2583-2

Workshop on Coherent Phenomena in Disordered Optical Systems

26 - 30 May 2014

Measurement of the Mobility Edge for 3D Anderson Localization

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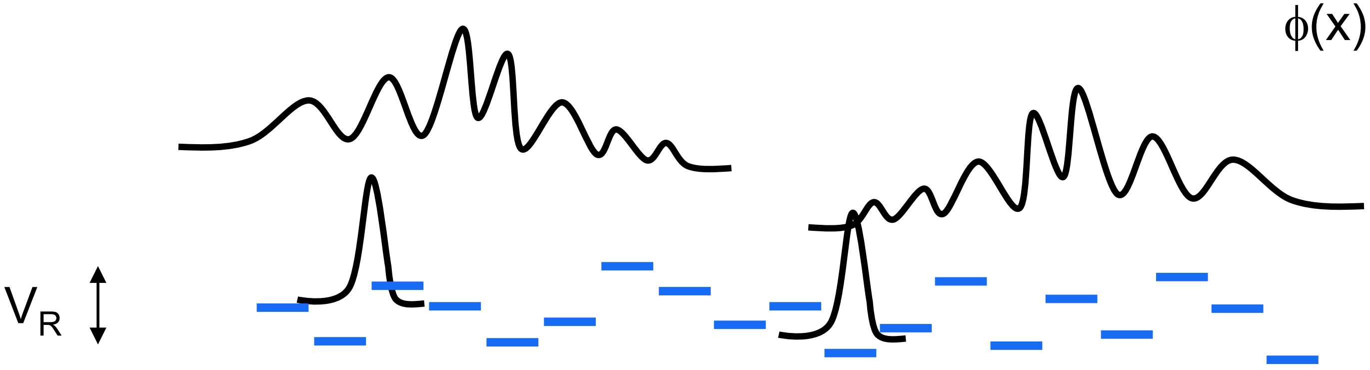
ICTP, May 2014

Anderson localization

$\phi(x) \sim \exp(-|x|/\xi)$



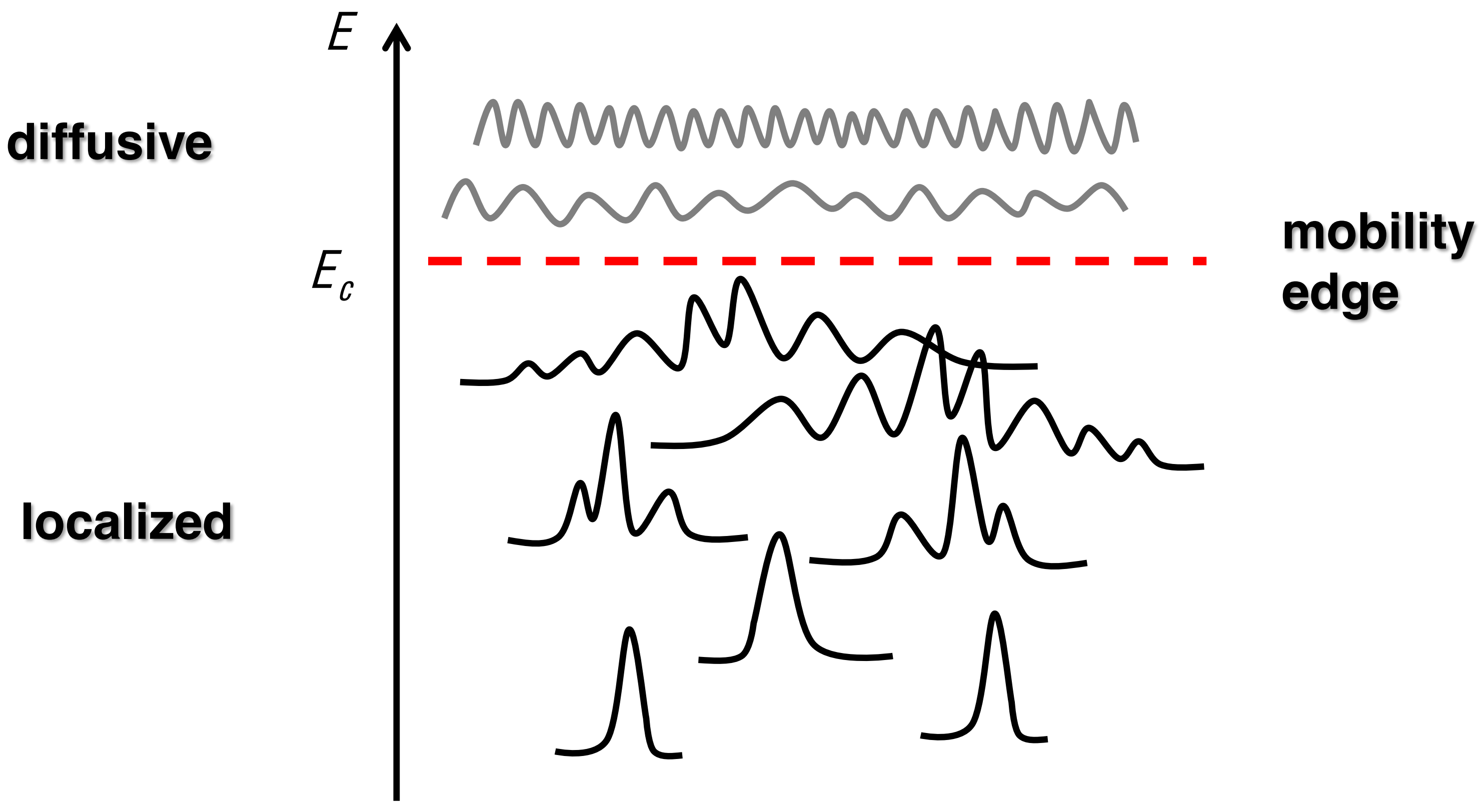
a disordered lattice: $\hat{H} = -J\sum_{\langle i,j\rangle} \hat{b}_i^{+}\hat{b}_j + \sum_i \varepsilon_i \hat{n}_i$

coherent backscattering leads to destructive interference and localization

In 3D there is a critical disorder strength (or energy) for localization

Anderson, Phys. Rev. 109, 1492 (1958); G. Feher, and E. A. Gere, Phys. Rev. 114, 1245 (1959).

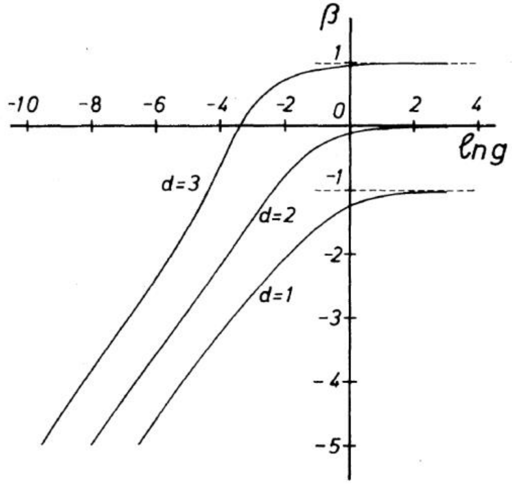
3D Anderson localization



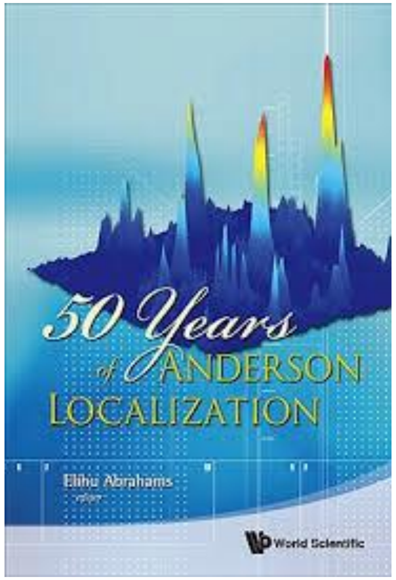
Only a single mobility edge is possible, since any isolated extended state would be unstable

50 years of theory

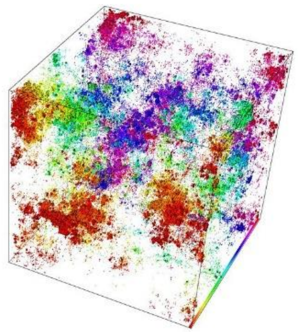
- A mobility edge exists for any d>2.
- The metal-insulator transition is a quantum phase transition (localization length, diffusion coefficient).
- Analytical methods (self-consistent theory), but require approximations, and depend on the disorder type.



Abrahams, Anderson, Licciardello, Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).



- Numerical simulations in 3D are possible but hard.
- Interactions change completely the problem.



Anderson localization in condensed matter

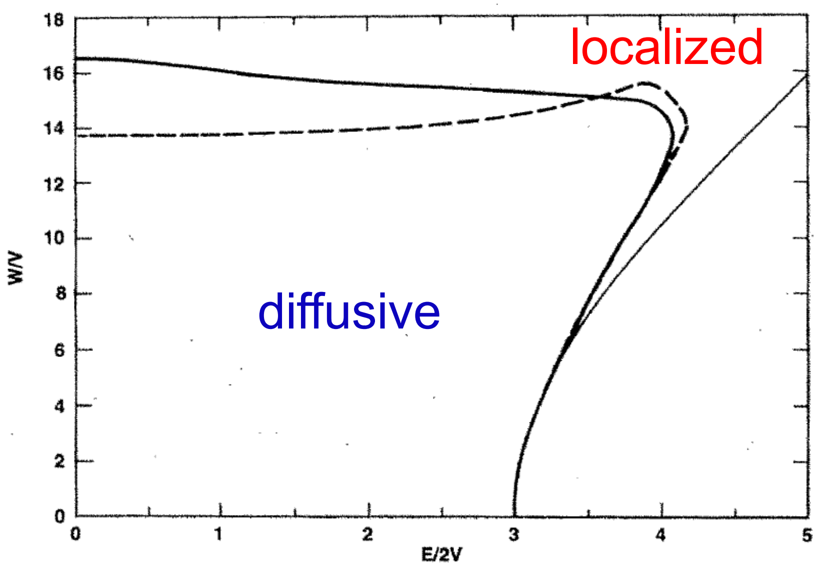


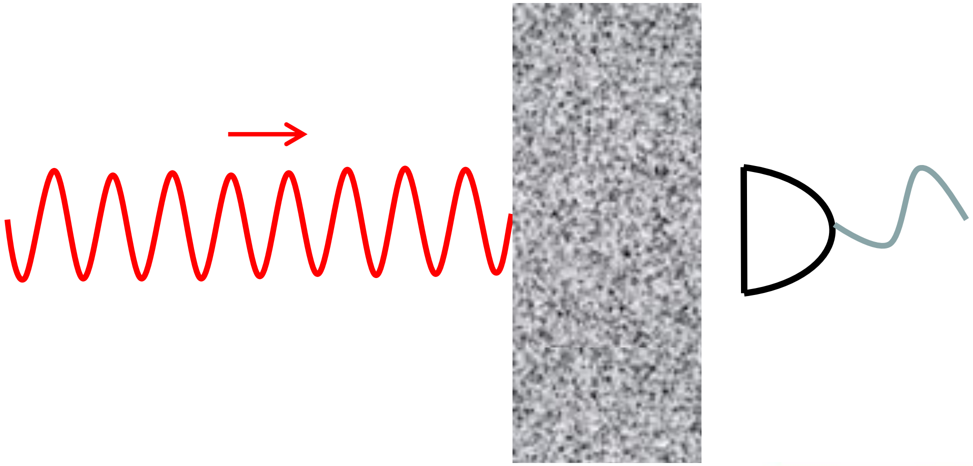
FIG. 5. Trajectory of the mobility edge for a simple cubic lattice as predicted by the present work (solid line) and by the $L(E)$ method (dashed line). The CPA band edges are also indicated (thin solid line).

The 3D Anderson model on a lattice is perfectly under control.

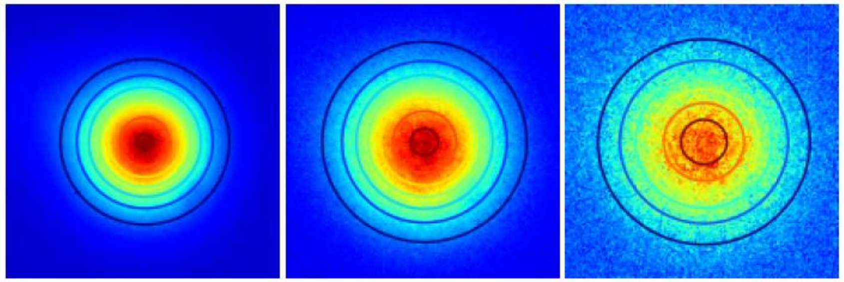
But the physics of electrons is enormously complicated by interactions!
Is there another physical system where to study the whole problem?

Vollhardt & Wölfle, Phys. Rev. Lett. 48, 699 (1982); Economou & Soukoulis, Phys. Rev. B 28.1093 (1983); Kramer & MacKinnon, Rep. Prog. Phys. 56, 1469-1564 (1993); Slevin & Ohtsuki, New J. Phys. 16, 015012 (2014).

Experiments: waves



Light or sound waves with controlled momentum



A test of the Ioffe-Regel criterion:
at the mobility edge, the mean free path is of the order of the wavelength.

$$k\ell \approx 1$$

Sperling, et al. Nat. Photonics (2012); Hu et al, Nat. Physics 4, 945 (2008).

