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Workshop on Coherent Phenomena in Disordered Optical Systems

26 - 30 May 2014

Nonlinear Excitations of Bose-Einstein Condensates with Higher-order Interaction

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Nonlinear excitations of Bose-Einstein condensates with higher-order interaction

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Trieste, May 26-30, 2014

Overview

- Introduction and theoretical results
  - ✓ The mean-field Gross-Pitaevskii (GP) equation?
  - ✓ Modifying the GP equation: The GP Eq. with higher-order nonlinearity
  - ✓ The quasi-1D Gross-Pitaevskii equation
- Stability analysis and analytical results
  - ✓ Linear stability analysis and instability criteria
  - ✓ Instability gain and diagram
  - ✓ Variational analysis
- Numerical results: effect of the higher-order interaction
  - ✓ Dynamical instability
  - ✓ Interplay between nonlinearity and dispersion:
    Generation of nonlinear localized excitations
- Conclusions and outlook

Motivation

- Is the standard Gross-Pitaevskii equation sufficient for studying the dynamics of BECs in higher-densities regime?
  What is the appropriate theoretical approach in such regime?

- Dynamical instability enhances higher-order effect in 1D systems.
  How does the higher-order interactions affect the generation of localized structures in BECs?

The mean-field Gross-Pitaevkii equation

Validity of the mean-field GPE

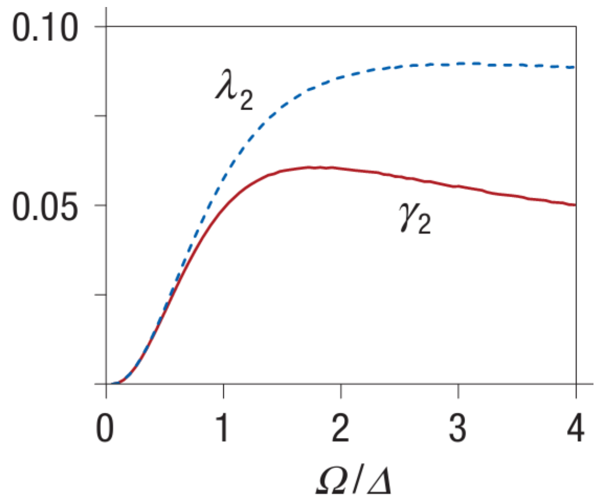
- Dilute systems, i.e. the mean interparticle distance is typically larger than the interaction range.
- Expansion parameter (n*a^3) and strength of confinement (a/L0) are very small
- Many-body encounters are scarce, and interactions are modeled by the shape-independent pseudopotential

High-density regime: Progress on atom chip and quantum computers involves strong compression, and an important increase in the densities. Then arise the problem of taking into account the three-body interactions.

Is the mean-field GPE valid in the limit of high densities?

NOT ENOUGH !

Strength of parameters that determine the three-body interaction along the line in parameter space with g=0, as a function of the Rabi frequency/detuning ratio.



1) The mean-field GPE with three-body interactions is an improvement.

The three-body interaction is tunable, independently on the two-body one.
So can be made bigger.

M. Brunner et al. Phys.Rev. Lett. 92, 078301 (2004)
M.P. Büchler et al., Nature Phys. 3, 726 (2007)

M.P. Büchler et al., Nature Phys. 3, 726 (2007)

The mean-field Gross-Pitaevkii equation

2) The quantum fluctuations

PRL 107, 135301 (2011) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2011

Dynamics and Thermodynamics of the Low-Temperature Strongly Interacting Bose Gas

Nir Navon,[1,*] Swann Piatecki,[2,†] Kenneth Günter,[1] Benno Rem,[1] Trong Canh Nguyen,[1]
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[1]*Laboratoire Kastler Brossel, CNRS, UPMC, École Normale Supérieure, 24 rue Lhomond, 75005 Paris, France*
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(Received 15 March 2011; revised manuscript received 21 June 2011; published 19 September 2011)

We measure the zero-temperature equation of state of a homogeneous Bose gas of ^{7}Li atoms by analyzing the *in situ* density distributions of trapped samples. For increasing repulsive interactions our data show a clear departure from mean-field theory and provide a quantitative test of the many-body corrections first predicted in 1957 by Lee, Huang, and Yang [Phys. Rev. **106**, 1135 (1957).]. We further

Such departure is due to the contribution of quantum fluctuations.

Beyond the mean-field Gross-Pitaevkii equation

The LHY Correction: Energy contribution of Quantum fluctuations

To understand the behavior of strongly interacting systems, LHY proposed an expansion of the interaction ground state energy per volume (energy and scattering amplitude are depdt):

$$E_{\mathrm{I}} = \frac{gn^2}{2}\left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na^3}\right)$$

Where the correction term is due to **quantum fluctuations** (QFs).

Indeed, at T=0, classical thermodynamics predicts the complete absence of excitations. However, quantum observables never fully come to rest (Heisenberg uncertainty principle); they exhibit QFs.

Evidence of Quantum (vacuum) fluctuations

QFs were detected indirectly, from their macroscopic consequences at the thermodynamic scale.

I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008) C. I. Sukenik, et al., Phys. Rev. Lett. 70, 560 (1993)

Direct microscopic observation of collective QFs in a ultracold atomic 1D cloud, dominantly on the form of quantum phonons:

- Insight into the quasi-long-range order regime
- The nonlocal analysis used reveals a clear deviation from a classical field theory

J. Armijo, Phys. Rev. Lett. 108, 225306 (2012)

Beyond the mean-field Gross-Pitaevkii equation

Quantum fluctuations and shape of the condensate

Although quantum fluctuations in Bose–Einstein condensates are usually small, it has been recently shown that under **dynamical instability** they can be amplified, for instance in the case of spinor BECs, agreeing with predictions of a beyond-mean-field theory.

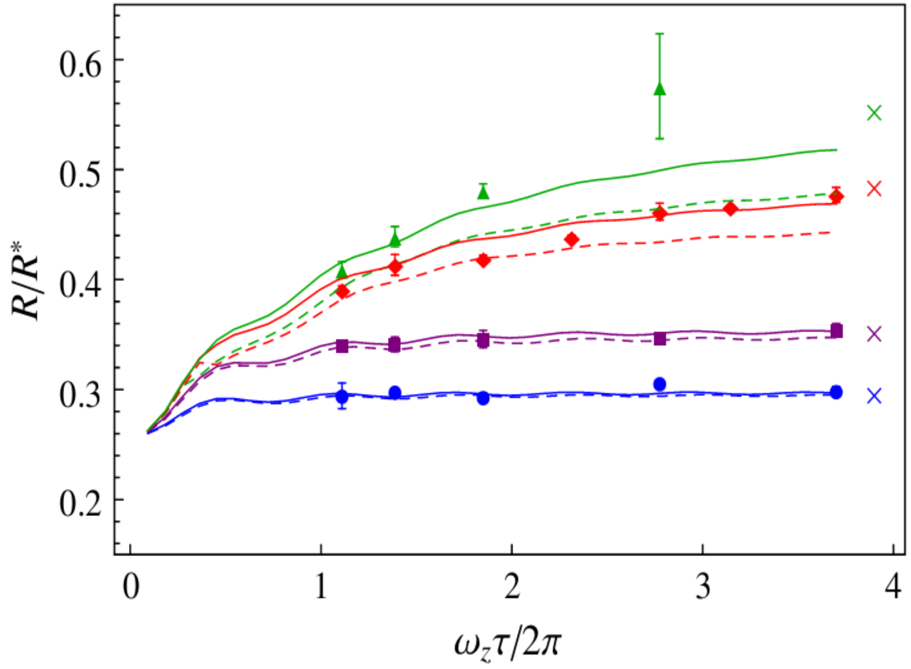

FIG. 3 (color online). Radius R of the Bose gas as a function of the duration τ of the interaction sweep. The radius R is normalized to the radius $R^* = a_{\rm ho}(15\lambda^2 N)^{1/5}$ [where $a_{\rm ho} = (\hbar/m\omega_z)^{1/2}$ and $\lambda = \omega_r/\omega_z$]. N is the measured atom number at the end of each sweep. The final values of a/a_0 are 380 (blue dots), 840 (purple squares), 2940 (red diamonds), and 4580 (green triangles). The solid (dashed) lines show the solution of a variational hydrodynamic approach (mean-field scaling solutions). The crosses show the predicted equilibrium beyond-mean-field radii.

