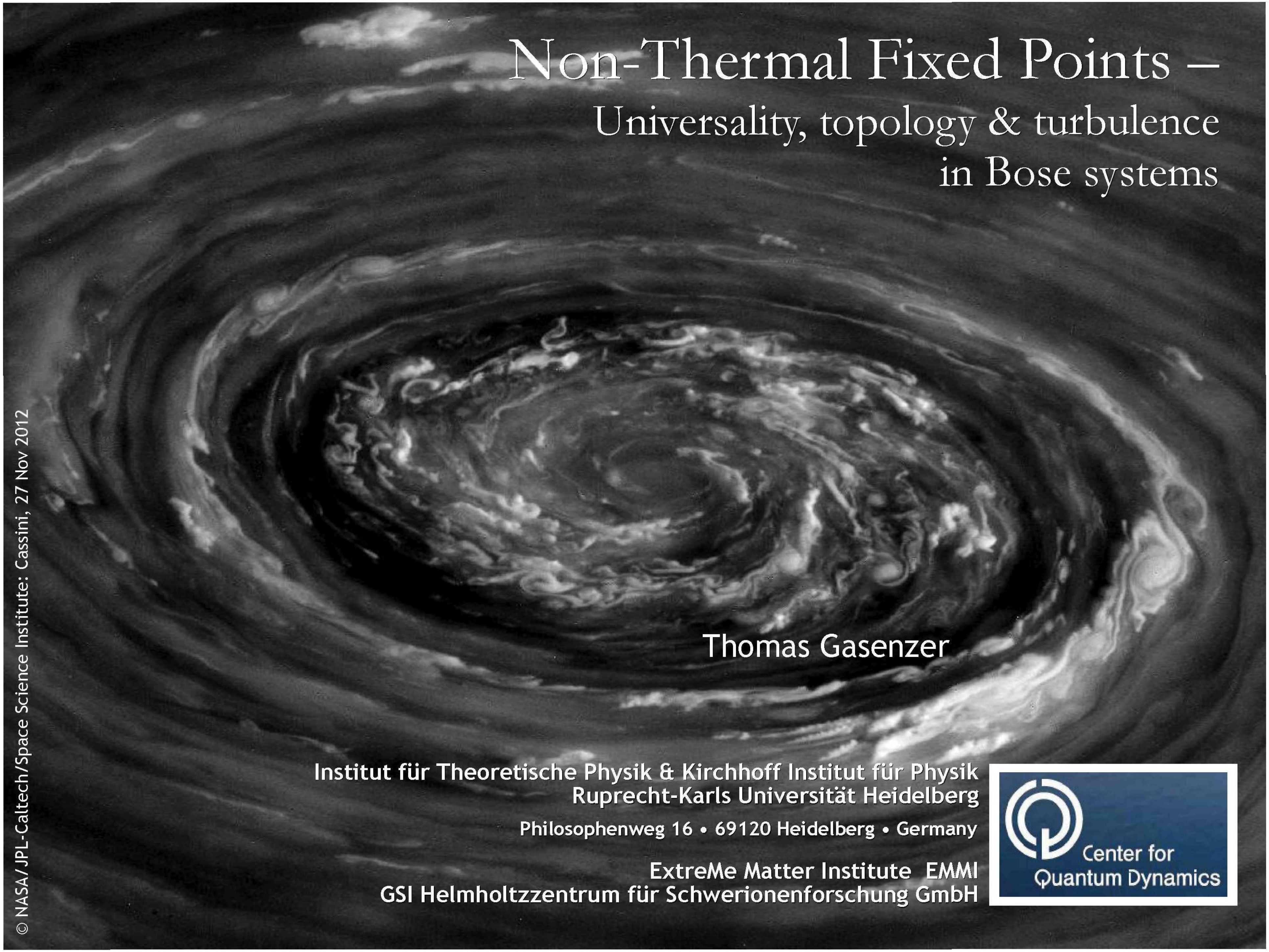

Non-Thermal Fixed Points –
Universality, topology & turbulence in Bose systems

Thomas Gasenzer

Institut für Theoretische Physik & Kirchhoff Institut für Physik
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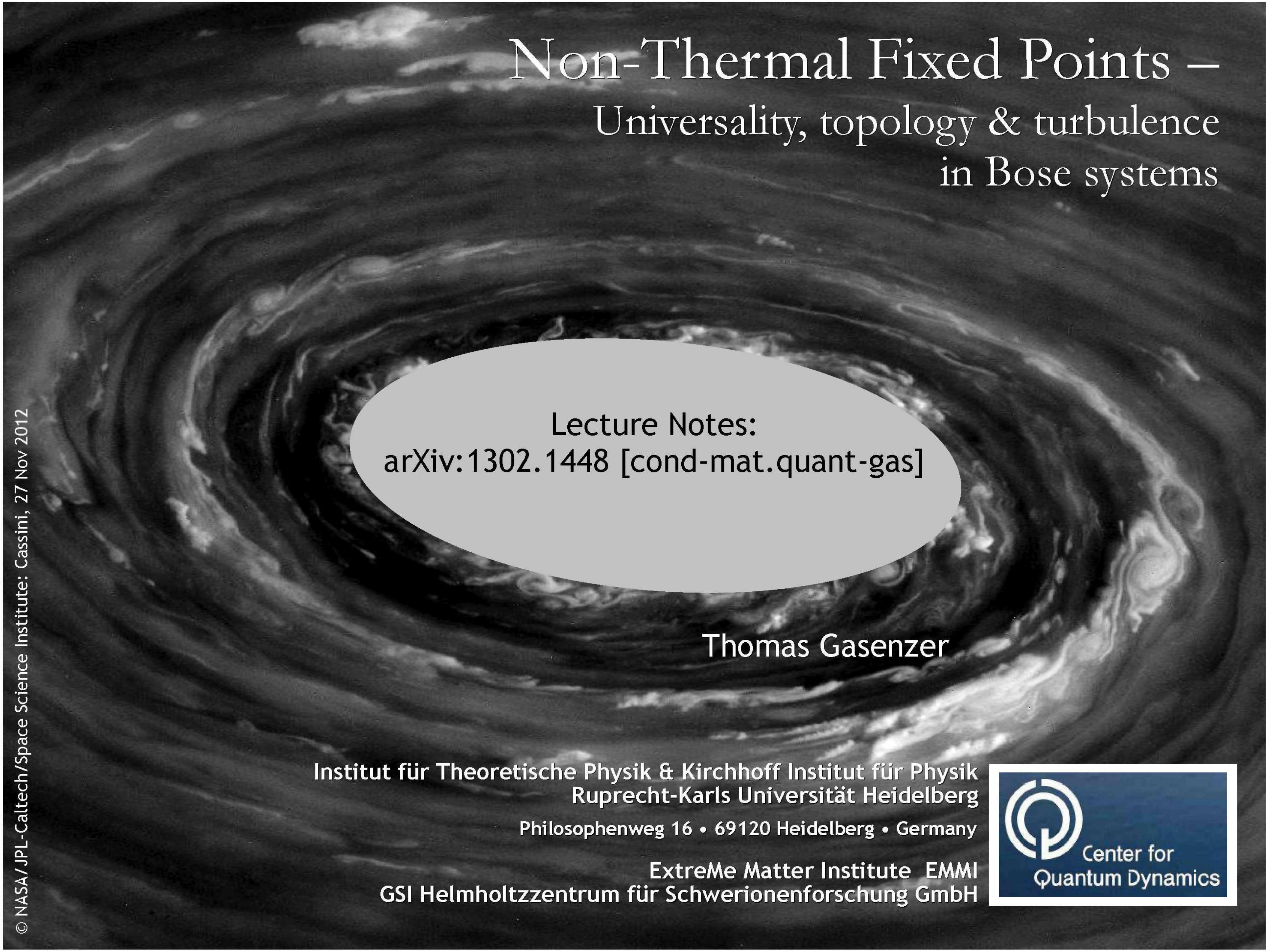
ExtreMe Matter Institute EMMI
GSI Helmholtzzentrum für Schwerionenforschung GmbH

Center for Quantum Dynamics



Non-Thermal Fixed Points –

Universality, topology & turbulence in Bose systems

Lecture Notes:
arXiv:1302.1448 [cond-mat.quant-gas]

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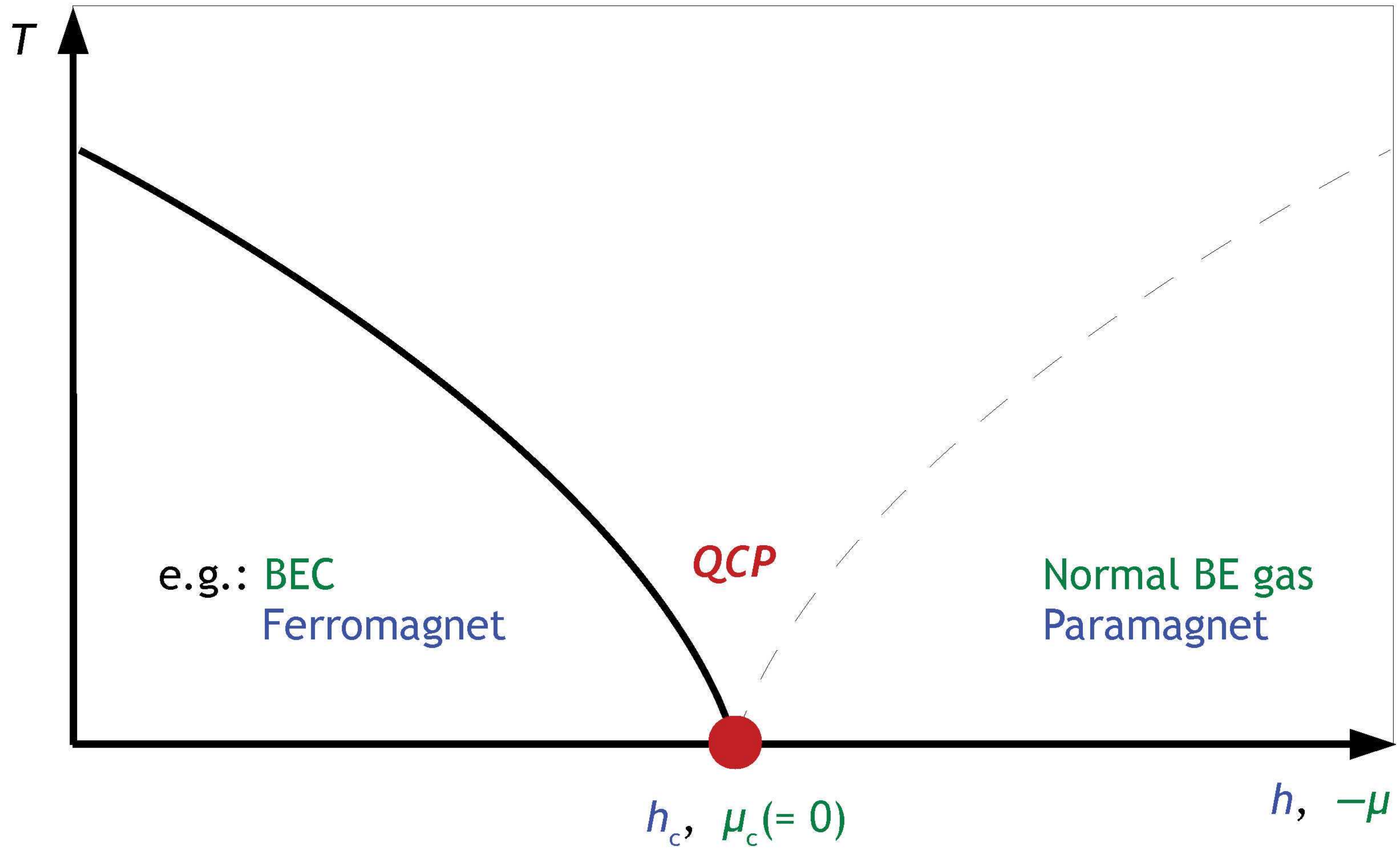
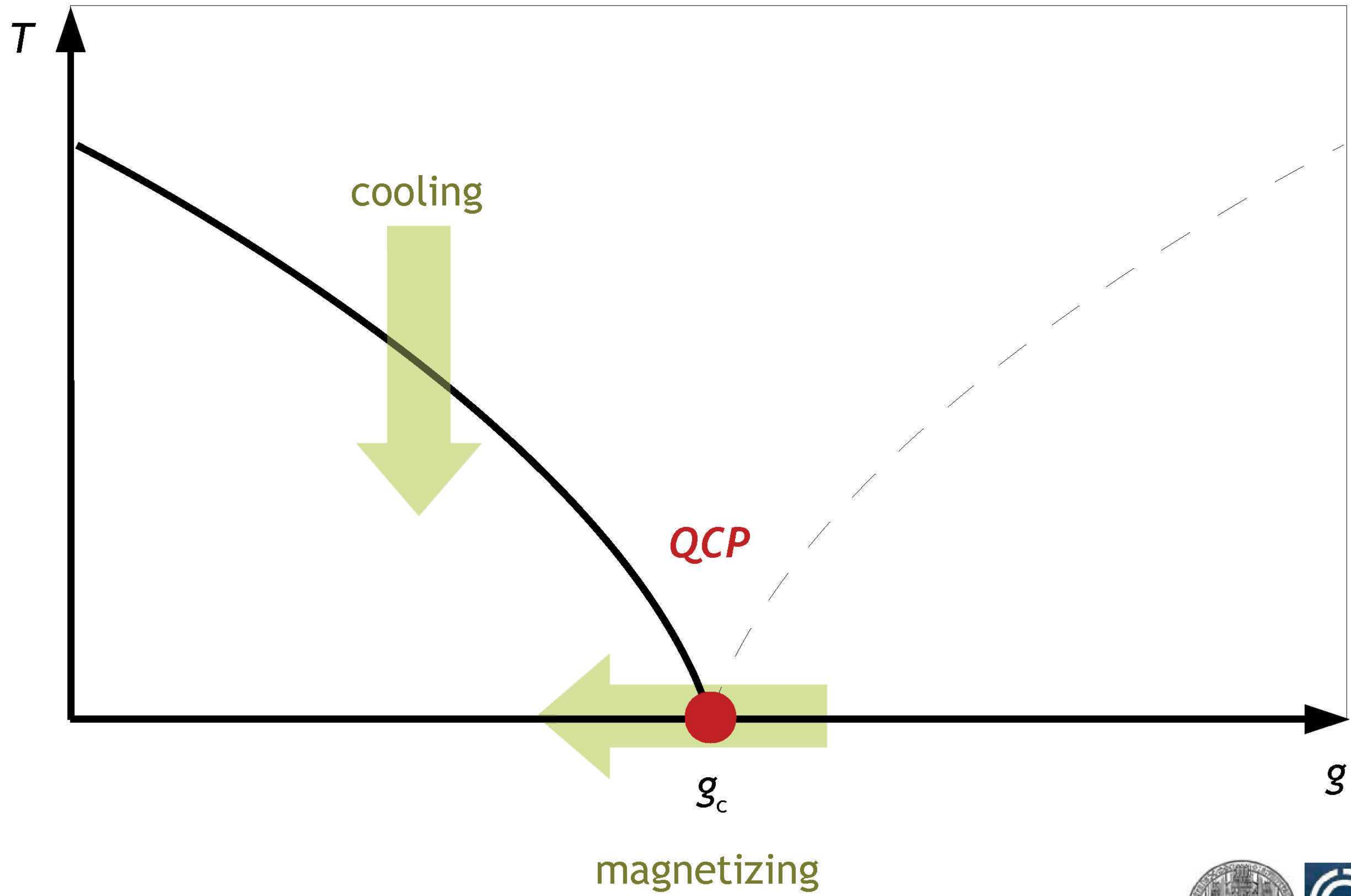
Quantum Phase Transition