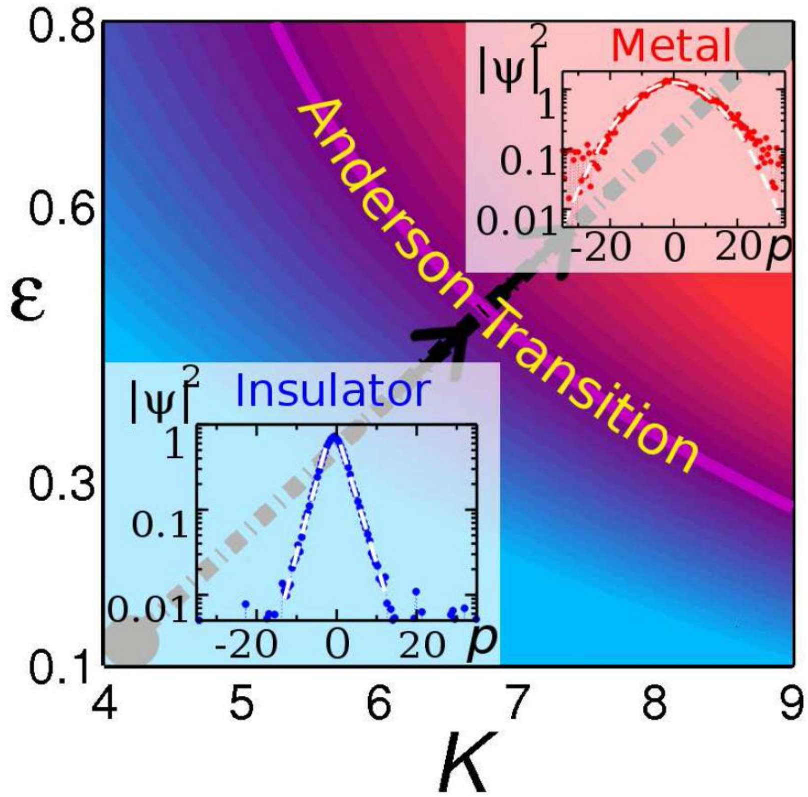
Experiments: waves

Kicked rotor: a controlled realization of the 3D Anderson Hamiltonian in momentum space

$H = p^2/2 + K\cos(x)[1+\varepsilon\cos(\omega_2 t)\cos(\omega_3 t)] \sum_n \delta(t-n)$

Very good for measuring critical properties,

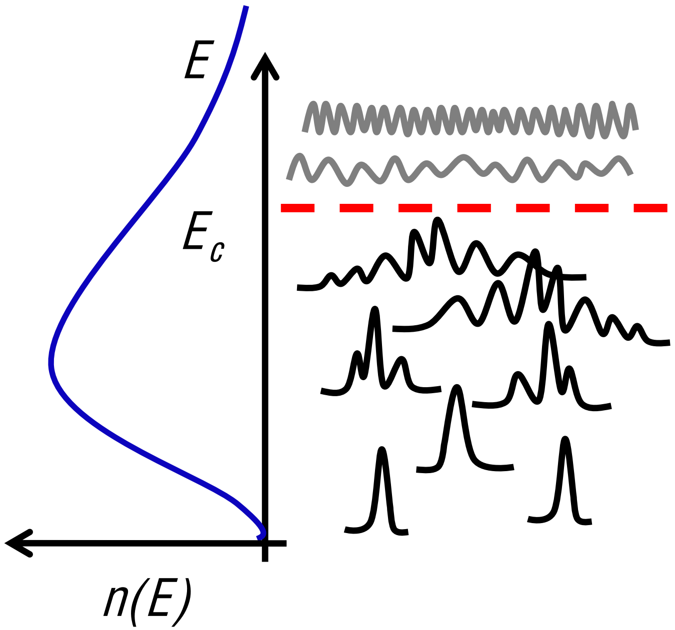
but condensed-matter studies are hard.

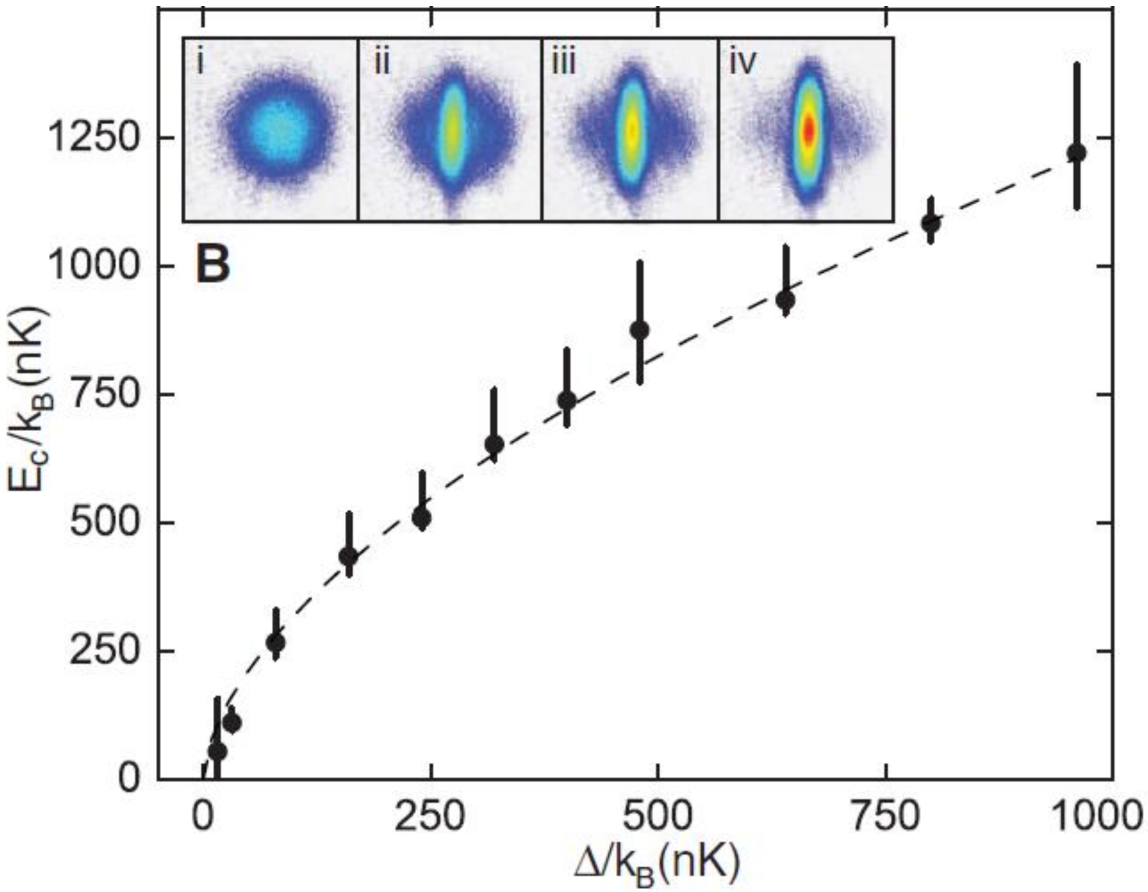


Chabé et al. Phys. Rev. Lett. 101, 255702 (2008).

Experiments: particles

Ultracold non-interacting fermions loaded adiabatically in laser speckles



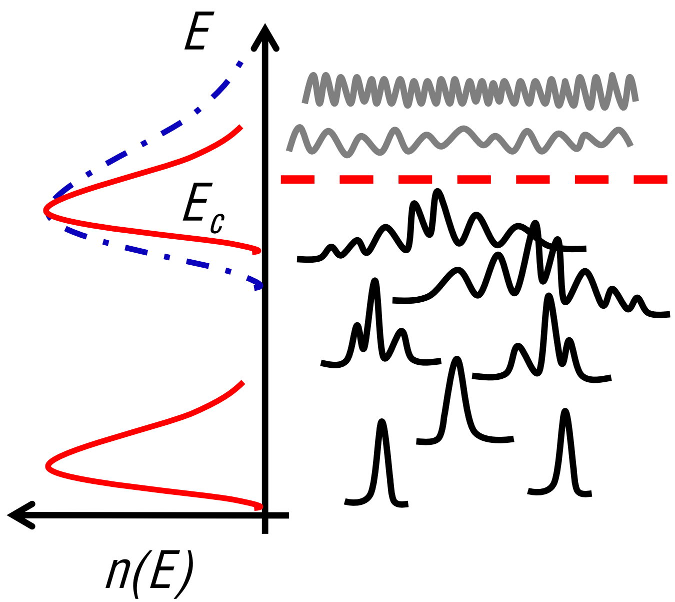


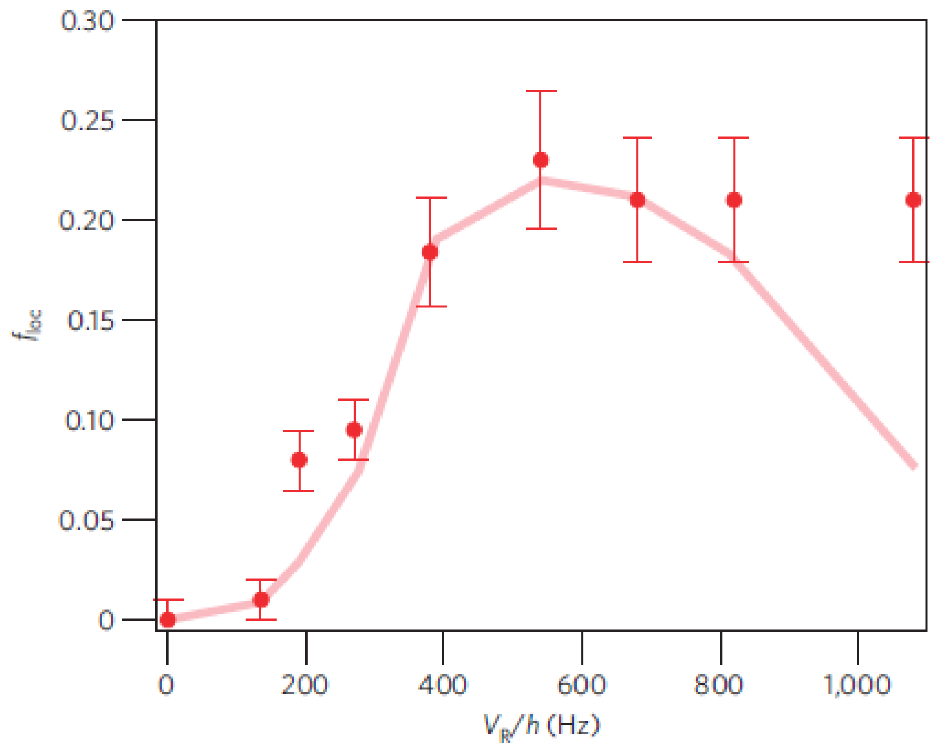
Kondov et al, Science 334,63 (2011)

Broad energy distribution (not affected by disorder?), fast dynamics, large mobility edge ($E_c \sim 2V_R$).

Experiments: particles

Bose-Einstein condensate loaded abruptly in laser speckles





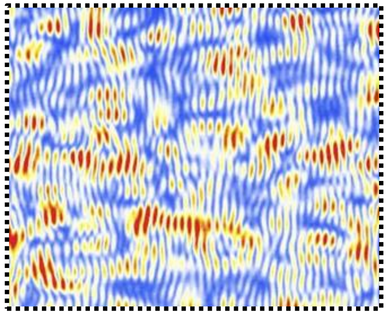
F. Jendrzejewski et al, Nat. Physics 8, 398 (2012)

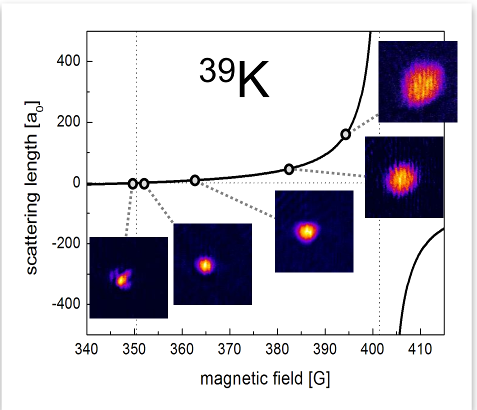
Very slow dynamics and careful determination of localized fraction;
The estimation of the mobility edge will require a comparison with the theoretical energy distribution.

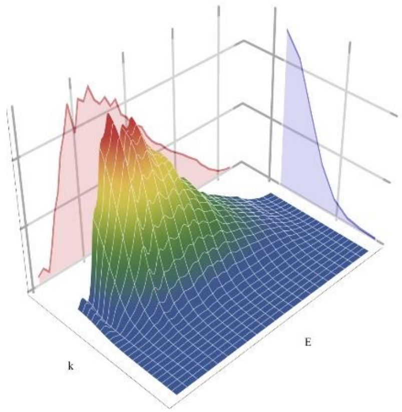
3D Anderson localization in Florence

Combining the best of all previous approaches, it is possible to measure Ec for a controlled, condensed-matter like system.

