2583-10

Workshop on Coherent Phenomena in Disordered Optical Systems

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Superfluid Stiffness of a Driven Dissipative Condensate with Disorder

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A. Janot, T. Hyart, P. R. Eastham, B. Rosenow, PRL **111**, 230403 (2013)

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ICTP, May 2014

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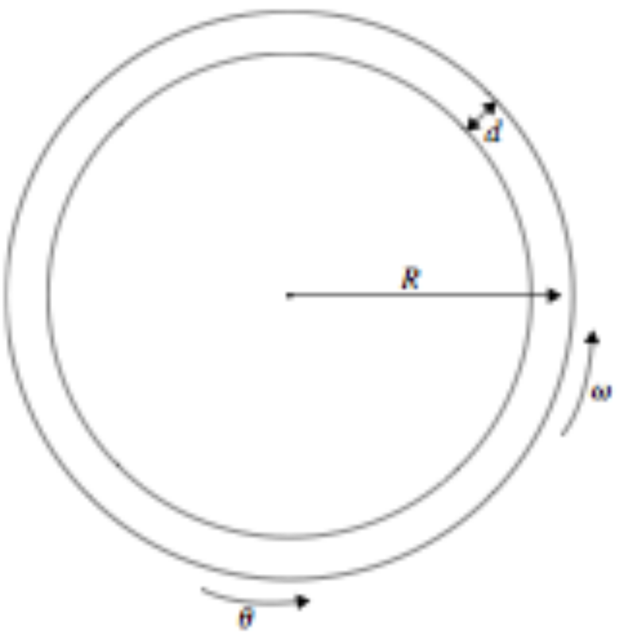
Superfluidity

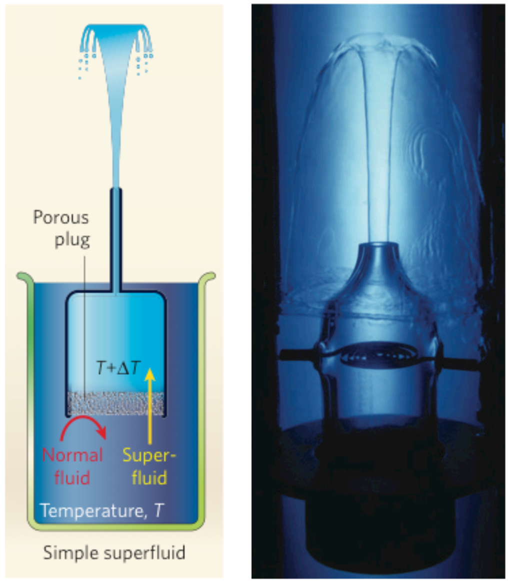
Rotating bucket experiment

- metastable superflow
- Hess-Fairbank effect

→ “frictionless flow“



taken from [A. J. Leggett, Quantum Liquids, Oxford University Press, 2006]

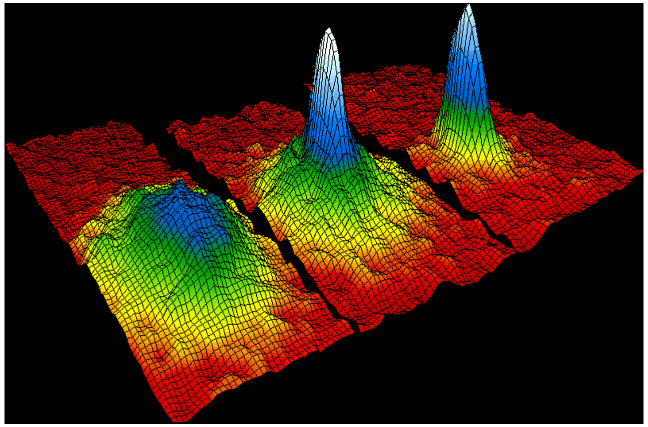


fountain effect

taken from [S. Fisher and G. Pickett, Nature **444**, 832 (2006), S. Balibar, Nature **464**, 176 (2010)]

Bose Einstein Condensate

- macroscopic occupation of the ground state
- off-diagonal long-range order



BEC of ultra-cold Rb-atoms

taken from [E. A. Cornell, C. E. Wieman and W. Ketterle, "Bose-Einstein Condensates in a Dilute Gas", Nobel lecture (2001)]

Condensate wavefunction

$$\boxed{\Psi = \sqrt{n}\, e^{\mathrm{i}\phi - \mathrm{i}\epsilon t}}$$

Superfluid velocity: $$v_\mathrm{s} \equiv \frac{\hbar}{m}\nabla\phi$$

Criteria for Superfluidity

Landau criterion

Creation of elementary excitations?
critical velocity:

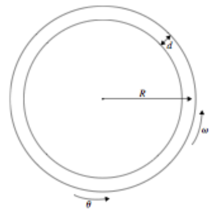
$$v_c = \min\left(\frac{\epsilon(p)}{p}\right)$$

Transverse Current response

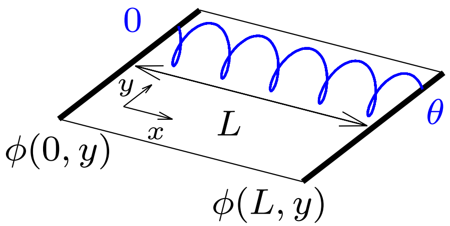
$$\vec{J} = \chi \vec{A} \sim \left((n_\mathrm{s} + n_\mathrm{n})\, P_{||} + n_\mathrm{n}\, P_\perp\right) \vec{A}$$

$\vec{A}$ – external vector field

χ – response function



Superfluid Stiffness – Phase twist



$$\text{tbc} \quad \Leftrightarrow \quad \text{pbc} \quad \text{“}+\text{“} \quad \vec{A}_\theta = \frac{\theta}{L}\vec{e}_\theta \quad \text{– twist current}$$

$$\text{tbc:} \quad \phi_\theta(\vec{x} + L\vec{e}_\theta) - \phi_\theta(\vec{x}) = \theta$$

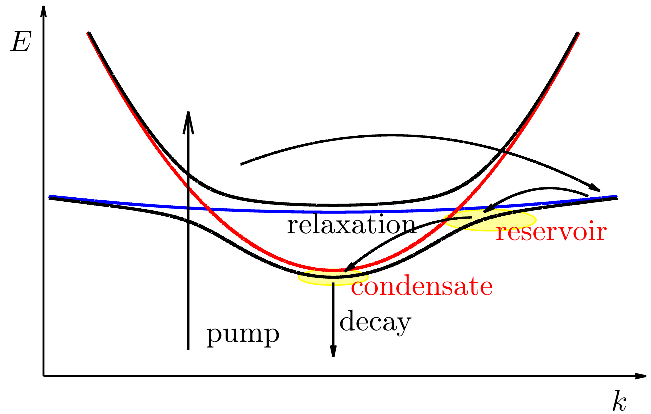
$$f_s = \frac{2mL^2}{\hbar^2} \lim_{\theta \to 0} \frac{\epsilon(\theta) - \epsilon(0)}{\theta^2}$$

[Leggett, RRL **25**, 1543 (1970); Fisher et al., RRA **8**, 1111 (1973); Roth and Burnett, PRA **67**, 031602(R) (2003)]

Polaritons – Driven Dissipative Condensate

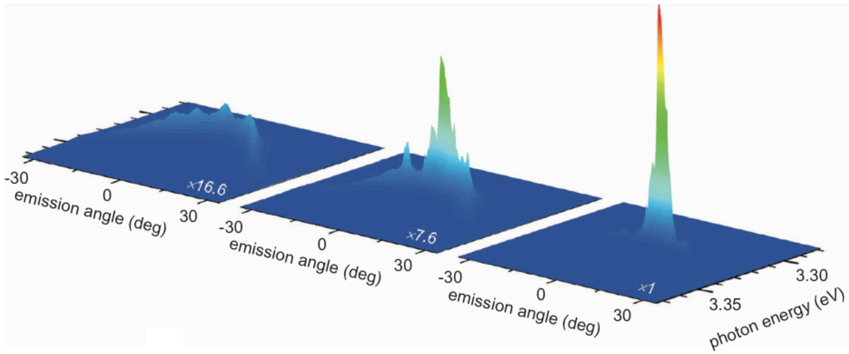
Polariton:

- mixed matter-light particle
- finite life-time → gain & loss physics “=“ driven dissipative
- boson



Condensate:

- macroscopic occupation of the ground state



[Franke et al., NJP **14**, 013037 (2012)]

