Introduction to space-time clustering

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School on Weather Regimes and Weather Types in the Tropics and Extra-tropics:
Theory and Application to Prediction of Weather and Climate

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23 October 2013

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Space-Time Clustering

This talk: Introduction to Hidden Markov Models (HMM)

Next talk: - Application of HMMs
- Introduction to non-stationary FEM clustering

Next week (workshop): Application of FEM clustering

Outline

- Motivation

- Markov Chain

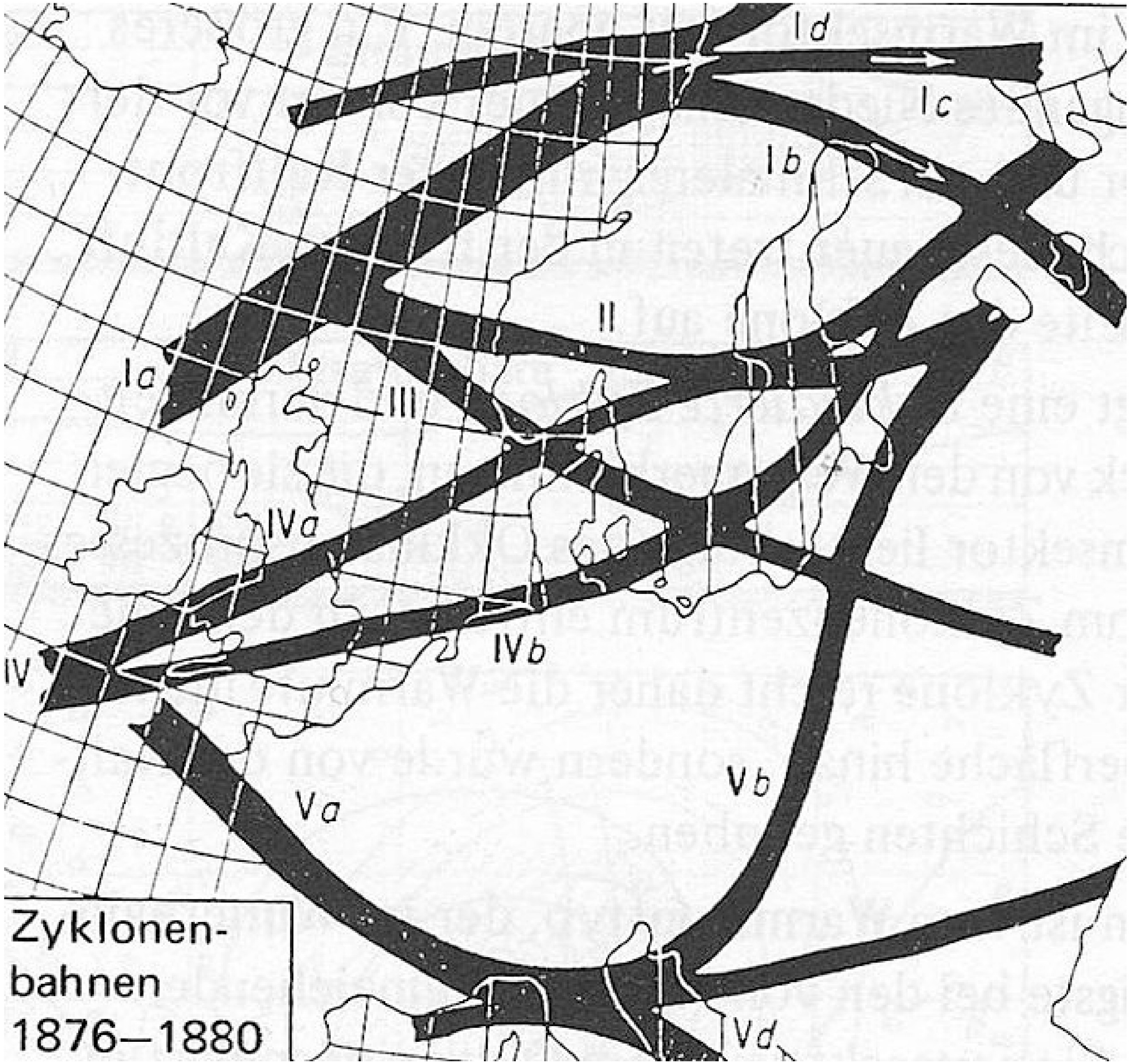
- Hidden Markov Models

- Gaussian Mixtures

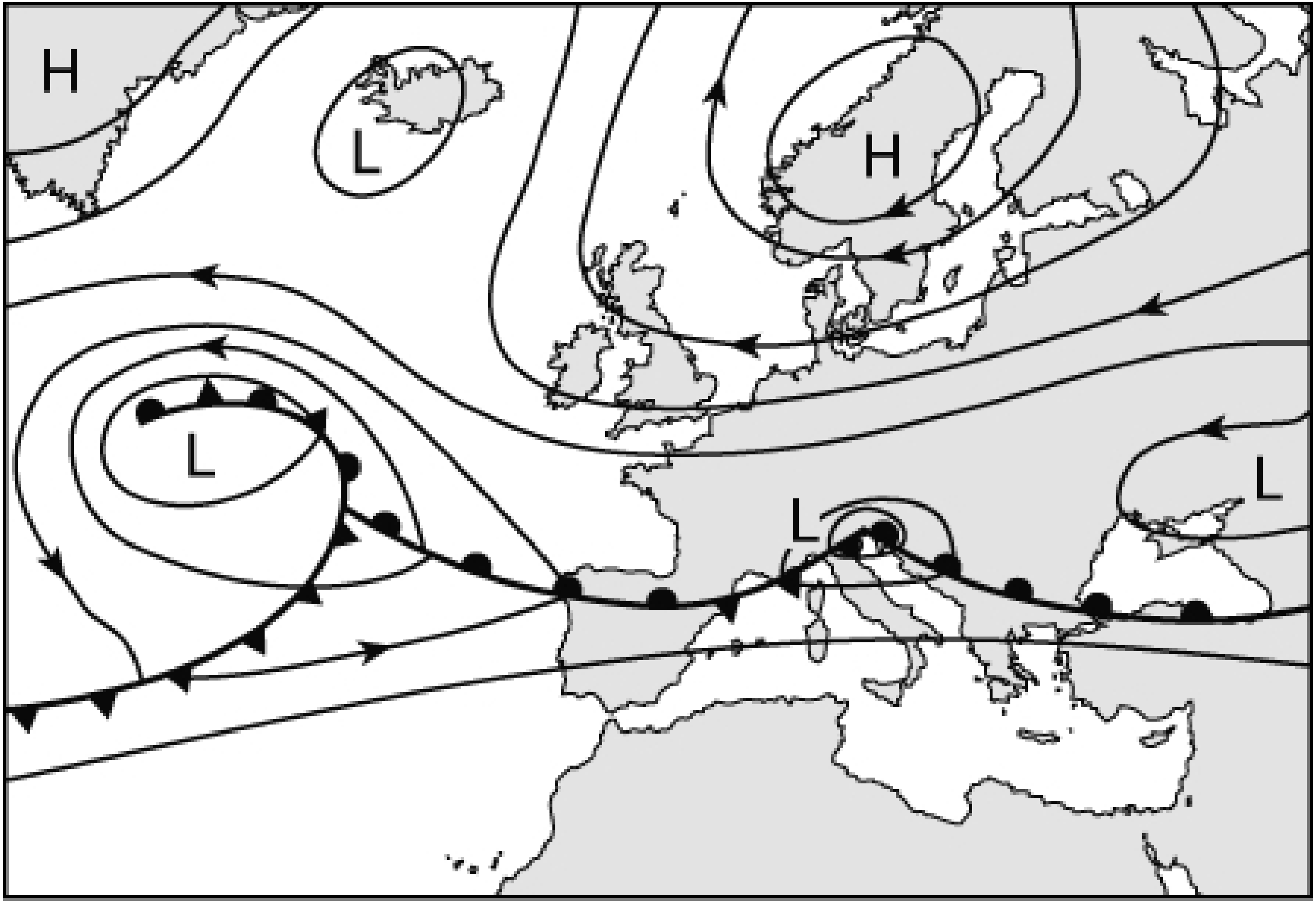
- Number of Regime States

- Markovianity

Grosswetterlagen



Source: W. J. Van Bebber 1891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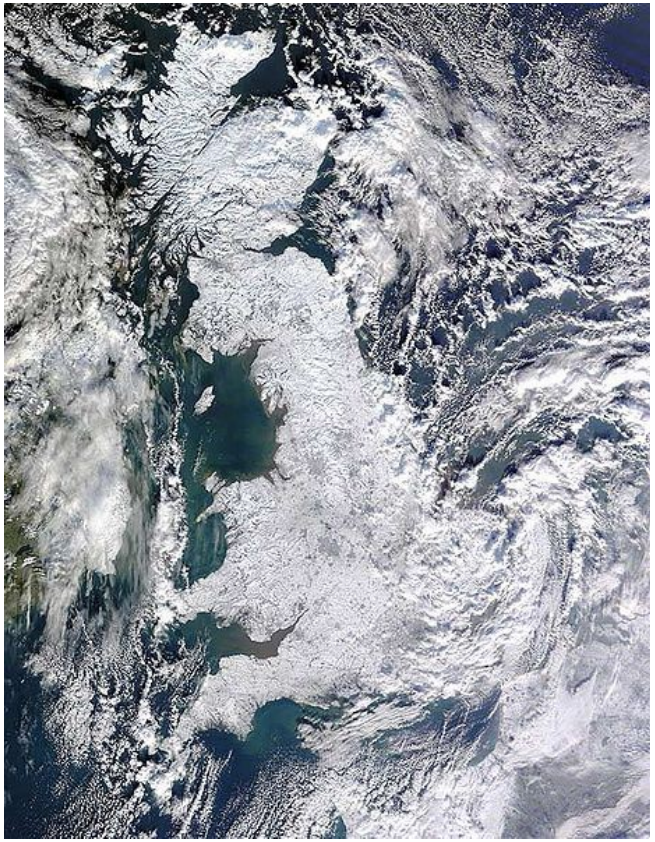
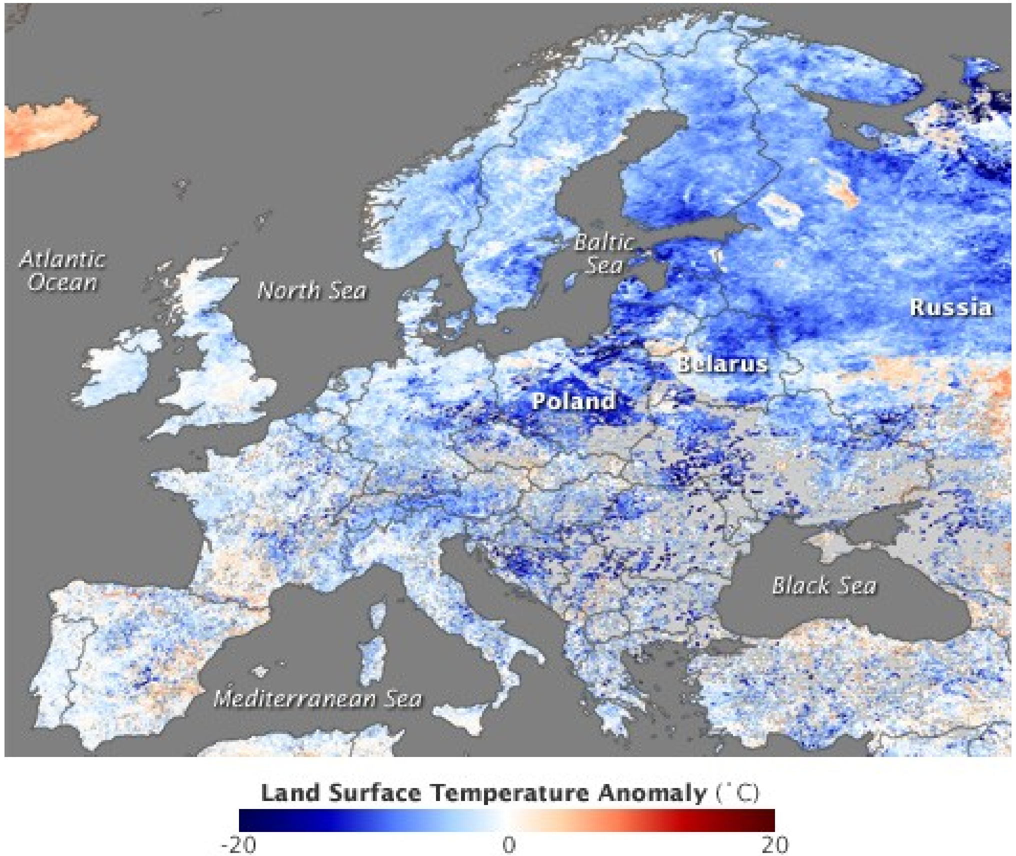
Impact of Blocking



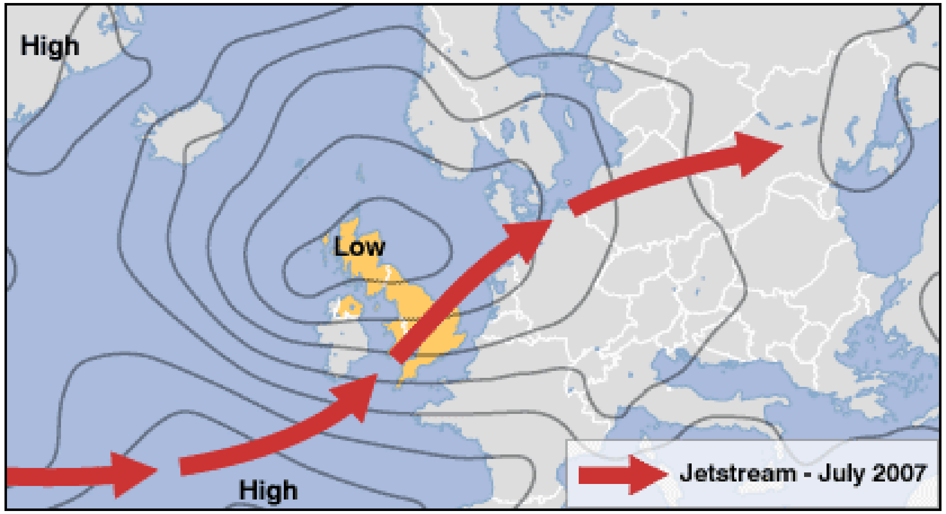
Snow cover across UK 7 January 2010
Source: NASA

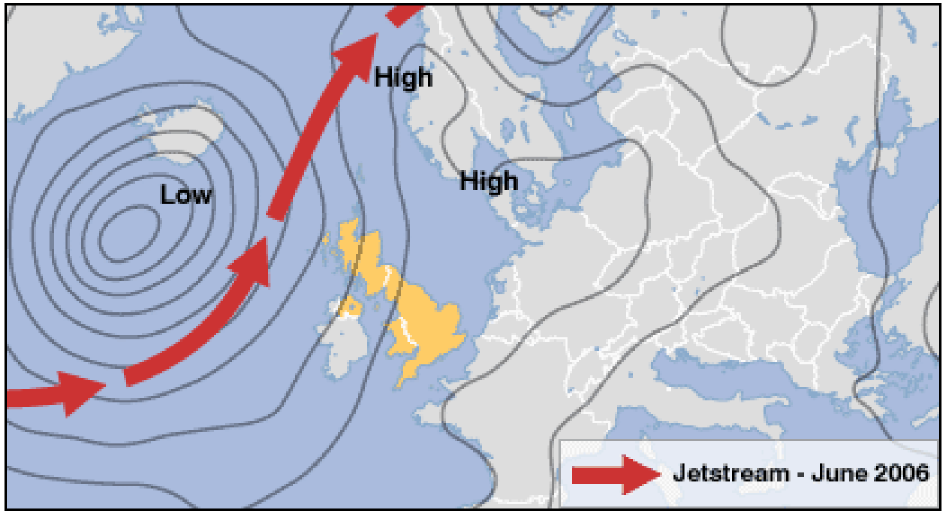


Difference of temperature between December 11–18, 2009 and the 2000–2008 average.
Source NASA