1) 3D isotropic disorder (crossed speckles as in Palaiseau)
2) Prepare a narrow wavepacket (a ^{39}K BEC with tunable interactions)
3) Measure its total energy
4) Excite it spectroscopically to the mobility edge

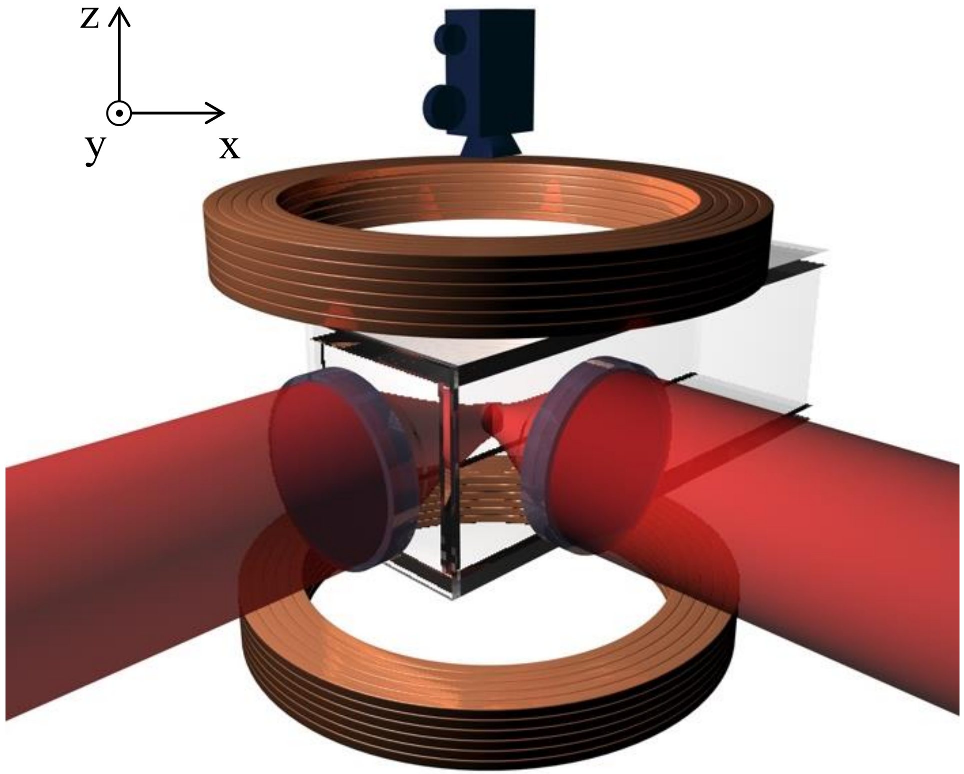




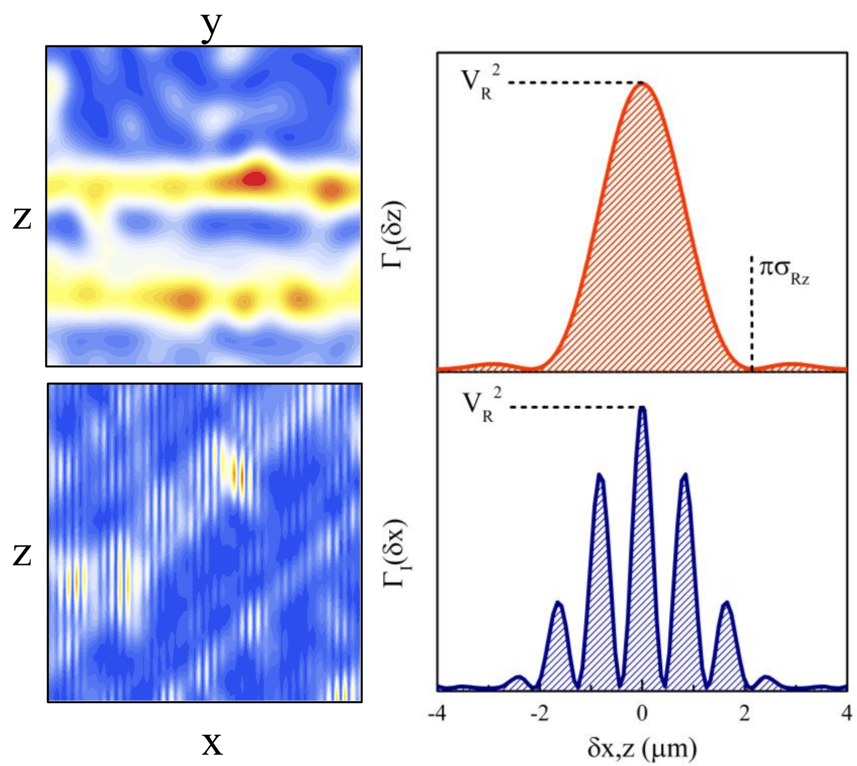


Semeghini, Landini et al., arXiv:1404.3528

3D speckles disorder

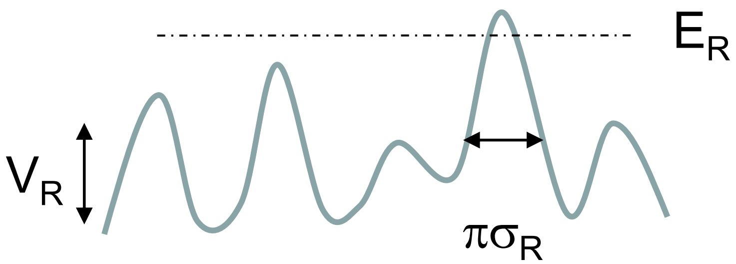


$g(x) = \langle V(r)V(r+x) \rangle$

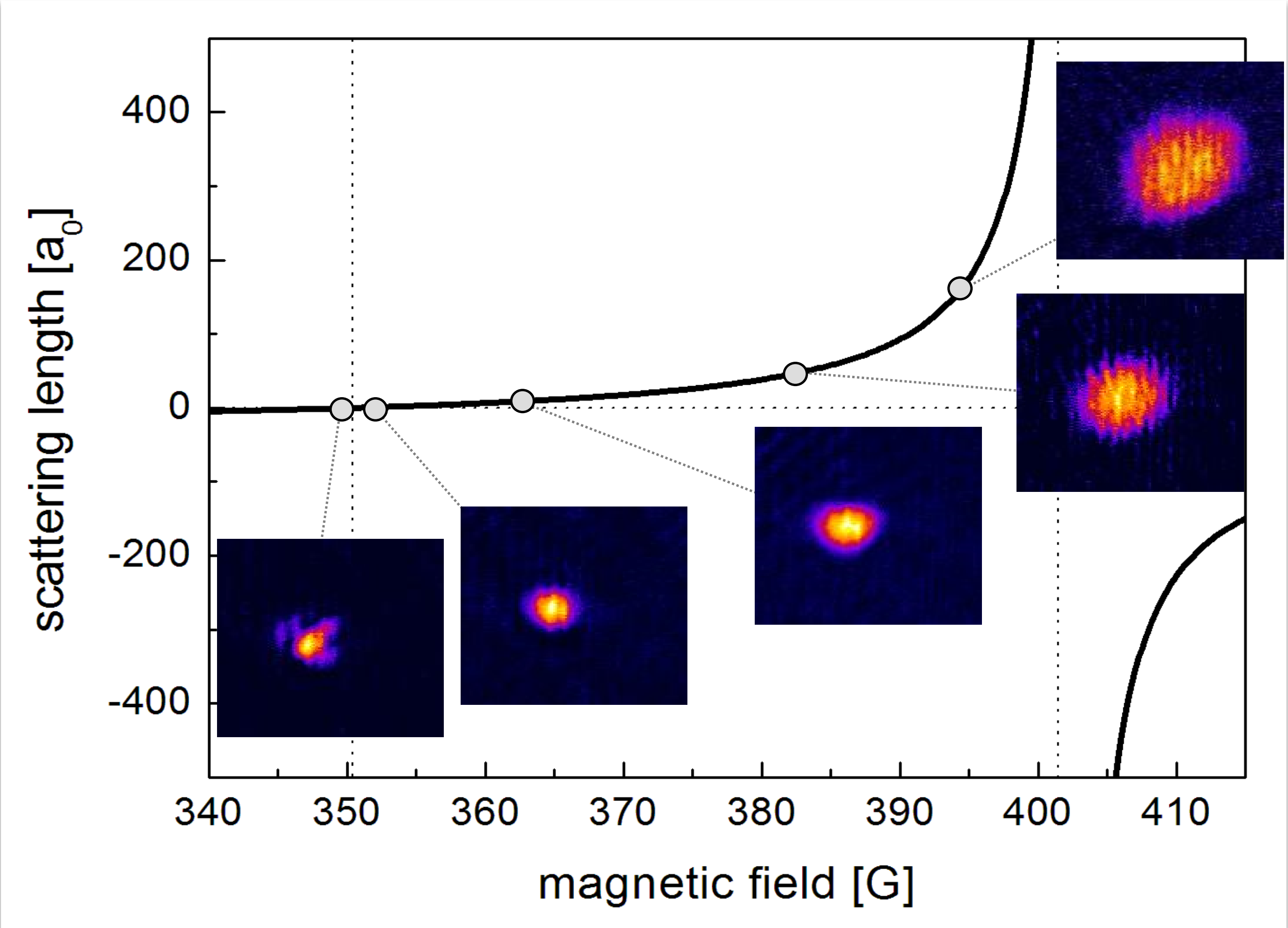


correlation energy: $E_R = \hbar^2/m\sigma_R^2 \approx 70\text{nK}$

same scheme as in Palaiseau
(thanks to V. Josse for suggestions)

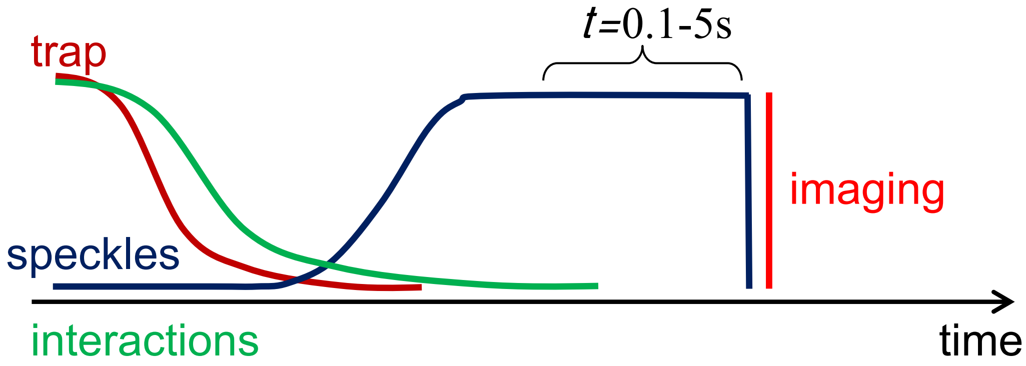


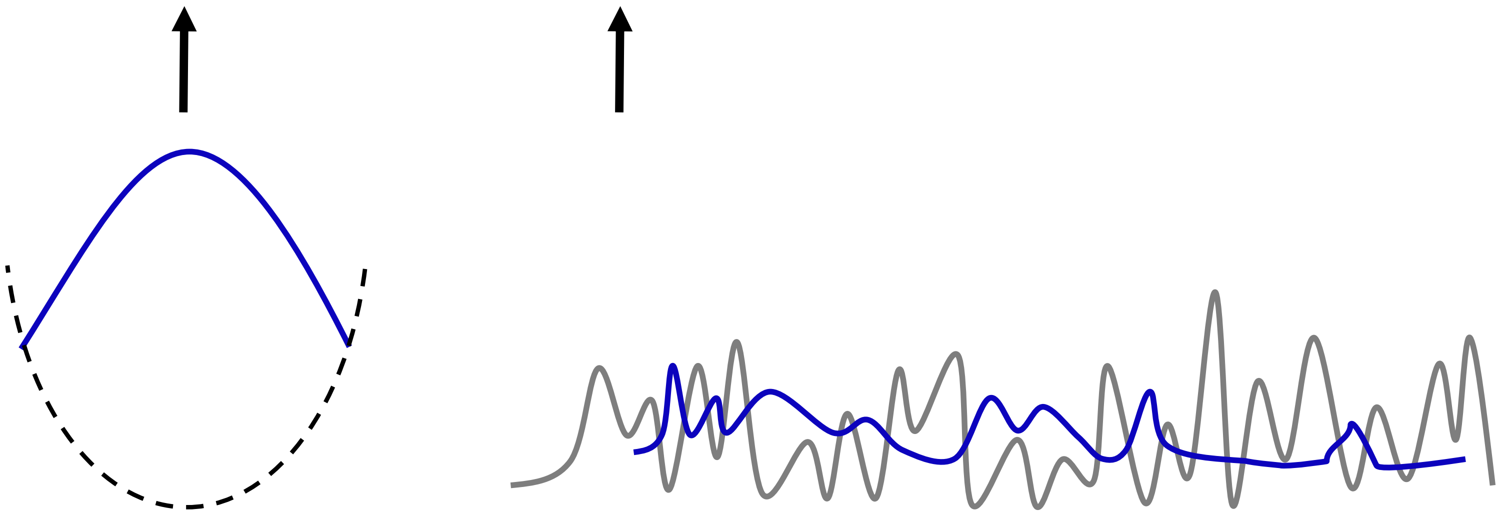
Tunable contact interactions via Feshbach resonances



