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Workshop on Coherent Phenomena in Disordered Optical Systems

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Strongly Disordered Superconducting Films: Tunneling Spectra & Electromagnetic Response

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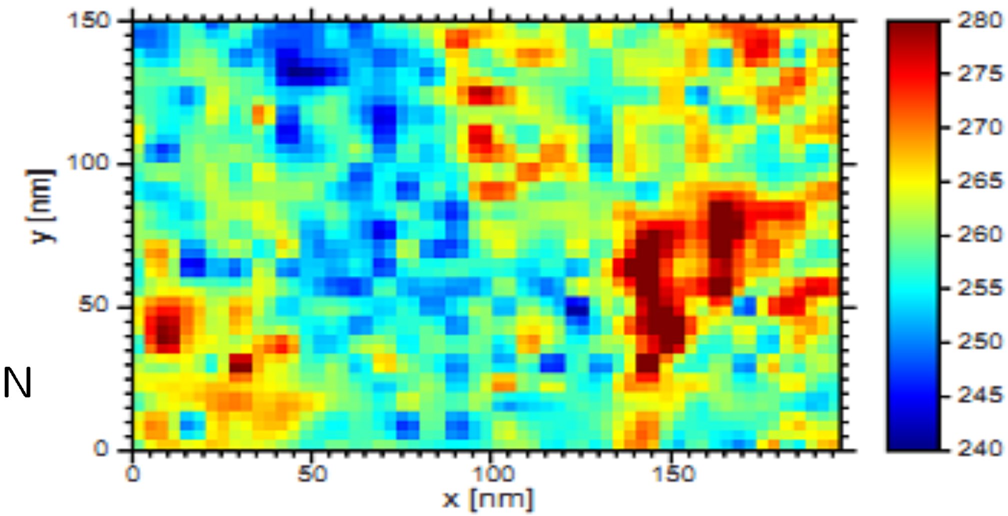
OF PHYSICS AND TECHNOLOGY

Coherent Phenomena in Disordered Optical Systems
ICTP, Trieste, 27.05.2014

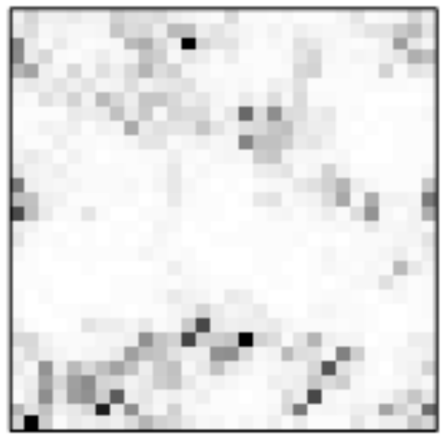
$\Delta(\mathbf{r})$



TiN

B. Sacepe *et al* (2008)

$\Delta(\mathbf{r})$



A. Ghosal *et al* (2001)

Electrodynamics of superconducting thin films

$$k_F l \gg 1$$

$$\sigma = 2e^2 N(0) D$$

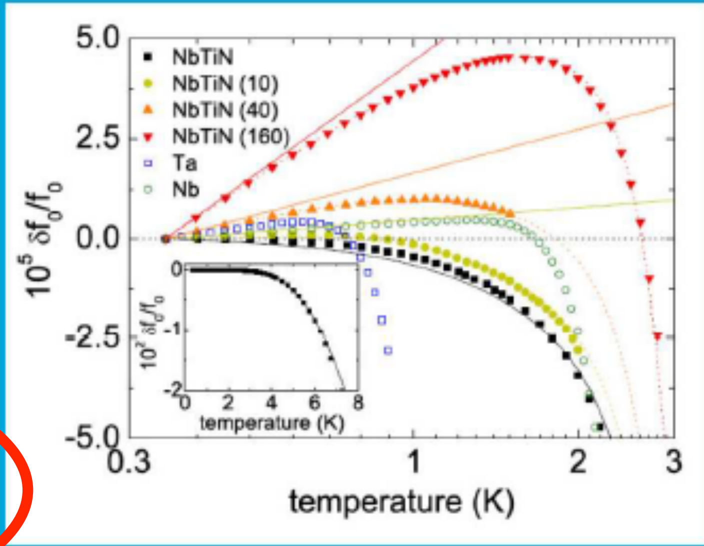
$$\frac{\sigma_1}{\sigma_N} = \frac{2}{\hbar\omega}\int_\Delta^\infty [f(E) - f(E+\hbar\omega)] g_1(E) dE + \frac{1}{\hbar\omega}\int_{\Delta-\hbar\omega}^{-\Delta} [1 - 2f(E+\hbar\omega)] g_1(E) dE$$

$$\frac{\sigma_2}{\sigma_N} = \frac{1}{\hbar\omega}\int_{\max(\Delta-\hbar\omega,-\Delta)}^{\Delta} [1 - 2f(E+\hbar\omega)] g_2(E) dE$$

$$g_1(E) = \left(1 + \frac{\Delta^2}{E(E+\hbar\omega)}\right) N_S(E) N_S(E+\hbar\omega)$$

$$g_2(E) = \frac{E(E+\hbar\omega)+\Delta^2}{\sqrt{(E+\hbar\omega)^2-\Delta^2}\sqrt{\Delta^2-E^2}} = -i g_1(E)$$

- Localization: D
- Correlations: N(0)



E to E+iΓ

$$\Gamma = 17\ \mu\mathrm{eV}$$

Dynes broadening parameter

Adapted from T. Klapwijk

PHYSICAL REVIEW B **88**, 180505(R) (2013)

Electrodynamic response and local tunneling spectroscopy of strongly disordered superconducting TiN films

P. C. J. J. Coumou,[1,*] E. F. C. Driessen,[2,†] J. Bueno,[3] C. Chapelier,[2] and T. M. Klapwijk[1,4]

For the least disordered film ($k_F l = 8.7$, $R_s = 13\ \Omega$), we find good agreement, whereas for the most disordered film ($k_F l = 0.82$, $R_s = 4.3\ \mathrm{k}\Omega$), there is a strong discrepancy, which signals the breakdown of a model based on uniform properties.

We model the microwave response using a description of the superconducting state, in which the superconductor is homogeneously weakened by the disorder-dependent pair-breaking parameter α, similar to the effect of magnetic impurities.[3] We assume homogeneous superconductivity and describe the superconducting state using the Usadel equation,

Abrikosov-Gorkov model of magnetic impurities

$$iE\sin\theta + \Delta\cos\theta - \underline{\alpha\sin\theta\cos\theta} = 0, \qquad (2)$$

where E is the quasiparticle energy, $\sin\theta$ and $\cos\theta$ are the quasiclassical Green's functions, and Δ is the pairing amplitude, which is determined self-consistently for each temperature and value of α.[10] The effect of the pair-breaking parameter is to smoothen the coherence peak in the quasiparticle density of states, as shown in the inset in Fig. 2.

Outline

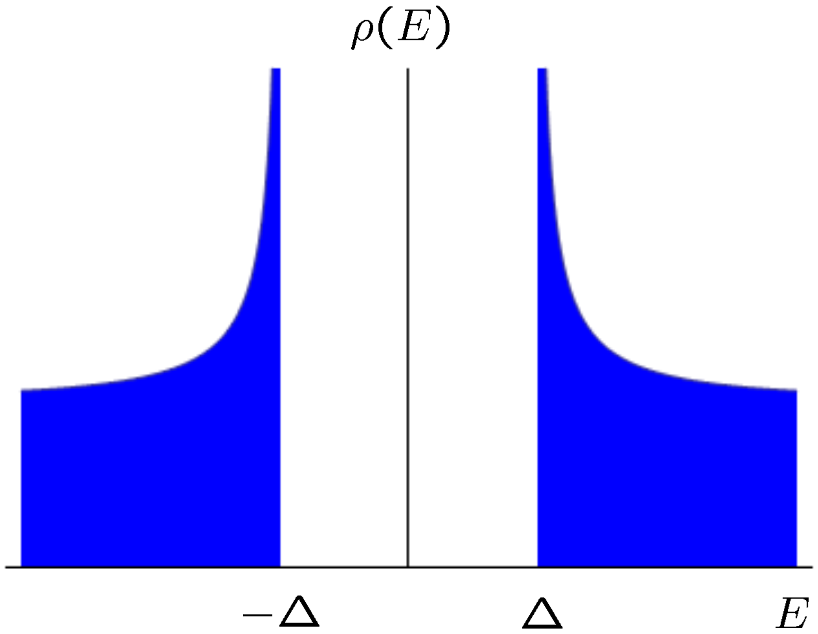
- DOS smearing in inhomogeneous superconductors
  - Models of disorder
  - Mesoscopic fluctuations on top of Coulomb repulsion
- Superfluid density in inhomogeneous superconductors