Quench across (Quantum) Phase Transition



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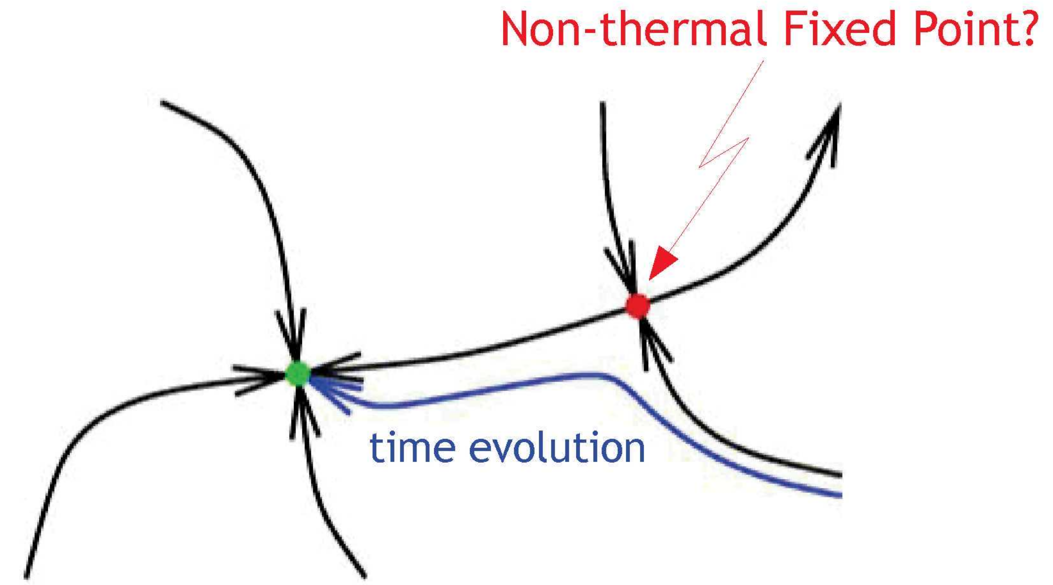
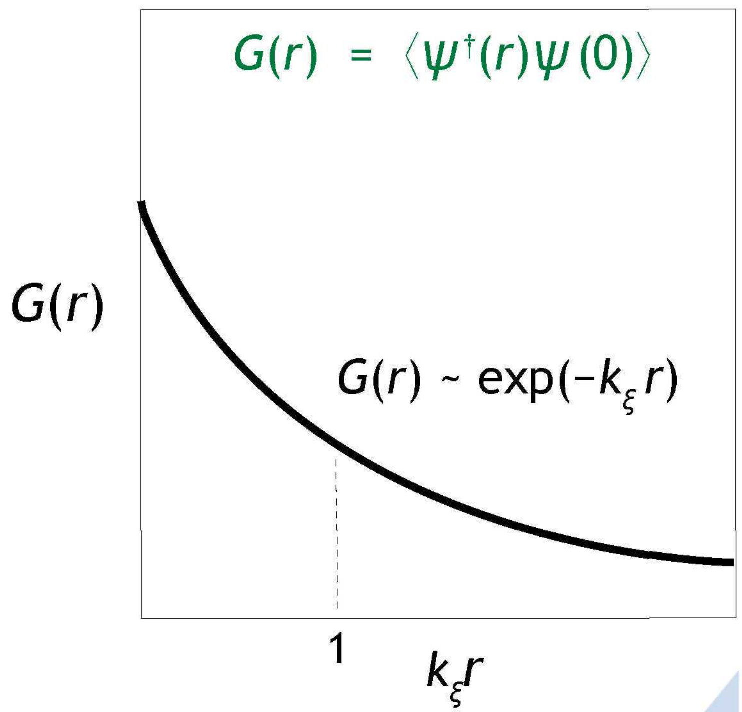
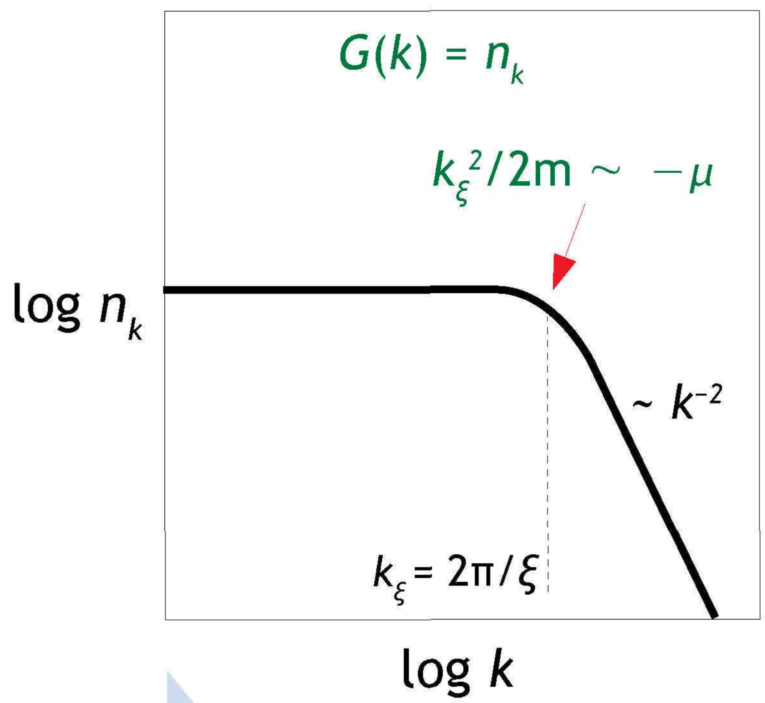
Universal dynamics



Correlation functions

e.g.: Bose gas above condensation temperature





Fourier transform

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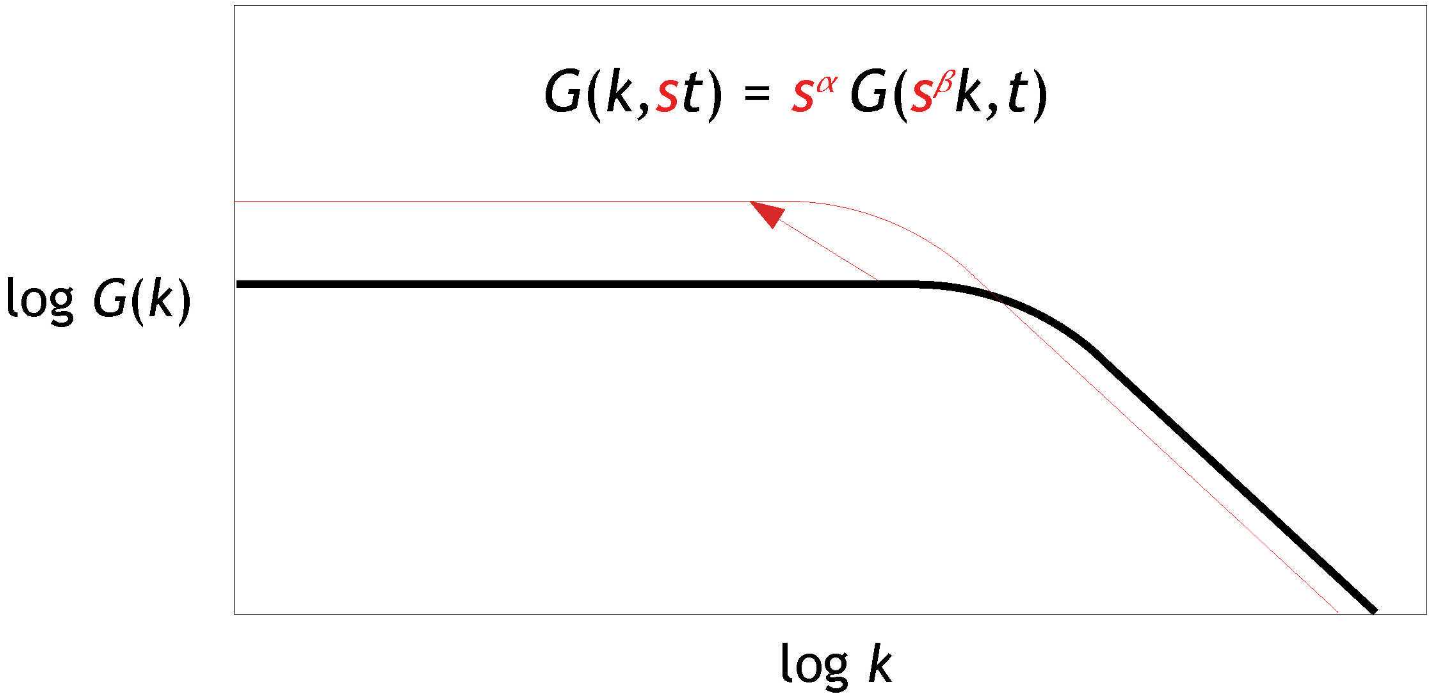
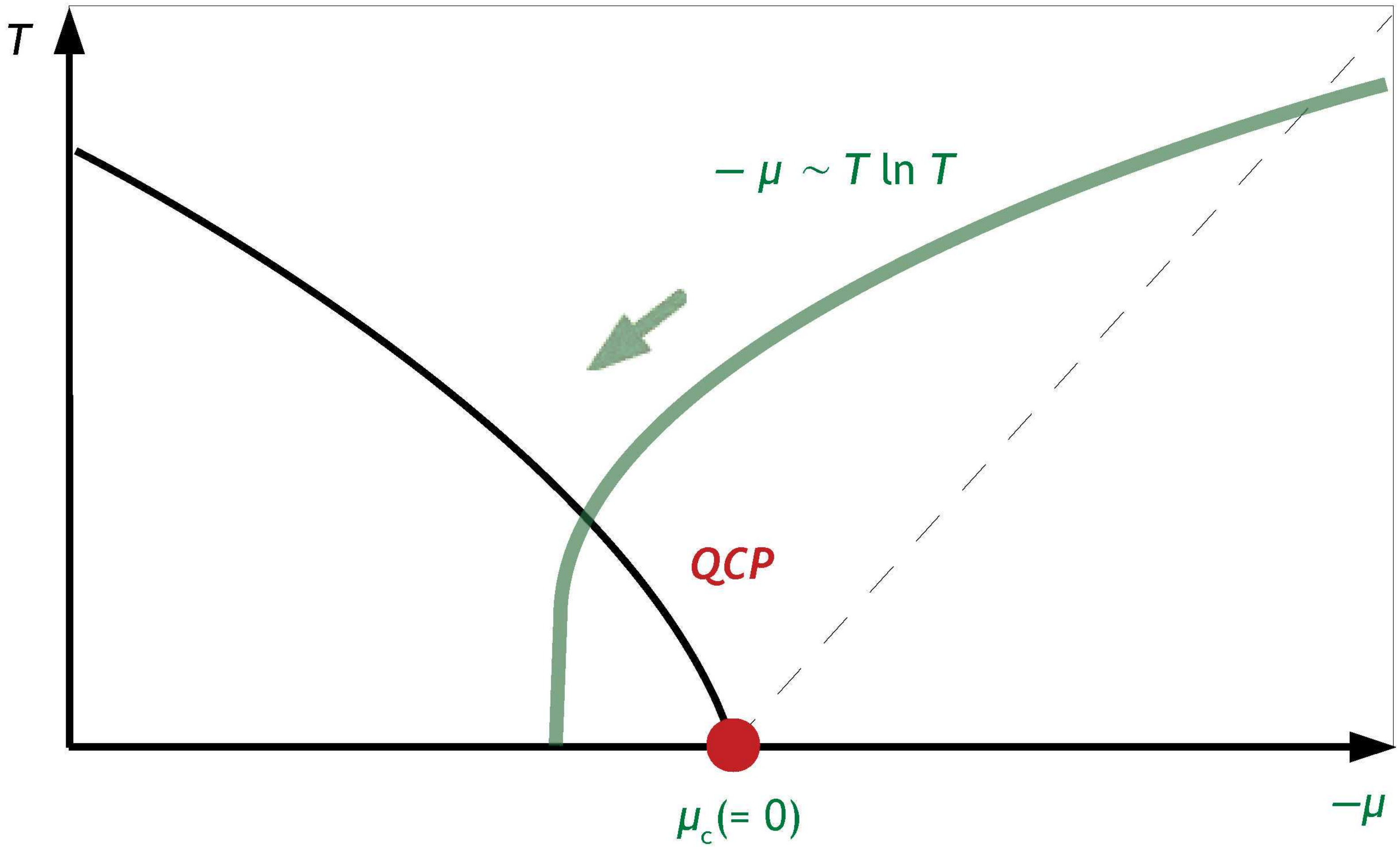
Time evolution ≙ Scaling transformation



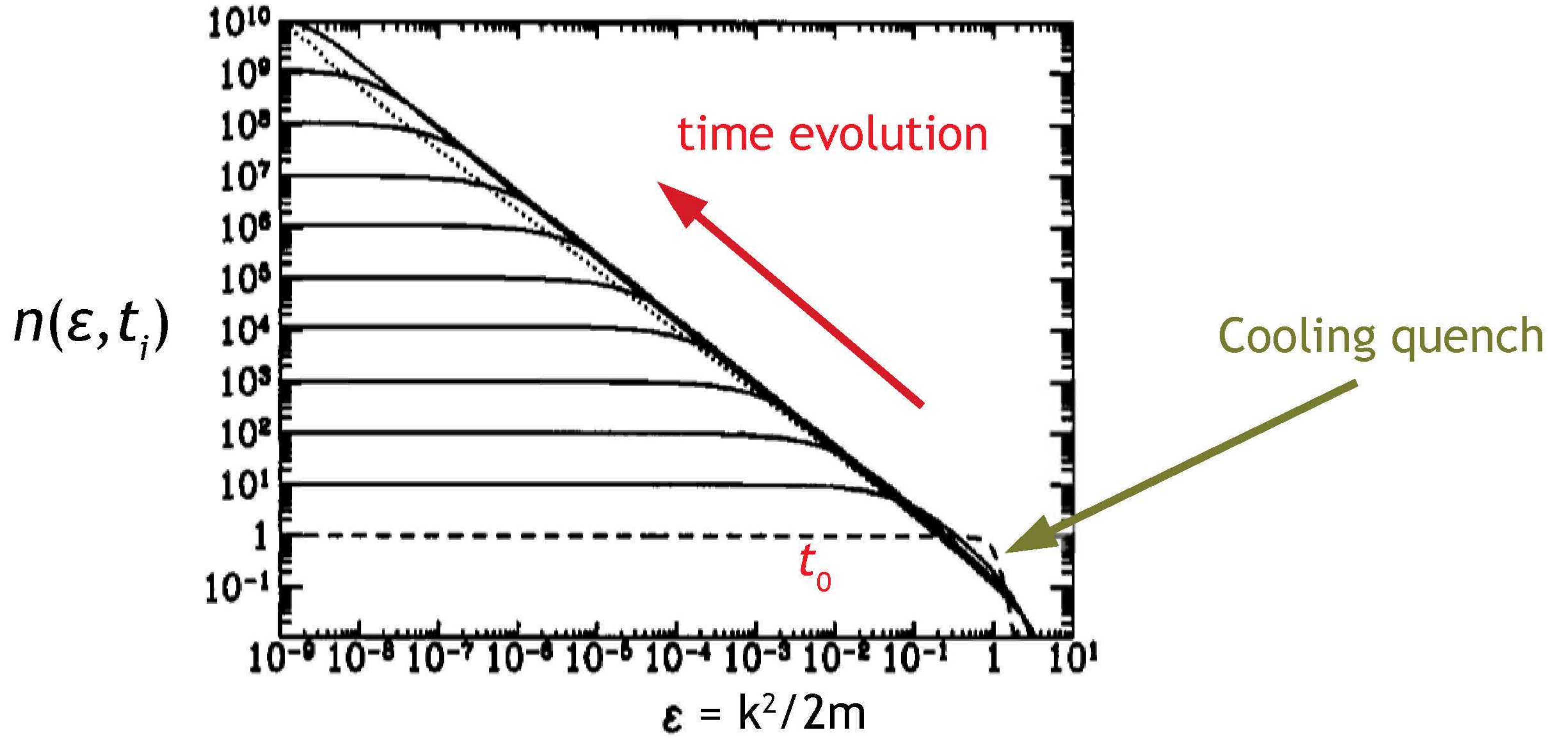
scaling form: $G(k,t) = (Qt)^{\alpha}\, G_s(\, k(Qt)^{\beta}\,)$

Bose condensation: equilibrium



Cooling Quench in a Bose Gas

Semikoz & Tkachev, PRD **55** (1995)
also: Svistunov (1991-)



Scaling form: $n(\varepsilon, t) = (Qt)^{\alpha}\ f_s(\varepsilon[Qt]^{\beta})$

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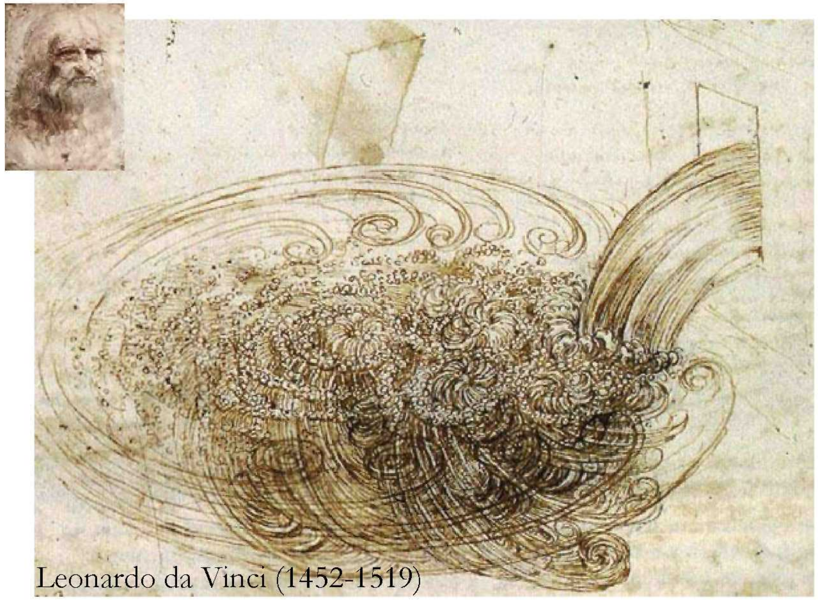
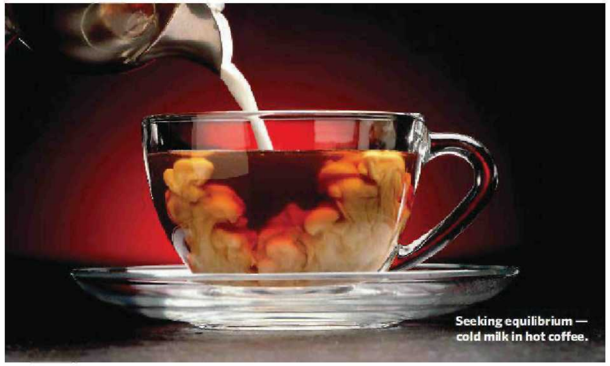
Classical Turbulence

