# Upward Mobility: Random Walks in the Sky (remembering previous steps)

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In collaboration with A. Paranjape and R. Sheth
(arXiv: 1201.3876, 1205.3401, 1305.0724, 1306.0551)

# Halo Mass Function and Bias

- How many halos of given mass (mass function)
- Correlation with underlying matter field (halo bias)

GOALS:

- Analytical halo statistics that reproduce N-body simulations
- Extract information on the initial conditions from real data

USE FOR COSMOLOGY:
Properties of Early Universe
$\Lambda$ vs Quintessence vs ModGrav

- CMB alone has told us everything. Need other probes

# Outline

Part ONE

- Progress on mass function from excursion sets
- Including non-Gaussianity

Part TWO

- Bias from excursion sets (straightforward!)

# Excursion set theory

- Halos from “dense enough” patches in the initial matter distribution $\delta_{in}$

- Mean density $\delta_R$ ≡ [ average of $\delta_{in}$ over volume $R^3$ ] ≥ threshold $B$

$$\delta_R(\mathbf{x}) \equiv \frac{1}{V_R}\int \mathrm{d}^3y\, W_R(\mathbf{y}-\mathbf{x})\delta(\mathbf{y})$$

- If $B$ from spherical collapse, then $B = \delta_c = 1.686$. But it may not be

- Halo mass $M$ proportional to the volume $V \sim R^3$ of the patch

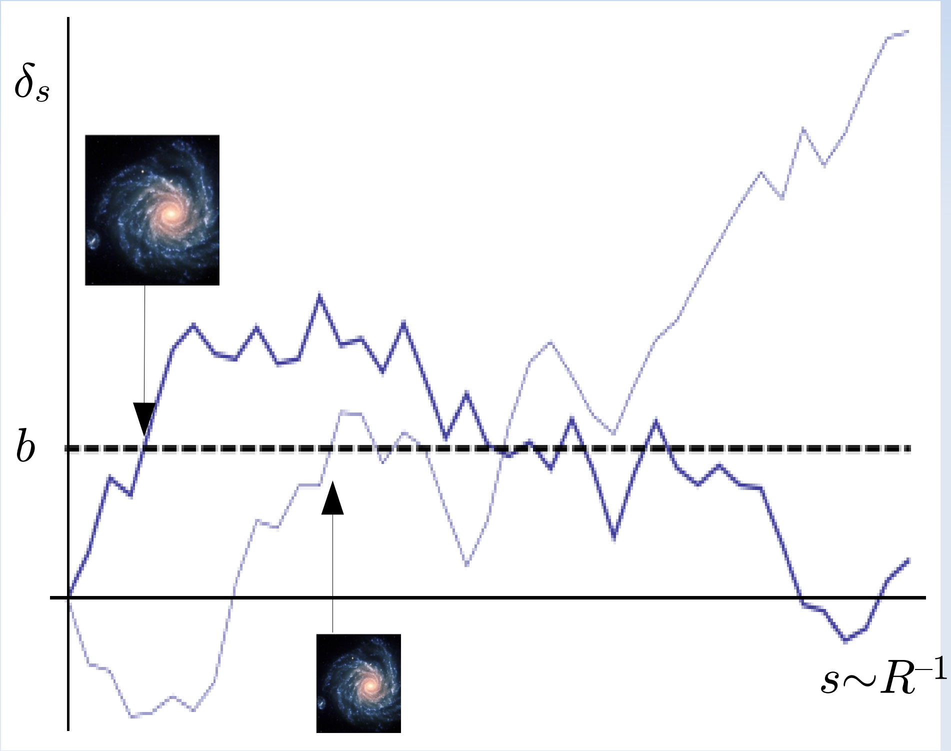
$$M = \bar{\rho}\, V_R \equiv \bar{\rho}\int \mathrm{d}^3y\, W_R(\mathbf{y})$$

# Excursion set theory

- Different locations realize different random walks: $s(M) \equiv \langle \delta_R^2(x) \rangle$

FIRST PASSAGE PROBLEM!

