Self-organizing maps (SOMs) and k-means clustering: Part 1

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Trieste, Italy, October 21, 2013

Teleconnection Patterns

- Atmospheric teleconnections are spatial patterns that link remote locations across the globe (Wallace and Gutzler 1981; Barnston and Livezey 1987)
- Teleconnection patterns span a broad range of time scales, from just beyond the period of synoptic-scale variability, to interannual and interdecadal time scales.

Methods for DeterminingTeleconnection Patterns

- Empirical Orthogonal Functions (EOFs) (Kutzbach 1967)
- Rotated EOFs (Barnston and Livezey 1987)
- One-point correlation maps (Wallace and Gutzler 1981)
- Empirical Orthogonal Teleconnections (van den Dool 2000)
- Self Organizing Maps (SOMs) (Hewiston and Crane 2002)
- k-means cluster analysis (Michelangeli et al. 1995)

Advantages and Disadvantages of various techniques

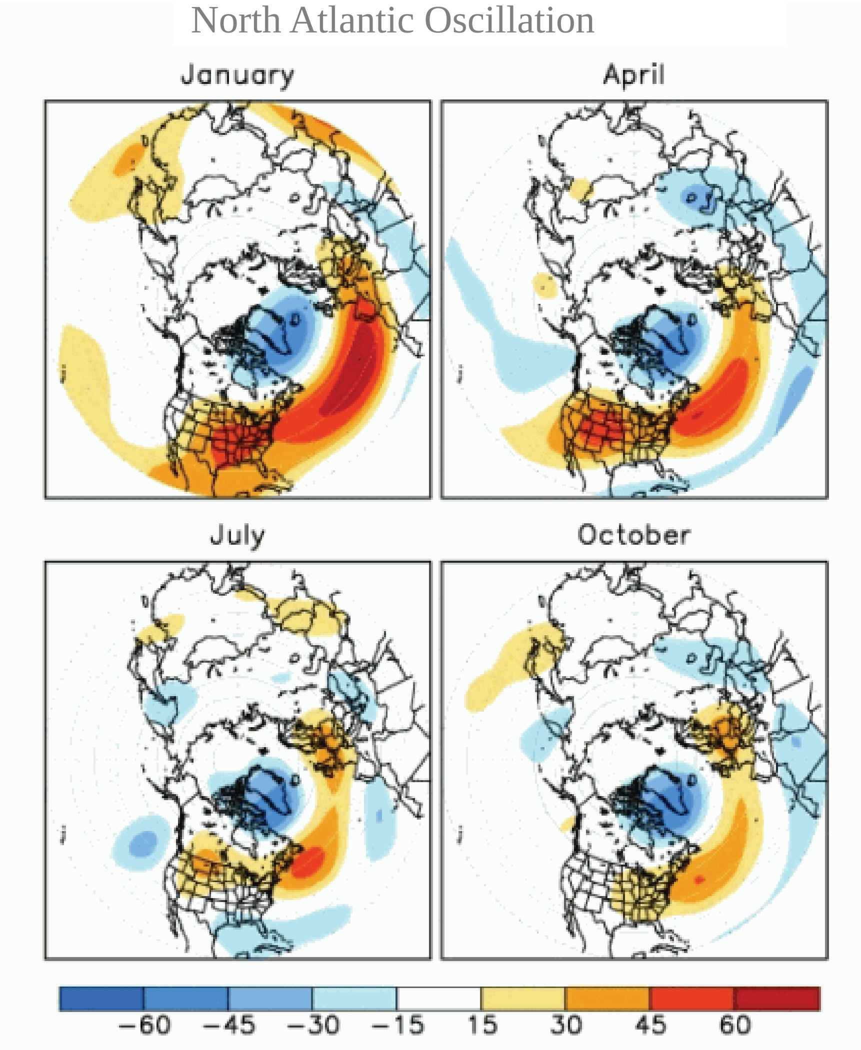
- Empirical Orthogonal Functions (EOFs): patterns maximize variance, easy to use, but patterns orthogonal in space and time, symmetry between phases, i.e., may not be realistic, can't identify continuum

