The Abdus Salam
International Centre
for Theoretical Physics
50th Anniversary 1964-2014

2583-1

Workshop on Coherent Phenomena in Disordered Optical Systems

26 - 30 May 2014

Momentum-space Signatures of Anderson Localization

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Univ. Konstanz, Dept. of Physics
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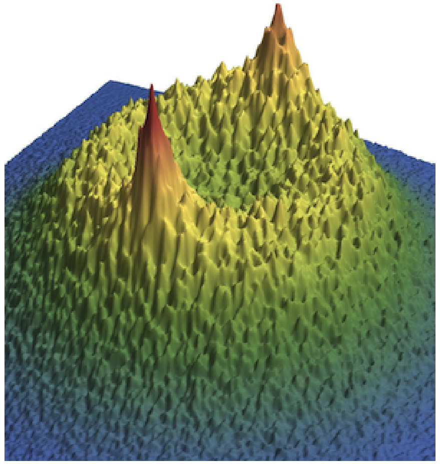
Welcome to Twin Peaks:
Momentum-space signatures of Anderson localization



Cord A. Müller

Universität Konstanz

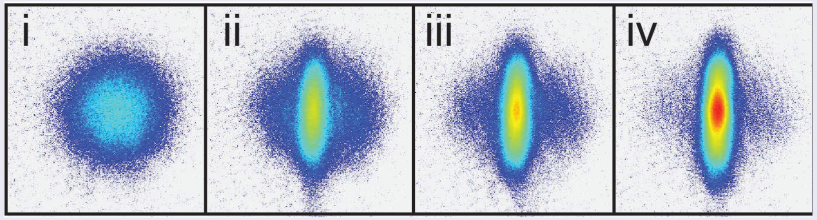
ICTP, Trieste, 27.05.2014

- Anderson localisation difficult to observe in pure form

  [Absorption, Decoherence, Interactions, . . .]

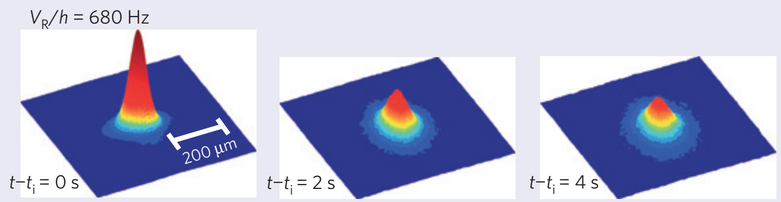
- Cold atoms, optical potentials: quantum simulation toolbox

  [Aspect, DeMarco, Esslinger, Hulet, Inguscio, Labeyrie, Rolston, Schneble, . . .]



$0 \longrightarrow$ disorder $\longrightarrow \infty$

[Kondov et al., Science (2011)]



$0 \longrightarrow$ time $\longrightarrow \infty$

[Jendrzejewski et al., Nat. Phys. (2012)]

- Which observable most suitable? “Absence of diffusion”?
- Empirical law: To every Claim, there is a Comment

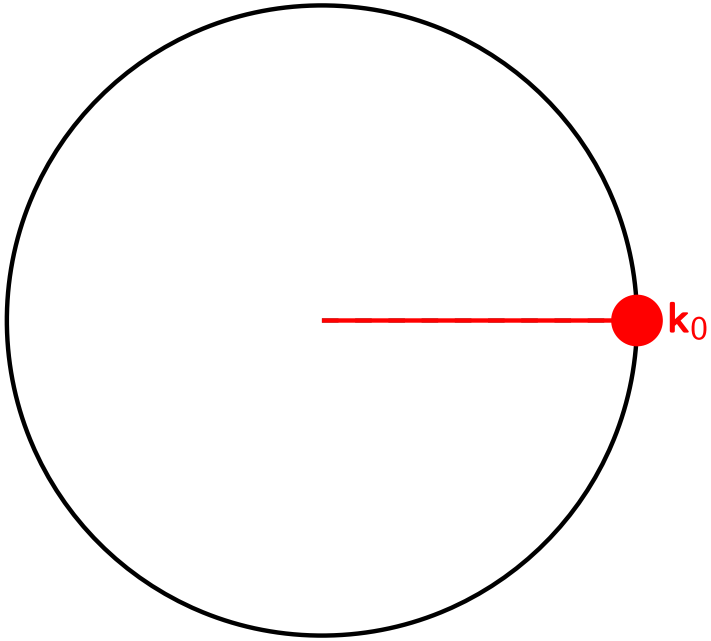
  [CAM & B. Shapiro, Comment submitted on DeMarco’s ‘Three-dimensional Anderson localization in variable scale disorder’ (PRL 2013)]

- How to prove phase coherence?
- ‘Smoking gun’ of Anderson localisation?

Our proposal: monitor momentum relaxation after quench

$$H_0 = \frac{\mathbf{p}^2}{2m} \quad \mapsto \quad H = \frac{\mathbf{p}^2}{2m} + V(\mathbf{r}), \qquad [r_\alpha, p_\beta] = i\hbar\delta_{\alpha\beta}$$

$(d = 2)$



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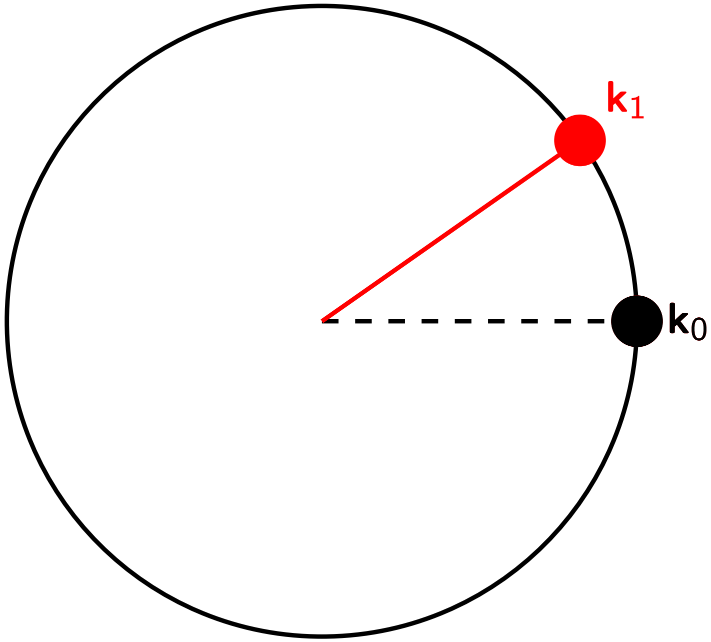

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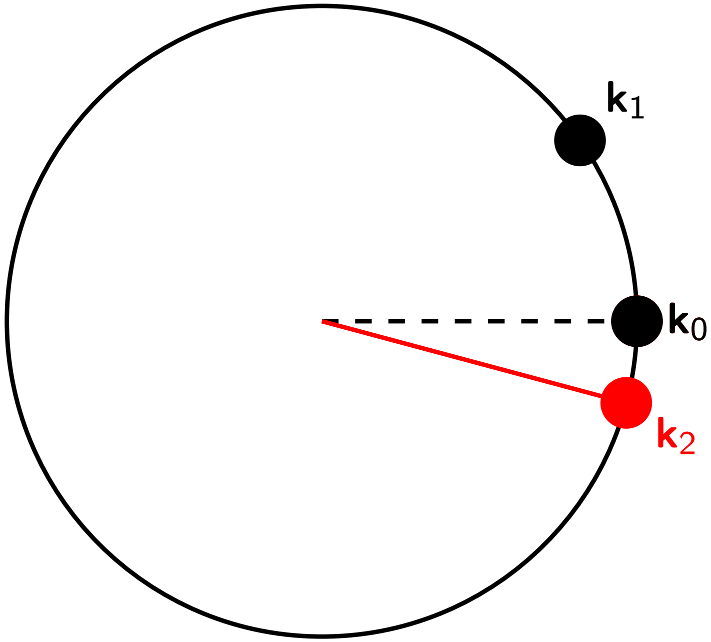

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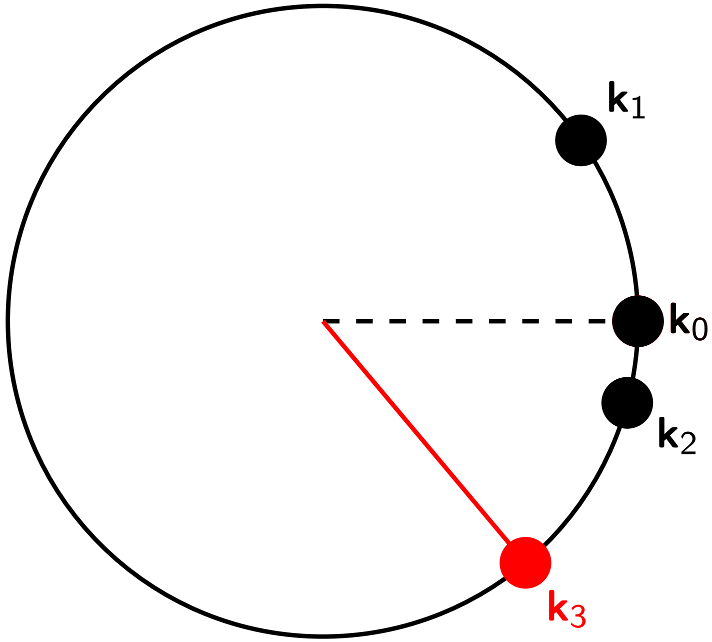