CORRELATED STEPS!
No known solution
NEED BETTER MATHS




- Abundance $n(M)$ $\longleftrightarrow$ first crossing probability $f(s)$ at scale $s(M)$

# First crossing distribution

- Probability of ANY crossing at $s$ :

$$f(s) = \frac{\mathrm{d}}{\mathrm{d}s} \langle \vartheta(\delta - b(s)) \rangle$$

Press & Schechter (1974)

- Not any, but **FIRST** crossing (cloud-in-cloud problem); solution only for Gaussian uncorrelated steps

$$f(s) = \frac{\langle \vartheta(b_1 - \delta_1) \ldots \vartheta(b_{N-1} - \delta_{N-1}) \vartheta(\delta_N - b_N) \rangle}{\Delta s}$$

Bond et al. (1991)

# First crossing distribution

- However: strongly correlated walks are less affected (less zig-zags)

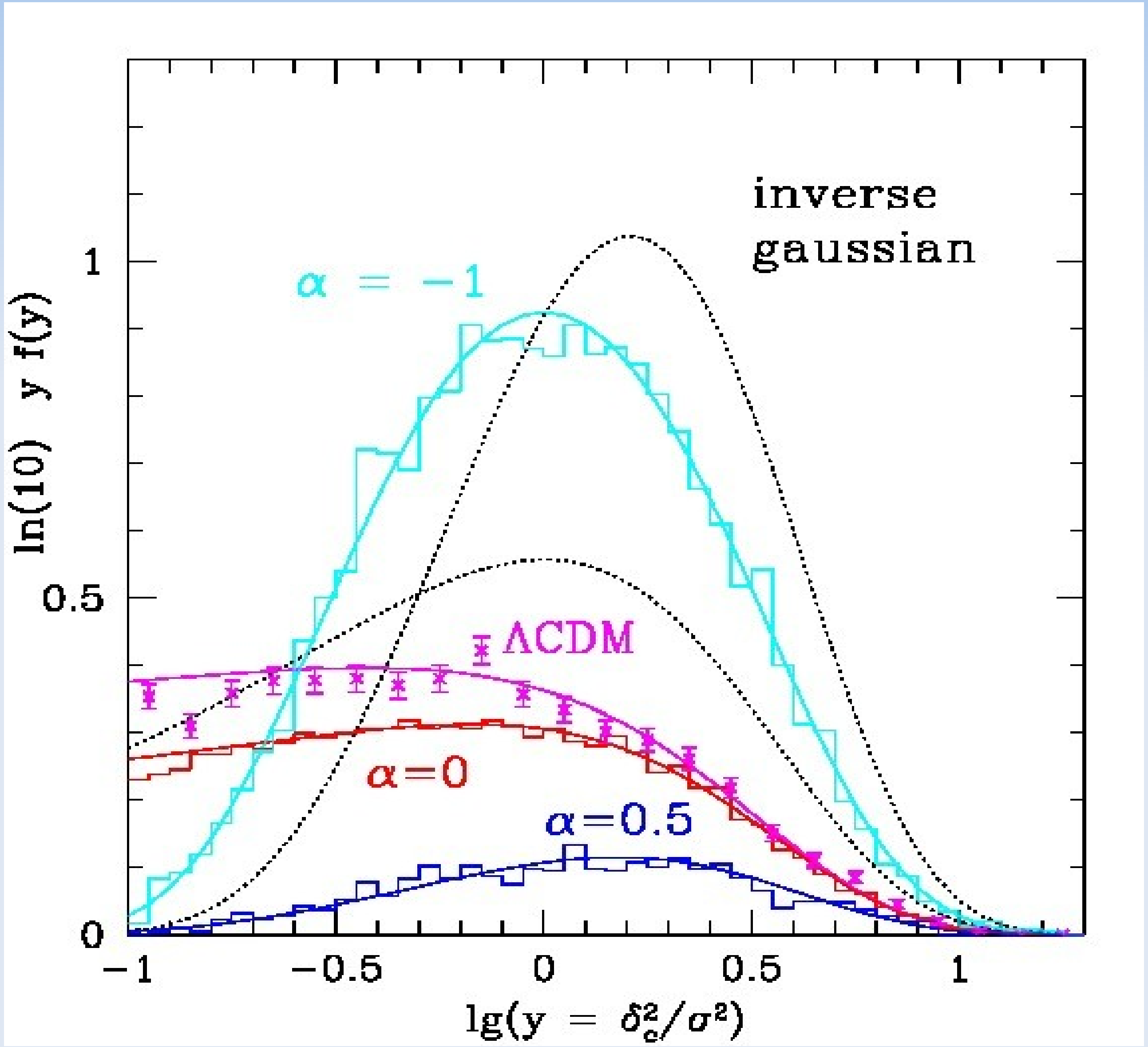
Paranjape, Lam & Sheth (2011)

