The Abdus Salam International Centre for Theoretical Physics
50th Anniversary 1964-2014

2583-13

Workshop on Coherent Phenomena in Disordered Optical Systems

26 - 30 May 2014

Non-equilibrium States of Coupled Cavity Arrays

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U.K.

Non-equilibrium states of coupled cavity arrays.

Jonathan Keeling

SUPA

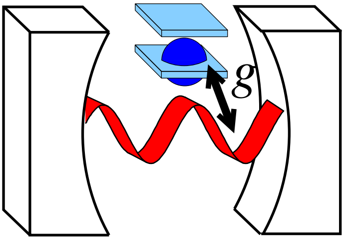
University of St Andrews
600 YEARS

ICTP, May 2014

Quantum Optics and cavity QED

- Quantum optics
- Cavity QED

$$H = \omega\psi^\dagger\psi + \frac{\omega_0}{2}\sigma^z + g\sigma^x(\psi + \psi^\dagger)$$



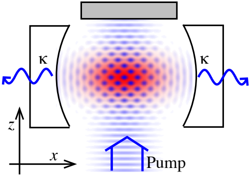
- Open system:

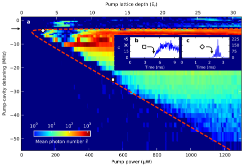
$$\partial_t\rho = -i[H,\rho] + \kappa\mathcal{L}[\psi] + \gamma\mathcal{L}[\sigma^-]$$
$$\mathcal{L}[X] = 2X\rho X^\dagger - X^\dagger X\rho - \rho X^\dagger X$$

- Rabi oscillations, collapse revival
- Fluorescence, Mollow triplet, power broadening, Purcell effect

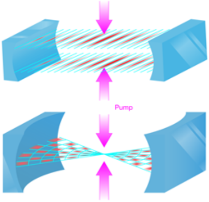
Many body cavity QED

Cold atoms in cavities



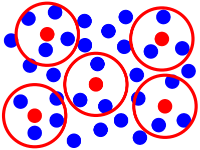


[Baumann *et al.* Nature '10]



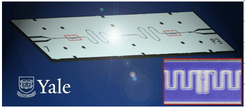
Multi-mode

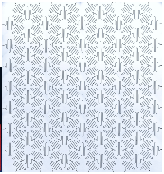
[Gopalakrishnan *et al.* Nat. Phys. '09]



Rydberg states

Superconducting qubits



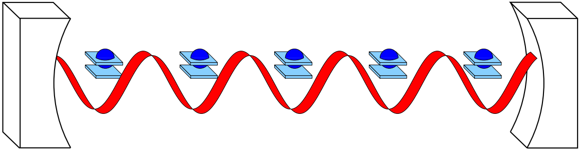


- 1 cavity many qubits
- Coupled cavity arrays

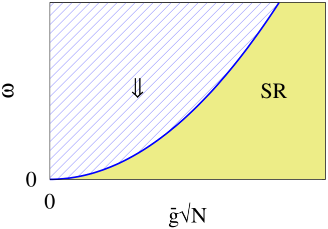
[Review: Houck *et al.* Nat. Phys. '12]

- Collective quantum optics
- Open system phase transitions

Many body quantum optics: Superradiance



$$H = \omega\psi^\dagger\psi + \sum_\alpha \frac{\omega_0}{2}\sigma^z_\alpha + g\left(\psi^\dagger\sigma^-_\alpha + \psi\sigma^+_\alpha\right)$$



- Coherent state: $|\Psi\rangle \to e^{\lambda\psi^\dagger + \eta\sum_\alpha \sigma^+_\alpha}|\Omega\rangle$
- Small g, min at $\lambda, \eta = 0$

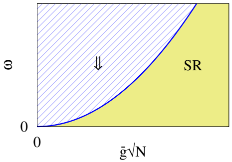
Spontaneous polarisation if: $Ng^2 > \omega\omega_0$

[Hepp, Lieb, Ann. Phys. '73]

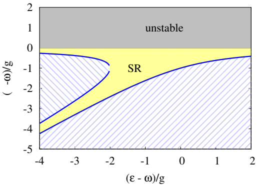
Dicke model and pumping

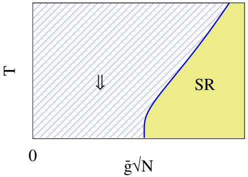
$$H_0 = \omega\psi^\dagger\psi + \sum_\alpha \frac{\omega_0}{2}\sigma^z_\alpha + g\left(\psi^\dagger\sigma^-_\alpha + \psi\sigma^+_\alpha\right)$$

- Ground state.



- Ground state - grand canonical, $H \to H - \mu N$



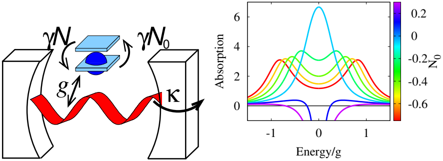


[Eastham and Littlewood, PRB '01]

Dicke model and pumping (continued)

$$H_0 = \omega\psi^\dagger\psi + \sum_\alpha \frac{\omega_0}{2}\sigma^z_\alpha + g\left(\psi^\dagger\sigma^-_\alpha + \psi\sigma^+_\alpha\right)$$

