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Joint ICTP-IAEA Workshop on Nuclear Data for Analytical Applications

21 - 25 October 2013

The interaction of charged particles with atomic nuclei

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WEDNESDAY, 23 OCTOBER 2013
09.00 - 10.30
The Denardo Lecture Hall

Workshop on Nuclear Data for Analytical Applications

The interaction of charged particles with atomic nuclei

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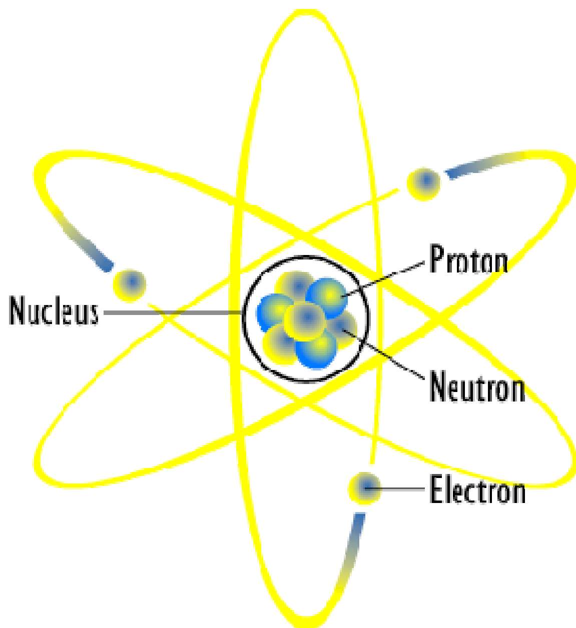
PLAN OF THE LECTURE

- **Nuclear physics fundamentals**
- **S-matrix formalism**
- **Projectile-nucleus interaction mechanisms**
- **Potential scattering**
- **Optical model**
- **Resonance interaction**
- **Compound nucleus**
- **R-matrix theory**
- **Proton induced reactions**
- **Deuteron Induced reactions**

ATOMIC NUCLEUS

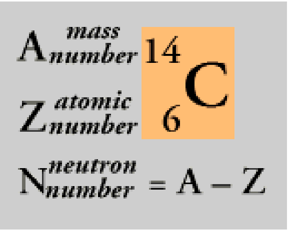


The atomic nucleus is the centre of an atom. The nucleus radius is much smaller than that of the atom. Thus, the nucleus occupies an extremely small volume inside the atom. Nuclei are composed of protons and neutrons. The number of protons in an atomic nucleus is called the atomic number, and determines which element the atom is. The number of neutrons determines the isotope of the element.

What else do we know about the nucleus?

The nucleus is identified by

- atomic number Z (*i.e.*, the number of protons)
- the neutron number, N
- the mass number, A, where $A = Z + N$.



The nucleus is also characterized by

- size
- shape
- binding energy $E_B = \Delta mc^2$, where $\Delta m = (Z \cdot m_p + N \cdot m_n) - M(Z,A)$
- angular momentum (spin)
- and (if it is unstable) half-life.

The nucleus is now understood to be a quantum system composed of protons and neutrons, particles of nearly equal mass and the same intrinsic angular momentum (spin) of ½.

The proton carries one unit of positive electric charge while the neutron has no electric charge.

The binding energy of a nucleus is the energy holding a nucleus together.

NUCLEAR REACTION

An example: $^{15}N + {}^{1}H \rightarrow {}^{12}C + \alpha + \gamma$ (4.965 MeV)

<u>Conservation laws:</u>

- Number of nucleons A
- Electric charge Z
- Energy
- Momentum
- Angular momentum

Application of the conservation of energy gives the Q value of the reaction (the reaction energy). It can be positive and negative.

The Q-value of the reaction

$$Q \equiv T_f - T_i$$

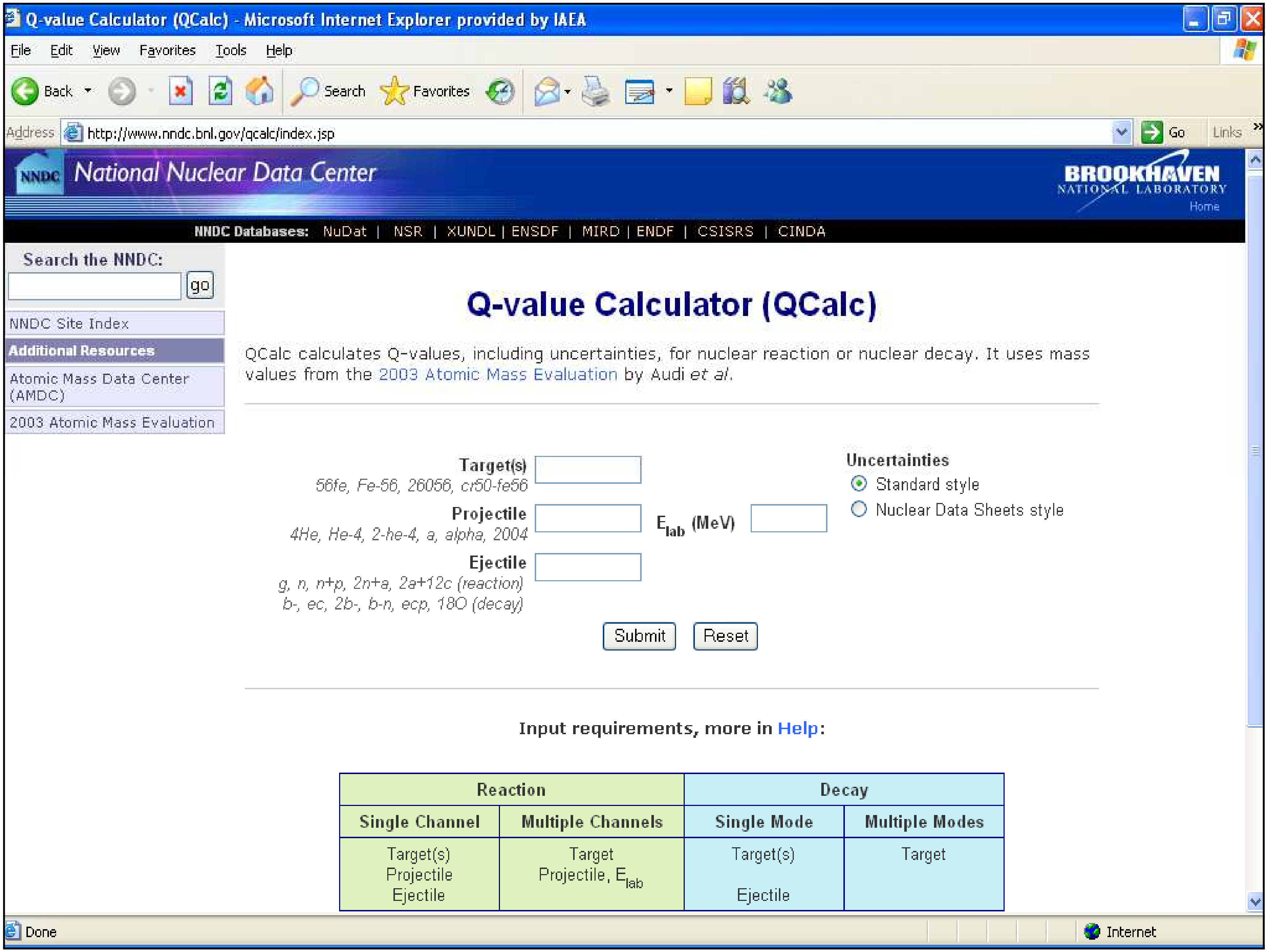
where T_i and T_f are the kinetic energies of the system in the initial and the final state, respectively

Q is the energy released by the reaction

If $Q > 0$, the reaction proceeds even if $T_i = 0$ (*exoergic reaction*)

If $Q < 0$, the reaction proceeds only if $T_i \geq |Q|$ (*endoergic reaction*)

$|Q|$ is the *threshold energy* of the reaction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Additional Resources
Atomic Mass Data Center (AMDC)
2003 Atomic Mass Evaluation
Q-value Calculator (QCalc)
QCalc calculates Q-values, including uncertainties, for nuclear reaction or nuclear decay. It uses mass values from the 2003 Atomic Mass Evaluation by Audi et al.
Target(s)
56fe, Fe-56, 26056, cr50-fe56
Projectile
4He, He-4, 2-he-4, a, alpha, 2004
Elab (MeV)
Ejectile
g, n, n+p, 2n+a, 2a+12c (reaction)
b-, ec, 2b-, b-n, ecp, 18O (decay)
Uncertainties
Standard style
Nuclear Data Sheets style
Submit
Reset
Input requirements, more in Help:
Reaction
Decay
Single Channel
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Multiple Modes
Target(s) Projectile Ejectile
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Target(s) Ejectile
Target
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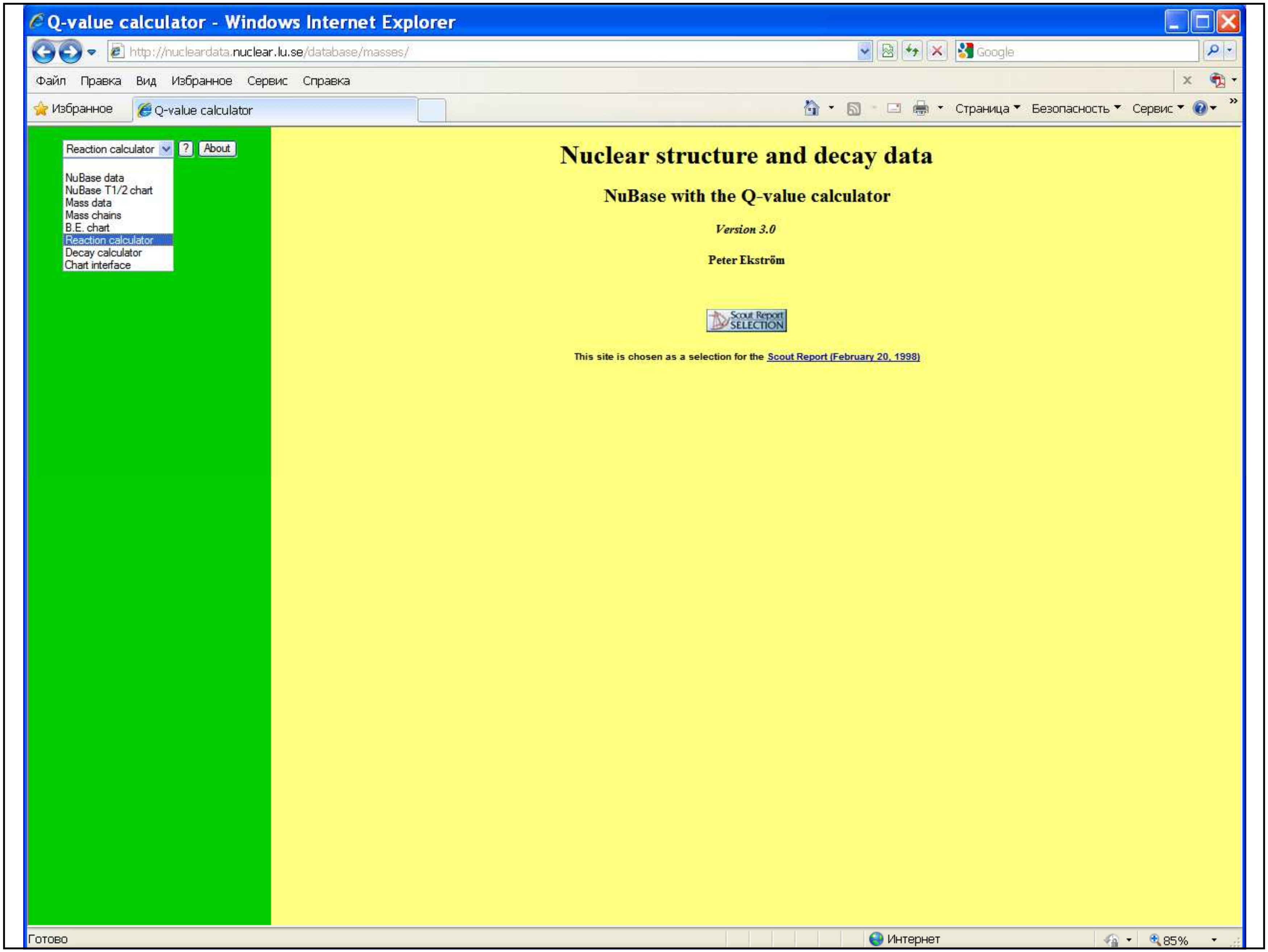
Internet