- Can relax FIRST into simply **UPWARDS**: $\delta = B$ ; $\delta' \geq B'$

$$f(s) = \left\langle \left[ \frac{\mathrm{d}}{\mathrm{d}s} \vartheta(\delta - B) \right] \vartheta(\delta' - B') \right\rangle = \int_{B'}^{\infty} \mathrm{d}v \, (v - B') \, p(B, v; s)$$

MM & Sheth (2012)

# First crossing distribution

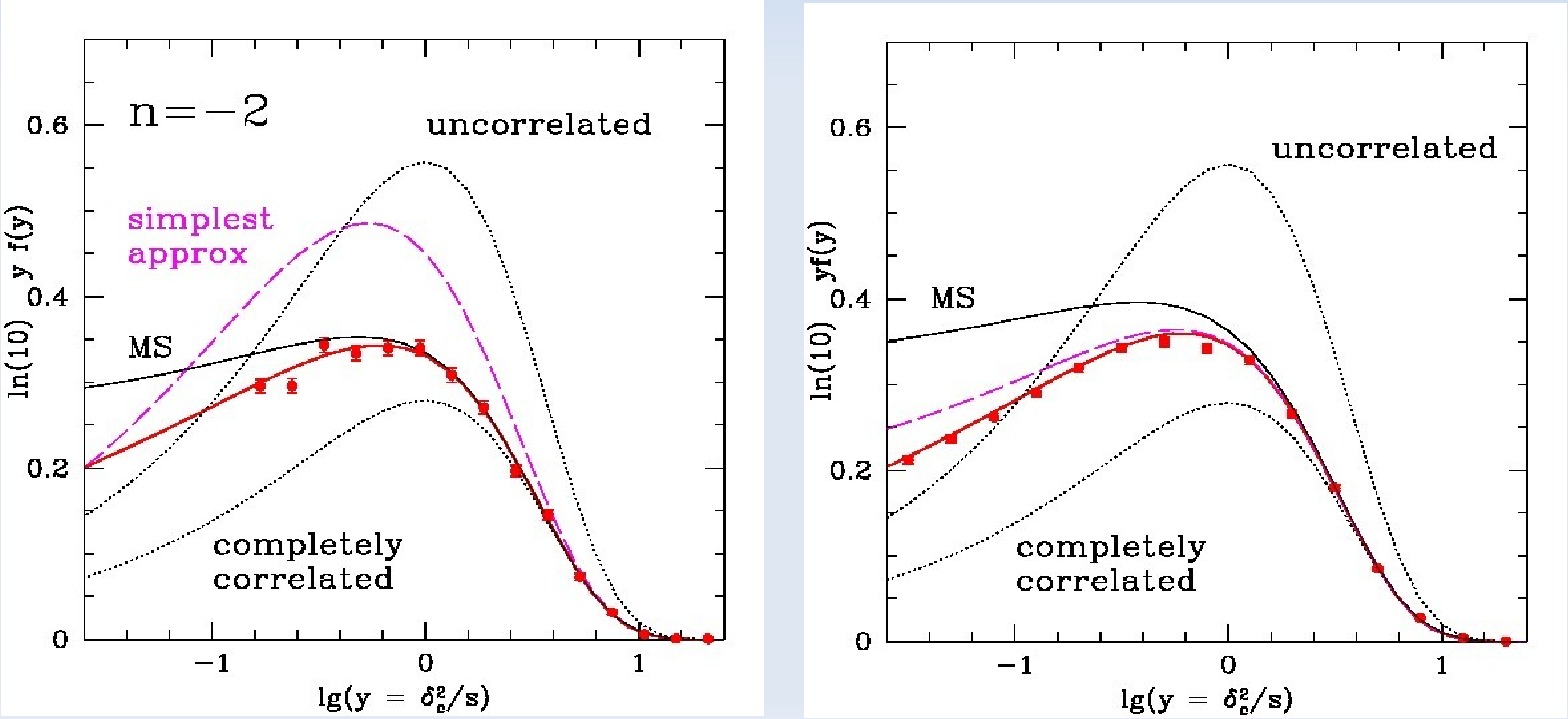



MM & Sheth (2012)

# Upward mobility, back-substitution

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s | \mathrm{first}, S)$$

- Do the **UPWARDS** approximation $\delta'(S) \geq b'(S)$ and solve for $f(S)$