The radius of the Bose gas depends on the scattering length.

N. Navon, et al., Phys. Rev. Lett. 107, 135301, (2011)

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qf/h=quadratic Zeeman shift at the end of the magnetic field ramp.

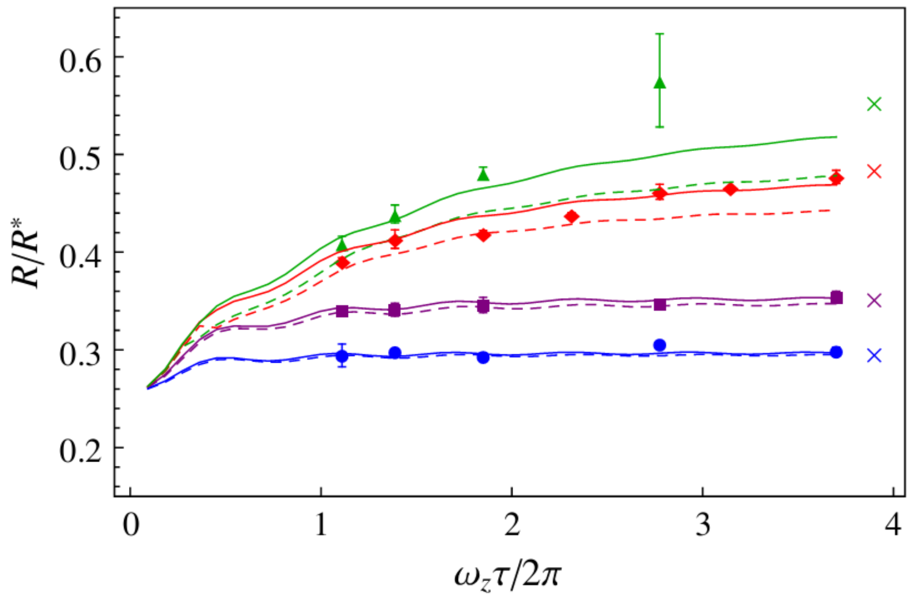

FIG. 3 (color online). Radius R of the Bose gas as a function of the duration τ of the interaction sweep. The radius R is normal-

N. Navon, et al., Phys. Rev. Lett. 107, 135301, (2011)

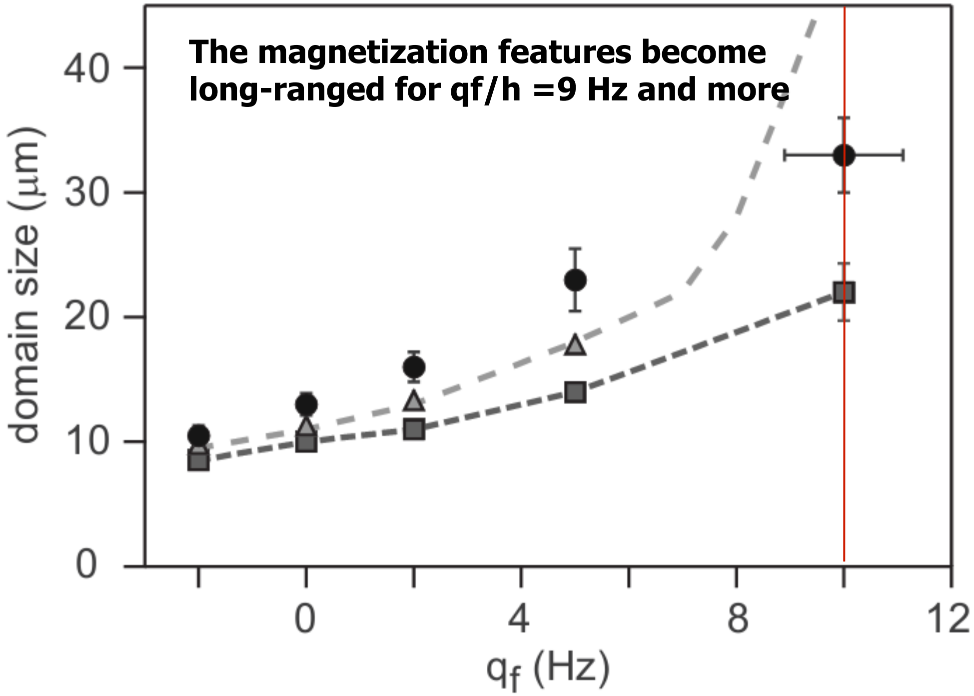

FIG. 2. Characteristic domain size after 87 ms of amplification at variable q_f. Data (circles) are averages over five experimental repetitions; error bars are statistical. Horizontal error bar reflects systematic uncertainty in q_f. Predictions based on numerical simulations for $|\Delta a| = 1.45 a_B$ [7] (squares) and $1.07 a_B$ [24] (triangles) are shown, with error bar reflecting systematic uncertainty in the atomic density.

S.R. Leslie, et al., Phys. Rev. A 79 (2009) 043631

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S.R. Leslie, et al., Phys. Rev. A 79 (2009) 043631

Quantum fluctuations and dimensionality

In low dimensions, QFs are dramatically enhanced. In particular for 1D systems, QFs may destroy long-range order and prevent Bose-Einstein condensation even at T=0.

I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008) D. Petrov, D. Gangardt, and G. Shlyapnikov, J. Phys. IV 116 , 5 (2004)

Some effects of Quantum fluctuations

In matter fields, QFs: - govern the correlations properties at low temperature
- cause quantum depletion in Bose-Einstein condensates, **bringing corrections to their equation of state**.

N. Navon, et al., Phys. Rev. Lett. 107, 135301 (2011)
T. D. Lee, K. Huang, and C. N. Yang, Phys. Rev.106, 1135 (1957)

Hence, including corrections to the GPE to take into account QFs becomes important, in particular, for **1D Bose systems**.

Beyond the mean-field Gross-Pitaevkii equation

2) Shape-dependent confinement

The shape-**in**dependent approximation becomes less valid under strong confinement.

H. Fu, Y. Wang, and B. Gao, PRA 67, 053612 (2003)

Contributions from higher partial waves may be included through the addition of higher-derivative terms, as described for Fermi systems in [R. Roth and H. Feldmeier, PRA.64.043603].

