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Exploring the scale dependence of Lagrangian bias

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Motivation

Scale dependence of halo bias can be a powerful probe of cosmology, potentially testing the initial conditions *(e.g., Dalal et al. 08; Slosar et al. 08; many others)* or the nature of gravity *(e.g., Parfrey, Hui & Sheth 11; Lam & Li 12).*

Problem is, halo bias in vanilla LCDM is *also* scale dependent due to the nature of Einstein/Newtonian gravity *(e.g., Chan et al. 12; Baldauf et al. 12)*.

Analytical predictions traditionally rely on excursion set approach and "peak-background split" argument. Recent work on excursion sets with correlated steps (see Marcello's talk) shows that scale-dependence is expected even in the *initial* conditions.

We explore this here.

Caveat: Everything in this talk is Lagrangian!

Outline

- Fourier space versus real space
- Scale dependence from random walks
- (Detour: Excursion Set Peaks)
- Numerical tests and comparisons with *N*-body simulations

Traditional estimates of bias

$$b_1^2(k) \equiv \frac{P_{\rm hh}(k)}{P_{\rm mm}(k)} \quad \text{or} \quad b_1(k) \equiv \frac{P_{\rm hm}(k)}{P_{\rm mm}(k)}$$

where $P_{\rm hh}(k) = \langle \delta_{\rm h}^2 \rangle, P_{\rm mm}(k) = \langle \delta^2 \rangle$ and $P_{\rm hm}(k) = \langle \delta_{\rm h} \delta \rangle$.

These typically show scale-dependence ~ k^2 with **constant** value at **large scales**, usually compared with "peak-background split"
(Kaiser 84; BBKS 86; Mo & White 96)

$$b_{n0} = f^{-1} \left(-\frac{\partial}{\partial \delta_{\rm c}} \right)^n f \,,$$

with halo mass function $f(\delta_{\rm c}; m)$.

Estimates beyond linear bias require measuring higher order correlations (e.g., quadratic bias requires bispectrum) and/or assumptions regarding locality/stochasticity/scale dependence.

From Fourier space to Real space

Peak-bg split argument works in real space. Not immediately obvious why/how this should be compared with Fourier space measurement.

Consider a toy example:

$$\delta_0(\mathbf{k}) = \delta(\mathbf{k})W(kR_0) \quad ; \quad \delta_\mathrm{h}(\mathbf{k}) = b(k|m)\delta(\mathbf{k})W(kR)$$

with $m \propto R^3$ and $W(y) = \mathrm{e}^{-y^2/2}$.

Natural real-space definition of linear bias could be $b(m, R_0) = \langle\, \delta_\mathrm{h}\delta_0 \,\rangle / \langle\, \delta_0^2 \,\rangle$. Then,

$$b(k|m) = b_{10}(m) \implies b(m, R_0) = (S_\times/S_0)b_{10}$$

$$\left[S_0 = \int \mathrm{d}\ln k\, \Delta(k)W(kR_0)^2 \ ; \ S_\times = \int \mathrm{d}\ln k\, \Delta(k)W(kR)W(kR_0)\right]$$

AP & Sheth 12

So even constant Fourier-space bias will lead to scale dependence in real space.

Real-space bias from random walks

Excursion set approach makes the peak-bg argument rigorous *(Mo & White 96)*.

Calculation proceeds by writing a conditional mass function

$$f(\delta_{\rm c}, m|\delta_0, R_0) = f(\delta_{\rm c}, s|\delta_0, S_0)$$

and then Taylor expanding. $\left[s \equiv \int \mathrm{d}\ln k\, \Delta(k) W(kR)^2 \;;\; m \propto R^3\right]$

For random walks with uncorrelated steps $f(\delta_{\rm c}, s|\delta_0, S_0) = f(\delta_{\rm c} - \delta_0, s - S_0)$ so Taylor expansion precisely recovers peak-bg results provided $S_0 \ll s$.

Two issues:

(1.) We'd like to compute cross-correlations, not Taylor coefficients.
(2.) We'd like to use random walks with correlated steps.

It's possible to do (1.) alone.
But it's easier to do (1.) and (2.) simultaneously.