- build up of long-range spatial correlations

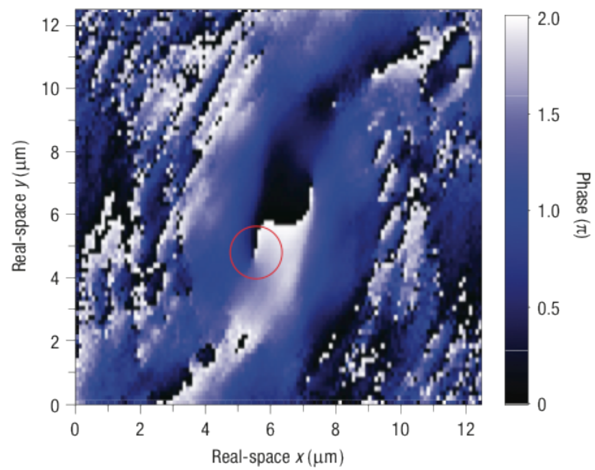
[Kasprzak et al., Nature **443**, 409 (2006), …]

See also: [Deng et al., Rev. Mod. Phys. **82**, 1489 (2010); Carusotto and Ciuti, Rev. Mod. Phys. **85**, 299 (2013)]

Polariton Condensate – Superfluidity?

Experimental Signatures

- suppression of scattering from defects
  [Amo et al., Nat. Phys. **5**, 805 (2009)]
- quantized vortices
  [Lagoudakis et al., Nat. Phys. **4**, 706 (2008); Sanvitto et al., Nat. Phys. **6**, 527 (2010)]



Vortex visible in a real space condensate phase profile
[Lagoudakis et al., Nat. Phys. **4**, 706 (2008)]

Driven vs. Equilibrium condensate

Effects of quantum and classical fluctuations

Similarities

- perturbatively in condensate fluctuations: correlation functions agree with equilibrium results (d=2,3)
  → long-range order (?)
  [Chiocchetta and Carusotto, Europhys. Lett. **102**, 67007 (2013); Roumpos et al., Proc. Natl. Acad. Sci. **109**, 6467 (2012)]
- low-frequency and long-wavelength effective thermalization, (d=3)
  [Sieberer et al., PRL **110**, 195301 (2013)]

Differences

- universal low-frequency decoherence,
  new dynamical critical exponent, (d=3)
  [Sieberer et al., PRL **110**, 195301 (2013); Sieberer et al., PRB **89**, 134310 (2014); Täuber and Diehl, PRX **4**, 021010 (2014)]
- no algebraic long-range order, exponential long-distance correlation function (d=2)
  [Altman et al., arXiv 1311.0876 (2013)]

- Experiments suggest superfluid behavior of polariton condensate
  [Lagoudakis et al., Nat. Phys. **4**, 706 (2008); Amo et al., Nat. Phys. **5**, 805 (2009); Sanvitto et al., Nat. Phys. **6**, 527 (2010)]
- Experiments involve a significant amount of disorder
  [Franke et al., NJP **14**, 013037 (2012)]
- Equilibrium: Superfluid—Bose-glass phase transition
  [Fisher et al., PRB 40, 546 (1989)]
- A driven dissipative condensate is different
  [Sieberer et al., PRL **110**, 195301 (2013), Altman et al., arXiv 1311.0876 (2013)]

Is a Driven Quantum Condensate in a Disordered Environment a Superfluid?

Superfluidity in a Driven Condensate

Steady state condensate wavefunction: $\Psi = \sqrt{n}\, e^{\mathrm{i}\phi - \mathrm{i}\omega t}$, $\vec{v}_s = \frac{\hbar}{m}\nabla\phi$

Landau criteria

excitations: $\delta\Psi_k \sim e^{\pm \mathrm{i}\omega_k t/\hbar - \gamma t}$

$$\omega_k = \sqrt{\frac{\hbar^2}{2m}k^2\left(\frac{\hbar^2}{2m}k^2 + 2n_0U\right) - (\hbar\gamma)^2}$$

→ Diffusive mode: $v_c = 0 \quad ???$

- generalized Landau criteria:
  → Superfluid behavior
- onset of drag force at

$$v_c \approx \sqrt{\frac{n_0 U}{m}}$$

[Wouters and Carusotto, PRL **105**, 020602 (2010)]; Amo et al., Nat. Phys. **5**, 805 (2009)]

Transverse Current response

Greens function:

$$G_k \sim \frac{1}{k^2} \qquad \text{for} \quad k \ll 1$$

→ expect superfluidity in principle

[Keeling, PRL **107**, 080402 (2011)]

Superfluid Stiffness

$$f_s = \frac{2mL^2}{\hbar^2}\lim_{\theta\to 0}\frac{\omega(\theta)-\omega(0)}{\theta^2}$$

“our aim”

Synchronization & Desynchronization

Driving & Disorder

Multi mode condensate

- Desynchronization
  [Wouters, PRB **77**, 121302 (2008);
  Eastham, PRB **78**, 035319 (2008)]

