac{4f^2}{35} + \frac{3f^3}{35} + \left(\frac{2}{7} + \frac{4f}{21} + \frac{2f^2}{15}\right) b_1(k)\right] R_2(k) \\ & + \left[\frac{f}{3} + \frac{f^2}{5} + \left(1 + \frac{f}{3}\right) b_1(k)\right] Q_1(k) \\ & + \left[\frac{f}{3} + b_1(k)\right] Q_2(k), \end{aligned} \tag{147}$$

$$\begin{aligned}\Delta p_2(k) = {} & 4f\left[\frac{1}{3} + \frac{4f}{7} + \frac{5f^2}{21} + \left(\frac{2}{3} + \frac{2f}{7}\right) b_1(k)\right] R_1(k) \\ & - f\left[\frac{8}{21} + \frac{32f}{49} + \frac{11f^2}{21} + \left(\frac{16}{21} + \frac{13f}{21}\right) b_1(k)\right] R_2(k) \\ & + 2f\left[\frac{1}{3} + \frac{2f}{7} + \frac{1}{3} b_1(k)\right] Q_1(k) + \frac{2f}{3} Q_2(k), \end{aligned} \tag{148}$$

$$\begin{aligned}\Delta p_4(k) = {} & 16f^2\left[\frac{2}{35} + \frac{3f}{77} + \frac{1}{35} b_1(k)\right] R_1(k) \\ & - \frac{4f^2}{35}\left[\frac{16}{7} + \frac{26f}{11} + b_1(k)\right] R_2(k) + \frac{8f^2}{35} Q_1(k), \end{aligned} \tag{149}$$

$$\Delta p_6(k) = \frac{32f^3}{231} R_1(k) - \frac{8f^3}{231} R_2(k). \tag{150}$$



Higher-order nonGaussianity

- Higher-order analysis
  - (Yokoyama & TM 2012, Yokoyama, Matsubara & Taruya 2013)
  - Contributions from gNL (prim. trispec)
  - Analysis of bispectrum

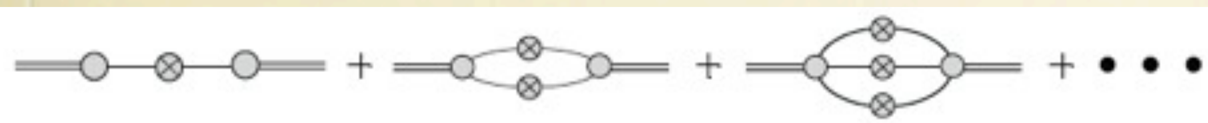


FIG. 5: Pure Gaussian case

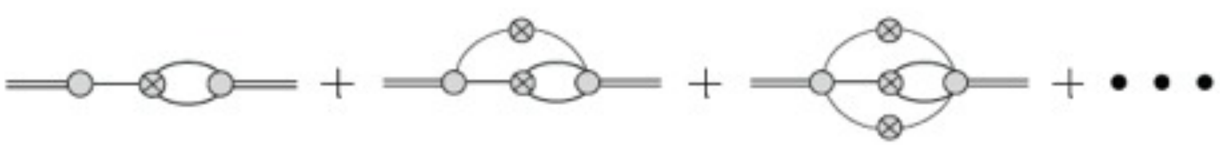


FIG. 6: Diagrams linearly proportional to the primordial bispectrum.