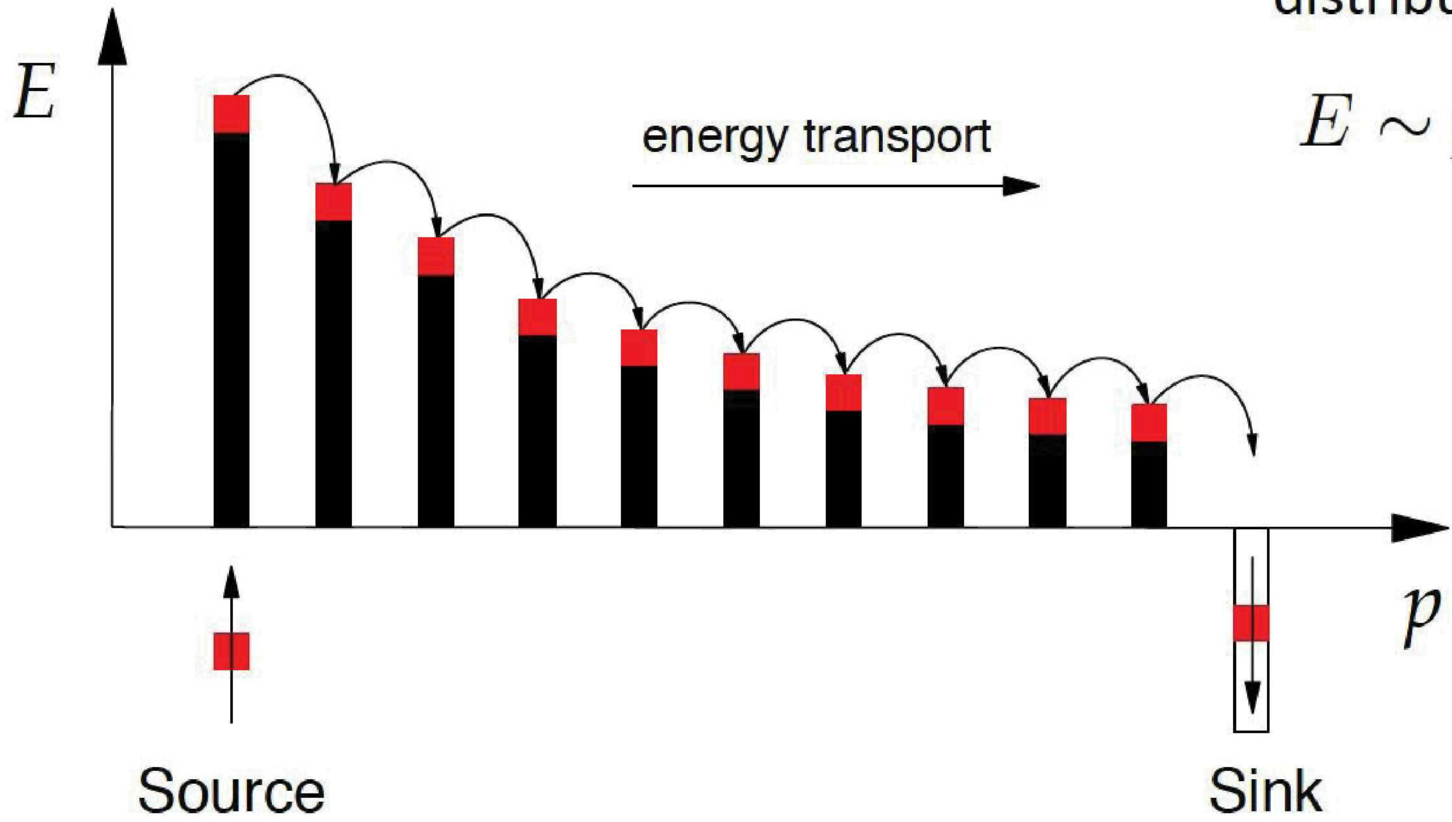


Leonardo da Vinci (1452-1519)

Kolmogorov (1941) $E(k) \sim k^{-5/3}$ (for incompressible fluids)



Turbulent Cascade



Stationary distribution

$$E \sim p^{-\kappa}$$

[Sketch by Asier Pineiro]

Transport in momentum space

Radial transport equation: local conservation

$$\partial_t n(k) = -\partial_k Q(k)$$

Stationary distribution $n(k,t) \equiv n(k)$ if $Q(k) \equiv Q$

Transport in momentum space

Radial transport equation (Boltzmann):

$$\partial_t n(k) = -\partial_k Q(k) \sim k^{d-1} J(k)$$

$$= k^{d-1} \, d\Omega_k \int d^d p \, d^d q \, d^d r \, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r}) \delta(\omega_\mathbf{k}+\omega_\mathbf{p}-\omega_\mathbf{q}-\omega_\mathbf{r})$$

coupling *mom. conservation* *energy conservation*

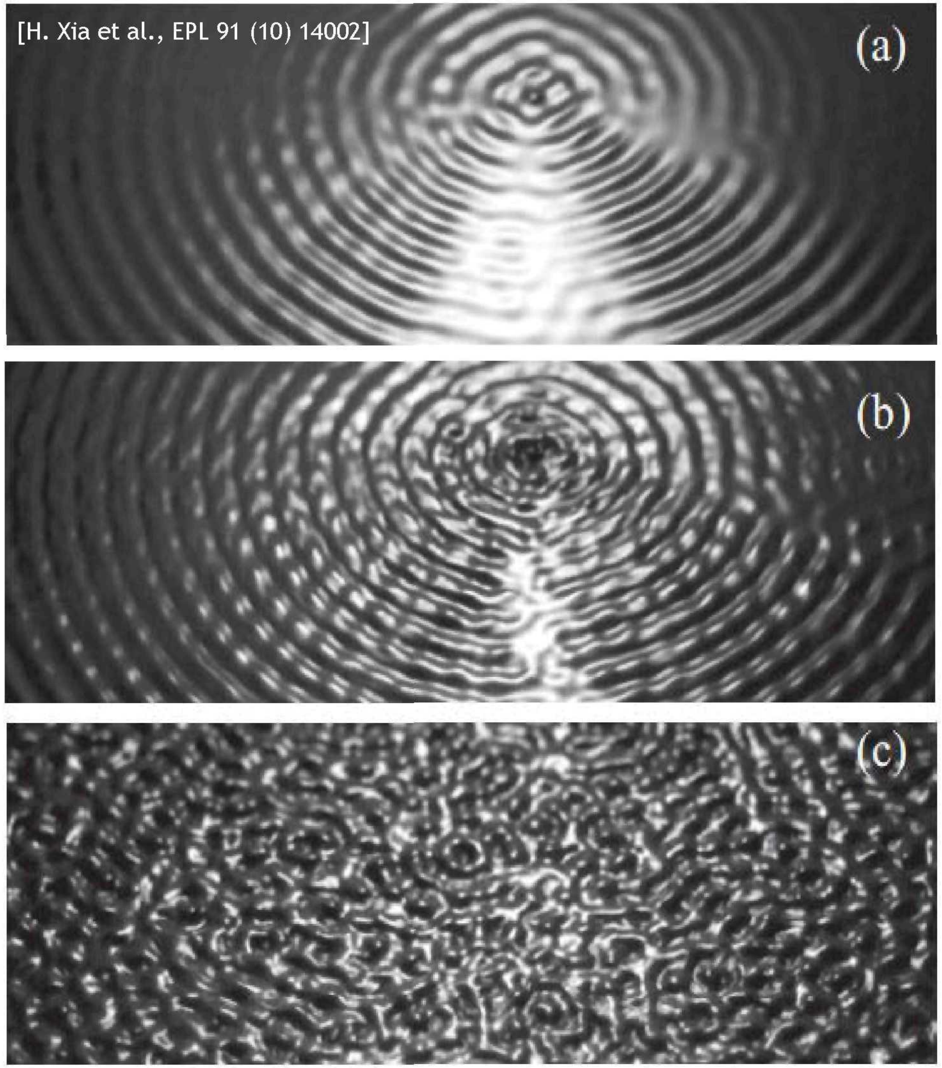
$$\times [(n_\mathbf{k}+1)(n_\mathbf{p}+1)n_\mathbf{q} n_\mathbf{r} - n_\mathbf{k} n_\mathbf{p}(n_\mathbf{q}+1)(n_\mathbf{r}+1)]$$

in-scattering rate *out-scattering rate*

Stationary distribution $n(k,t) \equiv n(k)$ if $Q(k) \equiv Q$

This requires a particular scaling of $n(k) \sim k^{-\zeta}$

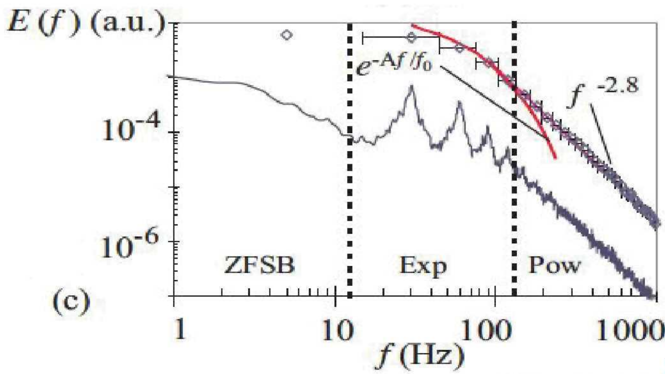
Wave Turbulence – e.g. on water



(kinetic) WT Theory prediction:

$$E_\omega \sim \omega^{-17/6}.$$

[Zakharov & Filonenko (67)]



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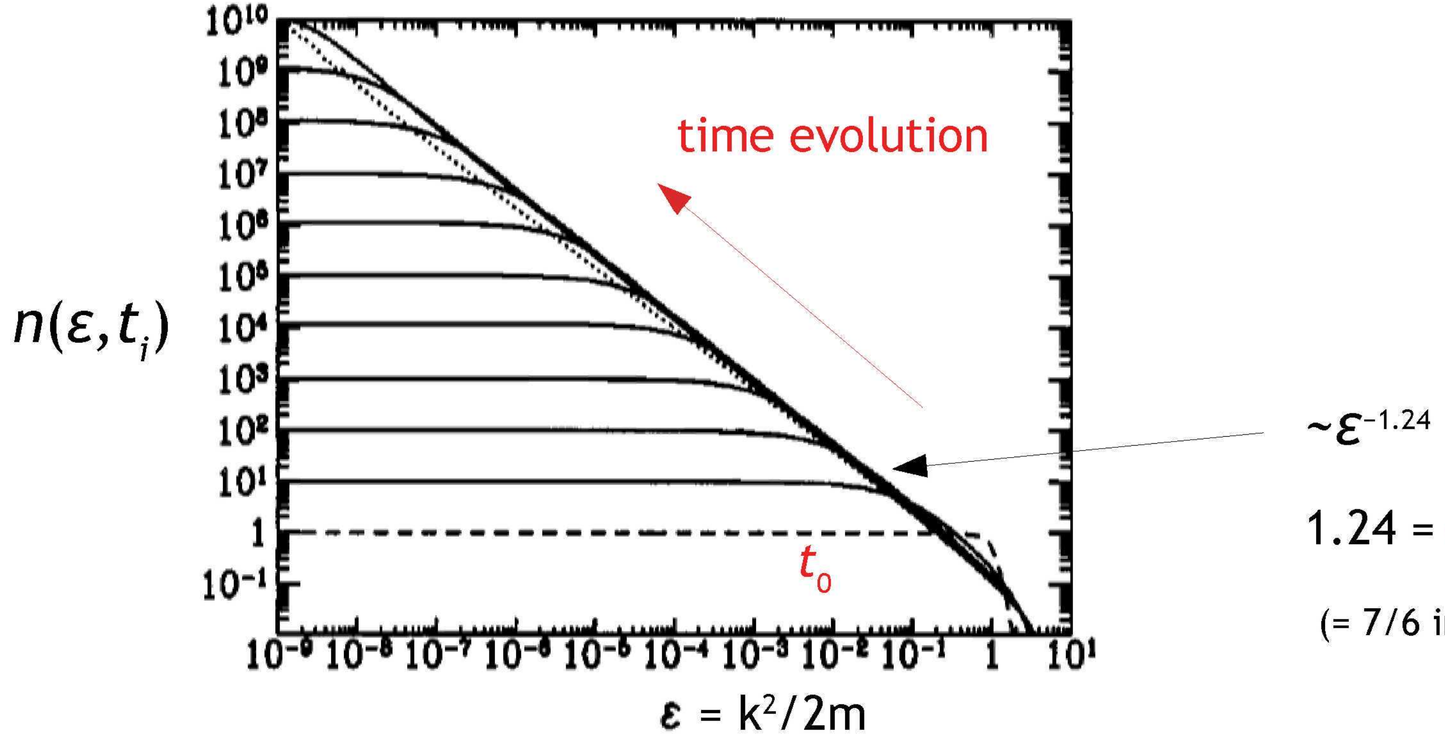
Cooling Quench in a Bose Gas

Semikoz & Tkachev, PRD **55** (1995)
also: Svistunov (1991-)



$1.24 = \alpha/\beta$

(= 7/6 in WWT)

Scaling form: $n(\varepsilon, t) = (Qt)^{\alpha} \ f_s(\varepsilon[Qt]^{\beta})$

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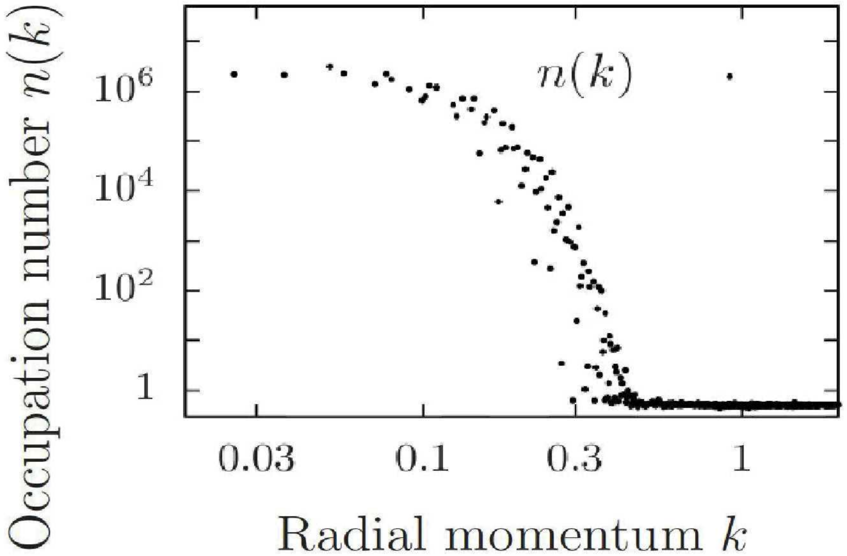
2D Bose gas: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)



"Initial" distribution

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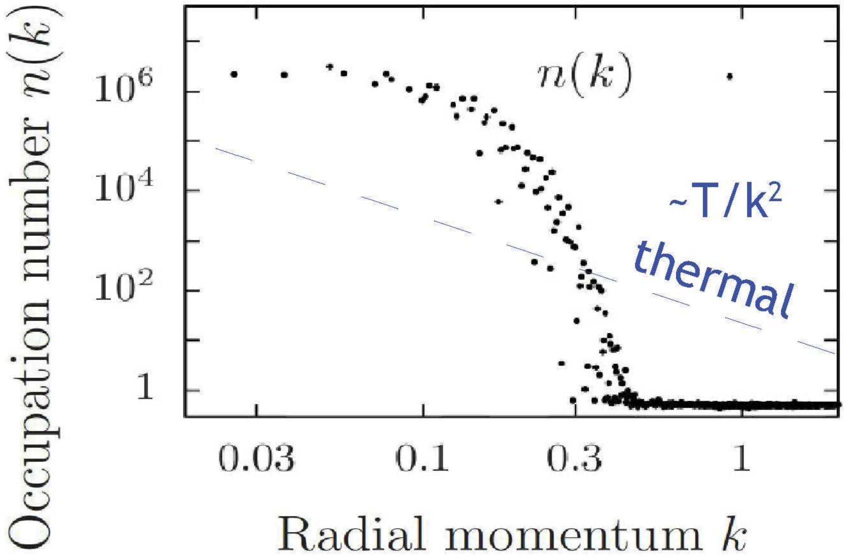
2D Bose gas: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)