A. Collin, et al., PRA 75, 013615 (2007)

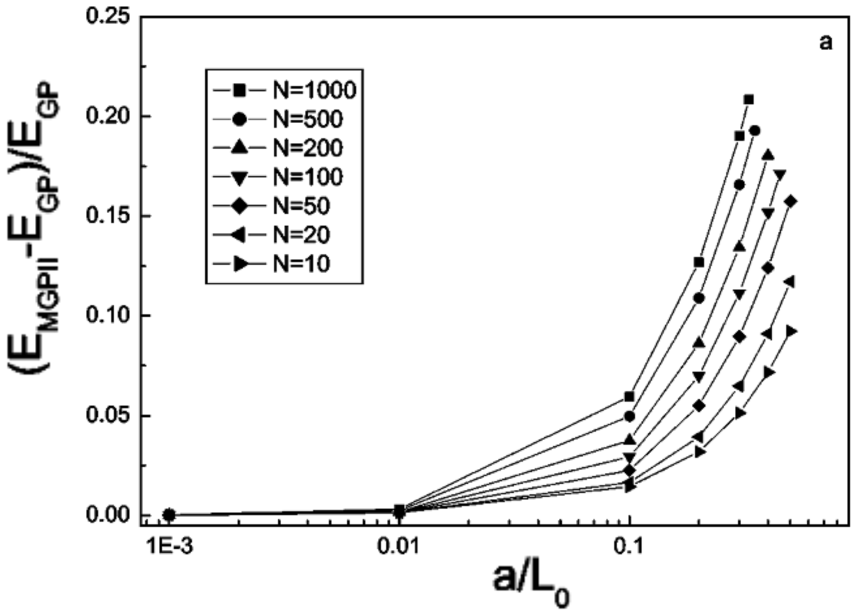
We may incorporate the shape dependence on the interaction potential via the effective range. The shape-dependent confinement correction and LHY correction are clearly from different origin.

3) A comparison MGPE Vs. GPE

Disagreement at high confinement strength.

The previous observations suggests an interest in the study the dynamical instability of **1D Bose systems through a modified GPE comprising the terms of QFs, shape-dependent confinement, and three-body interactions**.

From this, we may get the parameter domains where localized structures may appear in trapped Bose–Einstein condensates at high density in the presence of QFs.



H. Fu, Y. Wang, and B. Gao, PRA 67, 053612 (2003)

The GP Eq. with higher-order nonlinearity

- We start with the energy functional

In the ultracold regime (T<<Tc), a system may obey the T=0 formalism. The higher-order effects in the two-body scattering dynamics can be captured by

$$E[\Psi] = \int d\mathbf{r}\left[\frac{\hbar^2}{2m}|\nabla\Psi|^2 + V_{\text{ext}}(\mathbf{r})|\Psi|^2 + \frac{1}{2}g_0|\Psi|^4 + \frac{2}{5}q_0|\Psi|^5 + \frac{1}{2}p_0|\Psi|^2\nabla^2(|\Psi|^2) + \frac{1}{4}\chi_0|\Psi|^6\right]$$

Kinetic energy

External trapping potential

Two-body interaction

Higher-order correction to the two-body interaction due to quantum fluctuations

Three-body interaction

Higher-order correction to the two-body interaction Due to shape-dependent confinement

The GP Eq. with higher-order nonlinearity

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$$E[\Psi] = \int d\mathbf{r}\left[\frac{\hbar^2}{2m}|\nabla\Psi|^2 + V_{\text{ext}}(\mathbf{r})|\Psi|^2 + \frac{1}{2}g_0|\Psi|^4 + \frac{2}{5}q_0|\Psi|^5 + \frac{1}{2}p_0|\Psi|^2\nabla^2(|\Psi|^2) + \frac{1}{4}\chi_0|\Psi|^6\right]$$

The coefficients of nonlinear terms are:

$$g_0 = 4\pi\hbar^2 a/m,\ q_0 = 32 g_0 a^{3/2}/(3\sqrt{\pi})$$
$$p_0 = g_0 \times (\tfrac{1}{3}a^2 - \tfrac{1}{2}a r_e)$$

For a hard-sphere potential, *Re=2a/3* and *P0* is zero. The energy functional can be developed to obtain a generalization of the GP equation for the BEC wave function:

The 1D Gross-Pitaevskii equation

- Consider the full 3D GPE for BECs trapped in a purely parabolic potential

Very close to 0 K, the following mean-field Gross-Pitaevskii describes the dynamics of dilute 3D BECs:

$$i\hbar\Psi_t = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\mathbf{r})\Psi + g_0|\Psi|^2\Psi + q_0|\Psi|^3\Psi + p_0\nabla^2\left(|\Psi|^2\right)\Psi + \chi_0|\Psi|^4\Psi$$

The external potential reads:

$$V(\mathbf{r}) = \frac{1}{2}m\left(\omega_\perp^2\rho^2 + \omega_x^2 x^2\right) \text{ with } \rho = \sqrt{x^2+y^2}$$

How to get to the quasi-1D GPE in this context?

Experimental support: Make the radial trapping frequency very big compared to the axial one, to prepare a cigar-shaped condensate.

The quasi-1D Gross-Pitaevskii equation

- Reducing the full 3D Gross-Pitaevskii equation

Mathematical method:

1. Separate the condensate wf into radial and axial parts:

$$\psi(\mathbf{r},t)=\phi_0(\rho)\phi(x,t) \text{ where } \phi_0=\sqrt{\frac{1}{\pi a_\perp^2}}\times\exp(-\frac{\rho^2}{2a_\perp^2}),$$

$$\text{with } \rho=\sqrt{x^2+y^2} \text{ and } a_\perp=\sqrt{\hbar/m\omega_\perp},$$

And obtain the equations that describe the radial and axial condensate wavefunctions:

$$-\frac{\hbar^2}{2m}\nabla_\rho^2\phi_0+\frac{m}{2}\omega_\perp^2\rho^2\phi_0=\hbar\omega_\perp\phi_0$$

$$i\hbar\psi_t=-\frac{\hbar^2}{2m}\psi_{xx}+V(x)\psi+g_0|\psi|^2\psi+q_0|\psi|^3\psi+p_0\left(|\psi|^2\right)_{xx}\psi+\chi_0|\psi|^4\psi$$

$$\text{with } V(x)=\tfrac{1}{2}m\omega_x^2x^2$$

The quasi-1D Gross-Pitaevskii equation

- Reducing the full 3D Gross-Pitaevskii equation

2. Apply appropriate change of variables to the previous 1D GPE (rescaling the wavefunction to a dimensionless form)

$$t \longleftarrow \tfrac{1}{2}\omega_\perp t, \qquad x \longleftarrow a_\perp^{-1} x, \qquad \psi \longleftarrow \sqrt{a_\perp^{-3}}\,\psi$$

Normalized macroscopic wave-function

We get the dimensionless quasi-1D GPE

$$\boxed{\mathrm{i}\psi_t = -\psi_{xx} + V(x)\psi + g_0|\psi|^2\psi + q_0|\psi|^3\psi + p_0\left(|\psi|^2\right)_{xx}\psi + \chi_0|\psi|^4\psi}$$

where $V(x) = \alpha x^2$ and

$$\alpha = \frac{\omega_x^2}{\omega_\perp^2}, \quad g_0 \longleftarrow \frac{2a_\perp^3}{\hbar\omega_\perp} g_0, \quad q_0 \longleftarrow \frac{2a_\perp^{9/2}}{\hbar\omega_\perp} q_0, \quad \chi_0 \longleftarrow \frac{2a_\perp^6}{\hbar\omega_\perp}\chi_0, \quad p_0 \longleftarrow \frac{2a_\perp}{\hbar\omega_\perp} p_0.$$

In the rest of this talk, the above MGPE will be used for investigating the generation of localized excitations in the BECs.

