

The Abdus Salam
International Centre
for Theoretical Physics
50th Anniversary 1964–2014



2583-9

Workshop on Coherent Phenomena in Disordered Optical Systems

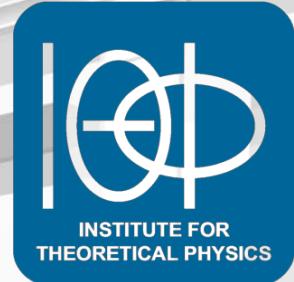
26 – 30 May 2014

New Insights on Coherent Wave Transmission through Disordered Systems

Stefan ROTTER
Vienna University of Technology, ITP
Vienna
Austria



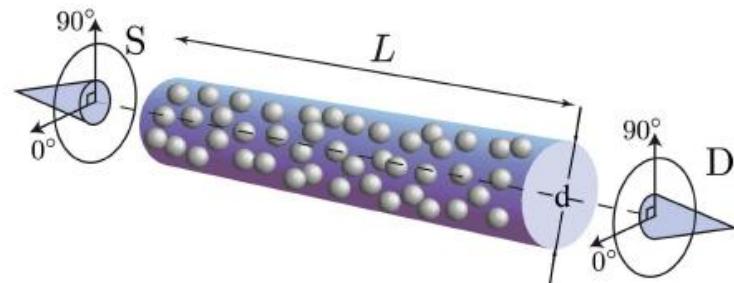
TECHNISCHE
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Vienna University of Technology



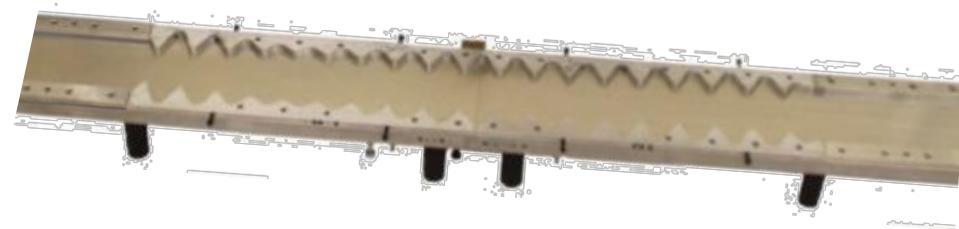
New Insights on Coherent Wave Transmission through Disordered Systems

Stefan Rotter
ICTP, May 2014

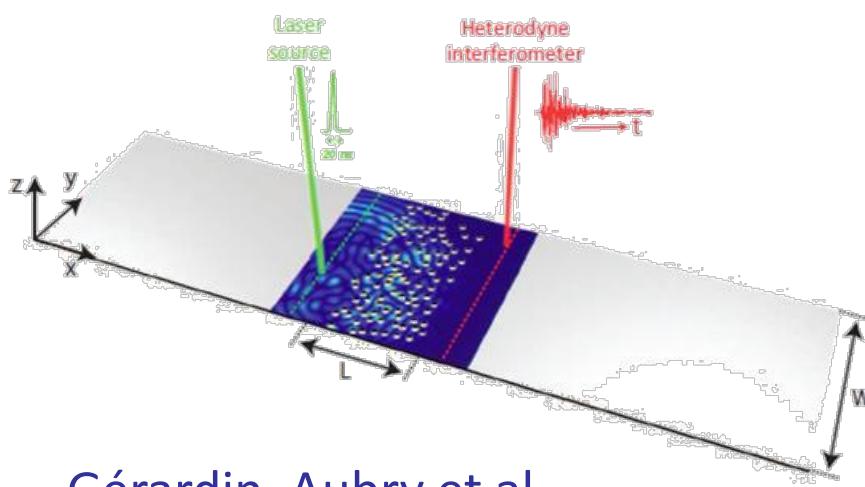
Experiments



Chabanov, Genack et al.,
PRL 92, 173901 (2004)



Kuhl, Stöckmann et al.,
PRB Rapid Comm. 86, 201106(R) (2012)



Gérardin, Aubry et al.,
arXiv:1404.2092 (2014)

Many more realizations in:

- Optical wave guides
- Acoustic wave guides
- Nanowires
- Polaritonic systems
- etc.

Transmission through a disordered waveguide

Relevant length scales:

Mean free path l^e

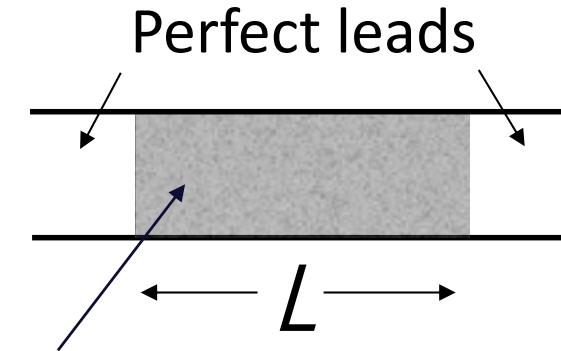
Localization length ξ

Regimes:

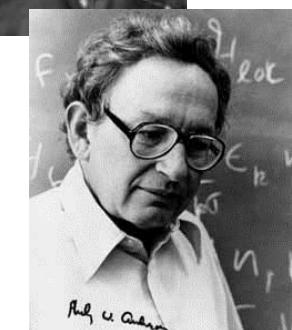
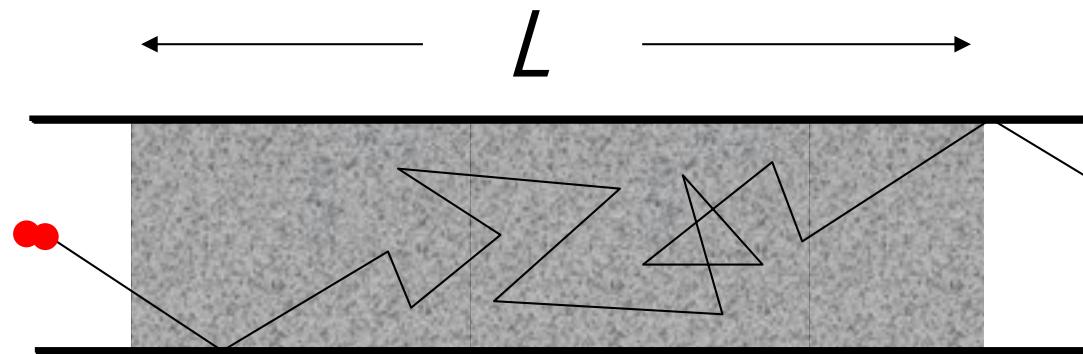
1. Ballistic $L < l^e$

2. Diffusive $l^e < L < \xi$

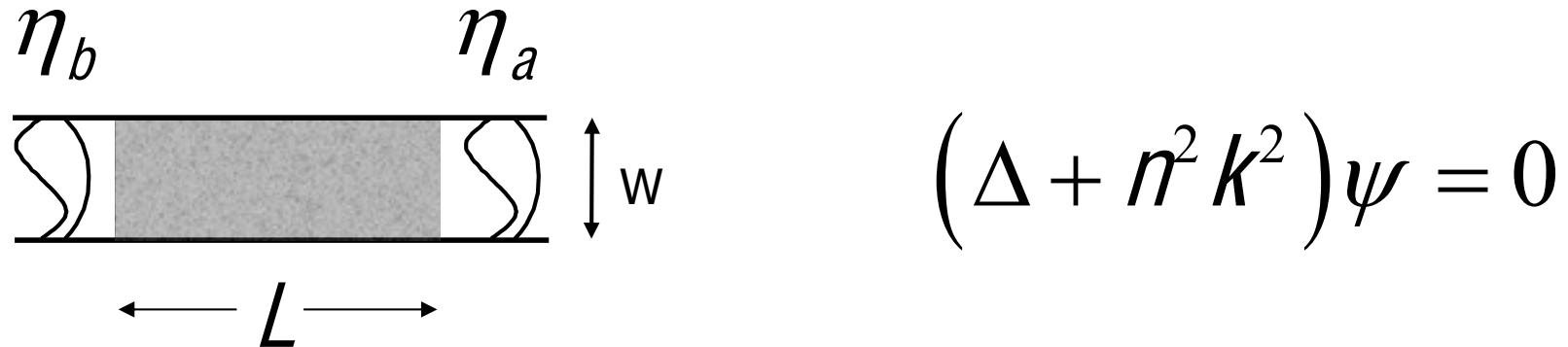
3. Localized $L > \xi$



Bulk disorder



Transmission through a disordered waveguide



$$t_{ab}(k) = -i\sqrt{k_x^a k_x^b} \int dy' \int dy \eta_a^*(y') G^+(y', y, k) \eta_b(y)$$

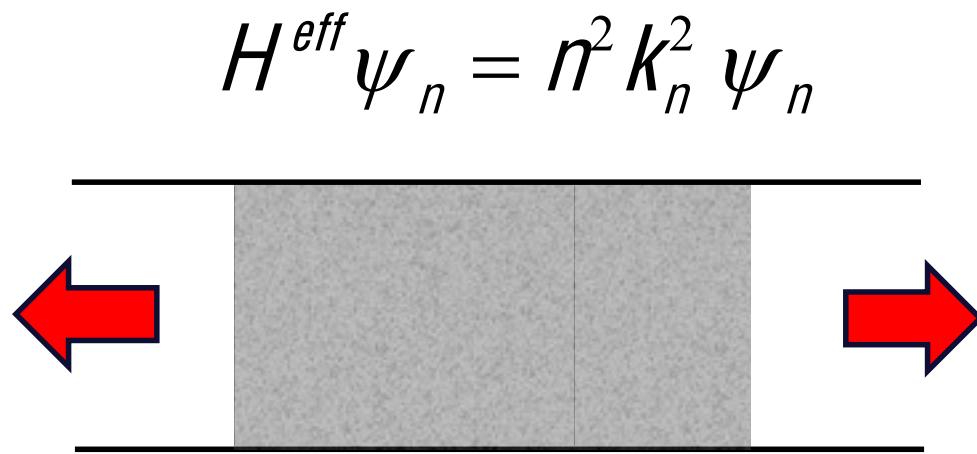
$$T = \sum_{b,a}^N |t_{ab}|^2, \quad R = \sum_{b,a}^N |r_{ab}|^2, \quad T + R = N$$