MM & Sheth (2013)

# Upward mobility, back-substitution

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S\, f(S) p(\delta \geq b, s | \mathrm{up}, S)$$

- Full understanding of the correlated random walk problem, with any power spectrum and barrier!

MM & Sheth (2013)

- Does this $f(s)$ reproduce simulations? Not really...

HOWEVER...

# Upward mobility, back-substitution

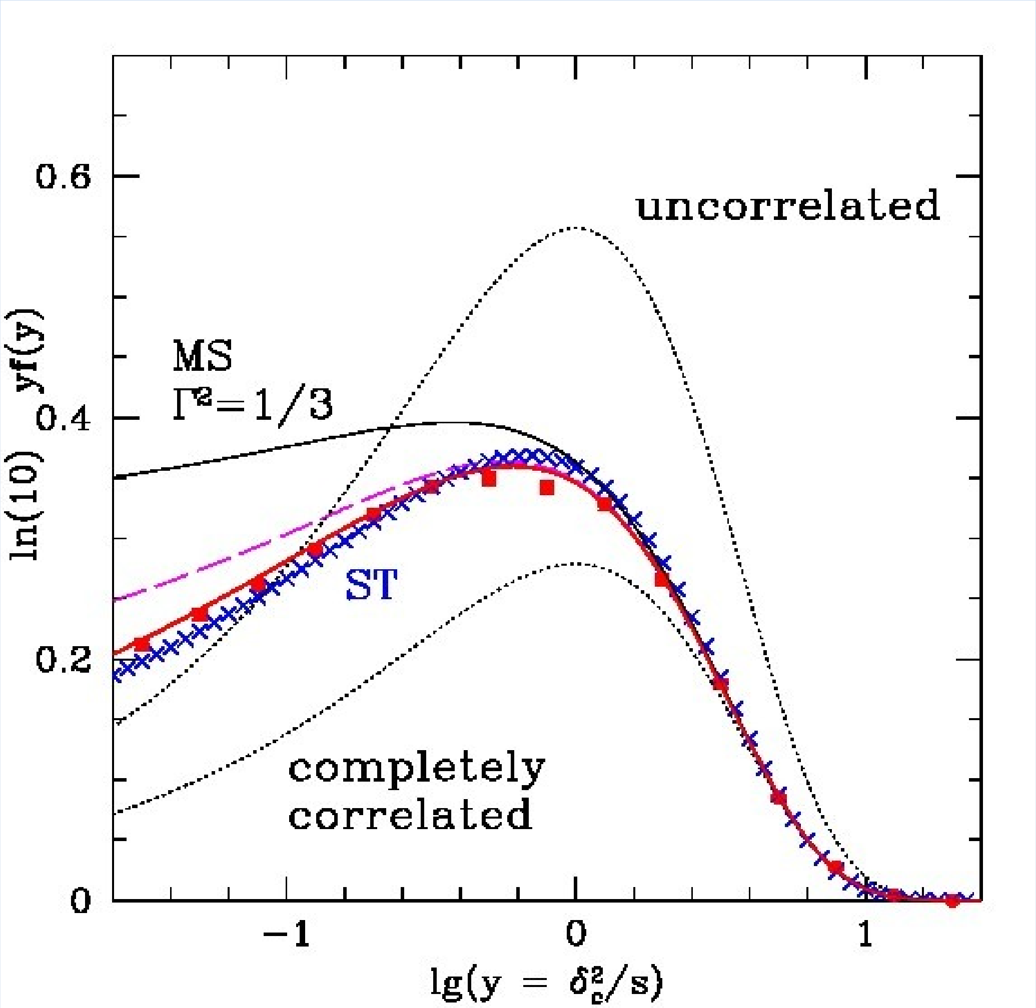
$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s | \mathrm{up}, S)$$