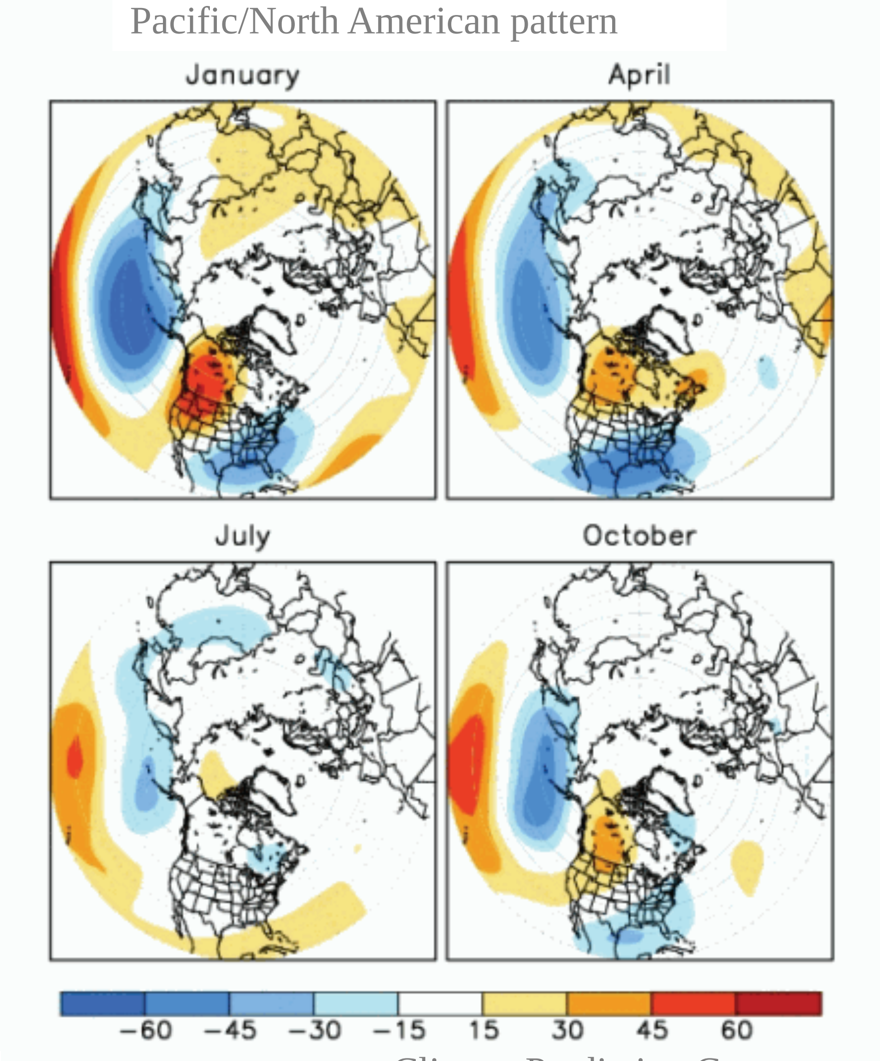
- Rotated EOFs: patterns more realistic than EOFs, but some arbitrariness, can't identify continuum

- One-point correlation maps: realistic patterns, but patterns not objective organized, i.e., different pattern for each grid point

- Self Organizing Maps (SOMs): realistic patterns, allows for a continuum, i.e., many NAO-like patterns, asymmetry between phases, but harder to use
- k-means cluster analysis: Michelangeli et al. 1995