- Transmission eigenvalues: $T = \text{Tr } t^\dagger t = \sum_a \tau_a$

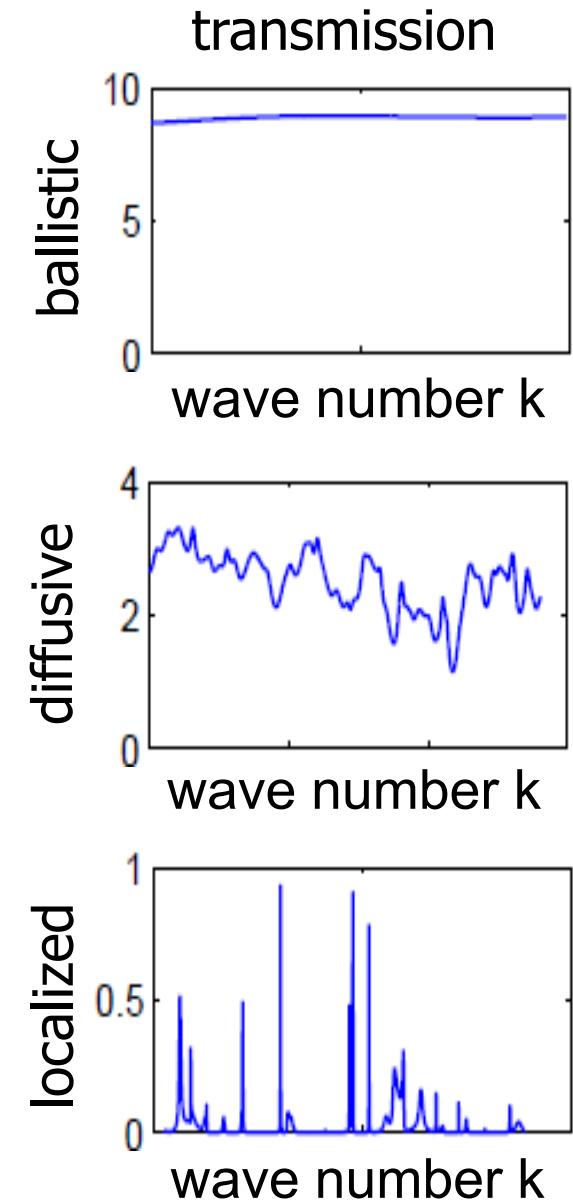
Disordered waveguides

$$G^+(y', y, k) = \sum_n \frac{\bar{\psi}_n(y') \psi_n(y)}{k^2 - k_n^2}$$

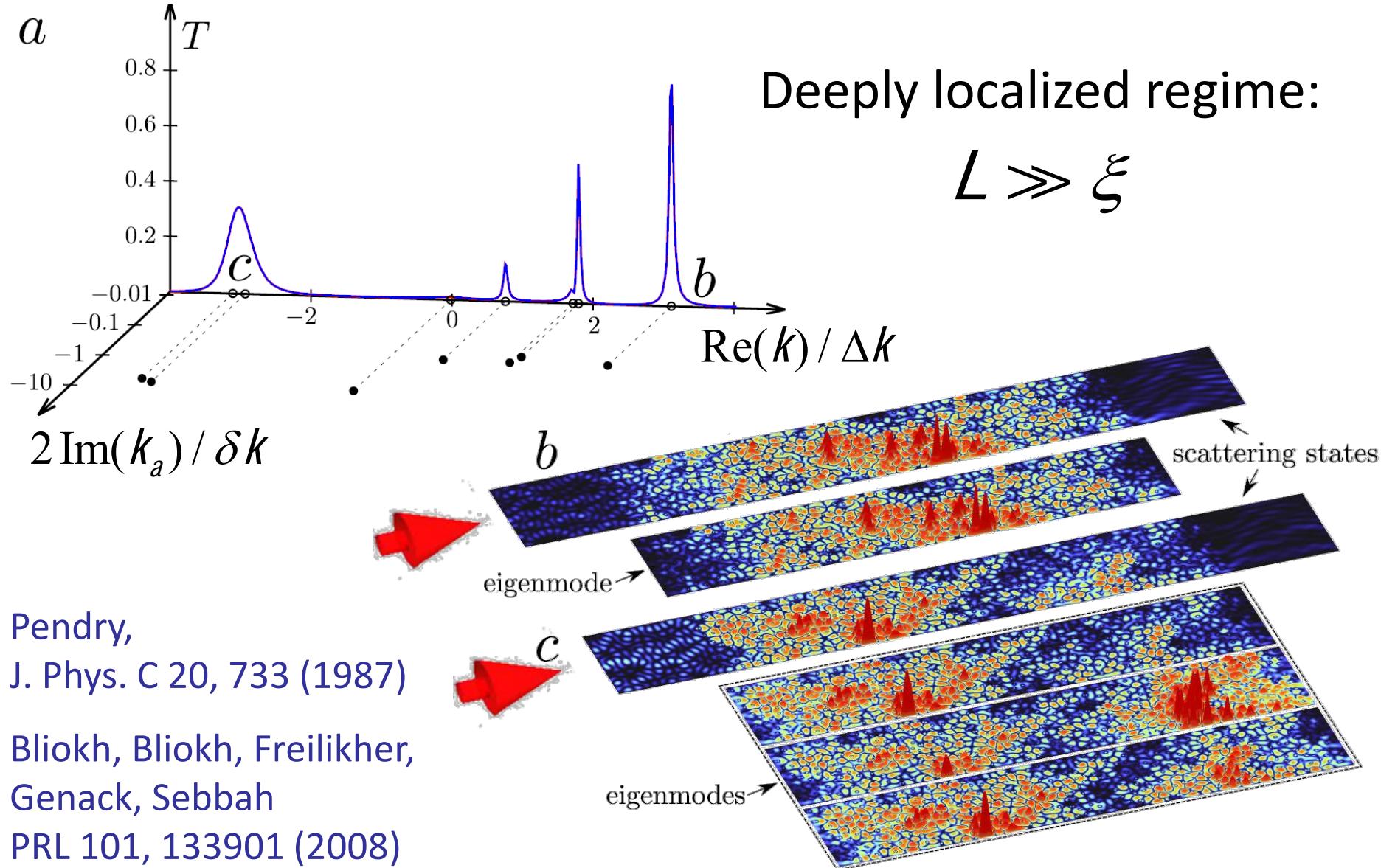
H. E. Türeci, L. Ge, S. Rotter, and A. D. Stone,
Science 320, 643 (2008).



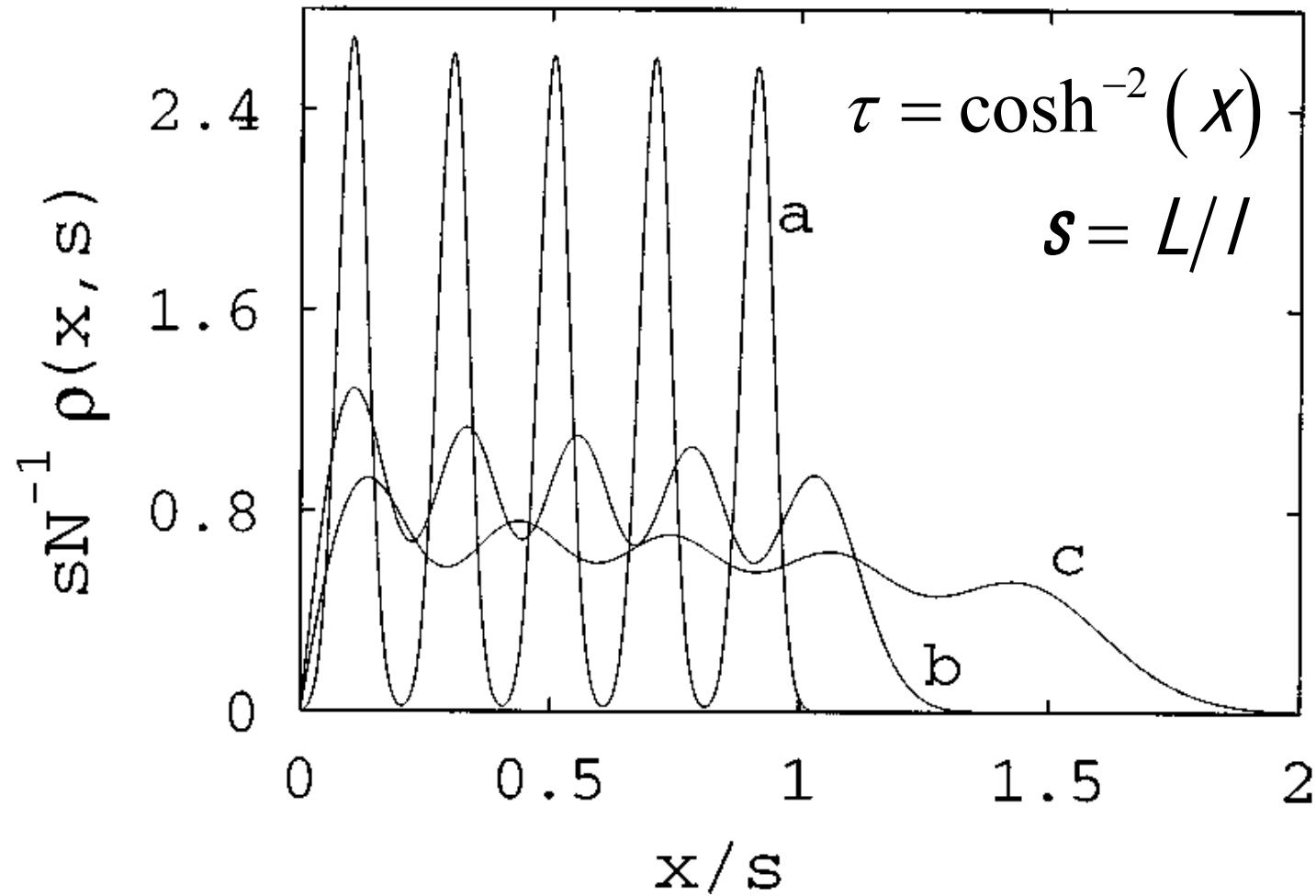
$$H^{eff} \psi_n = n^2 k_n^2 \psi_n$$



Scattering states and eigenstates of localized system

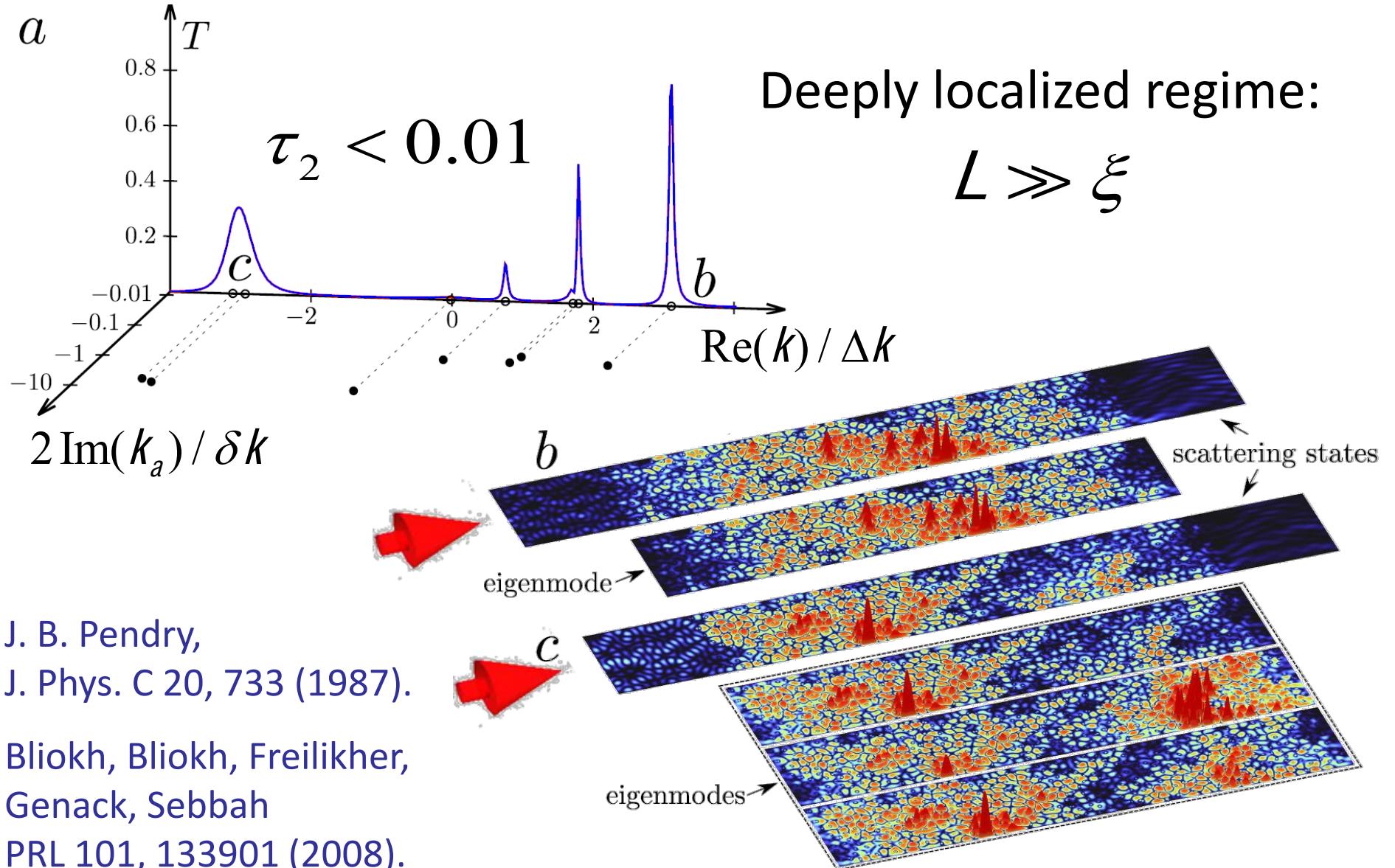


Crystallization of transmission eigenvalues



Frahm, Phys. Rev. Lett. 74, 4706 (1995)
Shi and Genack, Phys. Rev. Lett. 108, 043901 (2012)

Single channel regime



Transmission matrix

$$t = U \begin{pmatrix} \sqrt{\tau_1} & & 0 \\ & \ddots & \\ 0 & & \sqrt{\tau_N} \end{pmatrix} V, \quad tt^\dagger = U \begin{pmatrix} \tau_1 & & 0 \\ & \ddots & \\ 0 & & \tau_N \end{pmatrix} U^\dagger$$

$$T_{ab} = |t_{ab}|^2 = |U_{a1}|^2 \tau_1 |U_{1b}|^2$$

$$T_a = \sum_b T_{ab} = |U_{a1}|^2 \tau_1$$

$$T = \sum_a T_a = \sum_n \tau_n = \tau_1$$

Beenakker, Rev. Mod. Phys. (1997)

Statistics

Statistical moments:

$$s = \frac{T}{\langle T \rangle}, \quad s_a = \frac{T_a}{\langle T_a \rangle}, \quad s_{ab} = \frac{T_{ab}}{\langle T_{ab} \rangle}$$

Single-channel regime (random matrix theory):

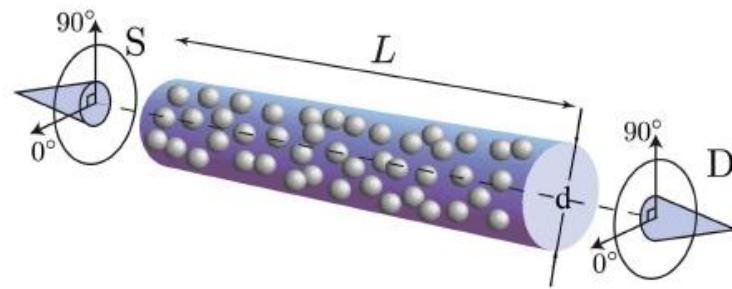
$$\langle s_a^n \rangle = n! \langle s^n \rangle$$

$$\langle s_{ab}^n \rangle = (n!)^2 \langle s^n \rangle$$

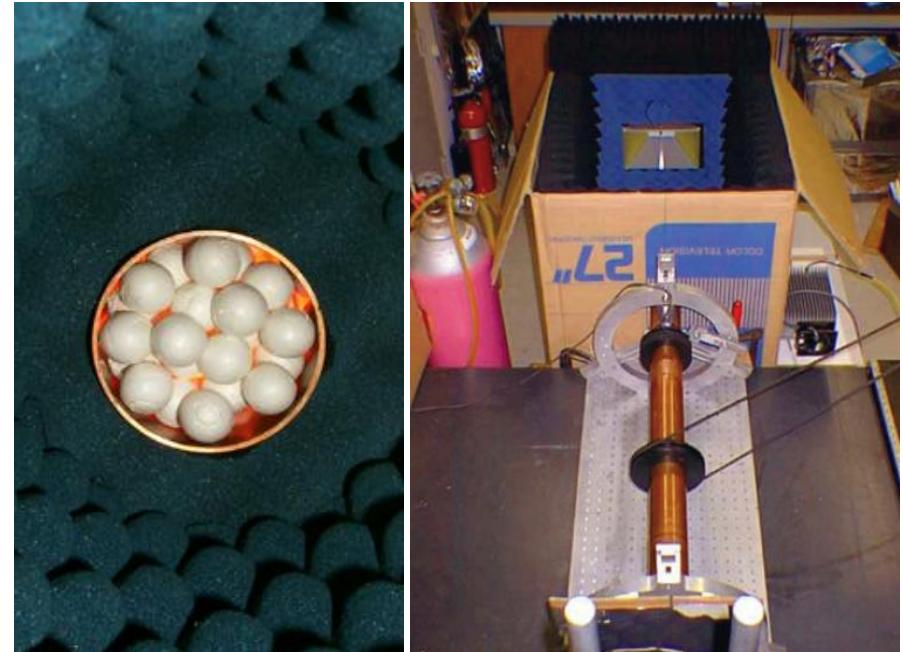
$$R = \frac{\langle s_{ab}^2 \rangle}{\langle s^2 \rangle} = 4$$

Microwave transmission measurements

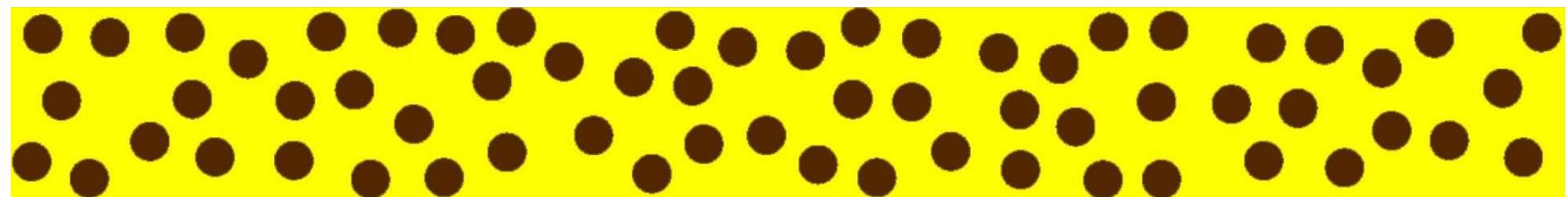
Microwave experiment
(disordered metal tube)



Chabanov et al., PRL 92, 173901 (2004)

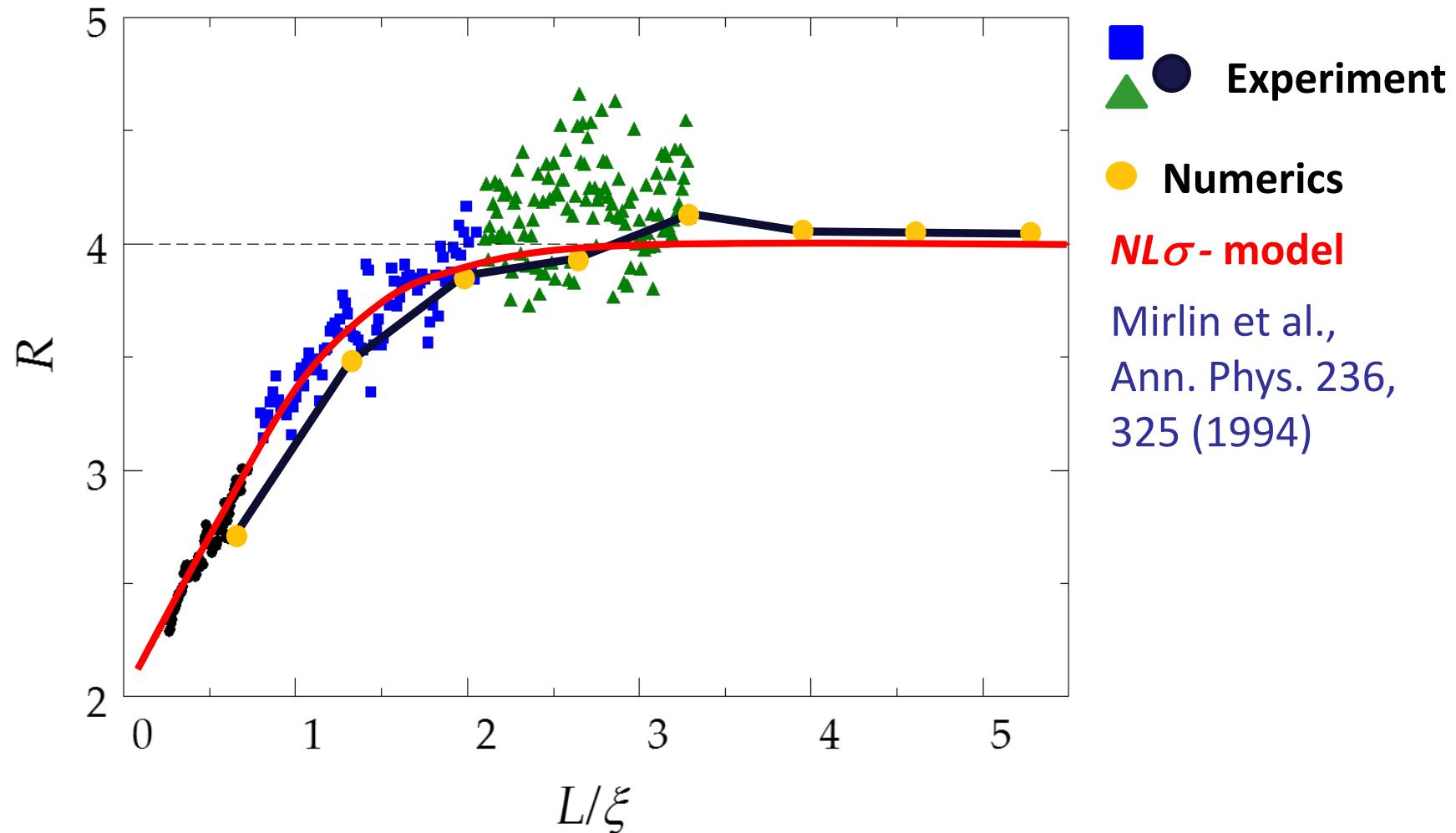


Numerical calculations (planar waveguide)

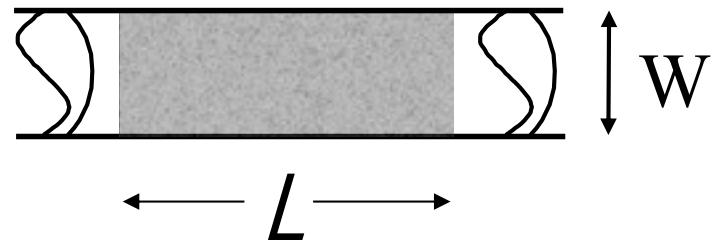


F. Libisch, S. Rotter, and J. Burgdörfer, New J. Phys. 14, 123006 (2012).

Entering the single channel regime of transport

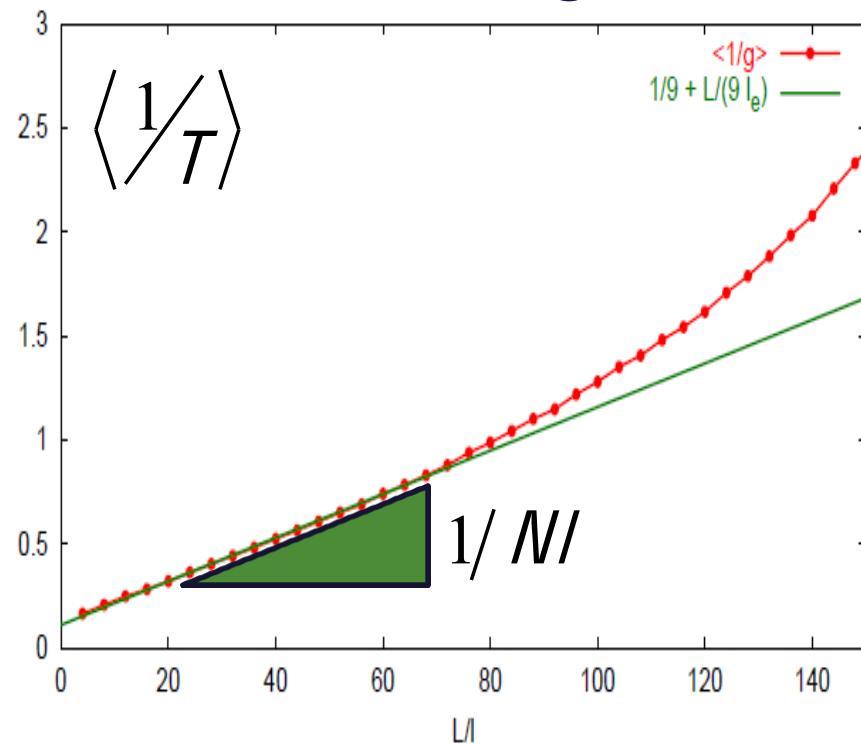


Renormalized localization length

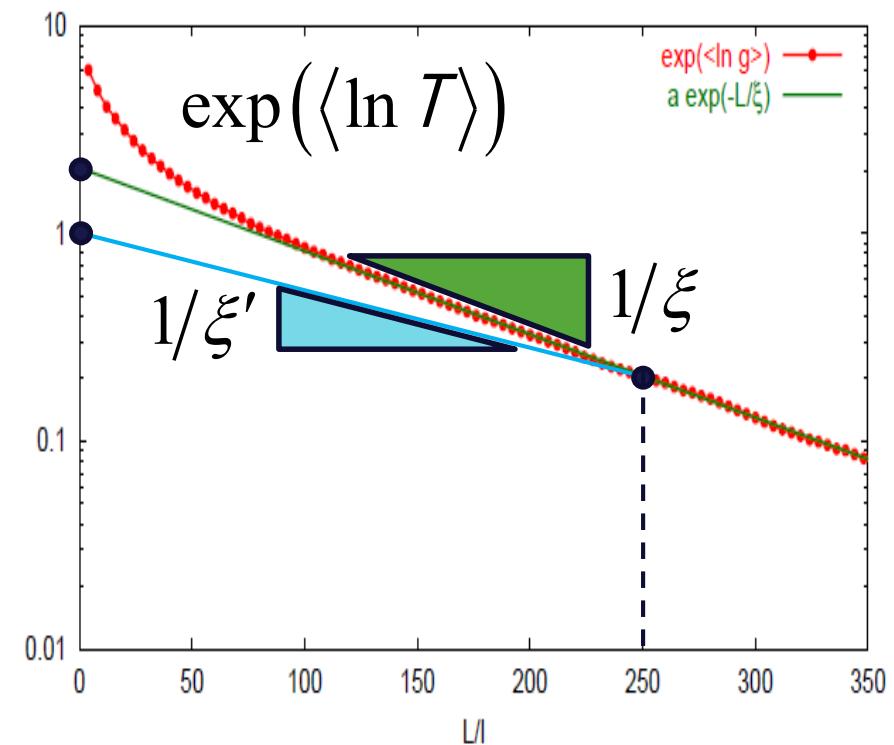


ξ' : renormalized
localization length

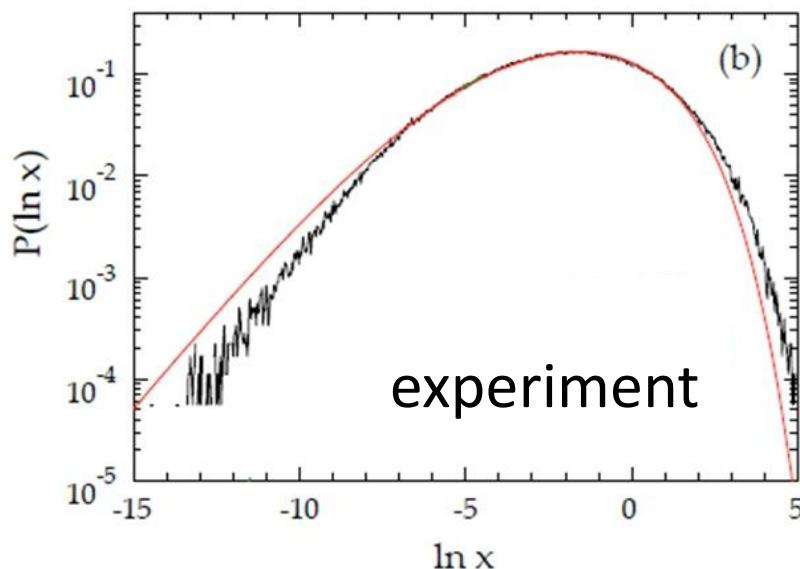
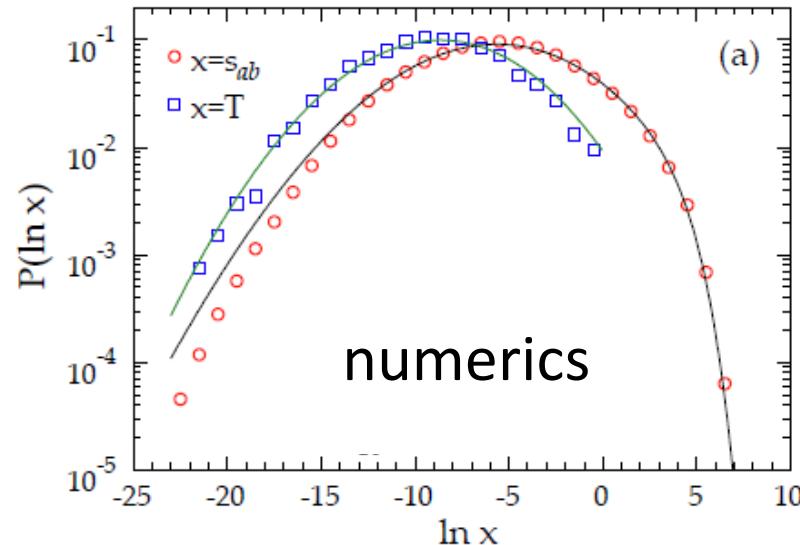
Diffusive regime



Localized regime



Distribution of log-intensity



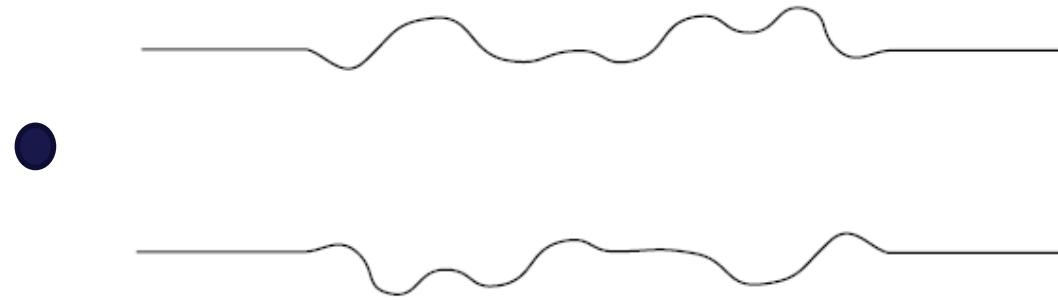
- $x=s_{ab}$ (num., $L/\xi=5.25$)
- $x=T$ (num., $L/\xi=5.25$)
- $x=s_{ab}$ (G&M, $L/\xi'=4.57$)
- $x=T$ (G&M, $L/\xi'=4.57$)

Gopar and Molina
Phys. Rev. B 81, 195415 (2010)

- $x=s_{ab}$ (exp., $L/\xi=2.52$)
- $x=s_{ab}$ (G&M, $L/\xi'=1.25$)
- $x=T$ (G&M, $L/\xi'=1.25$)

Peña, Girschik, Libisch, Rotter, Chabanov
Nature Communications 5, 3488 (2014)

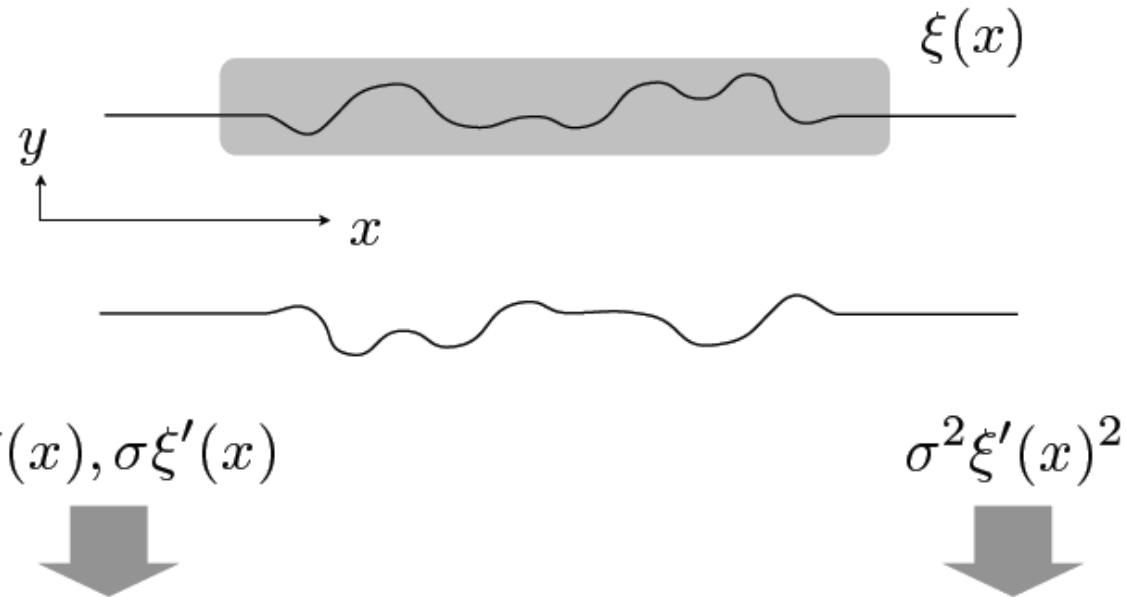
Surface-disordered waveguides



Transport regimes coexist

Sanchez-Gil, Freilikher,
Yurkevich, and Maradudin
Phys. Rev. Lett. 80, 948 (1998)

Scattering mechanisms

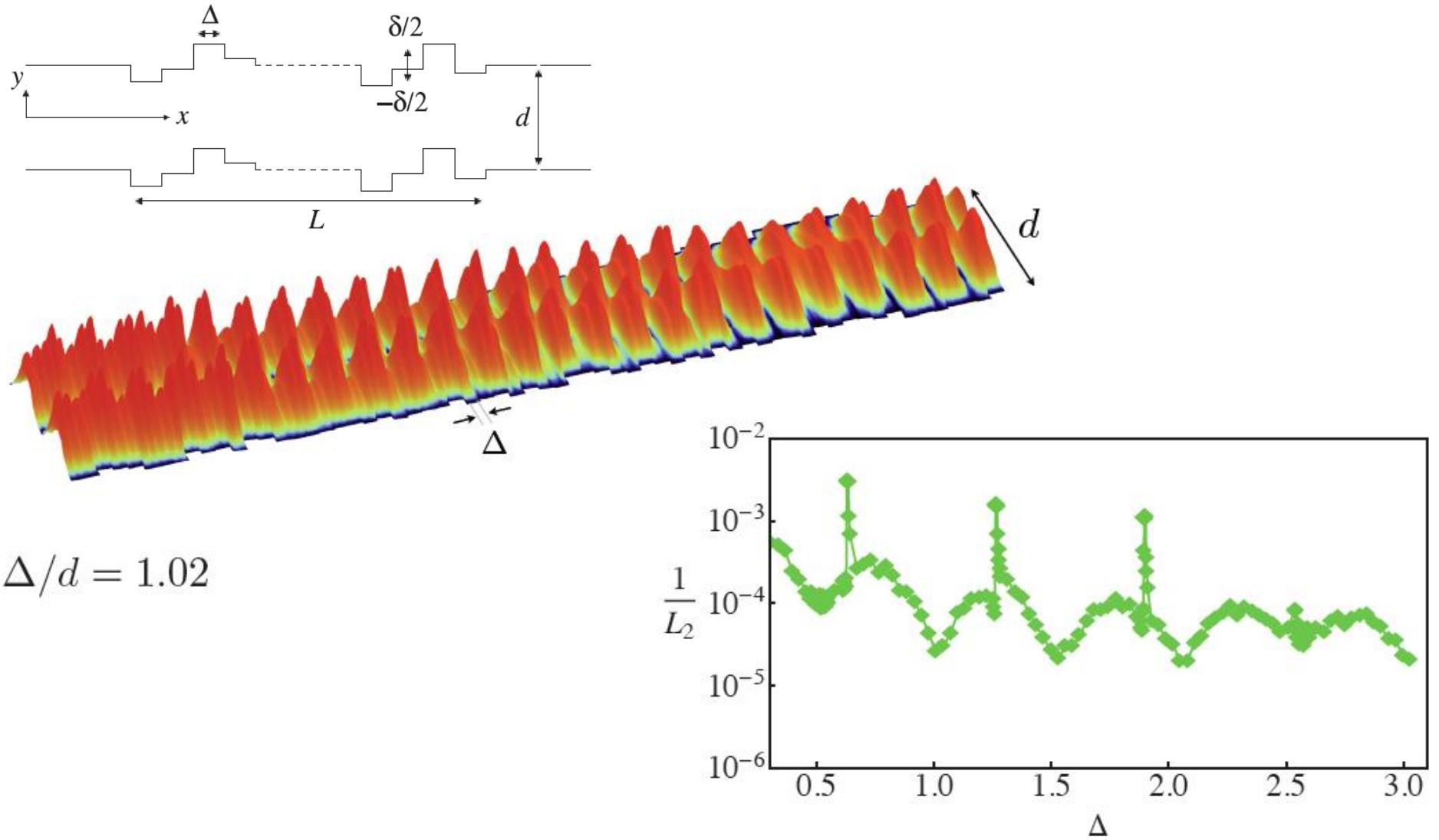


**Amplitude - gradient
scattering (AGS)**

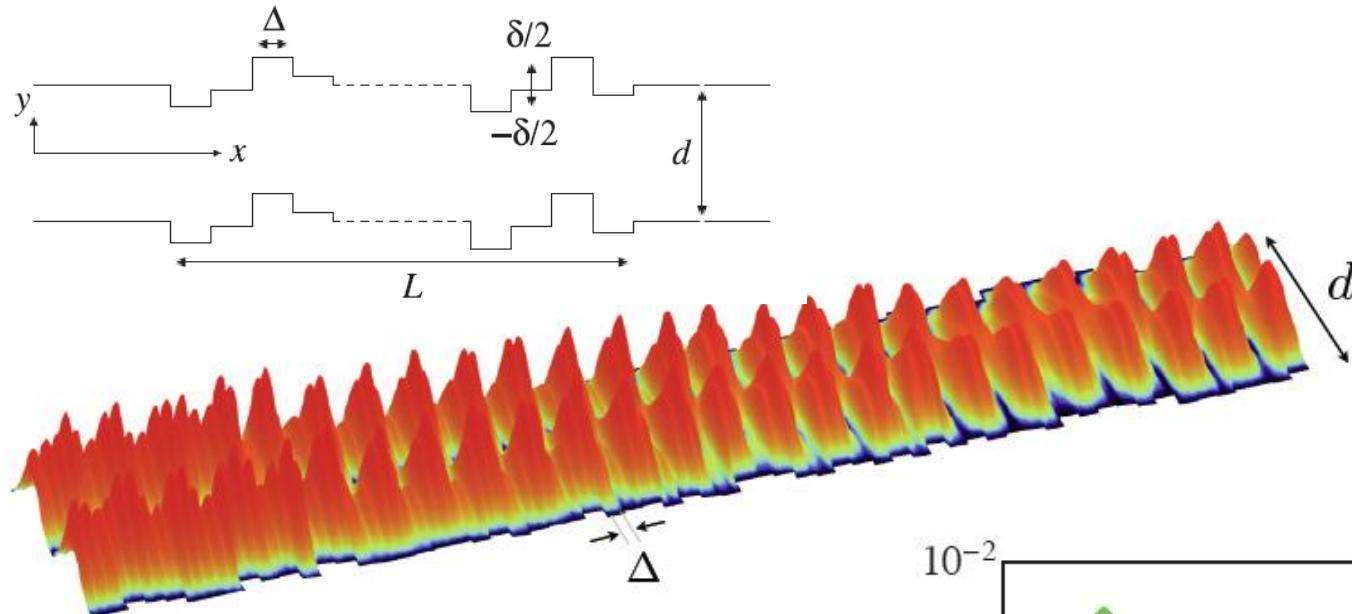
**Square - gradient
scattering (SGS)**

Rendon, Izrailev, Makarov,
Phys. Rev. B 83, 051124 (2011)

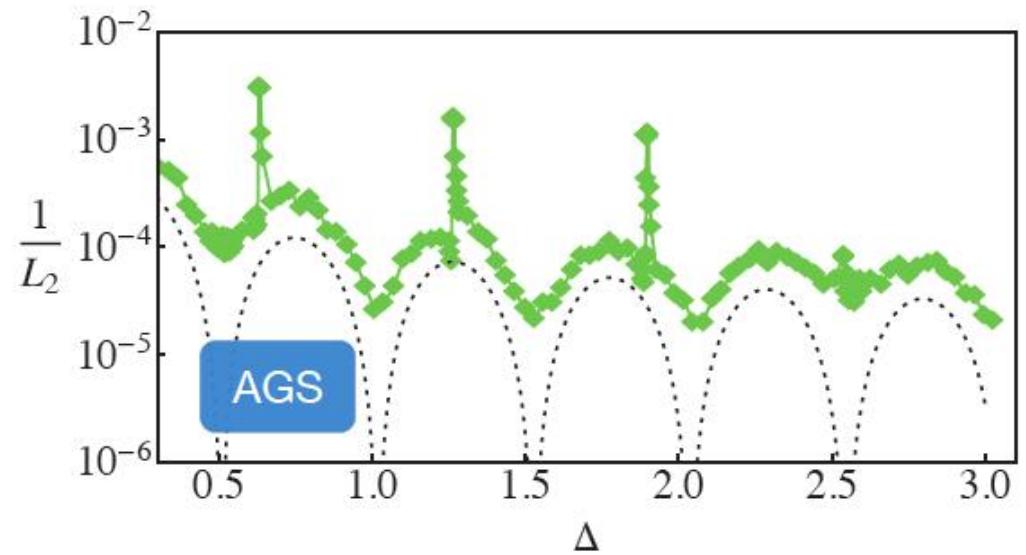
Results



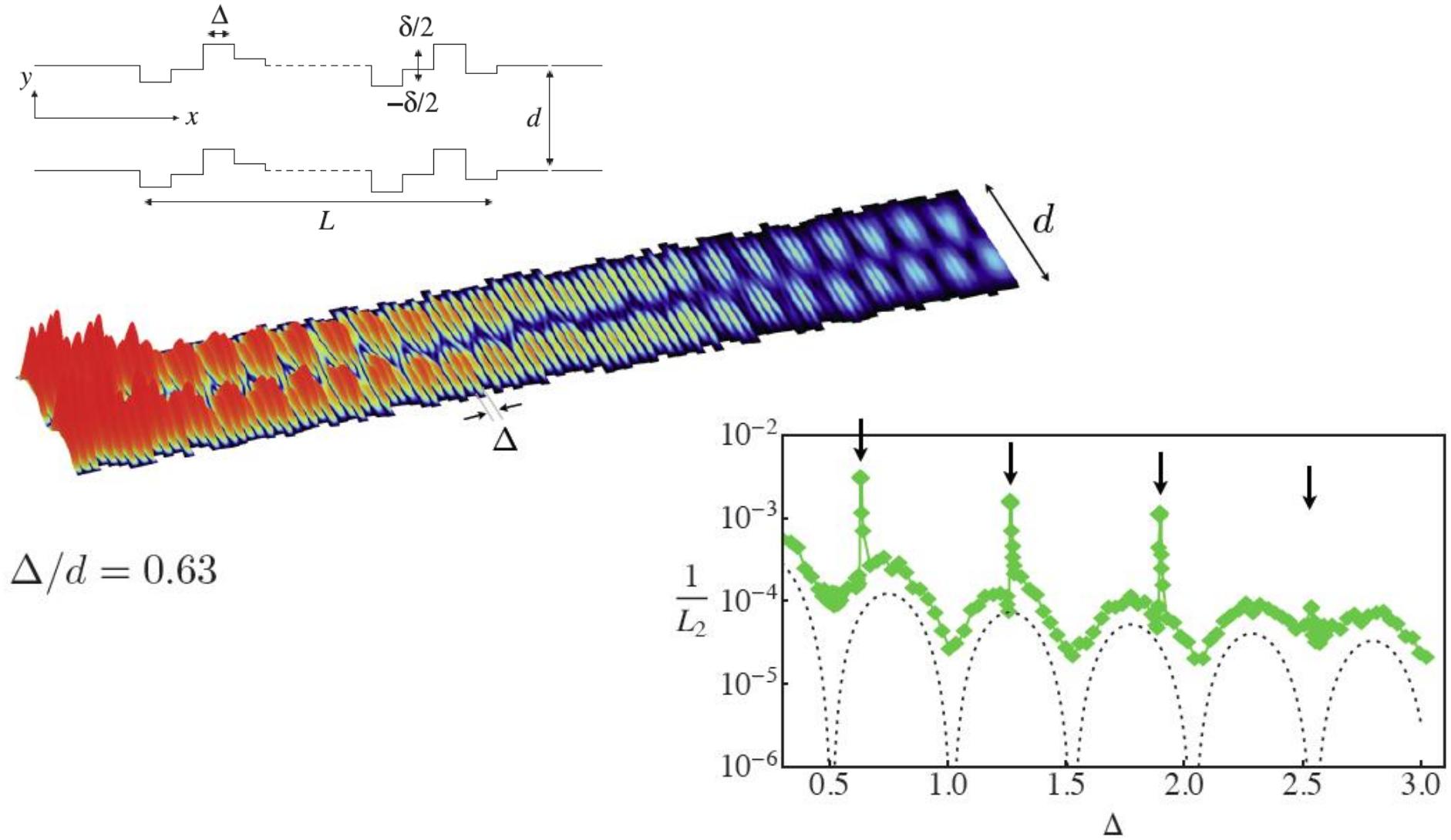
Results



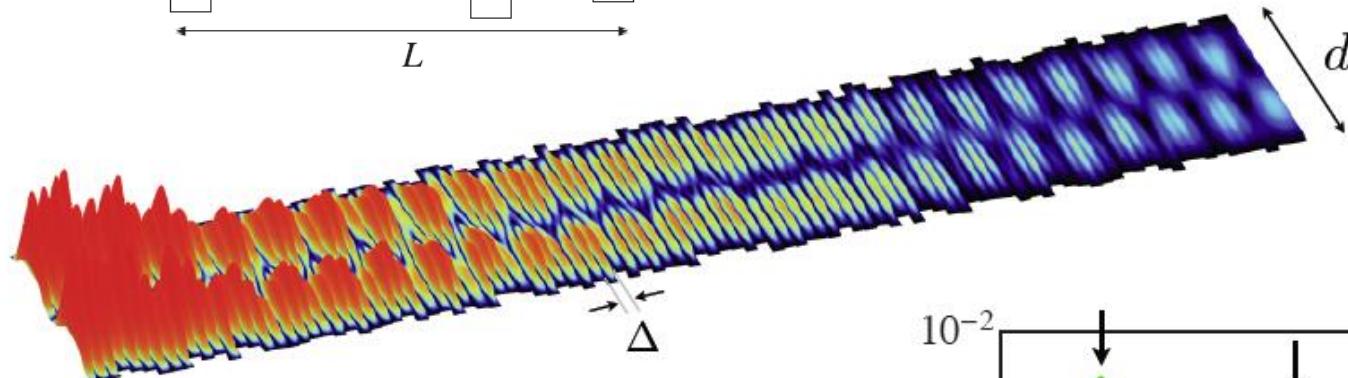
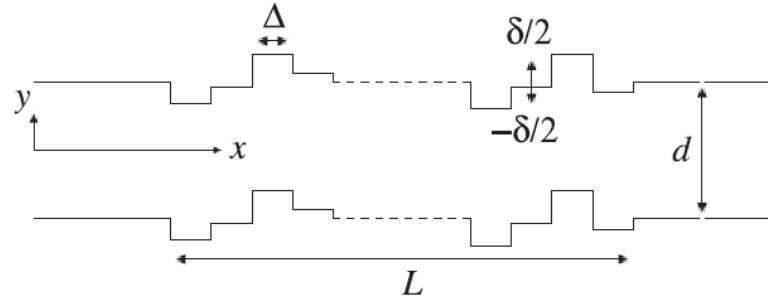
$$\Delta/d = 1.02$$



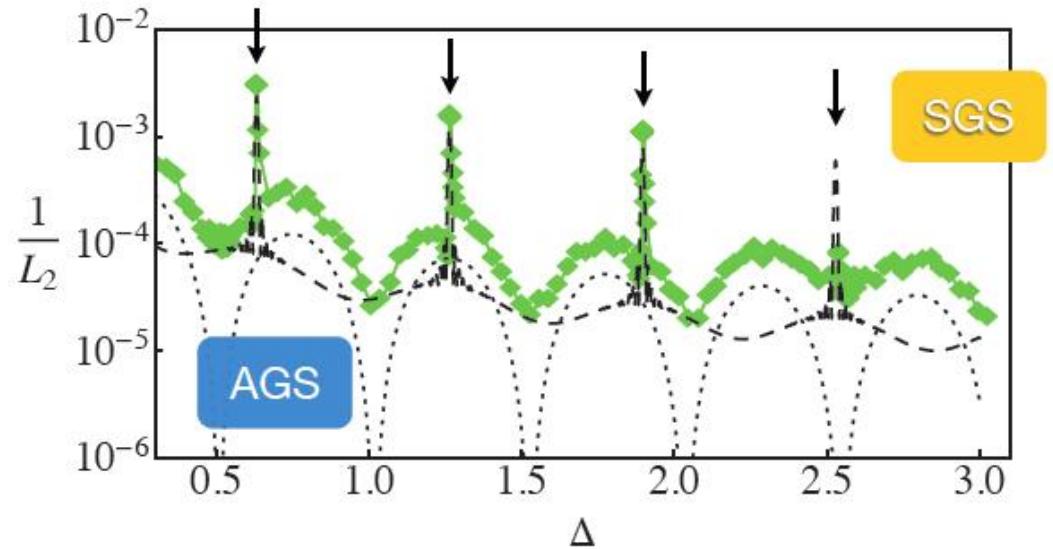
Results



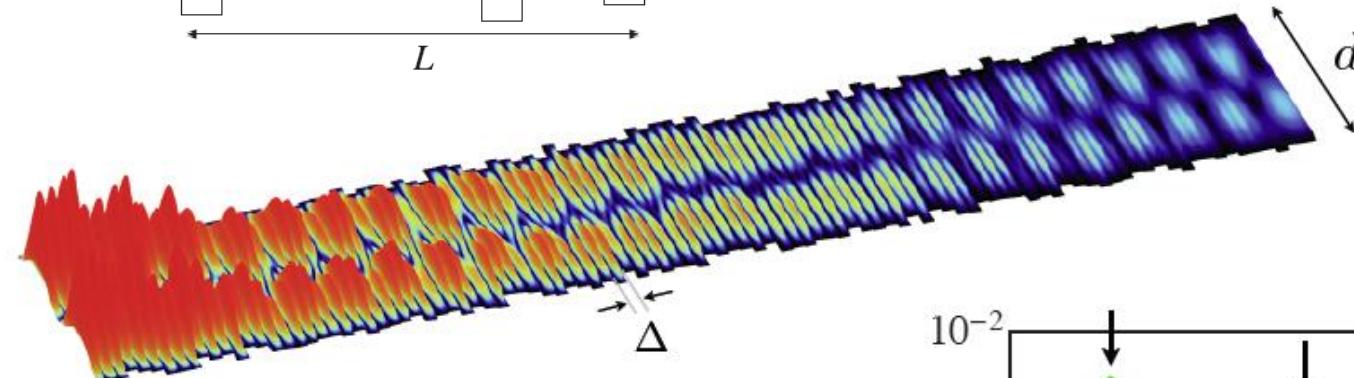
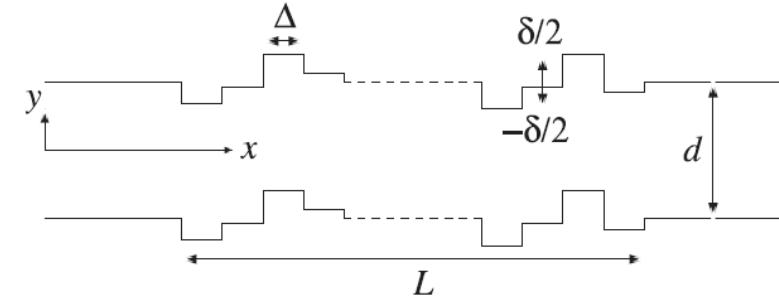
Reflection resonances



$$\Delta/d = 0.63$$

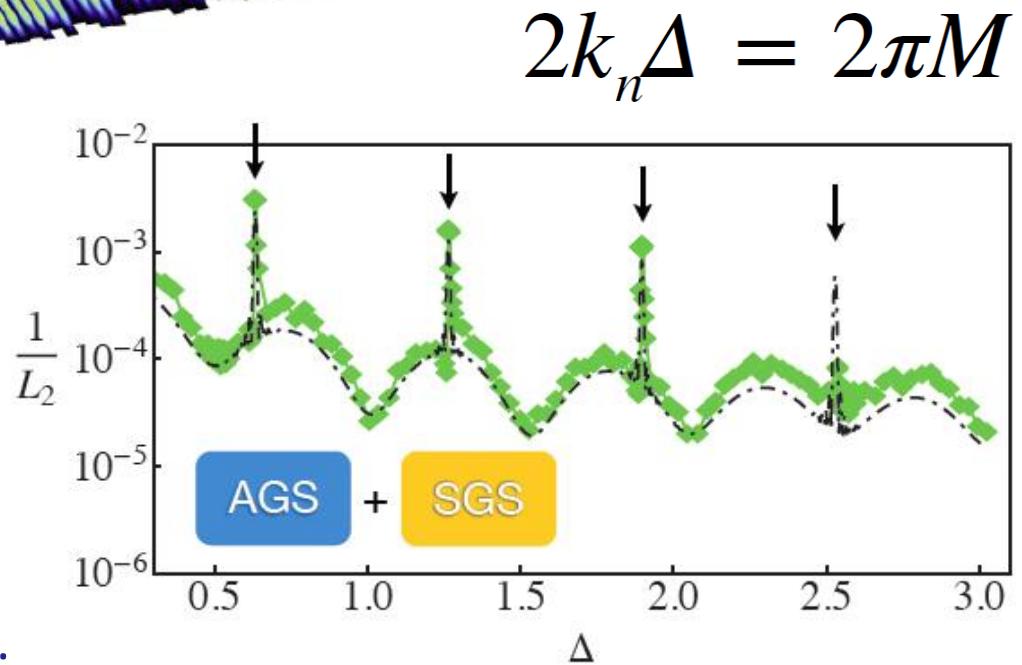


Quantitative agreement

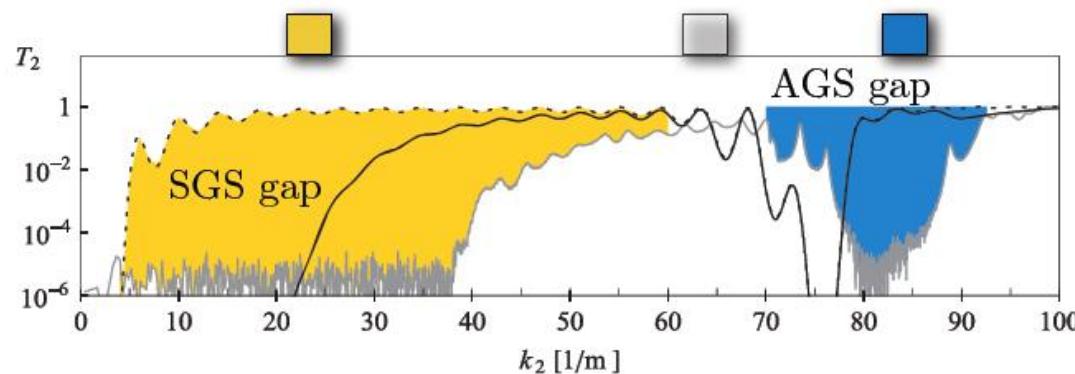
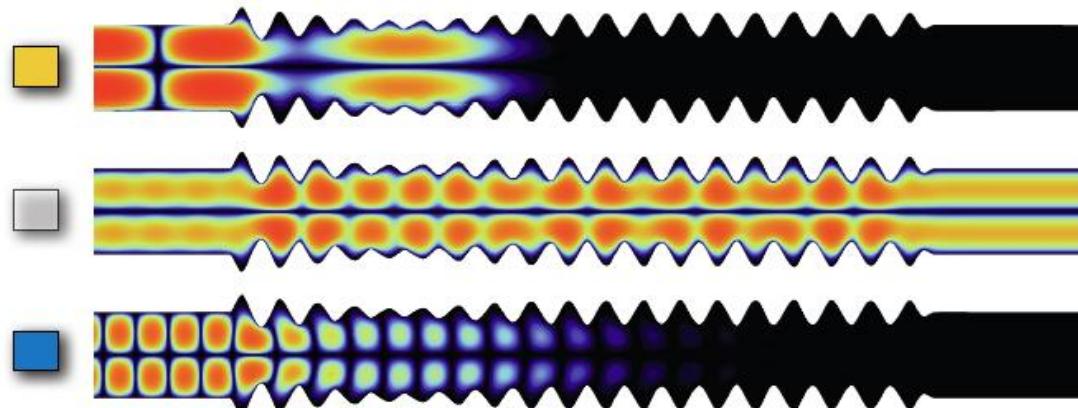


$$\Delta/d = 0.63$$

Doppler, Méndez-Bermúdez, Feist,
Dietz, Krimer, Makarov, Izrailev, and
Rotter, New J. Phys. 16, 053026 (2014).

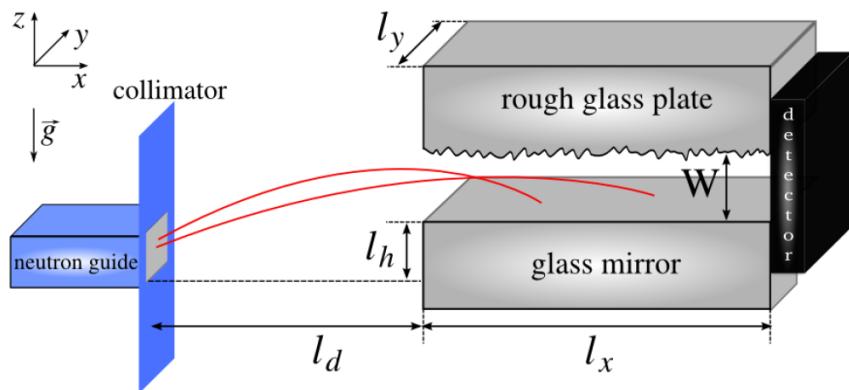


Engineering the surface profile

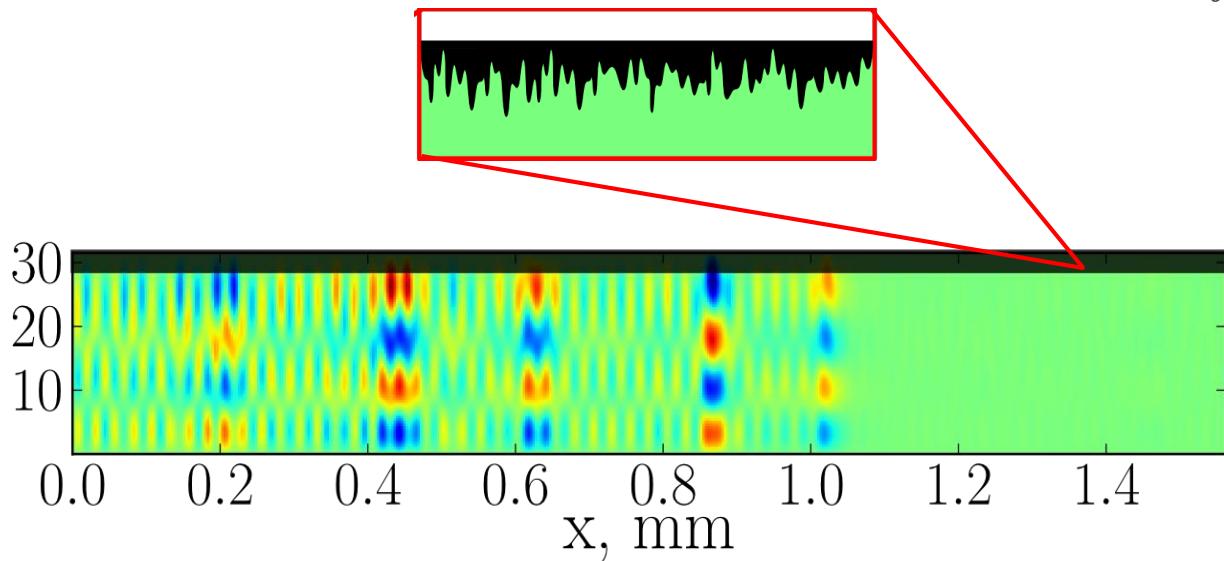
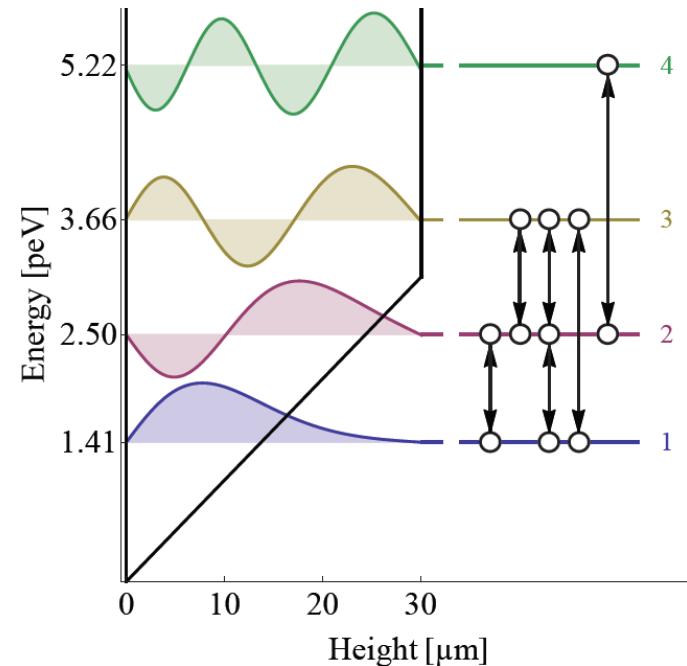


Dietz, Stöckmann,
Kuhl, Izrailev, Makarov,
Doppler, Libisch, Rotter
PRB Rapid Comm.
86, 201106(R) (12)

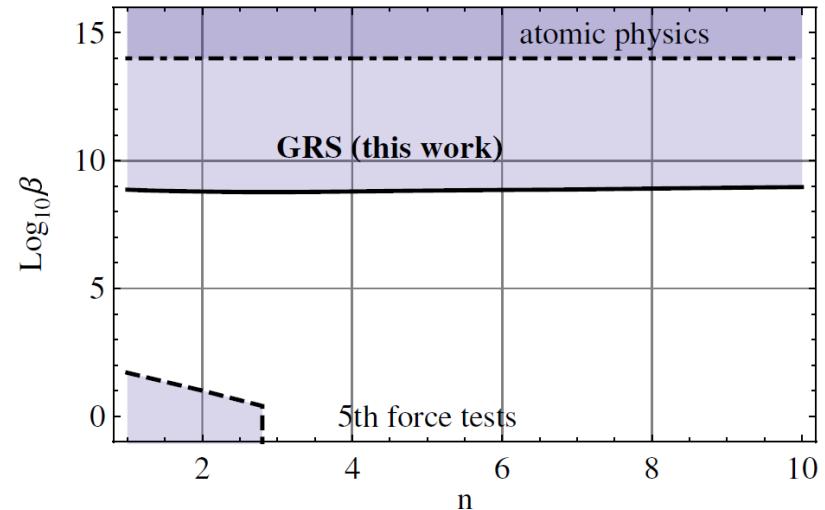
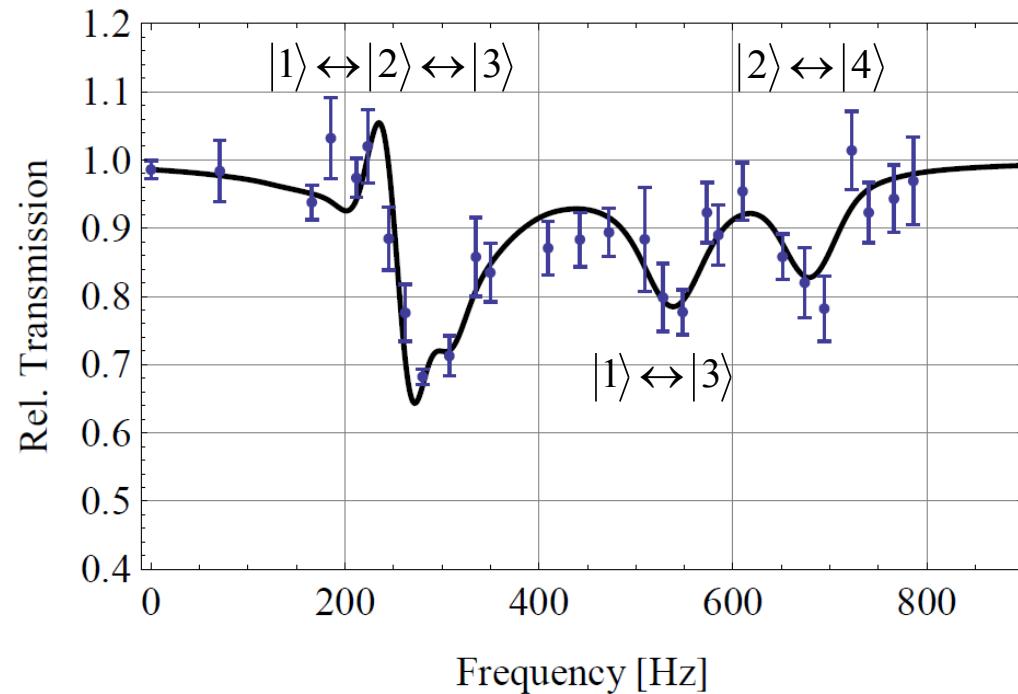
Gravity resonance spectroscopy



Jenke, Geltenbort, Lemmel, and Abele,
Nature Physics 7, 468 (2011)



Gravity resonance spectroscopy



Exclusion of dark energy
chameleon fields

Jenke, Cronenberg, Burgdörfer, Chizhova, Filter, Geltenbort, Ivanov, Lauer, Lins,
Rotter, Saul, Schmidt, and Abele, Phys. Rev. Lett. 112, 151105 (2014)



Selected for PRL:
Editors' Suggestion



Viewpoint

nature news

Random laser

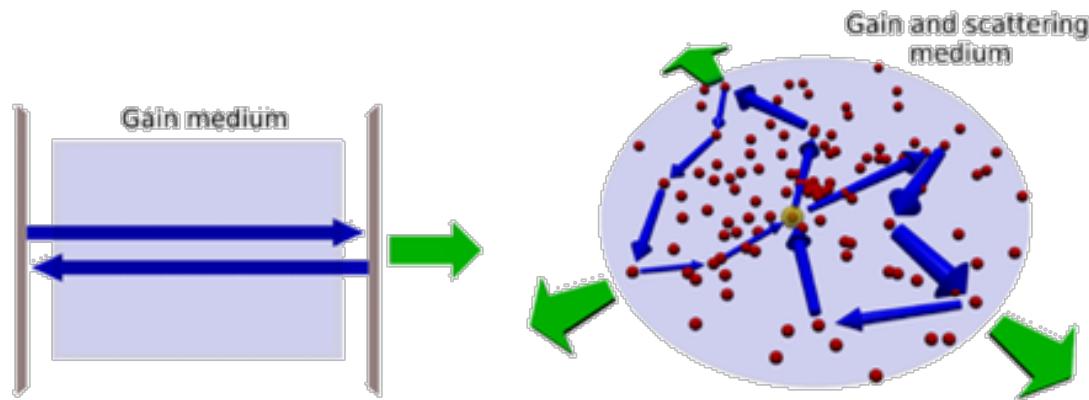
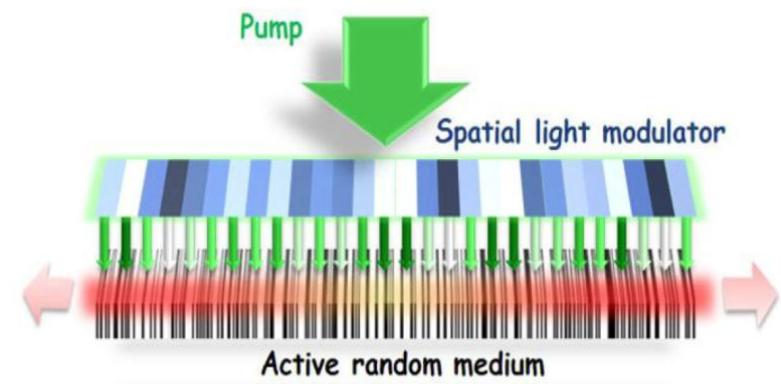
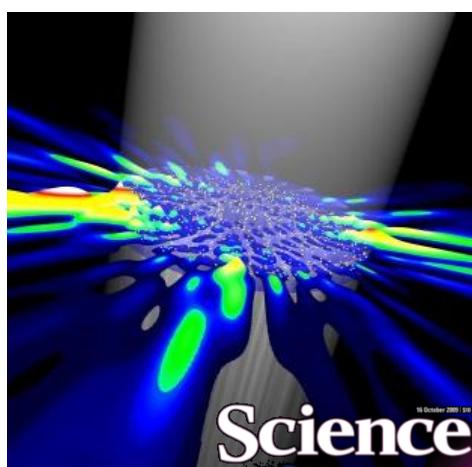
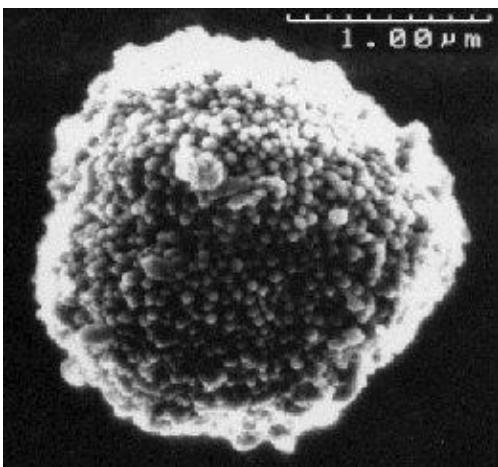


Image courtesy of Hui Cao

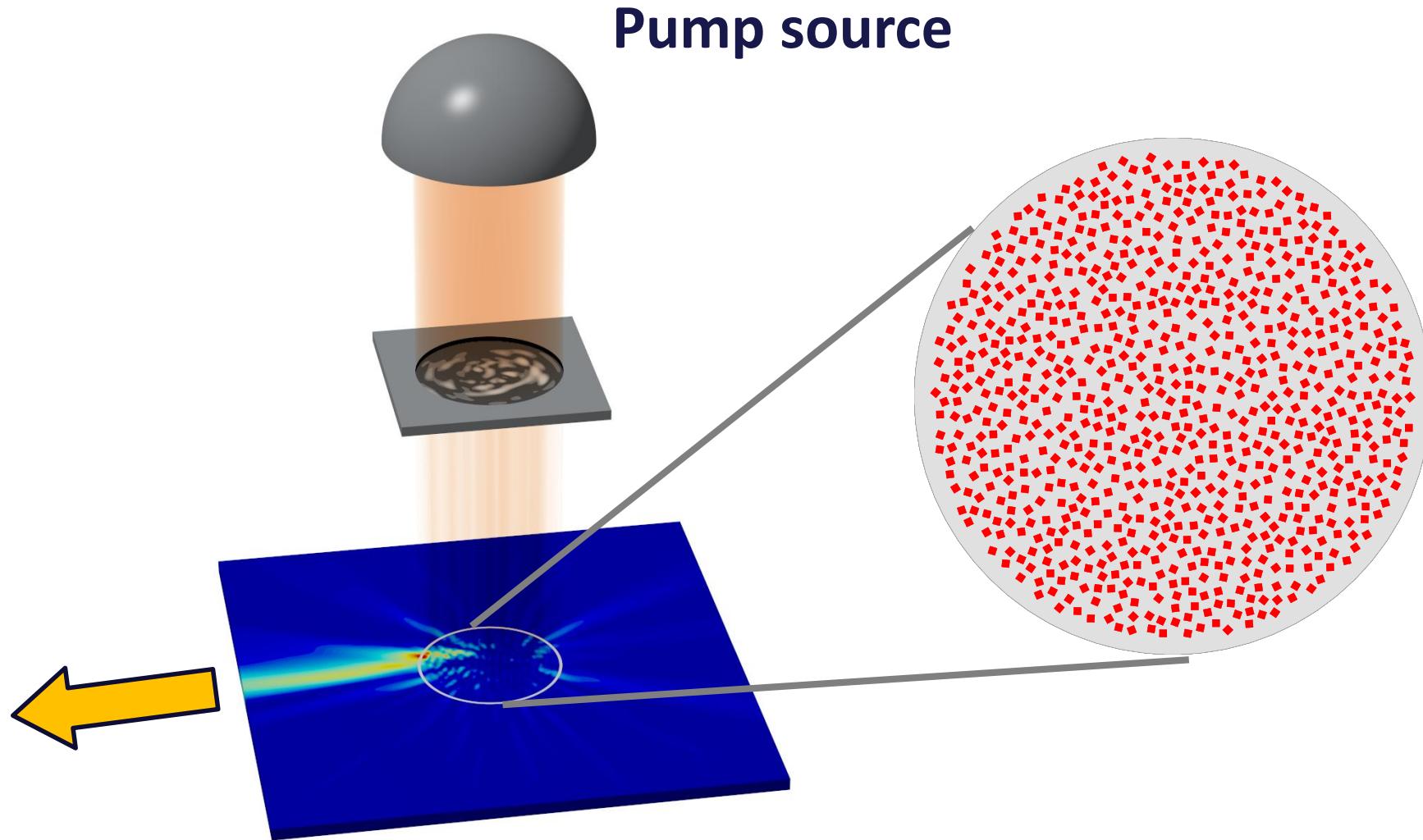


Bachelard, Andreasen,
Gigan, Sebbah
PRL 109, 033903 (12)

Bachelard, Gigan, Noblin,
Sebbah, Nature Physics (in press)

Türeci, Ge, Rotter, Stone, Science 320, 643 (08)
Andreasen et al., Adv. in Opt. Phot. 3, 88 (11)

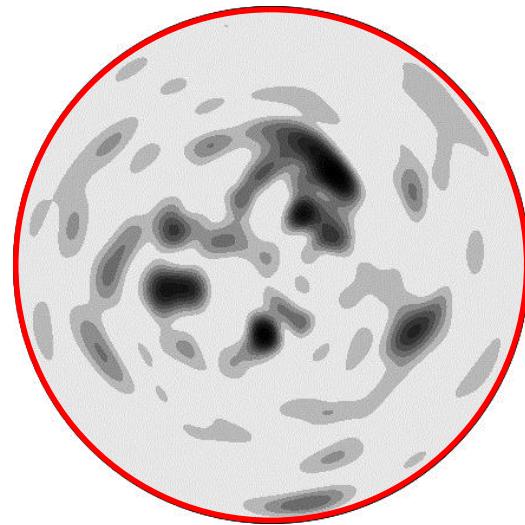
Directional output from random laser?



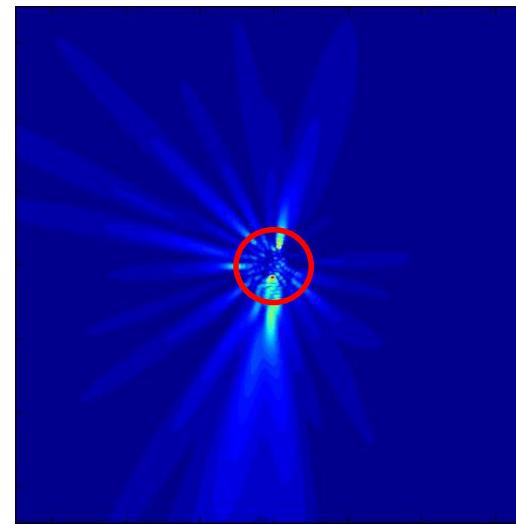
Results: Weakly scattering regime

Iteration: 0

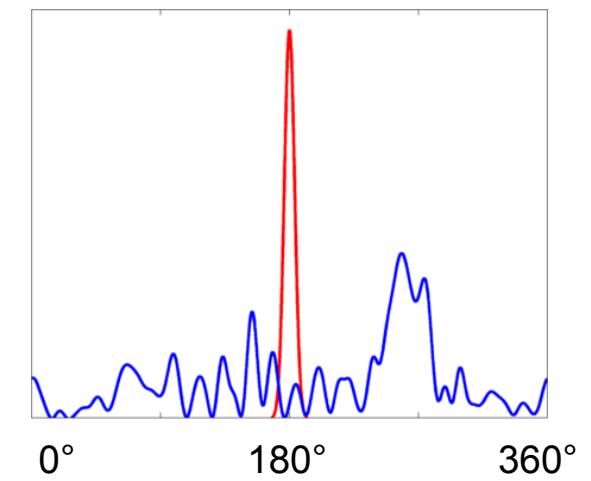
Pump profile



First laser mode



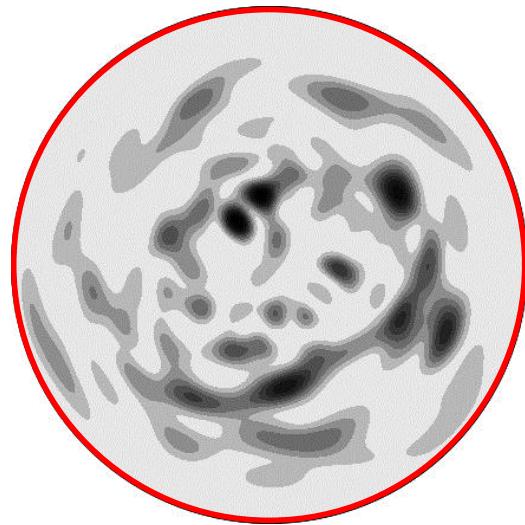
Emission pattern



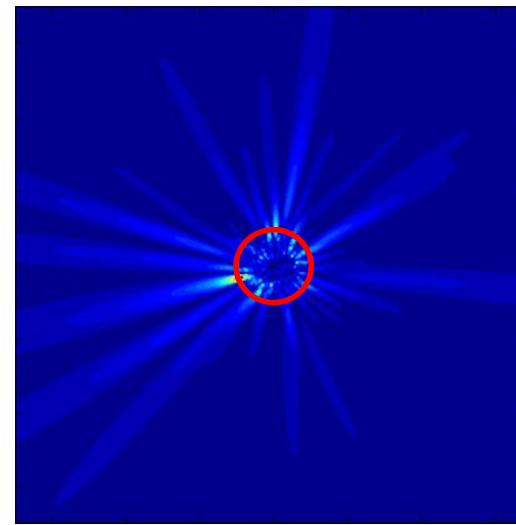
Results: Weakly scattering regime

Iteration: 4

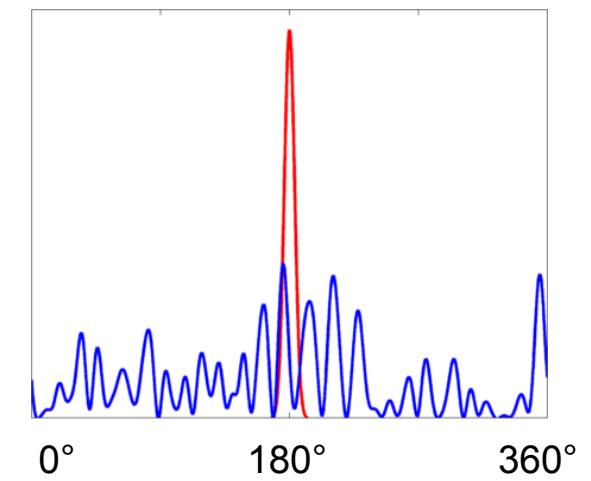
Pump profile



First laser mode



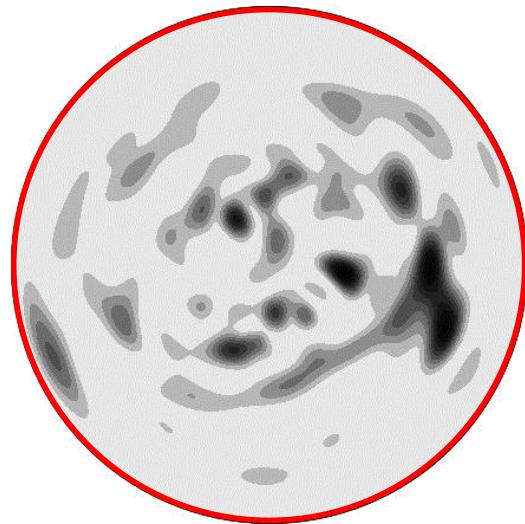
Emission pattern



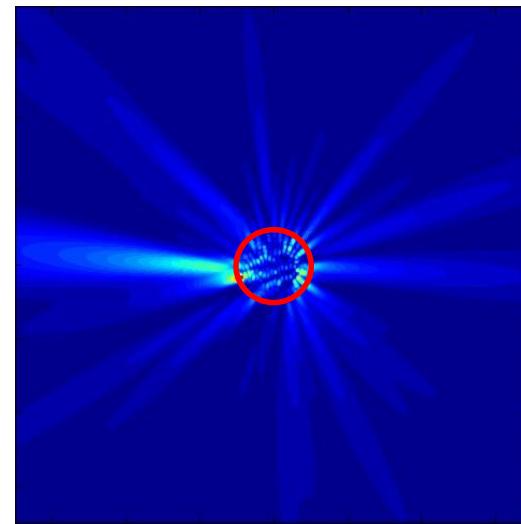
Results: Weakly scattering regime

Iteration: 8

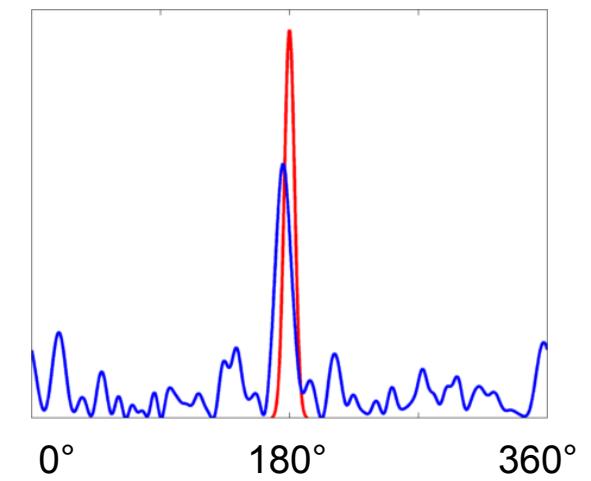
Pump profile



First laser mode



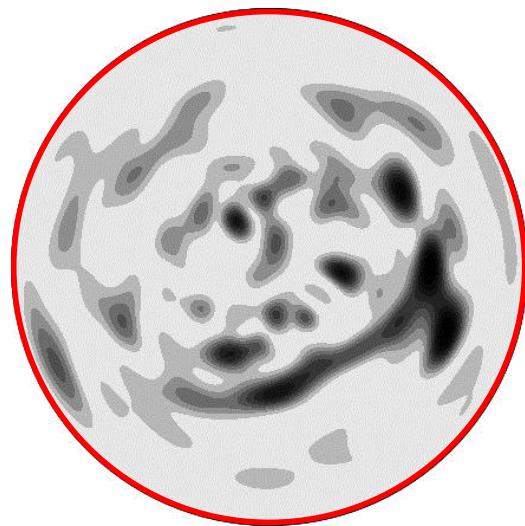
Emission pattern



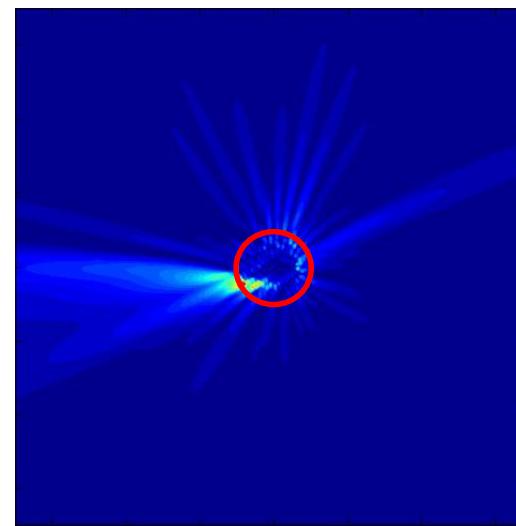
Results: Weakly scattering regime

Iteration: 12

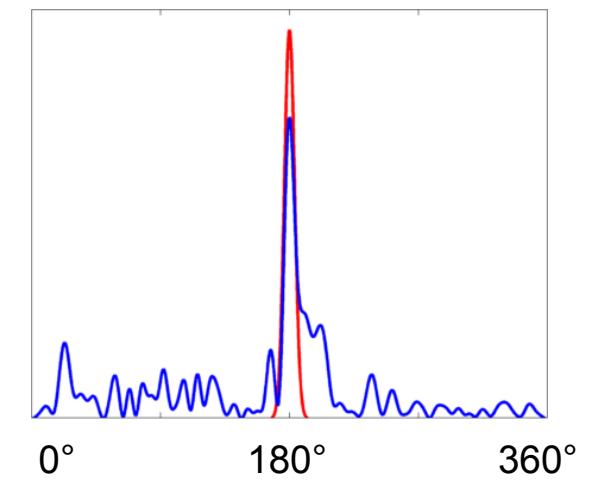
Pump profile



First laser mode



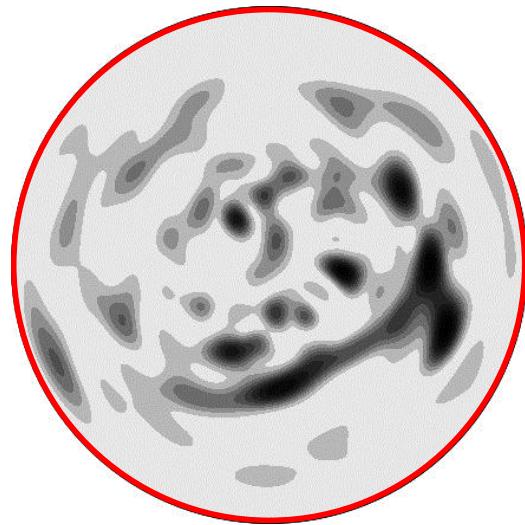
Emission pattern



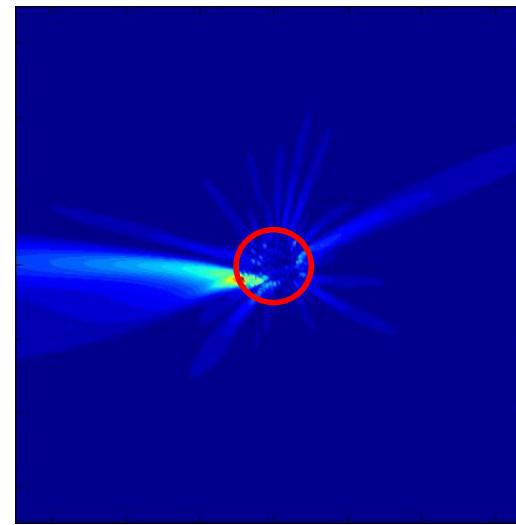
Results: Weakly scattering regime

Iteration: 16

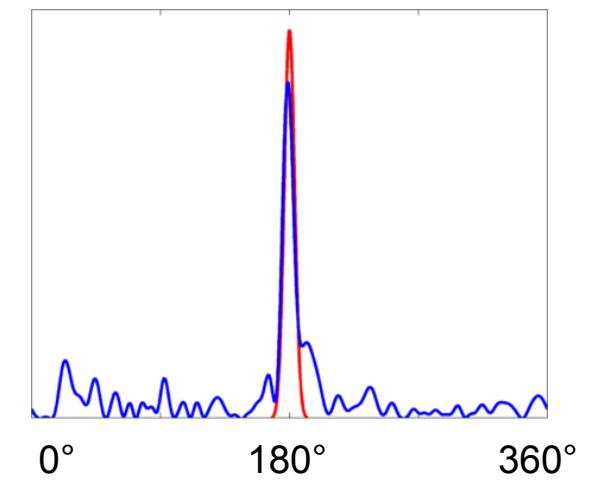
Pump profile



First laser mode



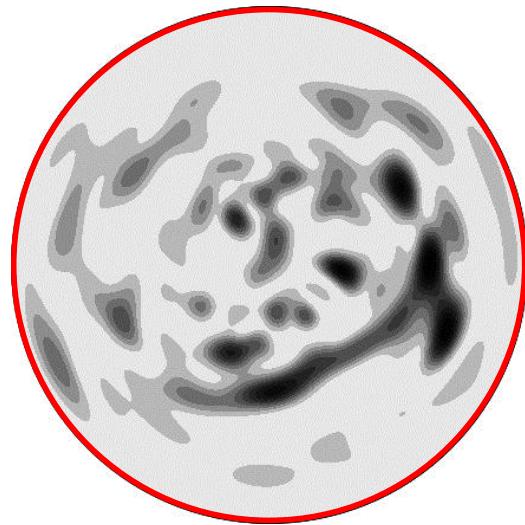
Emission pattern



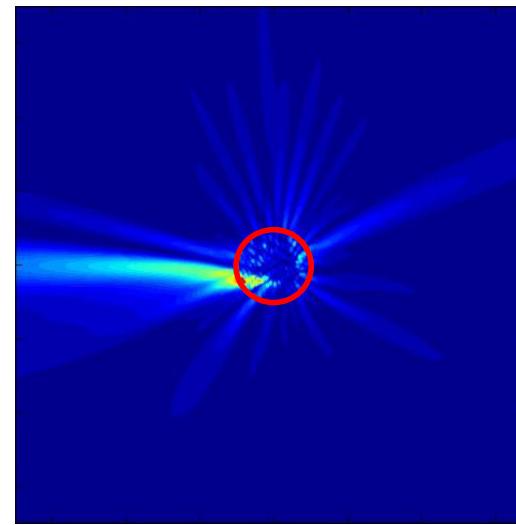
Results: Weakly scattering regime

Iteration: 25

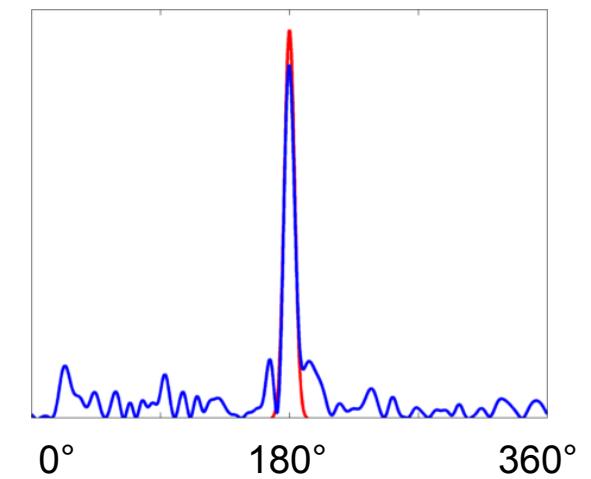
Pump profile



First laser mode



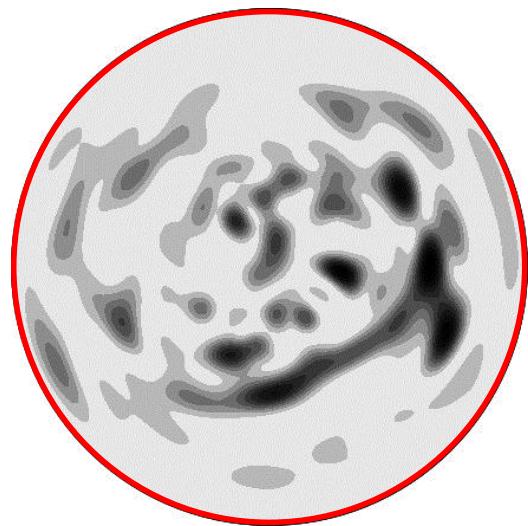
Emission pattern



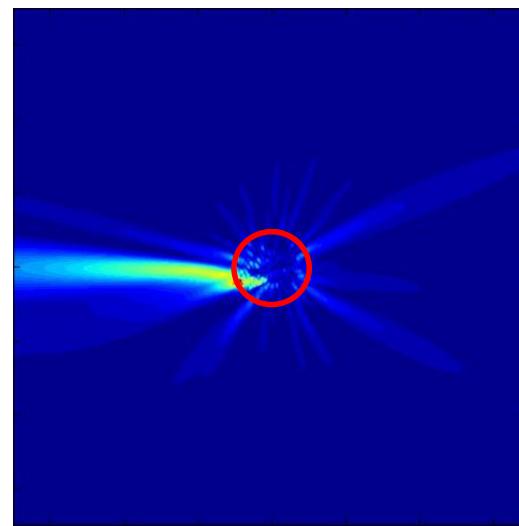
Results: Weakly scattering regime

Iteration: 42

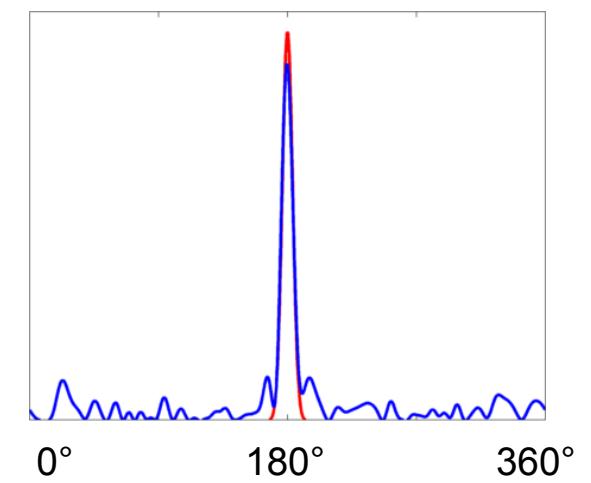
Pump profile



First laser mode

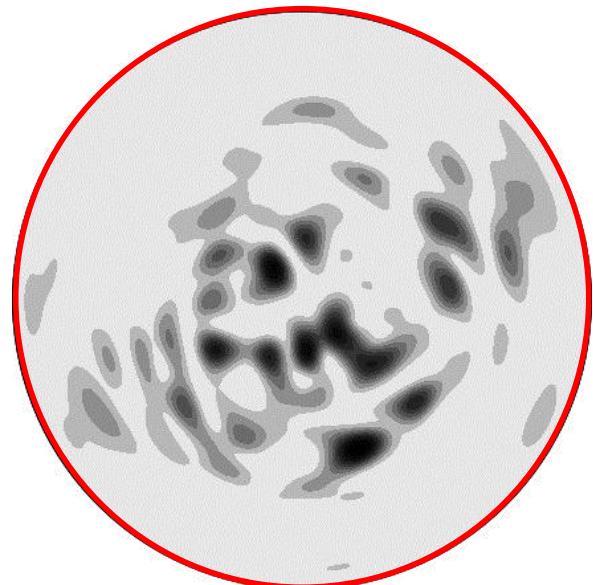


Emission pattern

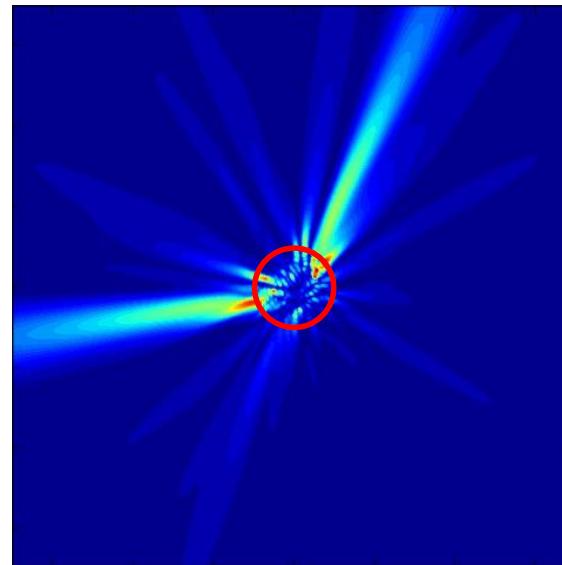


Optimization: Two directions

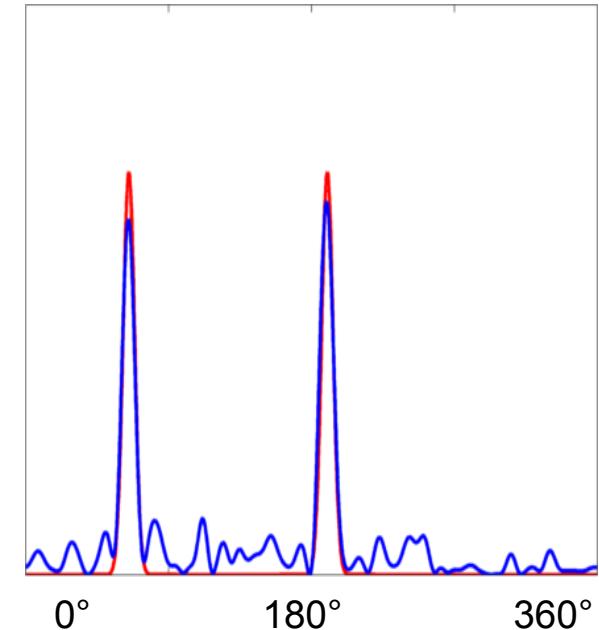
Pump profile



First laser mode



Emission pattern



Hisch, Liertzer, Pogany, Mintert,
Rotter, PRL 111, 023902 (13)



Selected for PRL:
Editors' Suggestion

Single-channel transport

A. Peña



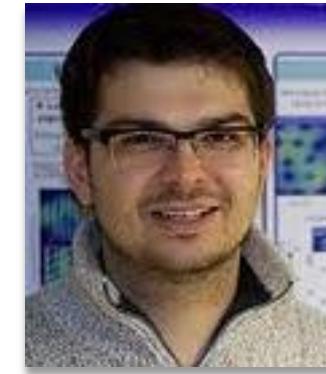
A. Chabanov



A. Girschik



F. Libisch



Surface-disordered waveguides

J. Doppler



F. Libisch



O. Dietz



U. Kuhl



H.-J. Stöckmann



J. Feist



A. Mendez-Bermudez



F. Izrailev



N. Makarov



D. Krimer



Neutron scattering

H. Abele



T. Jenke



H. Filter, M. Thalhammer,
G. Cronenberg



L. Chizhova



J. Burgdörfer



P. Geltenbort,
A. N. Ivanov,
T. Lauer, T. Lins,
H. Saul, U. Schmidt

Pump-shaping of random lasers

T. Hisch



M. Liertzer



F. Mintert



D. Pogany



H. E. Türeci



Li Ge



A. D. Stone



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AND TECHNOLOGY FUND



Thank you for your attention

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