Q-value calculator - Windows Internet Explorer
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3-body classical model
Two-nucleon transfer
Massive transfer
Decay of excited nuclei
Fragmentation
EPAX v.3
Break-up (DWBA)
Semiclassical model
LISE++
Fusion
Empirical model
Channel Coupling
Langevin equations
Experimental Data
σfus(E)
Driving potentials
Synthesis of SHE (movie)
All resources of the NRV Knowledge Base are free to use.
We, nevertheless, need a support of our project by official establishment
for further development of it. New models of nuclear dynamics and much
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Monte-Carlo
Experimental Data
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Radiative capture
Potential model
Experimental Data
NACRE
NACRE-II
Pre-equilibrium LP formation
4-body classical model
Semiclassical model
Moving sources
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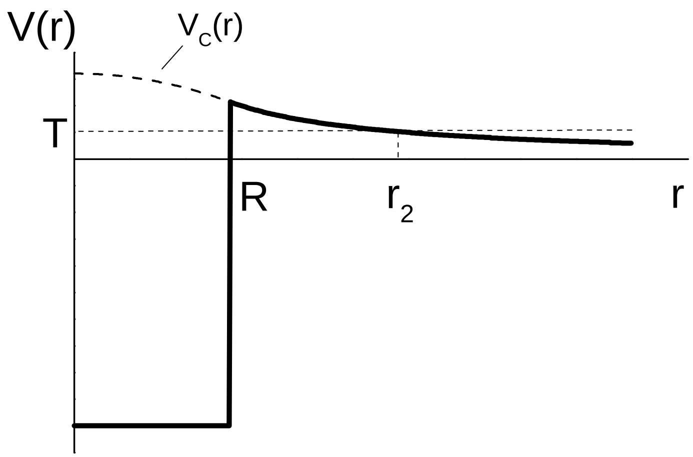
NUCLEAR AND ELECTRICAL FORCES

Nuclear forces	**Electrical forces**
Nuclear forces are: • *Strong* • *Attractive* • *Short-range* • *Non-central*	Can be represented as: $V_C(r) = \begin{cases} \dfrac{Zze^2}{r} & for \quad r \geq R \\ \dfrac{Zze^2}{2R}\left(3 - \dfrac{r^2}{R^2}\right) & for \quad r \leq R \end{cases}$ $R \approx (1.1 \div 1.5) \cdot A^{1/3}$ fm (1 fm = 10^{-13} cm)

COMPARISON OF THE FORCE FEATURES

Force properties	**Nuclear forces**	**Electrical forces**
Range	~1fm (10^{-13} cm)	Long
Electric charge	Non-sensitive	Sensitive
Saturation	Yes	No
Spin dependence	Yes	No

NUCLEAR + COULOMB POTENTIAL



The Coulomb repulsion changes to (nuclear) attraction at the distance *R*.

Nuclear potential is pictured in the form of a square potential well which is about 40÷50 MeV deep.

Coulomb potential barrier height is $$B_C = \frac{Zze^2}{R}$$

Transparency of the Coulomb barrier is $$D \approx e^{-\frac{2}{\hbar}\int_{r_1}^{r_2}\sqrt{2\mu(V_C - T)}\,dr}$$

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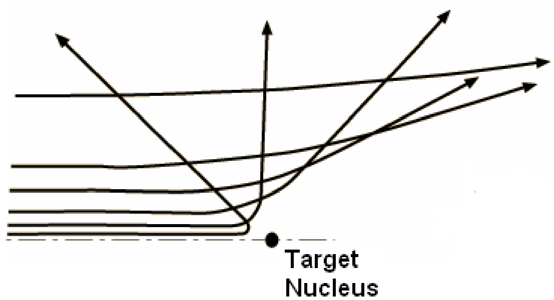
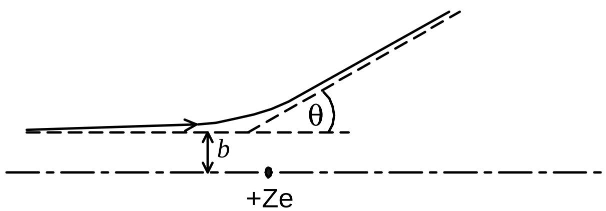
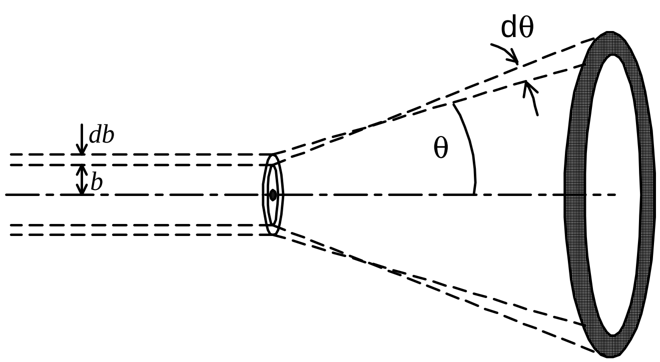
RUTHERFORD SCATTERING







Dependence of the force on the distance is known ⇒ analytical solution is possible

$$\tan\theta = \frac{2Zze^2}{m\upsilon^2 b}$$

$$d\sigma = \frac{dN}{N} = 2\pi b\, db$$

$$d\sigma = n\left(\frac{Zze^2}{m\upsilon^2}\right)\frac{d\Omega}{4\sin^4\frac{\theta}{2}}$$

Schrödinger equation

According to quantum mechanics a particle state is described by the wave function ψ, which is obtained as a solution of the wave equation.

For the case of elastic scattering of spinless non-identical particles the wave equation has a form of Schrödinger equation with a spherically symmetric potential $V(r)$

$$\Delta\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad ,$$

where in spherical coordinates Laplace operator Δ is

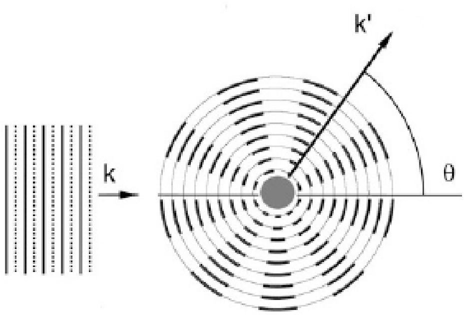
$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

NUCLEAR POTENTIAL SCATTERING

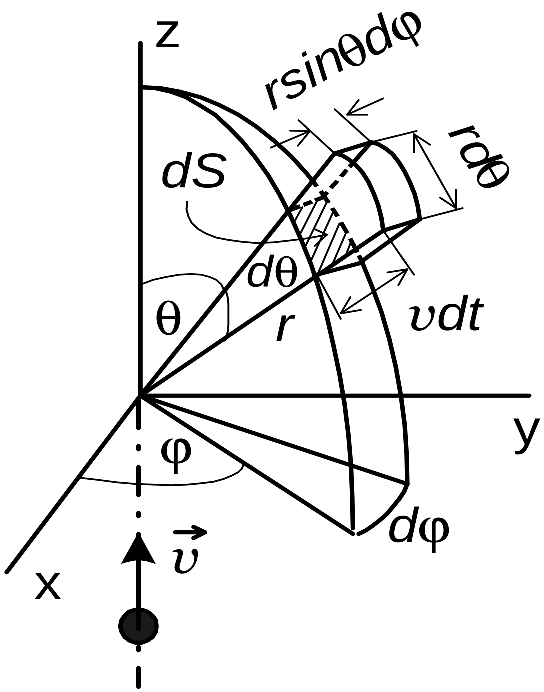
Prior to scattering	In the course of scattering	After scattering
Plane wave $\psi = e^{ikz}$ $k = \frac{p}{\hbar} = \frac{1}{\lambda}$	**Divergent wave arises** $\psi = f(\theta)\frac{e^{ikr}}{r}$	**Plane + Spherical waves** $\psi = e^{ikz} + \frac{e^{ikr} f(\theta)}{r}$

The angular distribution of the scattered particles is defined by the *f*(θ) function.

The differential cross-section is $\frac{d\sigma}{d\Omega} = |f(\theta)|^2$



DEFINITION OF THE DIFFERENTIAL CROSS-SECCTION



By definition the differential cross section $d\sigma/d\Omega$ is equal to the fraction dN/N of the projectiles scattered into the given solid angle $d\Omega$, where

$$d\Omega = \sin\theta d\theta d\varphi$$

Assuming a unity density of particles in the primary beam, the flux density is

$$N = v(\mathrm{s}^{-1}\cdot\mathrm{cm}^{-2}),$$

where v is the particle velocity. The number of particles dN traversing the surface element dS per time unit is determined by the probability of finding particles in the elementary volume

$$dV = vr^2\sin\theta d\theta d\varphi,$$

and the probability density is given by the square of the modulus of the scattered wave function:

$$dN = \left| f(\theta)\frac{e^{ikr}}{r}\right|^2 \upsilon r^2 \sin\theta\, d\theta\, d\varphi; \quad \frac{d\sigma}{d\Omega} = \left| f(\theta)\right|^2$$

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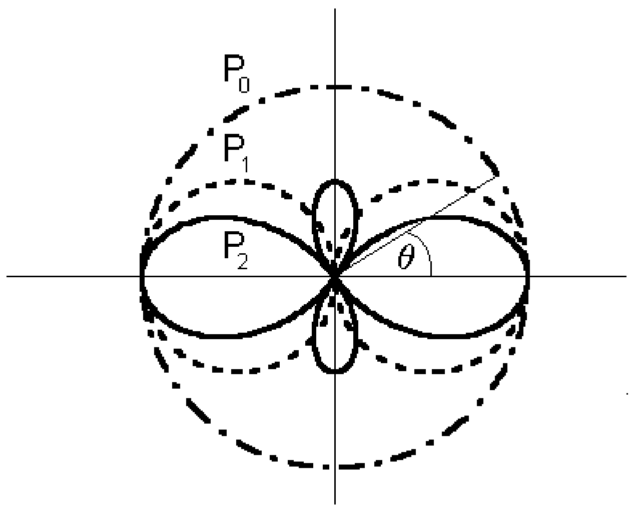
SOLUTION OF THE WAVE EQUATION

General solution is $$\psi = \sum_{l=0}^{\infty} A_l P_l(\cos\theta) R_{kl}(r) \quad ,$$

where $R_{kl}(r)$ is a radial wave function and $P_l(\cos\theta)$ are Legendre polynomials $P_0=1$, $P_1=\cos\theta$, $P_2=(3\cos^2\theta - 1)/2$, …



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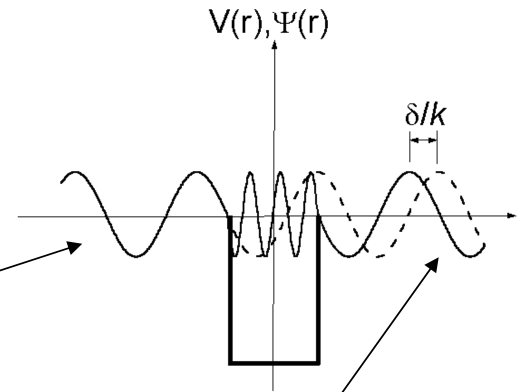
RADIAL WAVE FUNCTION ASYMPTOTIC

$e^{-i\left(kr-l\frac{\pi}{2}\right)}$ – convergent partial spherical wave

$e^{i\left(kr-l\frac{\pi}{2}\right)}$ – divergent partial spherical wave



Initial stage

$$R_{kl}(r) \sim e^{i\left(kr-l\frac{\pi}{2}\right)} - e^{-i\left(kr-l\frac{\pi}{2}\right)}$$

Final stage

$$R_{kl}(r) \sim e^{i\left(kr-l\frac{\pi}{2}+2\delta_l\right)} - e^{-i\left(kr-l\frac{\pi}{2}\right)} =$$

$$= S_l e^{i\left(kr-l\frac{\pi}{2}\right)} - e^{-i\left(kr-l\frac{\pi}{2}\right)}$$

$$S_l = e^{2i\delta_l}$$

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WAVE FUNCTIONS EXPANSION

Incident Wave:

$$e^{ikz} = \sum_{l=0}^{\infty} \frac{(2l+1)i^l}{2ikr} P_l(\cos\theta) \left[e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)} \right]$$

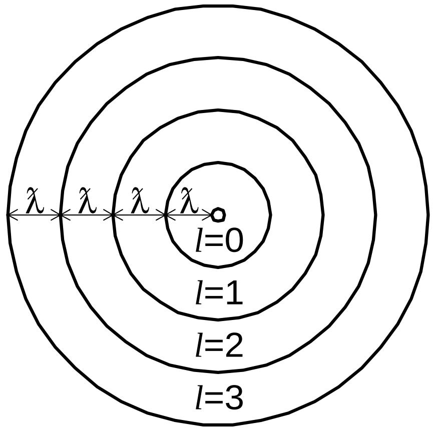
Incident + Divergent:

$$e^{ikz} + f(\theta)\frac{e^{ikr}}{r} = \sum_{l=0}^{\infty} \frac{(2l+1)i^l}{2ikr} P_l(\cos\theta) \left[S_l e^{i\left(kr - l\frac{\pi}{2}\right)} - e^{-i\left(kr - l\frac{\pi}{2}\right)} \right]$$

RELATION BETWEEN SCATTERING AMPLITUDE AND PHASES

$$f(\theta) = \frac{1}{2ik} \sum_l (2l+1)(e^{2i\delta_l} - 1) P_l(\cos\theta).$$

PARTIAL WAVES



The initial beam behaves as if it were subdivided in cylindrical zones

Suppose the projectile possesses kinetic momentum *p* and angular momentum *l*.