I

DOS smearing
in inhomogeneous superconductors

BCS coherence peak and Anderson theorem

$$\rho(E) = \rho_0 \ \mathrm{Re} \frac{E}{[E^2 - \Delta^2]^{1/2}}$$



Anderson theorem

Abrikosov & Gor'kov (1958)

T_c is insensitive
to nonmagnetic disorder

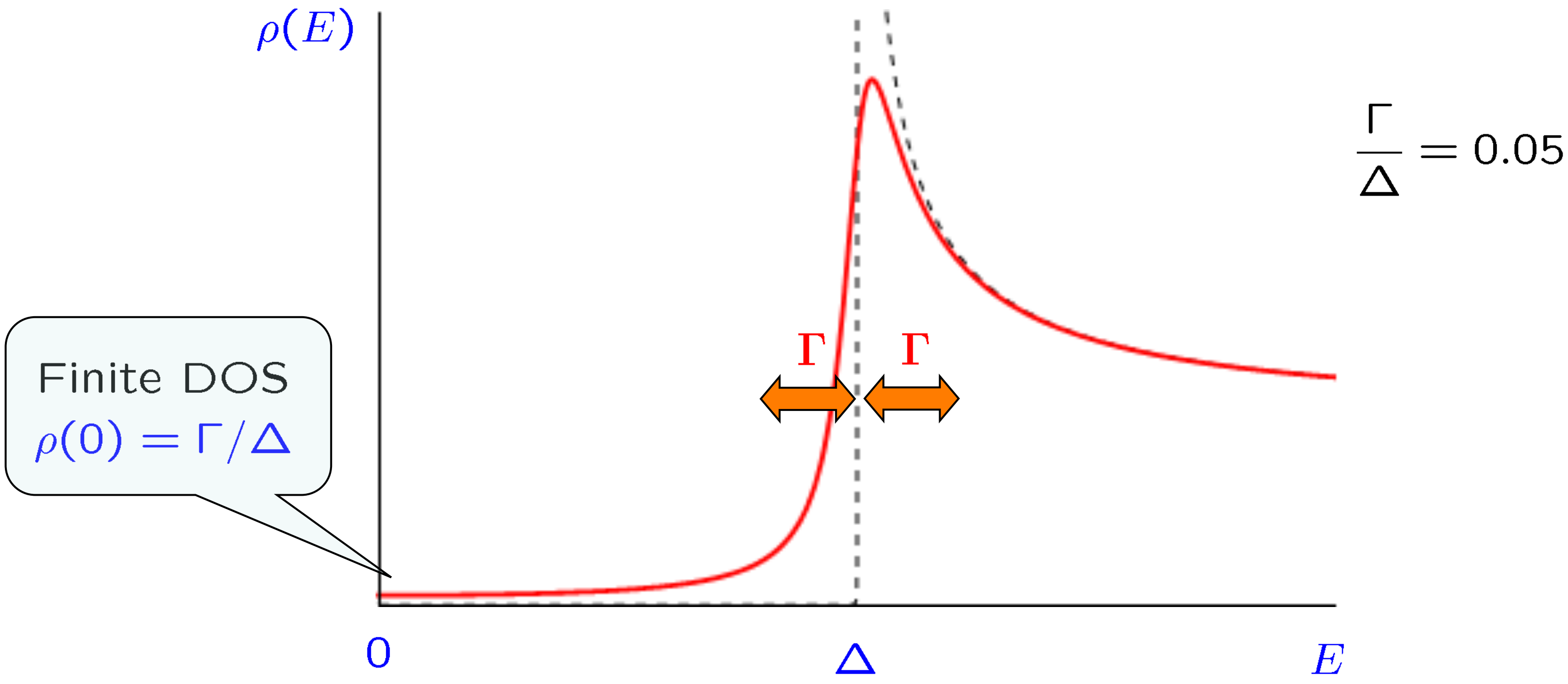
$$T_c = f(T_c\tau, E_F\tau)$$

Naïve smearing of the coherence peak

Dynes model

[R. C. Dynes *et al* (1978)]

$$\rho(E) = \rho_0 \ \mathrm{Re} \frac{E - i\Gamma}{[(E - i\Gamma)^2 - \Delta^2]^{1/2}}$$



Superconductor with paramagnetic impurities

Abrikosov & Gor'kov (1960)

$$\delta\mathcal{H} = \int d\mathbf{r}\, \psi^\dagger_\sigma(\mathbf{r})\mathbf{h}(\mathbf{r})\hat{\boldsymbol{\sigma}}\psi_\sigma(\mathbf{r})$$

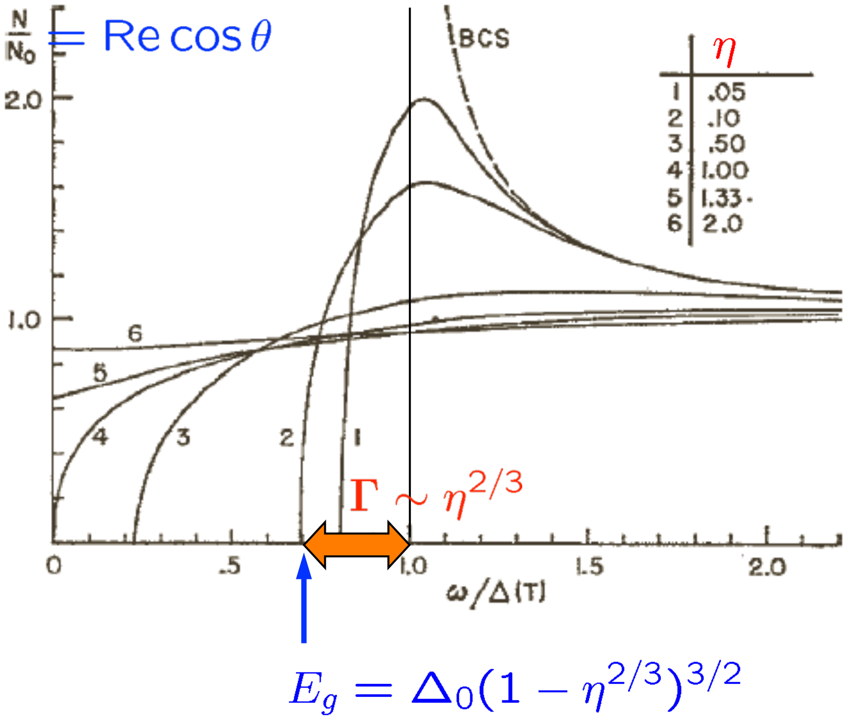
$$\langle h_i(\mathbf{r})h_j(\mathbf{r}')\rangle = \frac{\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')}{6\pi\nu\tau_s}$$

Usadel equation:

$$\eta = \frac{1}{\Delta_0\tau_s}$$

$$iE\sin\theta + \Delta_0\cos\theta - \underbrace{\Delta_0\,\eta\,\cos\theta\sin\theta}_{\text{depairing term}} = 0$$

$$E_g = \Delta_0(1-\eta^{2/3})^{3/2}$$



Two features of the AG model:

- "Minimal model" of depairing
- hard gap at $E = E_g$ (two solutions merging)

Models of inhomogeneous superconductors

- **Fluctuating coupling constant** $\lambda(\mathbf{r})$
  Larkin & Ovchinnikov (1972)
  Meyer & Simons (2001)

TR inv.