Contact interaction: $$U(r) \approx \frac{2\pi\hbar^2}{m} a\delta(r)$$

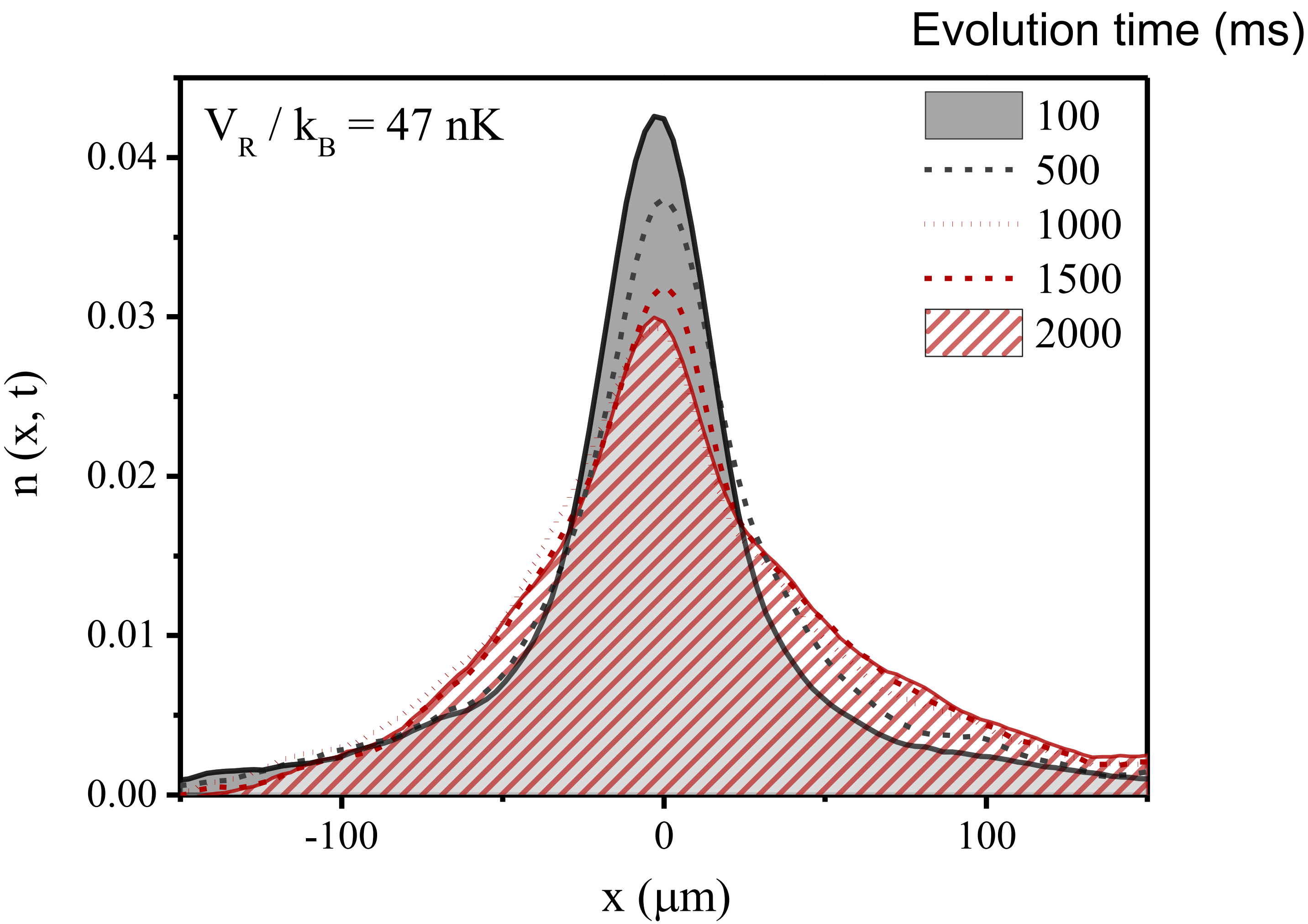
Quasi-adiabatic preparation





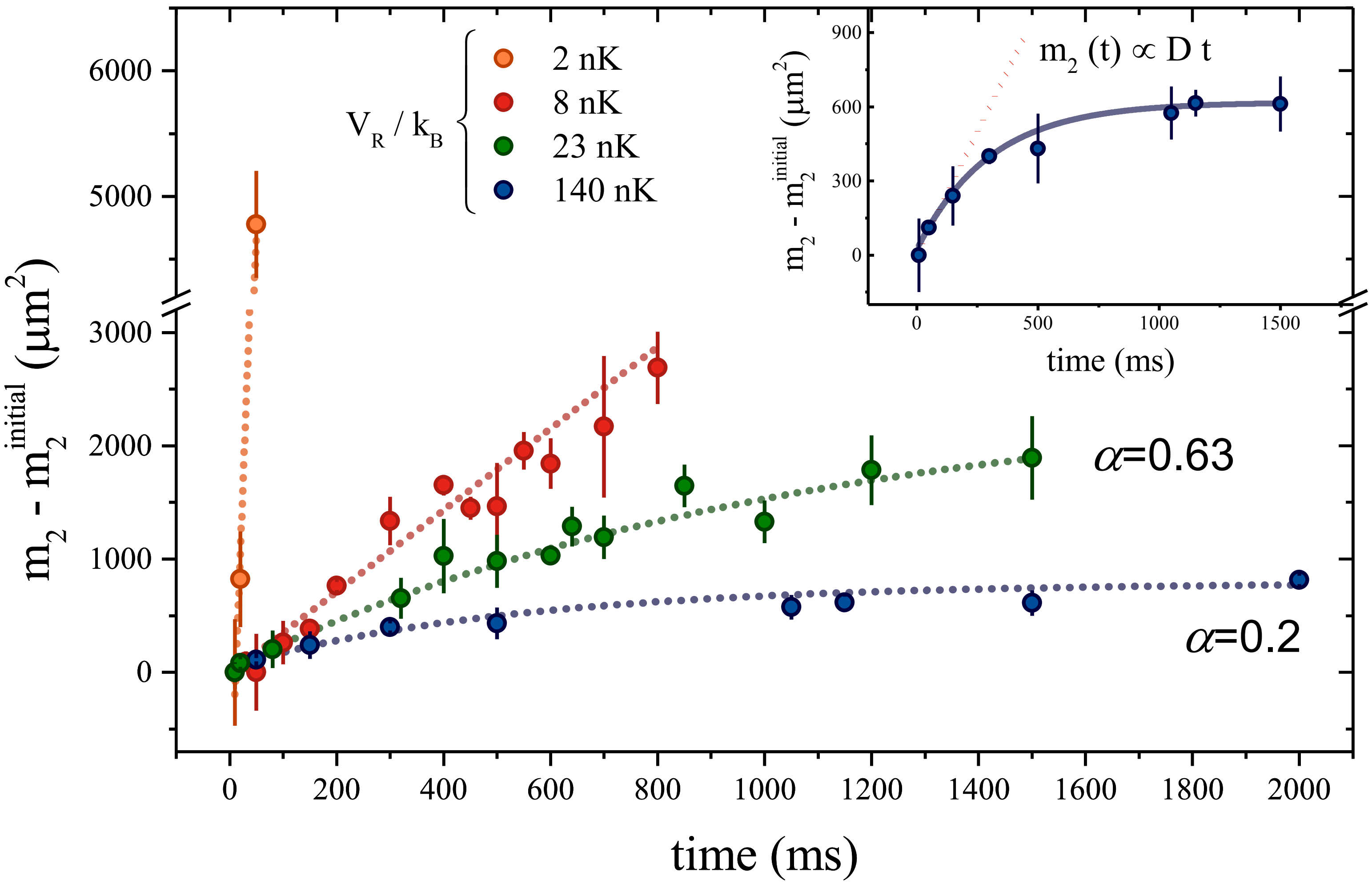
Optimized by minimizing the kinetic energy

Diffusion vs localization



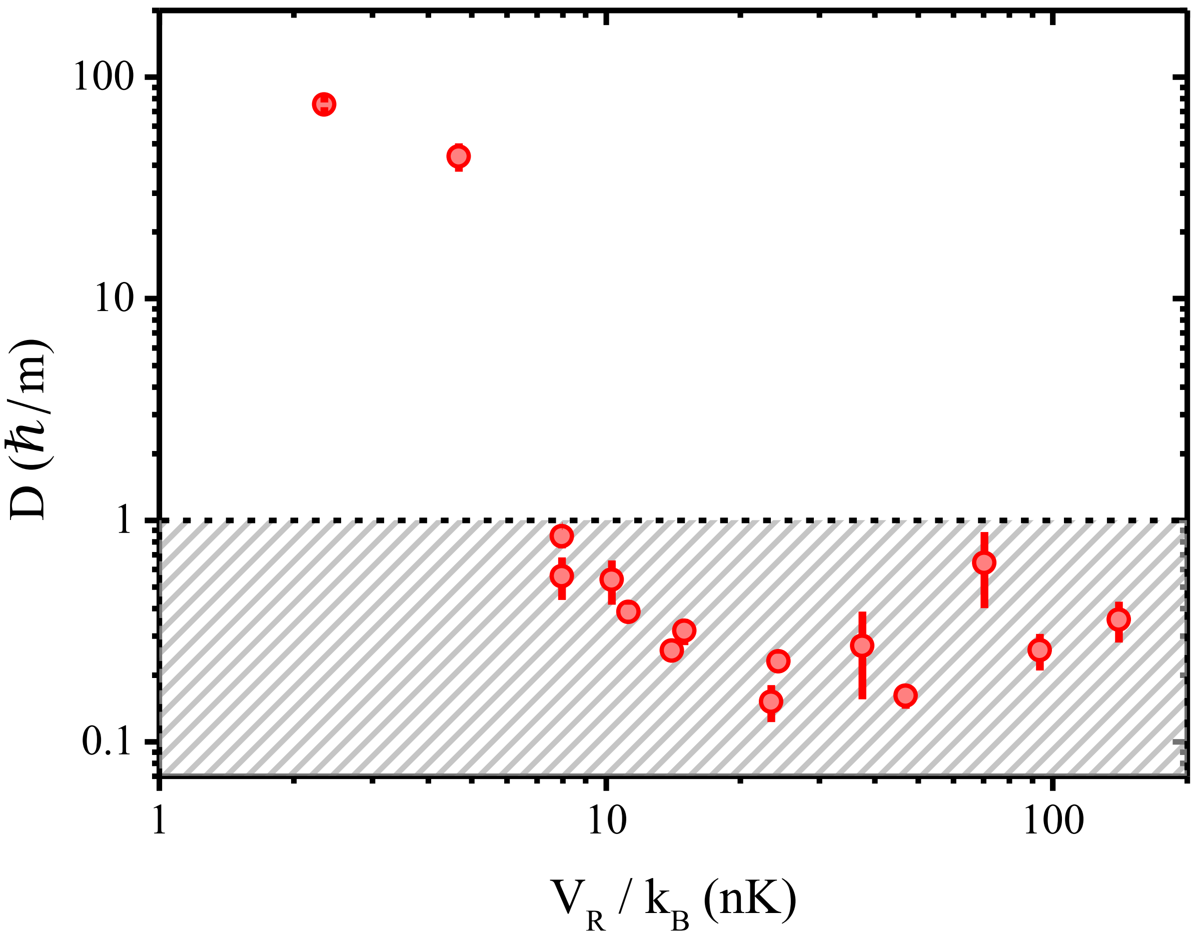
Doubly integrated spatial distributions

Diffusion vs localization



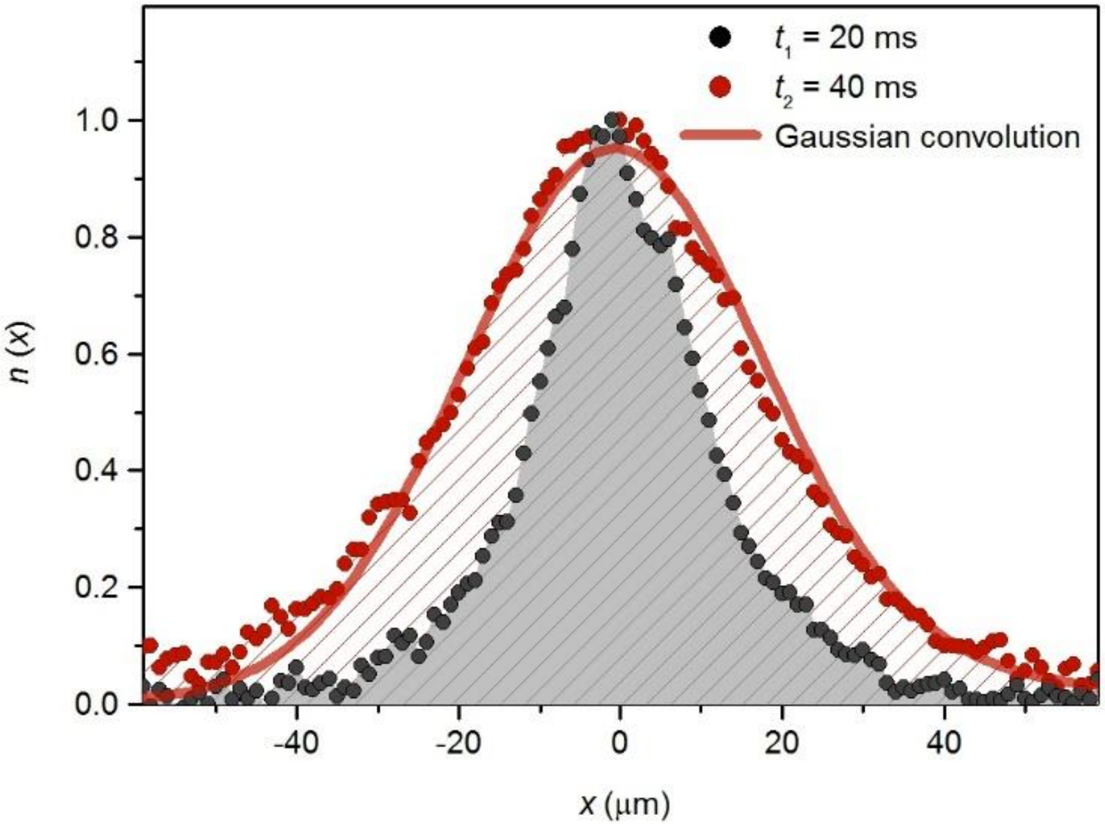
$m_2(t) = \widetilde{D} t^{\alpha}$ anomalous diffusion at criticality α =2/3

