

2499-14

**International Training Workshop on FPGA Design for Scientific
Instrumentation and Computing**

11 - 22 November 2013

**Digital CMOS Design
Coding - Representation of numbers**

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Outline

■ Digital CMOS design

- Boolean algebra
- Basic digital CMOS gates
- Combinational and sequential circuits
- **Coding - Representation of numbers**

Representing Numbers

How values can be coded ?

In a digital circuit each signal can take 2 values (0, 1) (Boolean world)

A vector of n bits can represent up to 2^n values



Representing Numbers

How values can be coded ?

What is the meaning of 0100 0110 ?

The character 'F'

The character 'Φ'

The number 46

The number 70

The number 123

Any symbol in a set where the *Card* = 256



Representing Numbers

How values can be coded ?

by itself a code has no signification



Representing Numbers

How values can be coded ?

arithmetic : dealing with numbers

How can I represent a number ?

Natural numbers

Relative numbers

Rational numbers

Real numbers



Representing Numbers

How can I represent a Natural number ?

I need at least n bits for a Natural ranging from 0 to $2^n - 1$

Standards



Representing Numbers

How can I represent a Natural number ?

Natural Binary Code :

The bits represent the successive powers of 2

$$\begin{array}{c} 0100\ 0110 \\ \nearrow \qquad \nwarrow \\ 2^7 \qquad \qquad 2^0 \end{array} = 2^1 + 2^2 + 2^6 = 70$$



Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal :

The bits represent the successive powers of 2

The quartets represent the successive powers of 10

$$\begin{array}{c} \text{0100 0110} = (2^1 + 2^2) \times 10^0 + 2^2 \times 10^1 = 46 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 2^3 \quad 10^1 \quad 10^0 \quad 2^0 \end{array}$$

packed



Representing Numbers

How can I represent a Natural number ?

Binary Coded Decimal - Unpacked :

The bits represent the successive powers of 2

The bytes represent the successive powers of 10

In each byte the 4 Msb are 0

0100 0110 = Illegal 0000 0110 = 6

10^0 2^0



Representing Numbers

How can I represent a Relative number ?

Sign + Value

The bits represent the successive powers of 2

The Msb represents the sign (1 means negative)



Representing Numbers

How can I represent a Relative number ?

Sign+Value :

$$+ \quad 0100 \ 0110 = 2^1 + 2^2 + 2^6 = 70$$

Diagram showing the binary representation 0100 0110. The first bit (0) is the sign bit. The remaining bits (100 0110) represent the value. Arrows point from the sign bit to a '+' sign and from the bits 1, 2, and 6 to the powers of 2 (2¹, 2², 2⁶).

$$- \quad 1100 \ 0110 = -1 \times (2^1 + 2^2 + 2^6) = -70$$

Diagram showing the binary representation 1100 0110. The first bit (1) is the sign bit. The remaining bits (100 0110) represent the value. Arrows point from the sign bit to a '-' sign and from the bits 1, 2, and 6 to the powers of 2 (2¹, 2², 2⁶).

Representing Numbers

How can I represent a Relative number ?

2's complemented

The bits represent the successive powers of 2

The Msb represents -2^{n-1}



Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$\begin{array}{c} \begin{array}{c} \nearrow \\ \nearrow \\ \nwarrow \end{array} \begin{array}{c} 0100 \\ 0110 \end{array} = 2^1 + 2^2 + 2^6 = 70 \\ \begin{array}{c} -2^7 \\ 2^6 \\ 2^0 \end{array} \end{array}$$

$$1100\ 0110 = 2^1 + 2^2 + 2^6 - 2^7 = -58$$

Representing Numbers

How can I represent a Relative number ?

2's complemented :

$$0100\ 0110 = 2^1 + 2^2 + 2^6 = 70$$

70

$$2^7 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 1$$

$$2^7 = 2^0 \quad + 2^3 + 2^4 + 2^5 \quad + 1 + 70$$

$$1011\ 1010 \quad -70 = 2^0 \quad + 2^3 + 2^4 + 2^5 \quad + 1 - 2^7$$

$$-70 = 2^1 \quad + 2^3 + 2^4 + 2^5 \quad - 2^7$$



Representing Numbers

How can I represent a Real number ?

Range
Precision



Representing Numbers

How can I represent a Real number ?

2's complement Fixed Point :

The bits represents the successive powers of 2

$$\begin{array}{cccc} & \nearrow & \nearrow & \nearrow & \nearrow \\ & -2^3 & 2^0 & 2^{-1} & 2^{-4} \\ 0100.0110 & = & 2^{-3} & + & 2^{-2} & + & 2^2 & = & 4.325 \end{array}$$

Representing Numbers

How can I represent a Real number ?