- **Magnetic disorder** $\mathbf{h}(\mathbf{r})$
  - Short-range
    Abrikosov & Gor'kov (1960)
    Balatsky & Trugman (1997)
    Lamacraft & Simons (2000)
    Silva & Ioffe (2005)
  - Long-range
    Ivanov, Fominov, MS & Ostrovsky (2009)

- **Universal mesoscopic disorder**
  Spivak & Zhou (1995)
  MS & Feigel'man (2005, 2012)

TR inv.

Random coupling constant (RCC) model

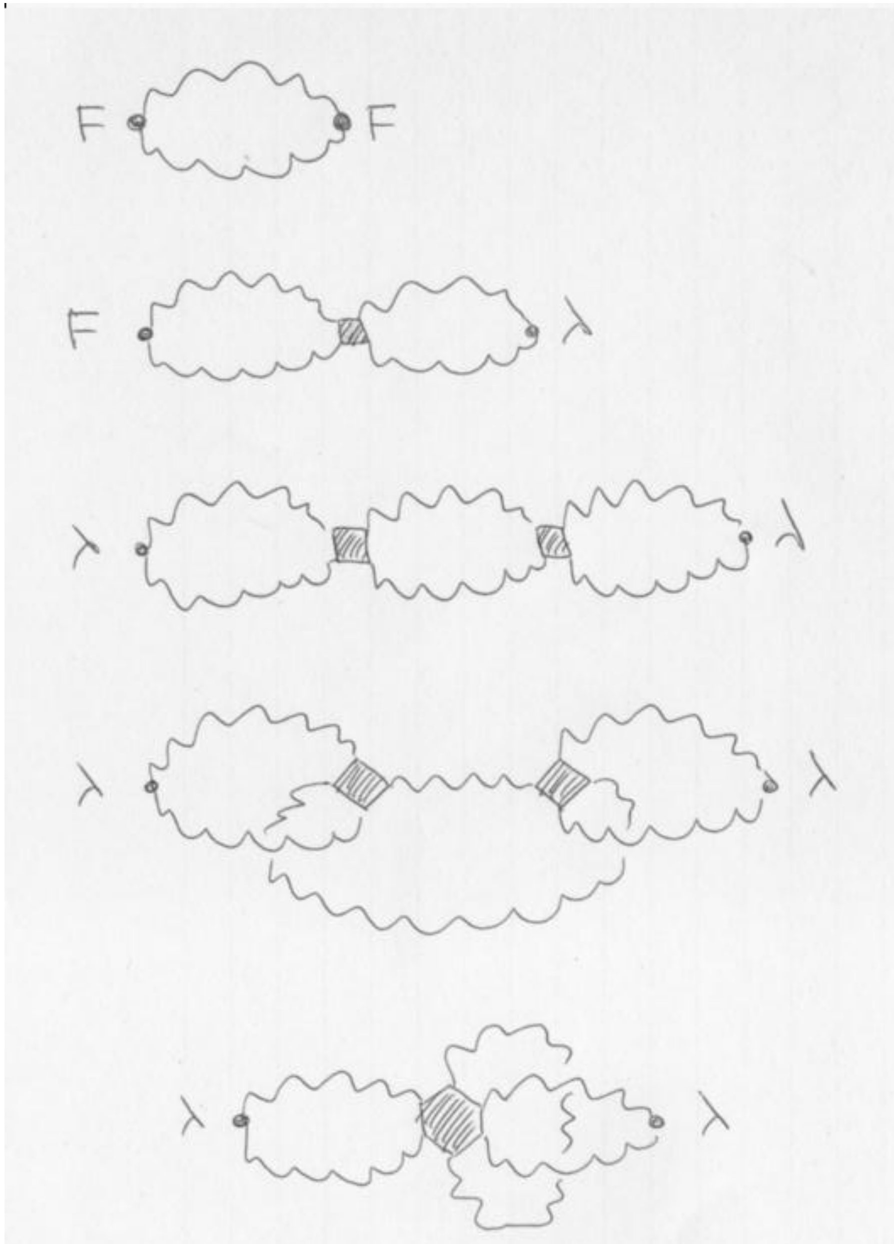
Larkin & Ovchinnikov (1972)

- Dirty limit ($\Delta\tau \ll 1$)
- Fluctuatuting coupling constant:

$$\frac{1}{\lambda(\mathbf{r})} = \frac{1}{\lambda_0} + u(\mathbf{r}), \qquad f_\lambda(\mathbf{r}) = \langle u(0)u(\mathbf{r})\rangle$$

- Phenomenological model
- Can be derived for mesoscopic fluctuations:

Random coupling constant (RCC) model



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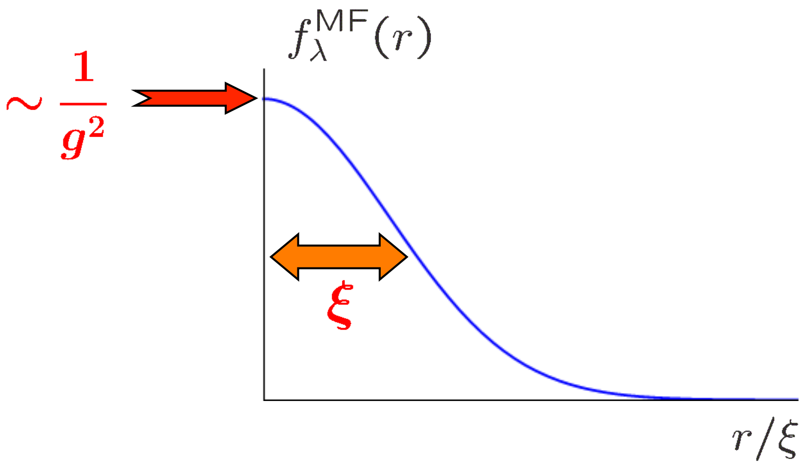


Random order parameter (ROP) model

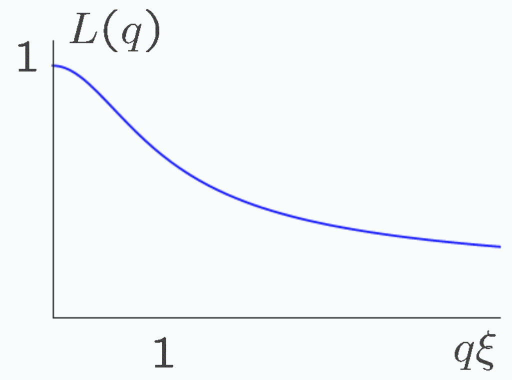
- **RCC model:** Fluctuations in $\lambda(\mathbf{r})$:

$$f_\lambda(q) = \langle \delta\lambda^{-1} \delta\lambda^{-1} \rangle_q$$

- **ROP model:** Fluctuations in $\Delta(\mathbf{r})$:

$$f_\Delta(q) = \langle \delta\Delta\, \delta\Delta \rangle_q = \Delta_0^2 L^2(q) f_\lambda(q)$$

fluctuation propagator



- **AG model:** Effective depairing and large-scale fluctuating $\Delta(\mathbf{r})$:

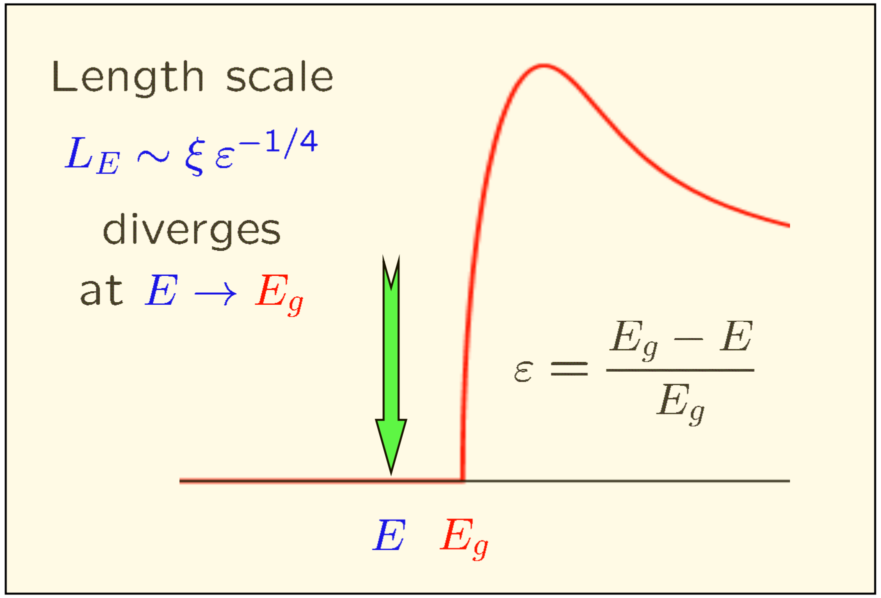
$$(D/2)\nabla^2\theta + iE\sin\theta + [\Delta_0 + \delta\Delta(\mathbf{r})]\cos\theta - \Delta_0\,\eta\cos\theta\sin\theta = 0$$