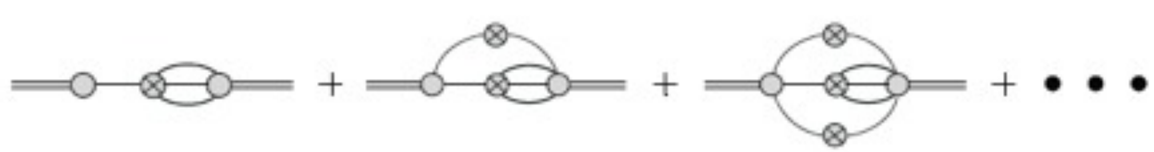
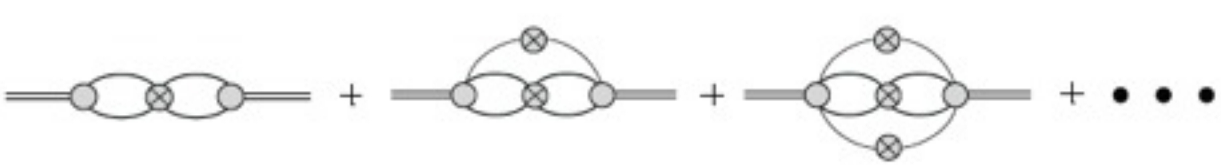