Musso, AP & Sheth 12

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Scale dependence from correlated steps

From Marcello's talk we know an accurate analytical first crossing distribution

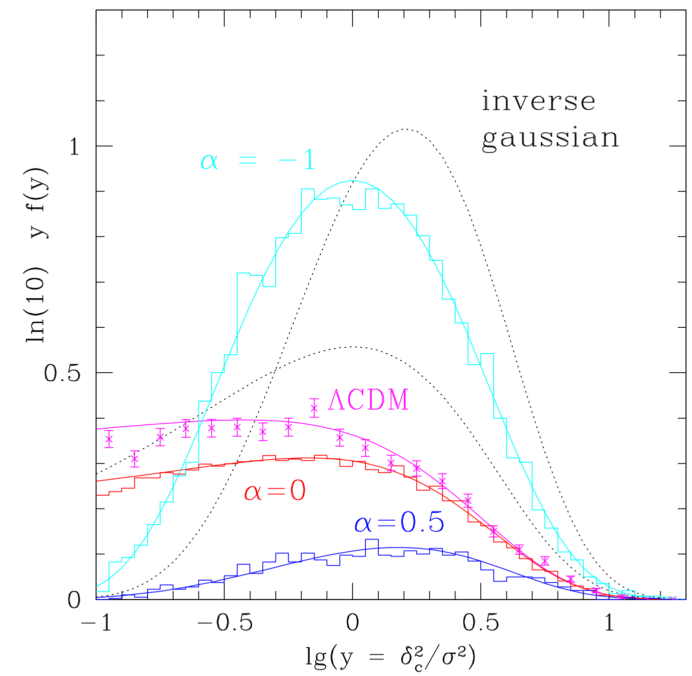
$\nu \equiv \delta_{\mathrm{c}}/\sqrt{s}\ ;\ x \sim$ walk slope

$\gamma = \langle\, x\, \nu\, \rangle \sim$ width of power spectrum

$p_{\mathrm{G}}(x-\mu;\Sigma^2):$ Gaussian with mean μ and variance Σ^2

$$f_{\mathrm{MS}}(\nu) = \frac{\mathrm{e}^{-\nu^2/2}}{\sqrt{2\pi}}\,\frac{1}{\gamma\nu}\int_0^\infty \mathrm{d}x\, x\, p_{\mathrm{G}}(x-\gamma\nu; 1-\gamma^2)$$
$$= \int_0^\infty \mathrm{d}x\,\frac{x}{\gamma\nu}\,p(x,\nu)$$

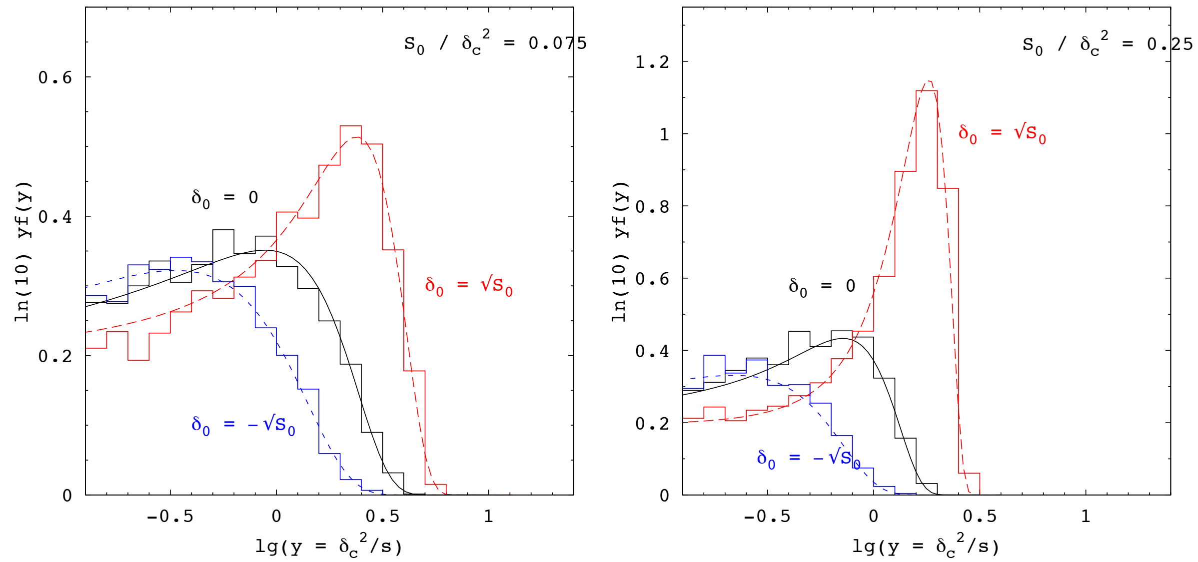
(Musso & Sheth 12)