Then from comparison between classical and quantum mechanical relations for the modulus of the angular momentum

$$\left|\vec{l}\right| = p\rho = \hbar\sqrt{l(l+1)}$$

follows that

$$\rho = \frac{\hbar}{p}\sqrt{l(l+1)} = ƛ\sqrt{l(l+1)}$$

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TAKING PROJECTILE CHARGE AND SPIN INTO ACCOUNT

IF PROJECTILE IS CHARGED THEN

$$f(\theta) = f_C(\theta) + \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)(S_l - 1)e^{2i\sigma_l}P_l(\cos\theta)$$

IF PROJECTILE HAS SPIN ½ THEN

$$d\sigma / d\Omega = \left|A(\theta)\right|^2 + \left|B(\theta)\right|^2,$$

$$A(\theta) = f_C(\theta) + \frac{1}{2ik}\sum_{l=1}^{\infty}[(l+1)S_l^+ + lS_l^- - (2l+1)]\exp(2i\sigma_l)P_l(\cos\theta);$$

$$B(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(S_l^+ - S_l^-)\exp(2i\sigma_l)P_l^1(\cos\theta)$$

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OPTICAL MODEL

In the so-called optical model nucleus is represented by means of a complex potential. The interaction of the projectile with the nucleus is then reduced to de-Broglie's wave refraction and absorption by a opaque sphere. The name of the model originates from the formal analogy with the light plane wave passing through a semitransparent sphere.

As well as refraction and absorption of the light is described by a complex index

$n = n_r + i\kappa_a$

the complex potential of the form

$U = V + iW$

is used to take into account scattering and absorption of the projectile by the nucleus. The real part of the potential is responsible for scattering whereas the imaginary part stands for absorption.

OPTICAL MODEL POTENTIAL

Complex potential: $U = V + iW$

The standard form of the potential: $U(r) = U_C(r) + U_R(r) + iU_1(r) + U_{so}(r)$

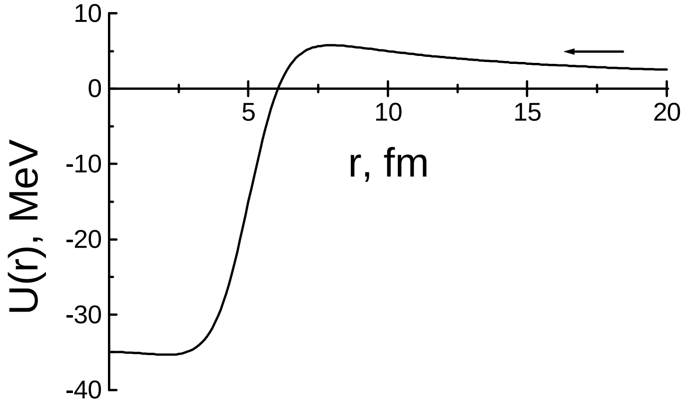
$$U_R(r) = -V_R f_R(r)$$

$$U_1(r) = 4a_i W_D \frac{df_i(r)}{dr}$$

$$U_{so} = \left(\frac{\hbar}{m_\pi c}\right)^2 V_{so} \frac{1}{r}\frac{df_{so}}{dr} \boldsymbol{l}\cdot\boldsymbol{s}$$

$$f_R(r) = \left[1 + \exp\left(\frac{r - R_x}{a_x}\right)\right]^{-1}$$

$$R_x = r_x A^{1/3}$$



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OPTICAL POTENTIAL PARAMETERS

<u>Low energy peculiarities</u>:

- The strength parameters often have strong energy dependence in the vicinity of the Coulomb barrier.
- The real potential radial dependence is of more complicated than Saxon-Woods form.
- The imagine part of potential reveals non-systematic dependence on nucleus mass number.
- The imagine part of potential is close to zero for light nuclei.
- Absorption is peaked at the nucleus surface.
- The radius of the imaginary potential diminishes with decreasing energy while its diffuseness increases.

<u>Conclusion</u>:

Global sets of parameters are inapplicable at low energy!

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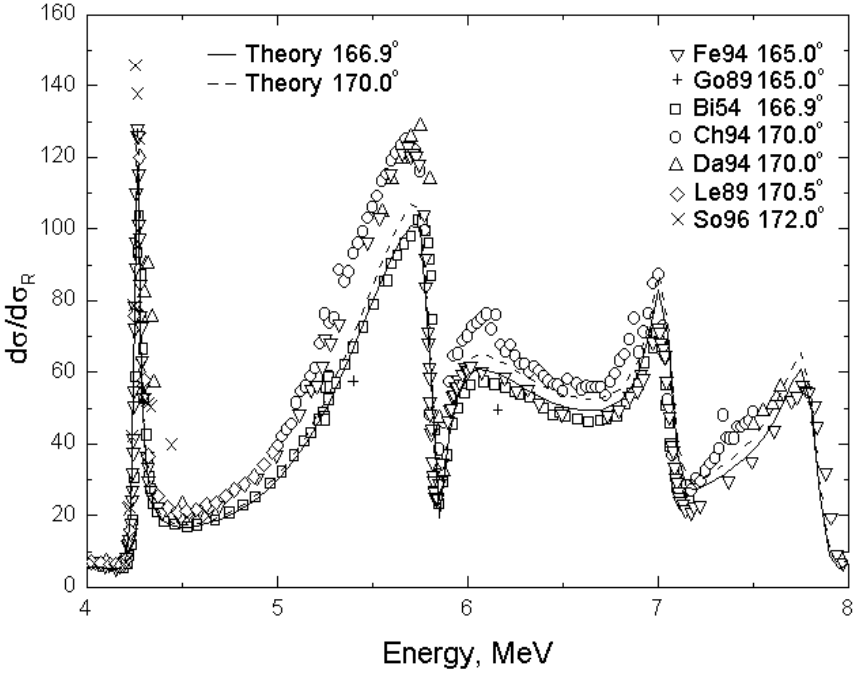
Modification of Saxon-Woods form-factor

For deformed nucleus channel coupling is essential. A simple way to take it into account is the modification of the real part of the optical potential by adding a surface term to the Saxon-Woods potential:

$$U_R(r) = -V_R f(r, r_R, a_R) + 4a_S V_S \frac{d}{dr} f(r, r_S, a_S)$$

Splitting V_R to take account of exchange mechanism in the case of complex particles scattering

The optimal potential parameters obtained for $^{12}C(^4He, {}^4He)\,^{12}C$ scattering								
$V_{l=0}$ MeV	$V_{l=1}$ MeV	$V_{l=2}$ MeV	$V_{l=3}$ MeV	$V_{l=4}$ MeV	$V_{l\geq 5}$ MeV	r_R fm	a_R fm	r_C fm
47.89	50.10	46.30	47.70	42.90	47.00	1.43	0.70	1.45



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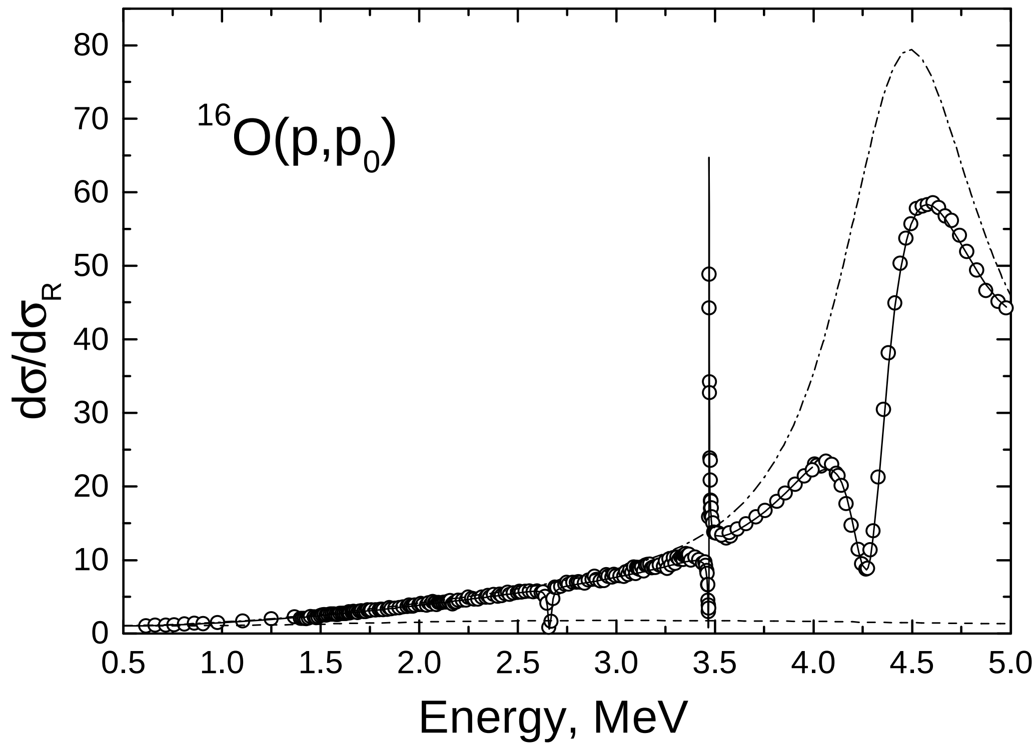
EFFECT OF SHAPE RESONANCE ON THE ELASTIC SCATTERING OF PROTONS FROM ^{16}O



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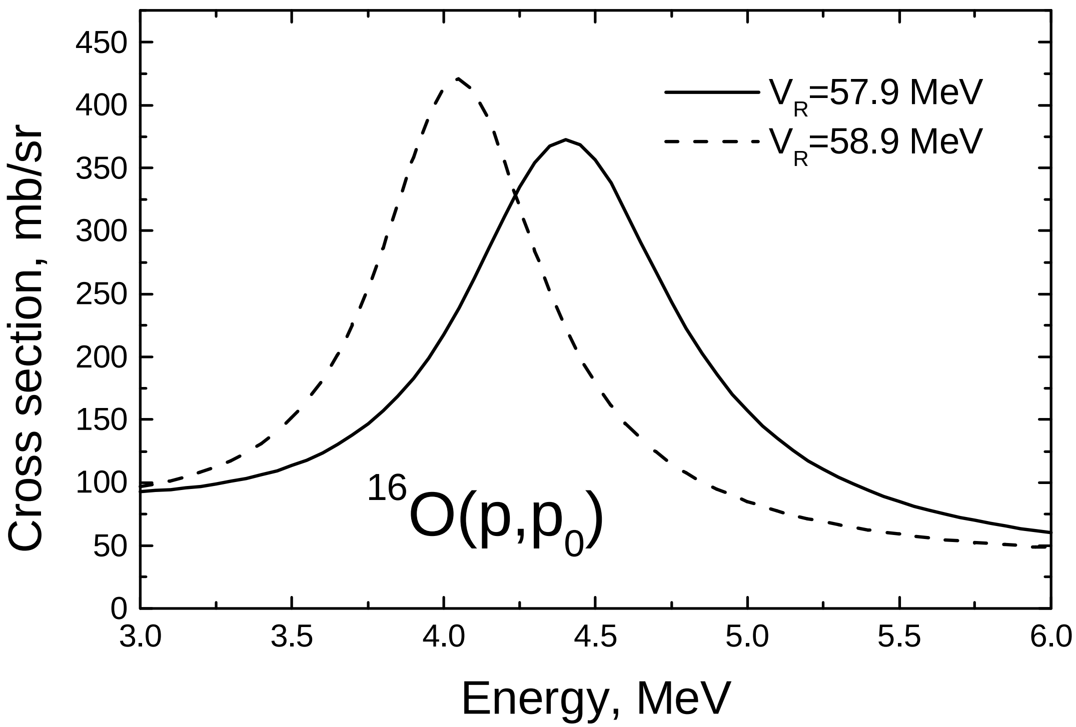
Shape resonance dependence on V_R



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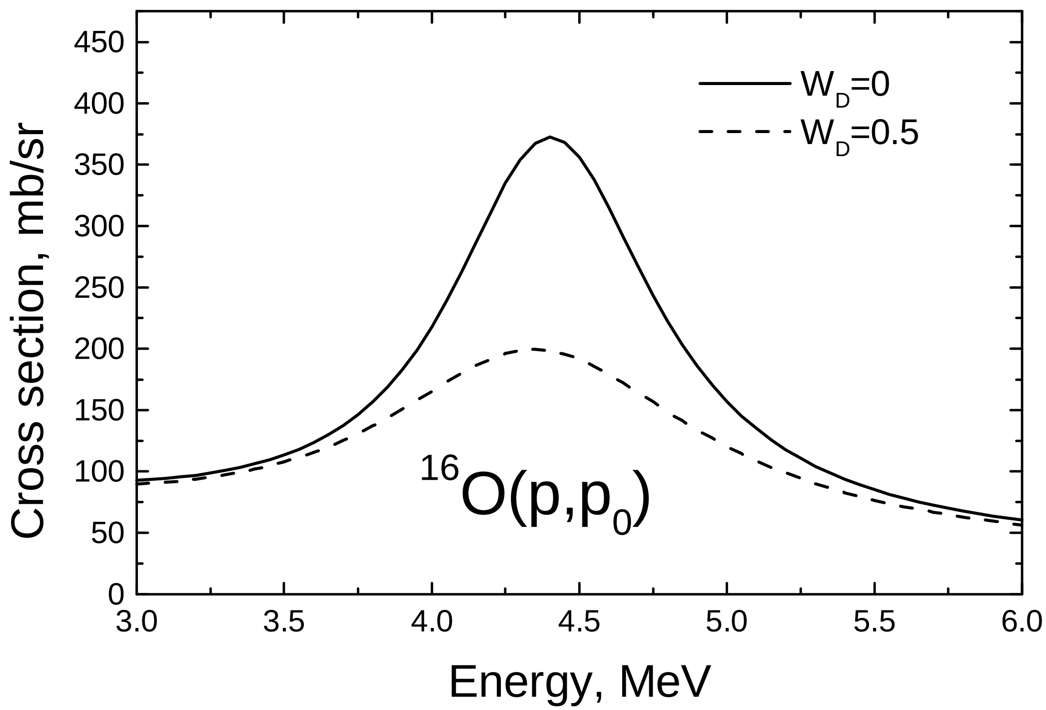
Shape resonance dependence on W_D