The dominant Northern Hemisphere teleconnection patterns



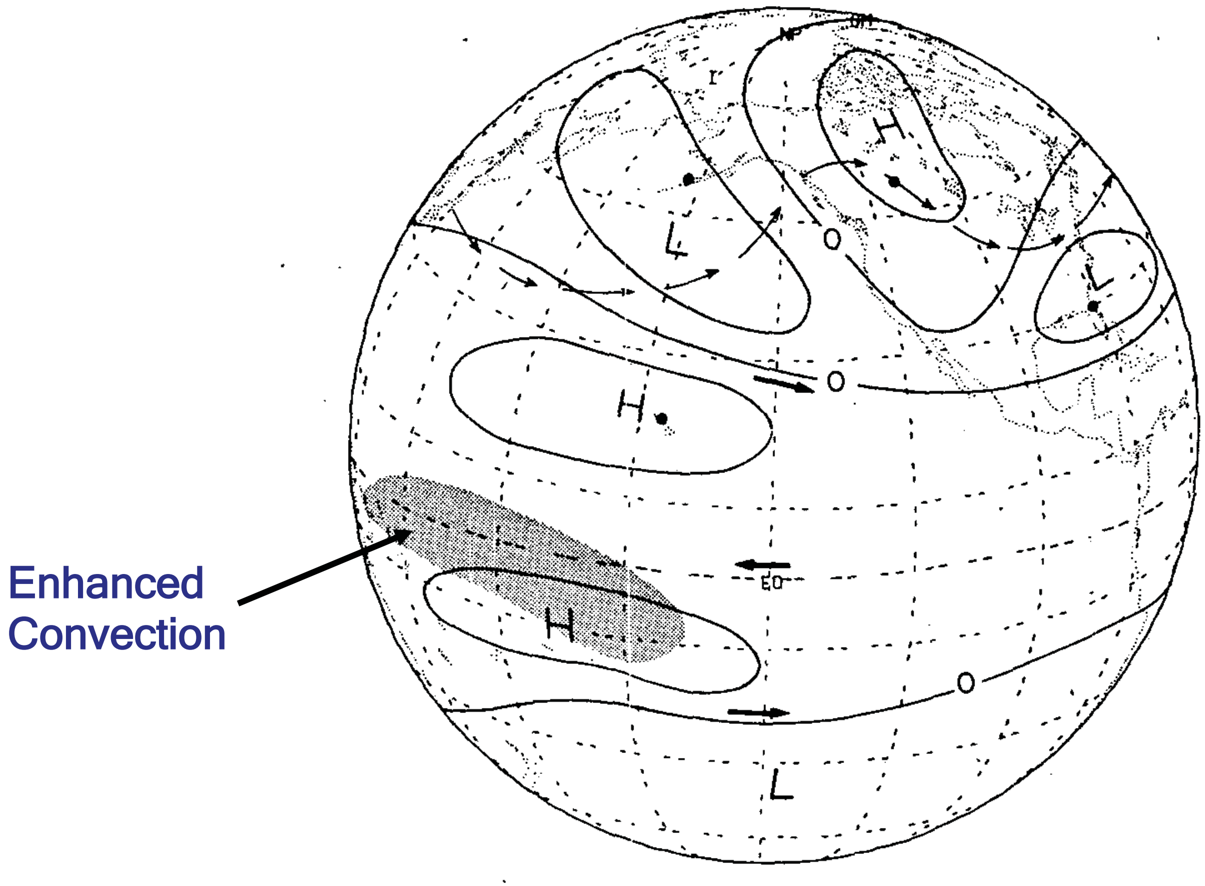


Climate Prediction Center

Aim of EOF, SOM analysis, and k-means clustering

- To reduce a large amount of data into a small number of representative patterns that capture a large fraction of the variability with spatial patterns that resemble the observed data