Impact of North Atlantic Jet Stream

2007 UK Floods

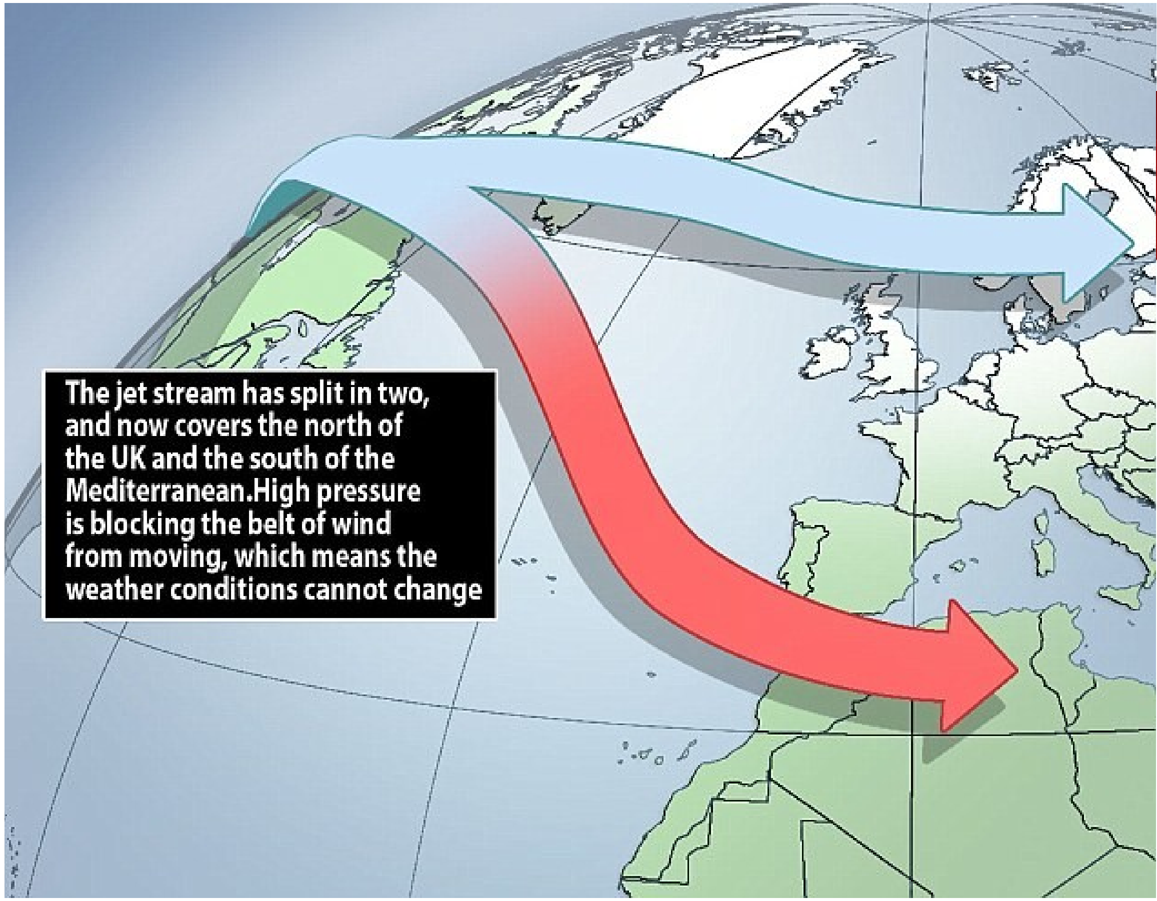




Source: MetOffice

Blackburn et al. 2008

Persistent Weather Events

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18 October 2012 Last updated at 18:16

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UK experiences 'weirdest' weather

By Roger Harrabin
Environment analyst

PA

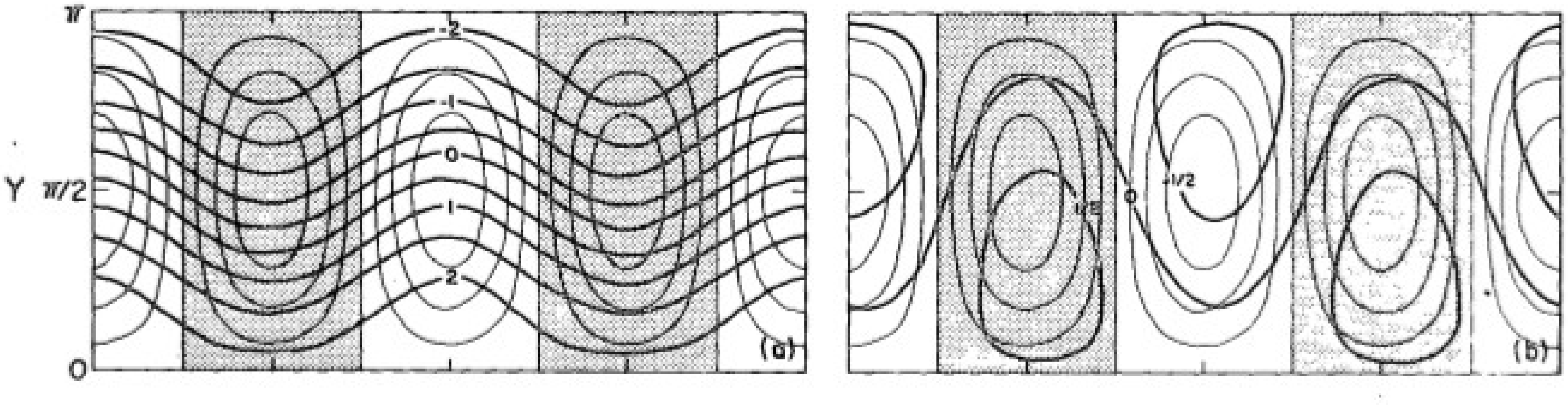
A dry spring gave way to wet conditions - a rapid transformation

The UK has experienced its "weirdest" weather on record in the past few months, scientists say.

Related Stories

Regimes as Steady State Solutions

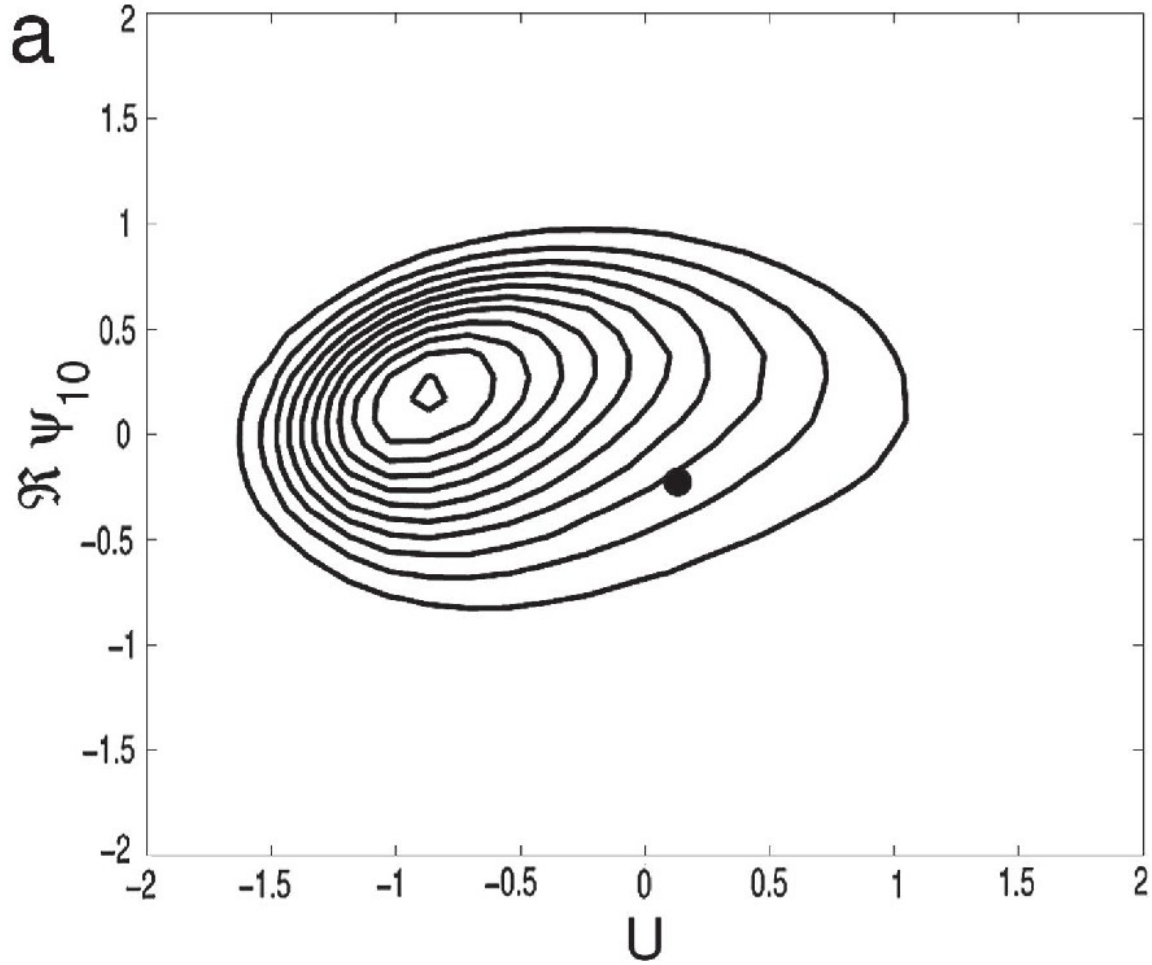
- Low-order truncation of barotropic model
  (only 3 Fourier modes; Charney-DeVore model)
- Topography and thermal forcing
- 2 types of steady states