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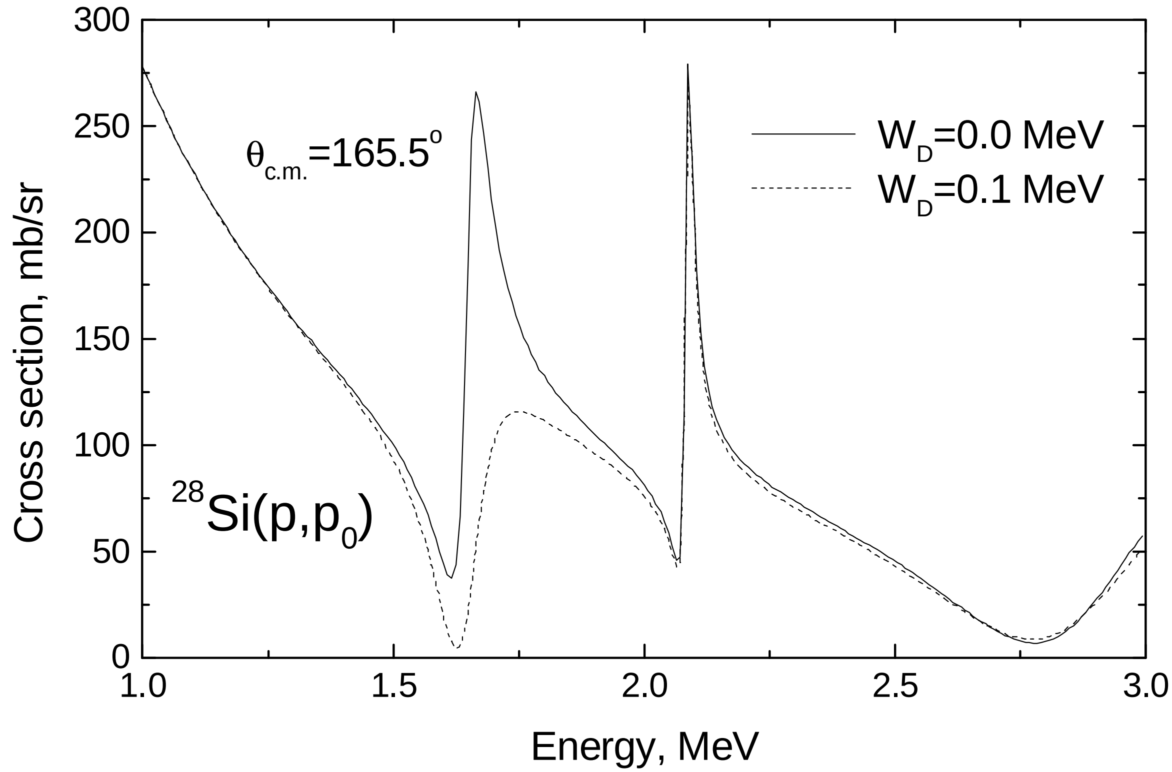
Shape resonance dependence on W_D



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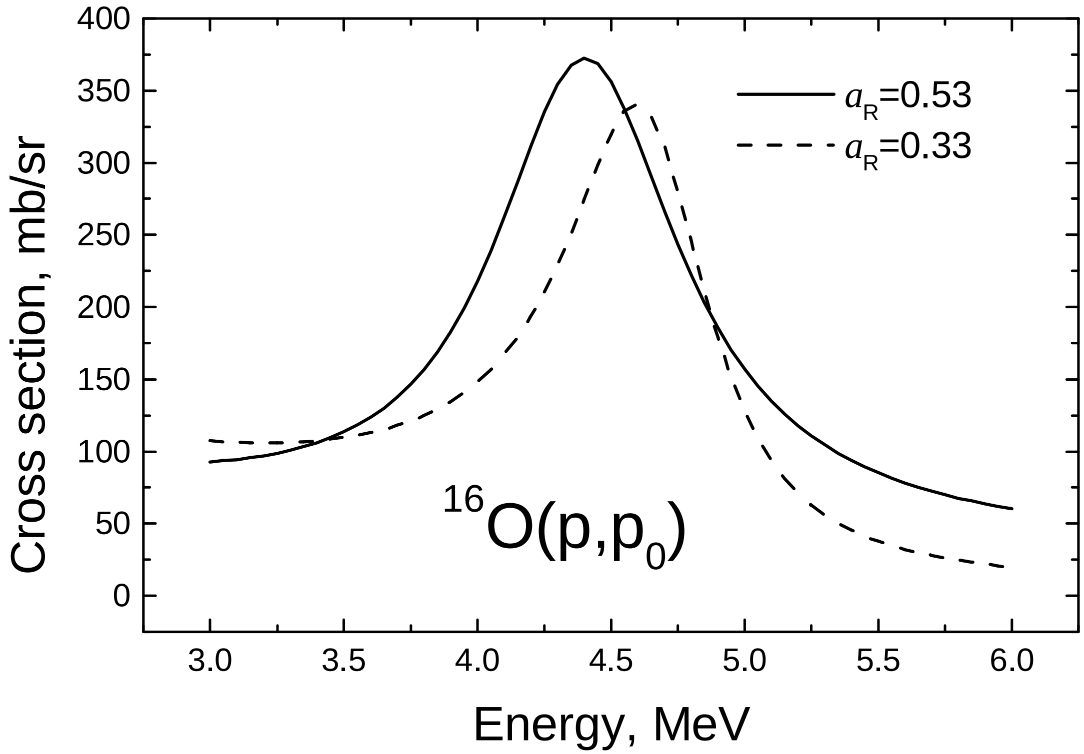
Shape resonance dependence on a_R



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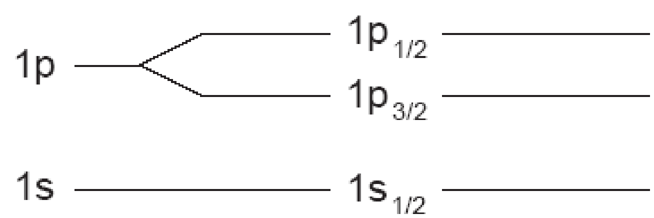
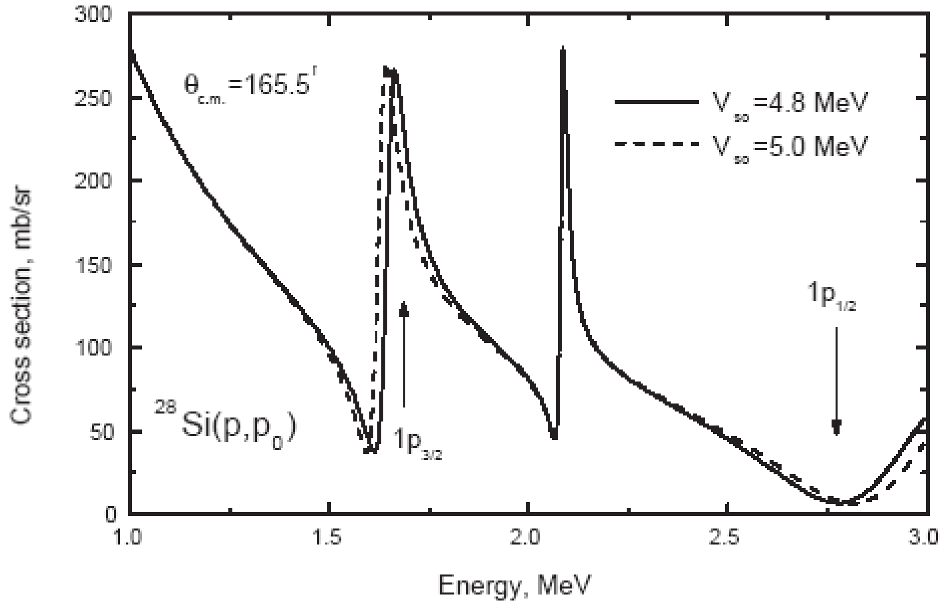
SPIN-ORBIT INTERACTION

Level splitting:



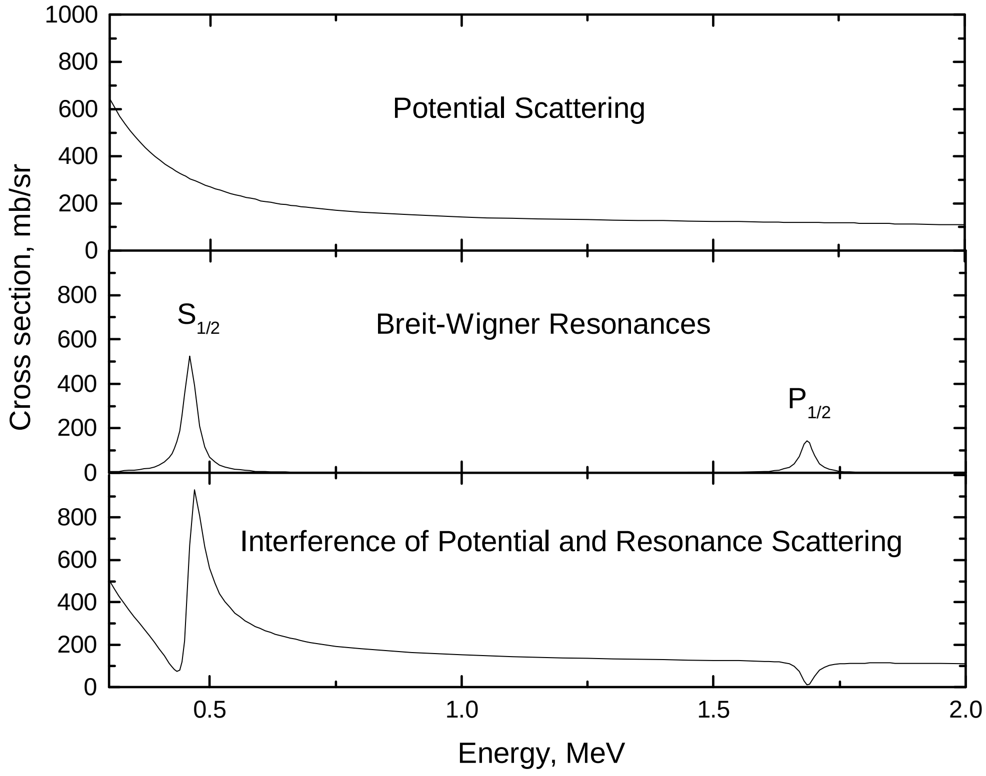
Resonances in proton elastic scattering:



RESONANCE SCATTERING

Modification of the scattering matrix element:

$$S_l^{\pm} = \exp(2i\lambda_l^{\pm})\left[\exp(-2\mu_l^{\pm}) + \exp(2i\phi_p)\frac{i\Gamma_p}{E_0 - E - \frac{1}{2}i\Gamma}\right]$$



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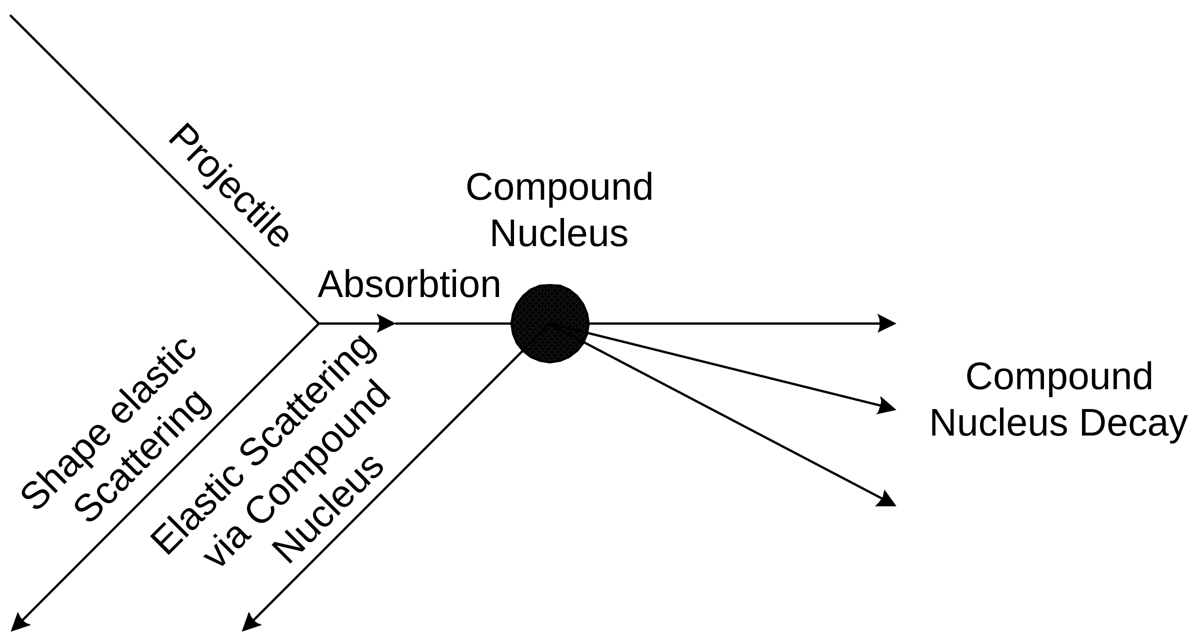
TYPES OF THE NUCLEAR INTERACTION MECHANISMS AT LOW ENERGIES



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DIFFERENT TYPES OF THE PROJECTILE – NUCLEUS INTERECTION

Elastic scattering: Outgoing particle = Ingoing particle

Outgoing particle energy = Ingoing particle energy

Direct reaction: Energy of the projectile is transferred to one nucleon or to a small group of nucleons.