HOWEVER:

- Toy model with Markovian velocities (not heights!)
- Rescale the barrier:

$$\delta_c \to \sqrt{0.7}\delta_c$$

IT WORKS!!




MM & Sheth (2013)

# Adding non-Gaussianity: MF

Same formalism for non-Gaussian initial conditions:

$$f(s) = \underbrace{\left[\frac{\mathrm{d}}{\mathrm{d}s}\int_{b(s)}^{\infty}\mathrm{d}\delta\, p(\delta;s)\right]}_{\text{old PS result}}\frac{1+\mathrm{erf}(X/\sqrt{2})}{2} - B'\,p(B;s)\left[\frac{e^{-X^2/2}}{2X\sqrt{2\pi}} + \dots\right]$$

full NG pdf

Gaussian factors

small!

- Non-perturbative in NG parameters: $p(\delta;s)$ is the exact pdf!

$$p(B;s) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{B^2}{2} + \mu\frac{B^3}{3!} + \dots\right] \qquad \left[\mu = \frac{\langle\delta^3\rangle}{s^{3/2}} \sim f_{\mathrm{NL}}\right]$$

- Residual NG corrections are small: OK as perturbations

MM & Sheth (2013)

# Light vs Mass: Halo Bias

- Relation between halo abundance and underlying dark matter density
- Usually, computed from $p(\delta;s|\delta_0;s_0)$ . Is there an easier way?

# Light vs Mass: Halo Bias

- Take the most generic dependence on the matter field:

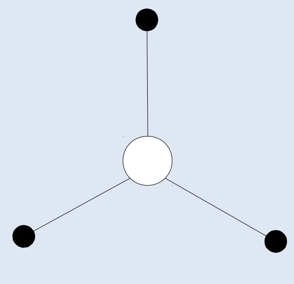
$$\delta_h(\mathbf{x}) = \sum_{k=0}^{\infty} \frac{1}{k!} \int \mathrm{d}^3 y_1 \ldots \mathrm{d}^3 y_k \, b_k(\mathbf{x}-\mathbf{y}_1, \ldots, \mathbf{x}-\mathbf{y}_k)\delta(\mathbf{y}_1)\ldots\delta(\mathbf{y}_k)$$

e.g. Matsubara (2011)

- Compute connected correlation function of $\delta_h$ and $\delta$ :

$$\langle \delta_h(\mathbf{x})\delta(\mathbf{z}_1)\ldots\delta(\mathbf{z}_n)\rangle_c = \int \mathrm{d}^3\mathbf{x}_1 \ldots \mathrm{d}^3\mathbf{x}_n \underbrace{\left\langle \frac{\delta^n \delta_h(\mathbf{x})}{\delta\delta(\mathbf{x}_1)\ldots\delta\delta(\mathbf{x}_n)} \right\rangle}_{\text{Bias functions}} \prod_{j=1}^{n} \langle \delta(\mathbf{x}_j)\delta(\mathbf{z}_j)\rangle$$

- $\delta_h$ acts as an effective vertex for $\delta$ :