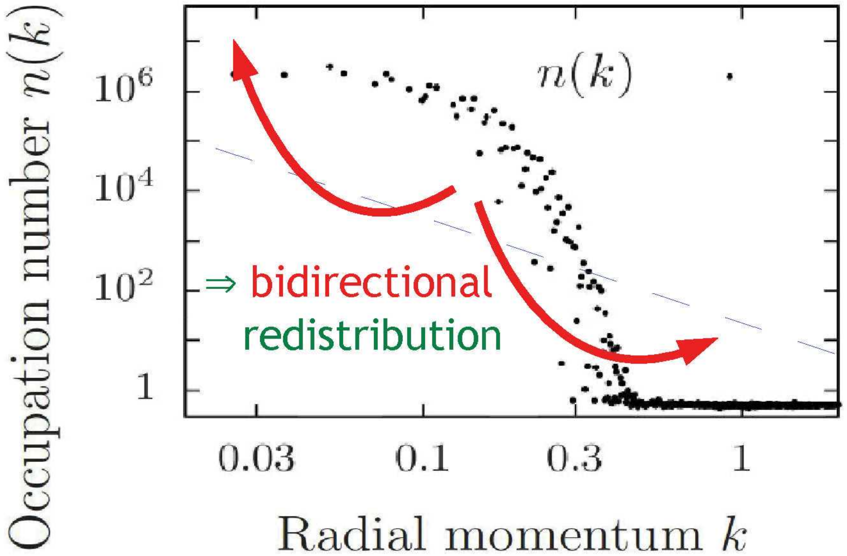
"Final" distribution

$$\frac{1}{e^{\omega/T}-1} \approx \frac{T}{\omega}$$

(Rayleigh-Jeans regime of BE distribution)

2D Bose gas: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)



$$\frac{1}{e^{\omega/T}-1} \approx \frac{T}{\omega}$$

2D Bose gas: Quench dynamics

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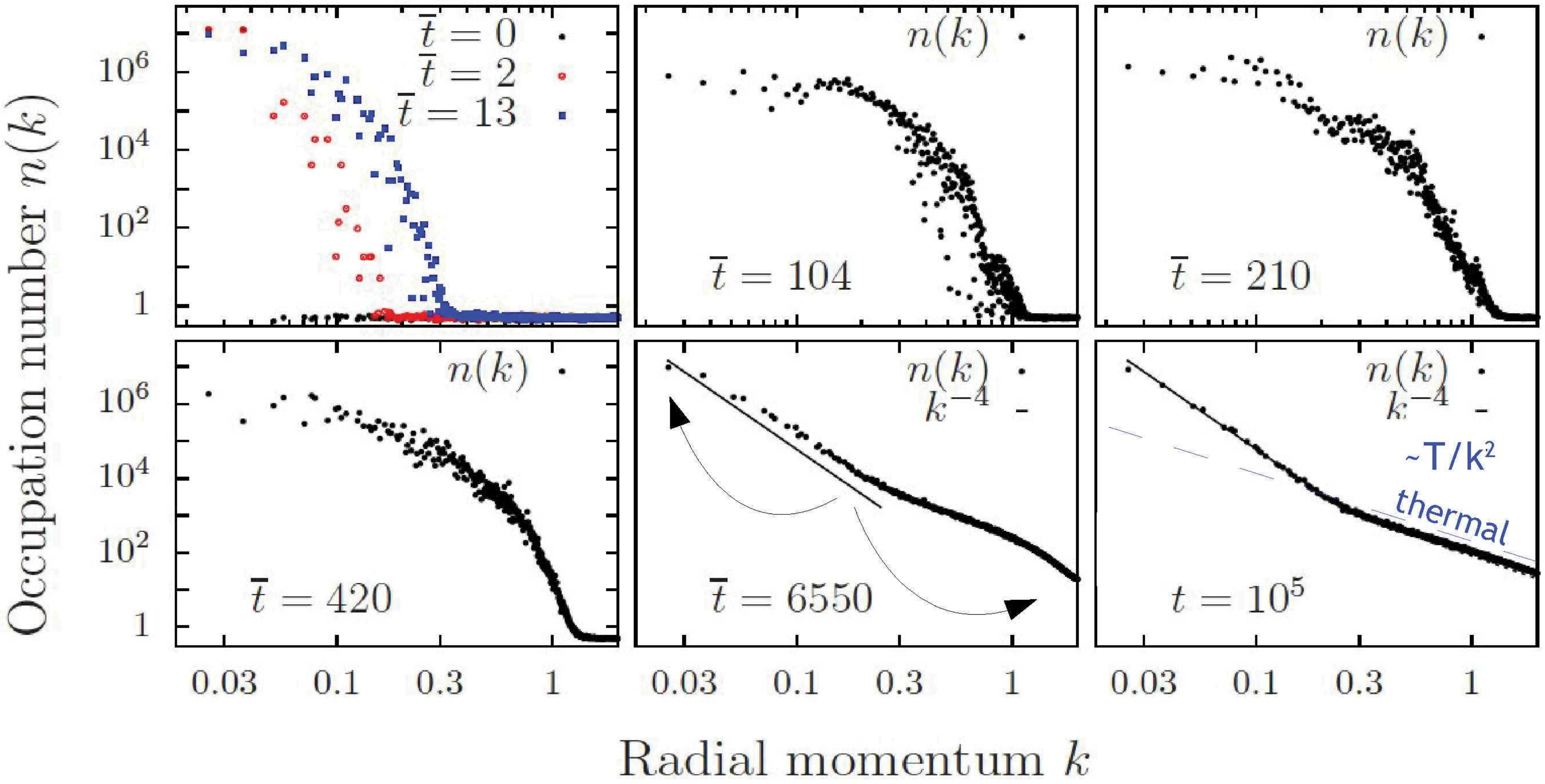

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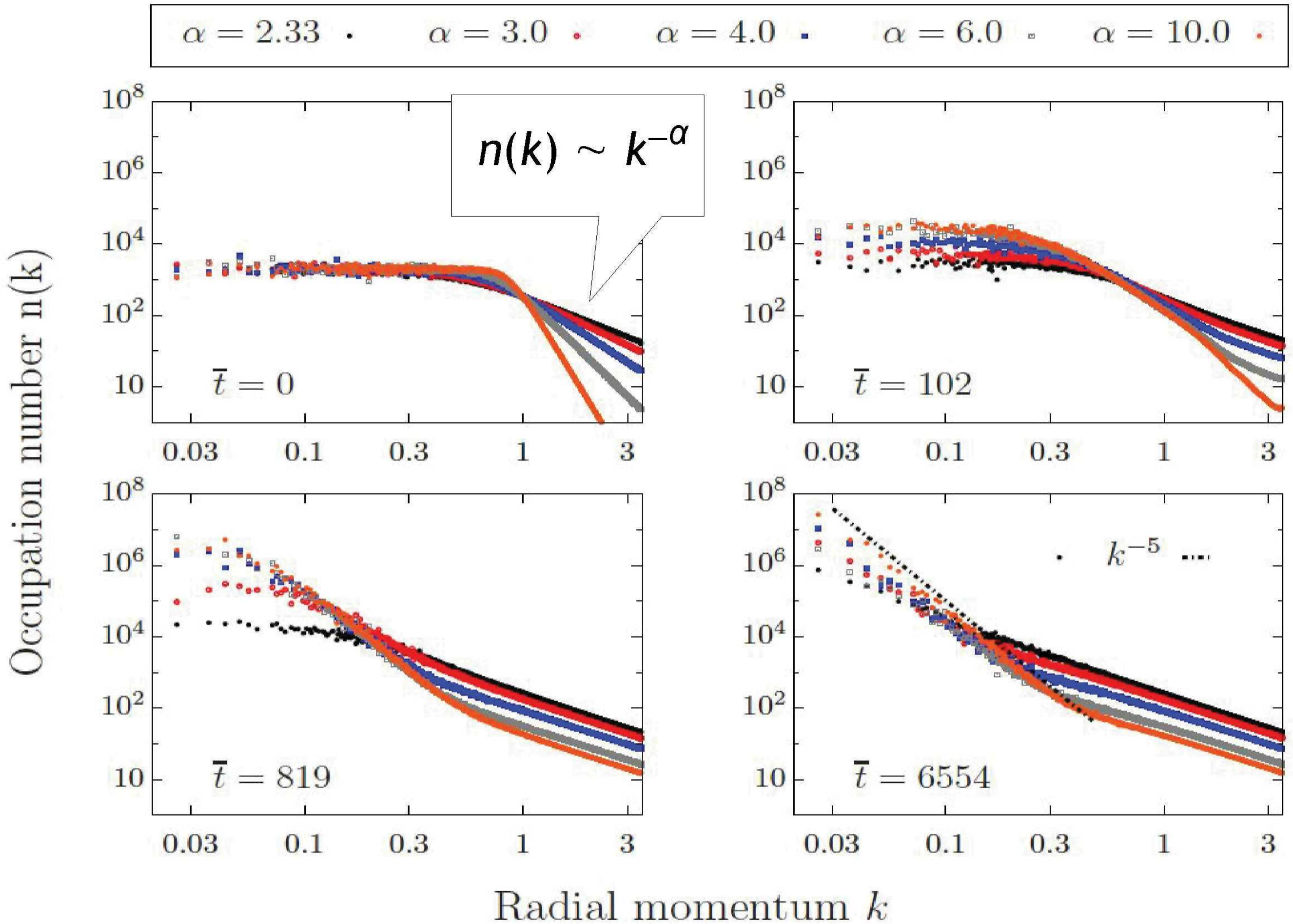
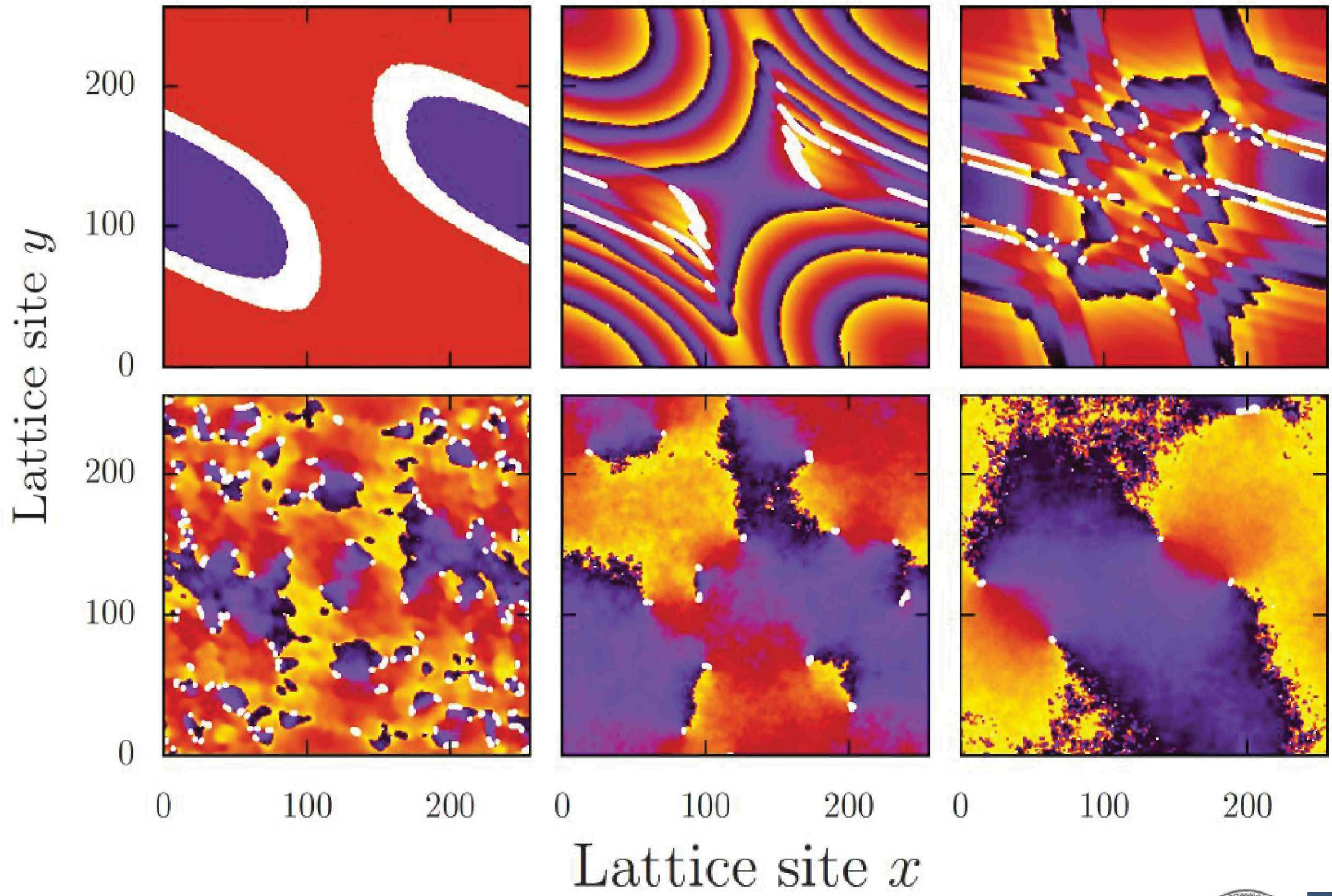
Scaling depends on quench



[B. Nowak, J. Schole, and TG, arXiv:1206.3181;
Scaling evolution: recent work by A. Pineiro & J. Berges, unpublished]

2D Bose gas: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)



with I. Chantesana, A. Pineiro & J. Berges

Transport in momentum space

Quantum Boltzmann breaks down for large ζ in $n \sim k^{-\zeta}$

$$\partial_t n(k) = -\partial_k Q(k) \sim k^{d-1} J(k)$$

$$= k^{d-1} d\Omega_k \int \mathrm{d}^d p\, \mathrm{d}^d q\, \mathrm{d}^d r\, |T_{\mathbf{kpqr}}|^2 \delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r})\,\delta(\omega_{\mathbf{k}}+\omega_{\mathbf{p}}-\omega_{\mathbf{q}}-\omega_{\mathbf{r}})$$

coupling *mom. conservation* *energy conservation*

$$\times \left[(n_{\mathbf{k}}+n_{\mathbf{p}})n_{\mathbf{q}}n_{\mathbf{r}} - n_{\mathbf{k}}n_{\mathbf{p}}(n_{\mathbf{q}}+n_{\mathbf{r}})\right]$$

in-scattering rate *out-scattering rate*

here: $T_{\mathrm{kpqr}} \equiv g = const.$

Cured by
effective many-body T-Matrix: $|T|^2 = g^2 \rightarrow |T_k^{MB}[n_k]|^2 \sim \frac{g^2}{1+C(gk^d n_k)^2}$

for $k \rightarrow 0$

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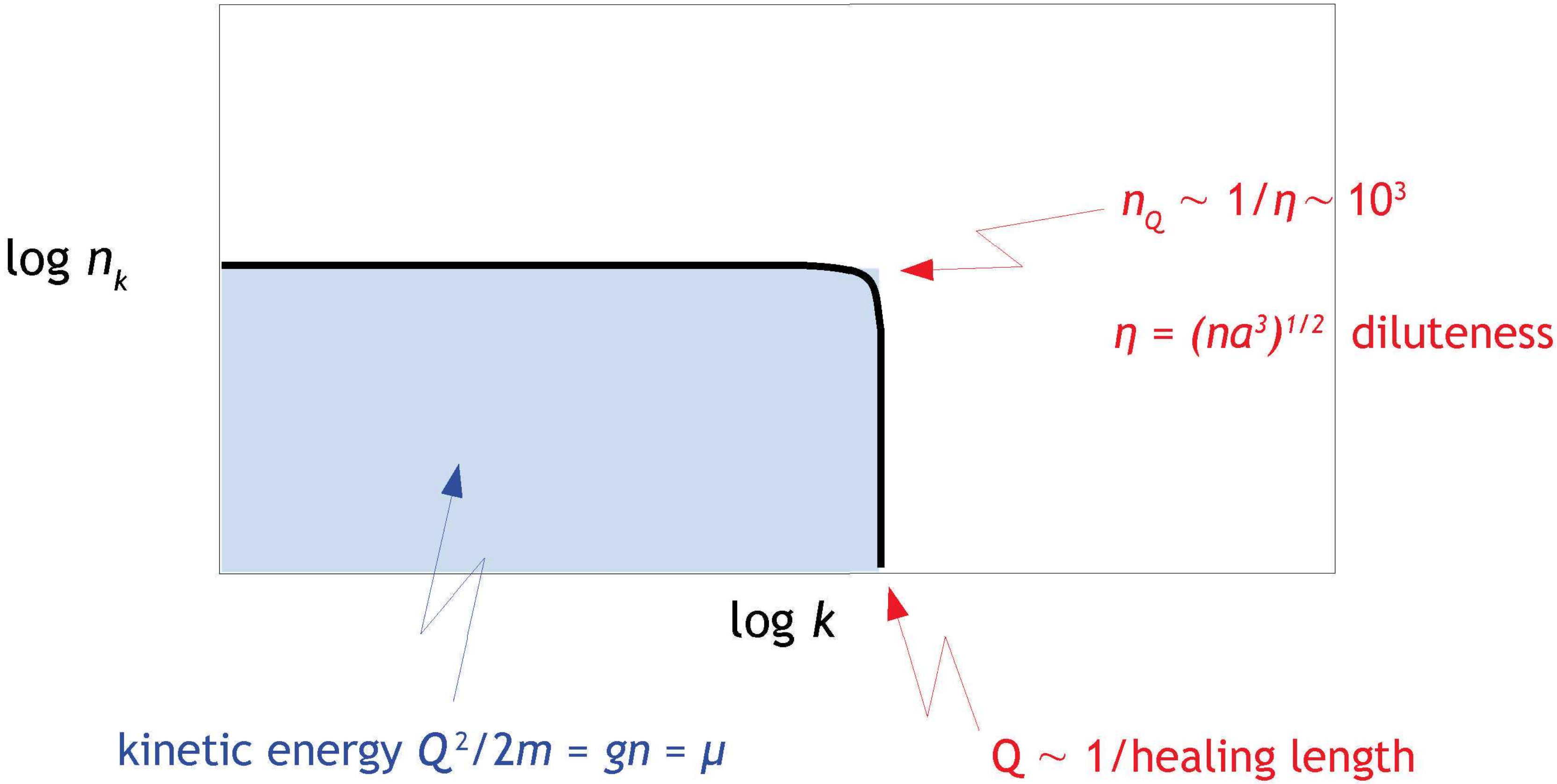
"Extreme" Initial Conditions