Link between the PNA and Tropical Convection

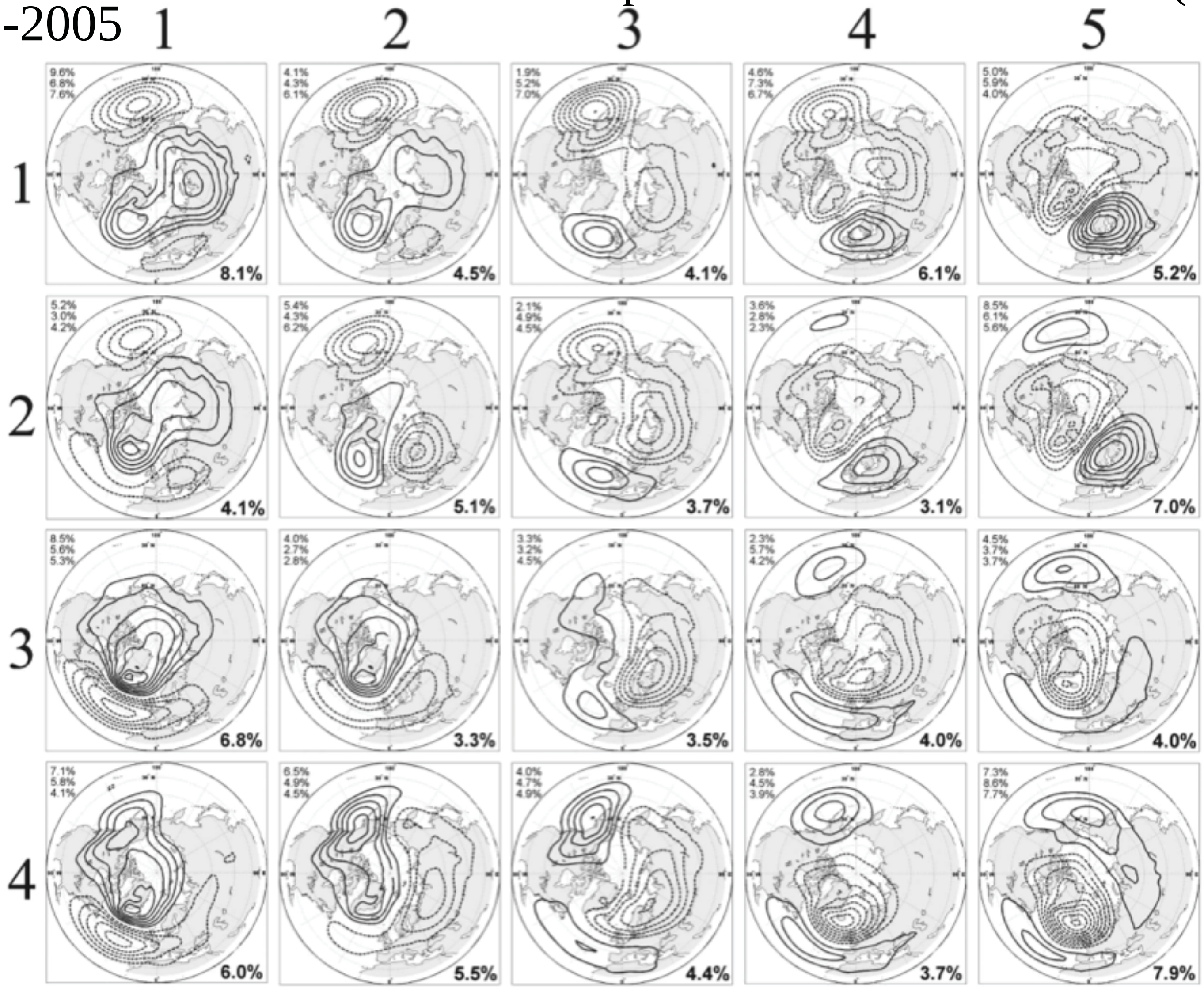


From Horel and Wallace (1981)