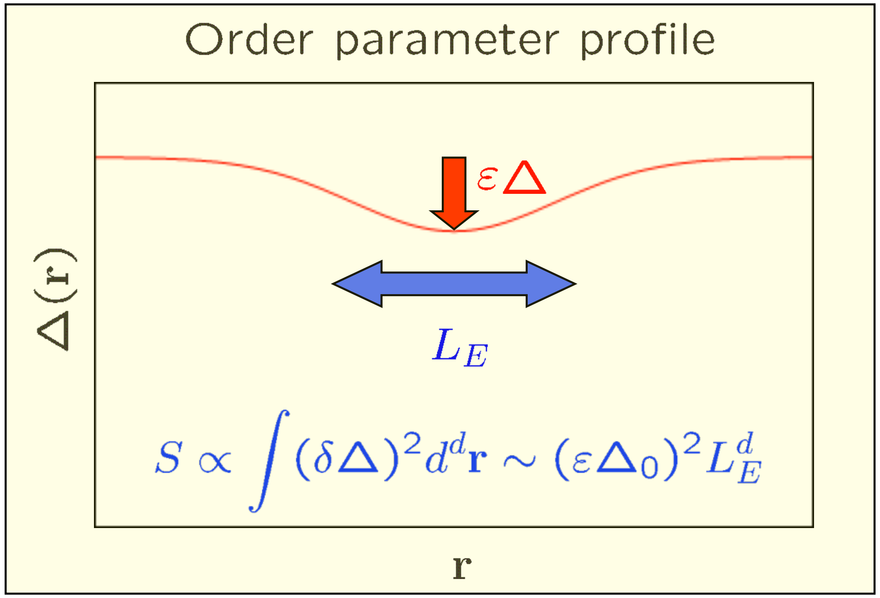
large scale fuctuations with $r \gg \xi$

effective depairing

$$\eta = \frac{2}{\Delta_0}\int \frac{\langle \Delta\Delta \rangle_q}{Dq^2} \frac{d^d q}{(2\pi)^d}$$

Optimal fluctuation





Effective depairing

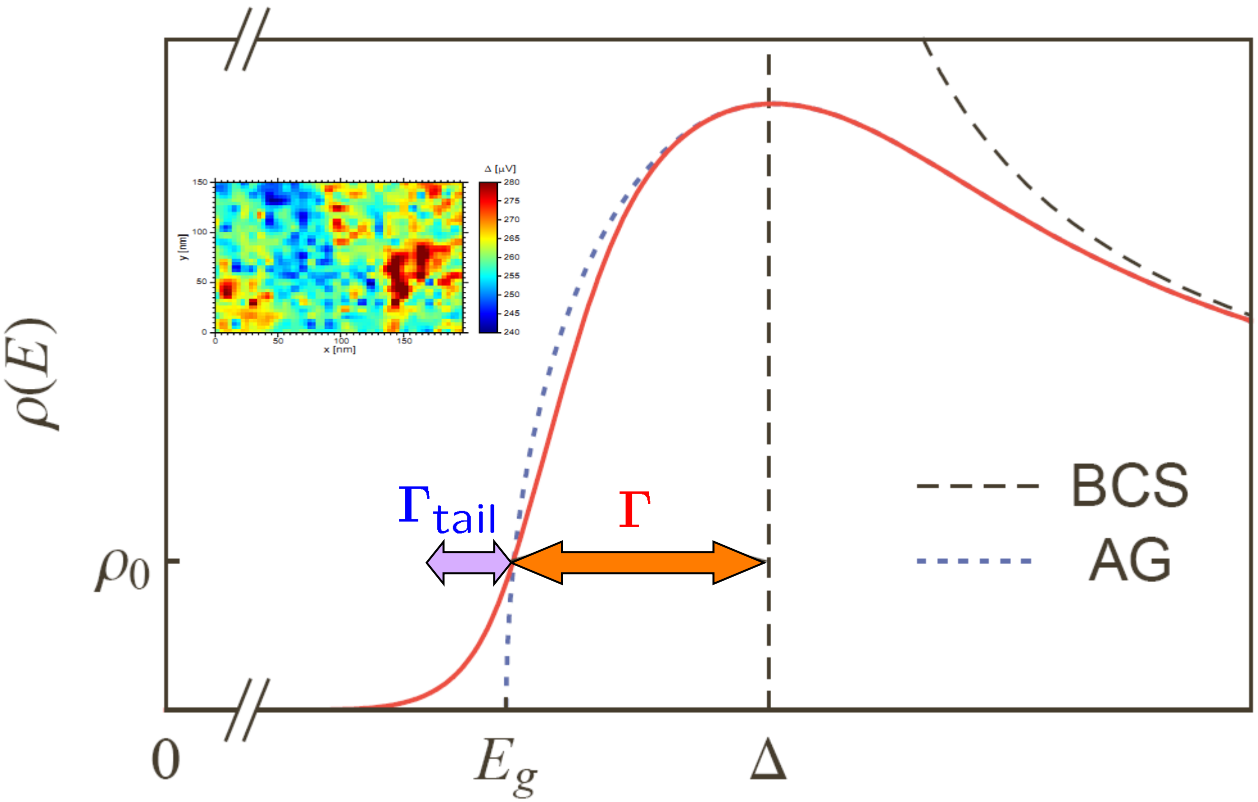
$$\eta = \frac{2}{\Delta_0}\int \frac{\langle\Delta\Delta\rangle_q}{Dq^2}\frac{d^d q}{(2\pi)^d}$$

DOS of the subgap states

$$\langle\rho(E)\rangle \propto \exp\left(-C\,\frac{\Delta_0^2\xi^d}{\langle\Delta\Delta\rangle_{q=0}}\,\varepsilon^{(8-d)/4}\right)$$

General summary on DOS smearing

- Roughly speaking, any disorder looks like magnetic impurities
- Subgap states are due to optimal fluctuations of $\Delta(r)$



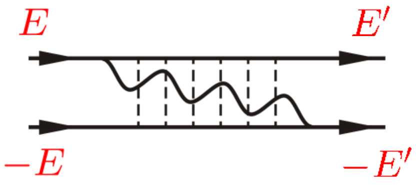
Universal mesoscopic disorder
+
Coulomb suppression of superconductivity

in superconducting films

homogeneously disordered films

moving from the metallic side ($g > 1$)

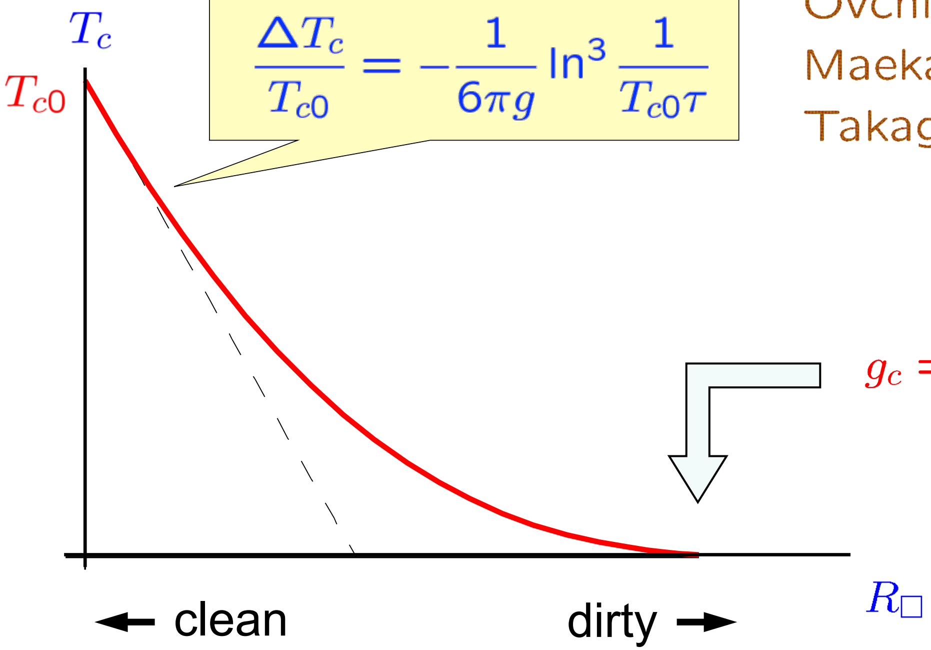
Coulomb suppression of T_c



$$\frac{\partial \lambda}{\partial \zeta} = \lambda^2 - \frac{1}{2\pi g}$$

$$T_c \tau = \left(\frac{1 - \frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T_{c0}\tau}}{1 + \frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T_{c0}\tau}} \right)^{\sqrt{\pi g/2}}$$