which also leads to an accurate conditional distribution

$$f(\nu|\delta_0) = \int_0^\infty \mathrm{d}x\,\frac{x}{\gamma\nu}\,p(x,\nu|\delta_0)$$

Musso, AP & Sheth 12



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Scale dependence from correlated steps

Musso, AP & Sheth 12

Define $\langle\, \rho_{\rm h} | \delta_0; S_0 \,\rangle \equiv f(\nu|\delta_0;S_0)/f(\nu)$

Cross-correlation $\langle\, \rho_{\rm h}\delta_0 \,\rangle = \int {\rm d}\delta_0\, p_{\rm G}(\delta_0;S_0)\,\delta_0\, \langle\, \rho_{\rm h}|\delta_0;S_0 \,\rangle$ is analytic, and has nice properties:

- ***Structure of linear bias***

$$b_1 \equiv \langle\, \rho_{\rm h}\delta_0 \,\rangle / S_0 = (S_\times/S_0)[b_{10} + \epsilon_\times b_{11}]$$

where $\epsilon_\times = 2\,{\rm d}\ln S_\times/{\rm d}\ln s \quad ; \quad b_{10} = -\partial\ln f/\partial\delta_{\rm c} \quad ; \quad \delta_{\rm c} b_{11} = \nu^2 - \delta_{\rm c} b_{10}$

- ***Interpretation of*** $\epsilon_\times$

Suppose that in Fourier space: $b_1(k) = b_{10} + (k^2 s/\sigma_1^2) b_{11}$ $\quad \left[\sigma_1^2 \equiv \int {\rm d}\ln k\, \Delta(k) k^2 W(kR)^2\right]$

Then in real space: $b_1 = (S_\times/S_0)[b_{10} + \epsilon_\times b_{11}]$

Scale dependence from correlated steps

Musso, AP & Sheth 12

Define $\langle \rho_{\rm h}|\delta_0; S_0 \rangle \equiv f(\nu|\delta_0; S_0)/f(\nu)$

Cross-correlation $\langle \rho_{\rm h}\delta_0 \rangle = \int {\rm d}\delta_0\, p_{\rm G}(\delta_0; S_0)\, \delta_0\, \langle \rho_{\rm h}|\delta_0; S_0 \rangle$ is analytic, and has nice properties:

- ***Extension to nonlinear bias***

$b_n \equiv S_0^{-n/2} \left\langle \rho_{\rm h} H_n(\delta_0/\sqrt{S_0}) \right\rangle$ has similar properties:

$b_n = (S_\times/S_0)^n \sum_{r=0}^{n} \binom{n}{r} b_{nr}\epsilon_\times^r$ where $b_{n0} = f^{-1}\left(-\frac{\partial}{\partial\delta_{\rm c}}\right)^n f,$ and linear relations between b_{nr}

- ***Suggests simple measurement prescription*** $\hat{b}_n = S_0^{-n/2} \sum_{i=1}^{N} H_n(\delta_{0i}/\sqrt{S_0})/N$

- ***Peak-background split*** is the large scale limit of Fourier-space bias, and can be recovered from *finite scale* measurement using linear relations.

- ***Extends to Excursion Set Peaks*** with same structure, different details.