Outgoing particle – any particle allowed by the conservation laws.

The interaction time is $\sim 10^{-22}$ s

Compound nucleus reaction:

Energy of the projectile is transferred to all the nucleons.

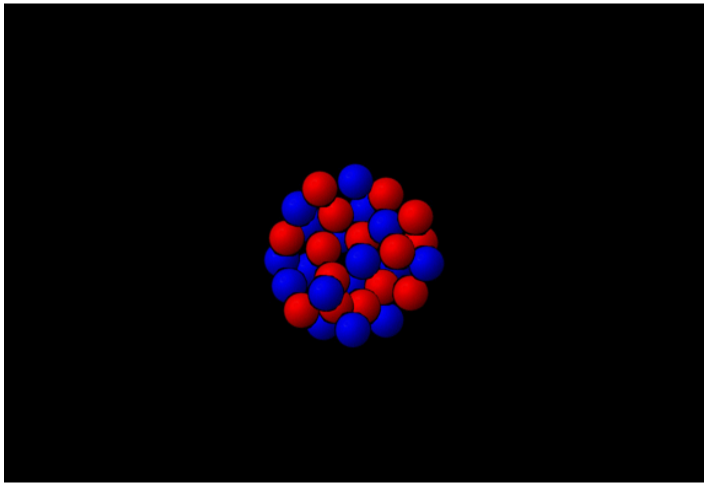
When particle is emitted the residual nucleus may stay both in ground state and in excited one.

The interaction time is $\sim 10^{-15}$ s.

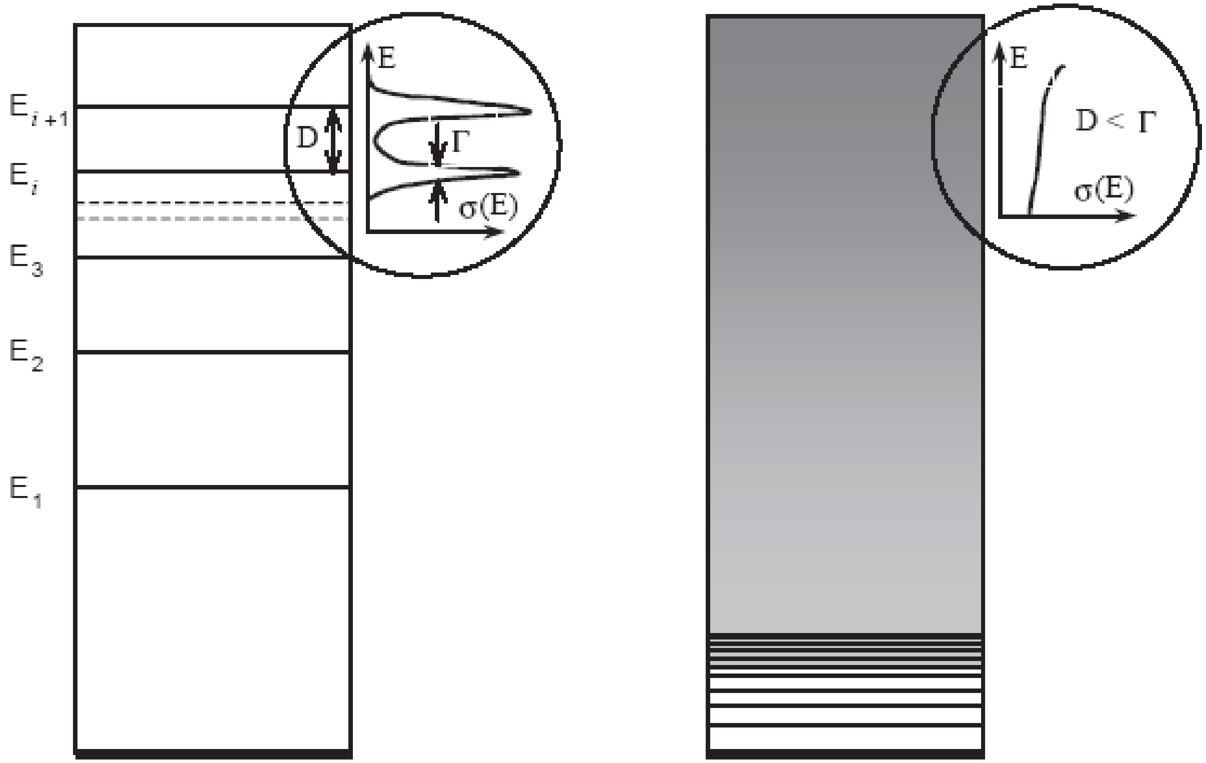
SCATTERING VIA COMPOUND NUCLEUS

An example: the (d,p)-reaction. The time the compound nucleus stays in an excited state is long as compared with the time needed for the projectile to traverse the domain occupied by the nucleus.

The reaction proceeds in two stages – the compound nucleus is created and in a while it decays.



NUCLEAR LEVELS DIAGRAM



Light nucleus

Heavy nucleus

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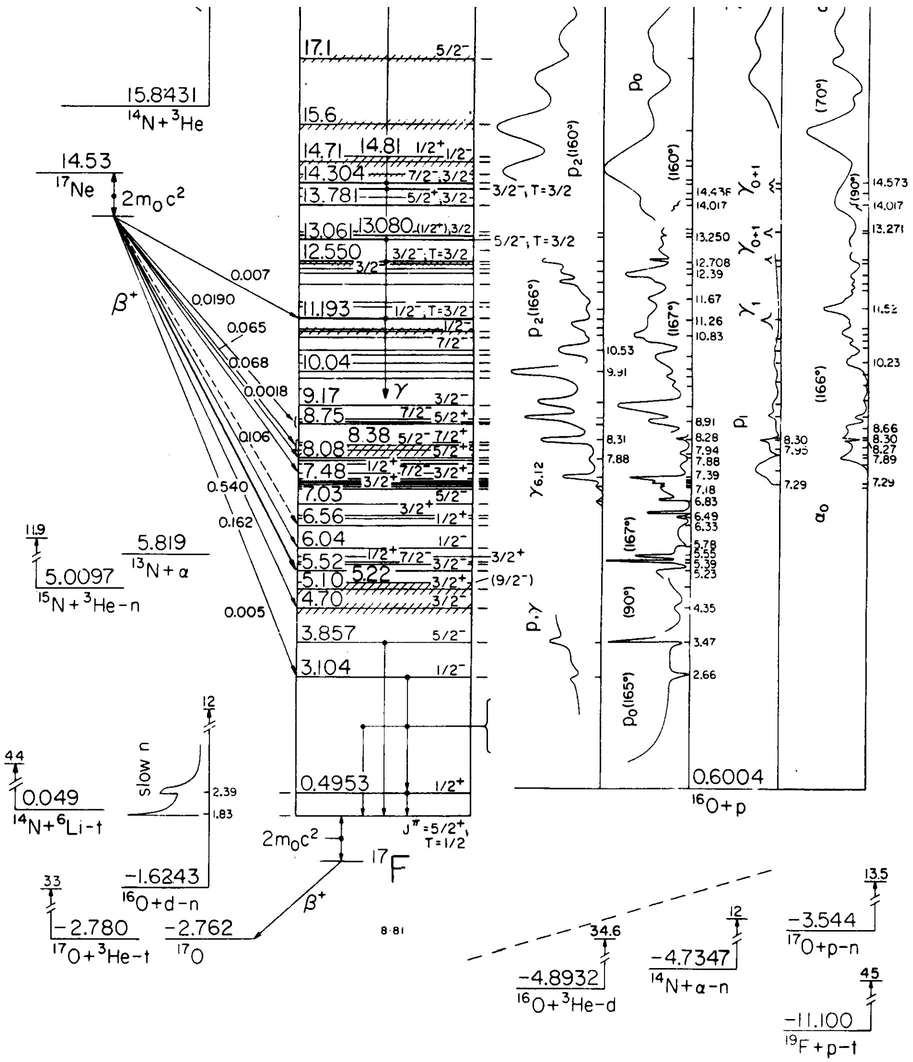
AJZENBERG-SELOVE'S COMPILATION

www.tunl.duke.edu/nucldata/fas/fas_list.shtml



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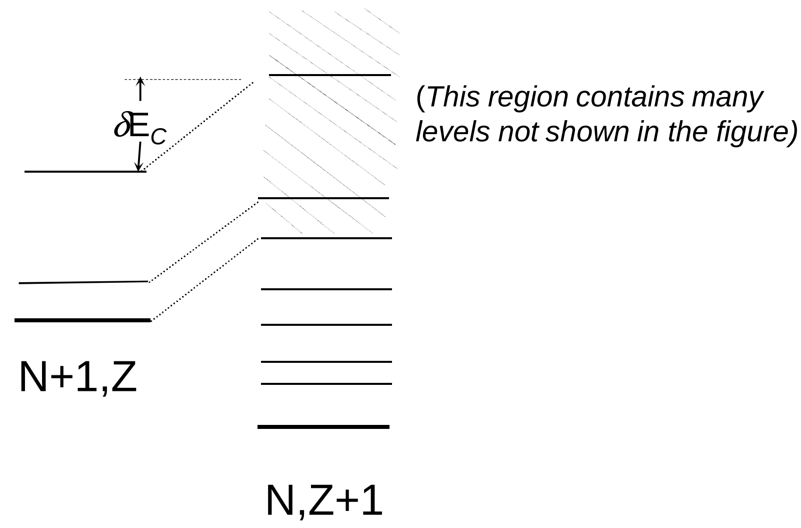
ISOBARIC ANALOG STATES

Isobare: Nuclide with the same masse (A) but different charge (Z)

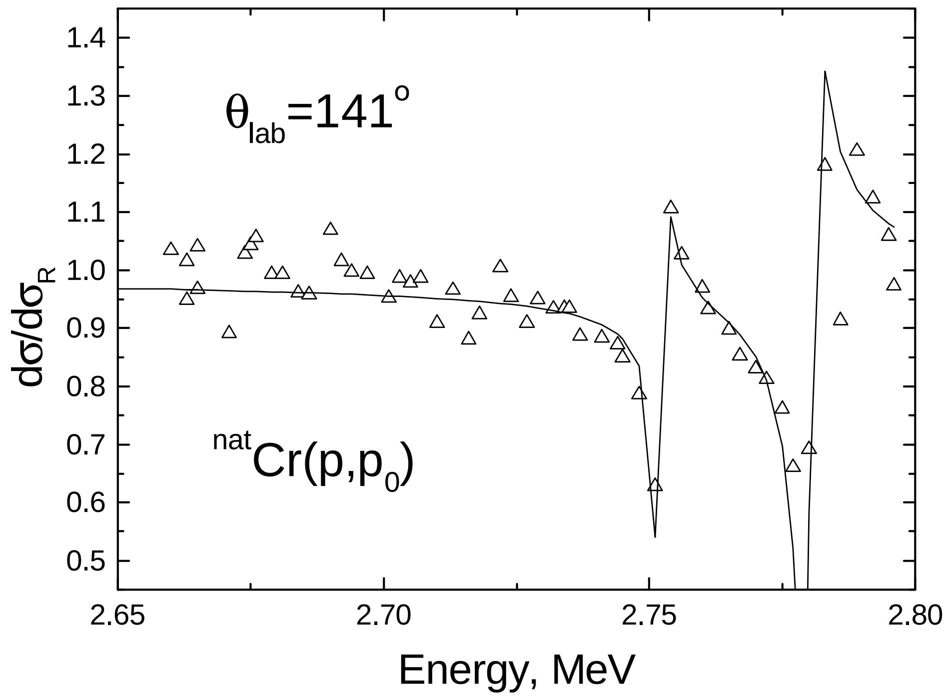
Except for electrical forces the n-n, n-p, and p-p interactions are similar

The energy levels of isobaric (equal A) nuclei are relatively insensitive toward the interchange of a proton and a neutron.