Single mode condensate

- Steady state (synchronized)
- Neglecting gain & loss:
  Superfluid—Bose-glass phase transition predicted for polaritons
  [Malpuech et al., PRL **98**, 206402 (2007)]

Model 1st part

Phenomenological Description of a Driven Dissipative Condensate

Extended Gross Pitaevskii Equation (eGPE):

$$\mathrm{i}\hbar\partial_t\Psi = \overbrace{\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{x}) + U\left|\Psi\right|^2\right)\Psi}^{\text{GPE – equilibrium condensate}} + \overbrace{\mathrm{i}\left(\hbar\gamma - \Gamma\left|\Psi\right|^2\right)\Psi}^{\text{Driving mechanism}}$$

condensate wavefunction $\Psi(\vec{x}, t)$

kinetic *mass*

external potential

interaction *onside potential constant*

gain *inscattering rate*

loss *gain depletion*

[Wouters and Carusotto, PRL **99**, 140402 (2007); Keeling and Berloff, PRL 100, 250401 (2008)]

Disorder Potential

Disorder potential:

- δ-correlated $\langle\langle V(\vec{x})\rangle\rangle = 0$
- Gaussian distributed $\langle\langle V(\vec{x})V(\vec{y})\rangle\rangle = V_0^2\ \delta\left(\frac{\vec{x}-\vec{y}}{\xi_V}\right)$

$$i\hbar\partial_t\Psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + \underbrace{V(\vec{x})}_{\text{disorder potential}} + U|\Psi|^2\right)\Psi + i\left(\hbar\gamma - \Gamma|\Psi|^2\right)\Psi$$

density	blue shift	healing length
$n_0 \equiv \frac{\hbar\gamma}{\Gamma}$	$n_0 U$	$\xi \equiv \sqrt{\frac{\hbar^2}{2m\ n_0 U}}$

Dimensionless eGPE

$$\mathrm{i}\partial_t \psi = \left(-\nabla^2 + \vartheta(\vec{x}) + |\psi|^2\right)\psi + \mathrm{i}\alpha\left(1 - |\psi|^2\right)\psi$$

- energy in units of blue shift
- length in units of healing length
- density in units of n_0

$$\psi(\vec{x},t) \equiv \frac{\Psi(\vec{x},t)}{\sqrt{n_0}} \;, \qquad \vartheta(\vec{x}) \equiv \frac{V(\vec{x})}{n_0 U}$$

$$\langle\langle \vartheta(\vec{x})\vartheta(\vec{y})\rangle\rangle = \kappa^2\ \delta\left(\vec{x} - \vec{y}\right)$$

Effectively two parameters

- disorder strength: $\kappa \equiv \dfrac{\xi_{\mathrm{V}} V_0}{\xi\, n_0 U}$
- non-equilibrium: $\alpha \equiv \dfrac{\Gamma}{U}$

Steady state ansatz: $\psi(\vec{x},t) = \sqrt{n(\vec{x})}\ e^{\mathrm{i}\phi(\vec{x}) - \mathrm{i}\omega t}$

Random sources and sinks

Hydrodynamic Equations

$$\omega = (\nabla\phi)^2 + \frac{1}{4}\frac{(\nabla n)^2}{n^2} - \frac{1}{2}\frac{\nabla^2 n}{n} + n + \vartheta$$

$$0 = \nabla(n\nabla\phi) + \alpha\ n(n-1)$$

condensate emission energy

continuity equation with sources and sinks

$n(\vec{x}) < 1$: local source

$n(\vec{x}) > 1$: local sink

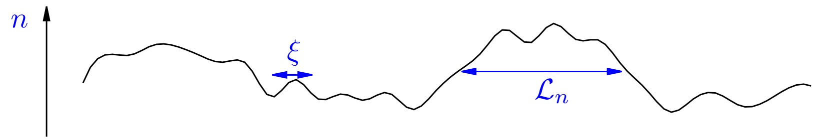
density fluctuations induce

random currents

Density and Phase Fluctuations

$\alpha = 0$

- Density fluctuations

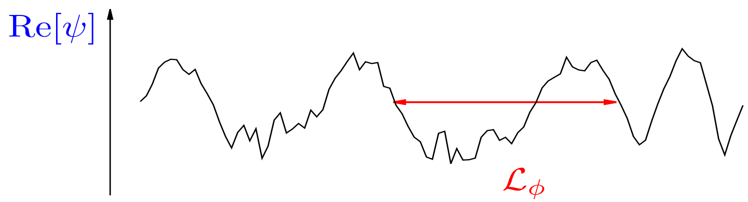


$$\text{Larkin length:} \quad \mathcal{L}_{\mathrm{n}} \sim \frac{1}{\kappa}$$

[Imry and Ma, PRL **35**, 1399 (1975); Nattermann and Pokrovsky, PRL **100**, 060402 (2008)]