(Bose gas in d = 3)

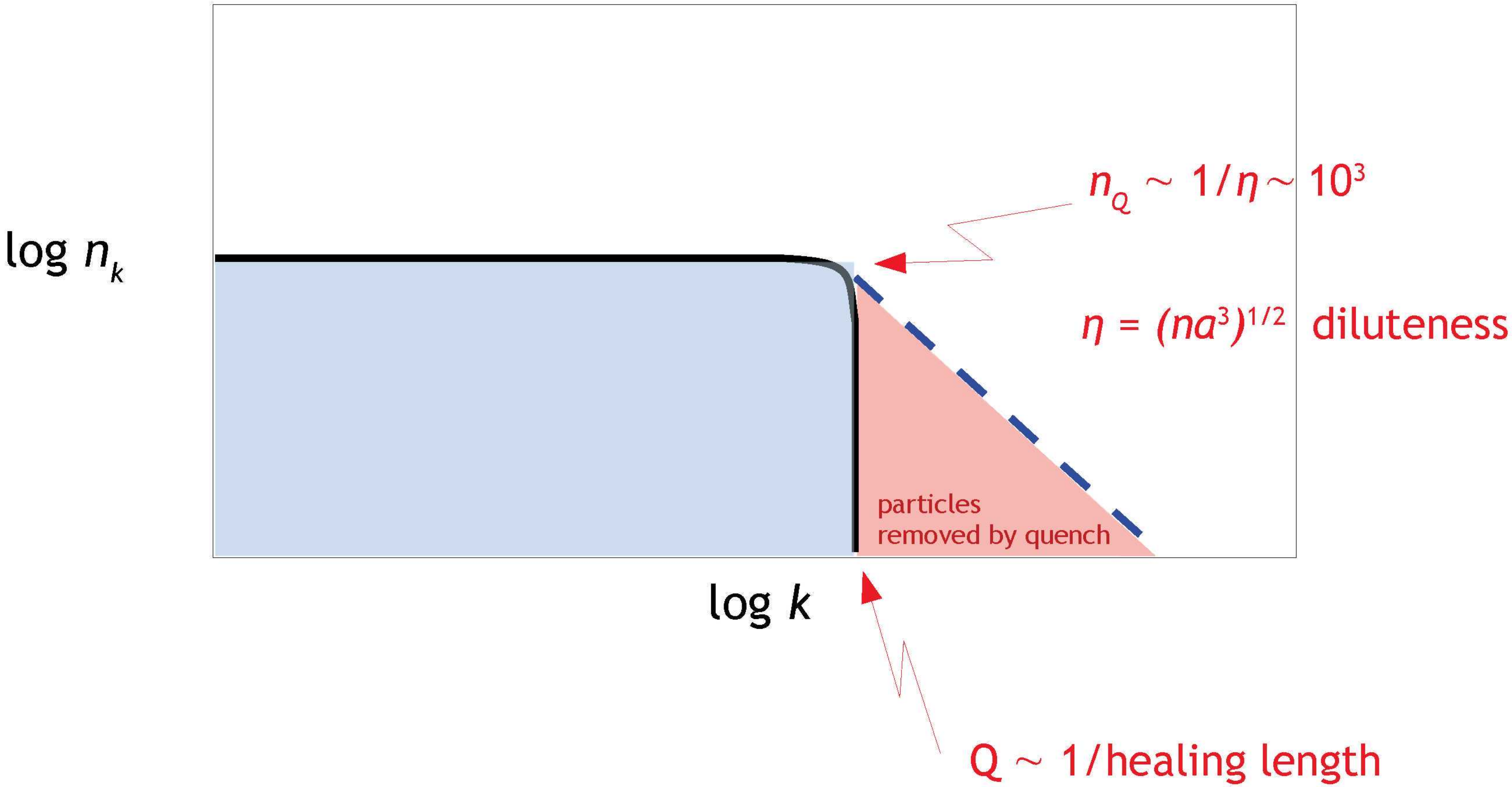
$$|T_k^{MB}[n_k]|^2 \sim \frac{g^2}{1+C(gk^3 n_k)^2}$$



[Discussion with J. Berges]

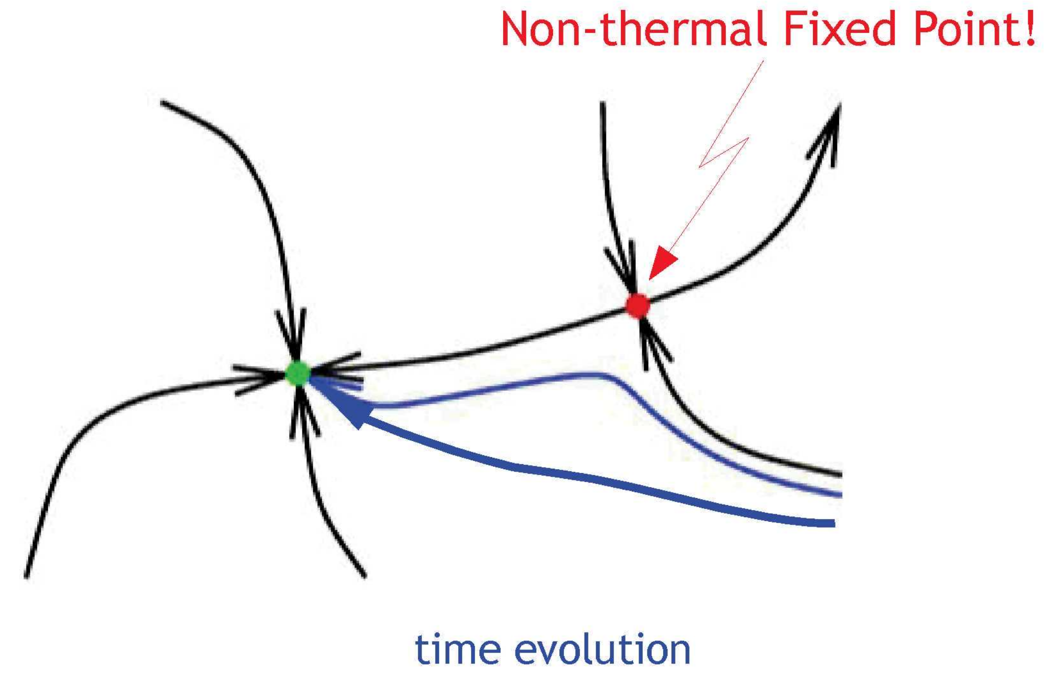
“Extreme” Initial Conditions

...obtained after cooling quench of system with $T/T_{crit} \sim \eta^{-1/3} \sim 10$



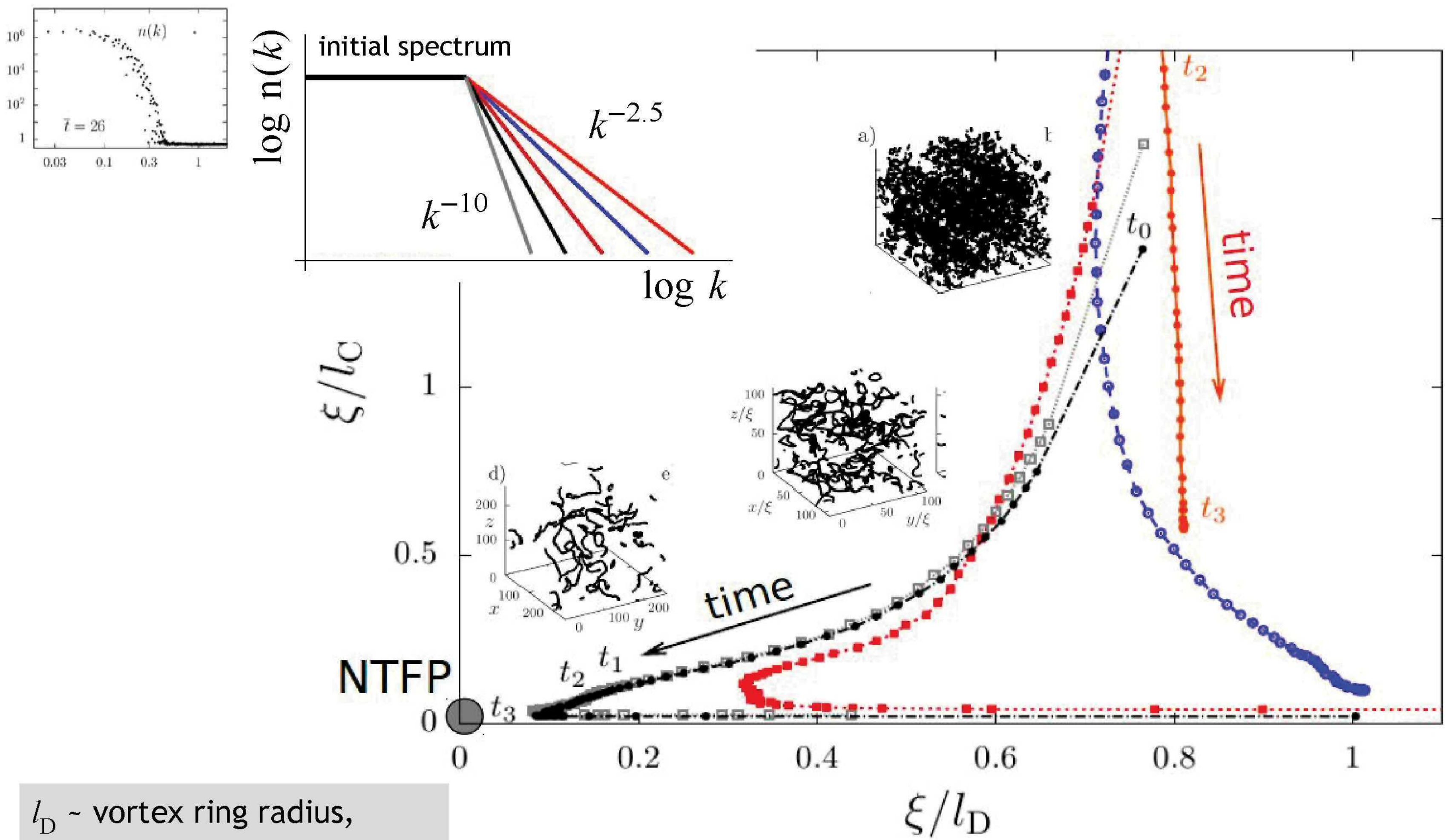
[Discussion with J. Berges]

Universal dynamics



Non-Thermal Fixed Point in 3D

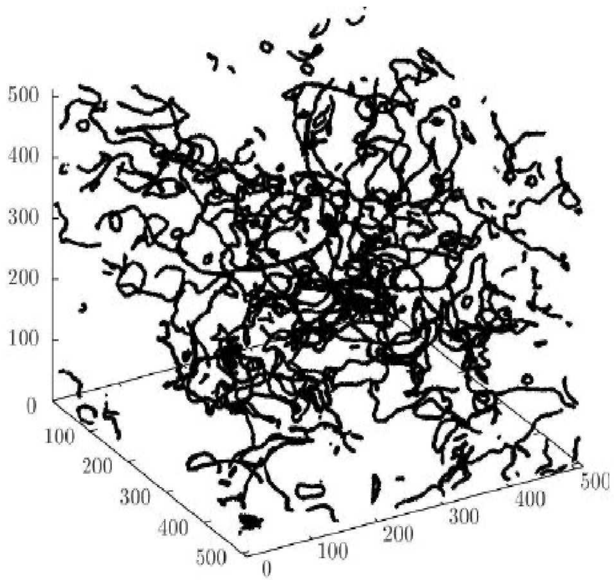
B. Nowak, J. Schole, TG,
arXiv:1206.3181v2
[cond-mat.quant-gas]

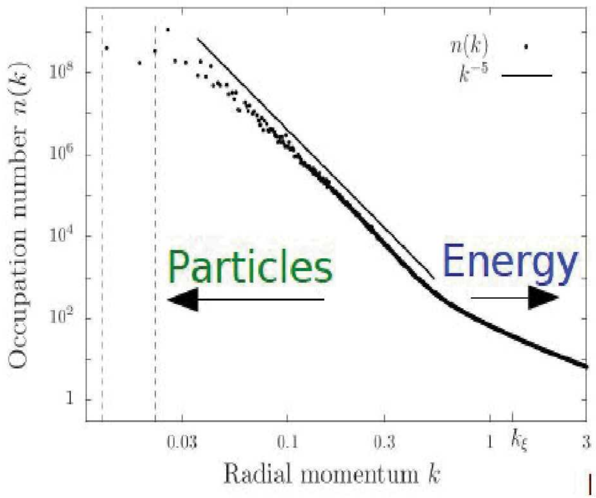


l_D ~ vortex ring radius,
l_C ~ phase coherence length

Dilution process: cf. also Kelvin-wave cascade Kozik & Svistunov (03-11)

Hydrodynamic = Strong Wave Turbulence



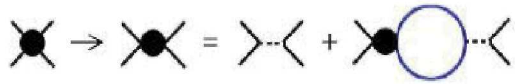


$n(k) \sim k^{-\zeta}$

strong scaling exponents

$\zeta_P = d + 2$

$\zeta_E = d + 2 + z$



Defects ← scaling correlations → QFT

Phase-ordering kinetics | Non-thermal Fixed Point

Pattern formation

Quantum Turbulence | Strong Wave Turbulence

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12);

J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603,
J. Berges, G. Hoffmeister, NPB **813** (09) 383,
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

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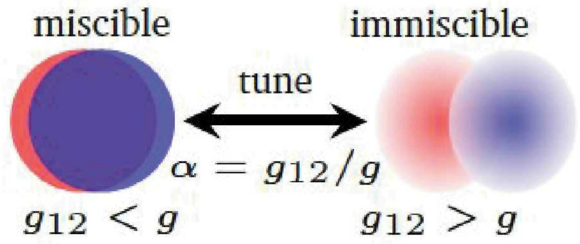
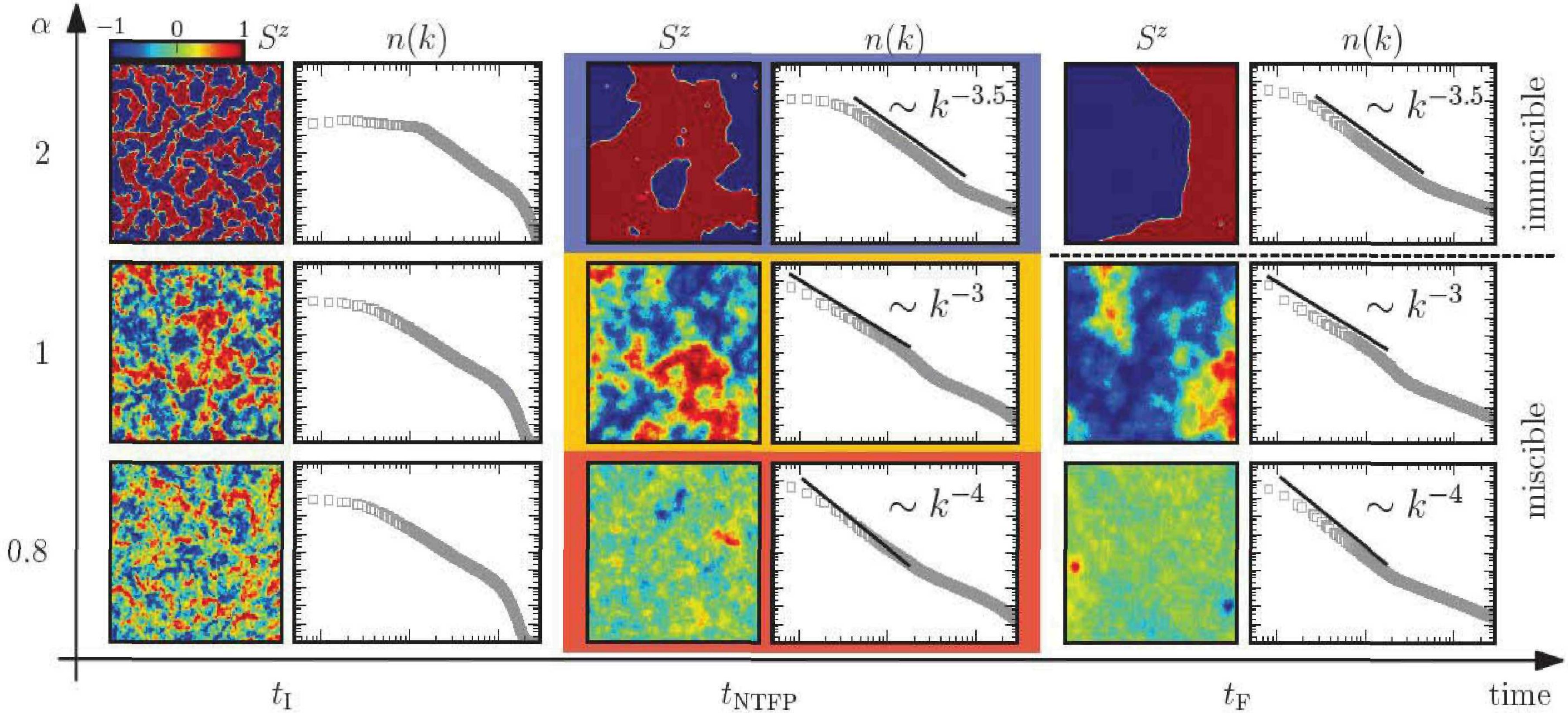
Coarsening dynamics
(2-component Bose gas)