Source: Charney and DeVore 1979

Steady States and PDF maxima

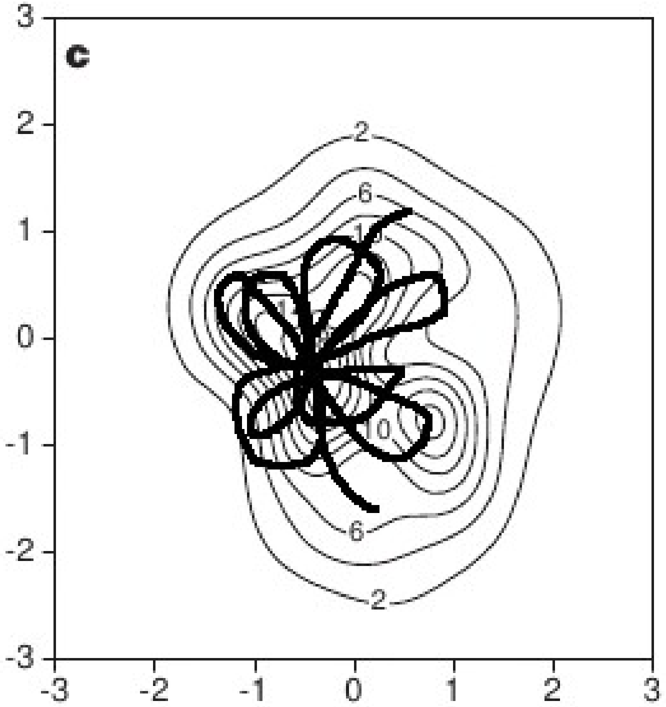
Charney-DeVore model with weather waves



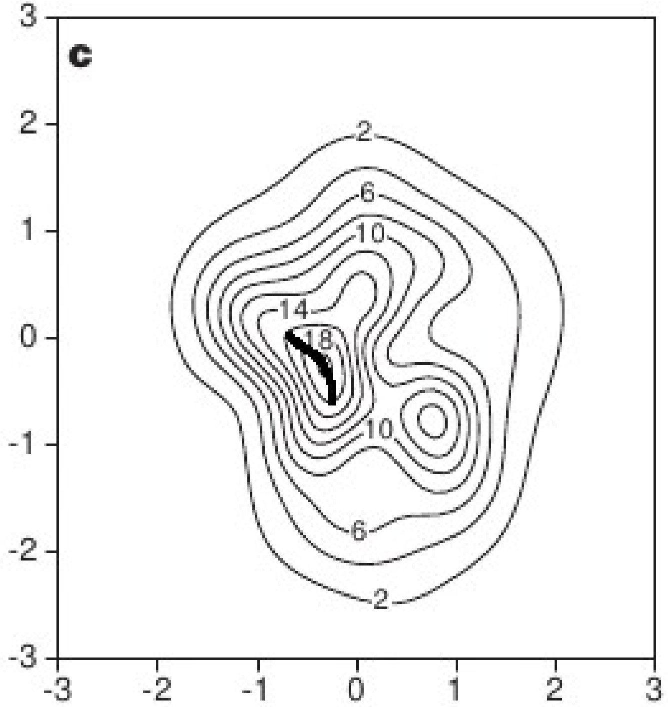
56 mode barotropic model; low-order truncation is equivalent to Charney-DeVore model
Majda et al. 2006, PNAS; Reinhold and Pierrehumbert 1982, Tung and Rosental 1985

Recurrent or Persistent?

Recurrent



Persistent



More Predictive Skill

Outline

- Motivation

- Markov Chain

- Hidden Markov Models

- Gaussian Mixtures

- Number of Regime States

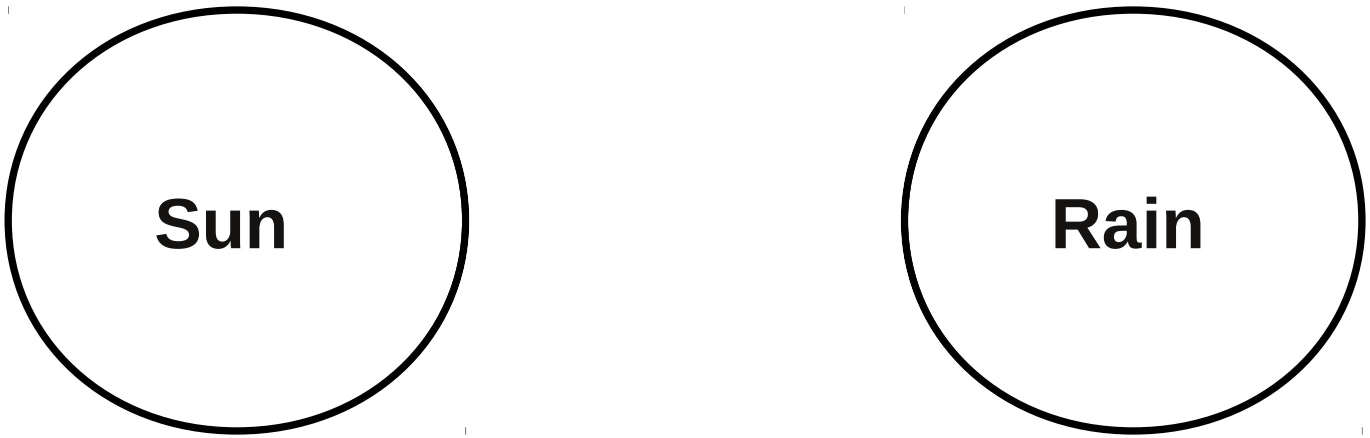
- Markovianity

Markov Chain

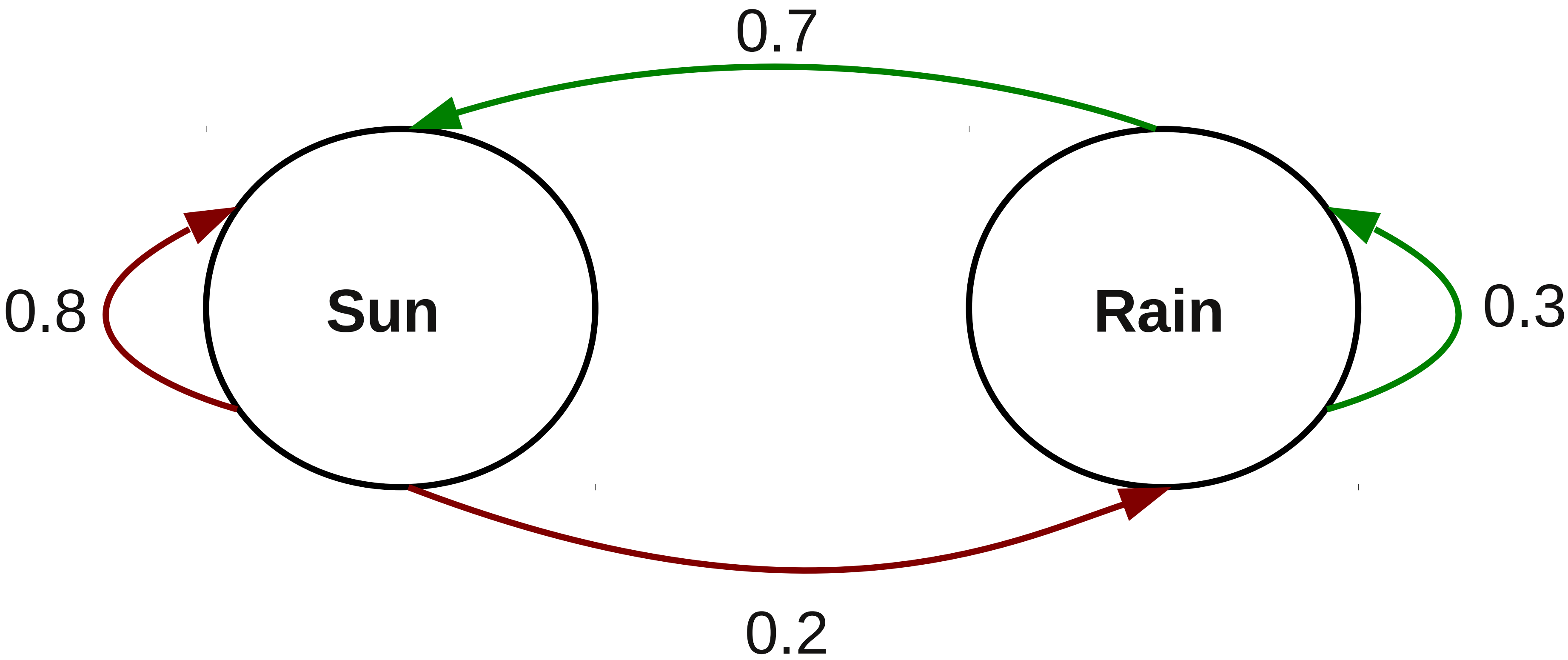
Weather Evolution in Trieste

Sun Rain Rain Sun Sun Sun Sun Rain Sun Sun Sun Sun Sun Sun
Sun Rain Rain Sun Sun Sun Rain Sun Sun Rain Rain Sun Sun Sun