$\alpha > 0$

- Phase fluctuations



$$\text{Phase correlation length:} \quad \mathcal{L}_\phi \sim \frac{1}{\alpha\kappa}$$

$$\text{Random currents:} \quad \nabla\phi \sim \frac{1}{\mathcal{L}_\phi}$$

Long-Range Order Destroyed by Phase Fluctuations

Condensate correlation function

$$\langle\!\langle \psi^*(\vec{x})\psi(\vec{0})\rangle\!\rangle \approx e^{-\frac{1}{2}\langle\!\langle\left(\phi(\vec{x})-\phi(\vec{0})\right)^2\rangle\!\rangle}$$

$$\sim e^{-\vec{x}^2/\mathcal{L}_\phi^2}$$

+ random sources and sinks
+ random currents
+ phase fluctuations

→ NO Long-Range Order

[AJ et al., PRL **111**, 230403 (2013)]

Stiffness – Perturbative results

Superfluid Stiffness

- probed by phase twist θ
- solve

$$\omega = (\nabla\phi)^2 + \frac{1}{4}\frac{(\nabla n)^2}{n^2} - \frac{1}{2}\frac{\nabla^2 n}{n} + n + \vartheta$$

$$0 = \nabla(n\nabla\phi) + \alpha\, n(n-1)$$

perturbatively

- evaluate stiffness

$$f_s = L^2 \lim_{\theta\to 0}\frac{\omega(\theta)-\omega(0)}{\theta^2}$$

Consider weak disorder limit

- κ << 1
- expansion of density, current and frequency in powers of κ

Stiffness – Perturbative results

Result:

$$f_s \approx 1 - \left\{c_1 + g_1(L)\,\alpha^2 + \left(g_2(L) + c_2 L^2\right)\alpha^4\right\}\kappa^2 + \mathcal{O}(\kappa^4)$$

$$\begin{aligned} c_1, c_2 &= \text{const} \\ g_1(L), g_2(L) &\sim -\log L \end{aligned}$$

$$f_s \approx 0 \quad \text{at} \quad \mathcal{L}_\mathrm{s} \sim \frac{1}{\alpha^2\kappa}$$

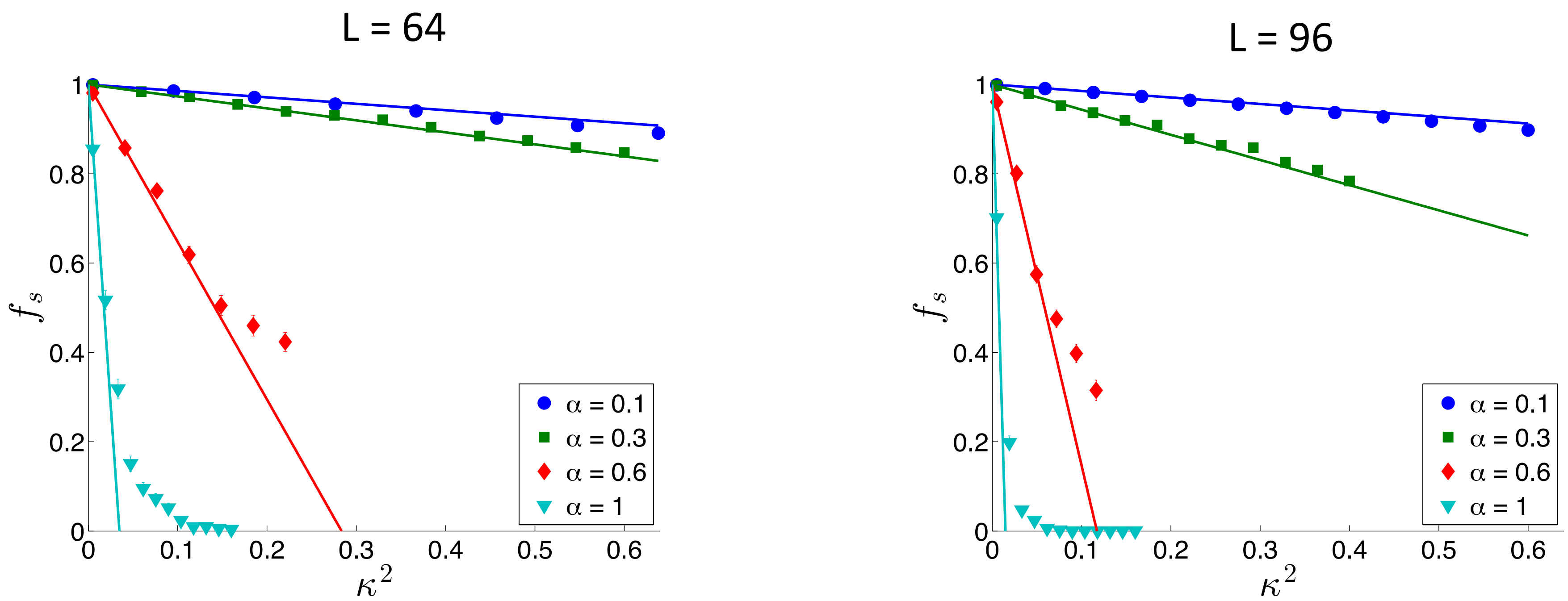
- reproduces equilibrium results for α = 0 [Meng, PRB **49**, 1205 (1994)]
- for finite α > 0, divergence in thermodynamic limit for any κ > 0