M. Karl, B. Nowak, TG, Sci. Rep. **3**, 2394 (2013); Phys. Rev. A **88**, 063615 (2013)





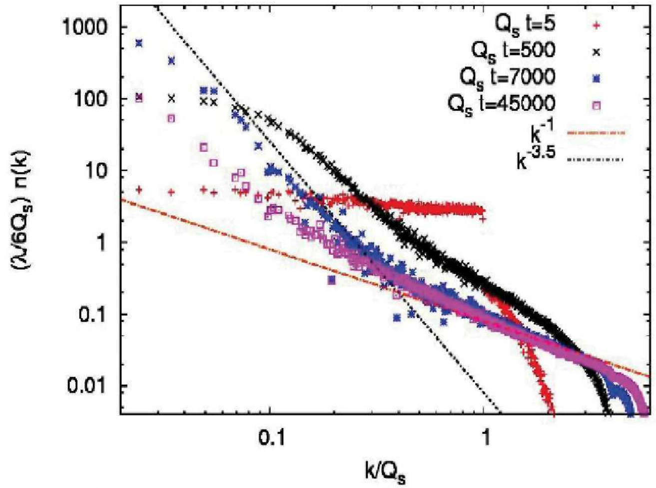
Simulations by **Markus Karl**

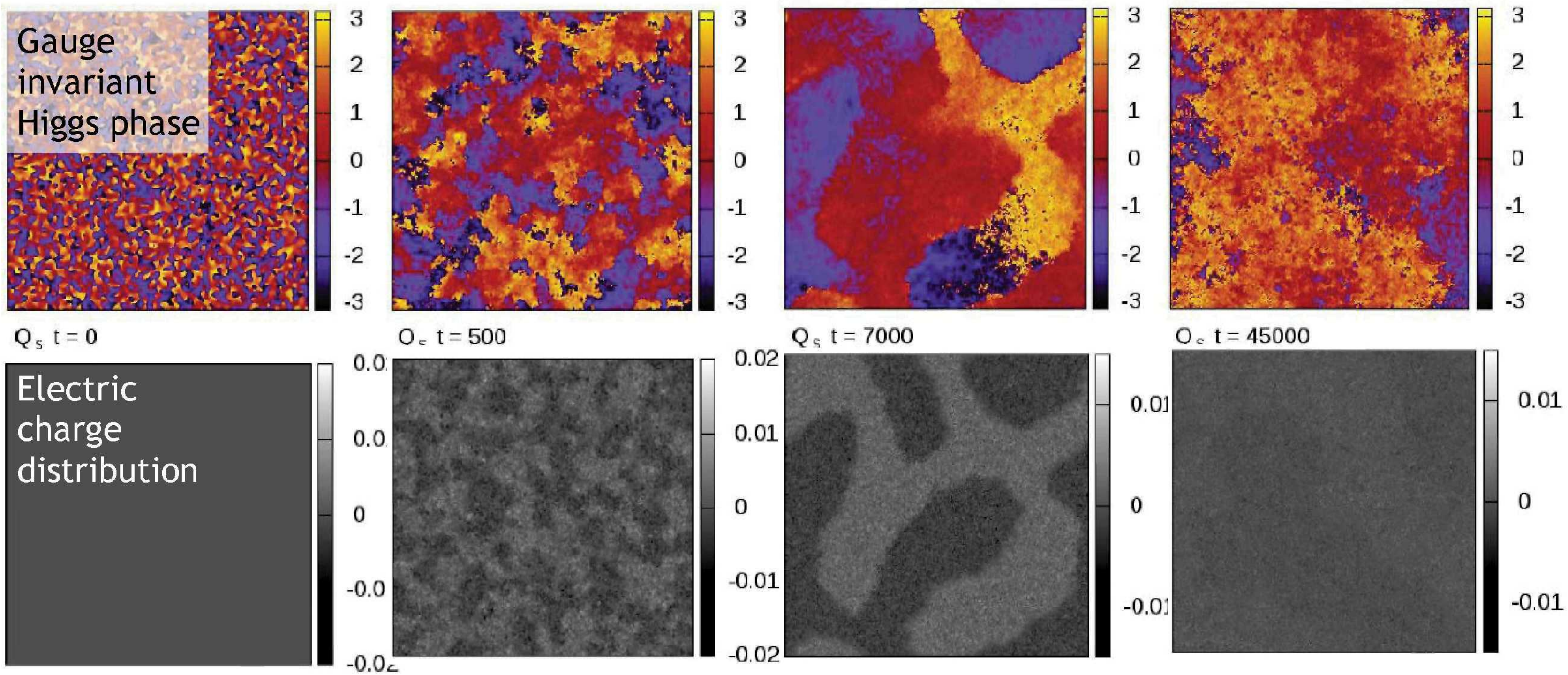
Gauge Turbulence

U(1) Anderson-Higgs (Ginsburg-Landau) model:

$$S[A_\mu, \phi] = -\int_x \left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^* D^\mu\phi + V(\phi)\right]$$

$$V(\phi) = \frac{\lambda}{4!}(\phi^*\phi - v^2)^2$$





TG, L. McLerran, J.M. Pawlowski, D. Sexty, 1307.5301 [hep-ph]

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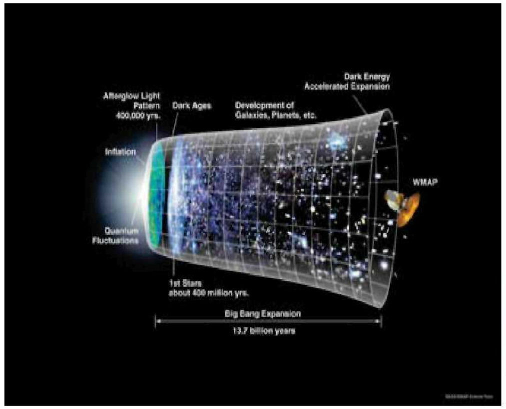
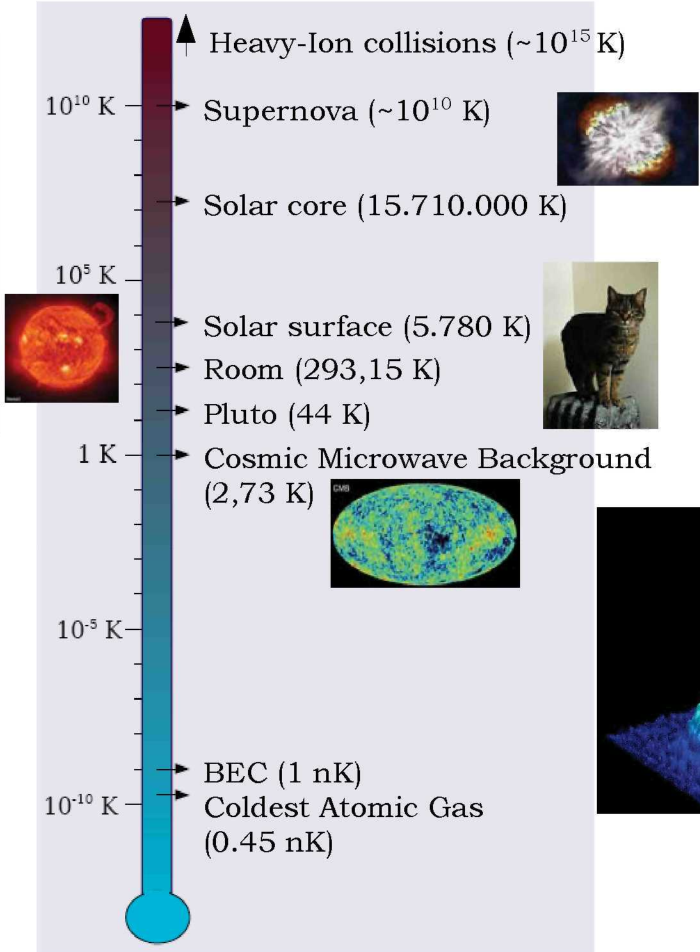
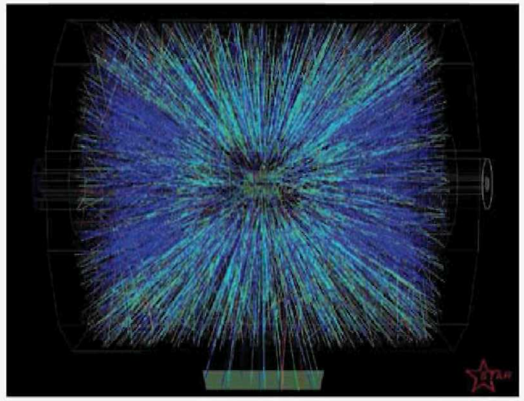
ULTRAcool Universality



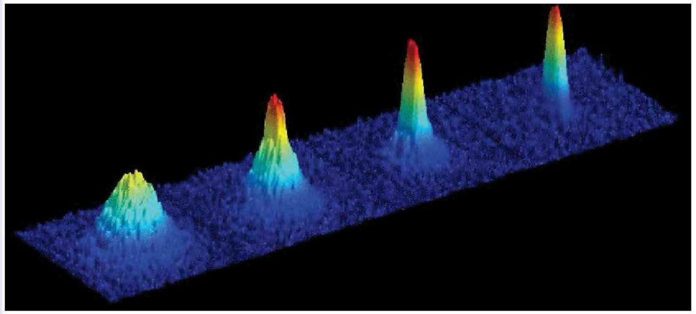
Early-universe:
Inflation & Reheating



Result of colliding two Gold nuclei (Relativistic Heavy Ion Collider, BNL):



Heavy-ion collisions



Ultracold atomic gases

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Thanks & credits

Linda Martini
Sebastian Erne
Markus Karl
Sebastian Heupts
Isara Chantesana
Steven Mathey
Sebastian Bock
Thorge Müller
Bart Andrews
Alexander Liluashvili

not in the picture:

Simon Sailer

Valentin Kasper
Asier Pineiro Orioli

(Collaboration with Jürgen Berges & Jan Pawlowski)

Collaboration with experimenters:

Wolfgang Müssel, Eike Nicklas, Helmut Strobel, Markus Oberthaler
Robert Bücker, Tim Langen, Wolfgang Rohringer, Jörg Schmiedmayer

€€€...

Alexander von Humboldt Stiftung / Foundation

Heisenberg-Programm
DFG Deutsche Forschungsgemeinschaft

RUPRECHT-KARLS-UNIVERSITÄT HEIDELBERG

LGFG BaWue

DAAD Deutscher Akademischer Austausch Dienst
German Academic Exchange Service

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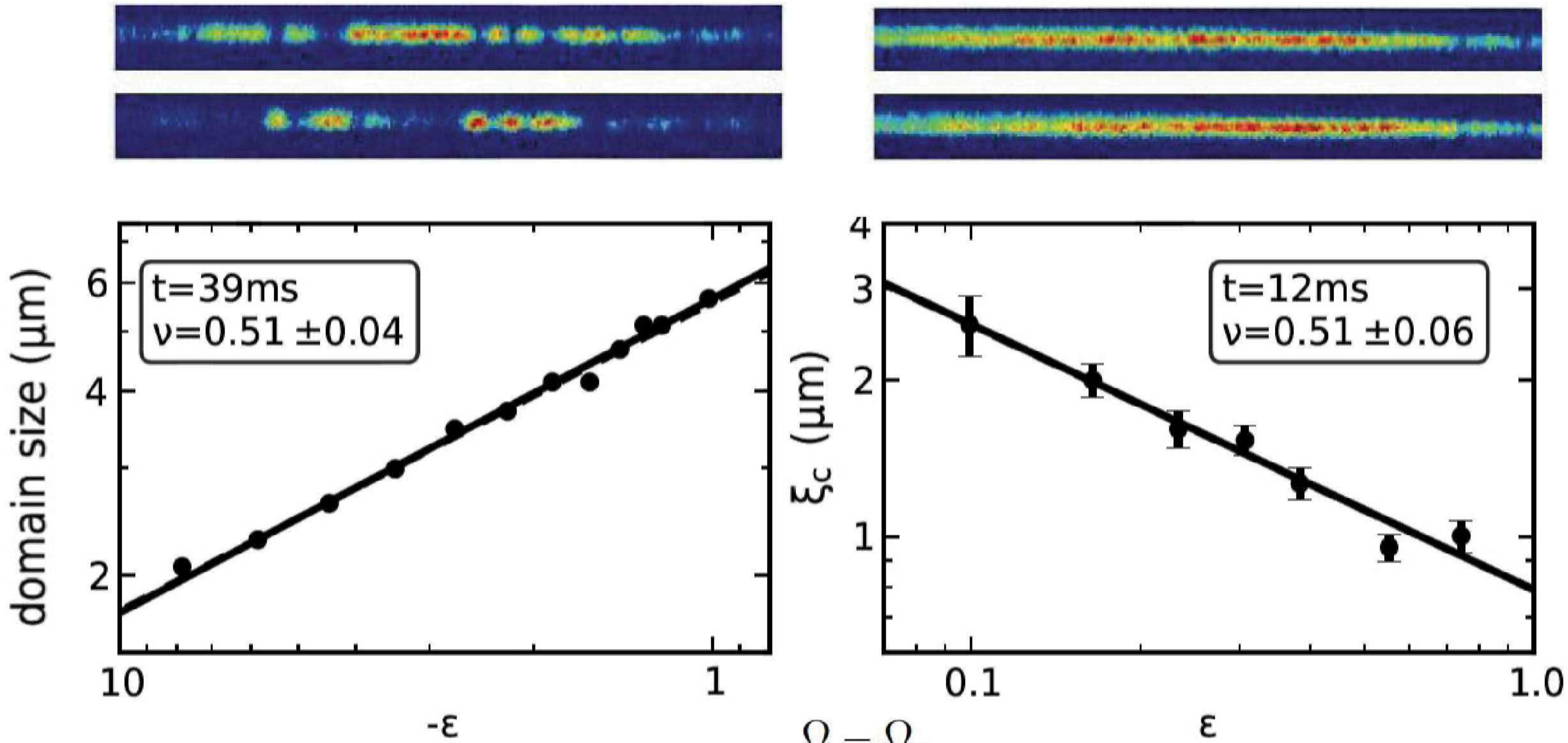
EMMI

HELMHOLTZ GEMEINSCHAFT

Center for Quantum Dynamics

Heidelberg Quantum Dynamics

[Johnson, Karl, Müssel, Nicklas, Strobel, TG, Bouchoule, Oberthaler, to be published]



$$\varepsilon = \frac{\Omega - \Omega_c}{\Omega_c}$$

Dynamical scaling: $G(s^{\nu}k, s^{-\nu z}t; s\varepsilon) = s^{-2\nu-\eta}\, G(k,t;\varepsilon)$