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- ❑ Stability analysis and analytical results
  - ✓ Linear stability analysis and instability criteria
  - ✓ Instability gain and diagram

- ❑ Numerical results: effect of the higher-order interaction
  - ✓ Dynamical instability
  - ✓ Interplay between nonlinearity and dispersion:
    Generation of nonlinear localized excitations

- ❑ Conclusions and outlook

Linear stability analysis and instability criteria

First, we introduce a modified lens-type transformation to get:

$$\mathrm{i}\phi_T = -\phi_{XX} + \left[g(T)|\phi|^2 + q(T)|\phi|^3 + p(T)\left(|\phi|^2\right)_{XX} + \chi_0|\phi|^4\right]\phi$$

The ansatz is taken to be:

$$\phi = (\phi_0 + \delta\phi)\exp\left[-\mathrm{i}\int_0^T \Omega(\upsilon)\,\mathrm{d}\upsilon\right]$$

$$g(T) = g_0(1 + 4\alpha T^2)^{-\frac{1}{2}}$$
$$q(T) = q_0(1 + 4\alpha T^2)^{-\frac{1}{4}}$$
$$p(T) = p_0(1 + 4\alpha T^2)^{\frac{1}{2}}$$
$$T(t) = \tan(2\sqrt{\alpha}t)/(2\sqrt{\alpha})$$

$$\Omega(T) = g(T)\phi_0^2 + q(T)\phi_0^3 + \chi_0\phi_0^4$$

Let the perturbation be the plane wave:

$$\delta\phi = \mathrm{Im}\left[U_2 \mathrm{e}^{\mathrm{i}(KX - \int_0^T \omega(\upsilon)\,\mathrm{d}\upsilon)}\right] + \mathrm{i}\,\mathrm{Re}\left[U_1 \mathrm{e}^{\mathrm{i}(KX - \int_0^T \omega(\upsilon)\,\mathrm{d}\upsilon)}\right]$$

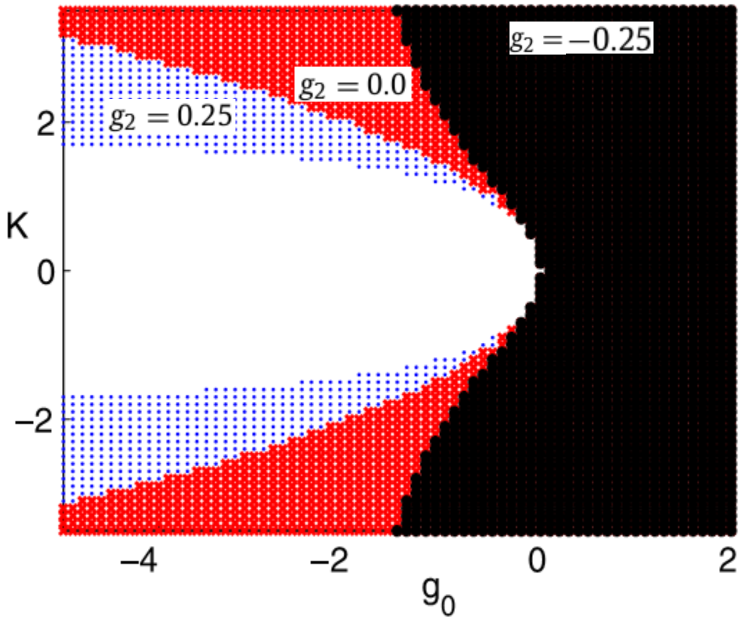
The dynamical instability is excited under the condition:

$$1 - 2g_0g_2(1 + 4\alpha T^2)^{\frac{1}{2}}\phi_0^2 + 2g_0(1 + 4\alpha T^2)^{-\frac{1}{2}}\left(\frac{\phi_0}{K}\right)^2 + 3g_0g1(1 + 4\alpha T^2)^{-\frac{1}{4}}\phi_0\left(\frac{\phi_0}{K}\right)^2 + 4\chi_0\phi_0^2\left(\frac{\phi_0}{K}\right)^2 < 0$$

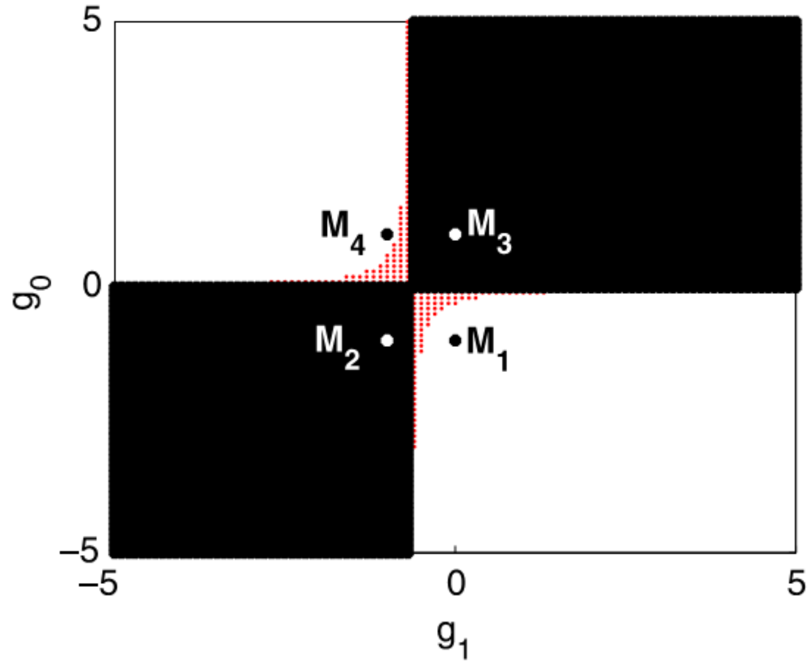
This condition may be used to obtain the instability diagram.

Instability diagram

The SD confinement shrinks the bandwidth of unstable modes, for attractive interaction



QFs changes the nature of interparticle interactions



Instability gain

Expression of the instability growth rate:

$$\mathrm{Im}\,\omega = K^2\left[-1+2p(T)\phi_0^2-2g(T)\left(\frac{\phi_0}{K}\right)^2-3\phi_0 q(T)\left(\frac{\phi_0}{K}\right)^2-4\chi_0\phi_0^2\left(\frac{\phi_0}{K}\right)^2\right]^{\frac{1}{2}}$$

The following Ref. deals with the effect of strength confinement on dynamical instability:

X.-Y. Qi et al, Phys Rev E 86 (2012) 017601

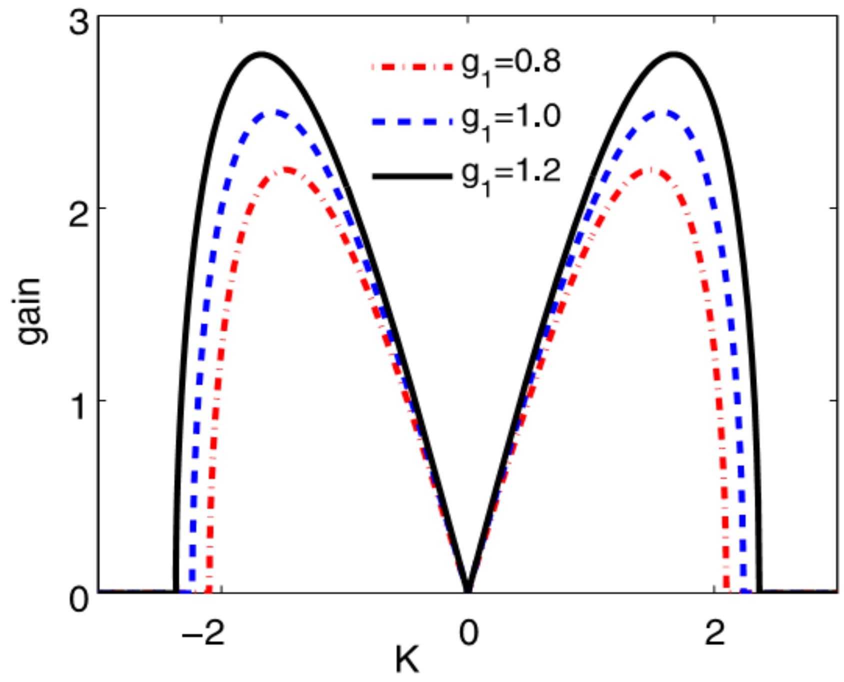
In the rest of the talk, we will mostly investigate the effects of quantum fluctuations and three-body interaction.