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Detour: Excursion Set Peaks

Mass Function

- ***Constant threshold:*** $B = \delta_c$

(Appel & Jones 90)

$$f_{\rm ESP}(\nu) = \frac{m}{\bar\rho V_*}\int_0^\infty {\rm d}x\,\frac{x}{\gamma\nu}\,F(x)\,p(x,\nu) \xrightarrow[\nu\gg1]{} \frac{m}{\bar\rho V_*}\gamma^3\nu^3 f_{\rm MS}(\nu)$$

x = peak curvature; F(x) = BBKS curvature weight; V_* = characteristic peak volume

AP & Sheth 12

- ***"Moving" threshold:*** $B(\sigma)$ $\left[\sigma^2 = s\right]$

$$f_{\rm ESP}(\nu) = \frac{m}{\bar\rho V_*}\frac{1}{\gamma\nu}\int_{\gamma B'}^\infty {\rm d}x\,(x-\gamma B')\,F(x)\,p(x,B/\sigma)$$

- ***"Ellipsoidal" threshold:*** $B = \delta_c + \beta\sigma$ with stochastic β

AP, Sheth & Desjacques 13

$$\begin{aligned} f_{\rm ESP}(\nu) &= \int {\rm d}\beta\,p(\beta) f_{\rm ESP}(\nu|\beta) \\ &= \frac{m}{\bar\rho V_*}\frac{1}{\gamma\nu}\int {\rm d}\beta\,p(\beta)\int_{\beta\gamma}^\infty {\rm d}x\,(x-\beta\gamma)\,F(x)\,p(x,\nu+\beta)\end{aligned}$$

Detour: Excursion Set Peaks

Bias

Bias calculation is identical to that for traditional excursion sets, but keeping track of barrier stochasticity.

E.g., linear bias is given by

$$\delta_{\mathrm{c}} b_1 = \left(\frac{S_\times}{S_0}\right) \frac{\int \mathrm{d}\beta\, p(\beta) \mathcal{B}_{1,\mathrm{ESP}}(\nu, \epsilon_\times | \beta)}{\int \mathrm{d}\beta\, p(\beta) f_{\mathrm{ESP}}(\nu|\beta)} \,,$$

where

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$$\begin{aligned}\mathcal{B}_{1,\mathrm{ESP}}(\nu, \epsilon_\times | \beta) \equiv \left(\frac{m}{\bar{\rho} V_*}\right) \frac{\mathrm{e}^{-(\nu+\beta)^2/2}}{\sqrt{2\pi}} \\ \times \frac{1}{\gamma\nu} \int_{\beta\gamma}^{\infty} \mathrm{d}x\, (x - \beta\gamma) F(x) p_{\mathrm{G}}(x - \beta\gamma - \gamma\nu; 1 - \gamma^2) \\ \times \left[\nu(\nu + \beta) - (1 - \epsilon_\times) \frac{\gamma\nu}{1 - \gamma^2} (x - \beta\gamma - \gamma\nu)\right]\end{aligned}$$