# Halo Bias from Excursion Sets

- Need a prediction for $\delta_h$. Can get it from excursion sets:

$$1+\delta_h(m) = \frac{\vartheta(B_1-\delta_1)\dots\vartheta(B_{N-1}-\delta_{N-1})\vartheta(\delta_N-B_N)}{\langle\vartheta(B_1-\delta_1)\dots\vartheta(B_{N-1}-\delta_{N-1})\vartheta(\delta_N-B_N)\rangle}$$

$$b_n(\mathbf{x}-\mathbf{y}_1,\dots,\mathbf{x}-\mathbf{y}_n) = \sum_{i_1,\dots,i_n}^{N}\left\langle\frac{\partial^n\delta_h(m)}{\partial\delta_{i_1}\cdots\partial\delta_{i_n}}\right\rangle W_{i_1}(\mathbf{x}-\mathbf{y}_1)\dots W_{i_n}(\mathbf{x}-\mathbf{y}_n)$$

$$\left\langle\frac{\partial^n\delta_h}{\partial\delta_{i_1}\cdots\partial\delta_{i_n}}\right\rangle = \frac{(-1)^n}{f(s)}\frac{\partial^n f(s)}{\partial B_{i_1}\cdots\partial B_{i_n}}$$

- Still a bit complicated...

$$\langle\delta_h\delta_0\rangle = \sum_{i=1}^{N}\left\langle\frac{\partial\delta_h}{\partial\delta_i}\right\rangle\langle\delta_i\delta_0\rangle \qquad \langle\delta_h\delta_0^2\rangle = \sum_{i,j=1}^{N}\left\langle\frac{\partial^2\delta_h}{\partial\delta_i\partial\delta_j}\right\rangle\langle\delta_i\delta_0\rangle\langle\delta_j\delta_0\rangle$$

# Halo Bias from Excursion Sets

- Use the UPWARDS approximation. Only two variables!

$$1+\delta_h = \frac{1}{f(s)}\left[\frac{\mathrm{d}}{\mathrm{d}s}\vartheta(\delta_s - B)\right]\vartheta(\delta_s' - B')$$

- The real space bias functions become easy:

$$b_1(\mathbf{x}-\mathbf{y}) = -\frac{1}{f(s)}\left[W_R(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial B} + W_R'(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial B'}\right]f(s)$$

$$b_n(\mathbf{x}-\mathbf{y}_1,\dots,\mathbf{x}-\mathbf{y}_n) = \frac{(-1)^n}{f(s)}\prod_{i=1}^{n}\left[W_R(\mathbf{x}-\mathbf{y}_i)\frac{\partial}{\partial B} + W_R'(\mathbf{x}-\mathbf{y}_i)\frac{\partial}{\partial B'}\right]f(s)$$

MM, Paranjape & Sheth (to appear)

# Halo Bias from Excursion Sets

- What should one measure? (see Aseem's talk)

$$\langle \delta_h \delta_0 \rangle = b_{10}^{(f)} \langle \delta\delta_0 \rangle + b_{11}^{(f)} \langle \delta' \delta_0 \rangle$$

$$\left\langle \delta_h \delta_0^2 \right\rangle_c = b_{20}^{(f)} \langle \delta\delta_0 \rangle^2 + 2b_{21}^{(f)} \langle \delta\delta_0 \rangle \langle \delta'\delta_0 \rangle + b_{22}^{(f)} \langle \delta'\delta_0 \rangle^2$$

MM, Paranjape & Sheth (2012)

- The coefficients are straightforward:

$$b_{nk}^{(f)} = \frac{(-1)^n}{f(s)} \frac{\partial^{n-k}}{\partial B^{n-k}} \frac{\partial^k}{\partial B'^k} f(s)$$

with $f(s) = \int_{B'}^{\infty} \mathrm{d}v (v - B') p(B, v)$

MM, Paranjape & Sheth (to appear)

# Halo Bias from Excursion Sets

- Linear bias in Fourier space:

$$b_1(k) = b_{10}^{(f)} \underbrace{W(kR)}_{\sim 1} + b_{11}^{(f)} \underbrace{2sW'(kR)}_{\sim k^2 R^2}$$

- Quadratic bias:

$$b_2(k_1, k_2) \simeq b_{20}^{(f)} + b_{21}^{(f)} (k_1^2 + k_2^2) R^2 + b_{22}^{(f)} k_1^2\, k_2^2 R^4$$