A SOM Example

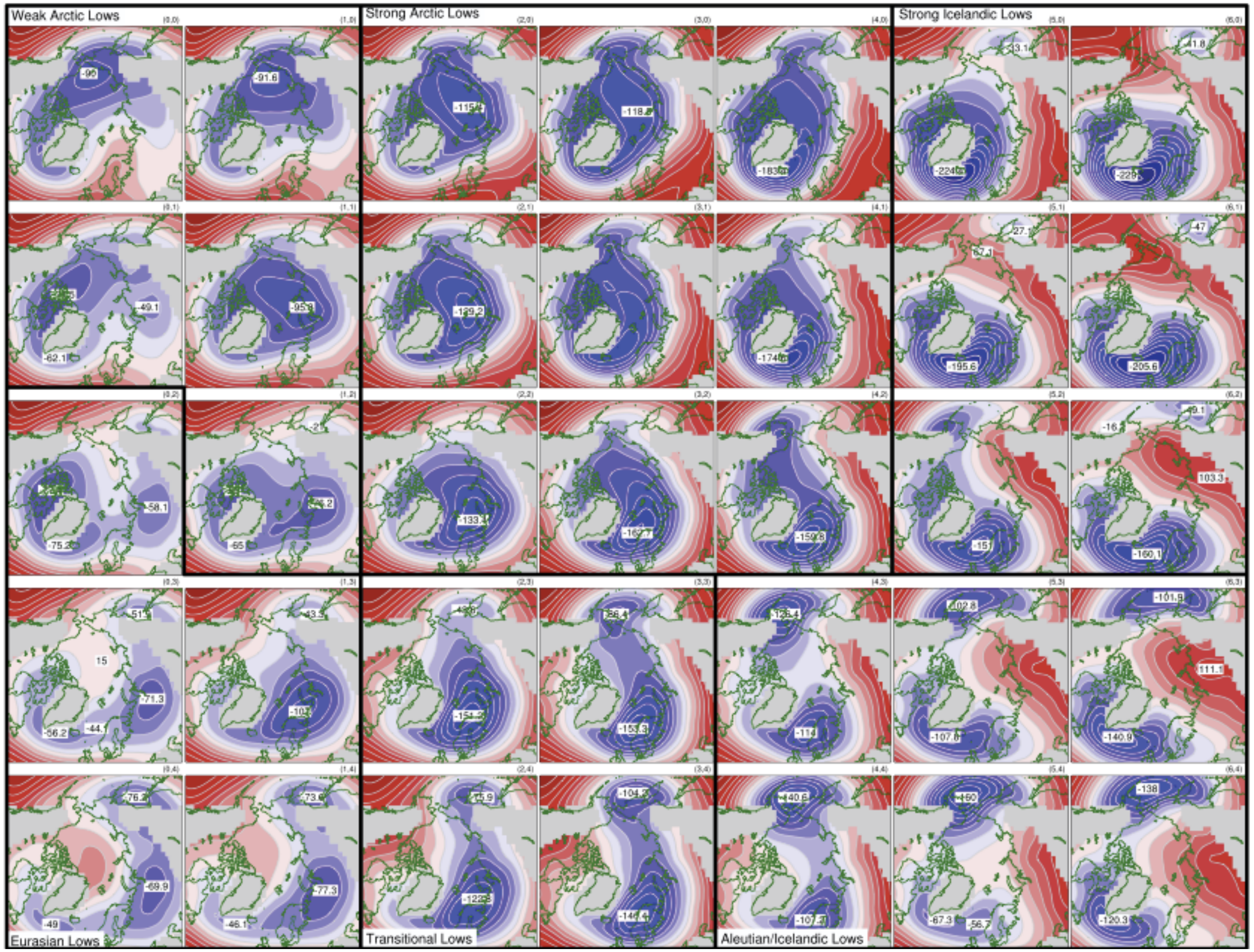
P1=1958-1977
P2= 1978-1997
P3=1998-2005

Northern Hemispheric Sea Level Pressure (SLP)



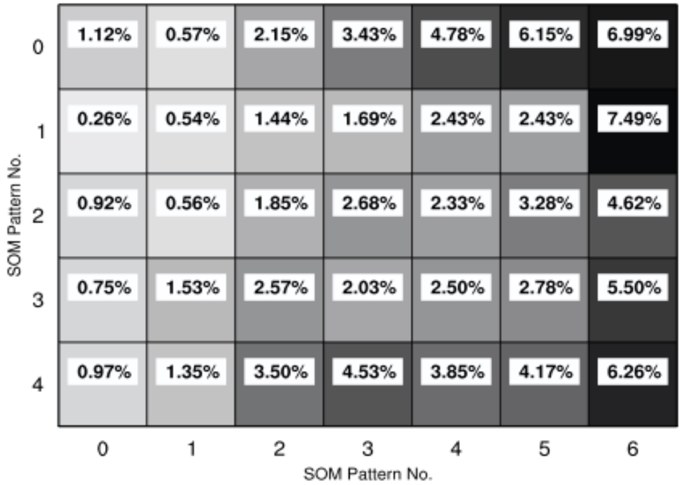
Another SOM Example (Higgins and Cassano 2009)