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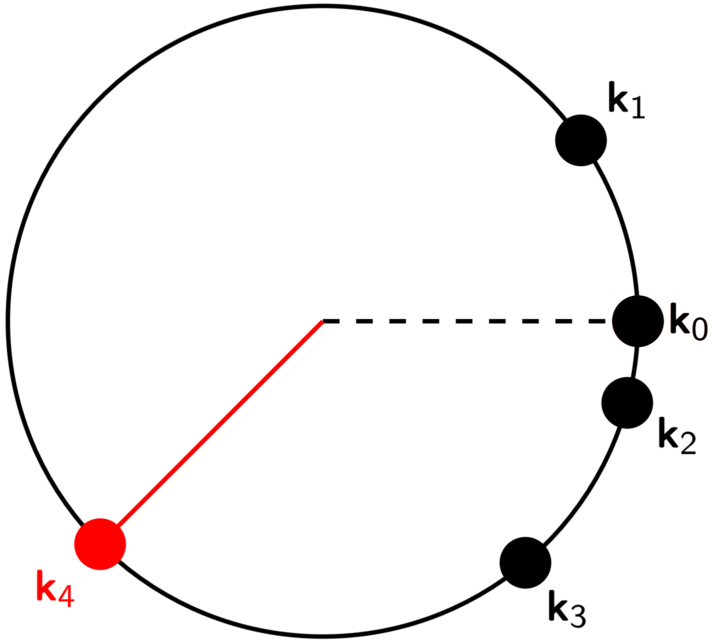

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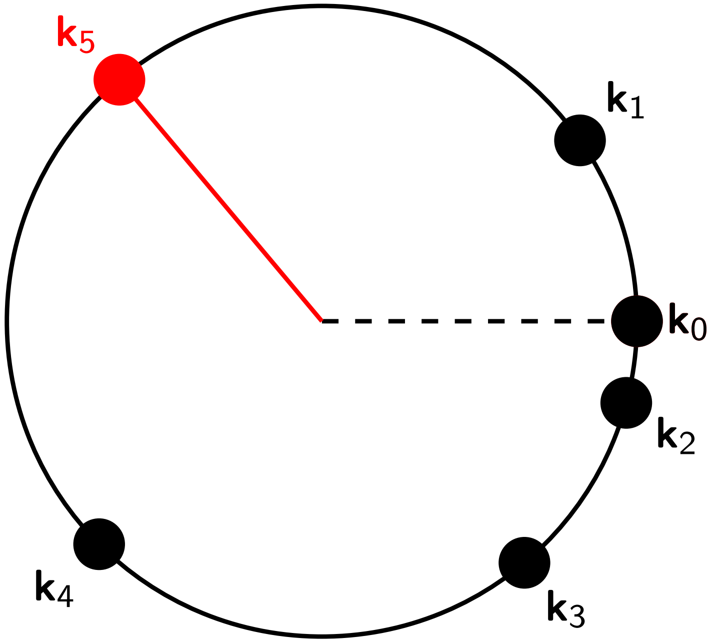

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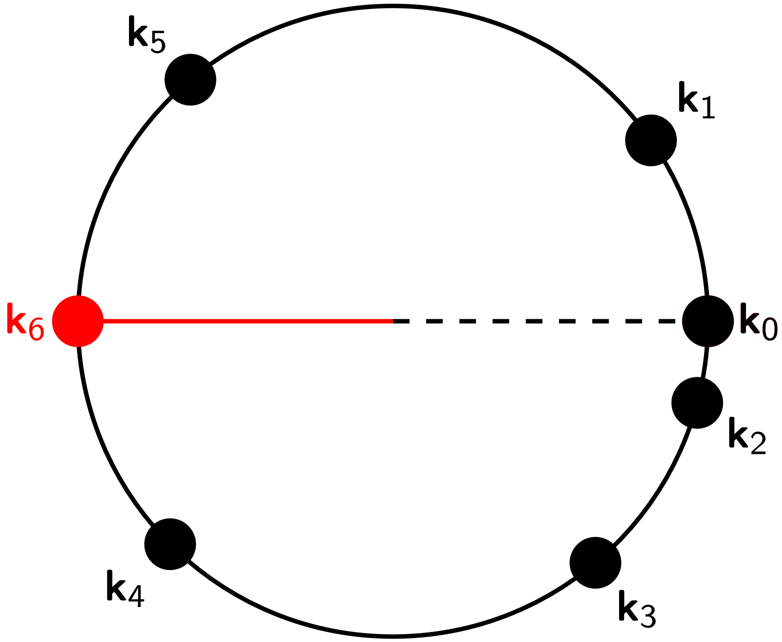

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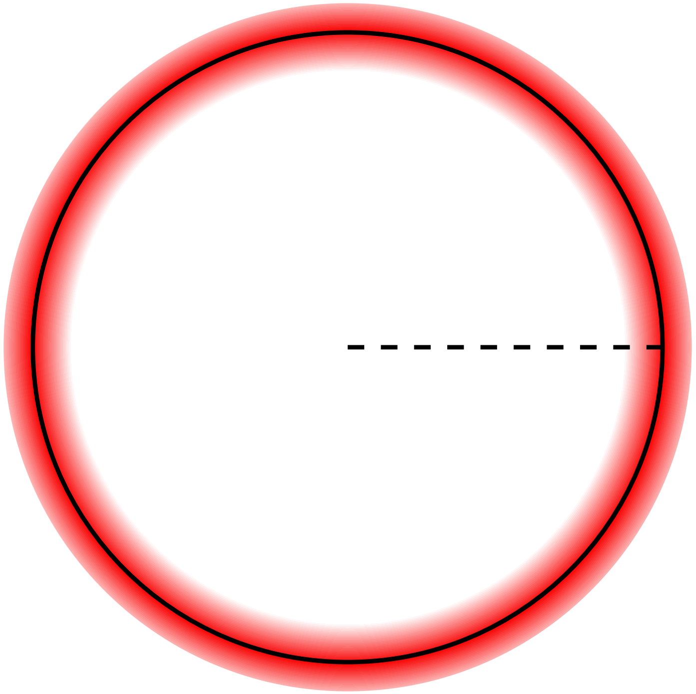
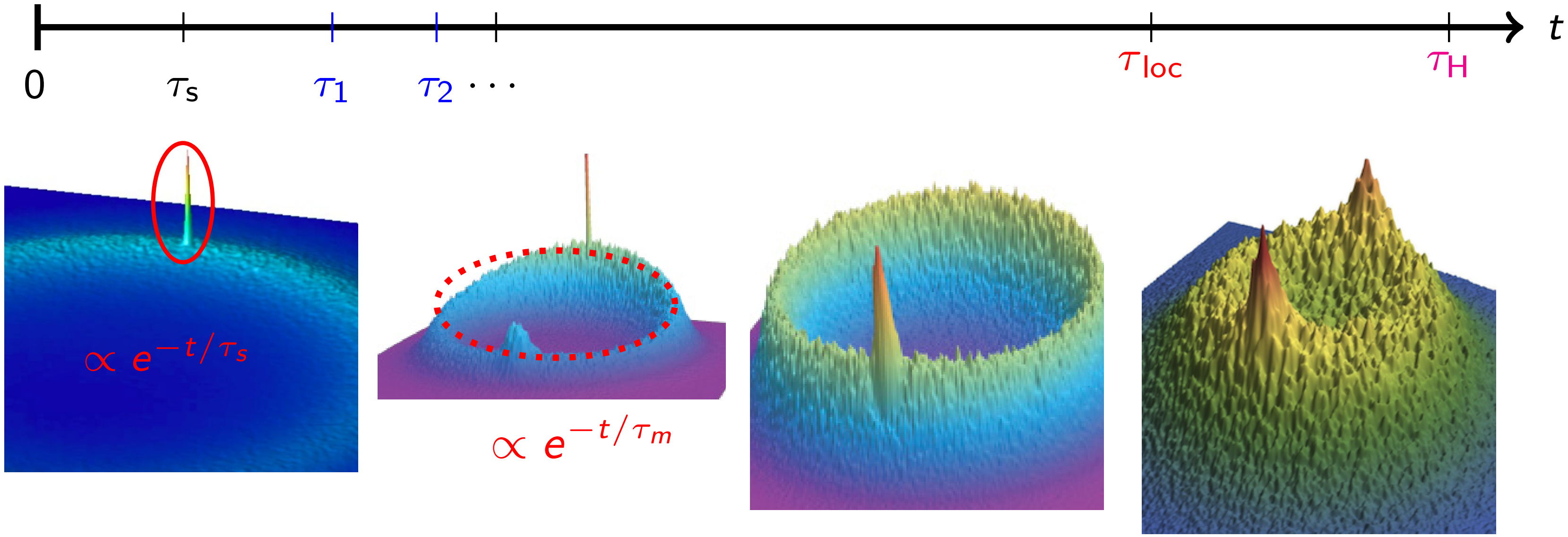


$n_{\mathbf{k}}(t) = \overline{|\psi_{\mathbf{k}}(t)|^2}$

Momentum isotropisation  ... or not?