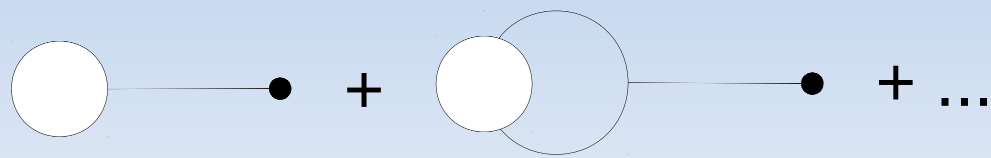
- Numerical predictions for the coefficients $b_{nj}(m)$ from $f(s)$ ; $b_{n0}$ is the same as in peak-background split

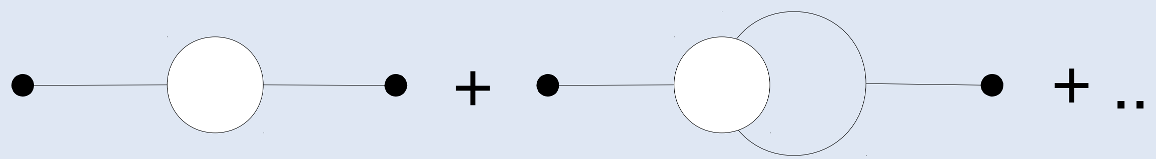
- $k$-dependence!

MM, Paranjape & Sheth (2012)

# Adding non-Gaussianity: Bias

- Expand halo-matter $n$-point functions in matter polyspectra

$\langle \delta_h(\mathbf{x})\delta(\mathbf{z}_1)\rangle_c =$ 

$\langle \delta_h(\mathbf{x})\delta(\mathbf{z}_1)\delta(\mathbf{z}_2)\rangle_c =$ 

- Generic excursion set bias for non-Gaussian walks:

$$\langle \delta_h \delta_0 \rangle = \sum_{i=1}^{N} \left\langle \frac{\partial \delta_h}{\partial \delta_i} \right\rangle \langle \delta_i \delta_0 \rangle + \frac{1}{2} \sum_{i,j=1}^{N} \left\langle \frac{\partial^2 \delta_h}{\partial \delta_i \partial \delta_j} \right\rangle \langle \delta_i \delta_j \delta_0 \rangle + \dots$$

# Adding non-Gaussianity: Bias

- With the two-step approximation:

$$\langle\delta_h\delta_0\rangle \simeq b_{10}^{(f)}\langle\delta\delta_0\rangle + b_{11}^{(f)}\langle\delta'\delta_0\rangle + \frac{1}{2}\left[b_{20}^{(f)}\,\langle\delta^2\delta_0\rangle + 2\,b_{21}^{(f)}\langle\delta'\delta\delta_0\rangle + b_{22}^{(f)}\,\langle\delta'^2\delta_0\rangle\right] + \dots$$

- Same definition of $b_{nj}$ as before, but now with non-Gaussian $f(s)$

$$\Rightarrow \quad \boxed{\Delta b_1(k) = \frac{2f_{\rm NL}^{\rm local}}{k^2T(k)}\left[s\,b_{20}^{(f)} + b_{21}^{(f)} + \left\langle(\delta')^2\right\rangle b_{22}^{(f)} + \mathcal{O}(k^2)\right]}$$

- All coefficients matter at small $k$. Possibly, also some effects at $k \sim R$ (where the leading term starts decaying). Equilateral NG?

MM, Paranjape & Sheth (to appear)

# Conclusions

- Accurate solution of first passage of correlated random walks
- Satisfactory mathematical understanding of the excursion set approach to structure formation
- Simple rescaling of the spherical collapse barrier reproduces correctly the Gaussian mass function
- Straightforward non-perturbative inclusion of NG (can do Eulerian field!)
- Predictions for bias functions and coefficients, new strategies to measure them in simulations (see Aseem's talk)
- To do: check against N-body simulations, generalization to excursion set theory of peaks
- Interesting possibilities (?) for primordial non-Gaussianity

Thanks!!

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**Bias Is Still Complicated!**

Thanks!!