Geopotential Height Anomaly at 1000 hPa

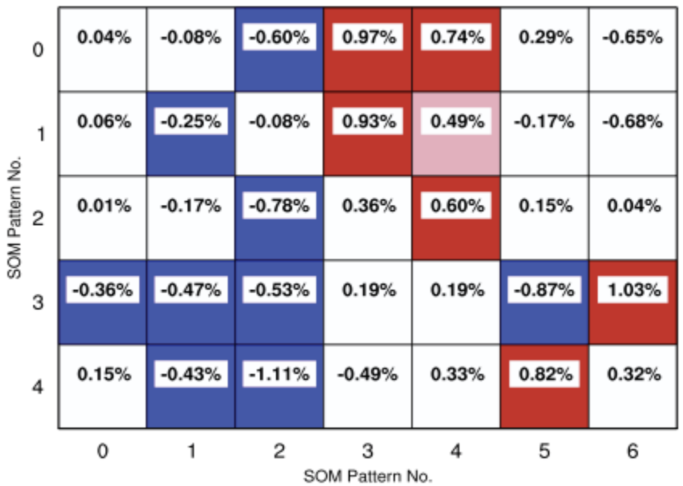


A third example

a) 20th Century NDJF SOM Pattern Frequency



b) 21st - 20th Century NDJF SOM Pattern Frequency



How SOM patterns are determined

- Transform 2D sea-level pressure (SLP) data onto an N-dimension phase space, where N is the number of gridpoints. Then, minimize the Euclidean between the daily data and SOM patterns

$$\|\mathbf{z} - \mathbf{m}_c^*\| = \min_i\{\|\mathbf{z} - \mathbf{m}_c^*\|\},$$

where $\mathbf{z}$ is the daily data (SLP) in the N-dimensional phase, $\mathbf{m}_c^*$ are the SOM patterns, and i is the SOM pattern number.