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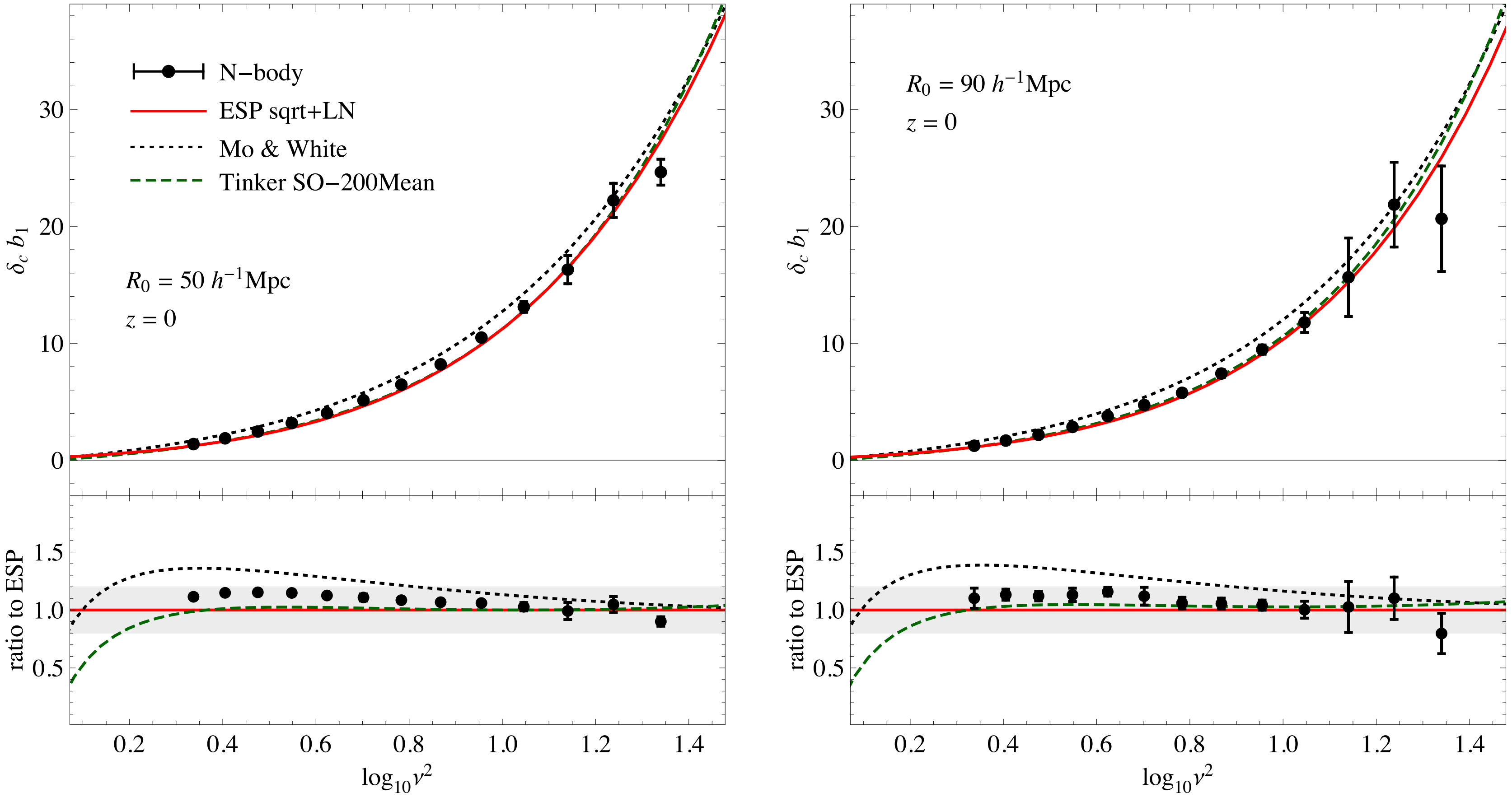
Numerical tests:
N-body simulations

$$b_n \equiv S_0^{-n/2} \left\langle \rho_{\rm h} H_n(\delta_0/\sqrt{S_0}) \right\rangle$$

Measurement:

- Find all *N* **halos** in mass bin $(m, m + \mathrm{d}m)$
- Choose S_0 (i.e., R_0)
- Estimate, e.g., $\hat{b}_1 = S_0^{-1} \sum_{i=1}^{N} \delta_{0i}/N$