A brief history of times . . .



1. Initial isotropisation [Plisson, Bourdel, CAM, EPJ ST (2012)]
2. Coherent back scattering (CBS)

   [Th: Cherroret, Karpiuk, CAM, Grémaud, Miniatura, PRA (2012)]

   [Exp: Jendrzejewski, Müller, Richard, Date, Plisson, Bouyer, Aspect, Josse, PRL (2012); Labeyrie, Karpiuk, Schaff, Grémaud, Miniatura, Delande, EPL (2012)]
3. Coherent forward scattering (CFS)

   [Karpiuk, Cherroret, Lee, Grémaud, CAM, Miniatura, PRL (2012)]

   [Micklitz, CAM, Altland, PRL (2014)]

   [Lee, Grémaud, Miniatura, `arXiv:1405.2979`]

1. Early times: elastic scattering

- Average momentum distribution $n_{\mathbf{k}}(t) = \overline{|\psi_{\mathbf{k}}(t)|^2}$:

$$n_{\mathbf{k}}(t) = \int \frac{\mathrm{d}E}{2\pi} n_{\mathbf{k}}(E,t) = \int \frac{\mathrm{d}E}{2\pi} \sum_{\mathbf{k}'} \Phi_{\mathbf{k}\mathbf{k}'}(E,t) n_{\mathbf{k}'}(0)$$

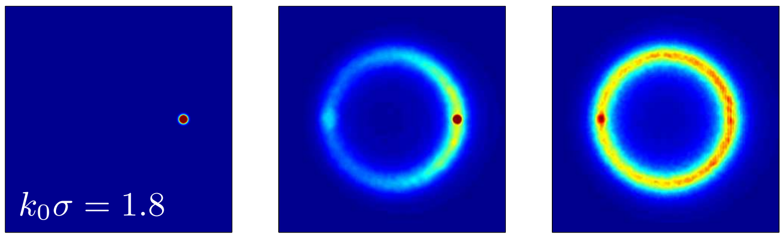
- Early times: Pauli master equation

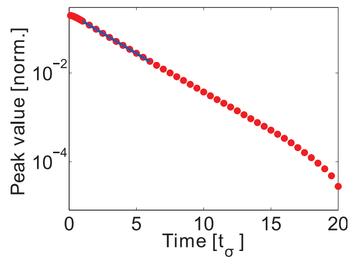
$$\partial_t n_{\mathbf{k}}(t) = \sum_{\mathbf{p}} \bar{U}_{\mathbf{k}\mathbf{p}} \left[n_{\mathbf{p}}(t) - n_{\mathbf{k}}(t) \right]$$

- Initially, incident mode $\mathbf{k}_0$ depopulates, $\tau_{\mathrm{s}}^{-1} = \sum_{\mathbf{p}} \bar{U}_{\mathbf{k}_0\mathbf{p}}$

$$n_{\mathbf{k}_0}(t) \approx e^{-t/\tau_{\mathrm{s}}} n_{\mathbf{k}_0}(0)$$

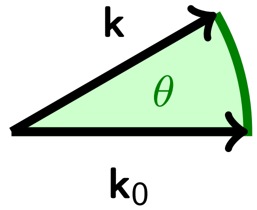
- Numerical simulation in 2D speckle (Thomas Plisson):





$\tau_{\mathrm{s}} = 2.4 t_\sigma$

$\overset{\mathrm{BA}}{=} 1.4 t_\sigma$



- On the elastic scattering circle $\mathbf{k} = k_0(\cos\theta, \sin\theta)$:

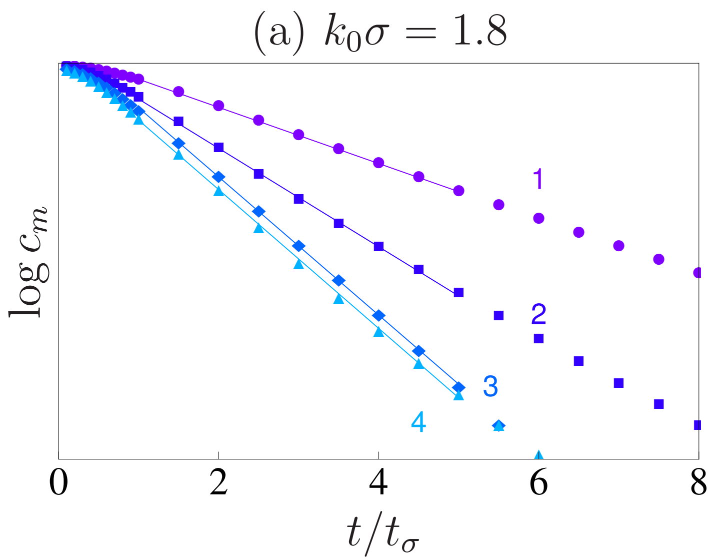
$$\partial_t n(\theta, t) = \tau_s^{-1} \int_0^{2\pi} \mathrm{d}\phi\, u(\phi - \theta) \left[n(\phi, t) - n(\theta, t) \right]$$

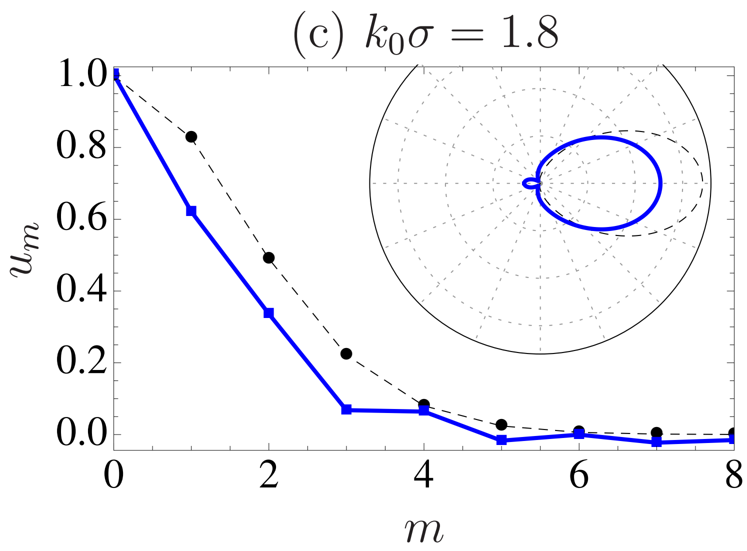
with phase function $u(\beta) = \bar{U}(\beta) / \int \mathrm{d}\phi \bar{U}(\phi)$

- Solution via Fourier analysis $n(\theta, t) = \sum_{m\in\mathbb{Z}} c_m(t) e^{im\theta}$

$$c_m(t) = e^{-t/\tau_m} c_m(0)$$

- Characteristic times: $\tau_m = \tau_s / [1 - \langle \cos m\theta \rangle_u]$
- Measure τ_s/τ_m to reconstruct $u(\beta)$:

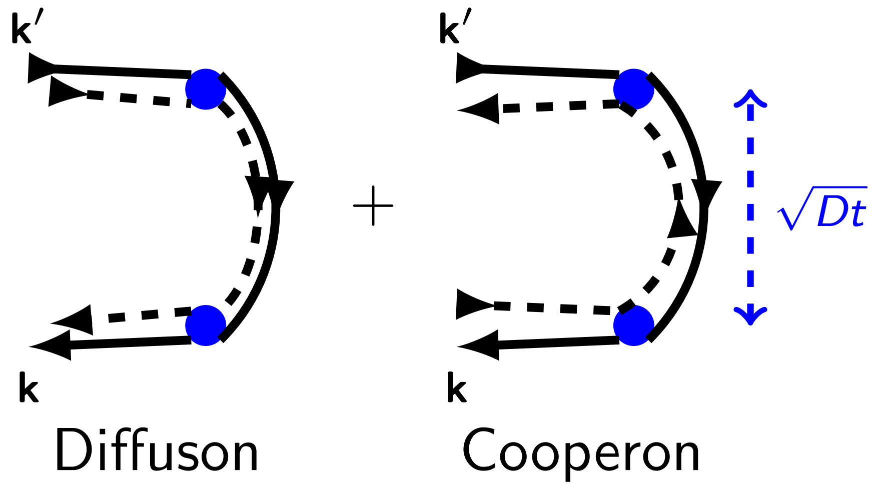




2. Diffusive momentum relaxation

$$n_{\mathbf{k}}(t) = \sum_{\mathbf{k}'} \Phi_{\mathbf{k}\mathbf{k}'}(t) n_{\mathbf{k}'}(0)$$

$$\Phi^{(D)}_{\mathbf{k}\mathbf{k}'} + \Phi^{(C)}_{\mathbf{k}\mathbf{k}'} =$$



[Gorkov, Larkin, Khmelnitskii (1979), Vollhardt & Wölfle (1980)]

Observation as Coherent Backscattering (CBS) of light

[van Albada & Lagendijk, Wolf & Maret (1985)]

Sensitive measure of dephasing

[G. Bergmann: "Weak localization in thin films — a time-of-flight experiment with conduction electrons", Phys. Rep. (1984)]

PHYSICAL REVIEW A **85**, 011604(R) (2012)

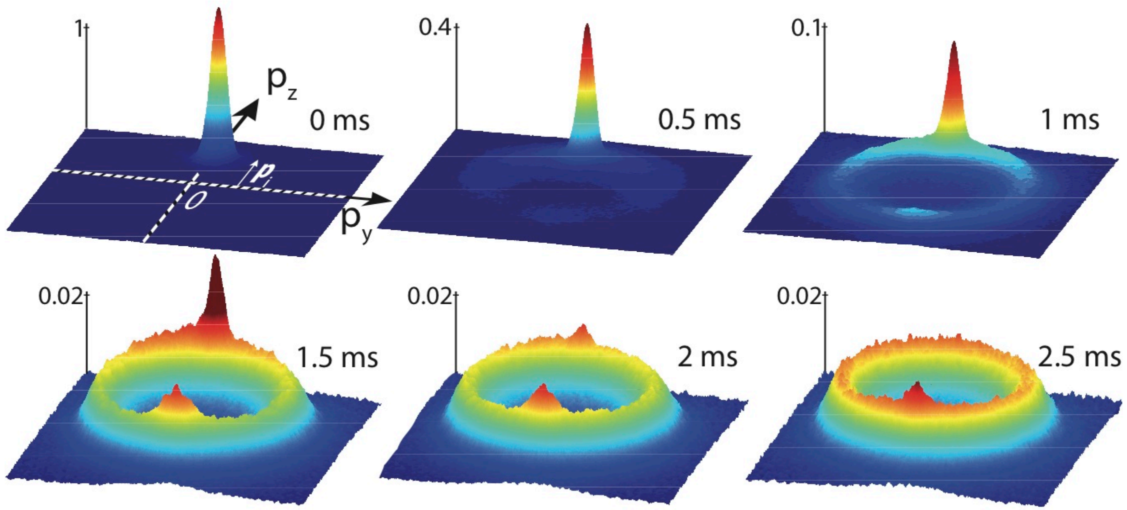
Coherent backscattering of ultracold matter waves: Momentum space signatures

Nicolas Cherroret,[1] Tomasz Karpiuk,[2,3] Cord A. Müller,[2] Benoît Grémaud,[2,4,5] and Christian Miniatura[2,4,6]
[1]*Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany*
[2]*Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore*

Selected for a Viewpoint in *Physics*
PRL **109,** 195302 (2012) PHYSICAL REVIEW LETTERS week ending 9 NOVEMBER 2012

Coherent Backscattering of Ultracold Atoms

F. Jendrzejewski,[1] K. Müller,[1] J. Richard,[1] A. Date,[1] T. Plisson,[1] P. Bouyer,[2] A. Aspect,[1] and V. Josse[1,*]
[1]*Laboratoire Charles Fabry UMR 8501, Institut d'Optique, CNRS, Univ Paris Sud 11, 2 Avenue Augustin Fresnel, 91127 Palaiseau cedex, France*



- ^{87}Rb BEC, 10^5 atoms
- $|F=2, m_F=-2\rangle$ paramagnetic
- $\Delta v_0 \approx 0.1\,\mathrm{mm/s}$
- $v_0 = \hbar k_0/m \approx 3.3\,\mathrm{mm/s}$

- CBS kernel ($\mathbf{q} = \mathbf{k} + \mathbf{k}'$) :

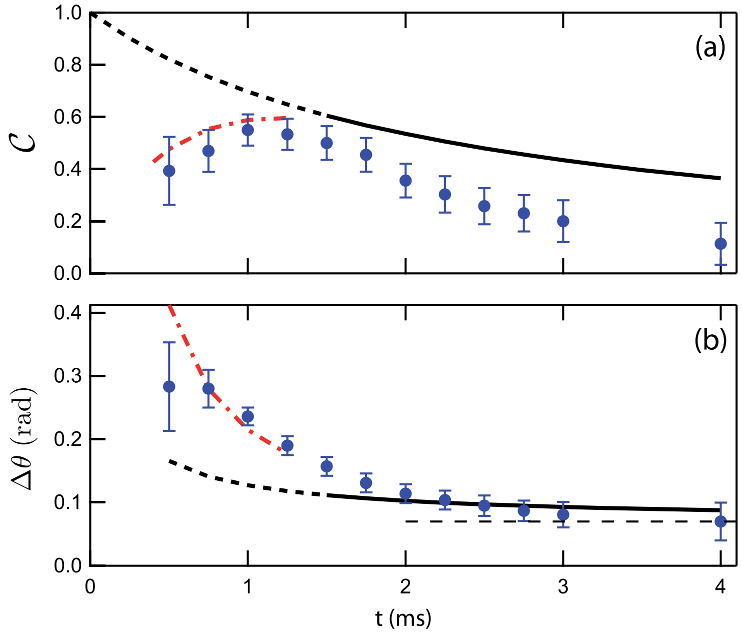
$$\int \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t} \frac{1}{-i\omega + D\mathbf{q}^2} = \Theta(t) e^{-D\mathbf{q}^2 t} : \qquad \Delta q^2(t) = [2Dt]^{-1}$$

- Convolution with initial distribution: $\Delta q^2 \mapsto \Delta q^2 + \Delta k^2$

$$\Delta\theta(t) = \Delta\theta_0 [1 + \Delta t/t]^{1/2}$$

Source coherence time: $\Delta t = [2D\Delta k^2]^{-1}$

- Contrast $C(t) = (1 + t/\Delta t)^{-d/2}$



3. Strong localization $t > \tau_{\mathrm{loc}}$

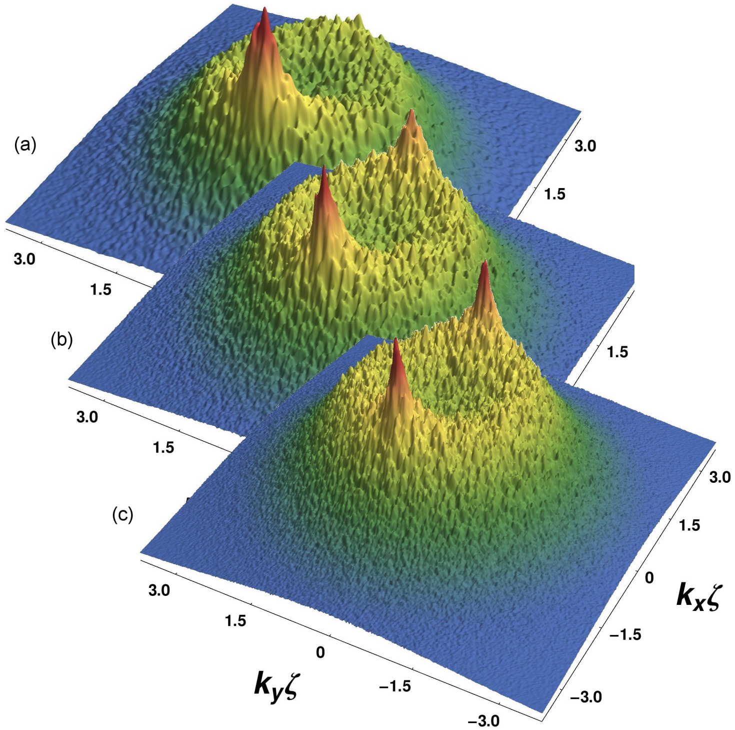
- Amplitudes feel strong localization at $\tau_{\mathrm{loc}} = \xi_{\mathrm{loc}}^2 / D_0$
- Scaling $D(\omega) \sim -\mathrm{i}\omega\xi_{\mathrm{loc}}^2$

$$\frac{1}{-\mathrm{i}\omega + D(\omega)q^2} \to \frac{1}{-\mathrm{i}\omega + 0} \times \frac{1}{1 + \xi_{\mathrm{loc}}^2 q^2}$$

- CBS signal should freeze:

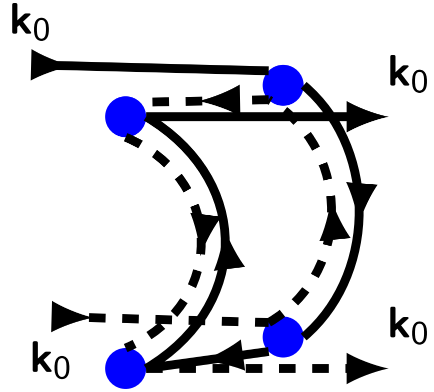
$$\int \frac{\mathrm{d}\omega}{2\pi} e^{-\mathrm{i}\omega t} \frac{1}{-\mathrm{i}\omega + 0} = \Theta(t)$$

- Peak still visible if $\Delta t \gg \tau_{\mathrm{loc}}$, i.e. $\Delta k^{-1} \gg \xi_{\mathrm{loc}}$



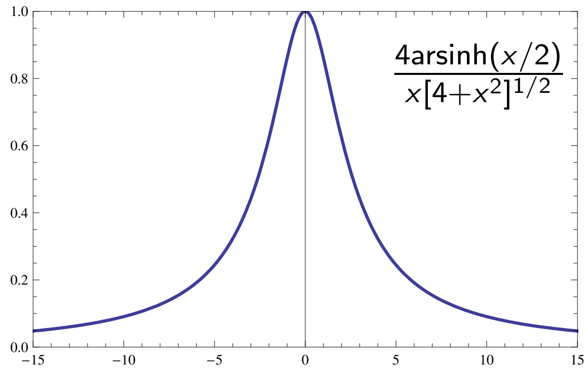
To our surprise: CFS!

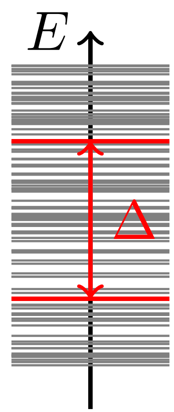
$\mathbf{k}_0 \to -\mathbf{k}_0 \to \mathbf{k}_0$



- Small correction of order $1/k_0\ell \ll 1$ in the WL regime. But in the AL regime:

$$C(\mathbf{q}, t) = F_d(q\xi_{\text{loc}})\frac{t}{\tau_{\text{H}}}$$





$\tau_{\text{H}} = \frac{h}{\Delta} = h\nu\xi_{\text{loc}}^d$: Heisenberg time of the localization volume.

[Hikami, "Anderson localization in a nonlinear-σ-model representation", PRB (1981)]

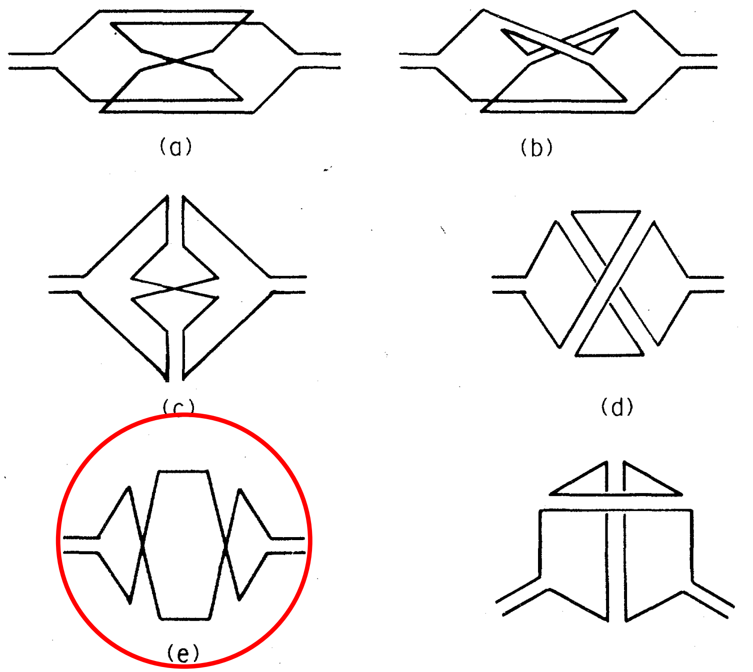


FIG. 5. Second-order diagrams for diffusion propagator.

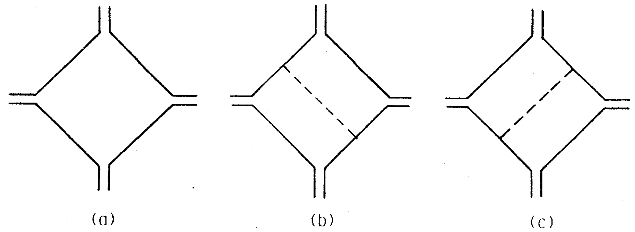


FIG. 6. Four-point vertex.

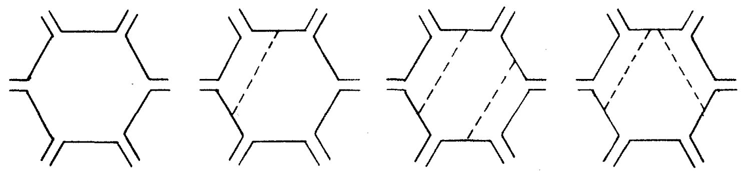
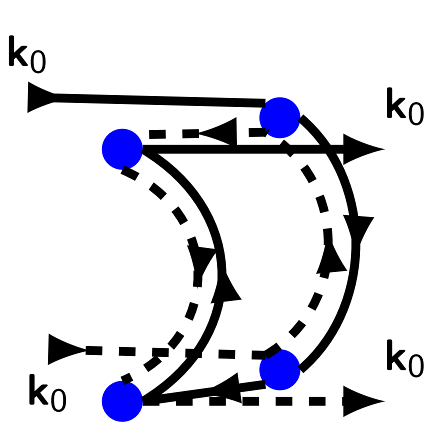


FIG. 7. Six-point vertex.

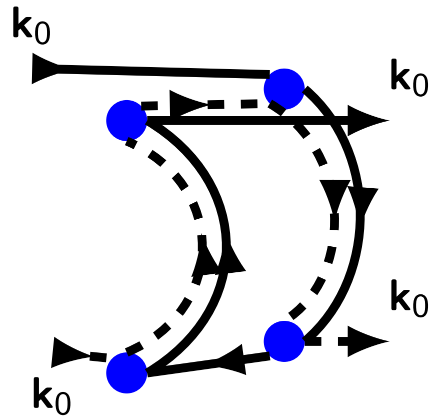
ings. These higher-order terms are derived by the correct calculation of the Green's function.[26] There are many diagrams. Examples of these diagrams are

Cooperon series $C + CC + CCC + \ldots$ impossible to sum

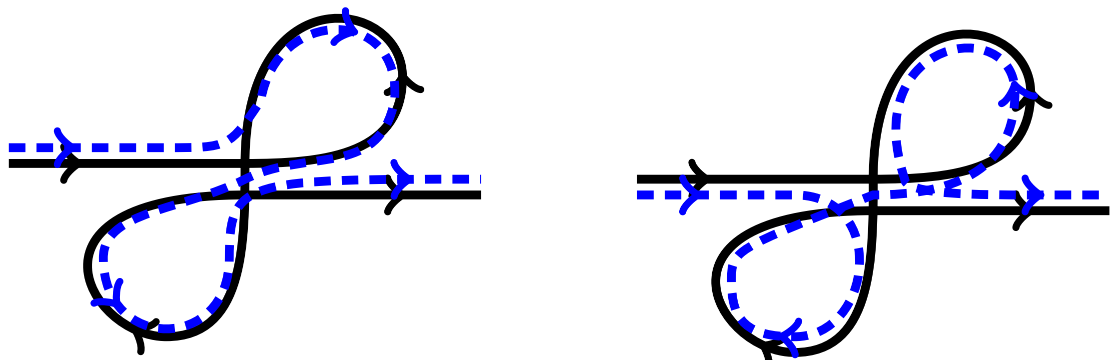
- Refined proposal: [with Tobias Micklitz & Alexander Altland]
  quasi 1D geometry + $U(1)$ gauge field (breaks T):



cooperons



diffusons



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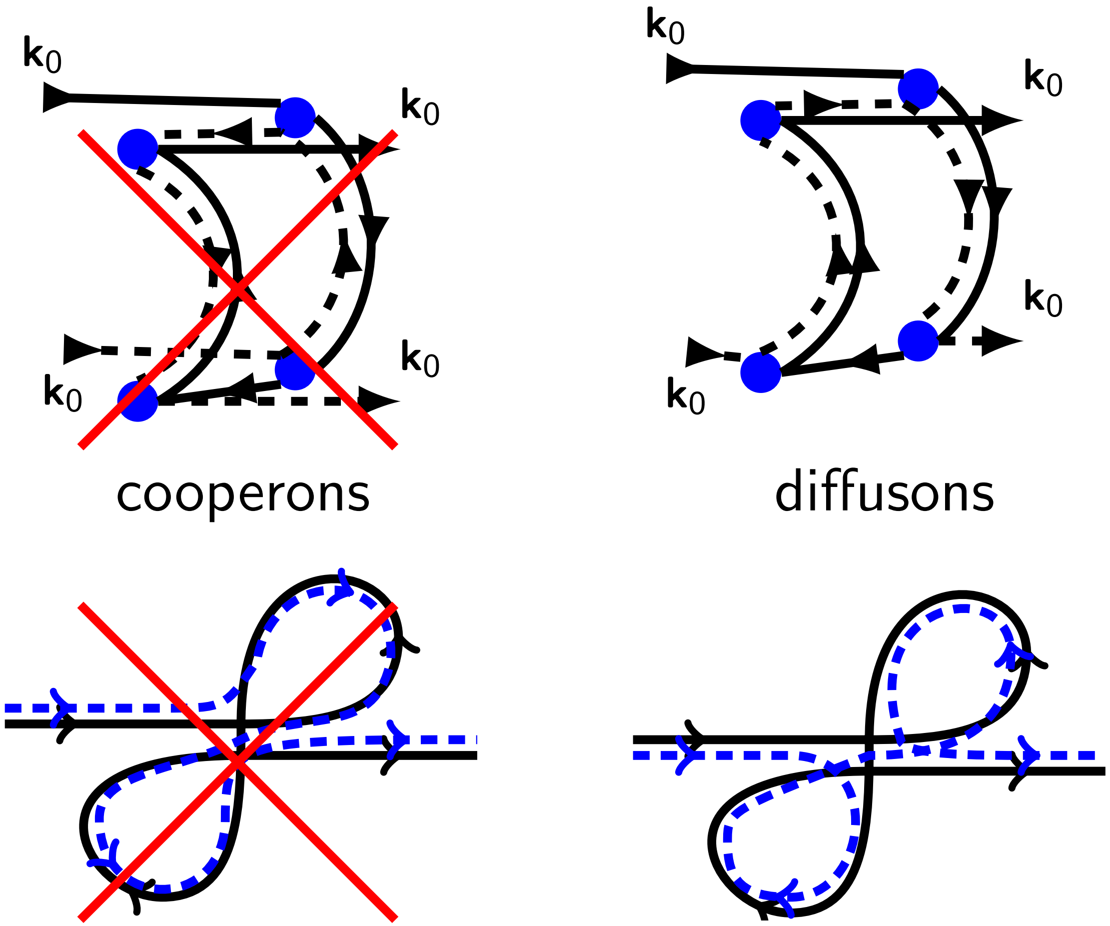


- Localization still effective, while GUE calculations simpler
  [Efetov, *Supersymmetry in Disorder and Chaos*, 1999]

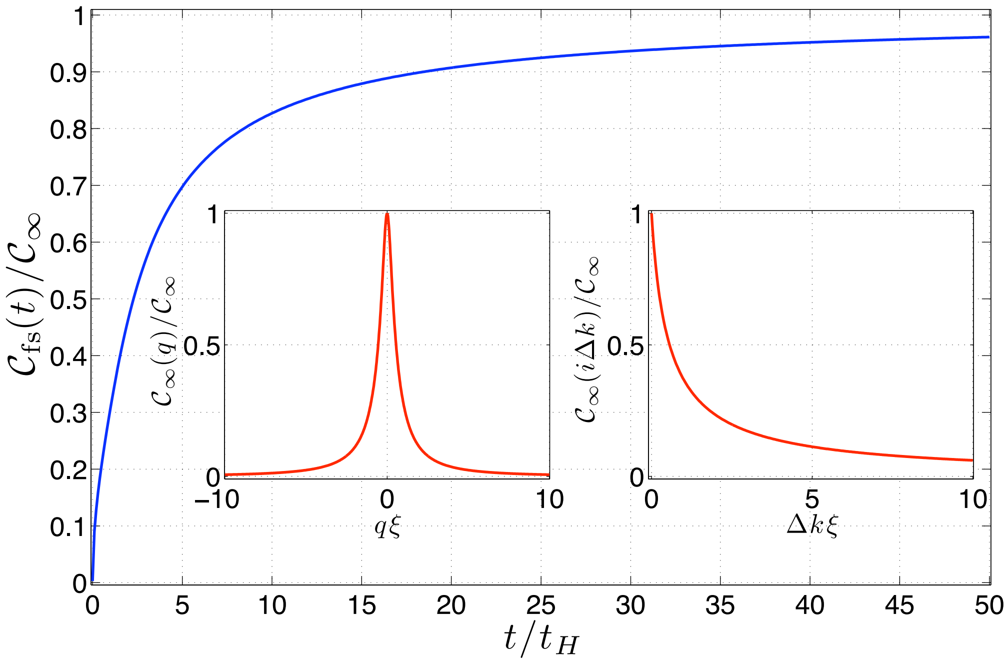
- Non-perturbative analysis with SUSY NLσM:

$$C(q,\omega) = \langle \mathrm{tr}\,(\mathcal{P}_{-+}\, Q(q)\, \mathcal{P}_{+-}\, Q(-q)) \rangle_{S_0}$$

$$S_0[Q] = \pi \nu S \int dx\, \mathrm{str} \left(i\omega Q \Lambda + \frac{D}{4} (\partial_x Q)^2 \right)$$

- Differential "transfer matrix" equations [Efetov & Larkin (1983)]
- Analytical solution by mapping to Coulomb problem

[Skvortsov & Ostrovsky, JETP Lett. (2007)]



$$\frac{C(0,t)}{C_0} = I_0\left(\frac{2\tau_H}{t}\right) e^{-2\tau_H/t}$$

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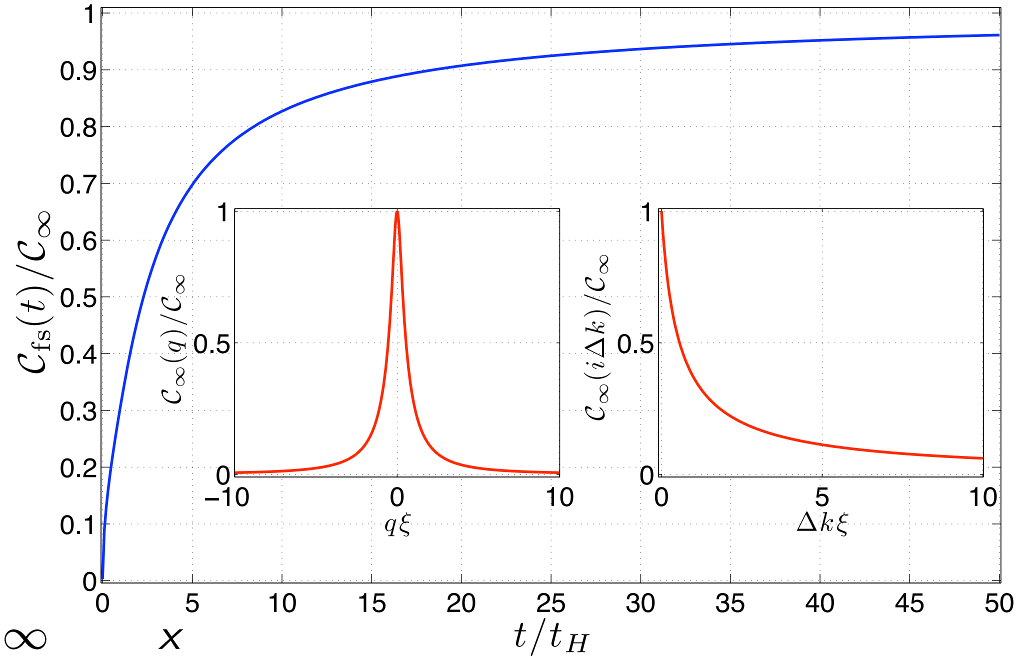
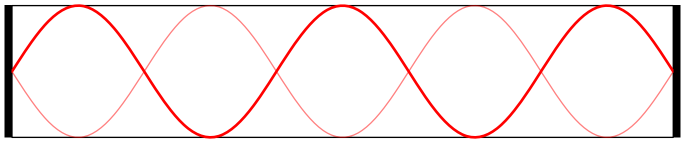


$$C(q,\infty) \propto \mathrm{Re}\int_0^\infty dx \int_0^x dy\, xK_1(x)K_{\sigma_q}(x)yK_1(y)I_{\sigma_q}(y), \quad \sigma_q = \sqrt{1-4iq\xi}$$

- Localization = no net transport:



$$\langle \mathbf{k} \rangle = \sum_{\mathbf{k}} \mathbf{k} n_{\mathbf{k}} = 0 \qquad \text{if} \qquad n_{\mathbf{k}} = n_{-\mathbf{k}}$$

- Localized eigenstates: $H|\alpha\rangle = \hbar\omega_\alpha|\alpha\rangle$

$$\psi_{\mathbf{k}}(t) = \sum_{\alpha} \langle \mathbf{k}|\alpha\rangle e^{-i\omega_\alpha t} \langle \alpha|\mathbf{k}_0\rangle$$

- Long-time limit $t \gg \overline{|\omega_\alpha - \omega_{\alpha+1}|^{-1}} \sim \tau_{\mathrm{H}}$:

$$n_{\mathbf{k}} = \sum_{\alpha} \overline{|\langle \mathbf{k}|\alpha\rangle|^2 |\langle \alpha|\mathbf{k}_0\rangle|^2}$$

- Coherence peaks due to $[\overline{x^2} \geq \overline{x}^2]$:

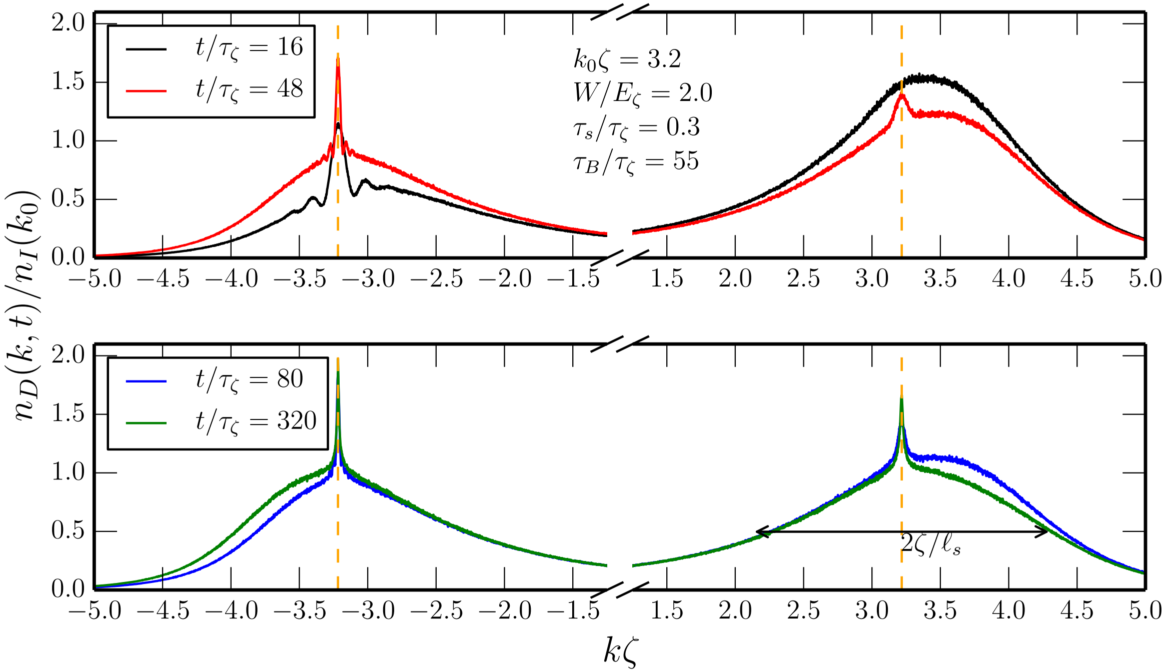
$$n_{\mathbf{k}_0} = \sum_{\alpha} \overline{|\langle \mathbf{k}_0|\alpha\rangle|^4} \geq \sum_{\alpha} \overline{|\langle \mathbf{k}_0|\alpha\rangle|^2} \times \overline{|\langle \mathbf{k}|\alpha\rangle|^2} = n_{\mathbf{k}\neq\mathbf{k}_0}$$

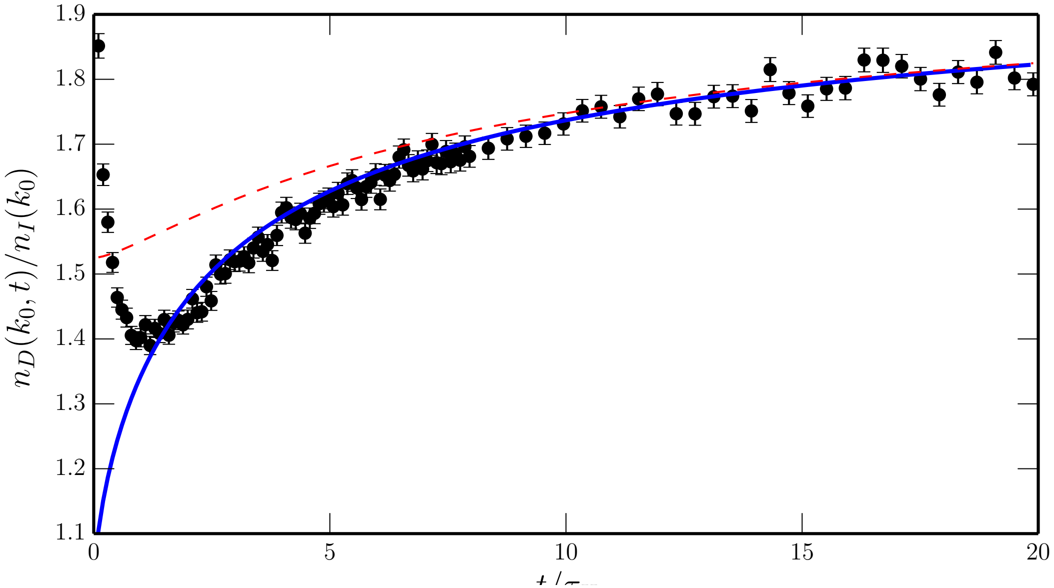
- and with time-reversal invariance $|\langle -\mathbf{k}|\alpha\rangle| = |\langle \mathbf{k}|\alpha\rangle|$.

1. arXiv:1405.2979 [pdf, ps, other]

Dynamics of localized waves in 1D random potentials: statistical theory of the coherent forward scattering peak

Kean Loon Lee, Benoît Grémaud, Christian Miniatura



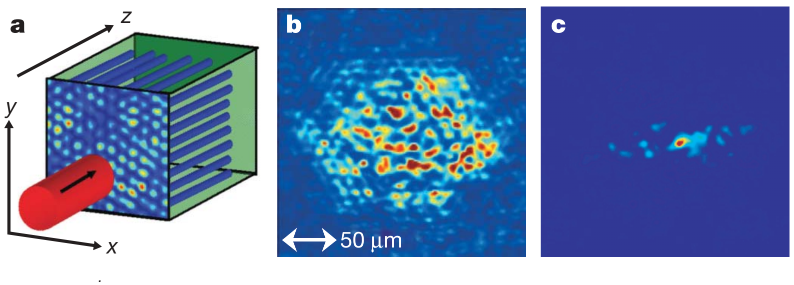


Conclusions, Outlook:

- "Welcome to Twin Peaks": [Karpiuk et al., PRL **109**, 190601 (2012)]

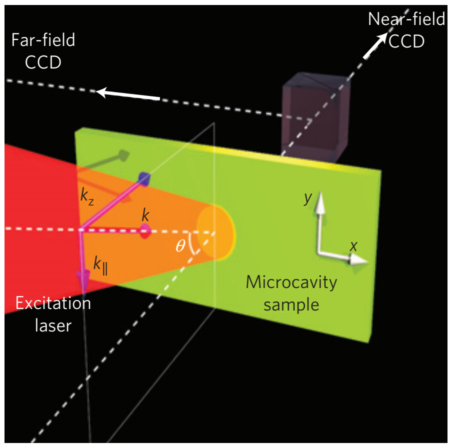
  AL's "smoking gun"
- Coherence peaks signal "Absence of Ergodicity"
- Fully analytical, time-resolved dynamics of strong localization [Micklitz, CAM, Altland, PRL **112**, 110602 (2014)]

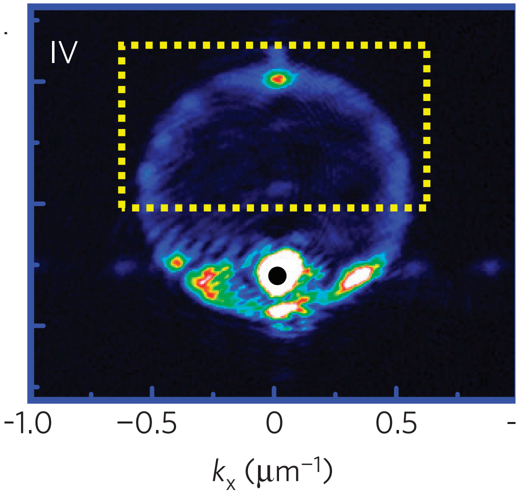
- Other natural candidate: Optical fibres



[Schwartz, Bartal, Fishman, Segev, Nature (2007)]

- Polariton condensates:

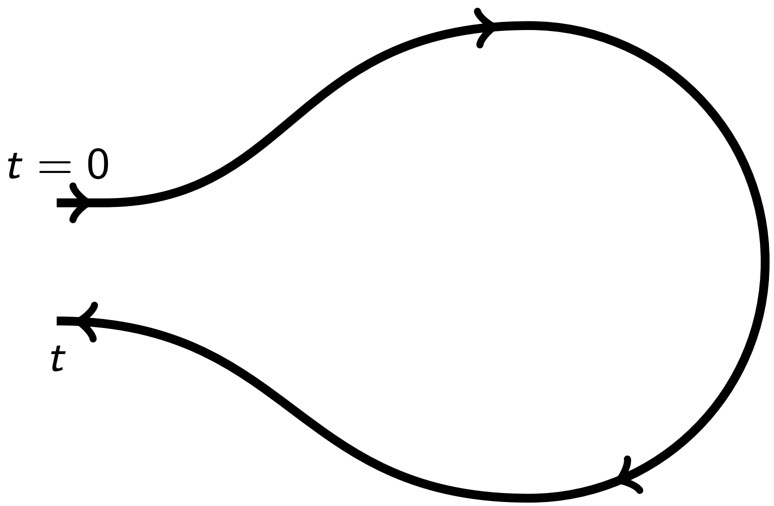




[Amo, . . . , Giacobino, Bramati, Nature Physics (2009)]

Controlled dephasing? Coherence Echoes!

- “Echo spectroscopy of Anderson localisation loops”
  with T. Micklitz (Rio de Janeiro), A. Altland (Cologne), and V. Josse, A. Aspect (Institut d’Optique, Palaiseau)
- One-loop Cooperon:



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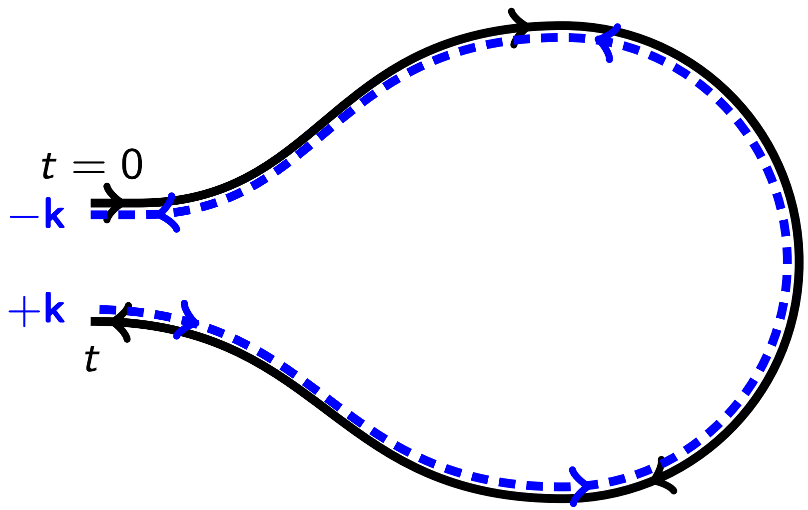

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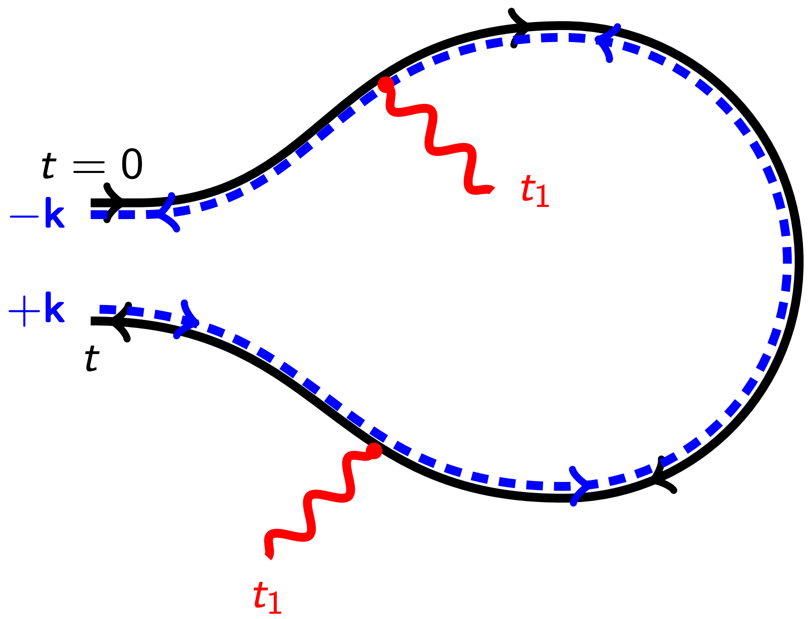

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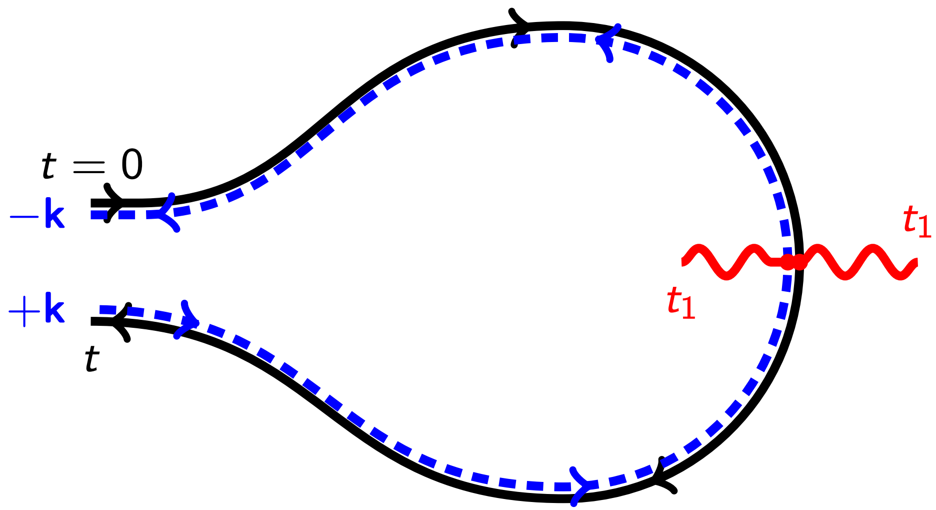

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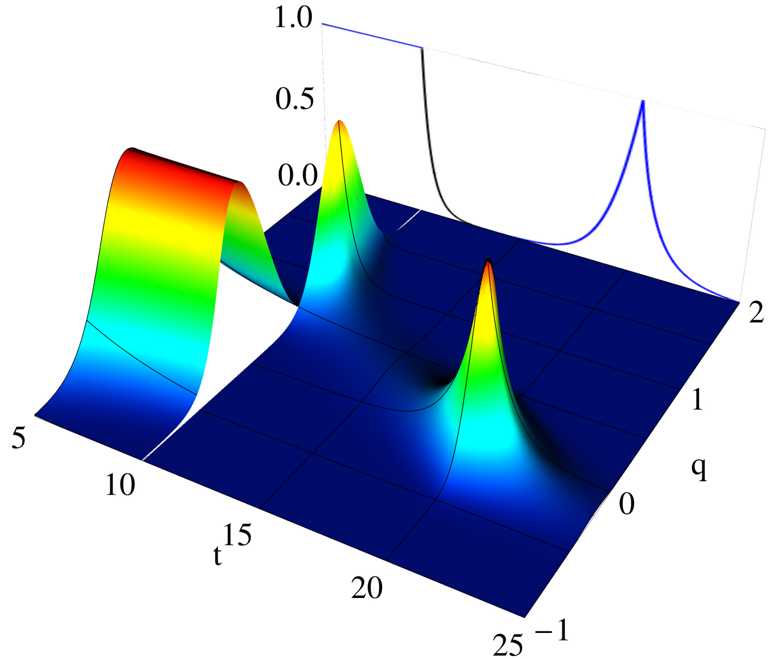

- Higher orders: use more pulses!

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- T. Karpiuk (Białystok, CQT)
- C. Miniatura, B. Grémaud (CQT)
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- T. Bourdel, V. Josse, A. Aspect (Palaiseau)