Diffusion vs localization



Weak-localization theory: at $D = \hbar/m$ interference effects become dominant

Momentum distribution



Essentially gaussian n(k), with small kinetic energy: ~10 nK

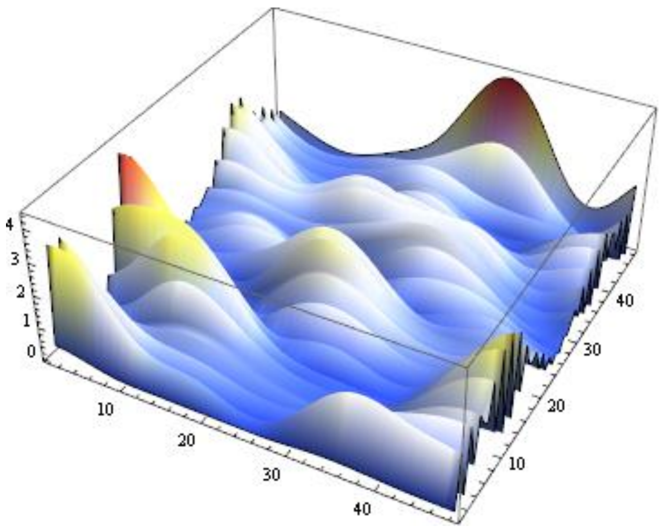
$$n(k) = \int \rho(E,k)\, f(E)\, dE \qquad\qquad n(E) = \int \rho(E,k)\, f(E)\, dk$$

$\rho\,(E,k)$ is the probability of having a momentum k at an energy E

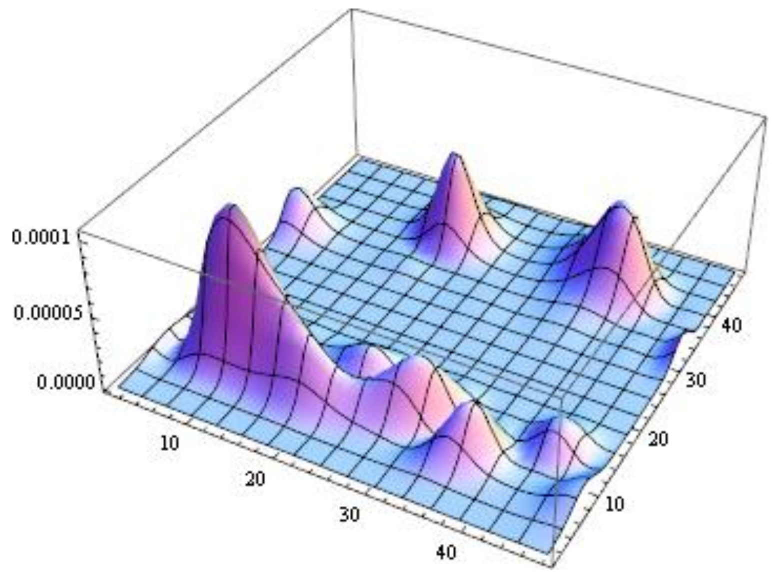
Numerical diagonalization of small systems

- L=12 μm
- dL=0.25 μm
- 300 eigenstates
- >50 different speckles

- eigenstates in real and momentum space
- partecipation ratio
- density of states

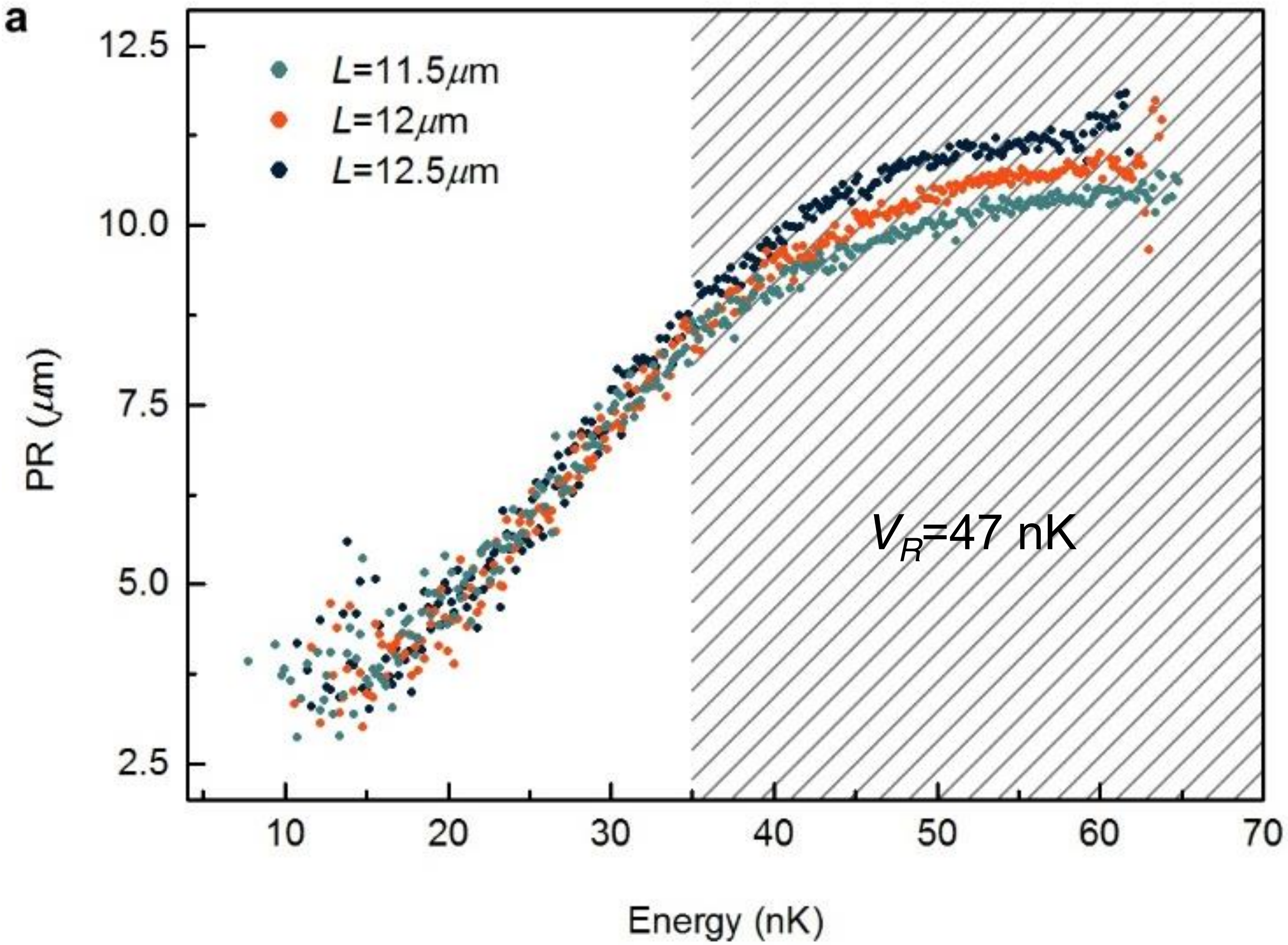


2D section of synthetic speckles



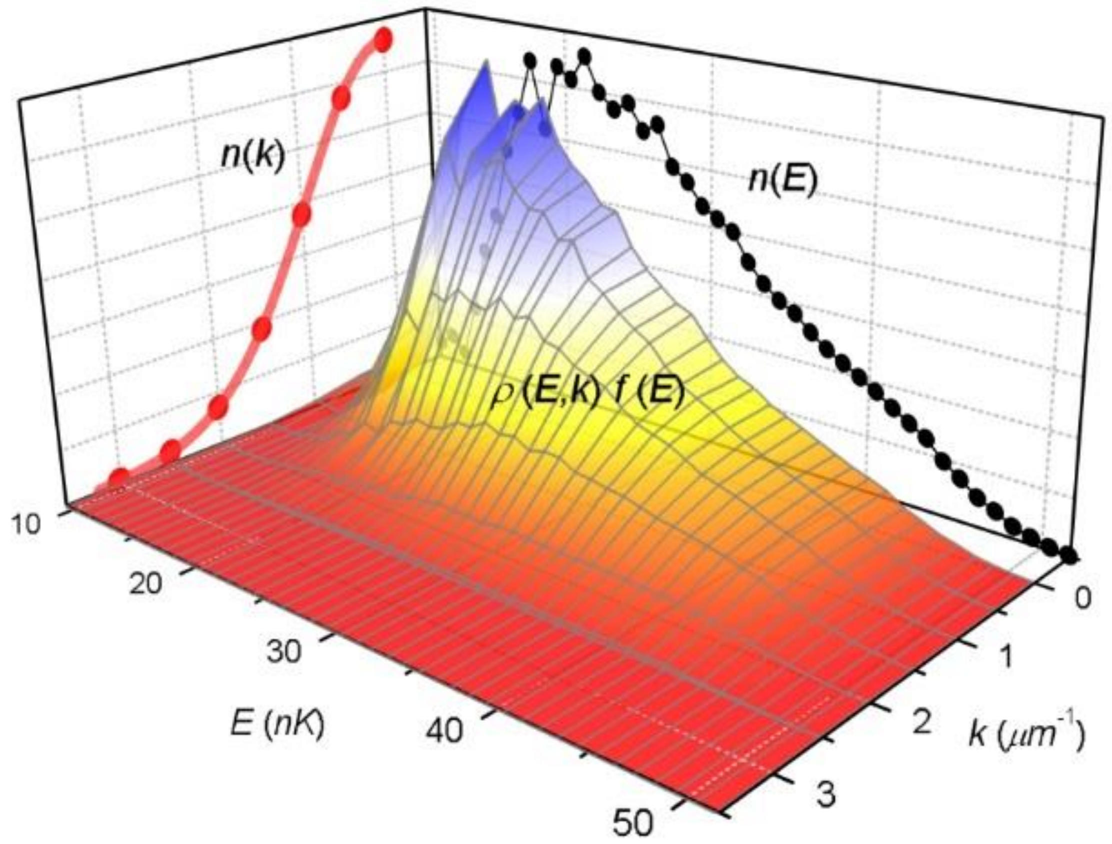
2D section of an eigenstate

Numerical diagonalization of small systems



Reliable only up to E∼0.75 V_R

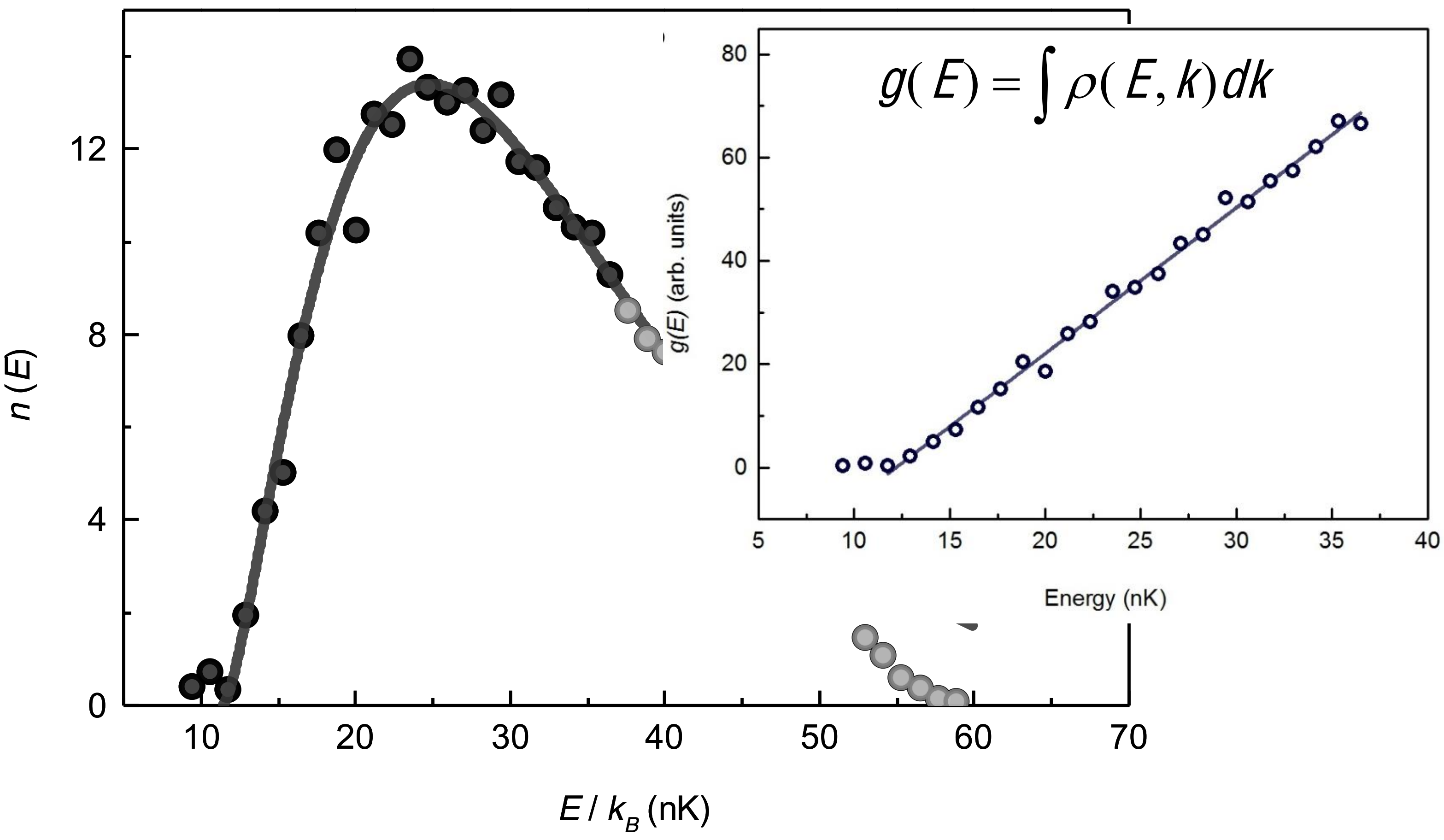
Energy distribution from momentum distribution