- fastest divergence sets superfluid depletion length

Expect:

- superfluid behavior for $L < \mathcal{L}_\mathrm{s}$
- no superfluid for $L \to \infty$

Stiffness – Perturbative vs Numeric results



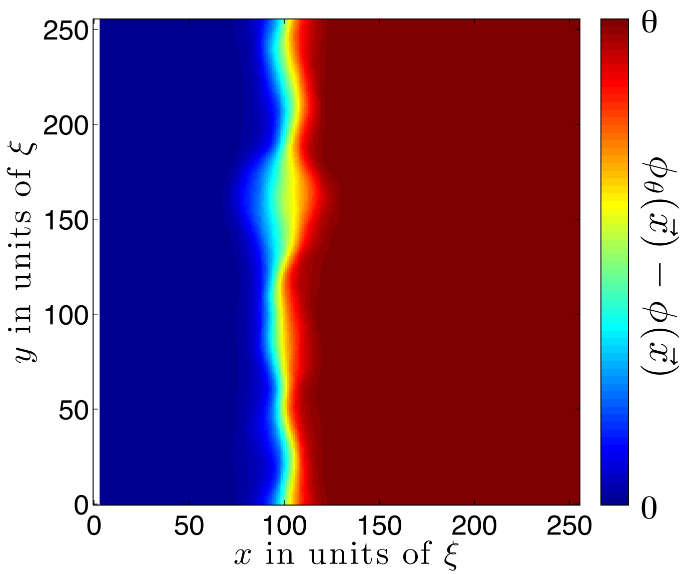
- for $f_s \lesssim 1$: perturbative result agrees with numerics for all α
- for $f_s \approx 0$: discrepancy for α ≈ O(1) even if κ << 1

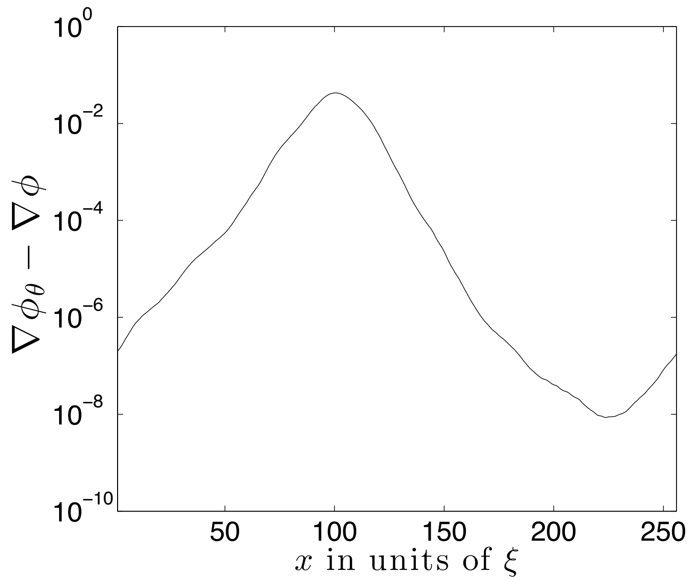
→ Condensate response for $\mathcal{L}_{\rm s} \gg L$?

→ Mechanism of superfluid depletion ?