Instability gain

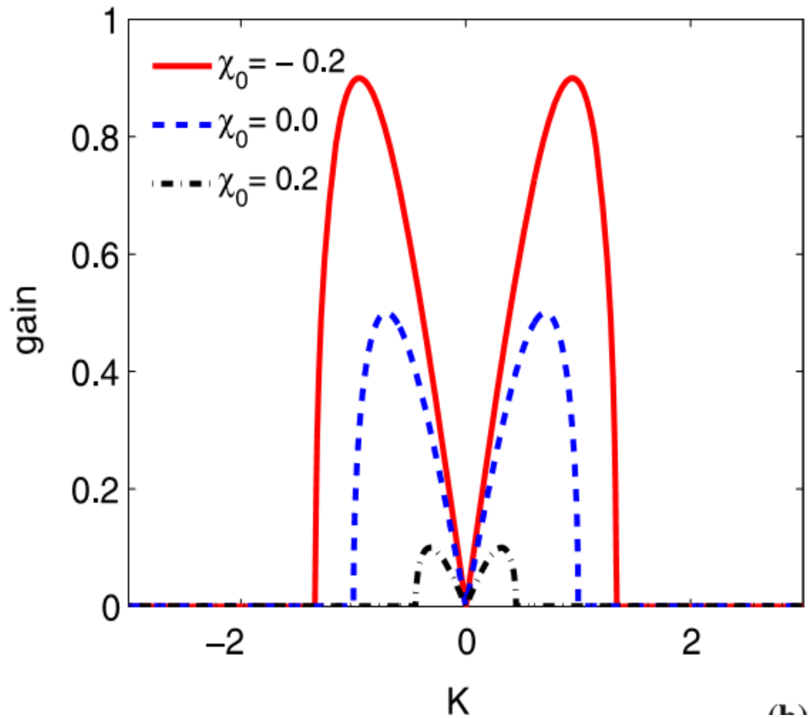
Expression of the instability growth rate:

$$\mathrm{Im}\,\omega = K^2\left[-1+2p(T)\phi_0^2-2g(T)\left(\frac{\phi_0}{K}\right)^2-3\phi_0 q(T)\left(\frac{\phi_0}{K}\right)^2-4\chi_0\phi_0^2\left(\frac{\phi_0}{K}\right)^2\right]^{\frac{1}{2}}$$

Effect of quantum fluctuations:



Effect of three-body interaction:



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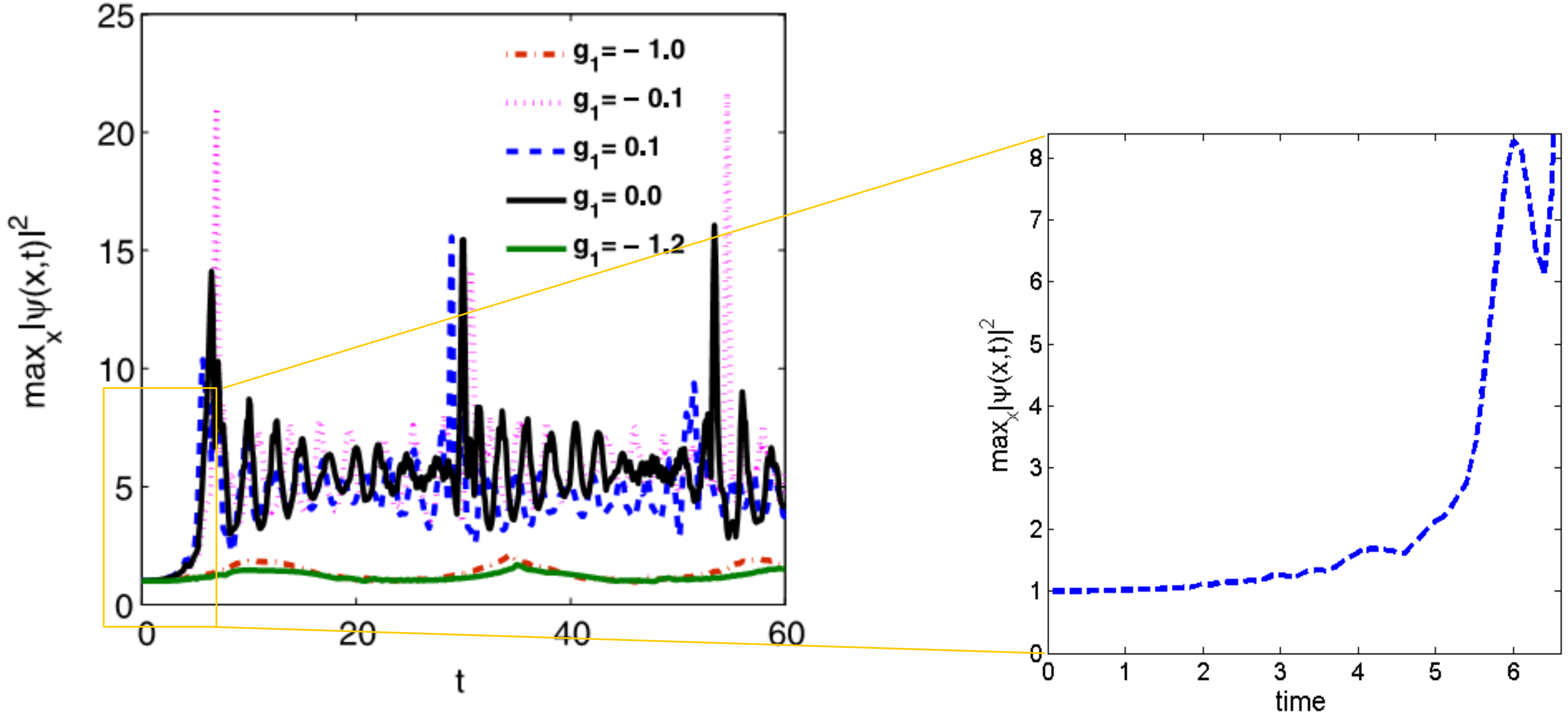
Excitation of Dynamical instability

Effect of quantum fluctuations

a) Two-body interaction is attractive



QFs may cause a BEC to exhibit dynamical instability in cases of both attraction and repulsion.