The isobaric analog state will have the same properties, but will have a higher energy, δE_C, because of the additional Coulomb energy associated with the extra proton, less the neutron-proton mass difference.



Isobaric Analog Resonances in the excitation function for proton elastic scattering



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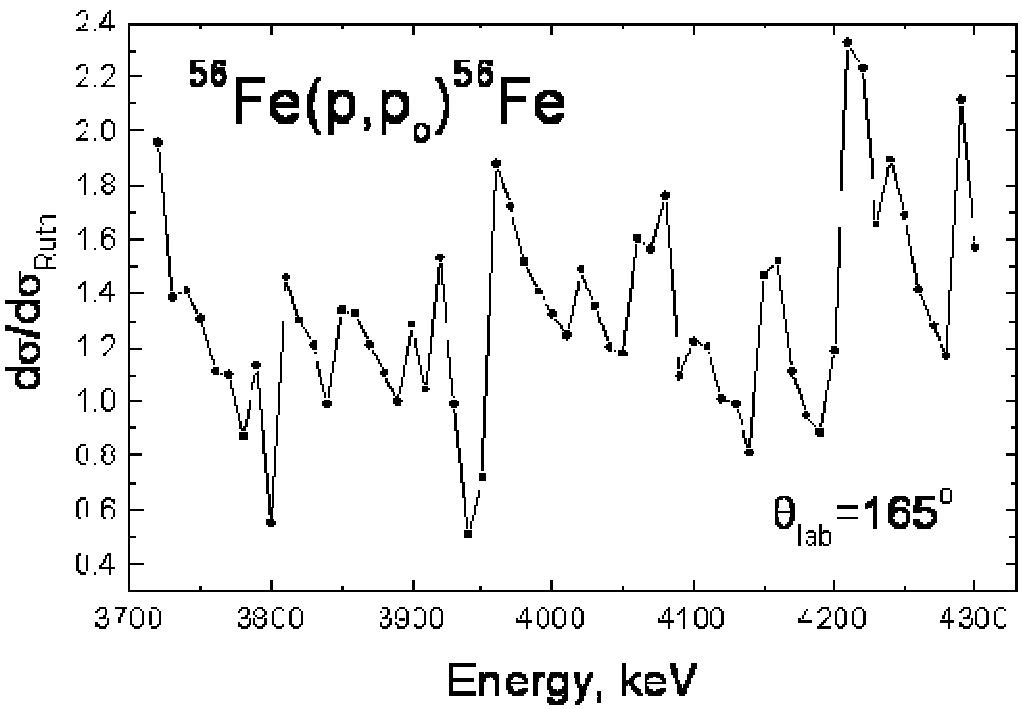
Ericson's fluctuations in the excitation function for proton elastic scattering from ^{56}Fe



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Statistical model calculations

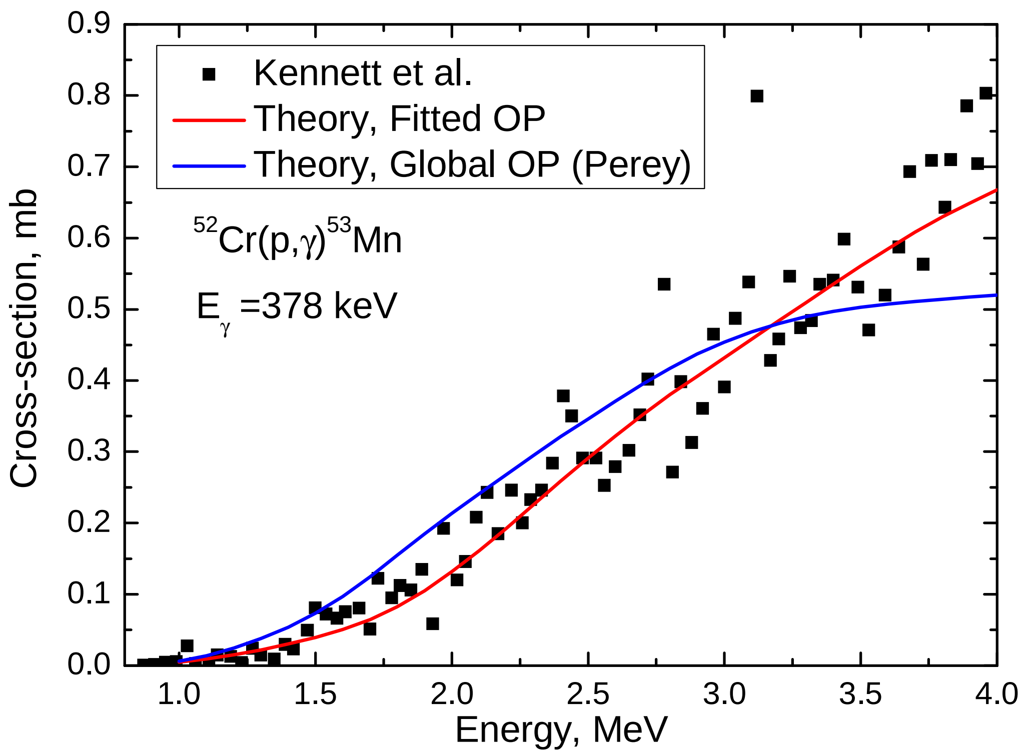
The cross-section for the formation of the compound nucleus:

$$\sigma_c(\varepsilon, I, P; U, J, \pi) = \frac{\pi}{k^2} \frac{2J+1}{(2I+1)(2i+1)} \sum_{S=|I-i|}^{|I+i|} \sum_{l=|J-S|}^{|J+S|} f_l(l, \pi) T_l^c(\varepsilon)$$

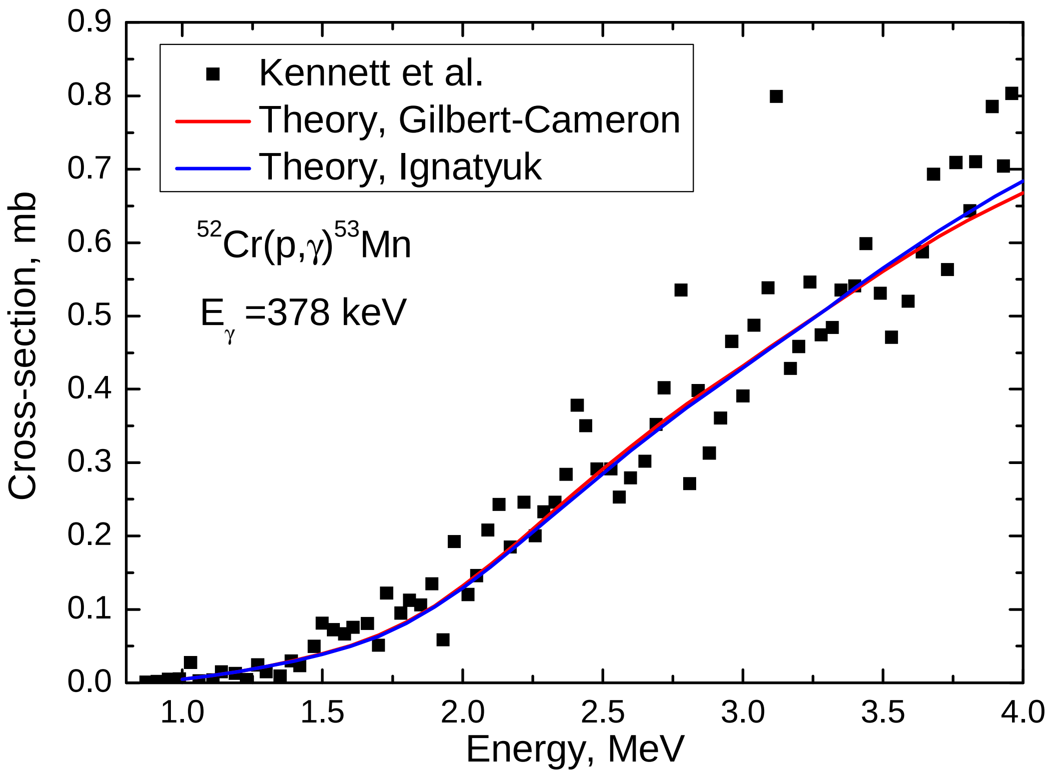
The cross-section for the decay of the compound nucleus:

$$\sigma_{c,c'}(\varepsilon, I, P; E', I', P') = \sum_{J,\pi} \sigma_c(\varepsilon, I, P; U, J, \pi) \frac{\Gamma_{c'}(U, J, \pi; E', I', P')}{\Gamma(U, J, \pi)}$$

Effect of the optical potential variation on the calculated cross-section



Effect of the level density model on the calculated cross-section



R-MATRIX THEORY

If a wave-function and its derivative are known at the boundary of the nucleus it can be found everywhere outside the nucleus

The *S* matrix is expressed through *R*-matrix which is defined to connect values u_l with its derivative at the nucleus boundary $u_l(a) = R_l a\left(\frac{du_l}{dr}\right)_{r=a}$

R_l can be expressed as $R_l = \sum_\lambda \frac{\gamma_{l,\lambda}^2}{E_\lambda - E}$

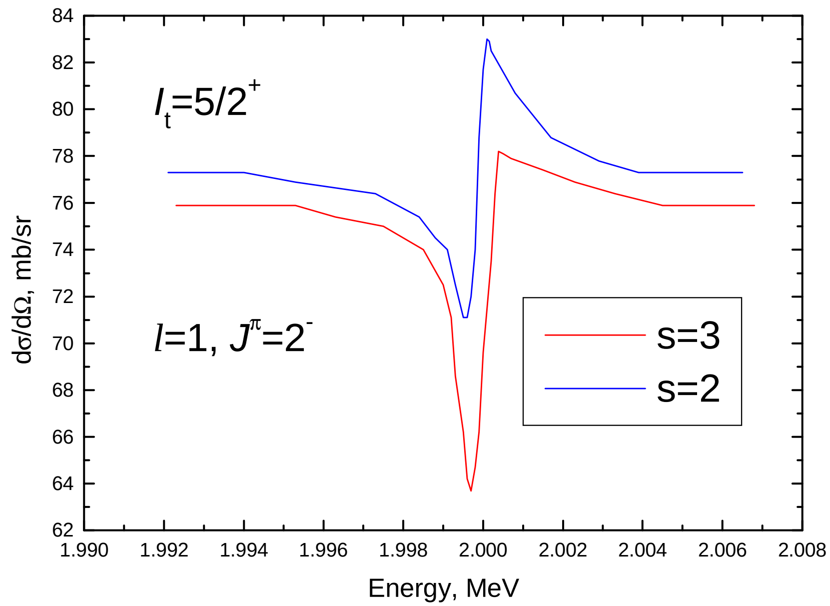
where $\gamma_{l,\lambda} = \left(\frac{\hbar}{2ma}\right)^{1/2} u_{l,\lambda}(a)$

The functions $u_{l,\lambda}$ correspond to actual states E_λ of the nucleus

The quantities $\gamma_{l,\lambda}$ are connected with energy width of states

The cross-section can be written in terms of the *R*-matrix

THE EFFECT OF THE CHANNEL SPIN ON A RESONANCE SHAPE



For a target of spin I_t and projectile of spin I_p the two spins are coupled to form a channel spin *s*. This channel spin is then combined with the relative orbital angular momentum *l* to form the spin of the compound nuclear state *J*.

Allowed combinations of quantum numbers for elastic proton scattering through the $J = 2^-$ level of the target with $I_t^\pi = 5/2^+$

l	*s*
1	2
1	3
3	2
3	3

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The Distorted Wave Born Approximation

The basic “ingredients” of a DWBA calculation:

1) Optical model potentials that describe the elastic scattering in the entrance and exit channels.

2) Wave functions and potentials that bind the transferred particle in the “donor” and the “acceptor” nuclei.