Finkelstein (1987)

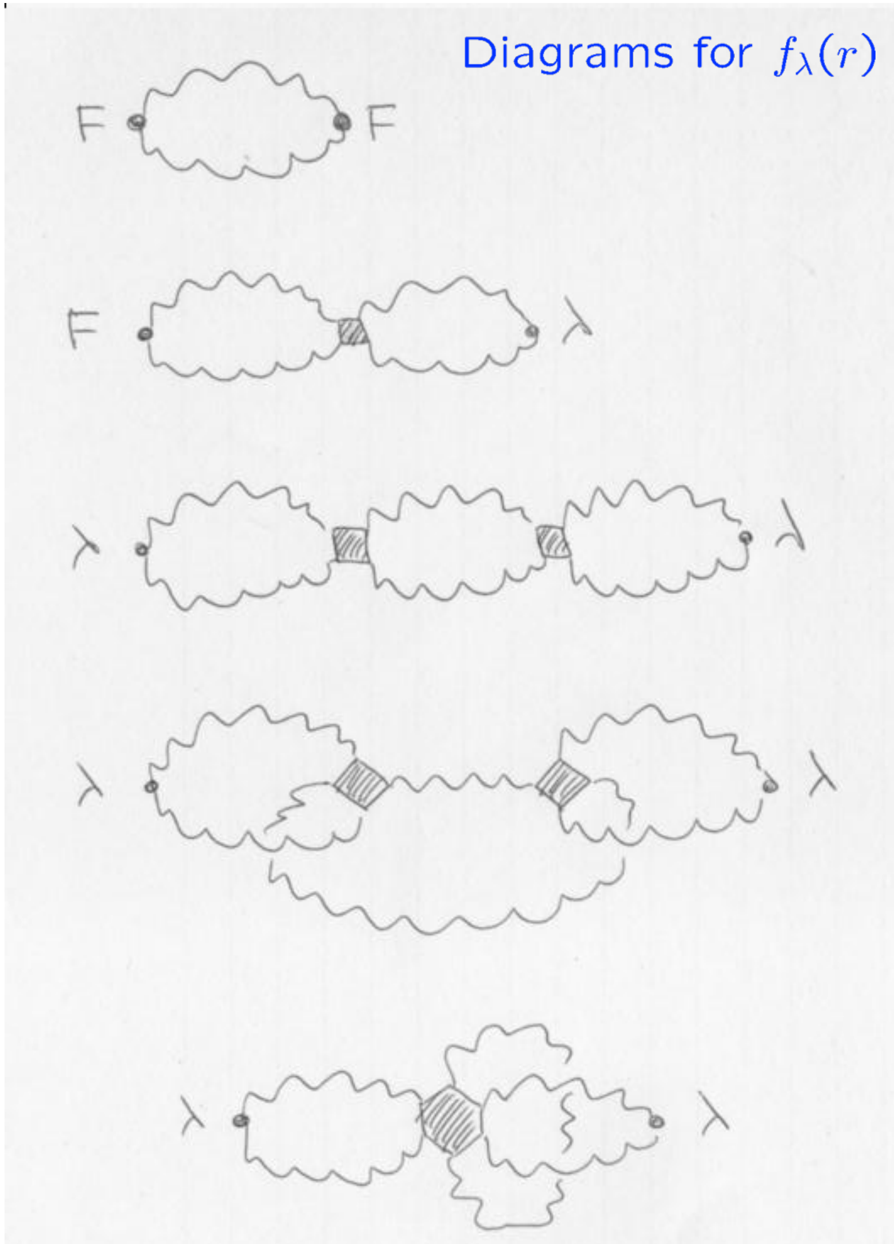


$$\frac{\Delta T_c}{T_{c0}} = -\frac{1}{6\pi g} \ln^3 \frac{1}{T_{c0}\tau}$$

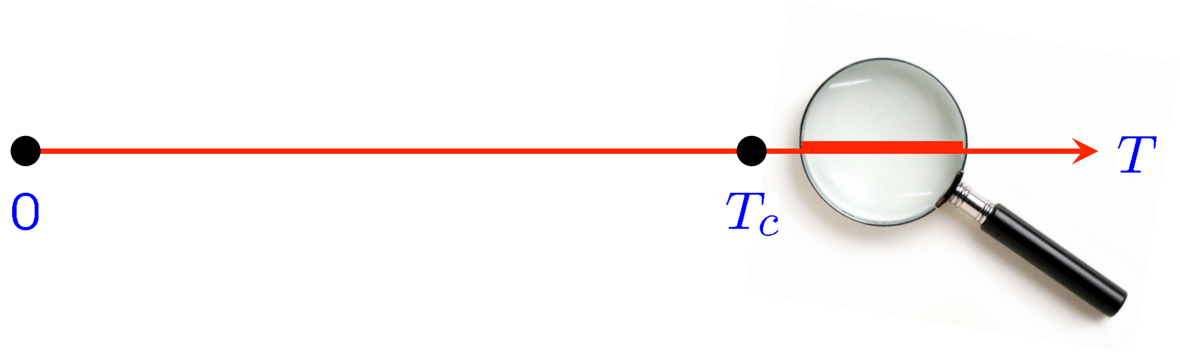
Ovchinnikov (1973)
Maekawa, Fukuyama (1982)
Takagi, Kuroda (1982)

$$g_c = \frac{1}{2\pi} \ln^2 \frac{1}{T_{c0}\tau}$$

Mesoscopic fluctuations



Vicinity of T_c

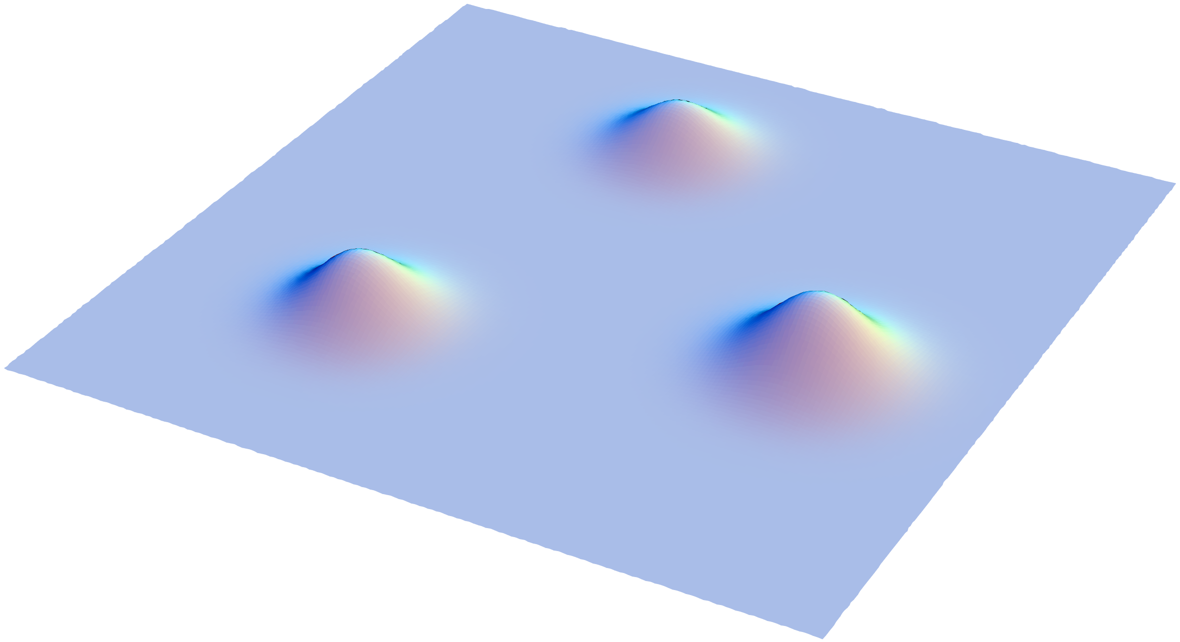


Vicinity of T_c

Superconductor with fluctuating T_c [Ioffe & Larkin (1981)]:

$$F = \int \left\{ \left[\alpha(T/T_c - 1) + \delta\alpha(\mathbf{r}) \right] |\Delta|^2 + \gamma |\nabla \Delta|^2 + \frac{\beta}{2} |\Delta|^4 \right\} d\mathbf{r}$$