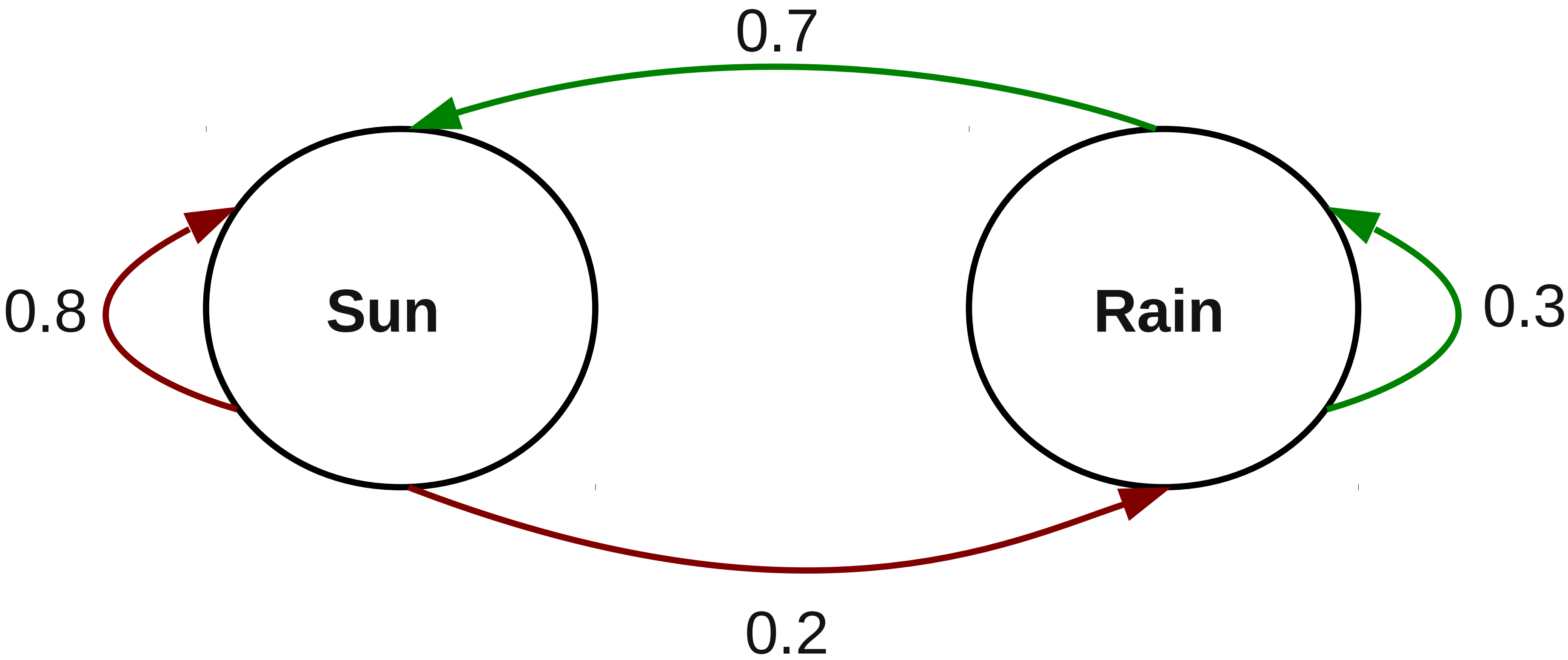
Markov Chain



Markov Chain



Markov Chain



$$x_{t+1} = P x_t ; P = \begin{bmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{bmatrix}$$

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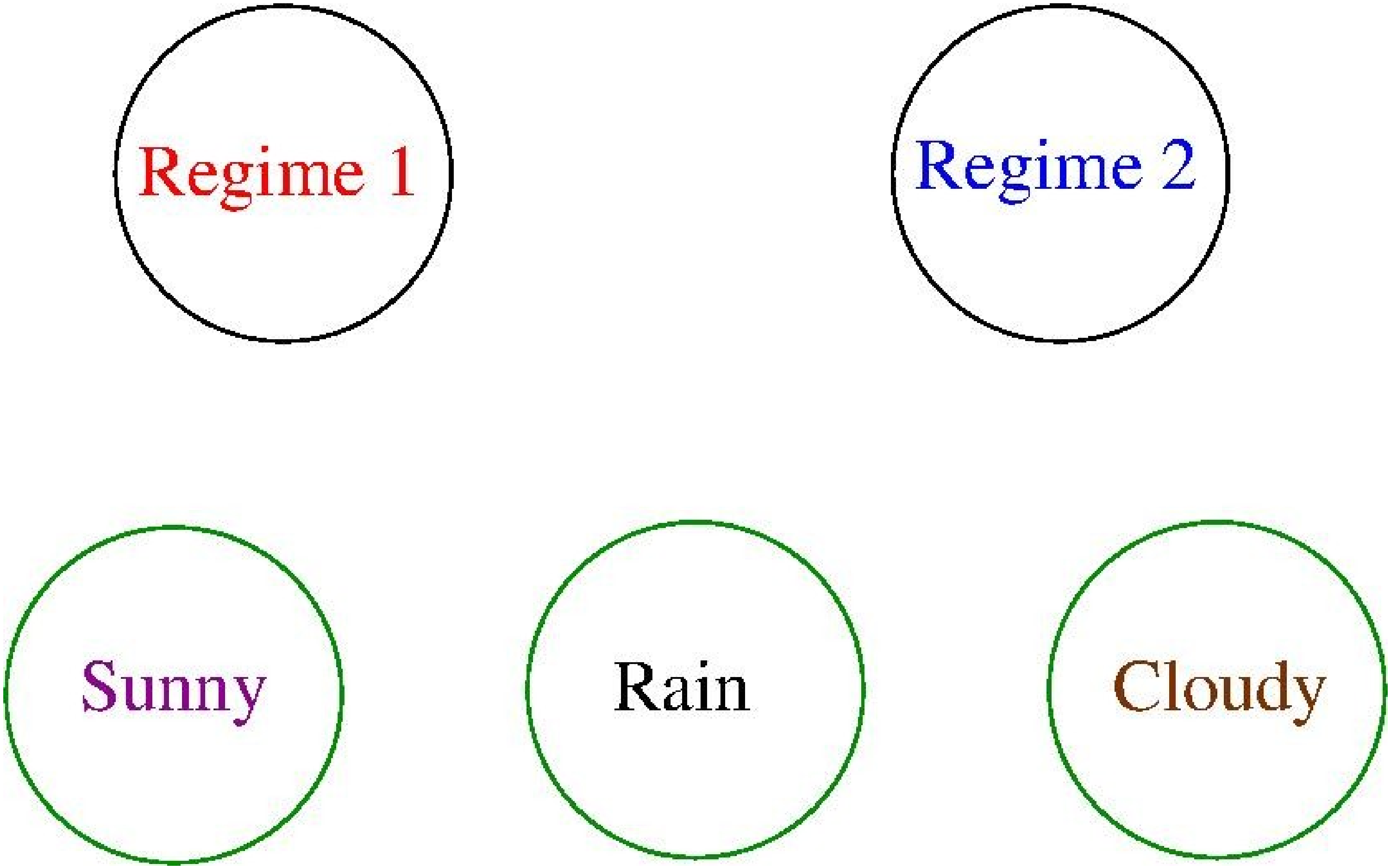
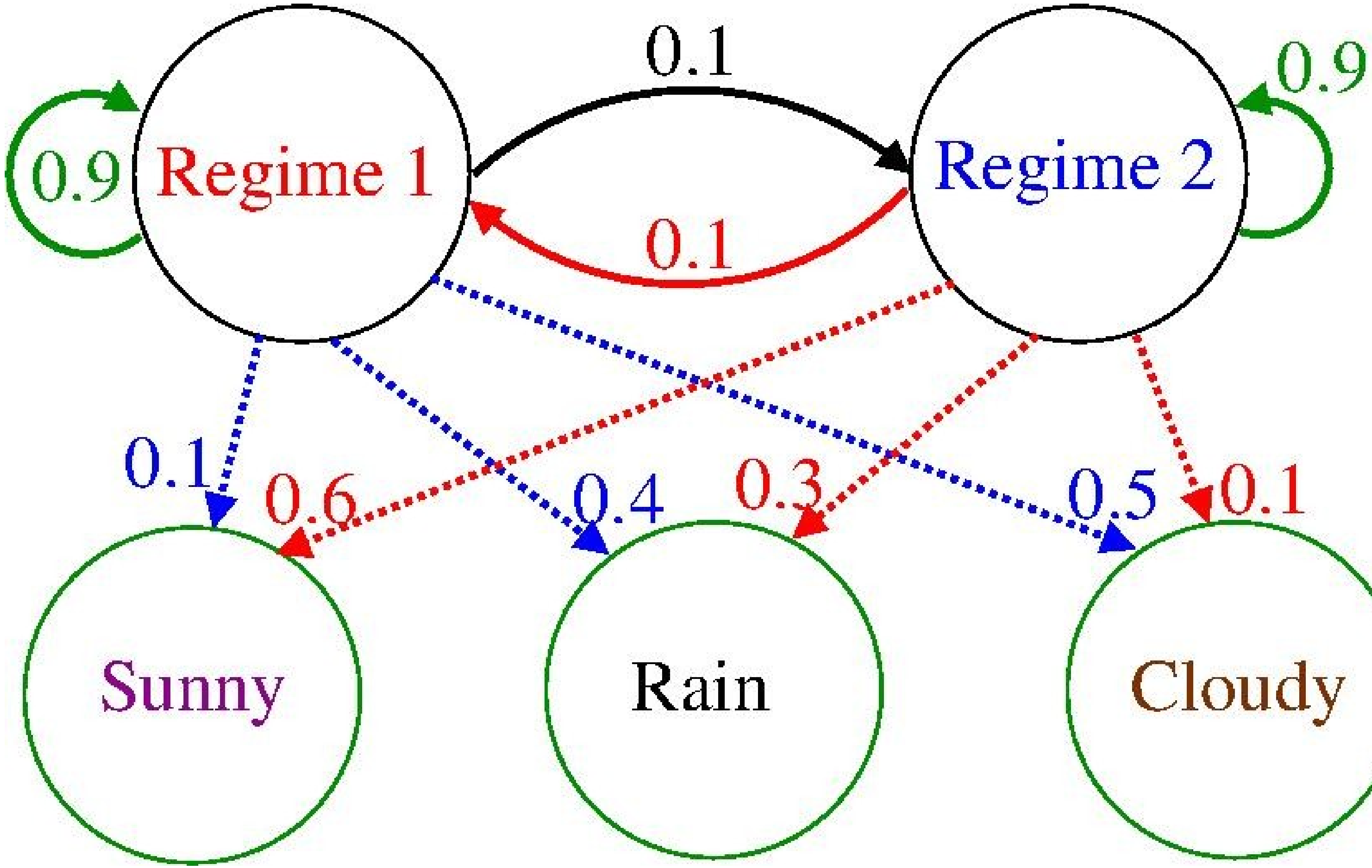
Hidden Markov Model