To calculate the internal wave functions some further information is needed:

1) The spin-parity (J_R^{π}) of the state of the “composite” nucleus

2) The angular momentum (L) of the transferred particle relative to the “core”

3) The number of nodes (N) in the radial wave function

DWBA

A transition amplitude for the reaction A(a,b)B is:

$$T = J \int d^3 r_b \int d^3 r_a \chi^{(-)}(\vec{k}_f, \vec{r}_b)^* \langle bB | V | aA \rangle \chi^{(+)}(\vec{k}_i, \vec{r}_a)$$

where

$\langle bB | V | aA \rangle$ is a form factor for the reaction (contains nuclear structure information)

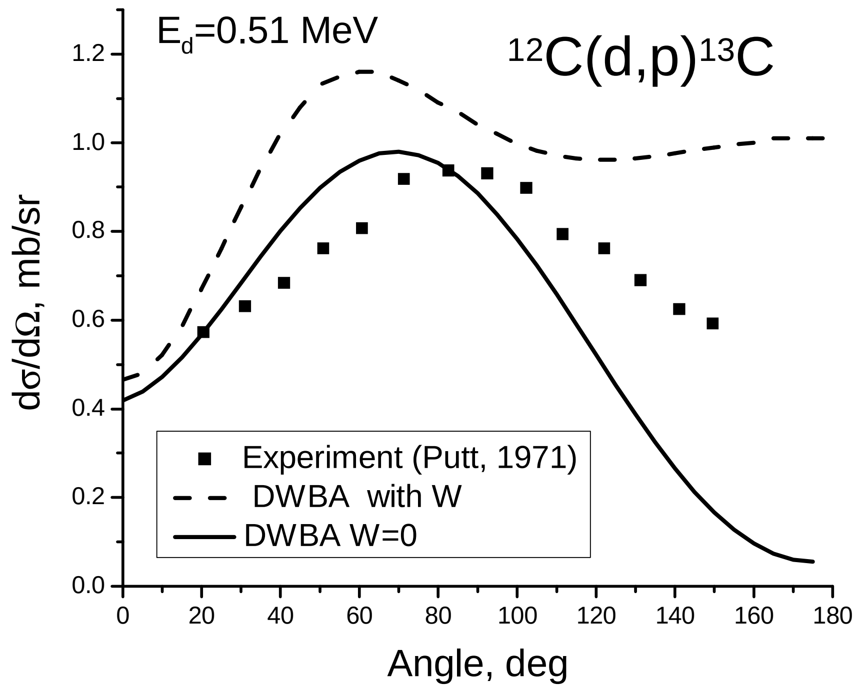
$\chi^{(+)}$ and $\chi^{(-)}$ are the distorted waves:

$$\chi^{(\pm)}(\vec{k}, \vec{r}) \rightarrow e^{i\vec{k}\cdot\vec{r}} + f(\theta)\frac{e^{\pm i\vec{k}\cdot\vec{r}}}{r}$$

The radial part of the distorted waves satisfies the equation

$$\left(\frac{d^2}{dr^2} + k^2 - \frac{L(L+1)}{r^2} - \frac{2\mu}{\hbar^2}\left[U(r) + U_C(r) + U_{Ls}(r)\mathbf{L}\cdot\mathbf{s} \right] \right) \chi_{JLs}(k,r) = 0$$

DWBA calculations



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PROTON INDUCED REACTIONS

• (p,α)-reaction

– are mainly exoergic (Q~1-3 MeV)
– the cross section is not large (because of high Coulomb barrier for α-partile)

• (p,n)-reaction

– are always endoergic (Q < -0.8 MeV)

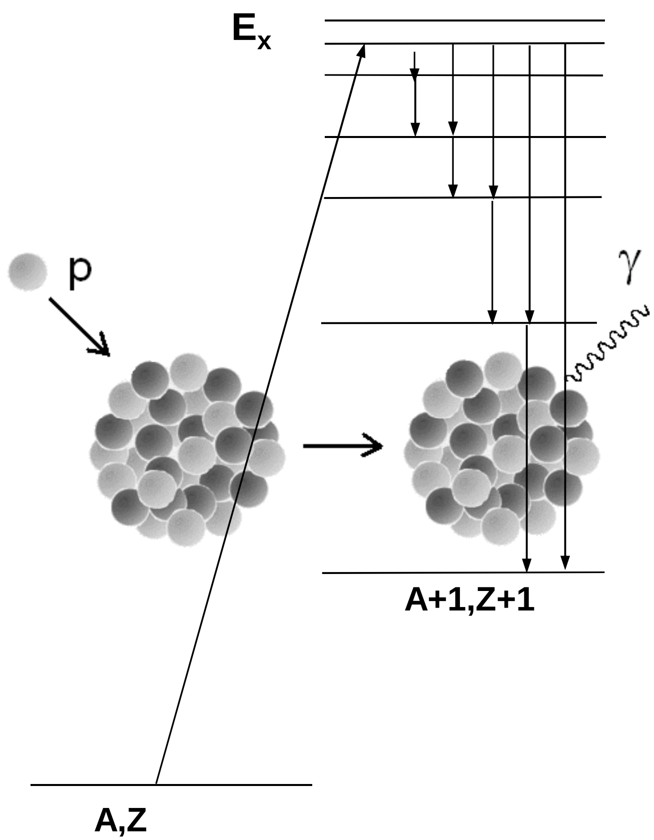
• (p,γ)-reaction

– γ-rays of different energy are emitted corresponding to transitions on different levels of the residual nucleus

• (p,p'γ)-reaction

–is only possible if the projectile energy exceeds the first level energy in the target nucleus

RADIATIVE CAPTURE OF PROTON



$$E_x = Q + \frac{M_A}{M_A + m_p} E_p$$

$$E_\gamma^{(i)} = E_x - E_{level}^{(i)}$$

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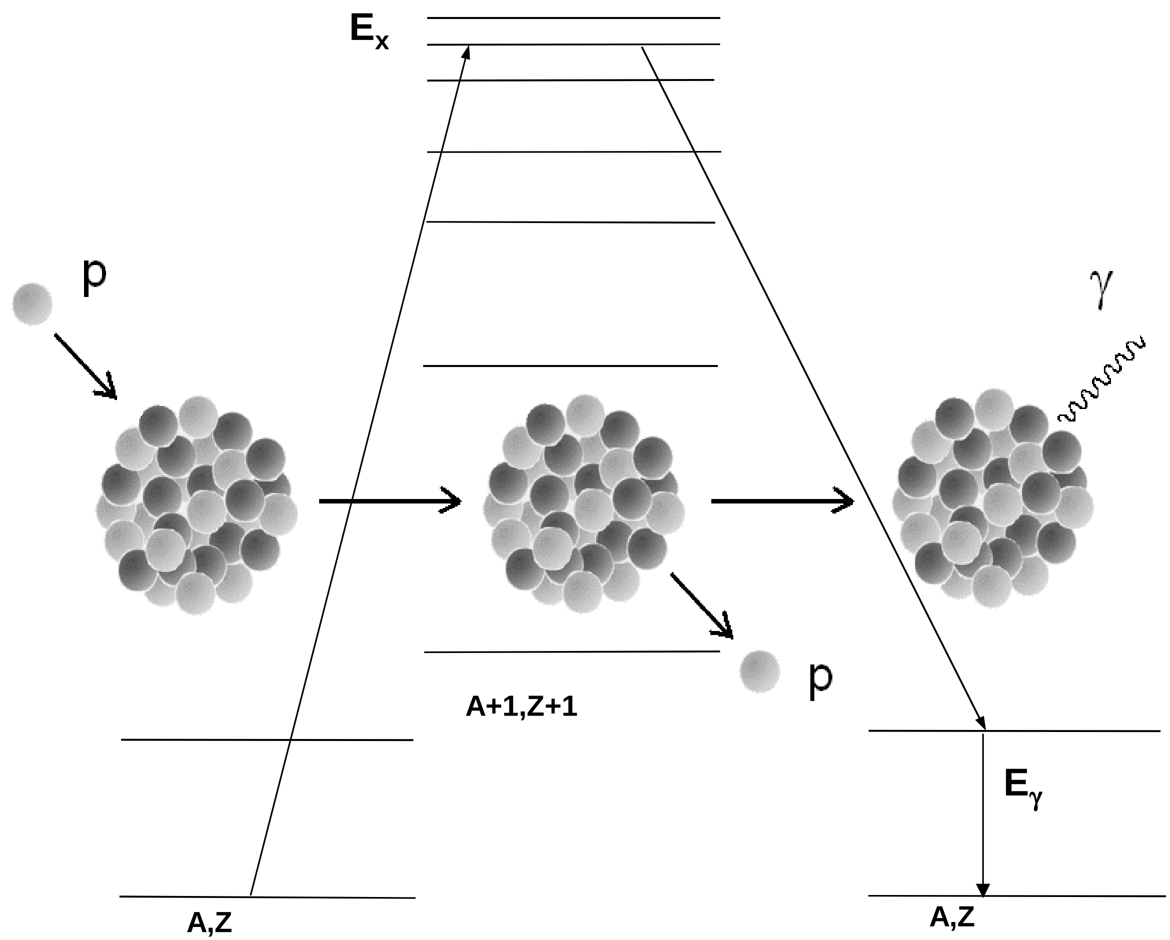
Scheme of the (p,p′γ)-reaction



Multipolarity

Multipolarity - Electric (E) or magnetic (M) classification of a gamma ray transition that carries of L units of angular momentum. Values vary from M1-M5 and E0-E5 where E0 transitions can only occur by internal conversion or pair production processes. Transitions of mixed multipolarity, e.g. M1+E2 are common.

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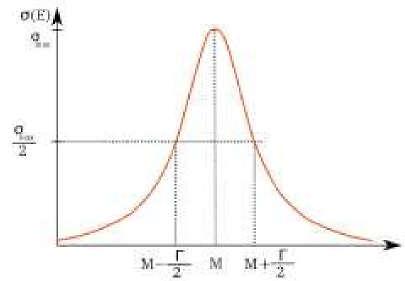
Isolated resonance – theoretical expressions for spin zero case

The resonance yield per unity solid angle and unity incident particles charge for prompt gamma-rays emission from homogenous target with energy thickness of ΔE_T is defined as

$$Y = \frac{N_0}{A} \int_{E-\Delta E_T}^{E} \frac{\sigma(p,\gamma)}{dE/dx} dE,$$

where N_0 - is Avogadro constant, A - is a molecular mass.



The cross-section for the transition between α and β states is

$$\sigma_{\alpha,\beta} = \pi \bar{\lambda}^2 (2J+1) \frac{\Gamma_{\alpha,l}\Gamma_{\beta,L}}{(E-E_0)^2 + \left(\frac{\Gamma}{2}\right)^2} = \pi \bar{\lambda}^2 \omega\gamma \frac{\Gamma}{(E-E_0)^2 + \left(\frac{\Gamma}{2}\right)^2},$$

where L – multipolarity, l – angular momentum of the projectile, and the resonance strength is defined as

$$\omega\gamma = (2J+1)\frac{\Gamma_{\alpha,l}\Gamma_{\beta,L}}{\Gamma}.$$

The resonance strength can be derived from a step Y_∞ in the thick target yield

$$\omega\gamma = \frac{2}{\lambda^2}\frac{M}{m+M}\left(\frac{dE}{dx}\right)Y_\infty$$

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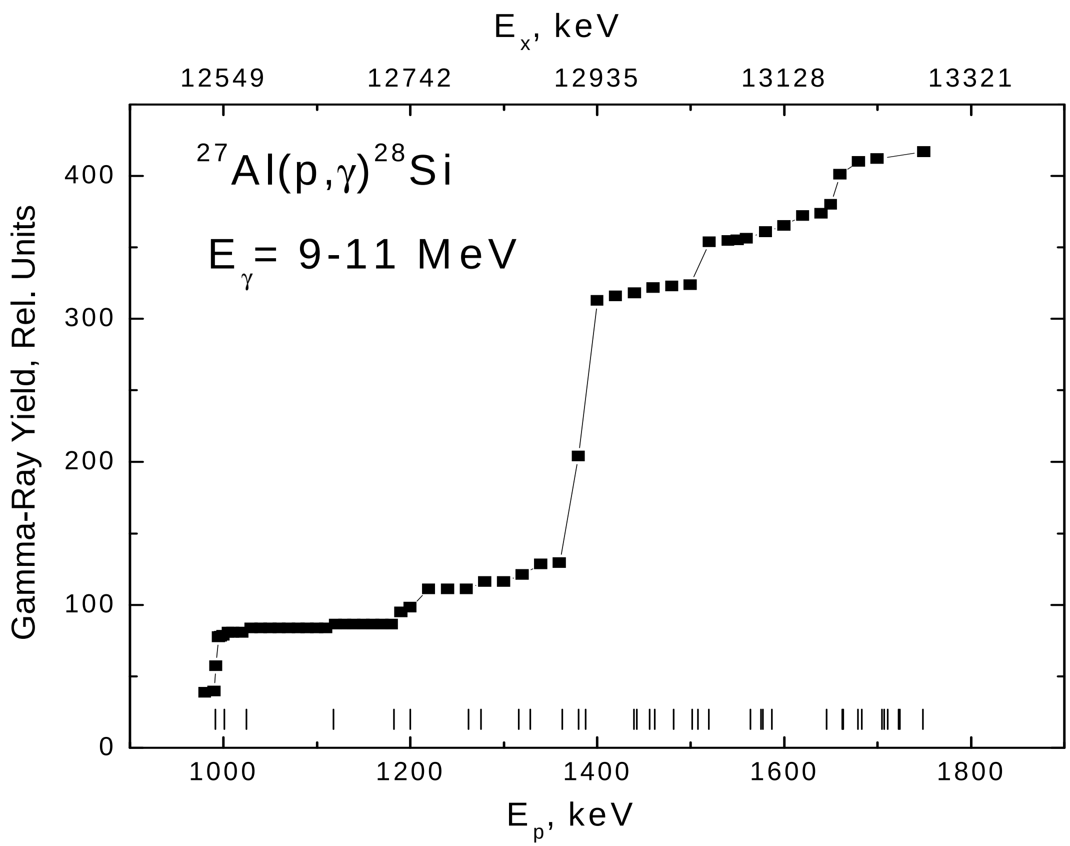
Thick-target yield of gamma rays from the $^{27}Al(p,\gamma)^{28}Si$ reaction



Resonance energies are indicated by bars

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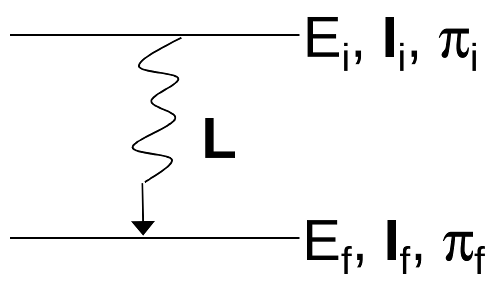
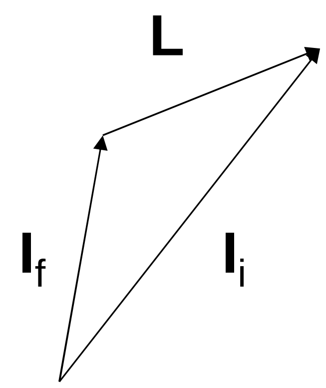
Selection rules

- Each photon carries a definite angular momentum **L**>0
- Dipole: L=1, quadrupole: L=2, ...