AP, Sefusatti, Chan et al. 13



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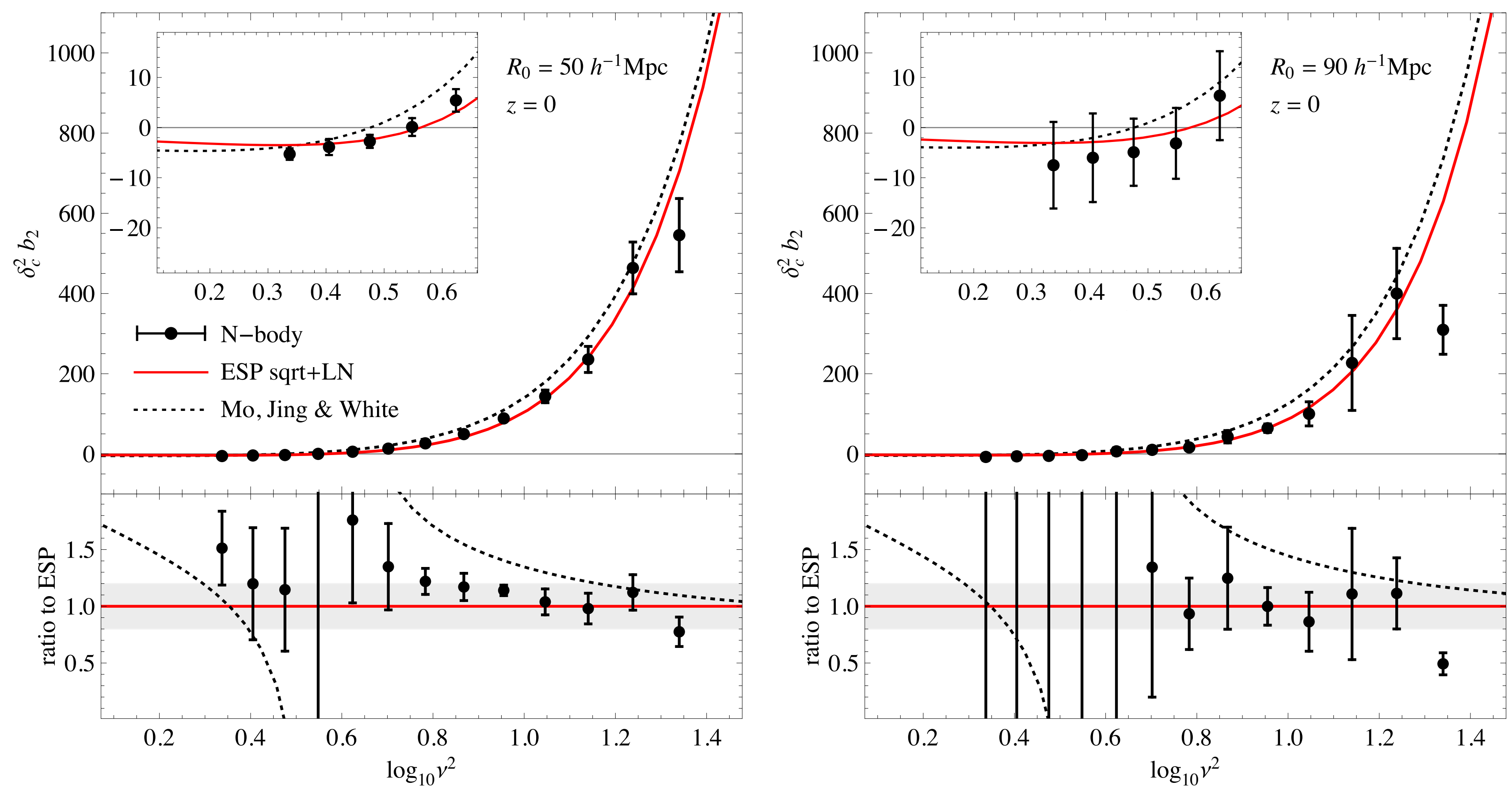
Numerical tests:
N-body simulations

$$b_n \equiv S_0^{-n/2} \left\langle \rho_{\rm h} H_n(\delta_0/\sqrt{S_0}) \right\rangle$$

Measurement:

- Find all *N* **halos** in mass bin $(m, m+\mathrm{d}m)$
- Choose S_0 (i.e., R_0)
- Estimate, e.g., $\hat{b}_2 = S_0^{-1} \sum_{i=1}^{N} (\frac{\delta_{0i}^2}{S_0} - 1)/N$

AP, Sefusatti, Chan et al. 13



Numerical tests:

N-body simulations

$$b_n \equiv S_0^{-n/2} \left\langle \rho_{\rm h} H_n(\delta_0/\sqrt{S_0}) \right\rangle$$

Measurement:

- Find all *N* **halos** in mass bin $(m, m+\mathrm{d}m)$
- Choose S_0 (i.e., R_0)
- Estimate, e.g., $\hat{b}_1, \hat{b}_2$ and reconstruct $\hat{b}_{10}, \hat{b}_{20}$