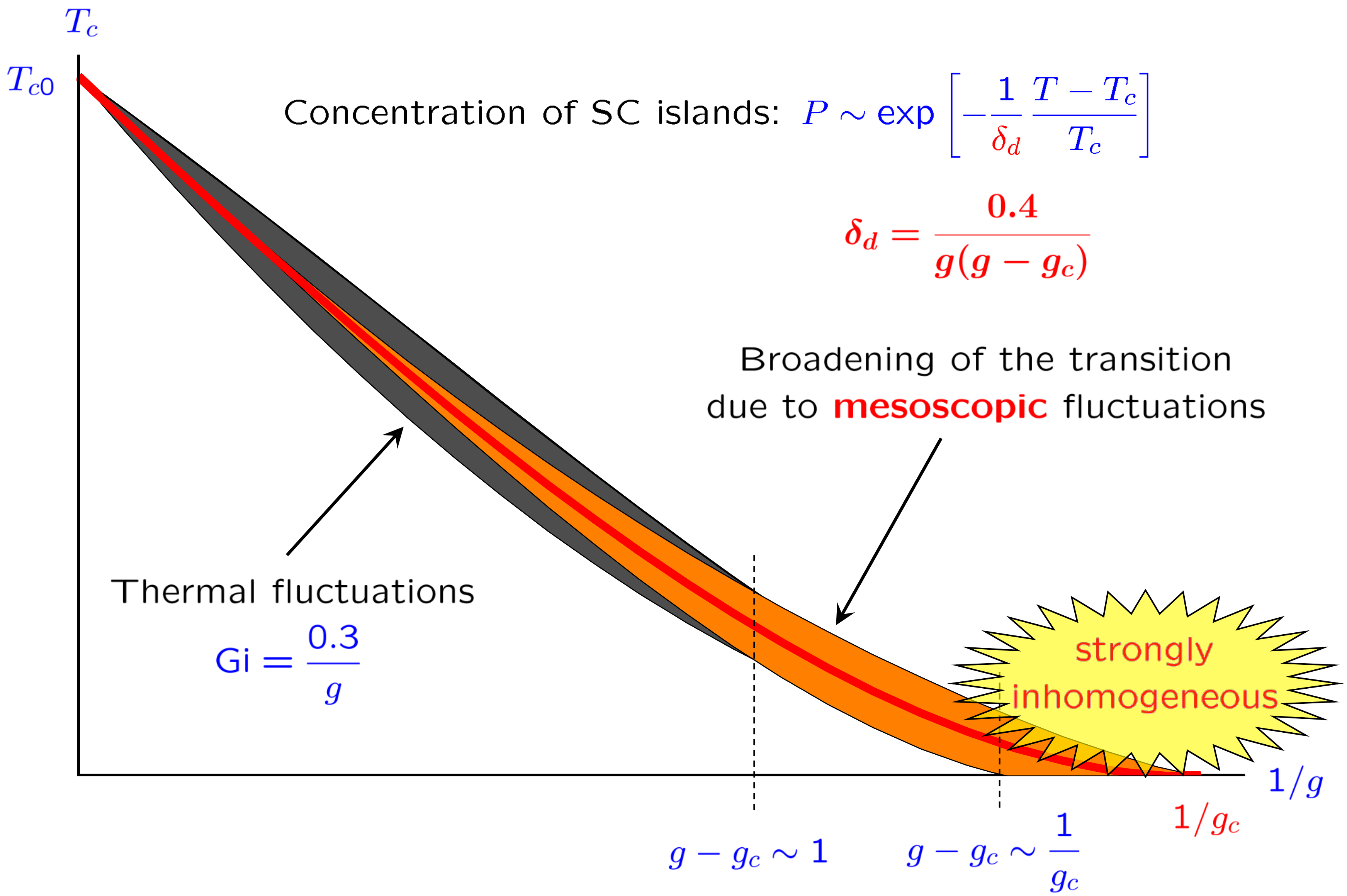
$$\langle \delta\alpha(\mathbf{r}) \delta\alpha(\mathbf{r}') \rangle = \frac{7\zeta(3)}{8\pi^4 DT} \frac{g}{g - g_c} \delta(\mathbf{r} - \mathbf{r}')$$



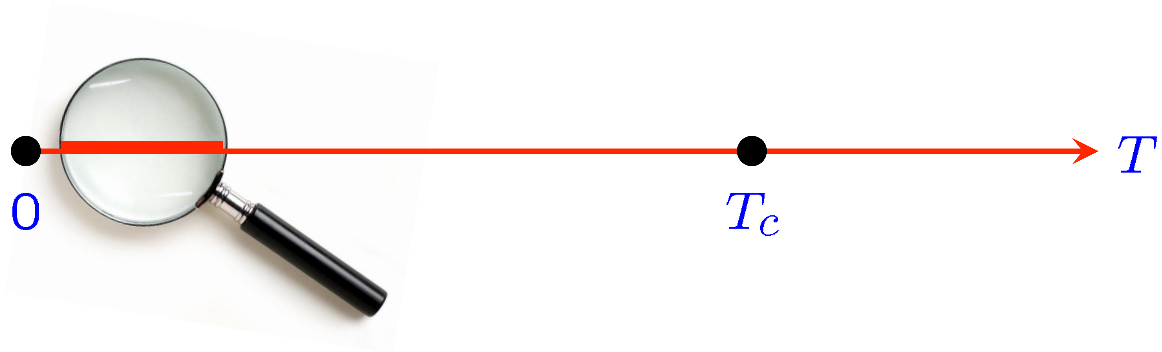
localized superconducting droplets form at $T > T_c$

Mesoscopic vs. thermal fluctuations near T_c

Skvortsov & Feigel'man (2005)