$\mathbf{I}_i = \mathbf{L}+\mathbf{I}_f$:



Selection rules:

$|I_i - I_f| \leq L \leq I_i + I_f$

$\Delta\pi$ = no: even electric (E2, E4), odd magnetic (M1,M3)

$\Delta\pi$ = yes: odd electric (E1,E3), even magnetic (M2,M4)

DEUTERON INDUCED REACTIONS

Three mechanisms:

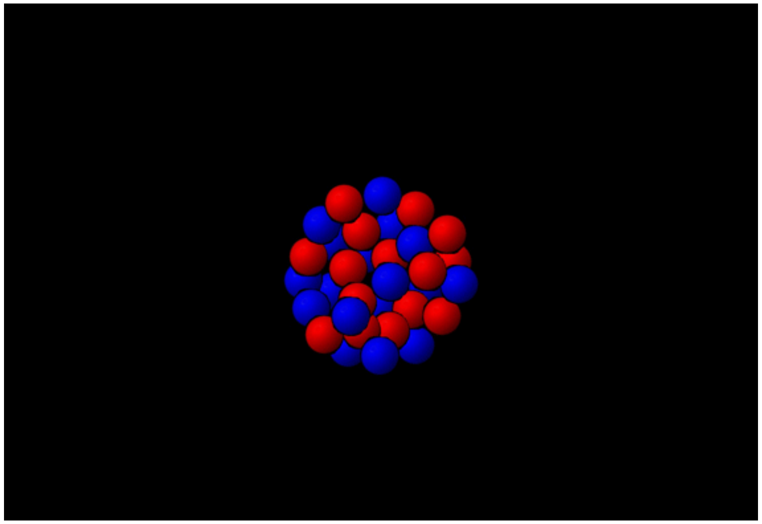
- Direct stripping (with amplitude of *D)*
- Resonant mechanism (with amplitude of *R)*
- Compound nucleus mechanism

The total amplitude of the process is $D + R$

DEUTERON INDUCED REACTIONS (STRIPPING)

For deuteron the p-n distance is ~$4 \cdot 10^{-13}$ cm (for the rest of nuclei ~ $2 \cdot 10^{-13}$ cm)

Due to electrical forces deuteron is oriented in such a way that proton is farther from the nucleus than neutron. Because deuteron binding energy is small, neutron may be absorbed by nucleus, while proton keeps moving



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DEUTERON INDUCED REACTIONS (VIA COMPOUND NUCLEUS)

- **Binding energy for deuteron is very low:**

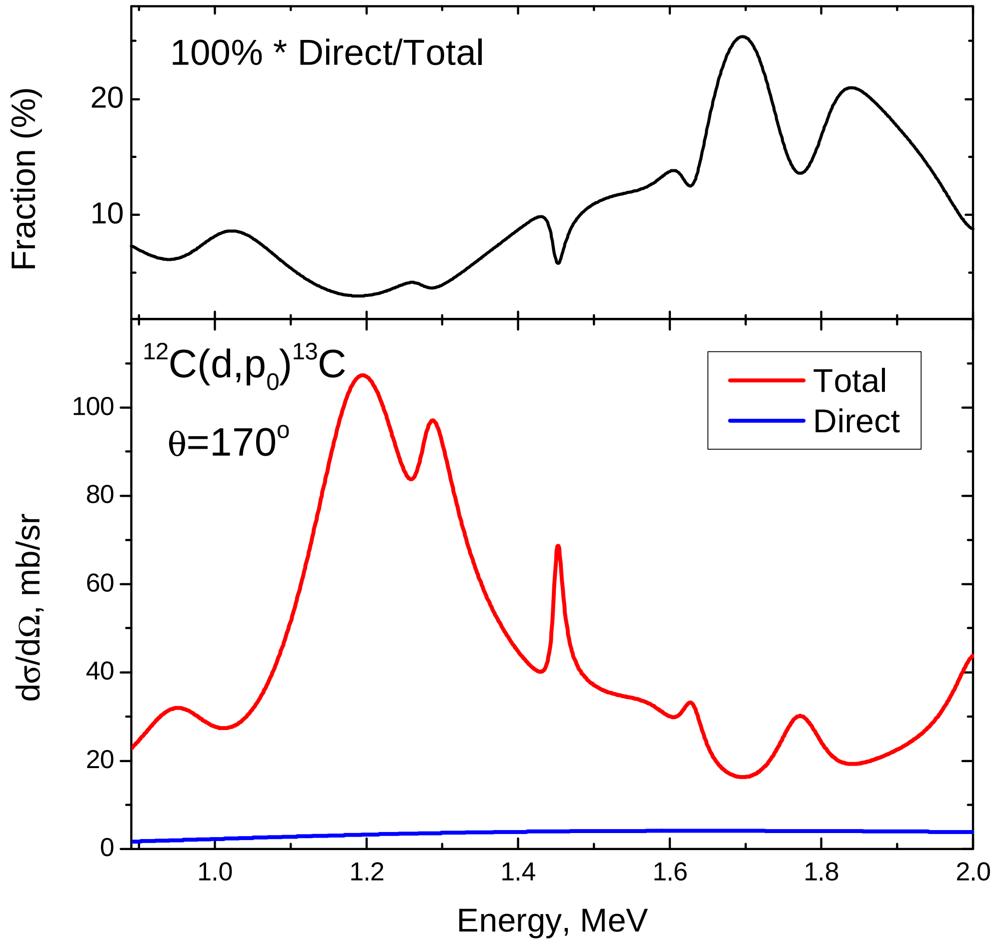
  $E_B \approx 2.2$ MeV -> 1 MeV/Nucleon (For the rest of nuclei ~8 MeV/Nucleon)

- **Since the binding energy of deuteron in a compound nucleus is**

  $E_B(A,Z)-E_B(A-2,Z-1)-E_B(d) \approx 8A - 8(A-2) - 2,2 \approx 14$ MeV

  the excitation energy of the compound nucleus is of order T+14 MeV and all the reactions (d, p), (d, n), (d, α) are possible and are highly exoergic.

Contribution of Direct Reaction Mechanism



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(p,α)-reactions

- The (p,α)-reactions are mainly exoergic (Q~1-3 MeV).
- Because of the high Coulomb barrier for a-particles the cross-section is not large
- For the (p,α)-reactions which lead to the excited states in the residual nucleus the cross-section for low energy protons is as a rule negligible.

Alphas as projectiles

- The gamma-emission mechanism resembles that for protons.
- The alpha-particle is a strongly bound system the reactions with nucleons in the exit channel are endoergic.
- The compound nucleus produced as a result of the capture of a low energy alpha particle can decay mainly back to the elastic channel or by emitting gammas.
- A threshold for the (α,p)-reaction is usually greater than 1 MeV and due to the Coulomb barrier the cross section for (α,p)-reaction is as a rule small.
- Different mechanisms contribute to the alpha elastic scattering cross section beyond the energy region where it follows the Rutherford law. These are direct (shape elastic) scattering, compound elastic scattering, resonance scattering of a different origin, and exchange processes which consist in exchange of nucleons between alpha-particle and target nucleus in the course of scattering.

Laboratory and centre of mass frames

The laboratory frame of reference is a frame where detector is placed.

In IBA experiments target nucleus is always in rest in this frame.

For a projectile of the mass M_1 moving along an x axis towards a target nucleus of the mass M_2 the point with a coordinate x_C defined as

$$x_C = \frac{M_1 x_1 + M_2 x_2}{M_1 + M_2}$$

where x_1 and x_2 are the projectile and the target coordinates respectively is a centre of mass (CM) of the system comprised of these two particles.

This point moves in the laboratory reference frame with the velocity

$$\vec{V}_C = \frac{M_1 \vec{v}_1}{M_1 + M_2}$$

where $\vec{v}_1$ is the projectile velocity. The centre of mass reference frame is defined as a frame with an origin fixed in the point x_C.

König's theorem

The kinetic energy of a system consisted of a projectile and a target is the kinetic energy associated to the movement of the center of mass and the kinetic energy associated to the movement of the particles relative to the center of mass.

For the projectile possessing the kinetic energy E_1 (in the laboratory frame) this means that

$$E_1 = \frac{(M_1 + M_2)V_C^2}{2} + E_{rel}$$

where E_{rel} is the kinetic energy of colliding particles in their relative motion in the CM system:

$$E_{rel} = \frac{M_2}{M_1 + M_2} E_1$$

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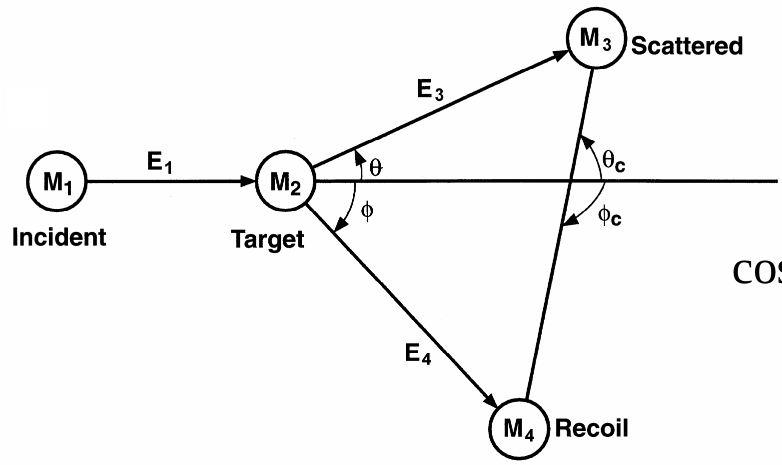
Kinematics of elastic scattering



$$\tan\theta_{lab} = \frac{\sin\theta_{cm}}{\gamma_3 + \cos\theta_{cm}}$$

$$\cos\theta_{cm} = [\cos^2\theta_{lab}(1-\gamma_3^2\sin^2\theta_{lab})]^{1/2} - \gamma_3\sin^2\theta_{lab}$$

$$\gamma_3 = \left(\frac{M_1M_3}{M_2M_4}\right)^{1/2}\left(1+\frac{M_1+M_2}{M_2}\frac{Q}{E_1}\right)^{-1/2}$$

$$K = \frac{E_3}{E_1} = \frac{\left\{(M_1/M_2)\cos(\theta) + [1-(M_1/M_2)^2\sin^2(\theta)]^{1/2}\right\}^2}{(1+M_1/M_2)^2}$$

Cross-section lab-c.m. transformation

The relation between the differential cross-sections expressed in the CM and laboratory frames is derived from the equality of the number of particles emitted in the corresponding solid angles in the two frames:

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} d\Omega_{lab} = \left.\frac{d\sigma}{d\Omega}\right|_{cm} d\Omega_{cm}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} \sin\theta_{lab} d\theta_{lab} d\varphi_{lab} = \left.\frac{d\sigma}{d\Omega}\right|_{cm} \sin\theta_{cm} d\theta_{cm} d\varphi_{cm}$$

Nuclear physics Internet resources relevant to IBA

The resource address	**Description**
http://www.nndc.bnl.gov/qcalc/	Q-value Calculator
http://nucleardata.nuclear.lu.se/database/masses/	Nuclear structure and decay data NuBase with the Q-value calculator
http://www.nndc.bnl.gov/ensdf/	Evaluated nuclear structure data file (ENSDF)
http://www.nndc.bnl.gov/nudat2/	Nuclear structure and decay data
http://nrv.jinr.ru/nrv/	Low energy nuclear knowledge base
http://www.nndc.bnl.gov/masses/mass.mas03	Atomic mass adjustment 2003
http://www.tunl.duke.edu/NuclData/	Energy levels of light nuclei, A=3-20
http://www-nds.iaea.org/ibandl/	IBA nuclear data library (IBANDL)
http://www-nds.iaea.org/exfor/	Experimental nuclear reaction data (EXFOR)
http://www.surreyibc.ac.uk/sigmacalc/	Evaluated differential cross sections for IBA (SigmaCalc)

Summary

<u>We have discussed:</u>

What nucleus is.

How a projectile interacts with a nucleus.

How quantum mechanics depicts the projectile-nucleus interaction.

What mechanisms of nuclear reactions exist.

How theoretical models are applied to describe the projectile-nucleus interaction.

Nuclear reaction kinematics