Domain Wall Formation

- twist θ along x-direction
- phase response: $\phi \xrightarrow{\vec{A}_\theta} \phi_\theta \ , \qquad \nabla\phi \xrightarrow{\vec{A}_\theta} \nabla\phi_\theta$
- one disorder realization with $\mathcal{L}_\mathrm{s} \ll L$





→ response exponentially localized

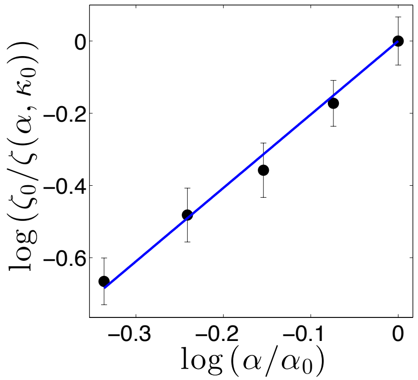
→ no superfluidity

Domain Wall Thickness

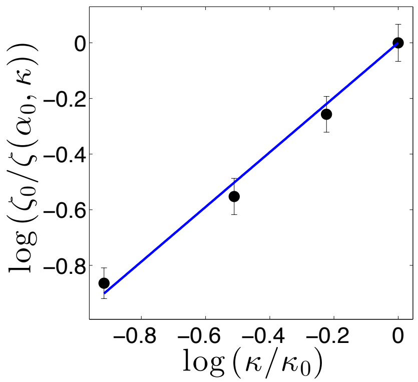
Ansatz: $$\nabla\phi_\theta - \nabla\phi = \frac{\theta}{2\zeta(1-e^{-\frac{L}{2\zeta}})} e^{-\frac{|x-x_0|}{\zeta}} \vec{e}_\theta = \begin{cases} \frac{\theta}{L}\vec{e}_\theta & \text{for } \zeta \to \infty \\ \theta\delta(x-x_0)\vec{e}_\theta & \text{for } \zeta \to 0 \end{cases}$$

decay length ~ thickness; x_0 – random position

scaling



$$\to \zeta \sim \frac{1}{\alpha^2}$$



$$\to \zeta \sim \frac{1}{\kappa}$$

$$\zeta \sim \frac{1}{\alpha^2\kappa} \sim \mathcal{L}_\mathrm{s}$$

Generic properties of the Domain wall:

- randomly pinned
- thickness is set by superfluid depletion length

Stiffness – Scaling

- vanishing stiffness explained by domain wall formation
- $\mathcal{L}_\mathrm{s}$ is only relevant length scale

→ $f_s \sim f_s(L/\mathcal{L}_\mathrm{s}) \quad \text{for} \quad L \gg \mathcal{L}_\mathrm{s}$?

Scaling ansatz:

$$f_s = e^{-c_2\,\alpha^4\kappa^2 L^2}(1-g) \sim e^{-L^2/\mathcal{L}_\mathrm{s}^2}$$