$$n(k) = \int \rho(E,k)\, f(E)\, dE \qquad n(E) = \int \rho(E,k)\, f(E)\, dk$$

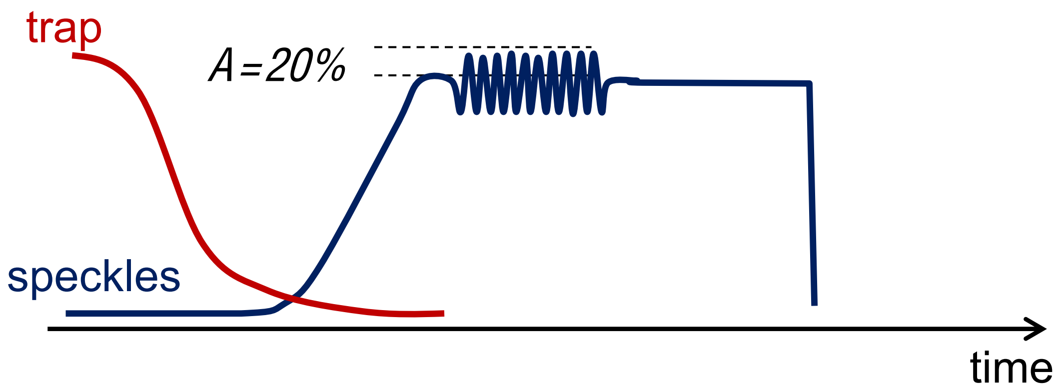
$$f(E) = \exp(-(E - E_0)/E_m)$$

Energy distribution from momentum distribution



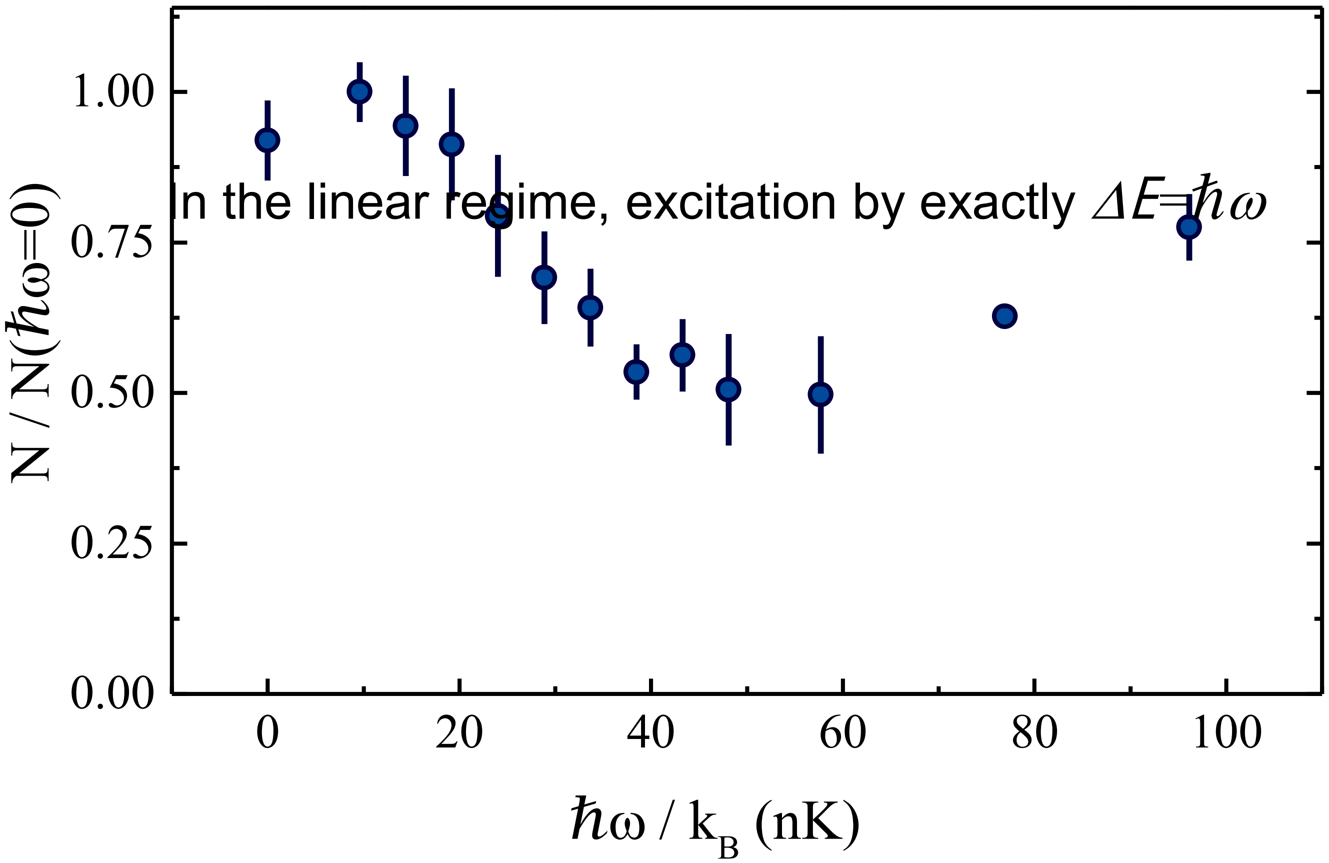
$$n(E) = g(E)\, f(E) = E^{\beta} \exp(-(E - E_0)/E_m) \qquad \beta \approx 1$$

Excitation spectroscopy

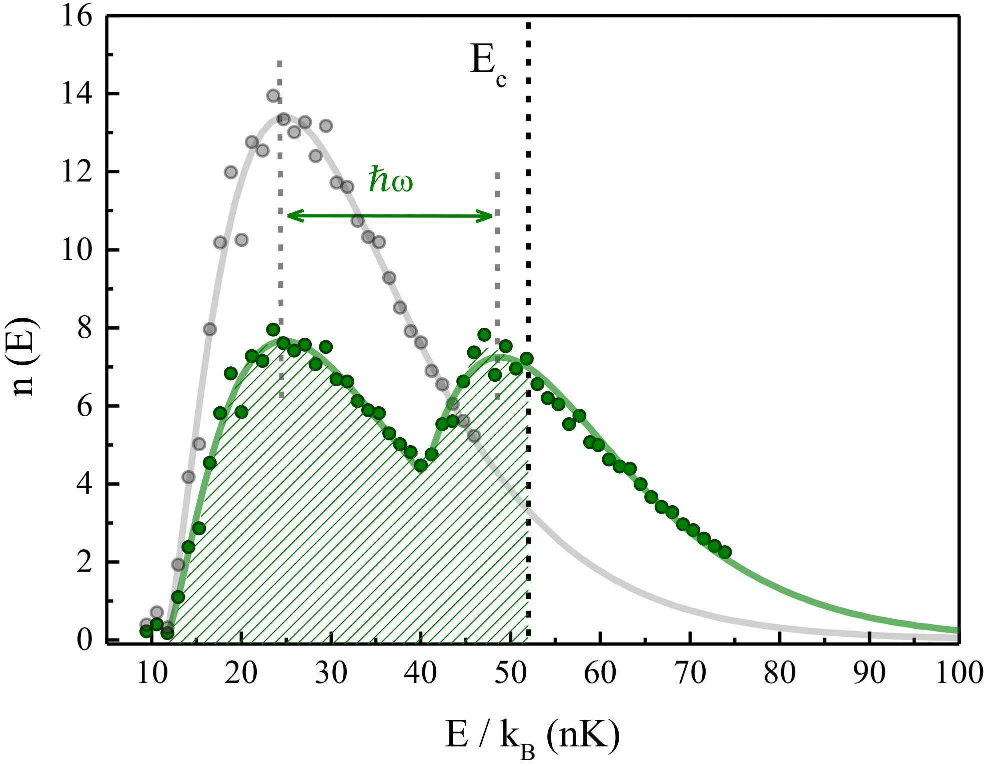


$$V_R(x,t) = V_R(x)(1 + A\cos(\omega t))$$

In the linear regime, excitation by exactly $\Delta E = \hbar\omega$



Excitation spectroscopy



Fitting model for the mobility edge:

$$N(\omega) = \int_{E_0}^{E_c} n(E,\omega)\, dE$$

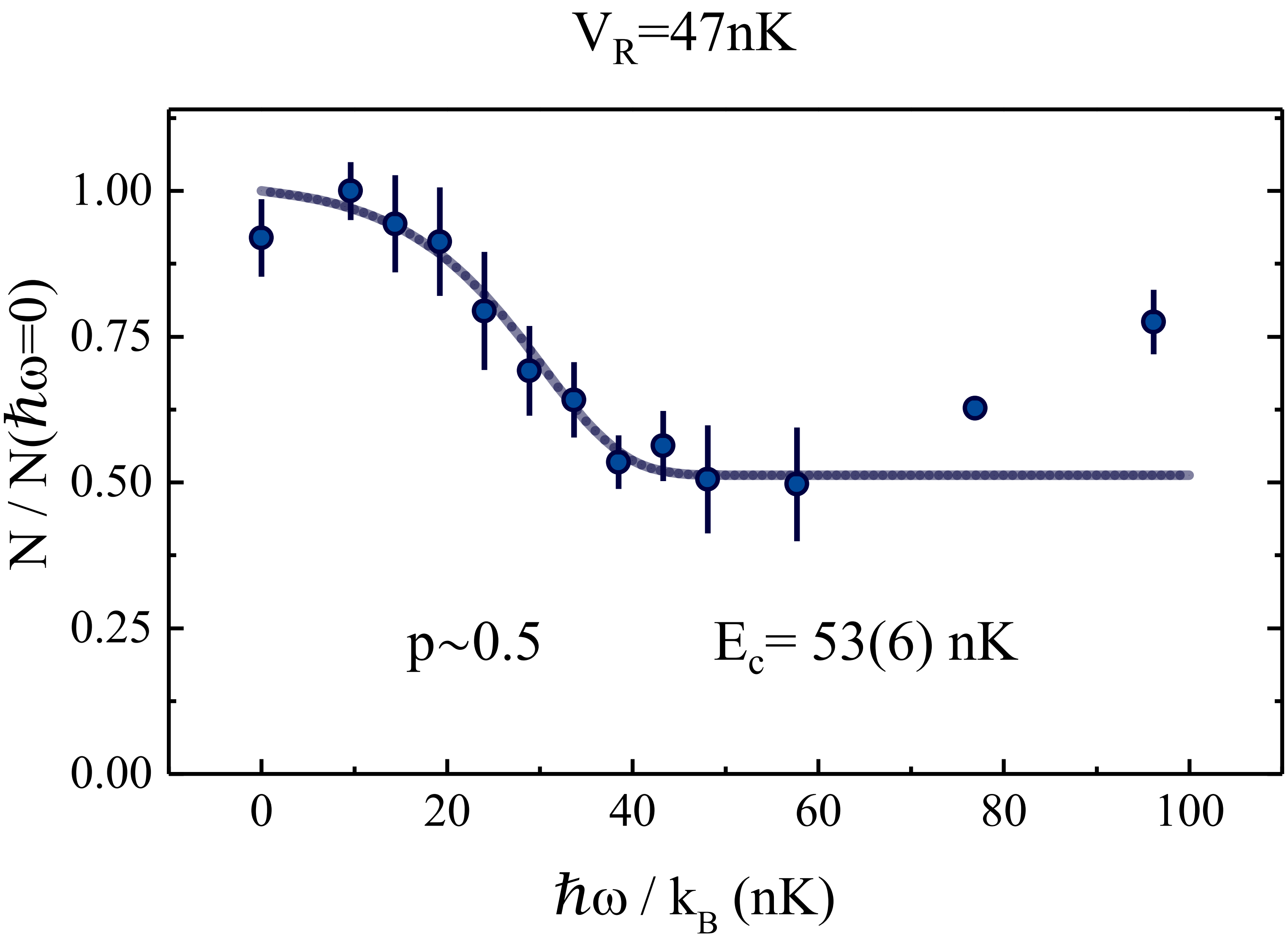
$$n(E,\omega) = (1-p)\, n_0(E) + p\, n_0(E - \hbar\omega)$$

p and E_c are fitting parameters

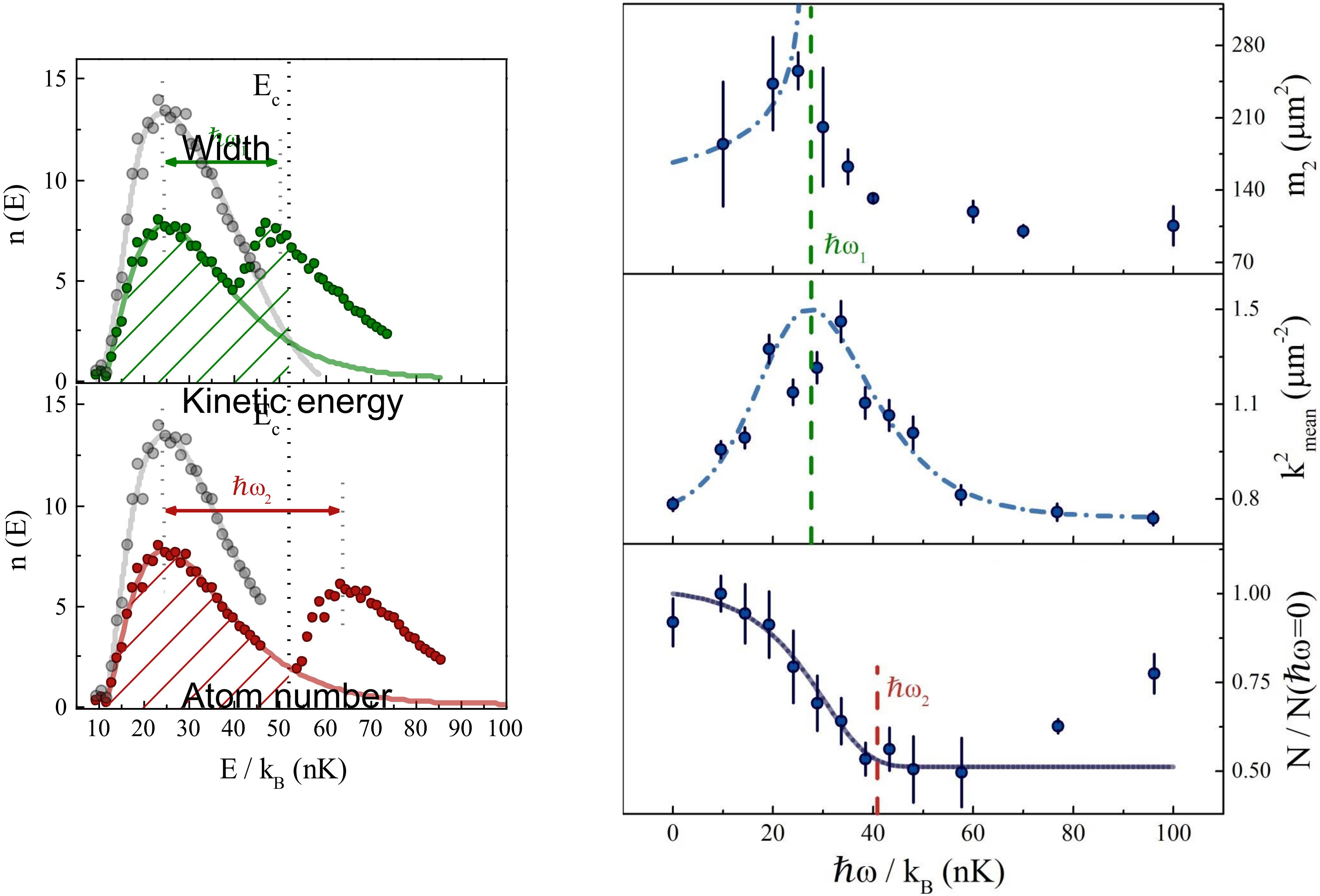
p can in principle be calculated, but its exact form is not crucial

$$p(E,\omega) = A^2 \sum_{i,f} \left|\left\langle f \middle| V(\mathrm{r}) \middle| i \right\rangle\right|^2 \delta(E_i - E)\,\delta(E_f - (E + \hbar\omega))$$