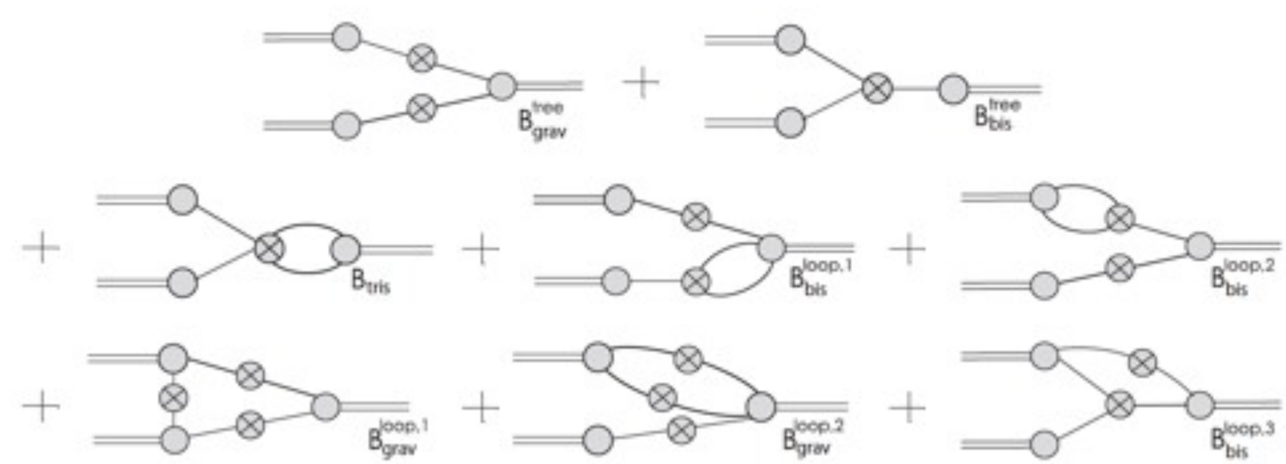
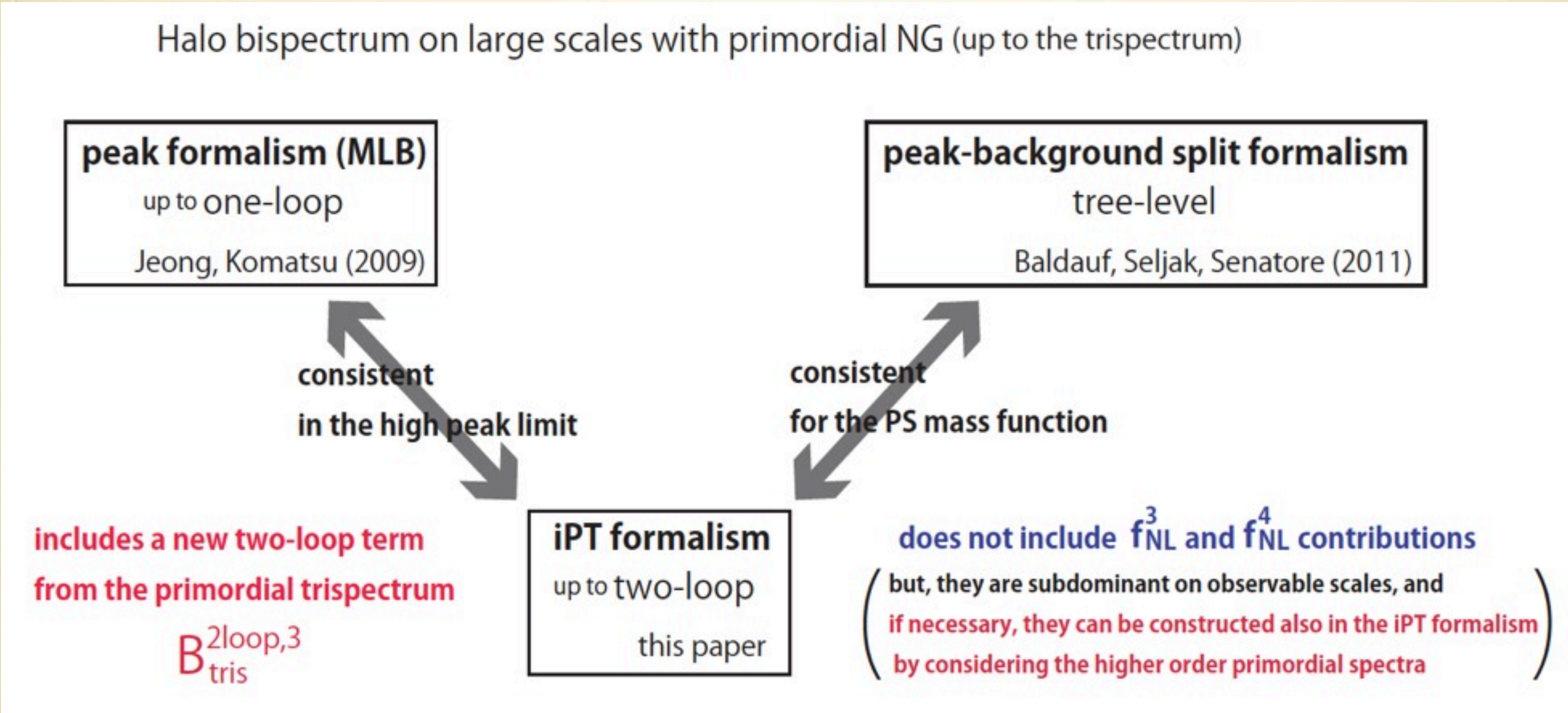