Excitation of Dynamical instability

Effect of quantum fluctuations

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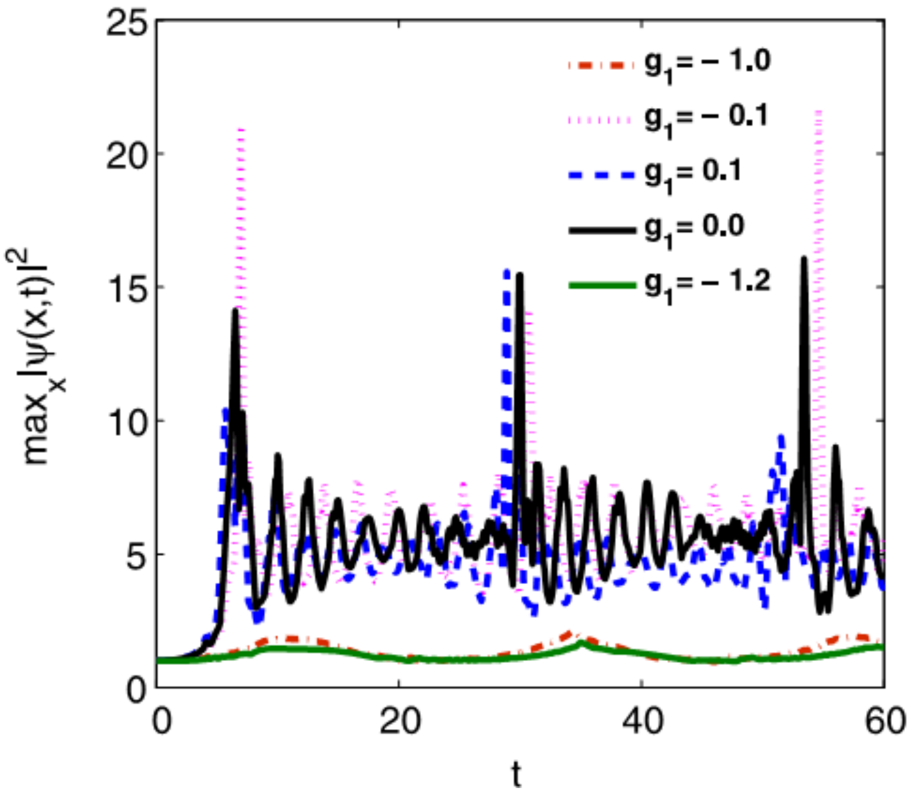
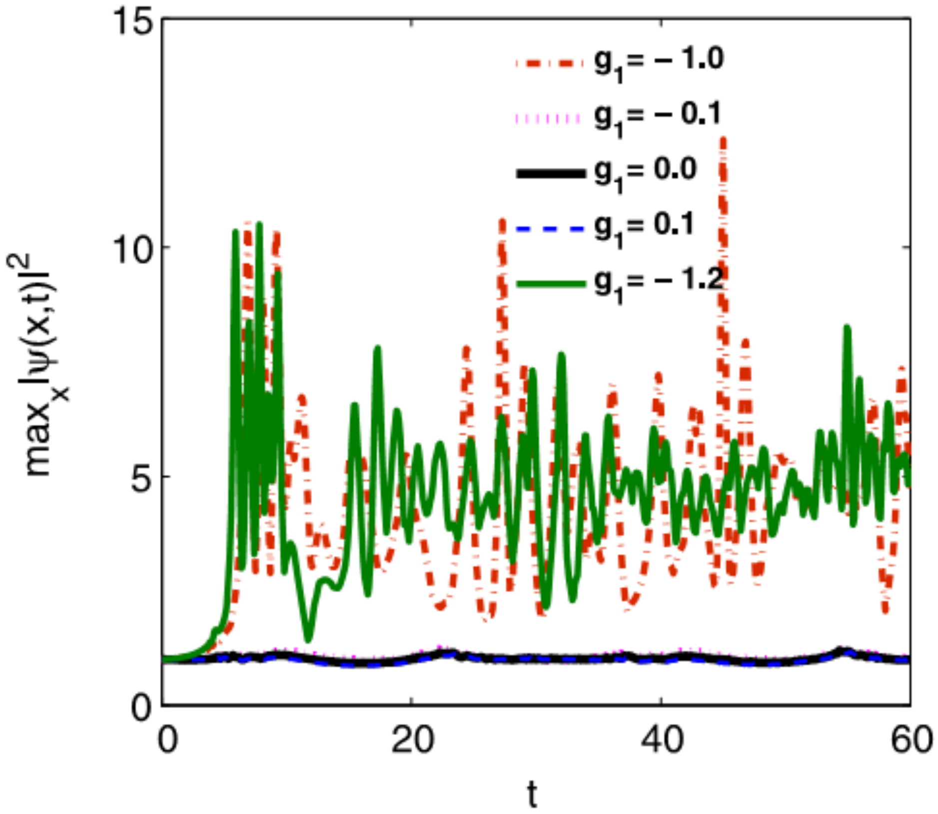


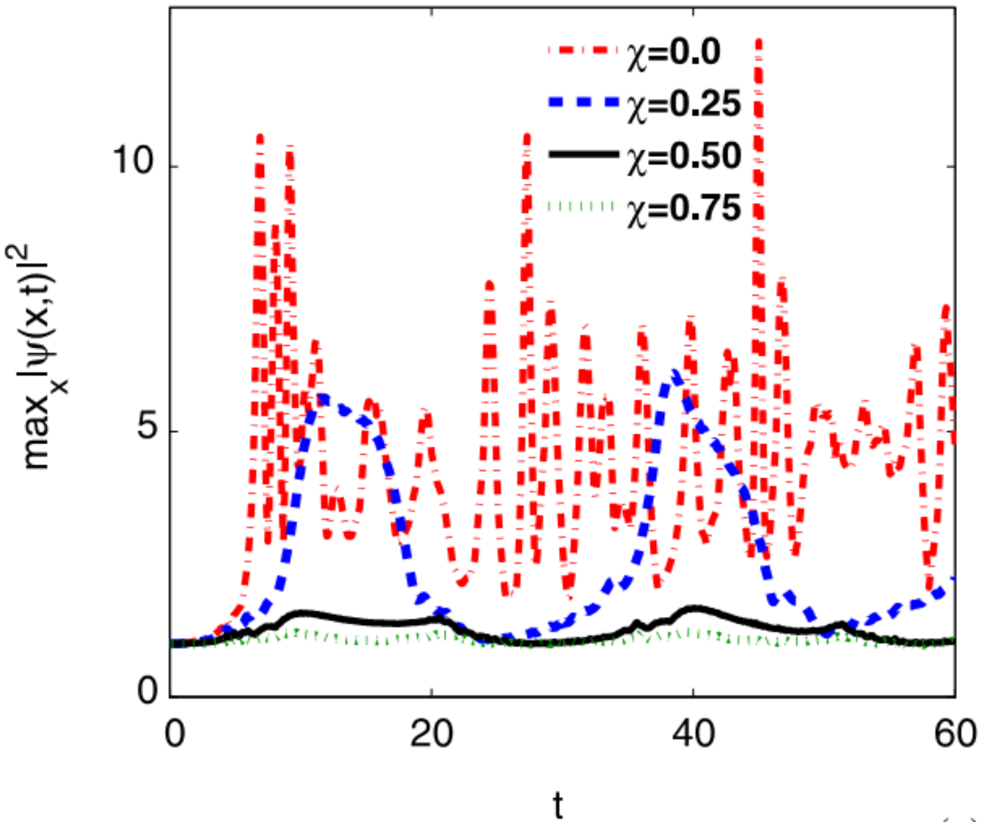
b) Two-body interaction is repulsive



QFs may cause a BEC to exhibit dynamical instability in cases of both attraction and repulsion.