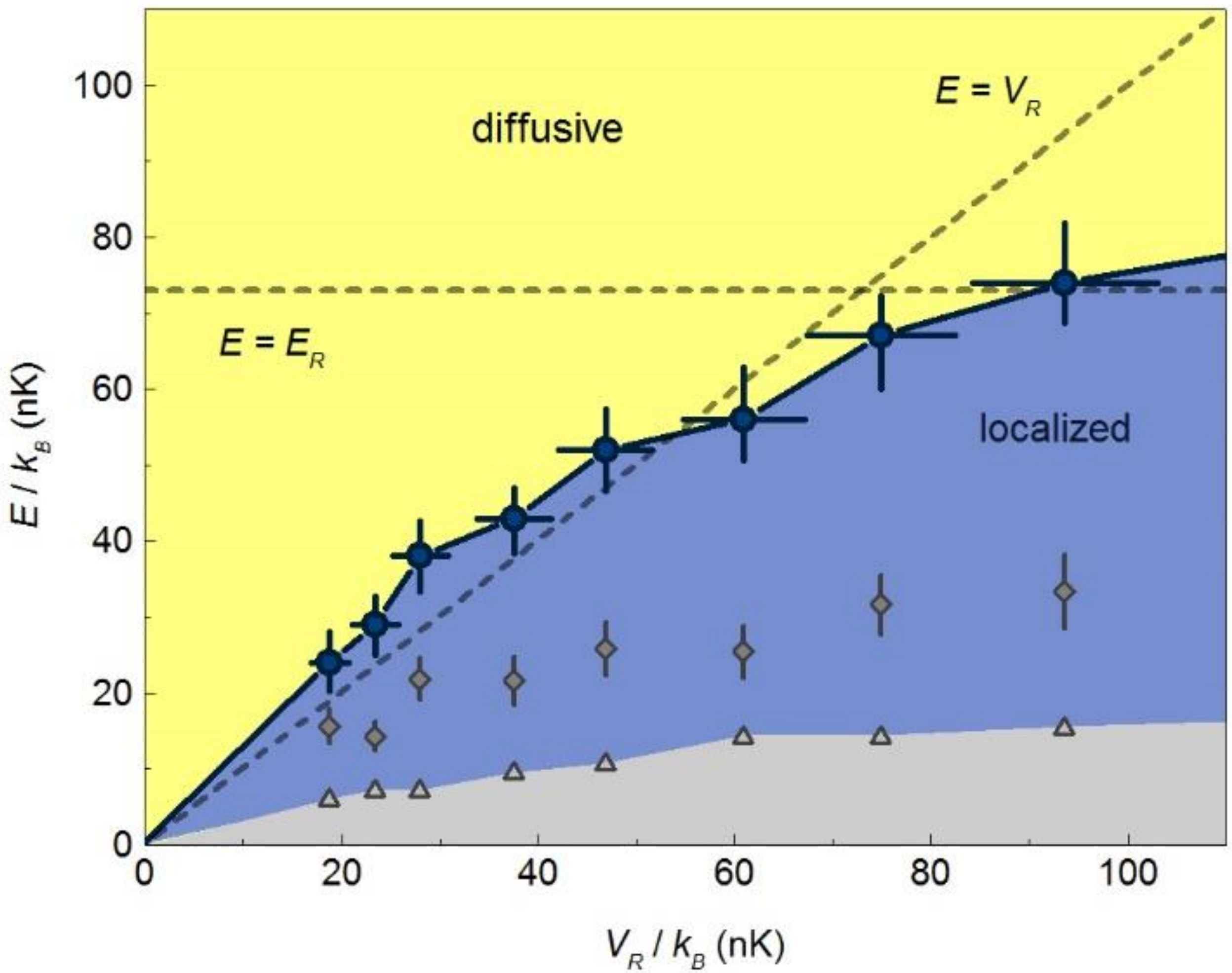
Excitation spectroscopy



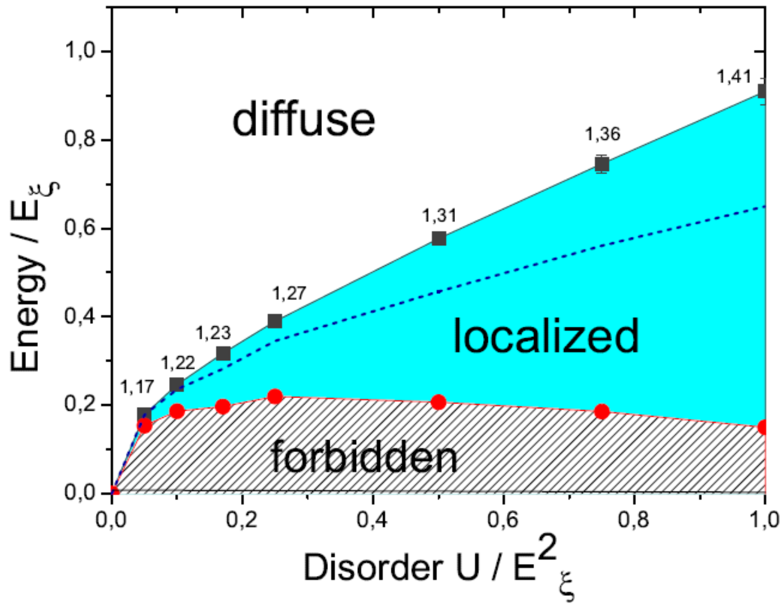
Excitation spectroscopy



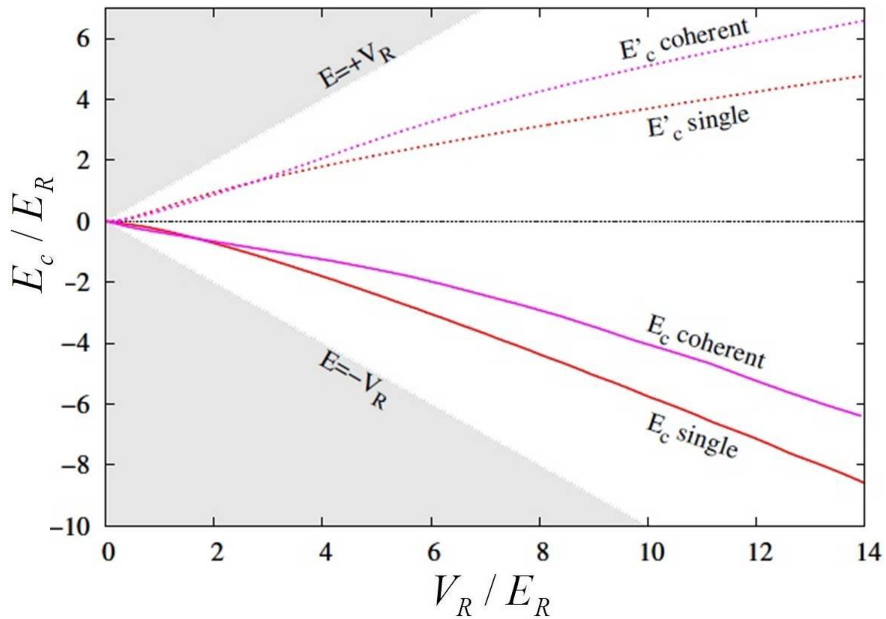
The mobility edge



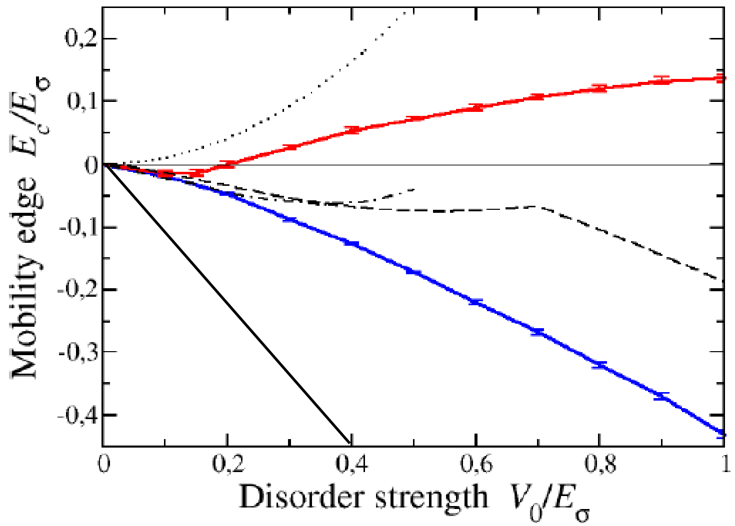
Self-consistent theories and exact calculations



Isotropic: Yedjour & Van Tiggelen, Eur. Phys. J. D (2009).

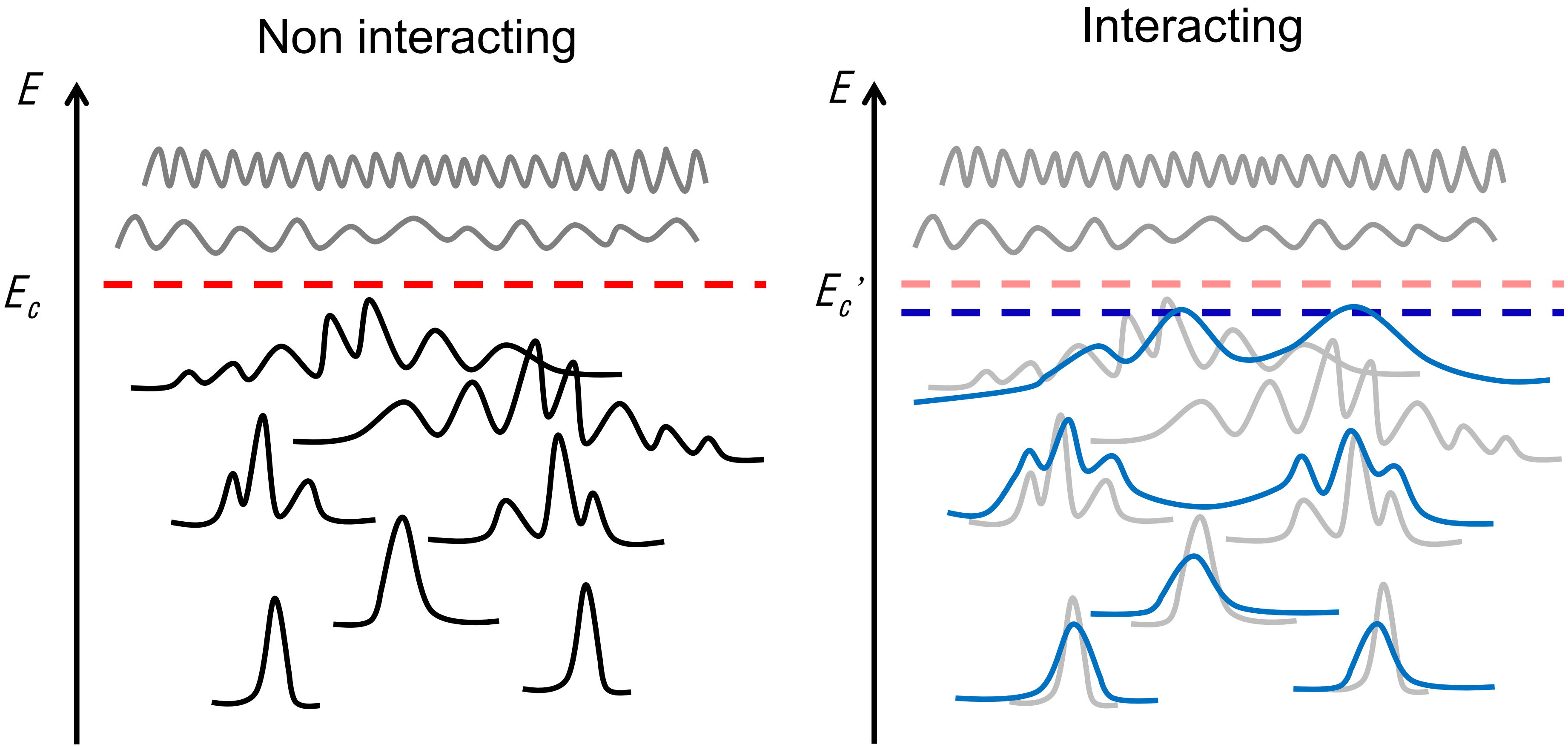


Anisotropic: Piraud, Pezzè and Sanchez-Palencia, EPL (2012).