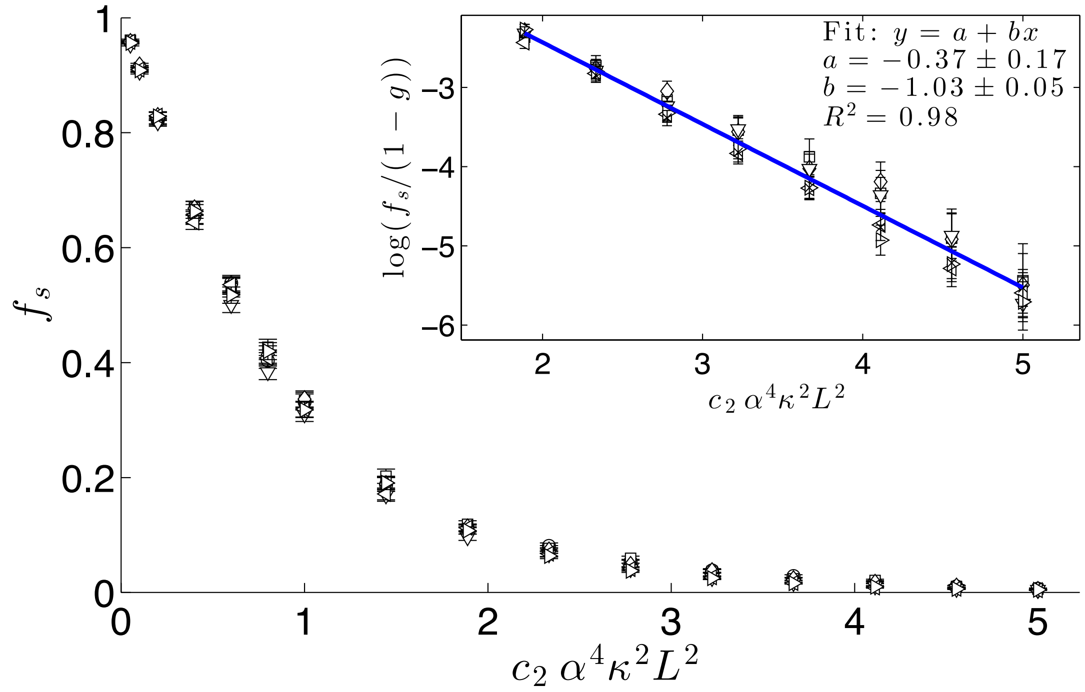
c_2 from perturbation theory

logarithmic correction $g(\alpha, \kappa, \log L)$

- reproduces perturbative results if expanded in κ

Stiffness – Scaling

Check ansatz by numerical simulations:



$$f_s = e^{-c_2\,\alpha^4\kappa^2L^2}(1-g) \sim e^{-L^2/\mathcal{L}_s^2}$$

✓

[AJ et al., PRL **111**, 230403 (2013)]

Generalization to arbitrary dimensions

Perturbatively calculated stiffness:

leading divergence

$$f_s \approx 1 - \ldots - c\ \alpha^4 \kappa^2 L^{4-d}$$

$$c\ \ -\mathrm{const.}$$

Superfluid depletion length:

$$\mathcal{L}_\mathrm{s} \sim \left(\frac{1}{\alpha^2 \kappa}\right)^{\frac{2}{4-d}}$$

- upper critical dimension $d_c = 4$
- expect depletion of superfluidity for all $d < d_c$

Summary

Driven Disordered Condensate

- no quasi long-range order

$$\langle\!\langle \psi^*(\vec{x})\psi(\vec{0})\rangle\!\rangle \sim e^{-\vec{x}^{\,2}/\mathcal{L}_\phi^2}$$

$$\mathcal{L}_\phi \sim \frac{1}{\alpha\kappa}$$

$\alpha -$ driving
$\kappa -$ disorder
$\psi -$ condensate

- no superfluid in the thermodynamic limit

- superfluid behavior for length scales

$$L \ll \mathcal{L}_\mathrm{s}$$

$$\mathcal{L}_\mathrm{s} \sim \frac{1}{\alpha^2\kappa}$$

- exponential depletion of stiffness

$$f_s \sim e^{-L^2/\mathcal{L}_\mathrm{s}^2}$$

- domain wall formation as response to phase twist

[AJ, Hyart, Eastham, Rosenow, PRL **111**, 230403 (2013)]

Acknowledgement

I like to thank …

… my co-workers

Timo Hyart

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… my institute

ITP

Institut für Theoretische Physik

UNIVERSITÄT LEIPZIG

… my graduate school

BuildMoNa

… and you!

Phase Fluctuations – Phase Correlation Length

Generalized Imry-Ma argument

- integration of non-equilibrium continuity equation over volume $\Omega_\phi \sim \mathcal{L}_\phi^d$

$$\int_{\Omega_\phi} \nabla(n\nabla\phi) = \alpha \int_{\Omega_\phi} n(1-n)$$

$$\int_{\partial\Omega_\phi} \cdot\, (n\nabla\phi) = \int_{\partial\Omega_\phi} \cdot\, \nabla\phi \approx \mathcal{L}_\phi^{d-1}\frac{1}{\mathcal{L}_\phi} \sim$$

fluctuation at scale ξ

$n \approx 1 + \mathcal{O}(\kappa)$

$\xi \ll \mathcal{L}_\phi$

$$\sim \alpha\kappa\sqrt{\left(\frac{\mathcal{L}_\phi}{\xi}\right)^d}$$

boundary current ξ = 1 (dim. less)

$$\mathcal{L}_\phi \sim \left(\frac{1}{\alpha\kappa}\right)^{\frac{2}{4-d}}$$

- gives phase fluctuation length
- result agrees with perturbation theory for κ << 1

Supplemental Material:

Simulation parameters

Domain wall, single snap shot, page 16: $L = 256\ a_L$; $\alpha = 0.5$; $\kappa = 0.5$; $\Theta = 1$; $a_L = \xi$

Domain wall scaling, page 17: $L = 256\ a_L$; $\alpha_0 = 0.7$; $\kappa_0 = 0.25$; $\Theta = 1$; $a_L = \xi$; 72 disorder realization

Stiffness scaling, page 19: $L = 64{,}96\ a_L$; $\alpha = 0.9, 1, 1.2$; $a_L = \xi$; up to 1320 disorder realization