FIG. 7: Diagrams linearly proportional to the primordial trispectrum.

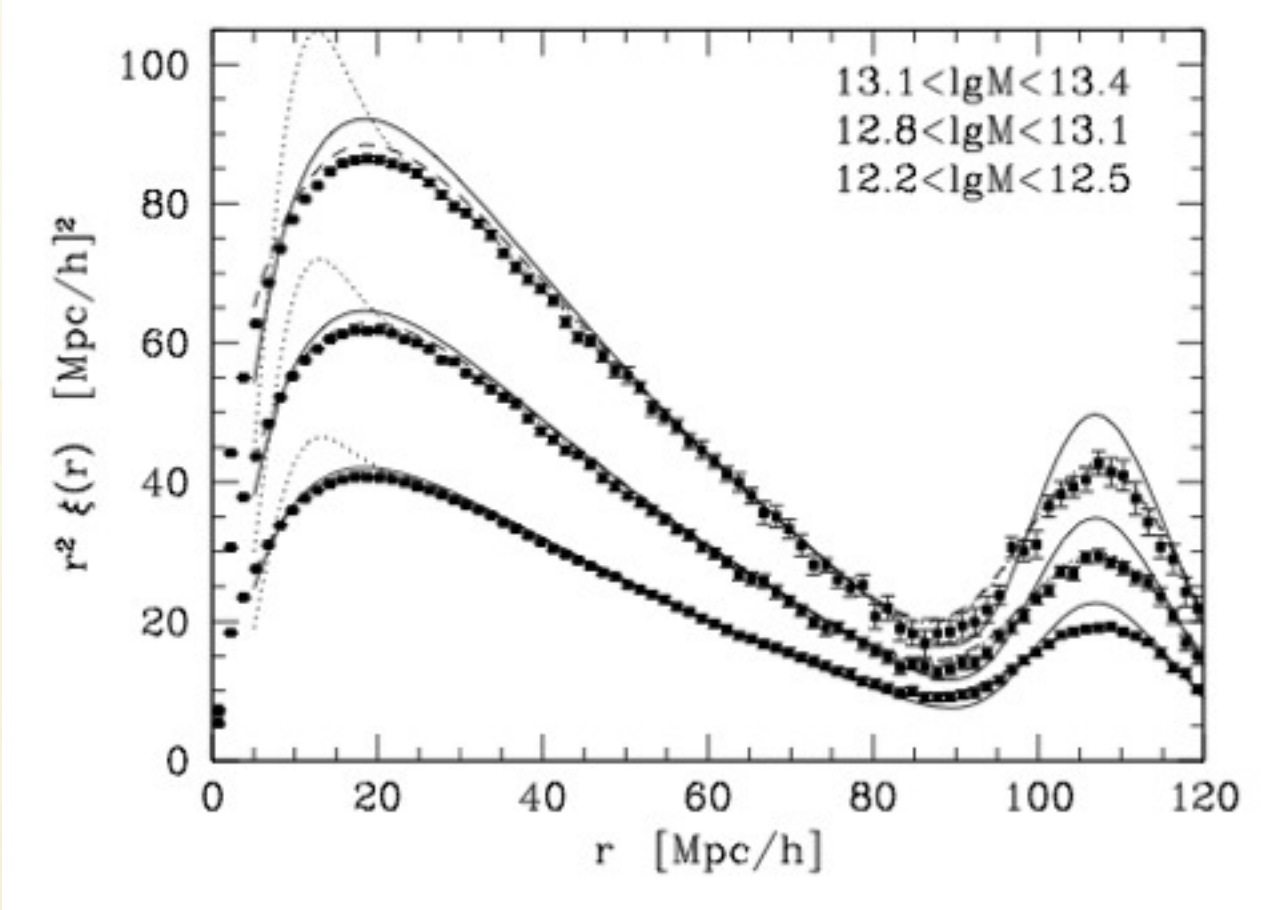


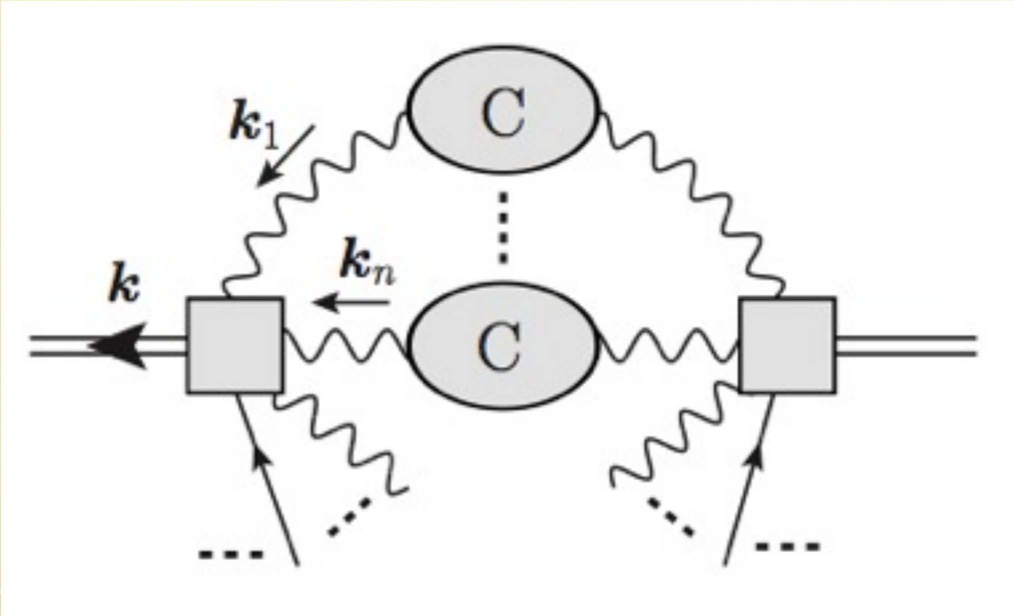


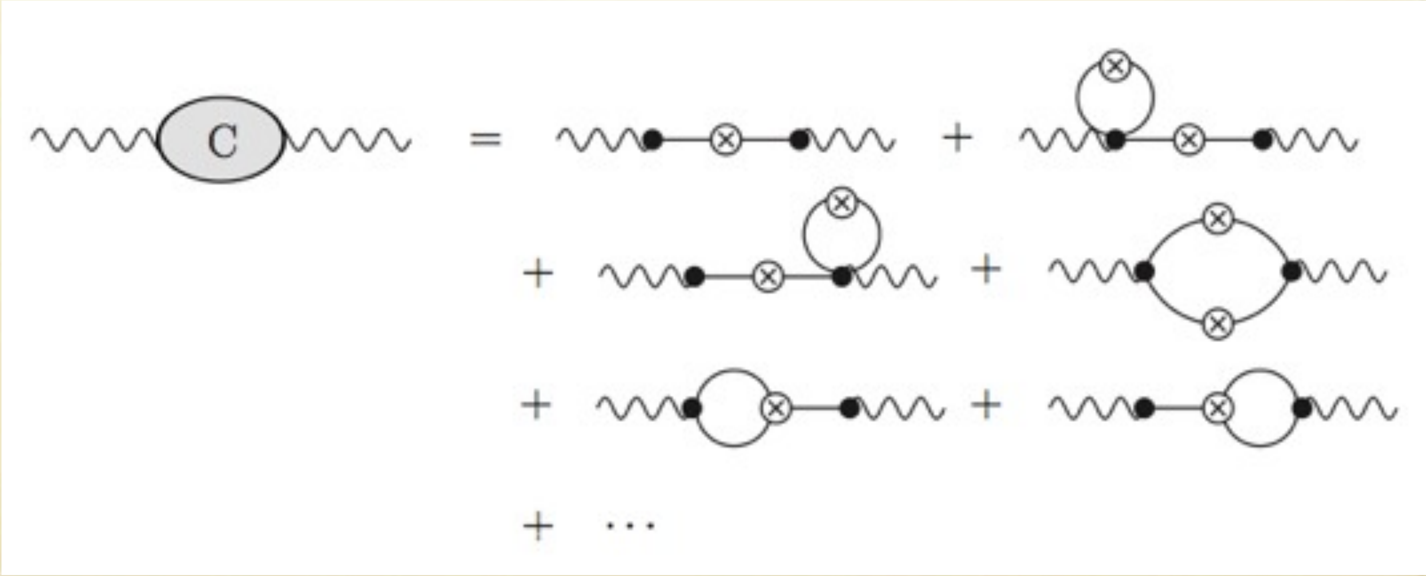

Halo bispectrum on large scales with primordial NG (up to the trispectrum)
peak formalism (MLB)
up to one-loop
Jeong, Komatsu (2009)
peak-background split formalism
tree-level
Baldauf, Seljak, Senatore (2011)
consistent
in the high peak limit
consistent
for the PS mass function
includes a new two-loop term
from the primordial trispectrum
$B_{tris}^{2loop,3}$
iPT formalism
up to two-loop
this paper
does not include f_{NL}^{3} and f_{NL}^{4} contributions
but, they are subdominant on observable scales, and
if necessary, they can be constructed also in the iPT formalism
by considering the higher order primordial spectra

Relation to CLPT

- Convolution Lagrangian perturbation theory (Carlson+ 13)
    - additional resummations on top of iPT
    - Diagrammatically, their method is equivalent resumming the following contributions:





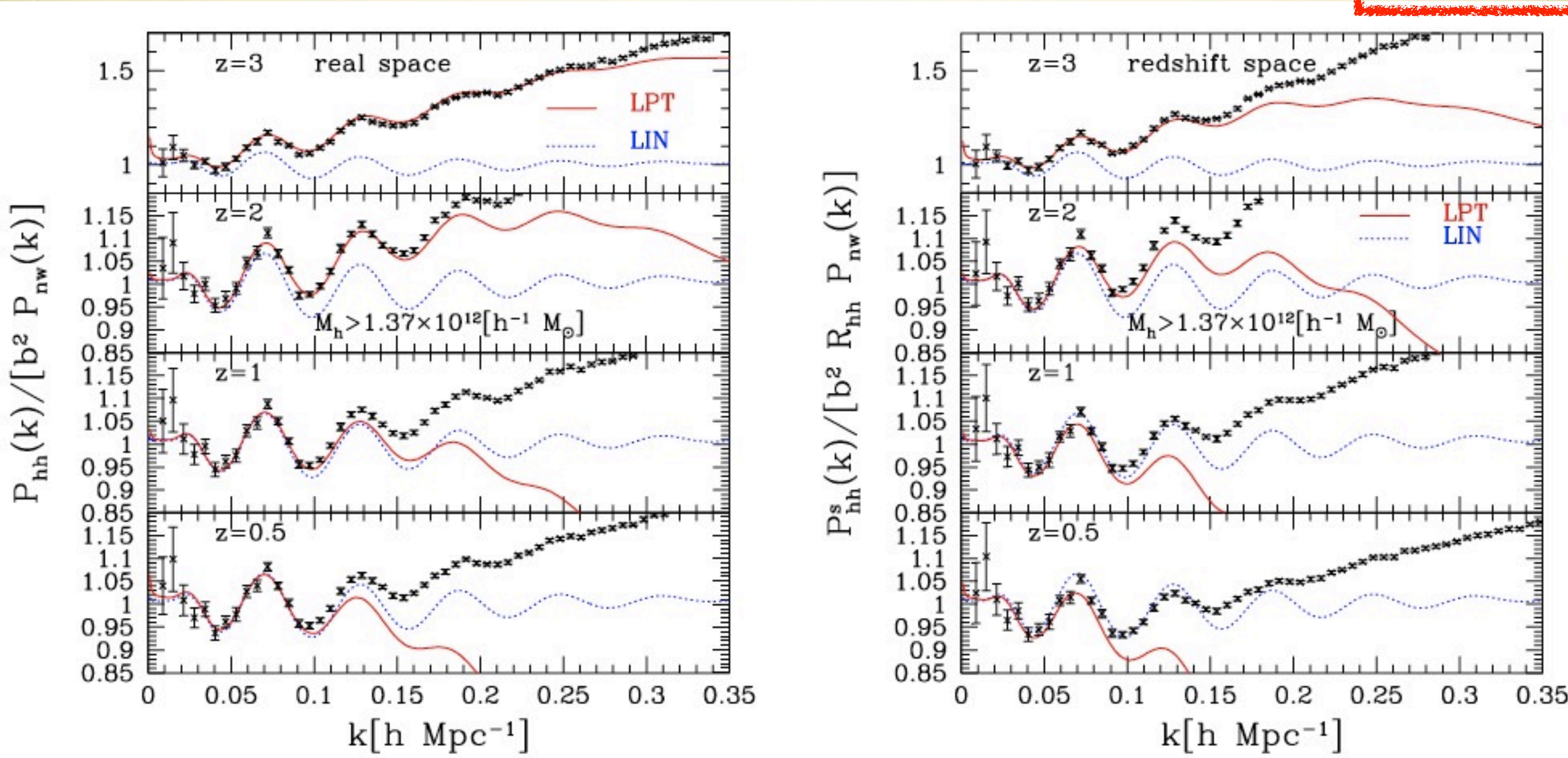


Summary

- Integrated perturbation theory
  - a consistent formulation of nonlinear perturbation theory, generally including nonlocal bias, RSD and nG
  - vertex resummations and bias renormalizations
- Scale-dependent bias from iPT
  - Previous formulas such as those of PBS are re-derived from the new formula by taking appropriate limits
- and many other applications for observables
- Future issue
  - Improved bias models beyond the simple halo approach
  - Higher-order corrections (2-loop and beyond)

Halo clustering: Lagrangian resummation & N-body

One-loop



No fitting parameter

Sato & TM (2011)