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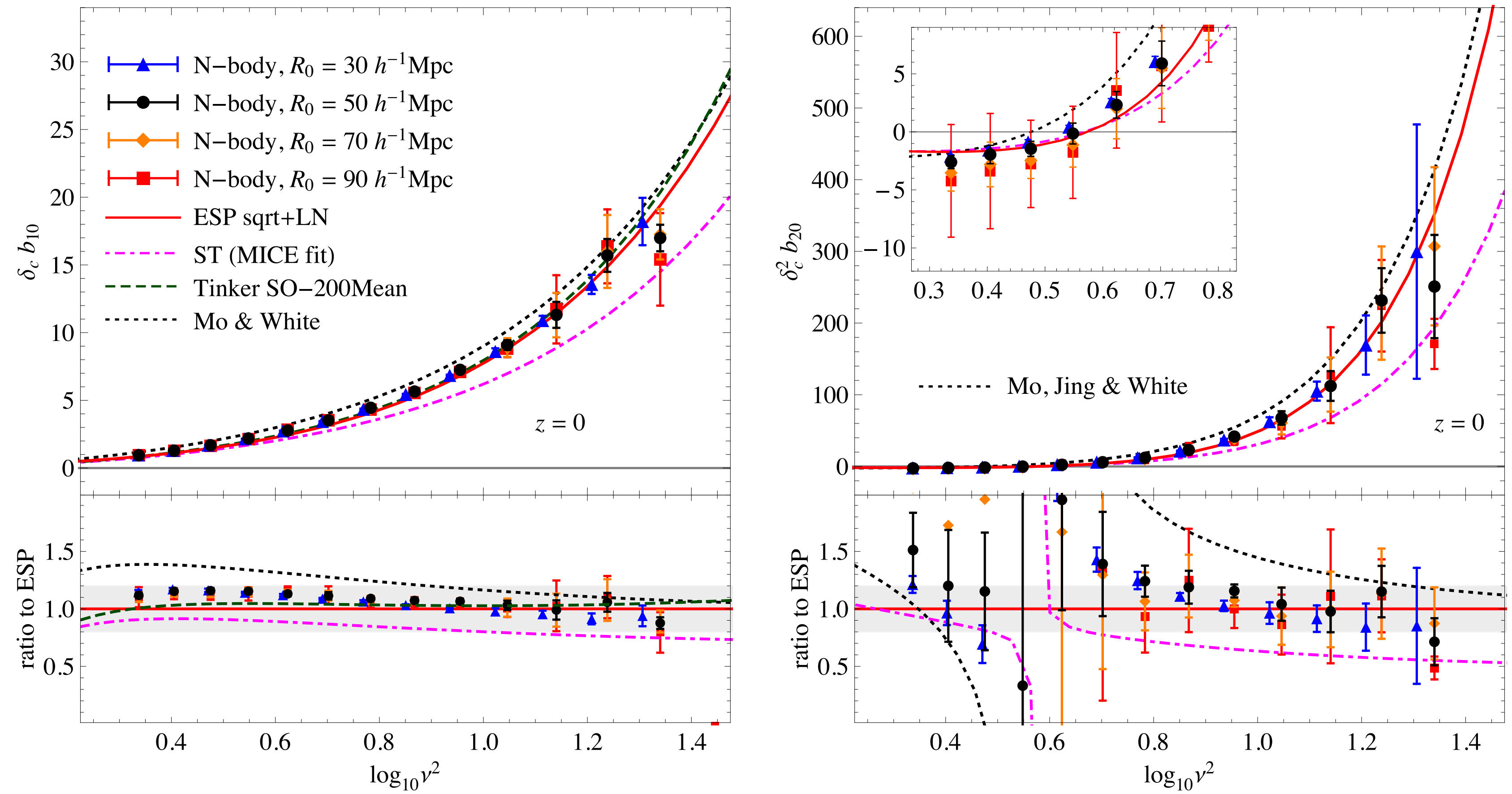