Wide range
High precision

Floating Point :
Logarithmic representation



Representing Numbers

How can I represent a Real number ?

$$R = (-1)^S \times M \times 2^E$$

Normalized scientific representation

S : Sign (1 if negative)

M : Mantissa ($\in [1, 2[$)

E : Exponent (Relative number)



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

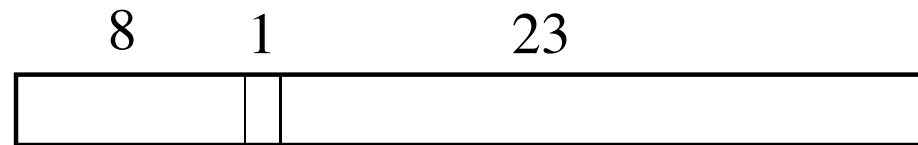
	Single Precision 32 bits	Double Precision 64 bits
S : Sign (1 if negative)	1 bit	1 bit
M : Mantissa ($\in [1, 2[$)	23 bits	52 bits
E : Exponent	8 bits	11 bits



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision :



Exponent

Number between
-127 and 127

The code 0000 0000
means -127

Natural Binary Code
by Excess of 127

Sign

Mantissa

Fixed point positive real number
1. . . .

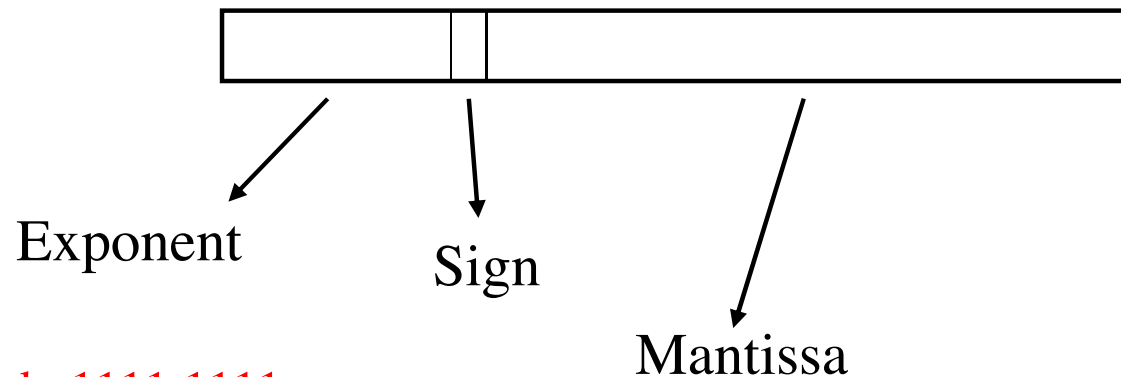
The 1 is not represented !!



Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



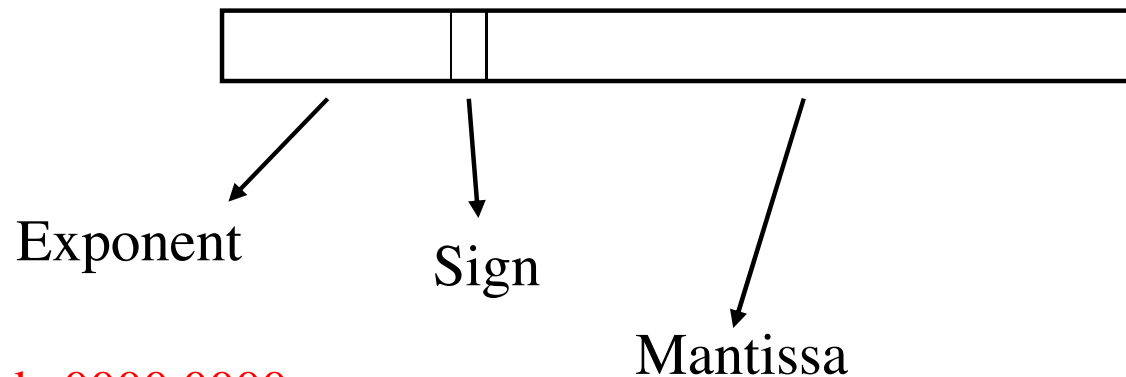
The code 1111 1111
(128) means $\pm\infty$ or
an error

.111 ... 111 means $\pm\infty$
other values mean error (NaN)

Representing Numbers

$$R = (-1)^S \times M \times 2^E$$

Single precision : Special cases



The code 0000 0000
(-127) indicates
denormalized
Mantissa

0.00 ... 000 means 0