Excitation of Dynamical instability

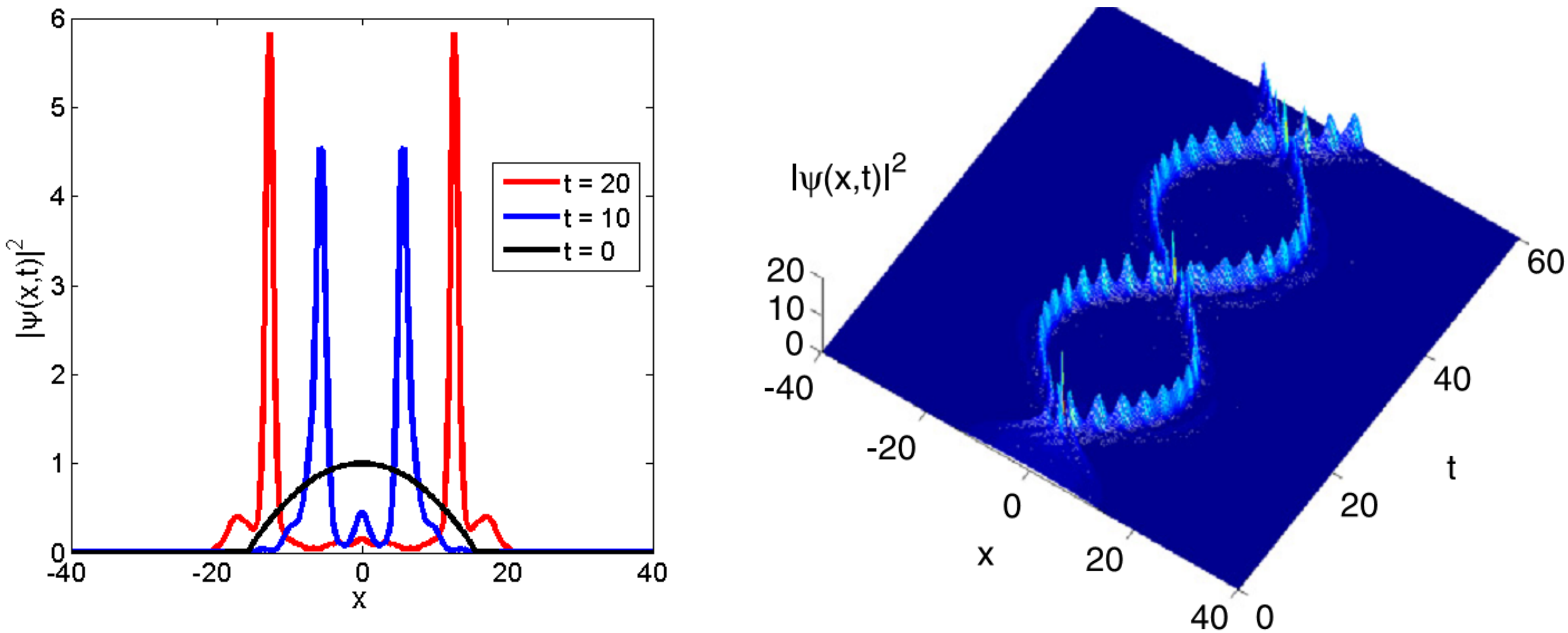
Effect of three-body interaction



Increasing the three-body interaction stabilizes the system.

Generation of nonlinear localized excitations

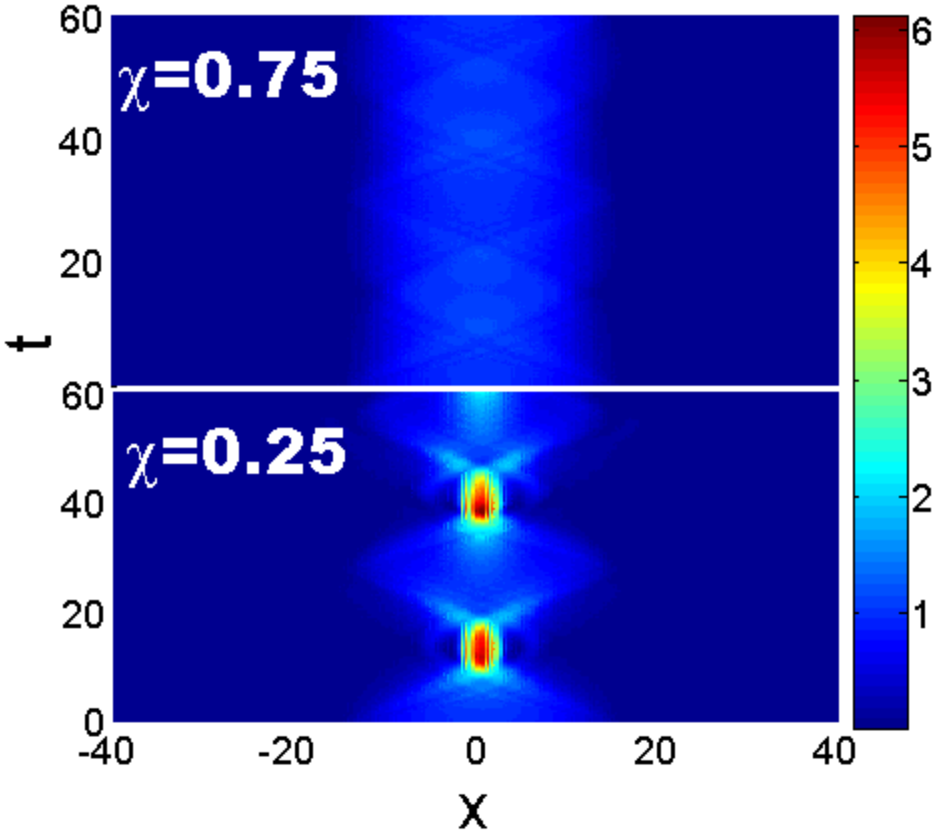
Effect of quantum fluctuations



It emerges that reducing the three-body interaction may allow to generate localized excitations.

Generation of nonlinear localized excitations

Effect of three-body interactions



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Conclusion and outlook

Summary of our results

The standard mean-field GPE is not sufficient for investigating the dynamics of BECs in a regime with high particle density and quantum fluctuations. The quartic, residual and quintic nonlinearities may be included to account for quantum fluctuations, shape-dependent confinement and three-body interactions among particles.

We obtained a criterion that defines the parameter domains where dynamical instability can be excited in such regime. In this domain, localized excitations have been generated in the condensate; their propagation has been portrayed.