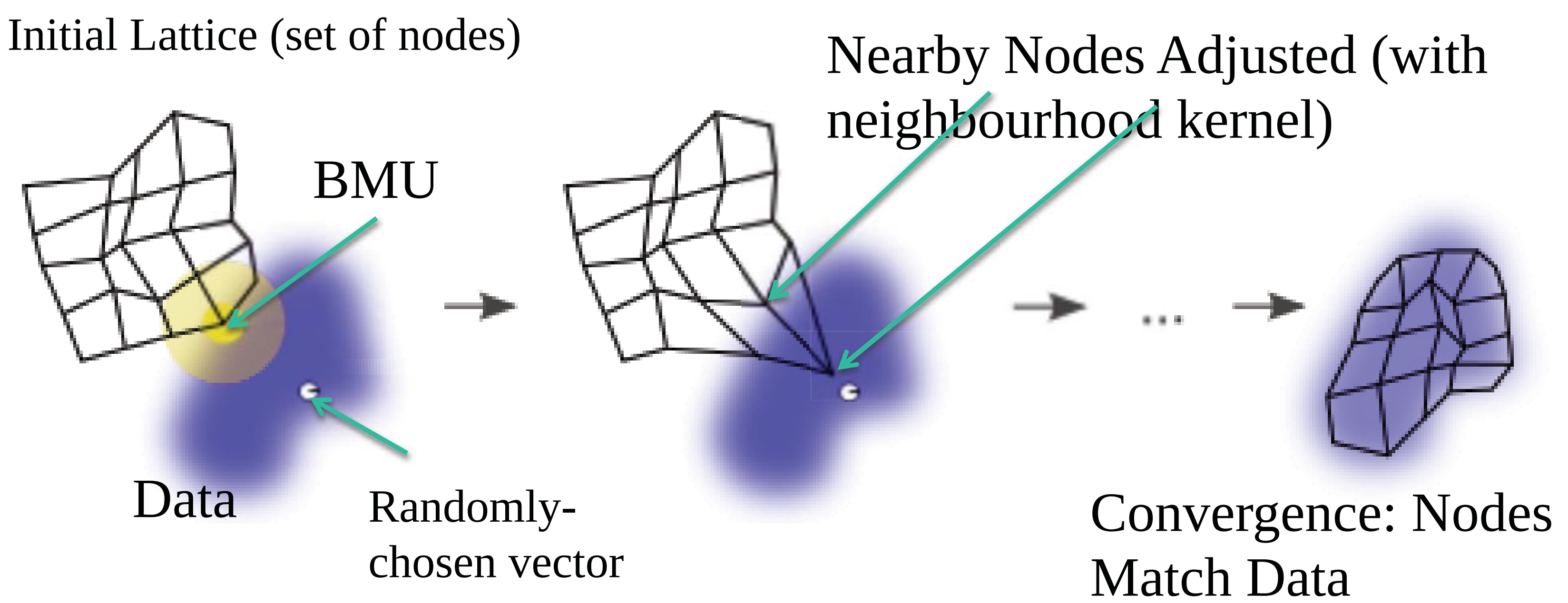
How SOM patterns are determined

- E is the *average quantization error*,

$$E = \frac{1}{N}\left(\sum_{t=1}^{N} \|\mathbf{z}_t - \mathbf{m}_c^*\|\right)$$

The $\mathbf{m}_c^*$ (SOM patterns) are obtained by minimizing E.

SOM Learning



SOM Learning

- 1. Initial lattice (set of nodes) specified (from random data or from EOFs)
- 2. Vector chosen at random and compared to lattice.
- 3. Winning node (Best Matching Unit; BMU) based on smallest Euclidean distance is selected.
- 4. Nodes within a certain radius of BMU are adjusted. Radius diminishes with time step.
- 5. Repeat steps 2-4 until convergence.

How SOM spatial patterns are determined

- Transform SOM patterns from phase space back to physical space (obtain SLP SOM patterns)
- Each day is associated with a SOM pattern
- Calculate a frequency, *f*, for each SOM pattern, i.e.,

$f(\mathbf{m}_c^*)$ = number of days $\mathbf{m}_c^*$ is chosen/total number of days

SOMs are special!

- Amongst cluster techniques, SOM analysis is unique in that it generates a 2D grid with similar patterns nearby and dissimilar patterns widely separated.

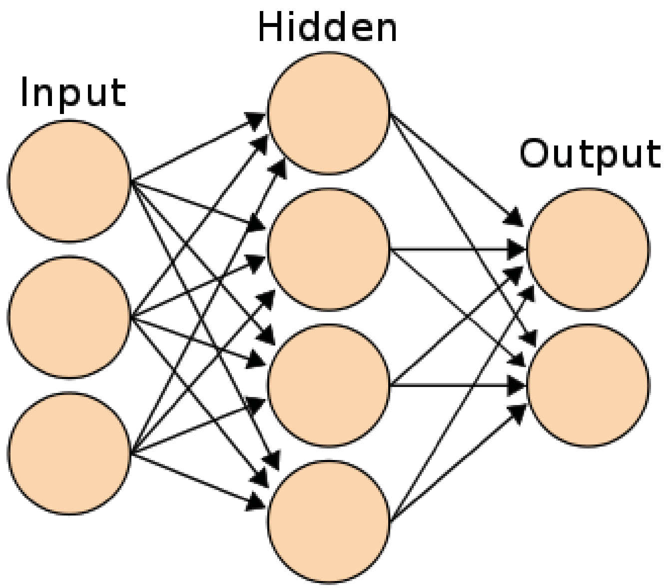
Some Background on SOMs

- SOM analysis is a type of Artificial Neural Network which generates a 2-dimensional map (usually). This results in a low-dimensional view of the original high-dimension data, e.g., reducing thousands of daily maps into a small number of maps.

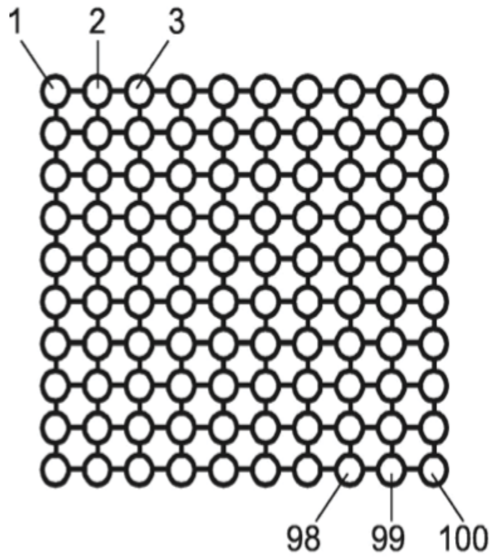
- SOMs were developed by Teuvo Kohonen of Finland.