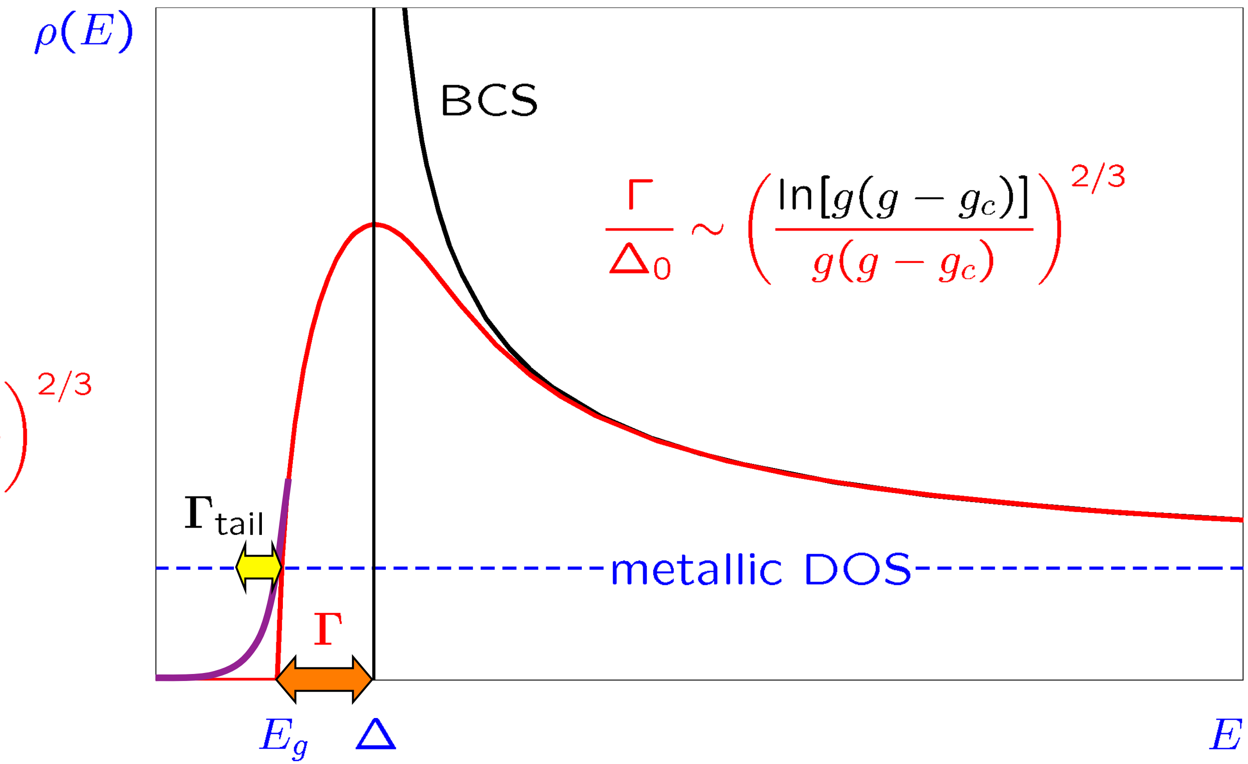


Low temperatures



Superconductor with fluctuating $\Delta(r)$

$$\frac{\Gamma_{\text{tail}}}{\Delta_0} \approx \left(\frac{0.4}{g(g-g_c)}\right)^{2/3}$$



Subgap states: $\langle\rho(E)\rangle \sim \exp\left\{-\left(\frac{E_g - E}{\Gamma_{\text{tail}}}\right)^{3/2}\right\}$

Feigel'man & Skvortsov (2012)

Summary on mesoscopic fluctuations + Coulomb

Homogeneously disordered superconducting films become strongly inhomogeneous at $g \to g_c$

(NB: mesoscopic disorder is *minimal* intrinsic disorder!)

M. A. Skvortsov and M. V. Feigel'man, Phys. Rev. Lett. **95**, 057002 (2005)
M. V. Feigel'man and M. A. Skvortsov, Phys. Rev. Lett. **109**, 147002 (2012)

II

Superfluid density
in disordered superconductors

Supefluid density

$$\mathbf{j}(\omega,\mathbf{q}) = -Q(\omega,\mathbf{q})\mathbf{A}(\omega,\mathbf{q})$$

Superfluid density: $Q = Q(0,0) = \dfrac{n_s e^2}{m}$

- **BCS, dirty limit:**

$$Q = 2\pi\sigma T\sum_{\varepsilon}\sin^2\theta_{\varepsilon} = 2\pi\sigma T\sum_{\varepsilon}\frac{\Delta^2}{\varepsilon^2+\Delta^2} = \pi\sigma\Delta\tanh\frac{\Delta}{2T}$$

$T=0$ limit: $\qquad Q_0 = \pi\sigma\Delta$

$$n_s/n \sim \Delta\tau \ll 1$$

WL correction to the SF density in SC films

Smith & Ambegaokar (1992)

$$Q_0 = \pi \left(\sigma - \frac{e^2}{2\pi^2} \ln \frac{1}{\Delta \tau} \right) \Delta$$

$$Q_0 = \pi \sigma_{\text{ren}} \Delta$$

SF density for the AG model

Skalski, Betbeder-Matibet & Weiss (1964)

Dirty limit with magnetic impurities

A la Mattis-Bardeen:

$$Q = 2\pi\sigma T \sum_{\varepsilon} \sin^2\theta_\varepsilon$$

with the modified spectral angle:

$$-\varepsilon\sin\theta_\varepsilon + \Delta\cos\theta_\varepsilon - \Gamma\cos\theta_\varepsilon\sin\theta_\varepsilon = 0$$

spin-flip rate
$\Gamma = 1/\tau_s$

$T = 0$ limit:

$$\boxed{Q_0 = \pi\sigma\Delta - \frac{4}{3}\sigma\Gamma}$$

$$\Delta = \Delta_0 - \frac{\pi^2}{4}\Gamma$$

SF density for the RCC model (1)

Fluctuatuting coupling constant: $f_\lambda(\mathbf{r}) = \langle \delta\lambda^{-1}(0)\delta\lambda^{-1}(\mathbf{r})\rangle$

Free energy $\langle\mathcal{F}\rangle$ calculated with the help of the replicated σ-model:

$$Q = \frac{e^2}{V}\,\frac{\partial\langle\mathcal{F}\rangle}{\partial\mathbf{A}^2}\bigg|_{A=0}, \qquad \langle\mathcal{F}\rangle = -\lim_{n\to 0}\frac{T}{n}\int e^{-S[Q]}DQ$$

constant vector potential $\mathbf{A}$

imaginary energy (Matsubara)

$$S = S_0 + S_A + S_\Delta + S_{\text{dis}}$$

$$S_0 + S_A = \frac{\pi\nu}{2}\int d\mathbf{r}\ \mathrm{tr}\left[D(\nabla Q + i\mathbf{A}[\tau_3, Q])^2 - 4(\varepsilon\tau_3 + \Delta\tau_1)Q\right]$$

$$S_\Delta + S_{\text{dis}} = \frac{2\nu}{\lambda_0 T}\int d\mathbf{r}\,|\Delta^a|^2 - \frac{2\nu^2}{T^2}\int d\mathbf{r}\,d\mathbf{r}'\ f_\lambda(\mathbf{r}-\mathbf{r}')|\Delta^a(\mathbf{r})|^2|\Delta^b(\mathbf{r}')|^2$$

perturbation theory in f_λ

SF density for the RCC model (2)

Assuming a uniform saddle point with $Q^{ab}_\varepsilon = \delta^{ab}\begin{pmatrix}\cos\theta_\varepsilon & \sin\theta_\varepsilon \\ \sin\theta_\varepsilon & -\cos\theta_\varepsilon\end{pmatrix}$ and $\Delta^a = \Delta_0$ we integrate out small fluctuations

$$\boxed{w = \begin{pmatrix} c \\ d \end{pmatrix} \begin{matrix} \longleftrightarrow \Delta_\parallel \ \text{(modulus)} \\ \longleftrightarrow \Delta_\perp \ \text{(phase)} \end{matrix} \qquad \Delta = \Delta_0 + \Delta_\parallel + i\Delta_\perp}$$

- **Cooperons and diffusons**:

$$S^{(2)}[c,d] = -\pi\nu \int (d\mathbf{q}) \sum_\varepsilon w^T_\varepsilon(-\mathbf{q})\Pi^{-1}_\varepsilon(\mathbf{q}) w_\varepsilon(\mathbf{q})$$

$$\Pi_\varepsilon(\mathbf{q}) = \begin{pmatrix} Dq^2 + 2\sqrt{\varepsilon^2+\Delta^2} + 4D\mathbf{A}^2\cos 2\theta_\varepsilon & -4iD\mathbf{A}\mathbf{q}\cos\theta_\varepsilon \\ 4iD\mathbf{A}\mathbf{q}\cos\theta_\varepsilon & Dq^2 + 2\sqrt{\varepsilon^2+\Delta^2} + 4D\mathbf{A}^2\cos^2\theta_\varepsilon \end{pmatrix}^{-1}$$

angle-dependent
mode mixing

SF density for the RCC model (3)