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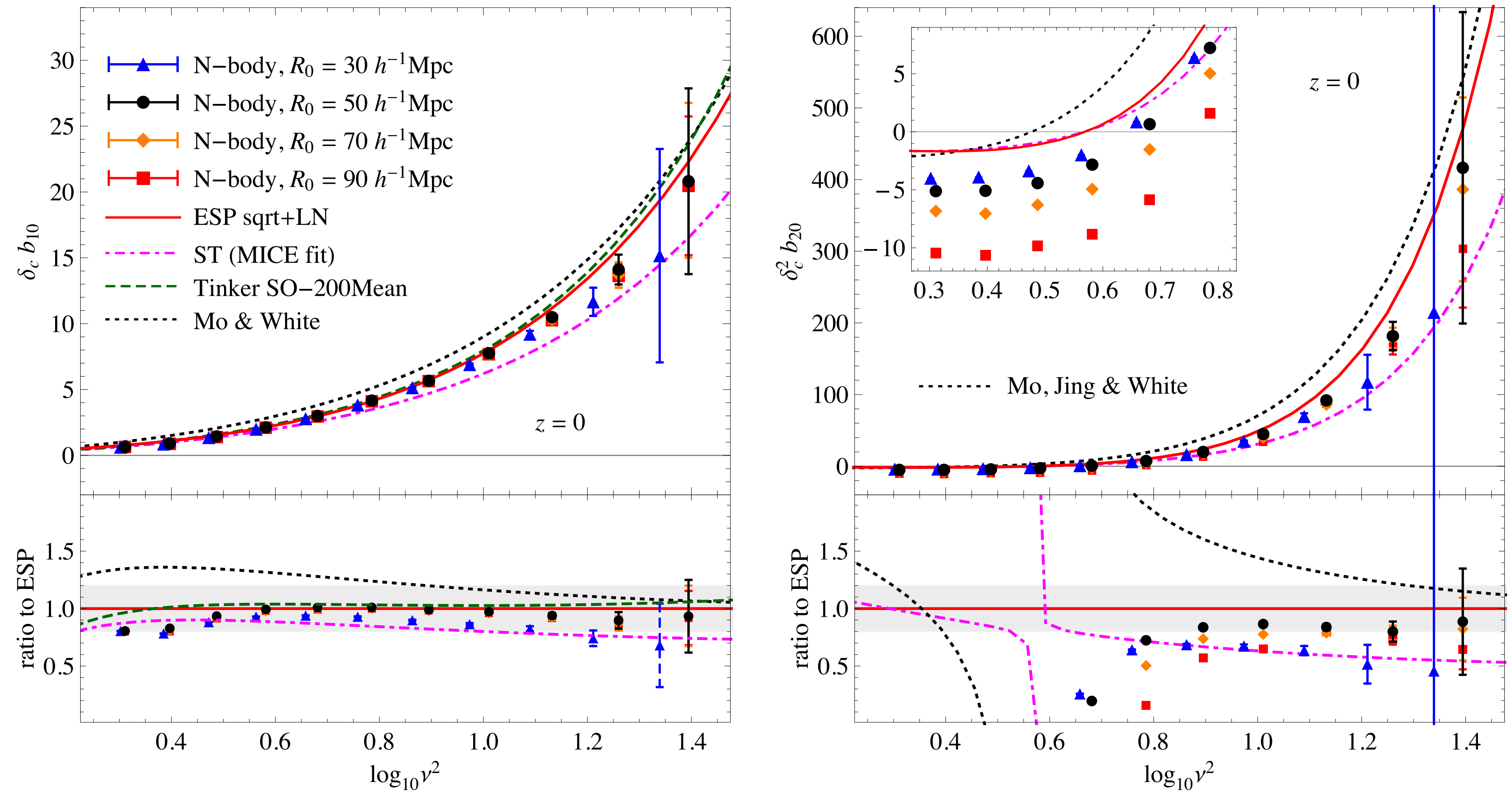
Numerical tests:
Pinocchio

$$b_n \equiv S_0^{-n/2} \left\langle \rho_{\rm h} H_n(\delta_0/\sqrt{S_0}) \right\rangle$$

Measurement:

- Find all *N* **halos** in mass bin $(m, m+\mathrm{d}m)$
- Choose S_0 (i.e., R_0)
- Estimate, e.g., $\hat{b}_1, \hat{b}_2$ and reconstruct $\hat{b}_{10}, \hat{b}_{20}$

AP, Sefusatti, Chan et al. 13



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Conclusions

- Random walks with correlated steps predict scale-dependent Lagrangian bias.
- This extends to excursion set peaks (ESP) calculations too.
- A simple prescription leads to measurements in simulations that are directly comparable to the analytical prediction.

$$\hat{b}_n = S_0^{-n/2} \sum_{i=1}^{N} H_n(\delta_{0i}/\sqrt{S_0})/N$$

- ESP makes accurate predictions for linear and quadratic bias coefficients.

- Measurement prescription is a fast, useful consistency check for "semi-analytic" algorithms (we tested PINOCCHIO).

Still to do:

- Scale-dependence beyond density
- Predictions for Eulerian bias
- Relevance for observations (?)

Thank you!