Artificial Neural Networks

- Artificial Neural Networks are used in many fields.
  They are based upon the central nervous system of animals.
- Input = Daily Fields
- Hidden = Minimization of
  Euclidean Distance
- Output = SOM patterns



A simple conceptual example of SOM analysis



Uniformly distributed data
between 0 and 1 in 2-dimensions

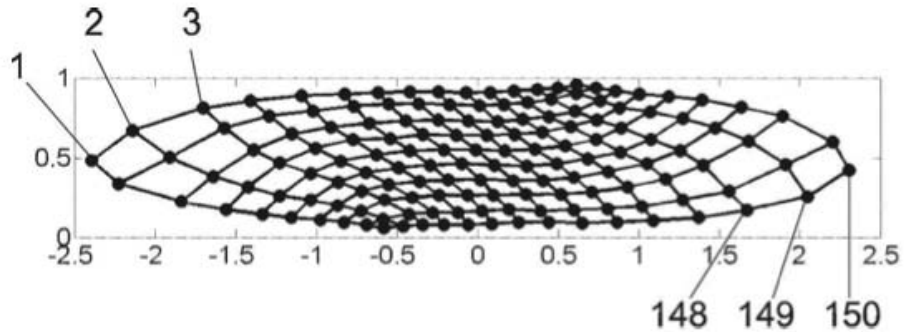
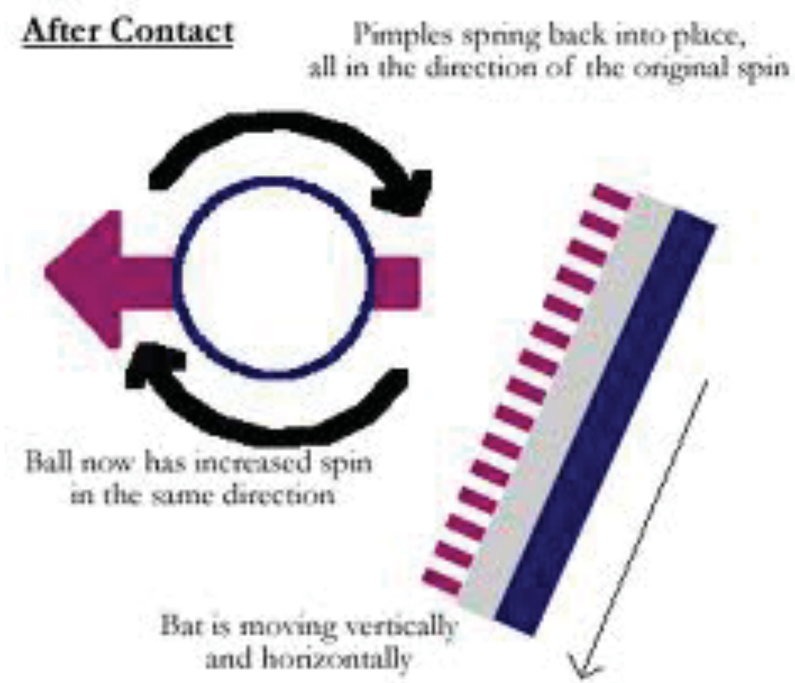


FIG. A2. Fine-tuned reference vectors of a 10 × 15 SOM: The analyzed dataset consists of 10 000 random two-dimensional data vectors with components normally distributed in *x* and uniformly distributed in *y*.

A table tennis example (spin of ball)

Spin occurs primarily along 2 axes of rotation. Infinite number of angular velocities along both axes components.



Joo SaeHyuk 주세혁

- Input - Three senses (sight, sound, touch) feedback as in SOM learning
- Hidden - Brain processes information from senses to produce output
- Output - SOM grid of various amounts of spin on ball.
- SOM grid different for every person