Useful references :

E. Wamba et al, Phys Rev E xx (2014) xxxxxx (In Press)

E. Wamba et al, Phys Lett A 377 (2013) 262

X.-Y. Qi et al, Phys Rev E 86 (2012) 017601

E. Wamba et al, Phys Rev E 77 (2008) 046216

H. Hu, Phys Rev A 67 (2003) 053612

Our findings may be useful to understand the dynamics of systems like

- Nonlinear optical media with spatially nonlocal non-linear response

  W. Królikowski, O. Bang, Phys. Rev. E 63 (2000) 016610
- Nematic liquid crystals with long-range molecular reorientational interactions

  A. Griesmaier, et al., Phys. Rev. Lett. 94 (2005) 160401

Conclusion and outlook

Outlook

In nonlinear optics, the propagation of light beams in self-defocussing nonlocal media with inhomogeneous nonlocality can be described by:

$$\mathrm{i}\psi_t + c\psi_{xx} - g\Delta n\psi = 0,$$

$$d(t)\Delta n_{xx} - \Delta n + |\psi|^2 = 0$$

$\psi(x,t)$ is the light field

$I(x,t) = |\psi(x,t)|^2$ is the beam intensity

$\Delta n(x,t) = s\int \mathcal{R}(x'-x)I(x',t)\,\mathrm{d}x'$ is the change in the refractive index

Combining the equation and considering the case of singular response

$\mathcal{R}(x) = \delta(x)$ $\qquad \Delta n(x,t) = s|\psi(x,t)|^2$

We obtain

$$\mathrm{i}\frac{\partial\psi}{\partial t} + c\frac{\partial^2\psi}{\partial x^2} - g|\psi|^2\psi - sgd(t)\frac{\partial^2|\psi|^2}{\partial x^2} = 0$$

Hence our model may be used to understand the dynamics of nonlinear optical media with spatially nonlocal non-linear response.


Thank you!

Obervation of Dynamical Instability

PHYSICAL REVIEW A **72**, 013603 (2005)

Unstable regimes for a Bose-Einstein condensate in an optical lattice

L. De Sarlo,* L. Fallani, J. E. Lye, M. Modugno,† R. Saers,‡ C. Fort, and M. Inguscio
LENS, Dipartimento di Fisica, and INFM, Università di Firenze via Nello Carrara 1, I-50019 Sesto Fiorentino (FI), Italy
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We report on the experimental characterization of energetic and dynamical instability, two mechanisms responsible for the breakdown of Bloch waves in a Bose-Einstein condensate interacting with a one-dimensional (1D) optical lattice. A clear separation of these two regimes is obtained by performing measurements at different temperatures of the atomic sample. The time scales of the two processes have been determined by measuring the losses induced in the condensate. A simple phenomenological model is introduced for energetic instability while a full comparison is made between the experiment and the 3D Gross-Pitaevskii theory that accounts for dynamical instability.

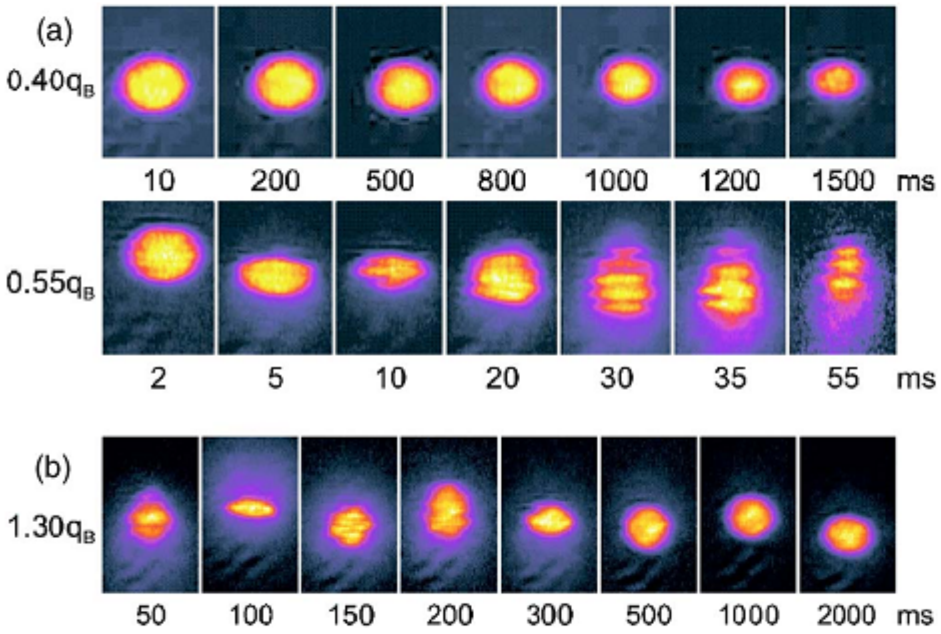

FIG. 9. (Color online) (a) Absorption images of the expanded condensate after different interaction times with a lattice with $s=1.15$ for two different values of quasimomentum. Note the sudden change of time scale crossing the threshold of dynamical instability at $q=0.525q_B$ and the appearance of structures in the density profiles for the unstable case ($q=0.55q_B$). (b) Reabsorption of excitations following 5 ms of interaction with the lattice and different times of evolution in the pure harmonic potential after switching off the lattice. In all these pictures the lattice moves from top to bottom.

Obervation of localized structures

Formation of a Matter-Wave Bright Soliton

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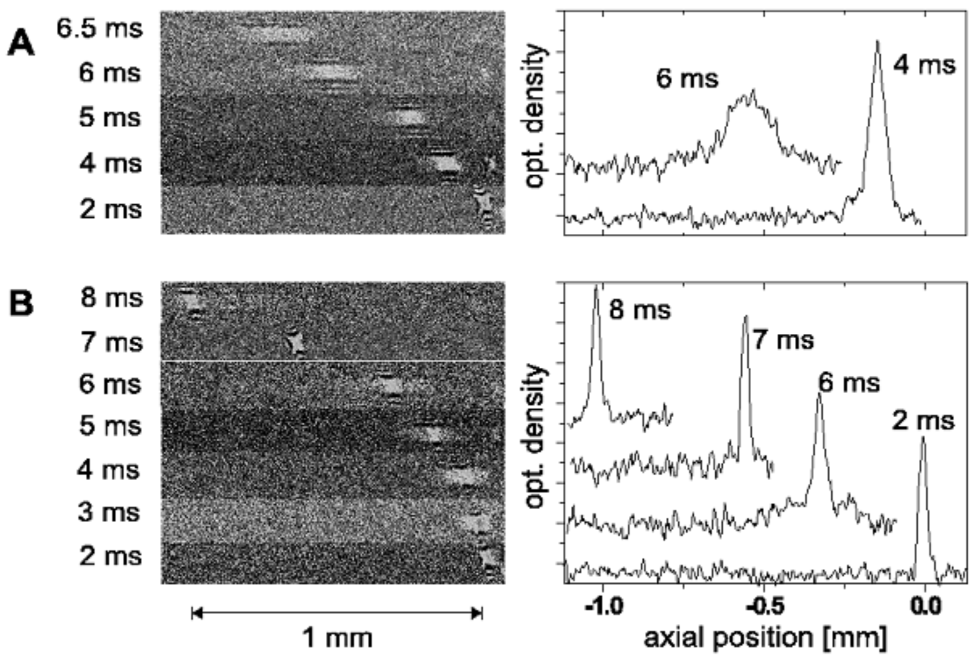

Abstract

We report the production of matter-wave solitons in an ultra-cold ^{7}Li gas. The effective interaction between atoms in a Bose-Einstein condensate is tuned with a Feshbach resonance from repulsive to attractive before release in a one-dimensional optical waveguide. Propagation of the soliton without dispersion over a macroscopic distance of 1.1 mm is observed. A simple theoretical model explains the stability region of the soliton. These matter-wave solitons open fascinating possibilities for future applications in coherent atom optics, atom interferometry and atom transport.

Figure 3: Absorption images at variable delays after switching off the vertical trapping beam. Propagation of an ideal BEC gas (A) and of a soliton (B) in the horizontal 1D waveguide in presence of an expulsive potential. Propagation without dispersion over 1.1 mm is a clear signature of a soliton. Corresponding axial profiles integrated over the vertical direction.