Isotropic: Orso & Delande, arXiv:1403.3821

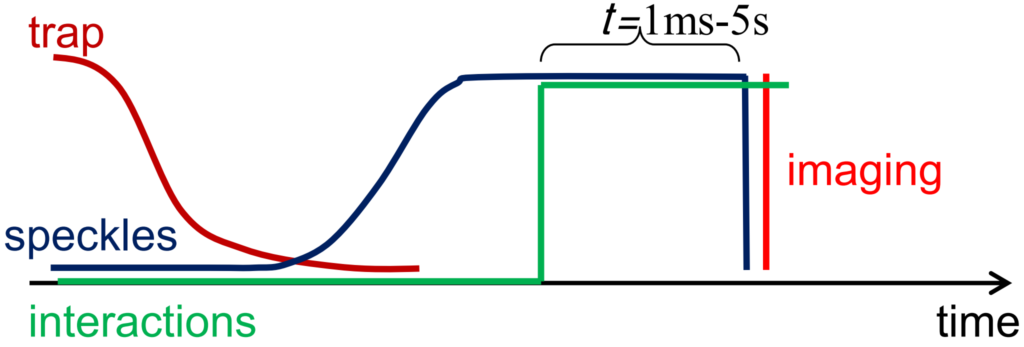
Anderson localization and interactions

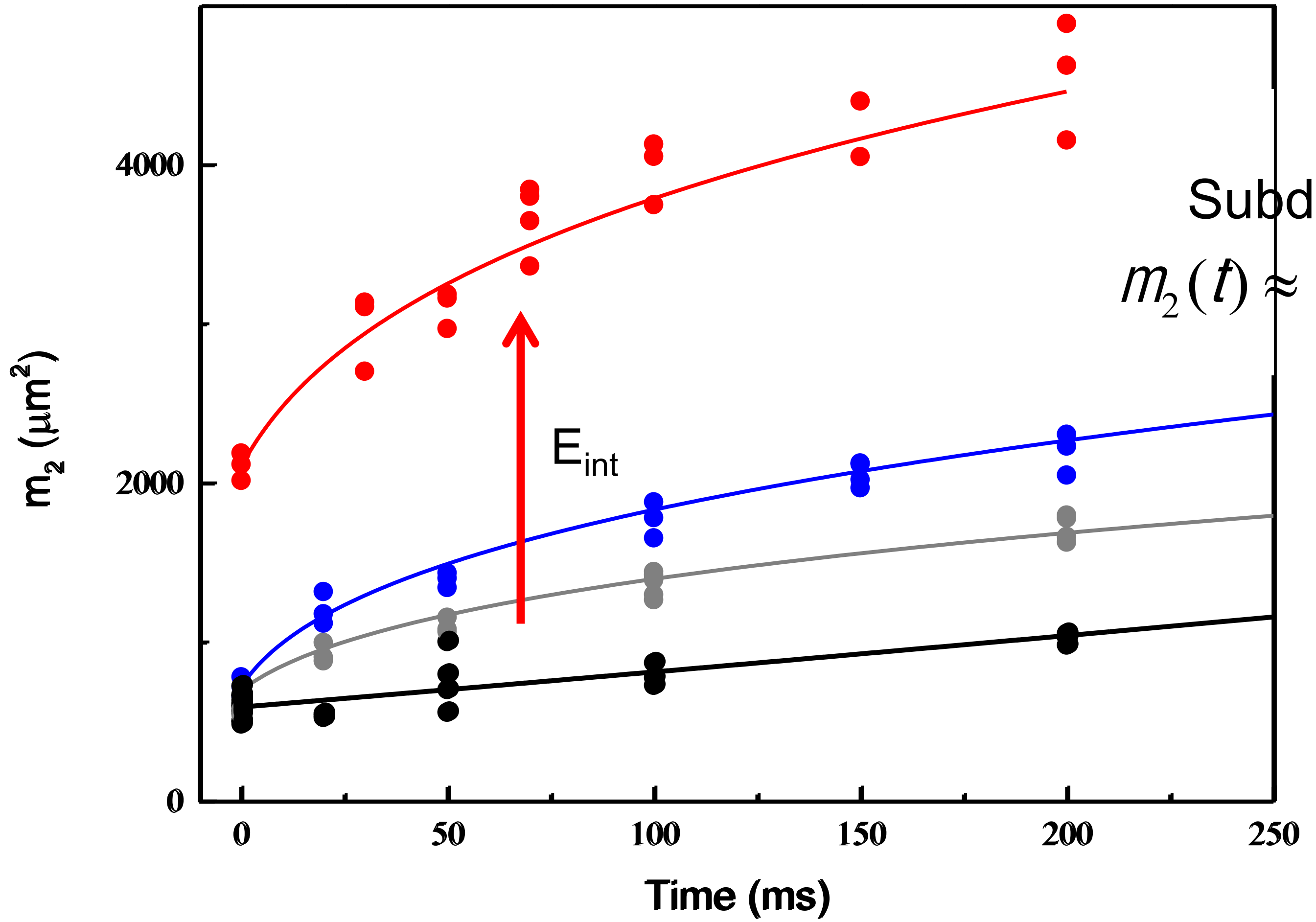


- Localized states are hybridized by interactions
- The many-body mobility edge is in principle shifted

A number of questions related to Bose-Einstein condensation, Anderson and many-body localization.

Interaction-induced dynamics

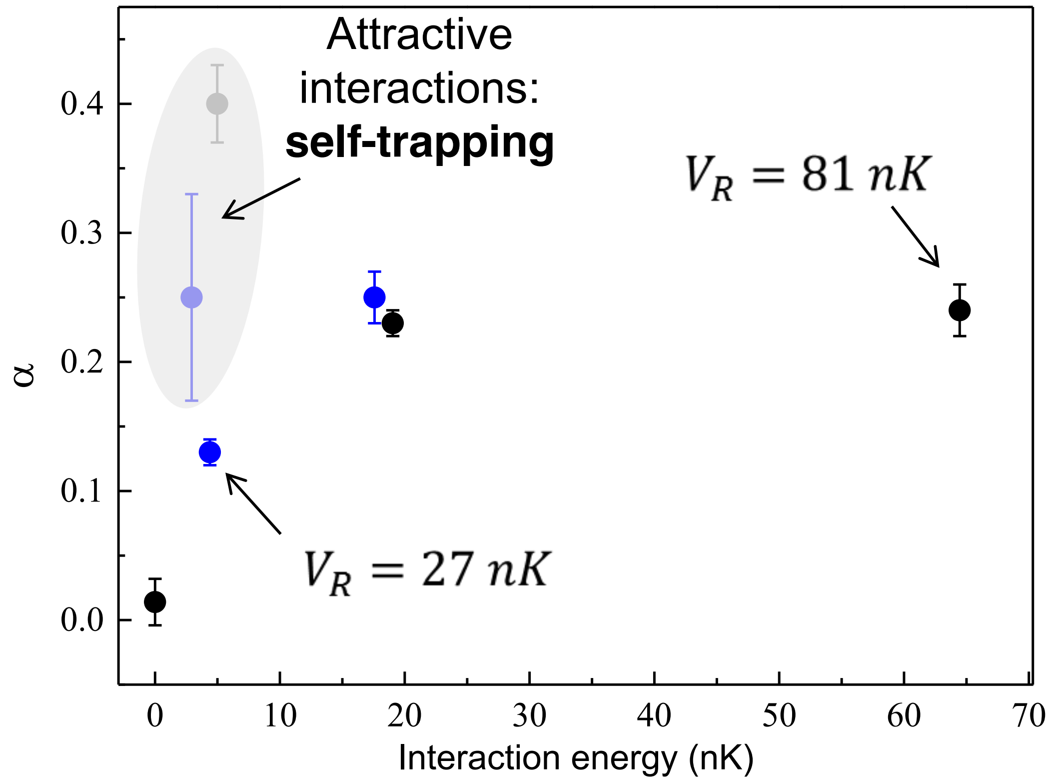




Subdiffusion:

$$m_2(t) \approx D(t)\ t = \tilde{D} t^{\alpha}$$

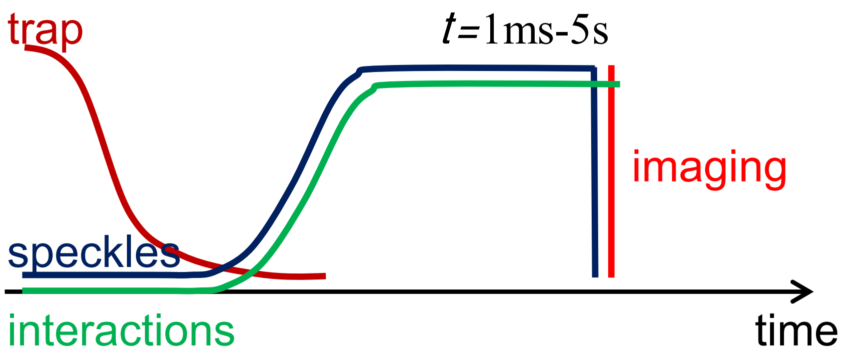
Interaction-induced dynamics



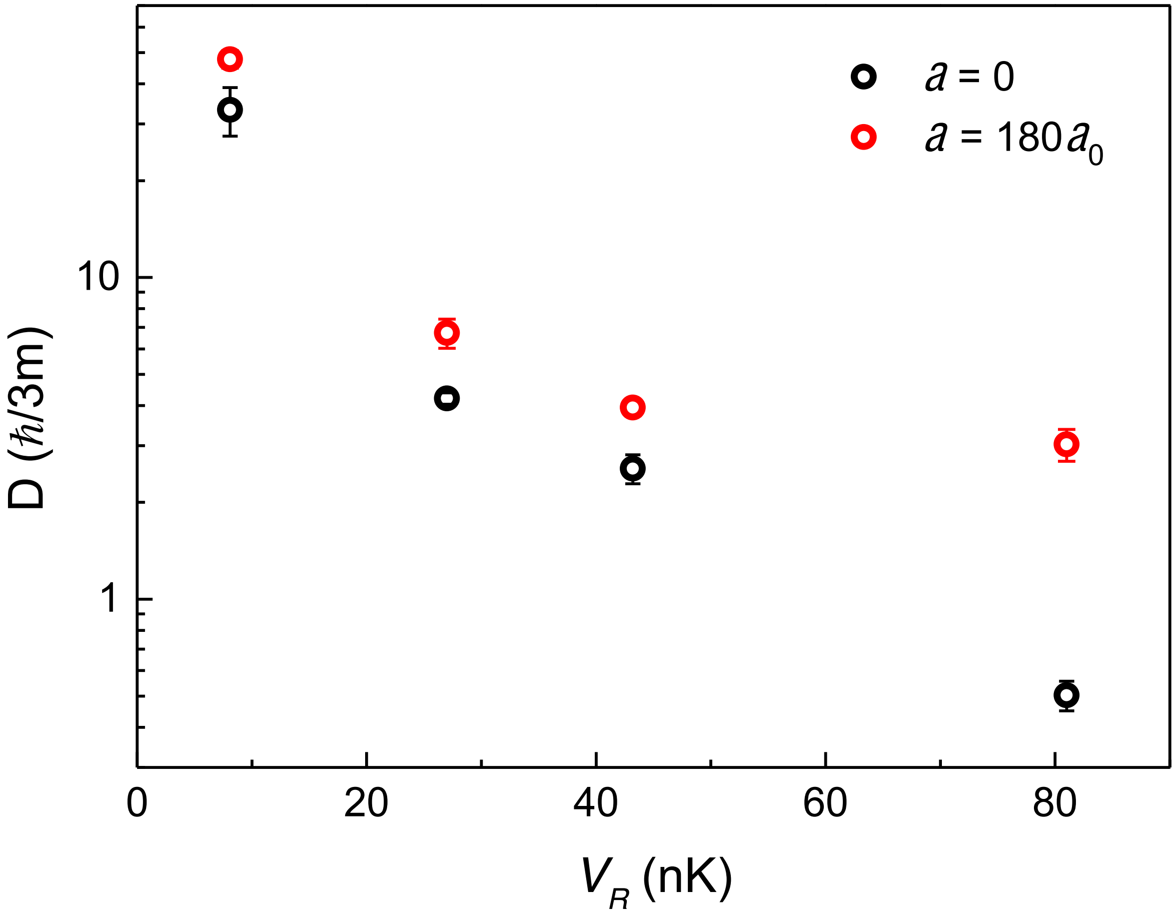
Few theories available.

Flach et al., Phys. Rev. Lett.102, 024101 (2009).
Cherroret et al. Phys. Rev. Lett. 112, 170603 (2014).

Diffusion and localization at the mobility edge



Interacting and non-interacting systems with the same energy



Conclusions and outlook

- A measurement of the mobility edge, in a system with controlled microscopic parameters.

- Narrower energy distributions: critical exponents.
- Evolution of the mobility edge with dimensionality at the 3D-2D crossover.
- Quantum simulation of the disordered interacting problem in 3D

The team

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useful discussions with many experts: Vincent Josse, Alain Aspect, Luca Pezzè, Pierre Lugan, Laurent Sanchez-Palencia, Cord Mueller, Antonello Scardicchio, Dominique Delande, ...