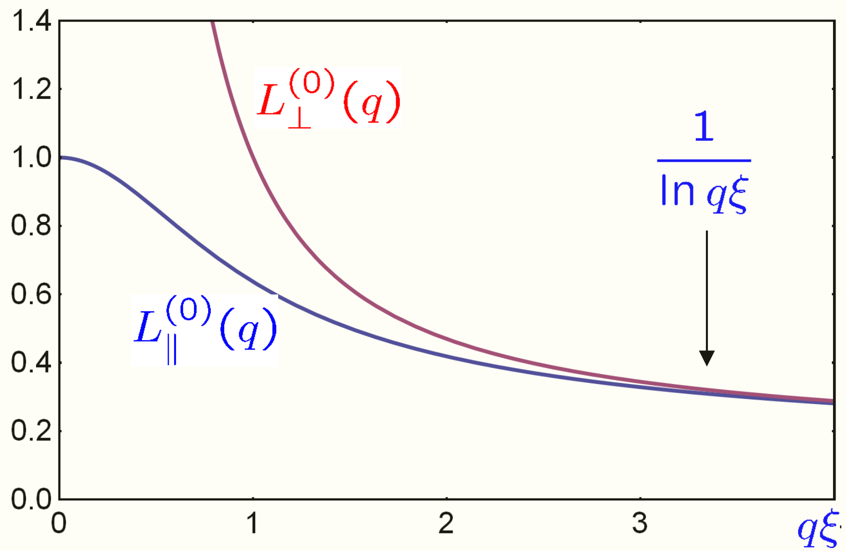
- **Order parameter fluctuations**:

$$S^{(2)}[\Delta_\parallel, \Delta_\perp] = \frac{2\nu}{T}\int (d\mathbf{q})\, \hat{\Delta}^T(-\mathbf{q}) L^{-1}(q) \hat{\Delta}(\mathbf{q}) \qquad \hat{\Delta} = \begin{pmatrix} \Delta_\parallel \\ \Delta_\perp \end{pmatrix}$$

Matrix fluctuation propagator:

$$L^{-1}(\mathbf{q}) = \frac{1}{\lambda_0} - 2\pi T \sum_\varepsilon \gamma_\varepsilon \underset{\uparrow}{\Pi_\varepsilon(\mathbf{q})} \gamma_\varepsilon \qquad \gamma_\varepsilon = \begin{pmatrix} \cos\theta_\varepsilon & 0 \\ 0 & 1 \end{pmatrix}$$

$\mathbf{A}$-dependent



Fluctuation propagators
in the absence of disorder
in $\lambda(\mathbf{r})$ (dirty BCS)
at zero temperature

SF density for the RCC model (4)

Fluctuatuting coupling constant: $f_\lambda(\mathbf{r}) = \langle \delta\lambda^{-1}(0)\delta\lambda^{-1}(\mathbf{r})\rangle$

$$S_{\text{dis}} = -\frac{2\nu^2}{T^2}\int d\mathbf{r}\, d\mathbf{r}'\, f_\lambda(\mathbf{r}-\mathbf{r}')|\Delta^a(\mathbf{r})|^2|\Delta^b(\mathbf{r}')|^2$$

$$\Delta_0 + \Delta_\parallel^a \qquad \Delta_0 + \Delta_\parallel^b$$

Averaging over fluctuations with $\langle \Delta_\parallel(-\mathbf{q})\Delta_\parallel(\mathbf{q})\rangle = \dfrac{T}{4\nu}L_\parallel(\mathbf{q})$

Resulting effective action:

$$S_{\text{eff}} = \frac{2\nu|\Delta_0|^2}{T}\int (d\mathbf{q}) f_\lambda(\mathbf{q}) L_\parallel(\mathbf{q})$$

SF density for the RCC model (5)

Integrating out fluctuations we arrive at the free energy $\mathcal{F}$
to be minimized with respect to θ_ε and Δ

$$\frac{\mathcal{F}}{V} = 4\pi\nu T \sum_{\varepsilon} \left[-\varepsilon \cos\theta_\varepsilon - \Delta_0 \sin\theta_\varepsilon + D\mathbf{A}^2 \sin^2\theta_\varepsilon \right] + 2\nu|\Delta_0|^2 \underbrace{\left(\frac{1}{\lambda_0} - \int (d\mathbf{q}) f_\lambda(\mathbf{q}) L_\parallel(\mathbf{q}) \right)}_{\text{effective } \lambda \text{ grows}}$$

$$\frac{\partial \Delta}{\partial f_\lambda} > 0$$

Superfluid response:

$$Q = 2\pi\sigma \left[T \sum_{\varepsilon} \sin^2\theta_\varepsilon - \frac{|\Delta_0|^2}{2\pi} \int (d\mathbf{q}) \frac{\partial [f_\lambda(\mathbf{q}) L_\parallel(\mathbf{q})]}{\partial (D\mathbf{A}^2)} \right]$$

A la Mattis-Bardeen

new term

Not described by Mattis-Bardeen!

Inhomogeneous supercurrent

$$\frac{\partial \langle L_{\parallel}(q)\rangle_{\mathrm{n}}}{\partial \mathrm{A}^2} = \frac{4D}{\Delta_0}[L_{\parallel}^{(0)}(q)]^2 \left[-P_2(q) + q^2 \underbrace{\boxed{L_{\perp}^{(0)}(q)}}_{\text{phase mode}} P_1^2(q)\right]$$

Seibold, Benfatto, Castellani, Lorenzana (2012)

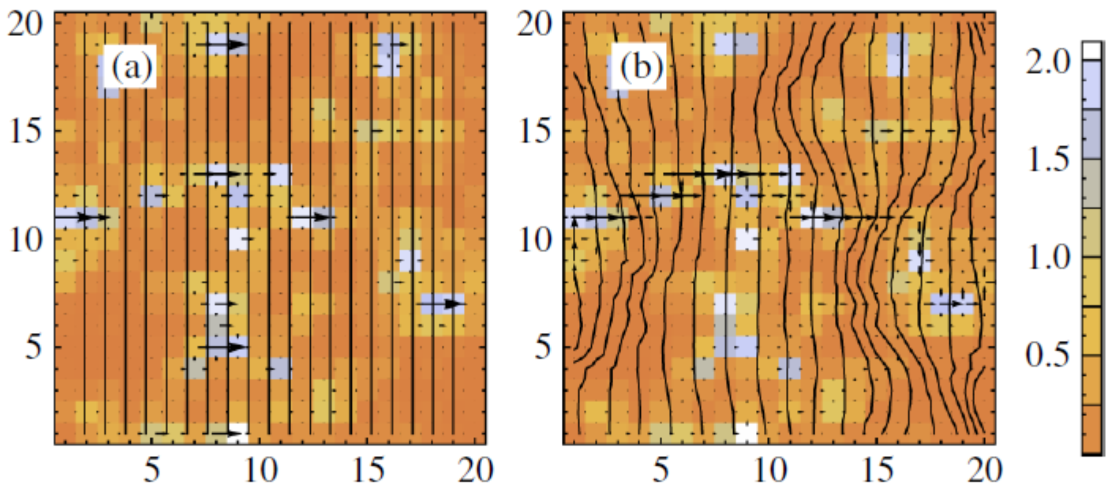


FIG. 1 (color online). Distribution of the local current (arrows) and lines of constant $\tilde{\theta}$ superimposed to the map of the order parameter, neglecting phase relaxation (a) and allowing for it (b).

SF density suppression by mesoscopic fluctuations

Superfluid response:

$$Q^{\text{MF}} \approx \pi\sigma\Delta\left[1 - \frac{1}{3g^2}\right]$$

Anticipated in the AG model:

$$Q^{\text{MF}} = \pi\sigma\Delta\left[1 - \frac{4}{3\pi}\eta_{\text{EM}}\right] \quad \Longrightarrow \quad \eta_{\text{EM}} \sim \frac{1}{g^2}$$

DOS smearing:

$$\eta_{\text{STM}} \sim \frac{\ln(6g^2)}{6g^2}$$

In inhomogeneous systems,
suppression of the SF density
is several times more effective
than the DOS smearing

Summary

- Superfluid response of a disordered superconductor is a delicate issue
- Mattis-Bardeen theory is no longer valid
- Result is strongly model-dependent, picking up all hidden diffusons and cooperons

$$Q^{\mathrm{MF}} \approx \pi\sigma\Delta\left[1 - \frac{1}{3g^2}\right]$$