Non-linear Heisenberg/sigma model: Z_2-symmetry breaking

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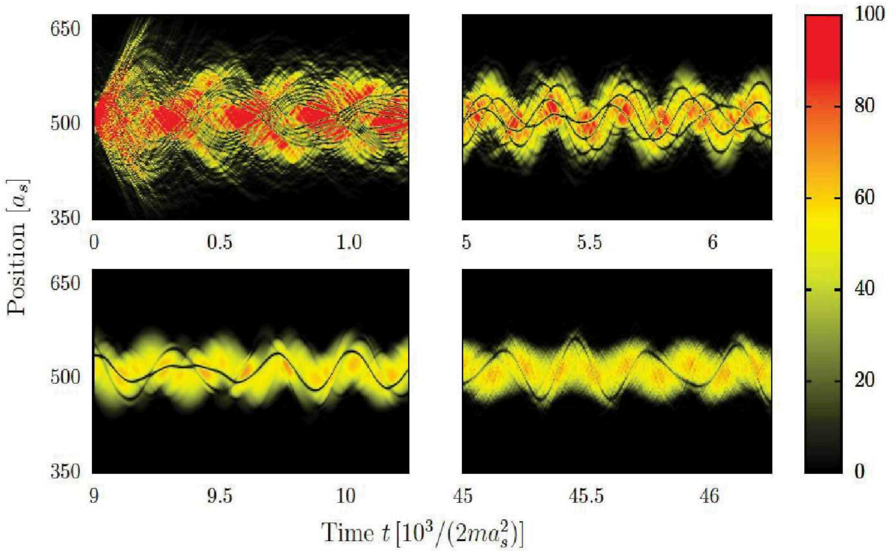
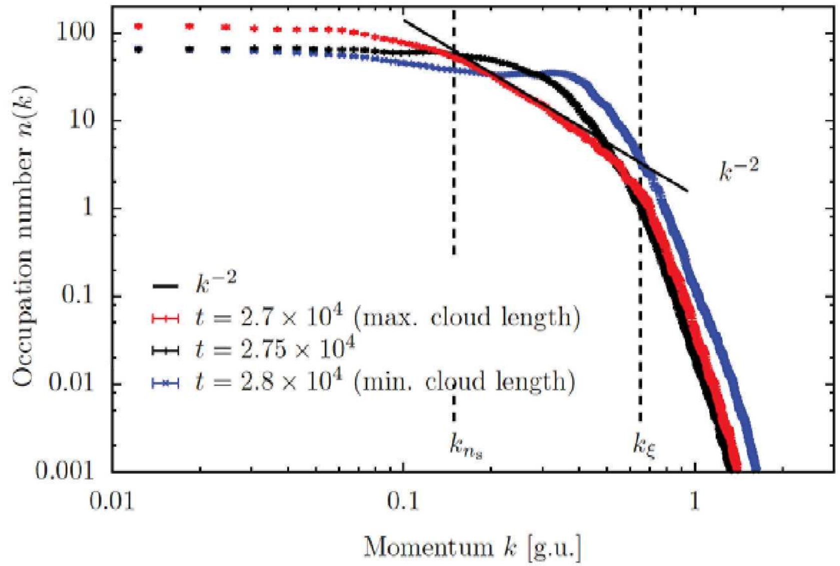
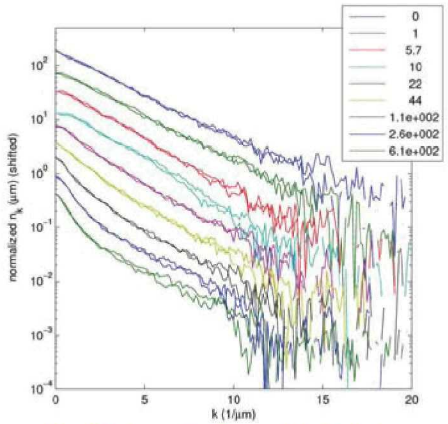
Solitons in 1 spatial dimension

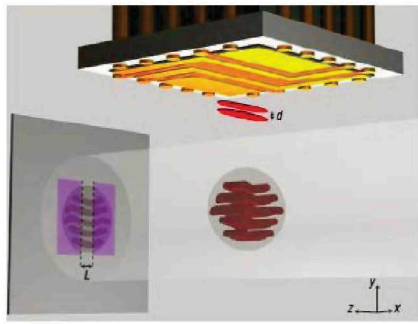




- Quasi stationary profile
- Scaling



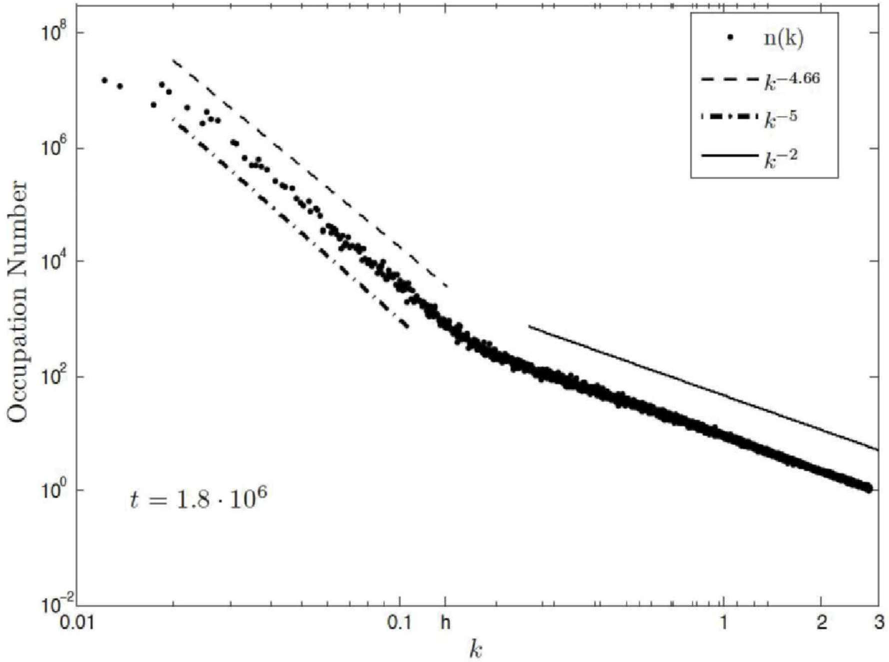
^{87}Rb-Experiment:
Schmiedmeyer group (Vienna)

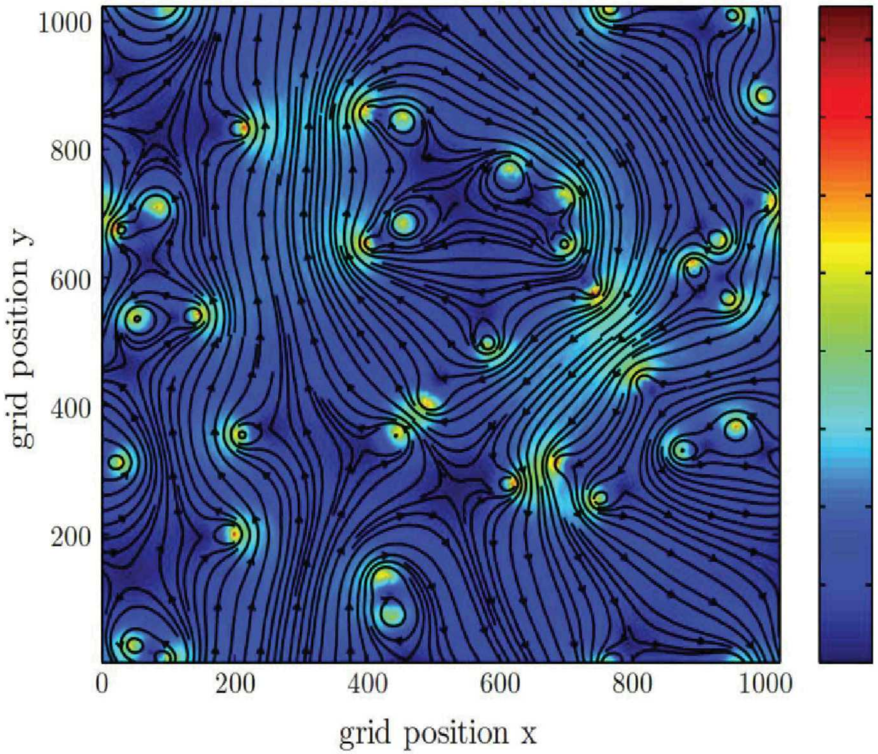


M. Schmidt, S. Erne, B. Nowak,
D. Sexty, and TG, NJP 14 (12) 075005

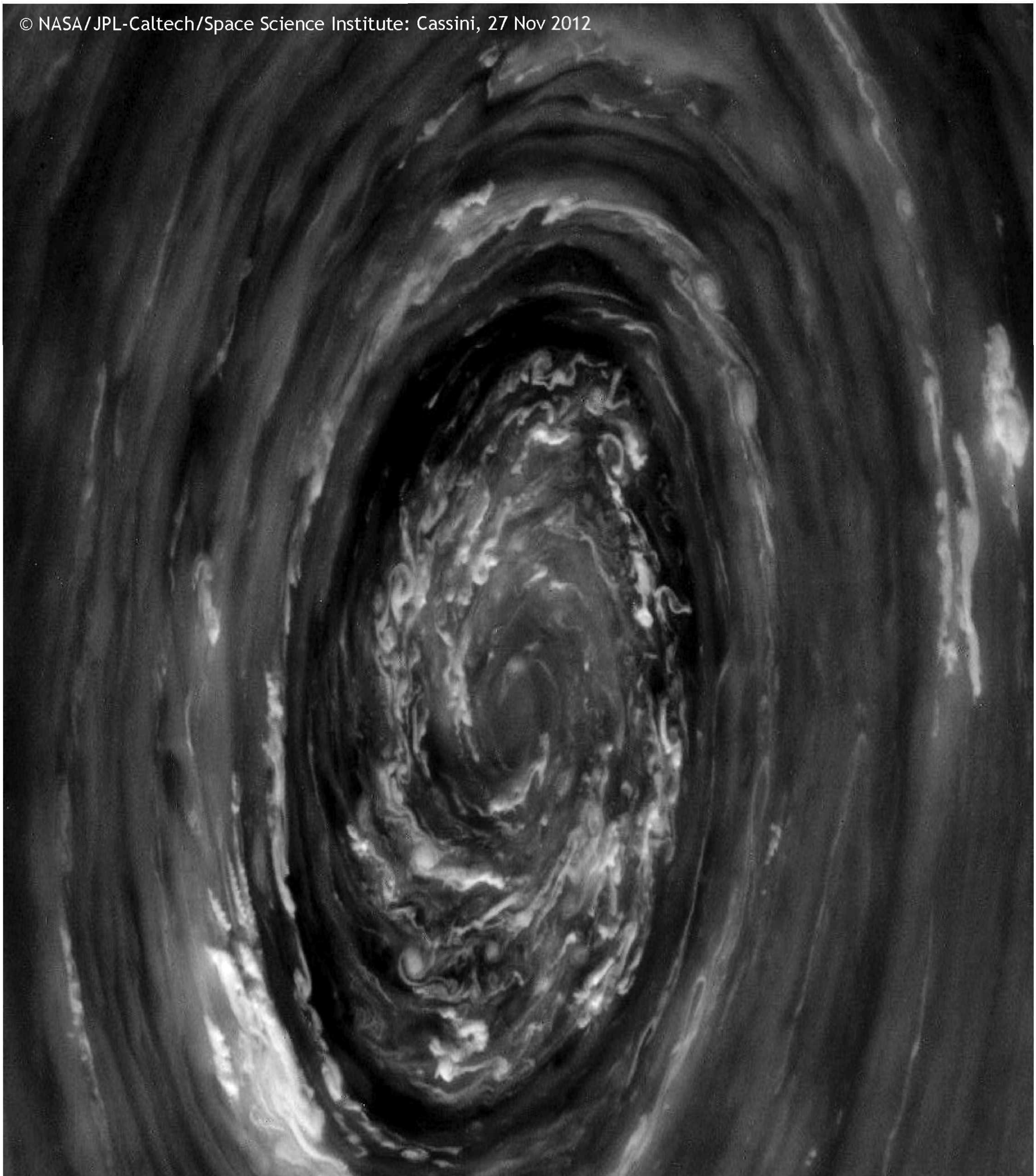
[R. Bücker, S. Erne, W. Rohringer, et al.]

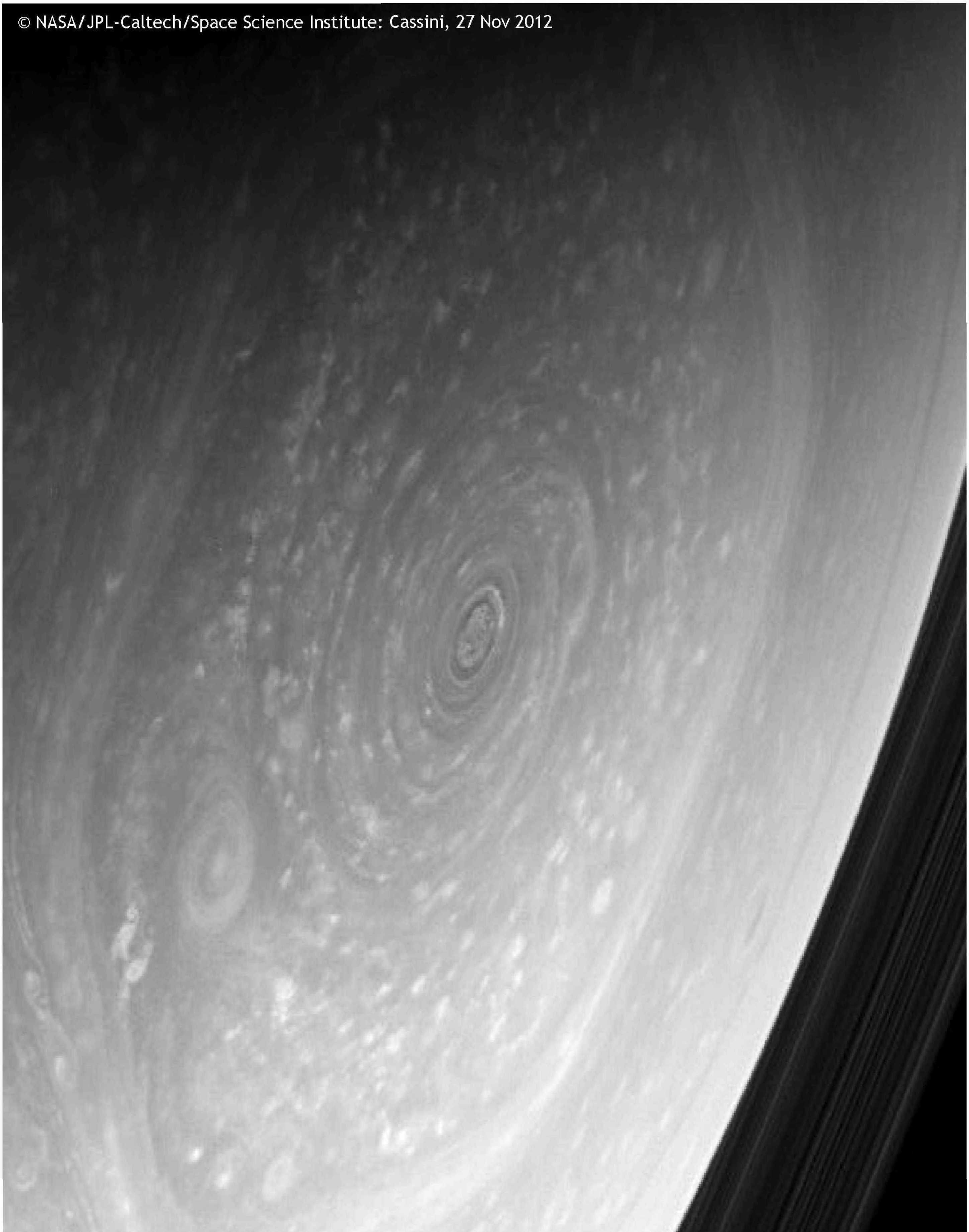
Kolmogorov-41 Scaling?