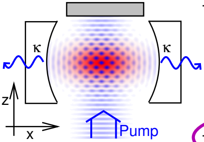
- Dissipative: Laser

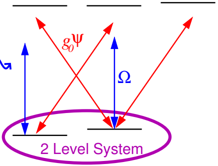


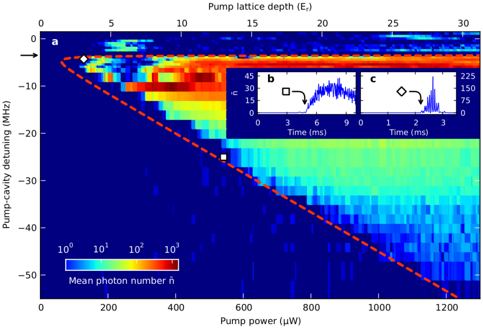
$$\dot\rho = -i[H,\rho] + i\kappa\mathcal{L}[\psi] + i\gamma_\downarrow\mathcal{L}[\sigma^-] + i\gamma_\uparrow\mathcal{L}[\sigma^+] + i\gamma_z\mathcal{L}[\sigma^z]$$

- Dissipative: coherent pumping
  $H = H + f(\psi + \psi^\dagger), \dot\rho = -i[H,\rho] + i\kappa\mathcal{L}[\psi]$
- Dissipative: Raman/Parametric pumping
  - Parametric pumping, $H = H + f(\psi\psi + \psi^\dagger\psi^\dagger), \dot\rho = -i[H,\rho] + i\kappa\mathcal{L}[\psi]$
  - Raman pumping ...

Self organisation and Dicke model







2 Level system,

$$\phi(x,z) \propto \begin{cases} 1 & \Downarrow \\ \cos(qz)\cos(qz) & \Uparrow \end{cases}$$

$$S = \sum_\alpha \sigma^z_\alpha/2$$

Feedback: $U \propto \dfrac{g_0^2}{\omega_c - \omega_a}$

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + U S_z \psi^\dagger\psi.$$

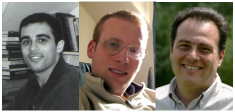
$$\partial_t\rho = -i[H,\rho] + \kappa\mathcal{L}[\psi]$$

[Dimer *et al.* PRA '07][Baumann *et al* Nature '10]

Outline

1. Many body cavity QED
   - Raman pumped Dicke model
   - From Dicke model to cavity Arrays
2. Cavity arrays: coherent pump
   - Fluoresence
   - Disorder
3. Cavity arrays: parametric pump
4. Future directions?
   - Collective dephasing

Dynamics of generalized Dicke model



1. Many body cavity QED
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Classical dynamics of the extended Dicke model

Open dynamical system:

$$H = \omega\psi^\dagger\psi + \omega_0 S^z + g(\psi + \psi^\dagger)(S^- + S^+) + US_z\psi^\dagger\psi.$$

$$\partial_t\rho = -i[H,\rho] + \kappa\mathcal{L}[\psi]$$

Classical EOM
($|\mathbf{S}| = N/2 \gg 1$)

$$\dot{S}^- = -i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$

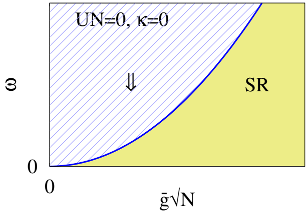
$$\dot{S}^z = ig(\psi + \psi^*)(S^- - S^+)$$

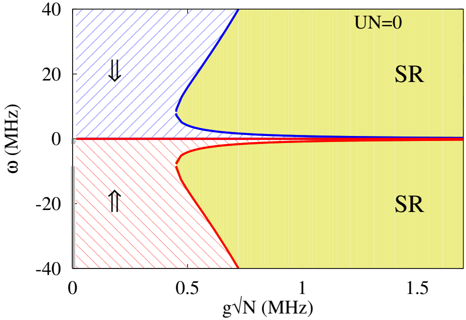
$$\dot{\psi} = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$

Equivalent to Maxwell-Bloch, $S^- \leftrightarrow P, S^z \leftrightarrow N$

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
$$0 = ig(\psi + \psi^*)(S^- - S^+)$$
$$0 = -[\kappa + i(\omega + US^z)]\psi - ig(S^- + S^+)$$





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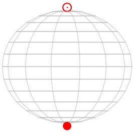
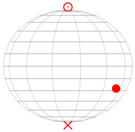


SR(A): $S_y = 0$



See also Domokos and Ritsch PRL '02, Domokos *et al.* PRL '10

Steady state phase diagram

$$0 = i(\omega_0 + U|\psi|^2)S^- + 2ig(\psi + \psi^*)S^z$$
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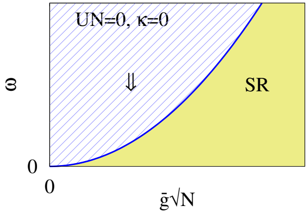

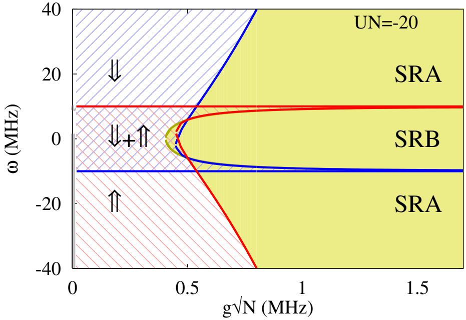

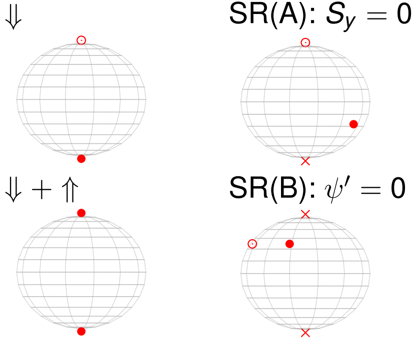

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Steady state phase diagram

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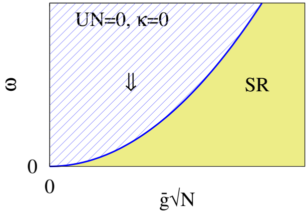
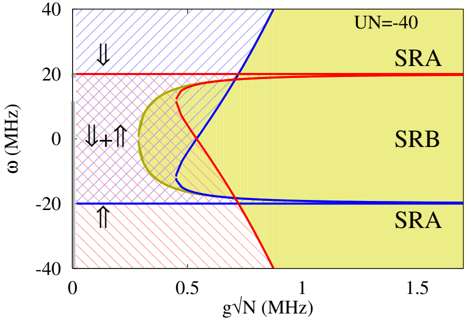


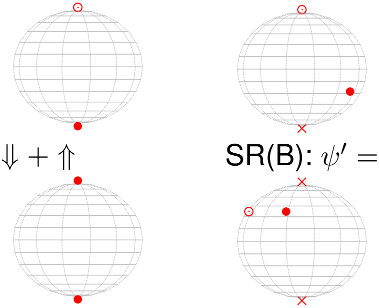


$\Downarrow$

SR(A): $S_y = 0$

$\Downarrow + \Uparrow$

SR(B): $\psi' = 0$



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Steady state phase diagram

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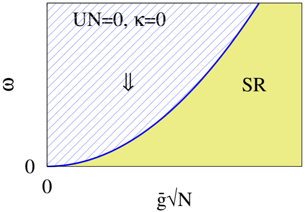
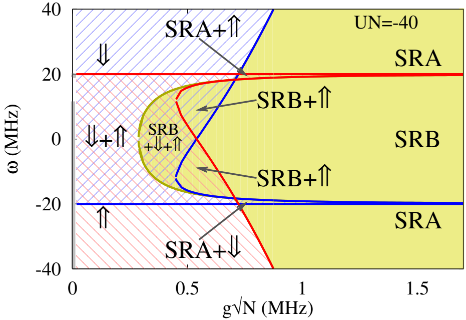
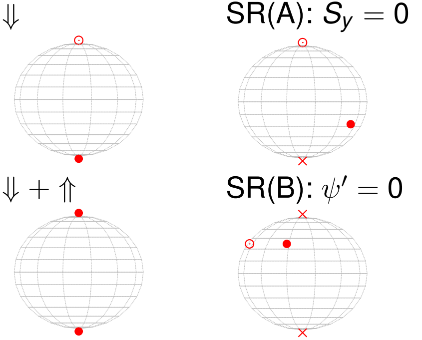


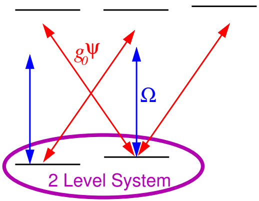




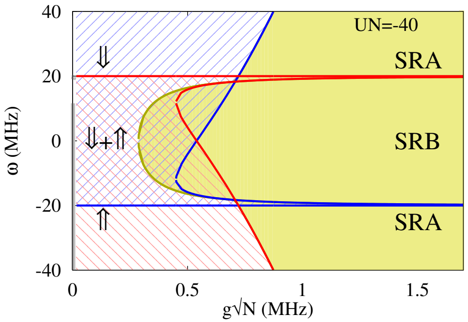
See also Domokos and Ritsch PRL ’02, Domokos *et al.* PRL ’10

Regions without fixed points

Changing U:



$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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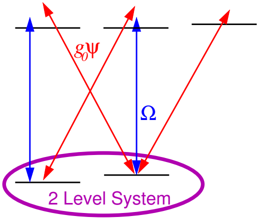
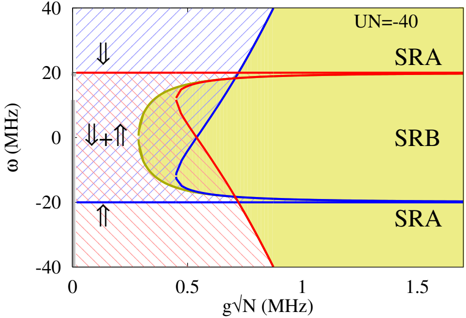


$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



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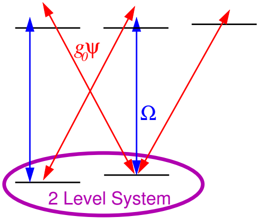
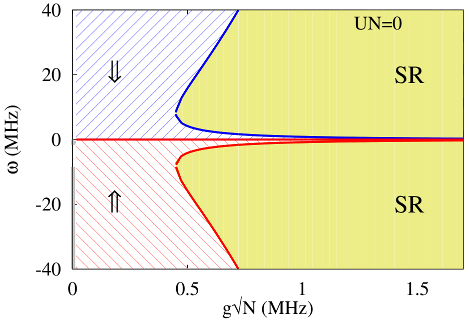
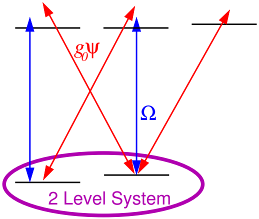


$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$



Regions without fixed points

Changing U:



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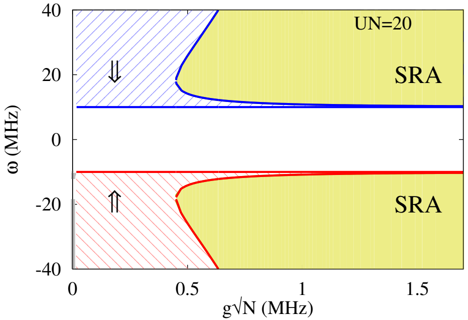
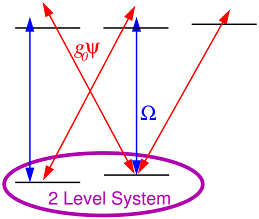
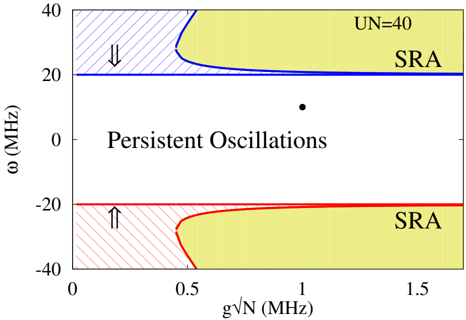


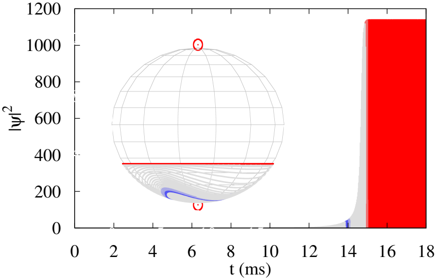
Regions without fixed points

Changing *U*:

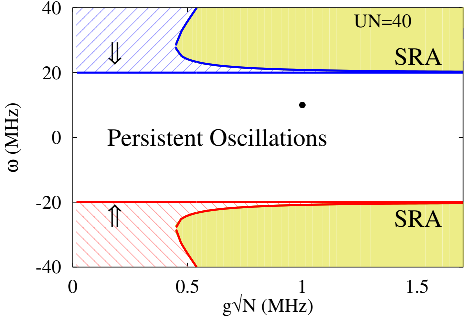


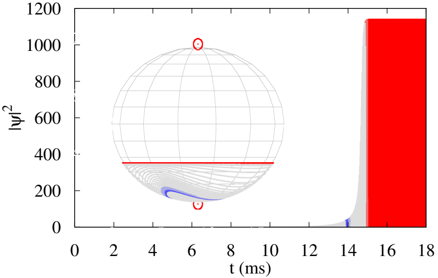
$$U \propto \frac{g_0^2}{\omega_c - \omega_a}$$

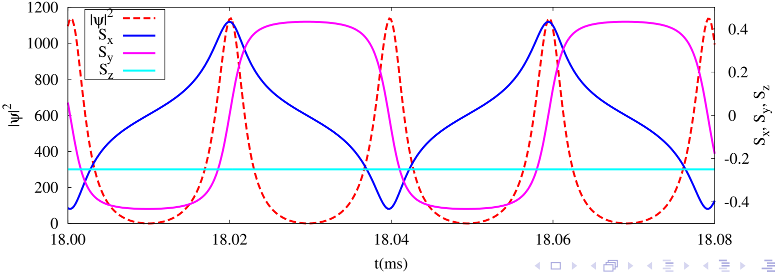




Persistent (optomechanical) oscillations







From Dicke model to Cavity Arrays

1. Many body cavity QED
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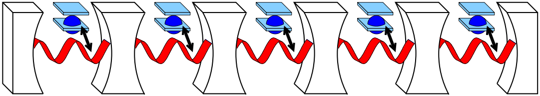
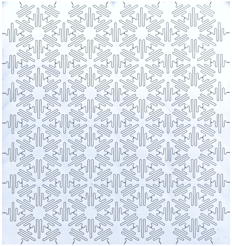
Coupled cavity arrays

- Control photon dispersion — lattice



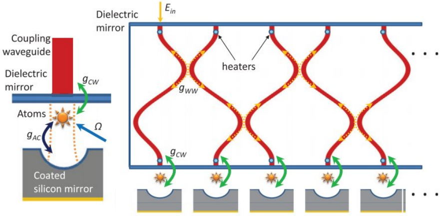
[Hartmann *et al.* Nat. Phys. '06; Greentree *et al.* Nat. Phys. 06; Angelakis *et al.* PRA '07]

- X-Hubbard Model [X=Bose, Jaynes-Cummings, Rabi, . . .]



Houck

[Underwood *et al.* PRA '12; Nat. Phys '12]

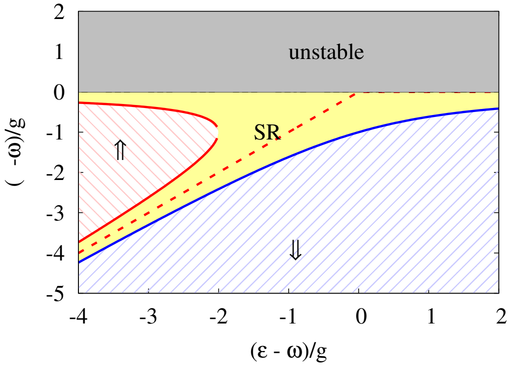


Hinds, Plenio

[Lepert *et al.* NJP '11; APL '13]

Equilibrium: Dicke model with chemical potential

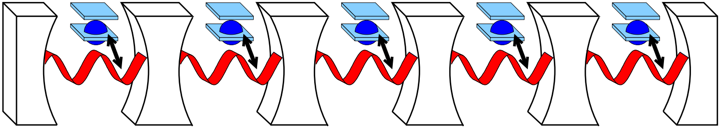
$$H - \mu N = (\omega - \mu)\psi^\dagger\psi + (\omega_0 - \mu)S^z + g\left(\psi^\dagger S^- + \psi S^+\right)$$



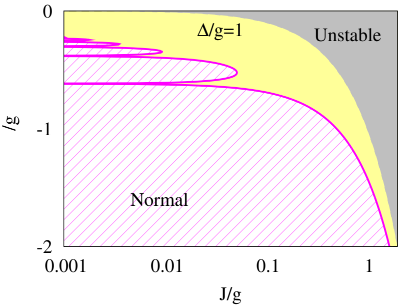
[Eastham and Littlewood, PRB ’01]

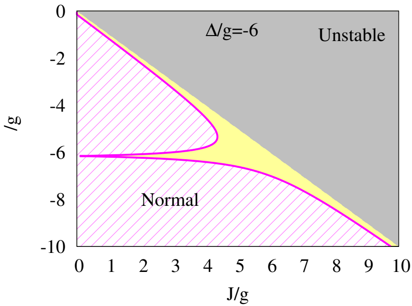
- Transition at:
  $g^2 N > (\omega - \mu)|\omega_0 - \mu|$
- Reduce critical g
- Unstable if $\mu > \omega$
- Inverted if $\mu > \omega_0$

Jaynes-Cummings Hubbard model



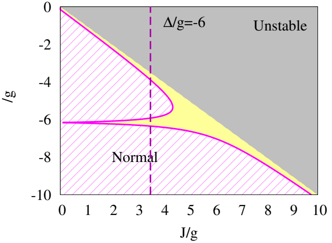
$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.})$$

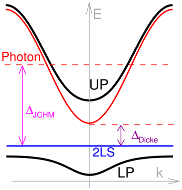




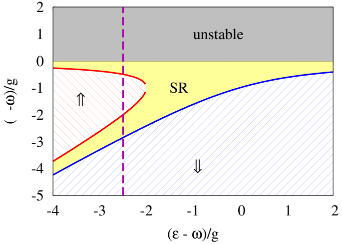
Dicke vs JCHM

JCHM





Dicke

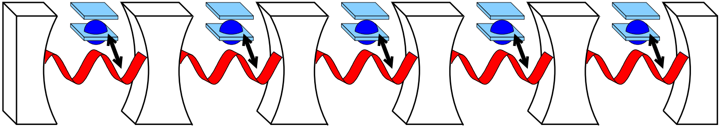


- $k = 0$ mode of JCHM $\leftrightarrow$ Dicke photon mode
- $\Uparrow \quad \leftrightarrow \quad n = 1$ Mott lobe

Cavity arrays: Coherent pump

1. Many body cavity QED
   - Raman pumped Dicke model
   - From Dicke model to cavity Arrays
2. Cavity arrays: coherent pump
   - Fluoresence
   - Disorder
3. Cavity arrays: parametric pump
4. Future directions?
   - Collective dephasing

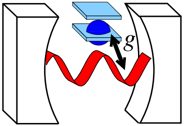
Coherently pumped JCHM



$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

$$\partial_t\rho = -i[H,\rho] + \frac{\kappa}{2}\mathcal{L}[\psi] + \frac{\gamma}{2}\mathcal{L}[\sigma^-]$$

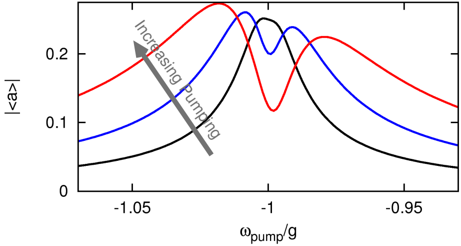
Coherently pumped single cavity [Bishop *et al.* Nat. Phys '09]



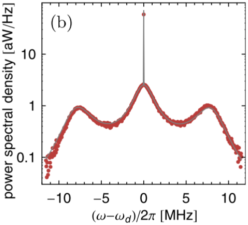
$$H = \frac{\Delta}{2}\sigma^z + g(\psi^\dagger\sigma^- + \text{H.c.}) + f(\psi e^{i\omega_{pump}t} + \text{H.c.})$$

$$\partial_t \rho = -i[H,\rho] + \frac{\kappa}{2}\mathcal{L}[\psi] + \frac{\gamma}{2}\mathcal{L}[\sigma^-]$$

- Anti-resonance in $|\langle\psi\rangle|$.
- Effective 2LS: $|\text{Empty}\rangle, |1\text{ polariton}\rangle$



- Mollow triplet fluorescence

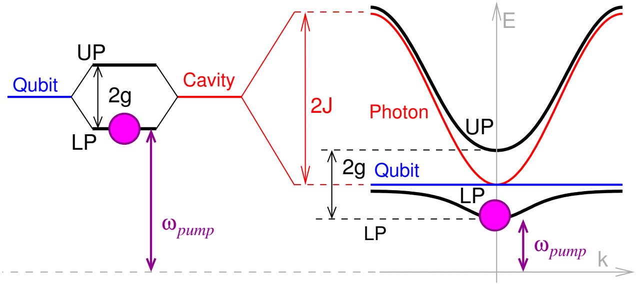


[Lang *et al.* PRL '11]

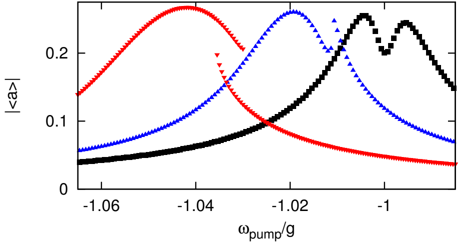
Coherently pumped dimer & array

Chose detuning *a la* Dicke model

Single cavity | Array



Evolution of anti-resonance vs J.



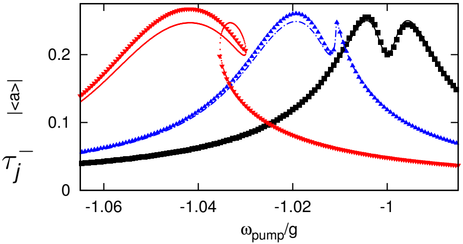
- Bistability at intermediate J
  - More/less localised states
  - Connects to Dicke limit

[Nissen *et al.* PRL '12]

Photon blockade picture $J \lesssim g$

- Polariton basis
- Nonlinearity $|\epsilon_2 - 2\epsilon_1| \propto g$.

$$H = \sum_i \left(\frac{\epsilon}{2} \tau_i^z + \tilde{f} \tau_i^x \right) - \frac{\tilde{J}}{z} \sum_{\langle ij \rangle} \tau_i^+ \tau_j^-$$

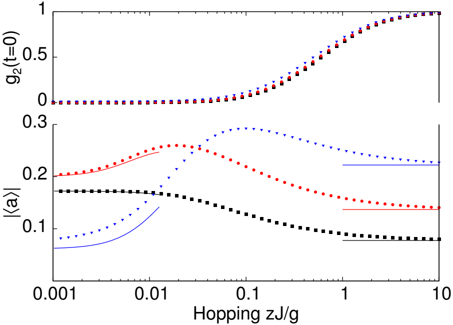


[Nissen *et al.* PRL '12]

- Decouple hopping: $\tau_i^+ \tau_j^- \to \langle \tau^- \rangle \tau^+ + \langle \tau^+ \rangle \tau^-$
- Bistability for

$$J > J_c = \frac{4}{\tilde{f}^2} \left(\frac{2\tilde{f}^2 + (\tilde{\kappa}/2)^2}{3} \right)^{3/2}$$

Coherently pumped array: correlations & fluorescence

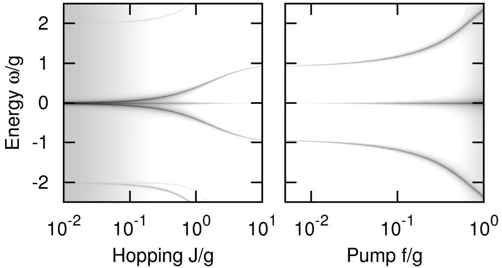


Correlations

- g_2 : $0 \to 1$ crossover.

Fluorescence

- Small J: Mollow triplet
- Large J: Off resonance fluorescence
  - Pump at collective resonance
  - Mismatch if $J \neq 0$.



Coherent pump with disorder

1. Many body cavity QED
   - Raman pumped Dicke model
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2. Cavity arrays: coherent pump
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   - Disorder
3. Cavity arrays: parametric pump
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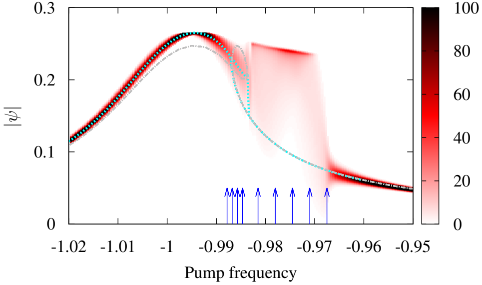
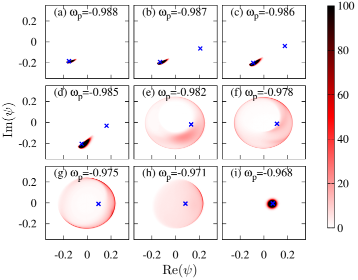
   - Collective dephasing

Coherent pumped array – disorder

$$H = -\frac{J}{z}\sum_{ij}\psi_i^\dagger\psi_j + \sum_i \frac{\Delta}{2}\sigma_i^z + g(\psi_i^\dagger\sigma_i^- + \text{H.c.}) + f(\psi_i e^{i\omega_L t} + \text{H.c.})$$

- Effect of disorder, $\Delta \to \Delta_i$
  - Distribution of ψ – Washes out bistable jump
- Bistability near resonance — phase of ψ depends on Δ_i
- Superfluid phases in driven system? [Janot *et al.* PRL '13]

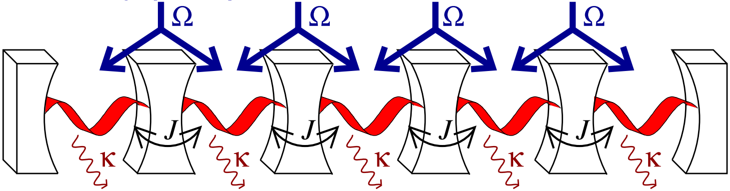




[Kulaitis *et al.* PRA, '13]

Cavity arrays: Parametric pump

Parametrically pumped JCHM



$$H = -\frac{J}{z}\sum_{<ij>}\psi_i^\dagger\psi_j + \sum_i \left[\omega_c\psi_i^\dagger\psi_i + U\psi_i^\dagger\psi_i^\dagger\psi_i\psi_i - \Omega\left(\psi_i^\dagger\psi_{i+1}^\dagger e^{-2i\omega_p t} + \text{H.c.}\right)\right]$$

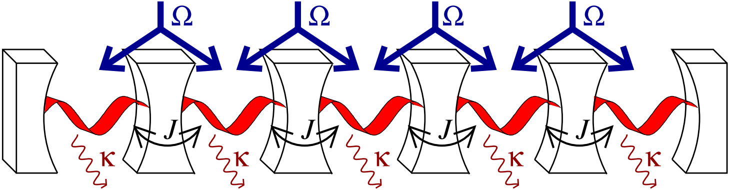
Rotating frame, blockade approximation, rescale:

$$H = -J\sum\left[\tau_i^+\tau_{i+1}^- + \tau_{i+1}^+\tau_i^- + g\tau_i^z + \Delta\left(\tau_i^+\tau_{i+1}^+ + \tau_{i+1}^-\tau_i^-\right)\right]$$

$$\partial_t\rho = -i[H,\rho] + \sum_i \kappa\mathcal{L}[\tau_i^-]$$

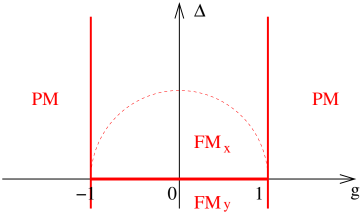
[Bardyn & Immamoglu, PRL '12]

Parametric pumping – equilibrium



$$H = -J \sum \left[\tau_i^+ \tau_{i+1}^- + \tau_{i+1}^+ \tau_i^- + g \tau_i^z + \Delta \left(\tau_i^+ \tau_{i+1}^+ + \tau_{i+1}^- \tau_i^- \right) \right]$$

- Equilibrium – transverse field Ising model
  - g – transverse field, $g_{\text{crit}} = 1$.
  - Δ – anisotropy.
    $\Delta = 0$: XY, $|\Delta| > 0$: Ising (X,Y).



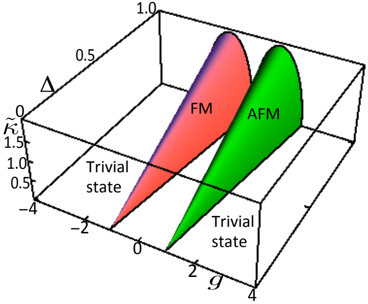
[Bardyn & Immamoglu, PRL '12]

Parametric pumping – open system

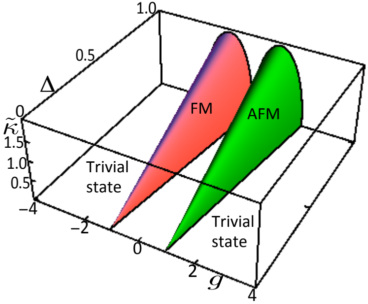
$$H = -J \sum \left[\tau_i^+ \tau_{i+1}^- + \tau_{i+1}^+ \tau_i^- + g \tau_i^z + \Delta \left(\tau_i^+ \tau_{i+1}^+ + \tau_{i+1}^- \tau_i^- \right) \right]$$

$$\partial_t \rho = -i[H, \rho] + \sum_i \kappa \mathcal{L}[\tau_i^-]$$

- Mean-field EOM: $\partial_t \langle \tau_i^\alpha \rangle = F_\alpha(\langle \tau_{i-1}^\beta \rangle, \langle \tau_i^\beta \rangle, \langle \tau_{i+1}^\beta \rangle)$
- Dynamical attractors, linear stability:



Why AFM/FM attractors



- Linear stability, fluctuation $\sim \exp(-i\nu_k t + ikr_i)$ Linear stability

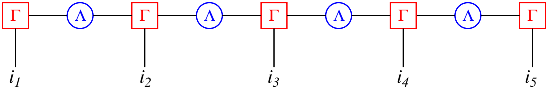
$$\nu_k = -i\kappa \pm 2J\sqrt{g^2 + 2g\cos k + (1-\Delta^2)\cos^2 k}$$

- $g \ll -1$, Dissipation matches ground state
  - Most unstable mode, $k = 0$
- $g \gg +1$, Dissipation matches max energy
  - Most unstable mode, $k = \pi$

[Joshi, Nissen, Keeling, PRA '13]

Beyond mean-field

- Matrix-product-operator representation of
  $\rho = \sum_{\{i_1, i_2, \ldots, i_N\}} c_{i_1, i_2, \ldots, i_N} \bigotimes_{j=1}^{N} \tau_j^{i_j}$

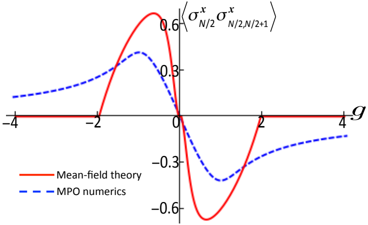


$$c_{i_1, i_2, \ldots i_N} = \sum_{\{\alpha_j\}} \Gamma^{[1] i_1}_{1, \alpha_1} \Lambda^{[1]}_{\alpha_1} \Gamma^{[2] i_2}_{\alpha_1, \alpha_2} \ldots \Gamma^{[N-1] i_{N-1}}_{\alpha_{N-2}, \alpha_{N-1}} \Lambda^{[N-1]}_{\alpha_{N-1}} \Gamma^{[N] i_N}_{\alpha_{N-1}, 1}.$$

Vidal, White, Schollwöck, *et al.* Density matrices: [Zwolak & Vidal, PRL '04]

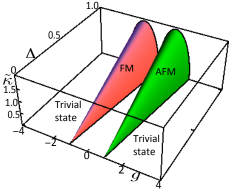
- Steady state only, 40 cavities, numerically converged
- Finite: no broken symmetry — correlators:

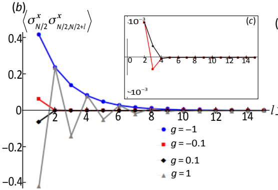
$\Delta = 1, \kappa = 0.5J$:

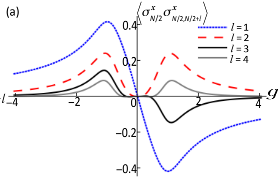


Correlations

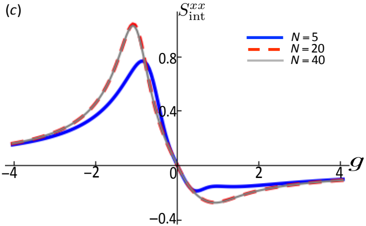
- AFM vs FM from sign of g ($\Delta = 1$)







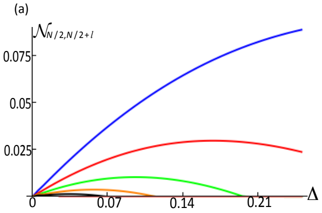
- Short range, finite susceptibility



- MFT - Correct nature of “order” but no phase transition.

Quantum Correlations

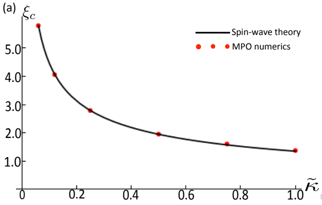
- Measures of entanglement: negativity $\mathcal{N}$



- $\Delta \to 0$, vanishing drive, XY, diverging range
- $\Delta \to 0$ analytic spin-wave theory:

$$\left|\langle \tau_i^- \tau_{i+l}^{\pm} \rangle\right| \propto \exp(-\xi_c l), \qquad \xi_c = -\ln Z_0$$

$\Delta = 0$



Future directions?

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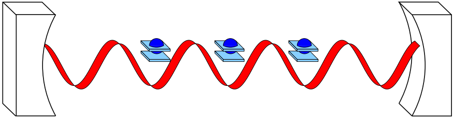
   - Collective dephasing

Collective effects and dissipation

- Real environment is not Markovian
  - [Carmichael & Walls JPA '73] Requirements for correct equilibrium
  - [Ciuti & Carusotto PRA '09] Dicke SR and emission
- Cannot assume fixed κ, γ
- Phase transition $\rightarrow$ soft modes
- Strong coupling $\rightarrow$ varying decay

Qubit Collective dephasing

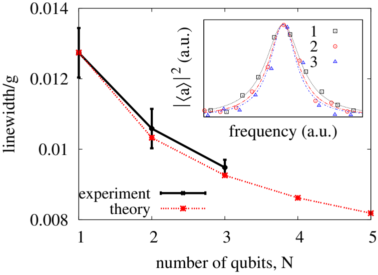
- Dicke model linewidth:



$$H = \omega\psi^\dagger\psi + \sum_{i=1}^{N} \frac{\epsilon_i}{2}\sigma_i^z + g\left(\sigma_i^+\psi + \text{h.c.}\right)$$

$$+\sum_i \sigma_i^z \sum_q \gamma_q \left(b_q^\dagger + b_q\right) + \sum_q \beta_q b_{iq}^\dagger b_q.$$

- Structured bath $\leftrightarrow \sigma^z$



[Nissen, Fink *et al.* PRL '13]

Acknowledgements

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topnes

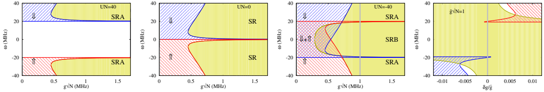
Topological Protection and Non-Equilibrium States in Strongly Correlated Electron Systems

EPSRC

Engineering and Physical Sciences Research Council

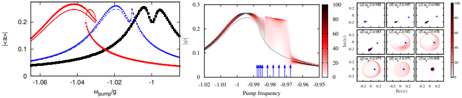
Summary

- Open system dynamics of Dicke model



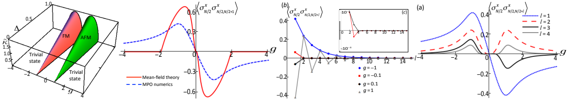
JK *et al.* PRL '10, Bhaseen *et al.* PRA '12

- Pumped coupled cavity array — bistability and disorder



Nissen *et al.* PRL '12

- Parametric pumping — non-equilibrium "phases" of transverse field Ising model



Joshi *et al.* PRA '13