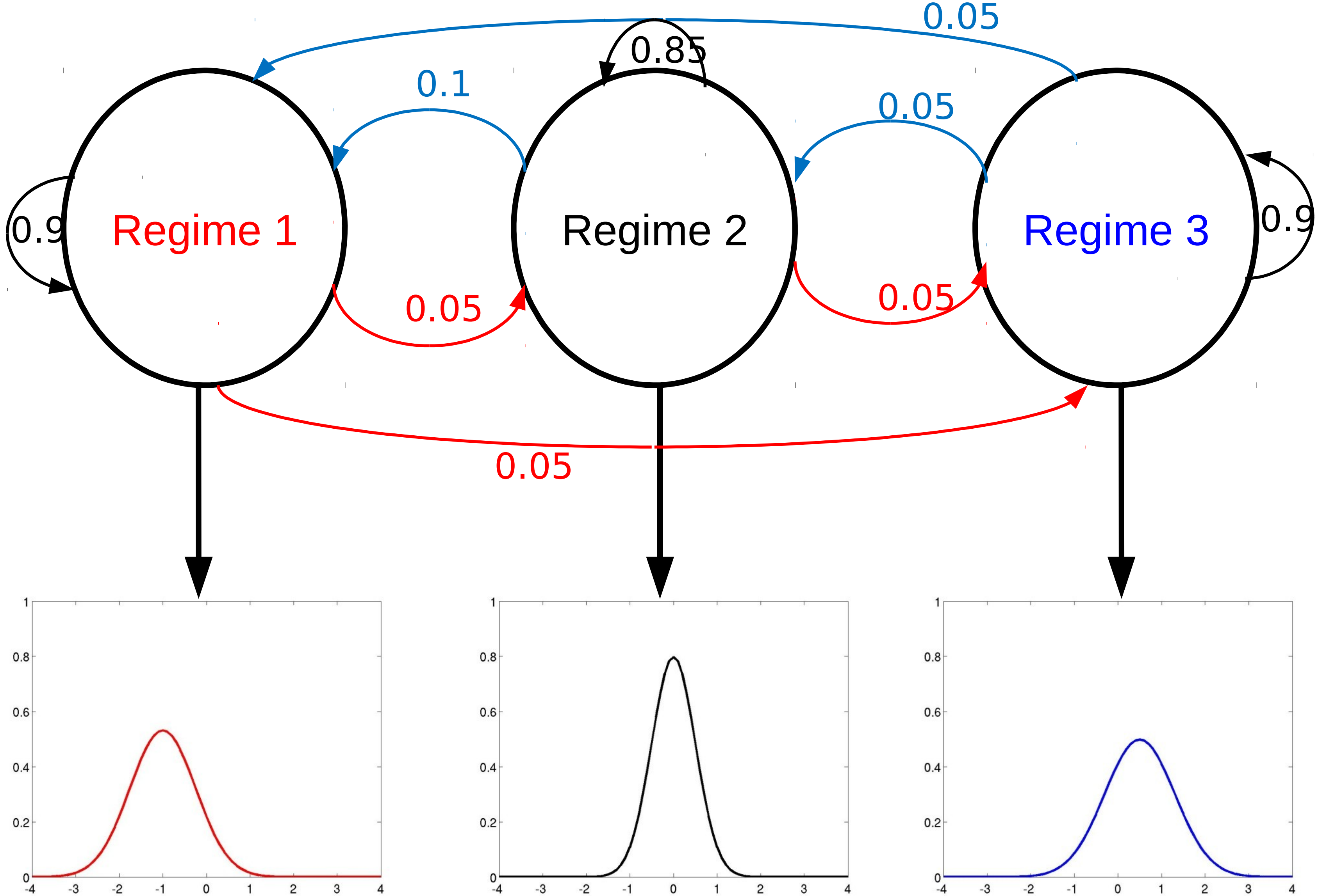
Hidden Markov Model



Hidden Markov Model



Hidden Markov Model



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- Markov Chain
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- Gaussian Mixtures
- Number of Regime States
- Check for Markovianity

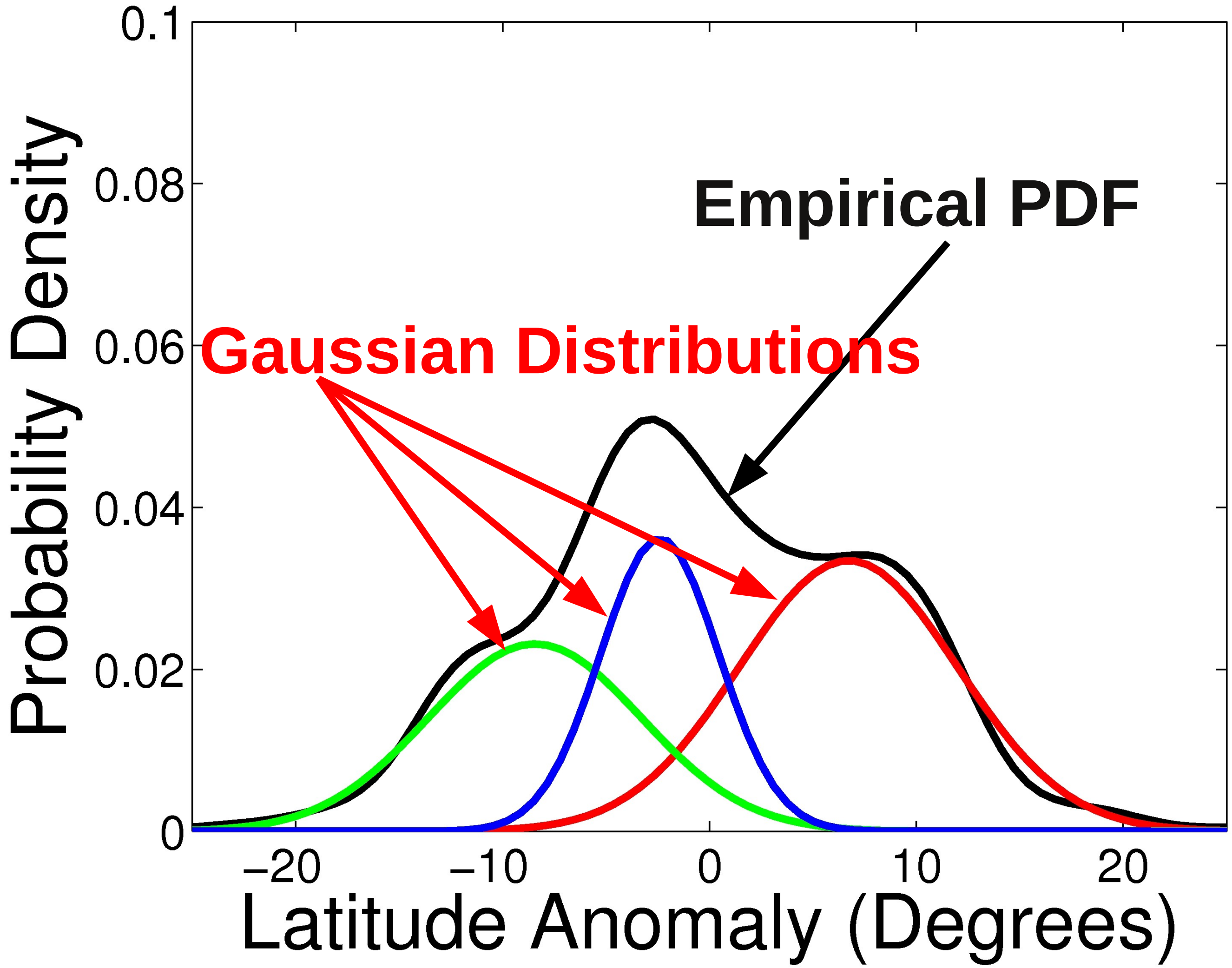
Gaussian Mixture Model

Weighted sum of N Gaussian densities:

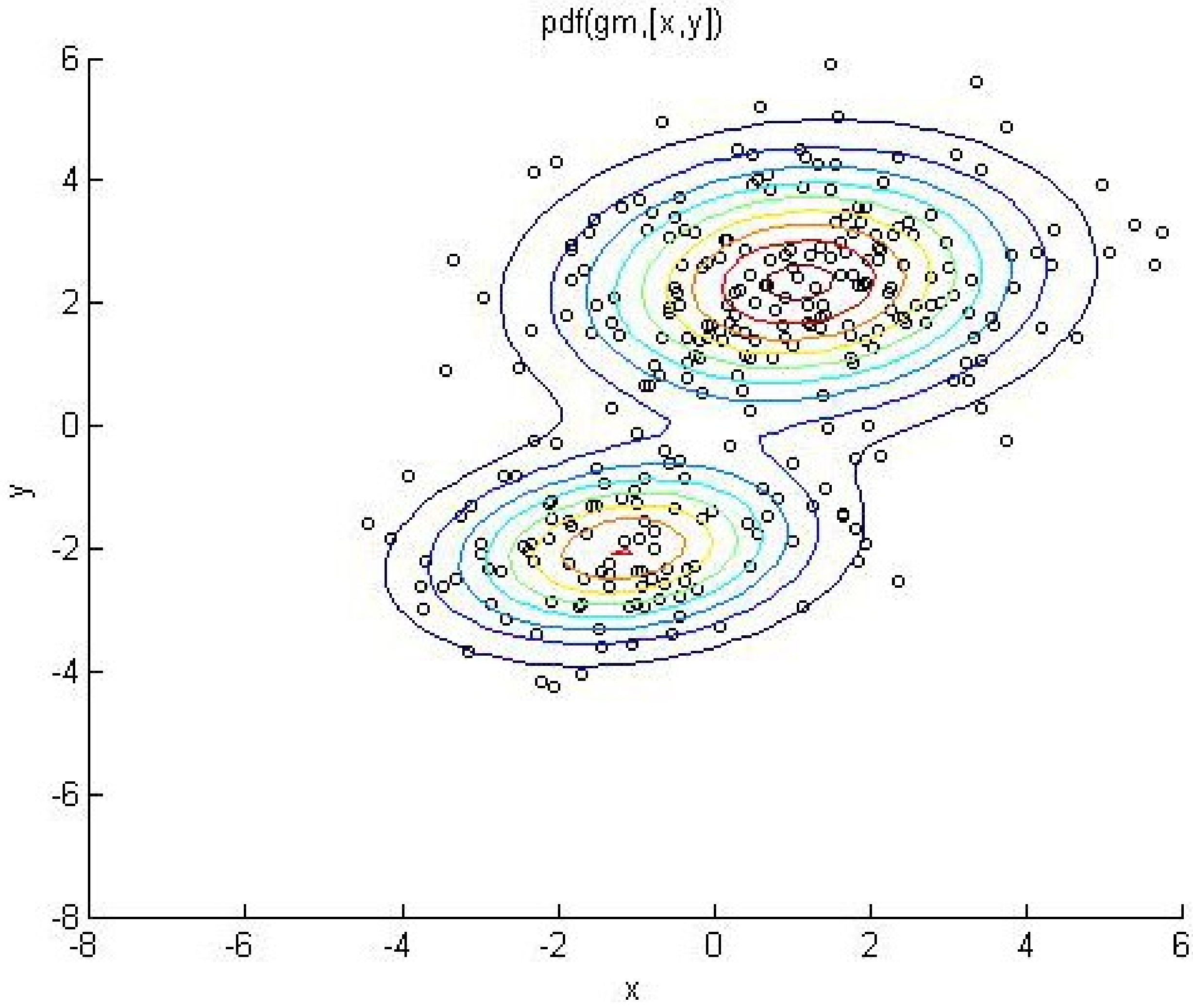
$$p(x|\lambda) = \sum_{i=1}^{N} w_i \, g(x|\mu_i, \Sigma_i)$$

$$g(x|\mu_i \Sigma_i) = \frac{1}{(2\pi)^{D/2} |\Sigma_i|^{1/2}.} \exp\left(-0.5 (x - \mu_i)' \Sigma^{-1} (x - \mu_i)\right)$$