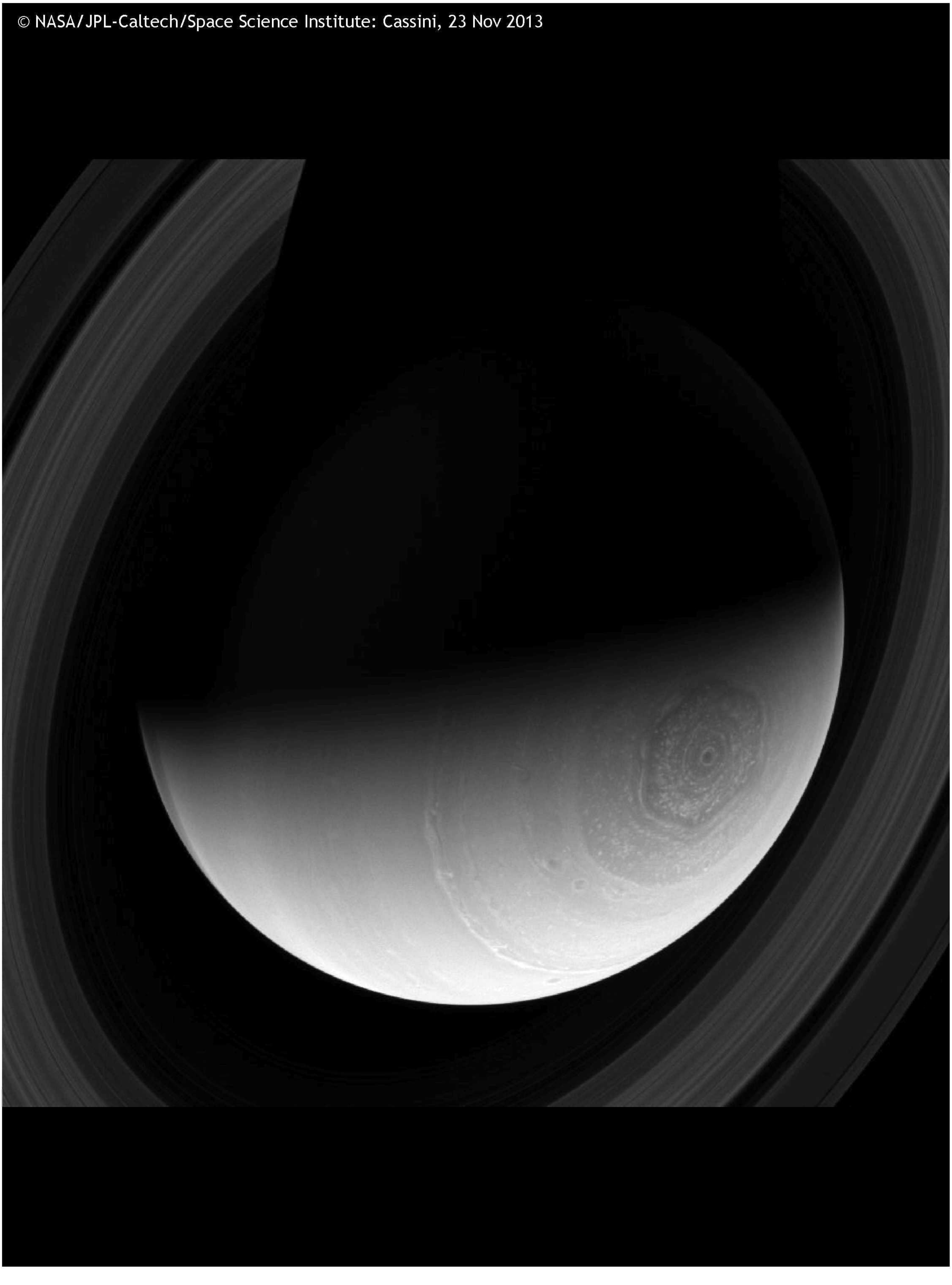




[T. Müller, M. Karl, and TG, to be published]

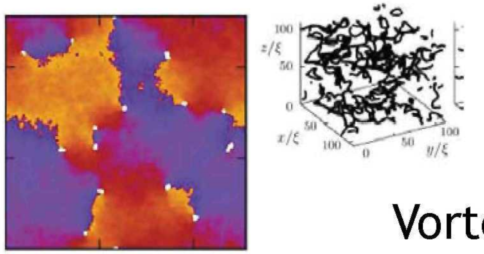

© NASA/JPL-Caltech/Space Science Institute: Cassini, 27 Nov 2012

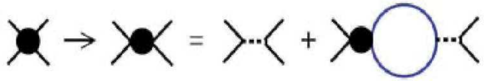

© NASA/JPL-Caltech/Space Science Institute: Cassini, 27 Nov 2012

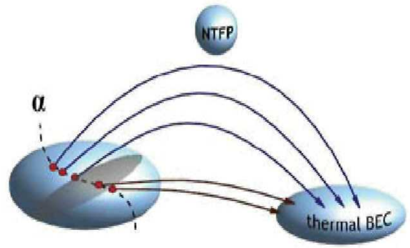


The End

Many other phenomena

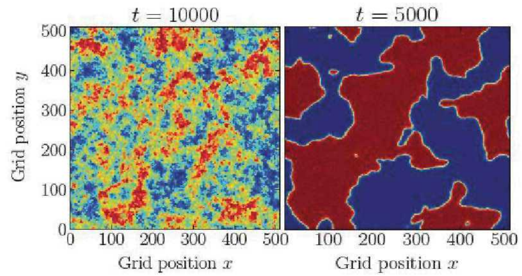




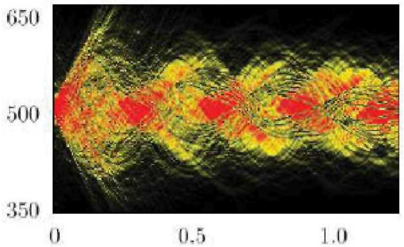


Non-Thermal Fixed Points

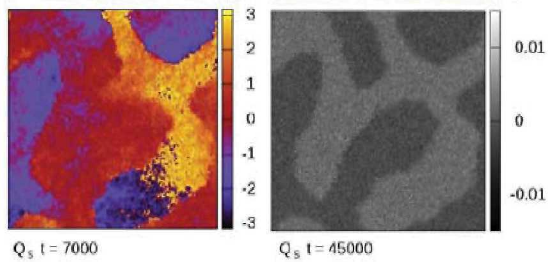
Vortex Dynamics in 2&3D



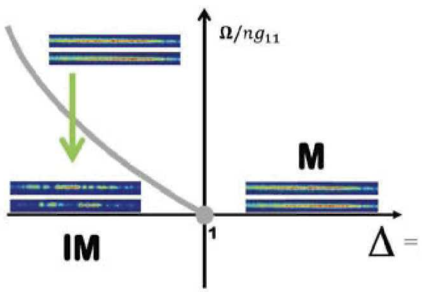
Pattern formation in Spinor gases



1D Soliton Gas



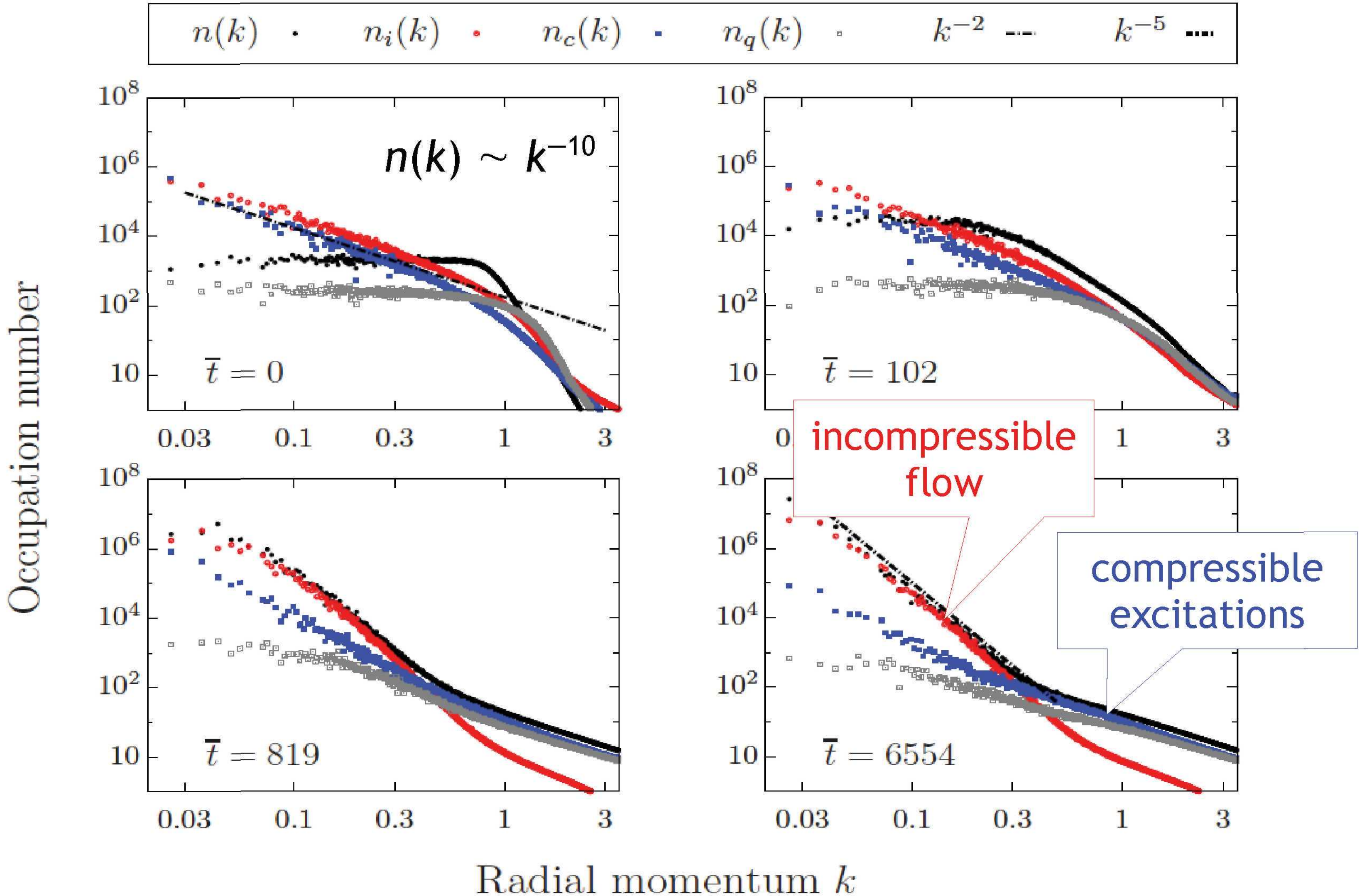
Charge confinement in Higgs models



"Kibble-Zurek", Exciton-Polaritons, Cosmology, QGP, ...

Supplementary slides

Hydrodynamic vs. kinetic Condensation



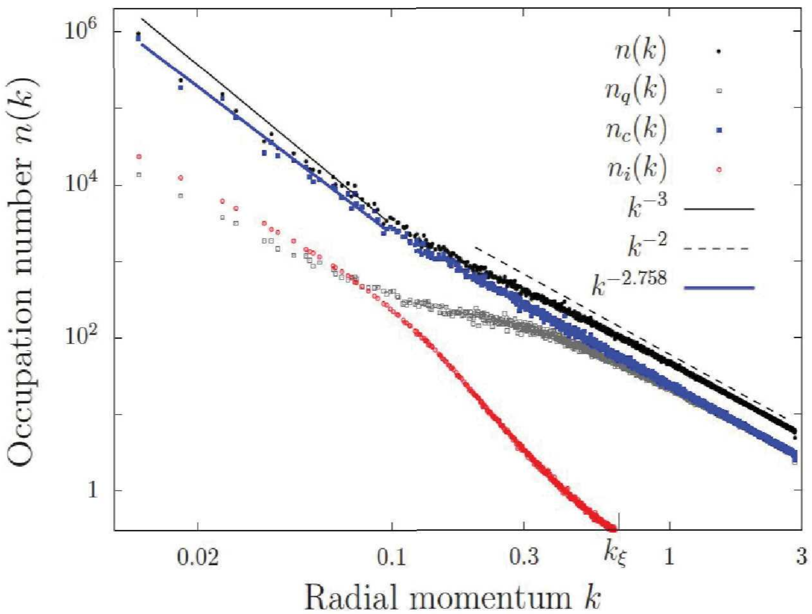
[B. Nowak, J. Schole, and TG, arXiv:1206.3181]

Center for Quantum Dynamics

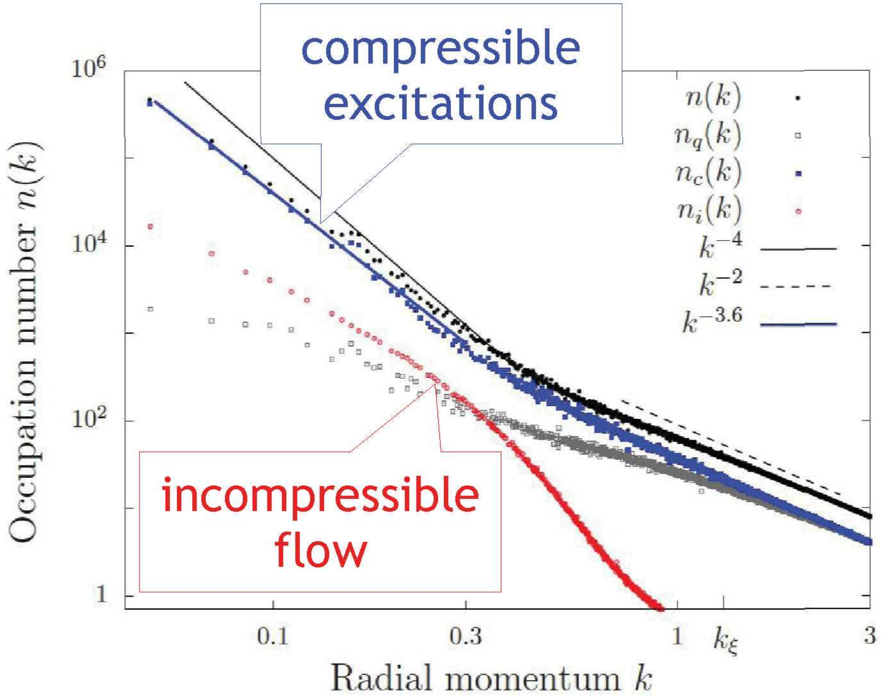
Anomalous Exponents

Kardar-Parisi-Zhang equation $\partial_t \theta(\mathbf{x},t) = \nu \nabla^2 \theta(\mathbf{x},t) + \frac{\lambda}{2} [\boldsymbol{\nabla} \theta(\mathbf{x},t)]^2 + \eta(\mathbf{x},t)$

$\leftrightarrow$ driven-dissip. phase dynamics of coherent Bose gas



d = 2



d = 3

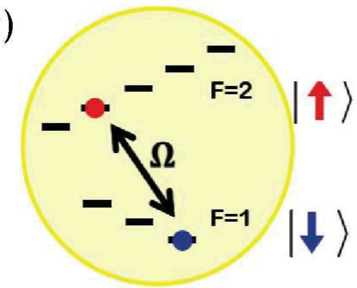
[S. Mathey, TG, and J. Pawlowski, arXiv:1405.7652]

Center for Quantum Dynamics

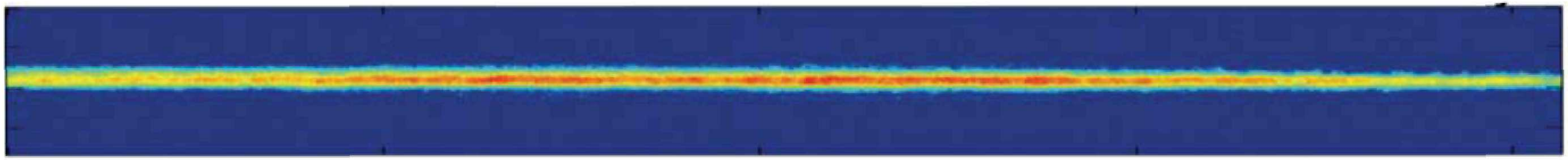
Heidelberg Quantum Dynamics

(Rubidium)

linear coupling:

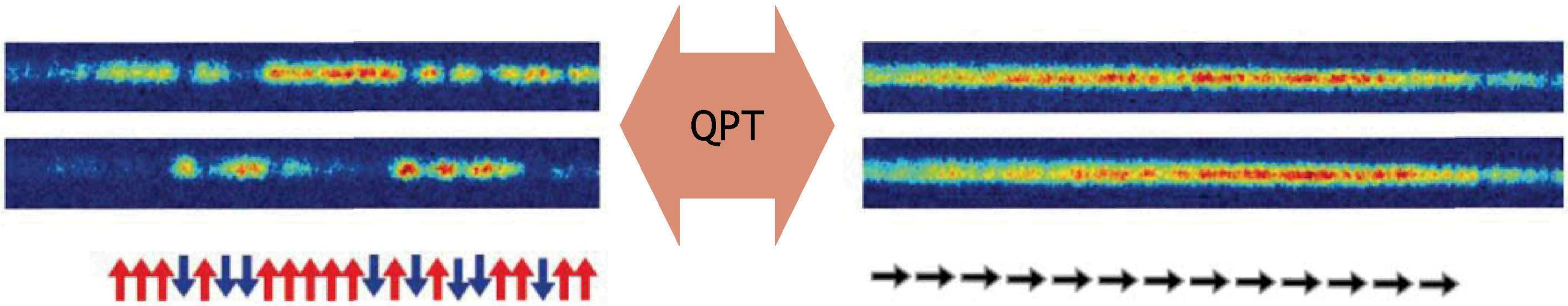


Density



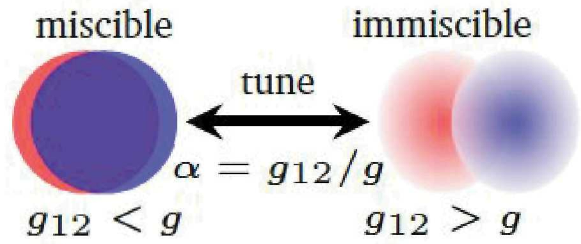
$\Omega < \Omega_{crit}$ immiscible

$\Omega > \Omega_{crit}$ miscible



Related: **Schmiedmayer**, TU Wien; **Spielman**, NIST Gaithersburg

Heidelberg Quantum Dynamics



(Rubidium)

linear coupling: