Universal Scaling of the Order-Parameter Distribution in Strongly Disordered Superconductors

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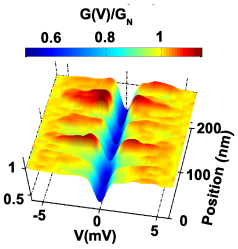
INTRODUCTION

Direct superconductor-insulator transition

Local tunneling spectra, NbN, T_c =1.65 K
also in TiN, Sacépé et al. (2011)

- The gap is always large.
- The height of the coherence peaks measures the local order parameter.

⇒ **Localization of preformed Cooper pairs**



Bosonic scenario of competition between local disorder and Cooper pair hopping

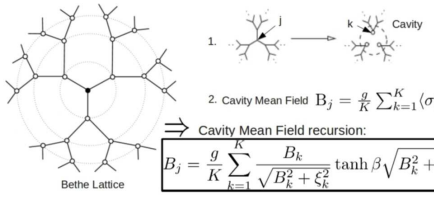
Model: $$H_I = -\sum_i \xi_i \sigma_i^z - \frac{g}{K}\sum_{\langle ij\rangle}\left(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y\right)$$

- Disorder $\xi_i \in [-1,1]$; $\sigma_i^z = \pm 1 \Leftrightarrow$ site occupied or not by a Cooper pair.
- Superconducting phase $\equiv$ spontaneous magnetization in the $x-y$ plane.

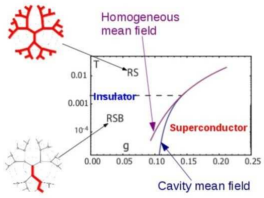
MOTIVATION

Cavity Mean Field approach

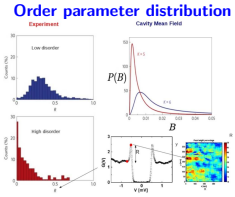
(Ioffe and Mézard, 2010; Feigl'man, Ioffe and Mézard, 2010)



Phase diagram



Order parameter distribution



(Sacépé et al., 2011)

Open questions

- Universality of the bosonic model?
- Bethe Lattice ≈ d= ∞ ⇒ does it describe 2D?

METHODOLOGY

Bogoliubov-de Gennes Mean Field Theory of the fermionic Hubbard model

(Ghosal, Randeria and Trivedi, 2001)

$$\mathcal{H}_{BdG} = -t\sum_{\langle ij\rangle,\sigma}(c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i,\sigma}(V_i - \mu + |U|\langle n_i\rangle/2)n_{i\sigma} + \sum_i[\Delta_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \Delta_i^* c_{i\downarrow}c_{i\uparrow}]$$

Solved self-consistently $\langle n_i\rangle = \sum_\sigma\langle c_{i\sigma}^\dagger c_{i\sigma}\rangle$ $\quad \Delta_i = -|U|\langle c_{i\downarrow}c_{i\uparrow}\rangle$

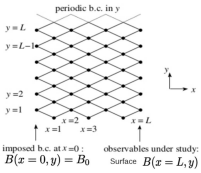
On-site disorder V_i box-distributed in $[-V_0, V_0]$; coupling $g \equiv \frac{t^2}{V_0 U}$.

Other approaches

- Inhomogeneous Mean field approach to the bosonic model.
- Quantum fluctuations using linear Spin-Wave theory (in progress).
- Quantum Monte-Carlo (in progress).

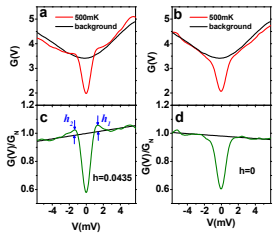
2D-Cavity Mean Field

(Monthus and Garel, 2010)



METHODOLOGY

Scanning tunneling spectroscopy measurements

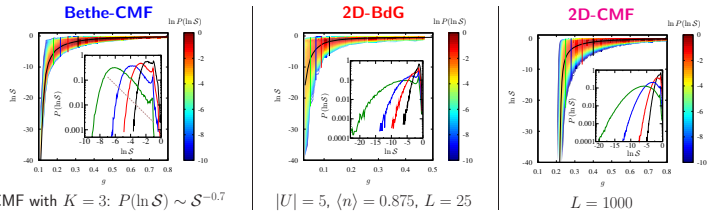


At $T = 500$ mK in a sample with $T_c = 1.65$ K

- **Background:** spatially averaged spectrum recorded at 8 K (V-shape, Altshuler-Aronov type interactions).
- Feature associated with superconductivity: divide the individual spectra at low temperatures by the background.
- The gap is always large.
- Averaged height of the coherence peaks: $h = \frac{h_1+h_2}{2} \Rightarrow$ measure of the local order parameter.

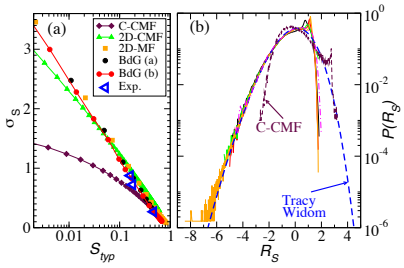
Local order parameter	CMF	BdG	STM experiments
	$\mathcal{S}_i \equiv \frac{B_i}{g}$	$\mathcal{S}_i \equiv \frac{1}{4}\sum_{k=1}^{4}\frac{2\Delta_k}{\lvert U\rvert}$	$\mathcal{S}_i \equiv \frac{h_i}{\mathrm{Max}[h]}$

NUMERICAL RESULTS



CMF with $K = 3$: $P(\ln\mathcal{S}) \sim \mathcal{S}^{-0.7}$ | $|U| = 5$, $\langle n\rangle = 0.875$, $L = 25$ | $L = 1000$

Universal scaling in 2D



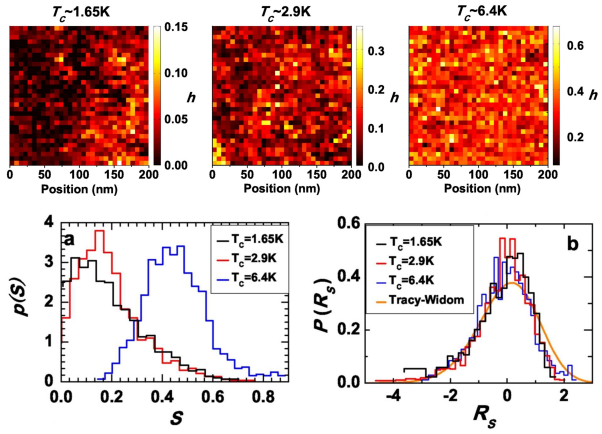
$$\mathcal{R}_\mathcal{S} = (\ln\mathcal{S} - \ln\mathcal{S}_{\mathrm{typ}})/\sigma_\mathcal{S}$$

with $\mathcal{S}_{\mathrm{typ}} = \exp\overline{\ln\mathcal{S}}$
and $\sigma_\mathcal{S}^2 = \overline{\ln^2\mathcal{S}} - \overline{\ln\mathcal{S}}^2$.

Parameters: (a) 2D-CMF, $L = 1000$; 2D-MF, $L = 120$; CMF, $L = 15$, $K = 3$; BdG (a), $|U| = 9$, $\langle n\rangle = 0.3$, $L = 25$; BdG (b), $|U| = 5$, $\langle n\rangle = 0.875$, $L = 25$. (b) same parameters as in (a) with in addition, 2D-CMF, $g = 0.4$; 2D-MF, $g = 0.2$; CMF, $g = 0.2$; BdG (a), $g = 0.1$, ie $V_0 = 1.1$; BdG (b), $g = 0.08$, ie $V_0 = 2.5$; magenta dashed dotted line, BdG with $|U| = 1.5$, $\langle n\rangle = 0.875$, $L = 25$, $g = 0.2$, ie $V_0 = 3.33$.

EXPERIMENTAL RESULTS

Universal scaling of the experimental distribution



CONCLUSION: **Universal behavior of the order parameter distribution.**
Bethe lattice ≠ 2D. Effects of quantum fluctuations?

G. Lemarié, A. Kamlapure, D. Bucheli, L. Benfatto, J. Lorenzana, G. Seibold, S. C. Ganguli, P. Raychaudhuri, and C. Castellani, *PRB* **87**, 184509 (2013).

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