Gaussian Mixture Model

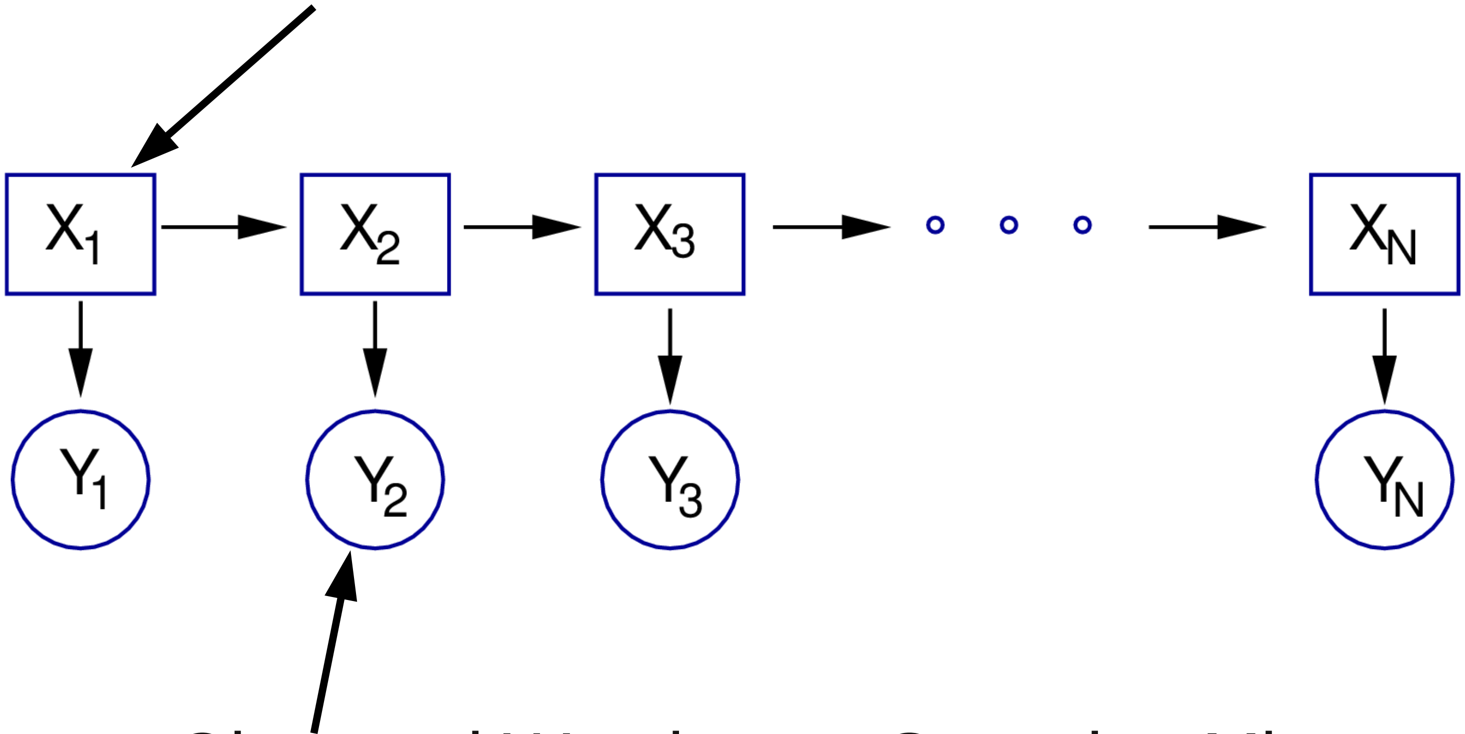


Gaussian Mixture Model in Higher Dimensions



Space-Time Modelling

Regime State evolution → Markov Matrix



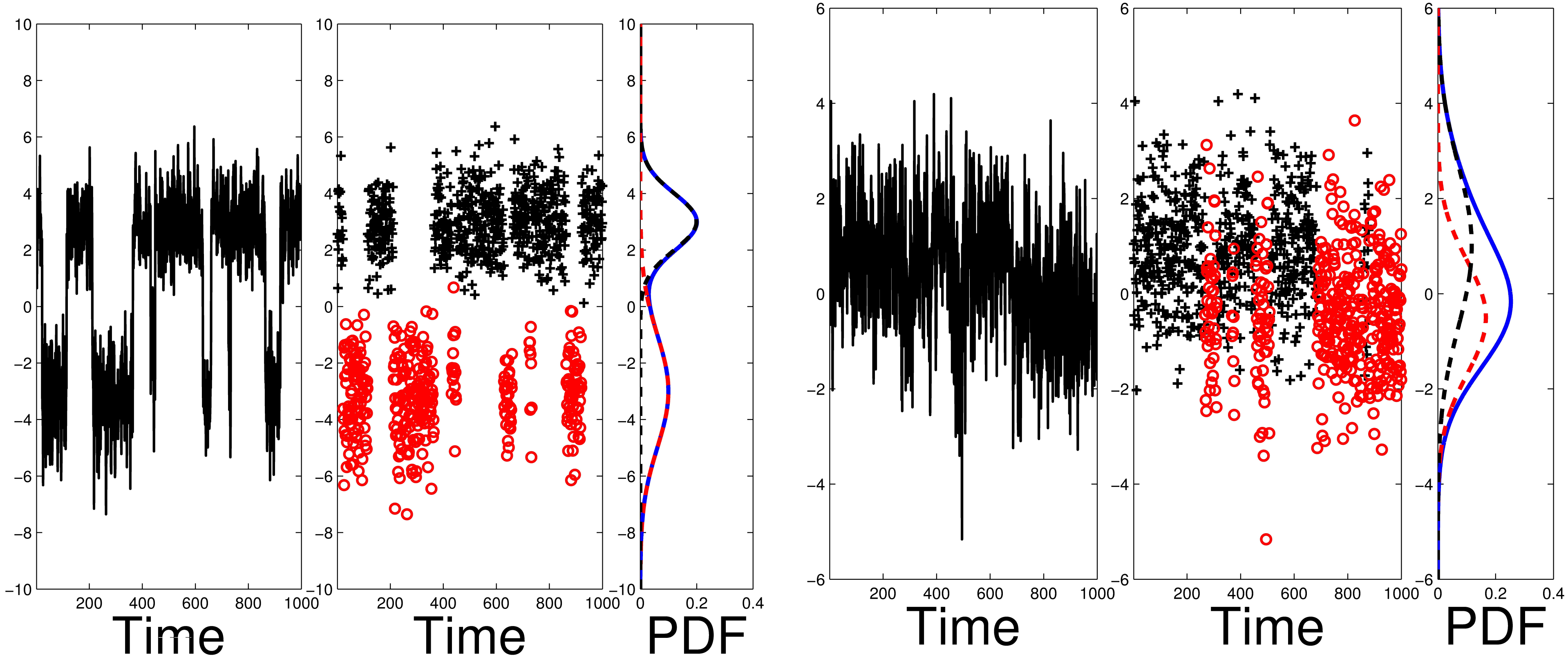
Observed Weather → Gaussian Mixture

HMM Parameter Estimation

Expectation-Maximization Algorithm (Dempster et al. 1977)
- Maximum Likelihood Estimate
- Estimates: Markov Transition Matrix
  Gaussian Mixtures
  Viterbi Path

Tutorial paper on HMM:
Rabiner, 1989: A tutorial on Hidden Markov Models and selected applications in speech recognition. Proc. IEEE, 77, 257-286.

HMM Example



$$A = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}$$

$$B_1 = N(3.0, 1.0); B_2 = N(-3.0, 2.0)$$

$$B_1 = N(1.0, 1.7); B_2 = N(-0.5, 1.2)$$

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Number of Regimes?

Transition Matrix:

$$A = \begin{bmatrix} 0.49 & 0.47 & 0.03 & 0.01 \\ 0.47 & 0.49 & 0.01 & 0.03 \\ 0.02 & 0.01 & 0.53 & 0.44 \\ 0.01 & 0.01 & 0.44 & 0.54 \end{bmatrix}$$

Eigenvalues: λ_1=1.0, λ_2=0.935, λ_3=0.096, λ_4=0.019

Reduced transition Matrix

$$A = \begin{bmatrix} 0.96 & 0.04 \\ 0.025 & 0.975 \end{bmatrix}$$

Eigenvalues: λ_1=1.0, λ_2=0.935

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Markovianity

- Model reduction leads to memory effects (Non-Markov)

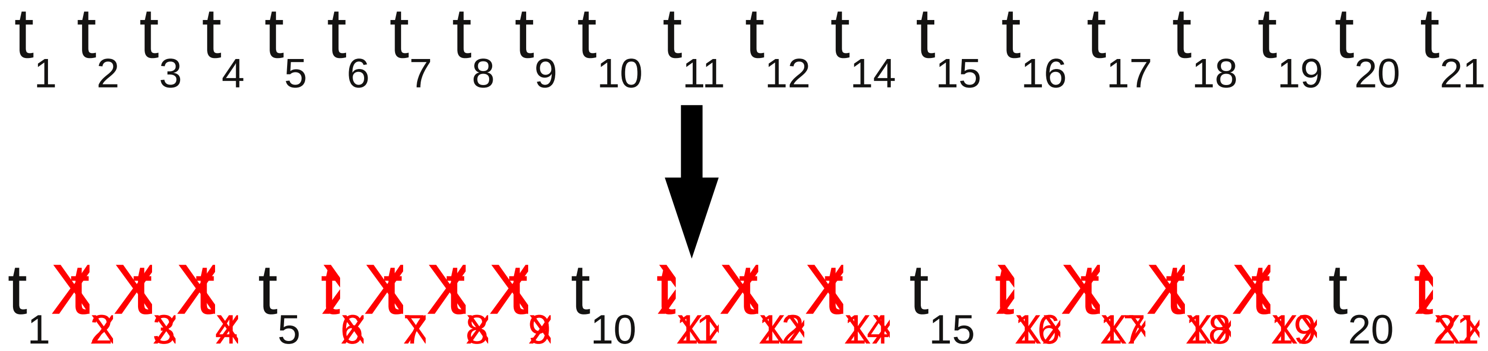
- Have to check whether Markov assumption still applies

- Can do this by
  a) Coarse graining time series
  b) Embedding (Broomhead and King 1986)

- Test significance against simple model (e.g. AR(p) model)

Markovianity

a) Coarse Graining



b) Embedding (Broomhead and King 1986)

$$x_{1,}\, x_{2,}\, x_{3,}\, x_{4,}\, x_{5,} \dots$$

$$\downarrow$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix}, \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}, \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix}, \dots$$

Summary

- Persistent regime detection with HMMs

- Regimes evolve according to a Markov chain

- Distribution of observed 'weather' depend on regime state

- Eigenvalue spectrum is used to determine number of persistent regime states

- Need to check for Markovianity of regime state sequence

Literature:

- Rabiner 1989: A tutorial on Hidden Markov Models and selected applications in speech recognition. Proc. IEEE, 77, 257-286.

- Dempster et al. 1977: Maximum likelihood from incomplete data via the EM algorithm. J. Roy. Stat. Soc. B, 39, 1-38.

- Charney and DeVore, 1979: Multiple Flow Equilibria in the Atmosphere and Blocking. J. Atmos. Sci., 36, 1205-1216.

- Broomhead and King, 1986: Extracting qualitative dynamics from experimental data. Physica D, 2, 217-236.

- Majda et al. 2006: Distinct Atmospheric Regimes despite nearly Gaussian Statistics - A Paradigm Model. Proc. Natl. Acad. Sci. USA, 103, 8309-8314.

- Franzke et al. 2008: A Hidden Markov Model Perspective on Regimes and Metastability in Atmospheric Flows. J. Climate, 21, 1740-1757.

- Franzke et al. 2011: Persistent Circulation Regimes and Preferred Regime Transitions in the North Atlantic. J. Atmos. Sci., 68, 2809-2825.

HMM source codes

- HMMTool http://iri.columbia.edu/our-expertise/climate/tools/
hidden-markov-model-tool/

- C GHMM http://ghmm.org/

-Matlab toolbox http://www.cs.ubc.ca/~murphyk/Software/HMM/hmm.html

- C++ HMMlib http://www.cs.au.dk/~asand/?page_id=152

- Java jahmm https://code.google.com/p/jahmm/

- R depmixS4 http://CRAN.R-project.org/package=depmixS4

- Java Metamacs http://www.mi.fu-berlin.de/w/CompMolBio/
SoftwareFramework#A_61Metamacs_61