Elementary Ideas- Advanced Understanding
Plasmas, Fusion, Confinement

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Very Beginning

Plasma is the 4th State

Plasma is an ionized gas

Mostly Ions and Electrons

Electromagnetic Forces are the determinants of dynamics

Mostly Classical Electromagnetism

Plasma Dynamics in a Nutshell

Our Basic Field Equations are Maxwell's:

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \mathbf{E} = 4\pi \rho \quad , \quad \nabla \cdot \mathbf{B} = 0 \qquad (1)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Given (1) → all matters e·m are done!

But where is Plasma Physics?
dynamics?

For that matter, barring a few cases (H. Energy physics), much of the physics is covered by (1)

Constitutive Relations

$$\boxed{\begin{aligned} \rho &= \rho(\underset{\sim}{E}, \underset{\sim}{B}) \\ \underset{\sim}{J} &= \underset{\sim}{J}(\underset{\sim}{E}, \underset{\sim}{B}) \end{aligned}}$$ Closure

$$J^{\mu} = (\rho, \underset{\sim}{J})$$

$$\underset{\sim}{E}, \underset{\sim}{B} \Longrightarrow F^{\mu\nu}$$

$$J^{\mu} = J^{\mu}(F^{\mu\nu}) \quad \text{Rel.}$$

Plasma dyamics is to simply to derive □

Then there is algebra!

Challenges : $J = J(F)$

In our standard practice we use :

single particle responses : orbits
Fluid descriptions +
Kinetic Theory

Any amount of smartness

Final Aim : To find $J(F)$ and stick it into Maxwell

In fact, it is more than that

In other fields (Condensed Matter)

You invoke Quantam Mechanical Models

: The holy grail is ever the same

Challenges $J = J(F)$

Maxwell wrote long long ago

The struggle of a current practitioner is to find the constitutive relations

An unwary practioner may not be fully cognizant

But this cognizance advances our understanding a great deal!

Why Plasma Physics

Fundamental Physics: Barring this unique planet, the Cosmos is in the plasma-state

Initial Plasma Physics Laboratory

Langmuir: Particle 'scattering' by plasma waves (e.m fields of) simulate

Momentum - Changing Collisions

Such collisions advance (the~~r~~mal) equilibration rates by several orders of magnitude

Waves and Instabitities - Consequences.

Fusion-Thermonuclear

Plasma physics was launched as a major physics discipline by the promise of thermonuclear fusion.

Fusion, powering the stars, takes place naturally in the celestial spheres

In Laboratory, fusion poses an formidable Challenge

Why

The Charged Particle High Temp. Gas must be **Confined**

EQUILIBRIUM

Plasma Physics – Special Challenge.

Necessity to establish a Confined Equilibrium
≡ CE is the hallmark of a plasma

It is also the biggest challenge
+ a headache to boot

Not just the difference between the stars and the lab.

It is what distinguishes plasma physics from other fields in physics:

What is the last time you heard a condensed matter physicist worrying about equilibrium?

Confined Equilibria

Stars confine thru gravitation

On earth we lack the mass!

⇒ Must Seek Magnetic Confinement

Requirement:

Confinement Radius: In Meters

Good News:

If we can make sufficiently strong B

+

The plasma transport is classical

⇒

Confinement radius could be ~10 Cm

We would have had Fusion.

Good Magnetic Confinement

(1) Equilibrium: Time Independent accessible solutions of the Maxwell - Plasma system

Crudely $S_0 = \underset{\sim}{B_0}, n_0, T_0, \beta_0 = \frac{n_0 T_0}{B_0^2/8\pi}$

$\Downarrow$ Create $\Rightarrow$ $\Uparrow$

(2) stability $\quad S = S_0 + \delta(t)$

$$\delta(t) = e^{-\gamma t + i\omega t}$$

$\omega \sim$ good or ok, $\quad \gamma > 0$ bad

(3) Transport:

Thermal Classical → Under control

Instibility Induced →

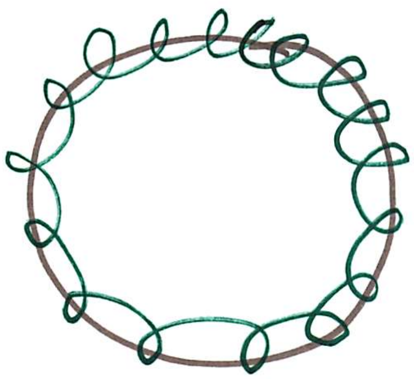
Long Scale and Fast: show-stopper

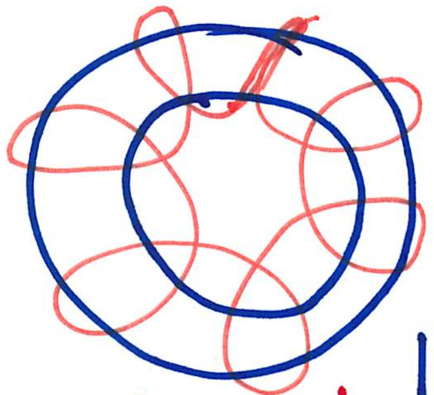
We have to learn to live with short-scale and slow transport

Magnetic Confinement

Uniform Magnetic field - (Q-d) Confined

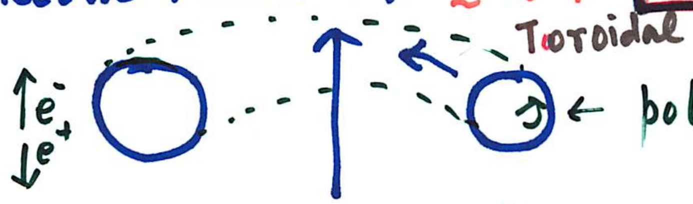
Close the Field Line





Geometry ⇒ Inhomogenous

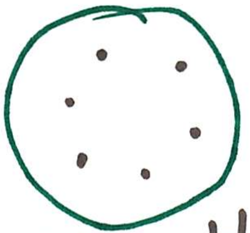
Inhom ⇒ ∇B drifts ⇒ charge separation ⇒ Electric Fields ⇒ $\underset{\sim}{E} \times \underset{\sim}{B}$ ⇒ plasma to the Wall

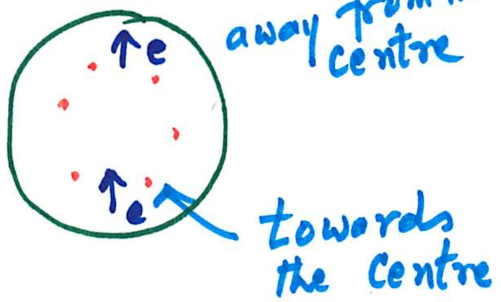


In μs

No Equilibrium from a purely toroidal field.

Magnetic Confinement – Helical Twist





The field lines do not bite their tail

$\underset{\sim}{B} = \underset{\sim}{B}_T + \underset{\sim}{B}_p$

No motion on the average!

$\underset{\sim}{B}_p \Rightarrow$ Twist $\equiv$ Rotational Transform $\equiv \iota$

Two different Methods of 'Twisting'

Two different fusion paths

Stellarators

ι from outside currents

No Current-driven Disruptions

the bane of a putative tokamak reactor!

Tokamaks

ι from the plasma

⇓

automatic heating (ohmic)

axi-symmetric

easier to build / theorize

A More Sophisticated View

The $|\nabla B|$ drifts $\longrightarrow$ $\perp$ Current $\underset{\sim}{J}_\perp$

To Avoid charge separation

$$\cancel{\frac{\partial \rho}{\partial t}} + \nabla \cdot \underset{\sim}{J} = 0$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{J} = \nabla_\perp \cdot J_\perp + \nabla_\parallel J_\parallel = 0$$

$\nabla_\perp \cdot J_\perp$ ⇓ not zero ⇒ $\nabla_\parallel J_\parallel$ ⇓ must be nonzero

Plasma Induced Current is the required $J_\parallel$ in a Tokamak

It is essential to understand:

The large B_T Needed for Gross level stability

Much Smaller B_p Equilibrium

Point to Ponder: When does stability is really equilibrium!

How do we find Equilibria

How do we calculate Gross Stability

For a closed system (tokamak, st..)

MHD - Magnetohydrodyamics - Good

$$\underset{\sim}{J} \times \underset{\sim}{B} = c \nabla p$$

$$\underset{\sim}{\nabla} \times \underset{\sim}{B} = \frac{4\pi}{c} \underset{\sim}{J}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{B} = 0$$

Challenge : Find $\underset{\sim}{B}$ such that

$\nabla p \neq 0$ is possible

That is the region of Confinement

$|\nabla p|$ is the figure of merit.

Variety of Equilibria – Interesting/Not So

Not all fields are confining

(1) Vaccum Field $\nabla \times B = 0 \Rightarrow J_{plasma} = 0$

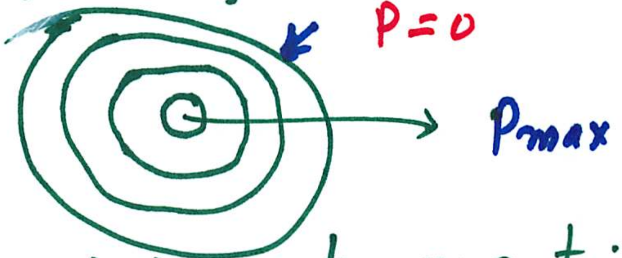
$$\nabla p \equiv 0$$

(2) Simple Force-Free Field

$$\nabla \times B = \lambda B \Rightarrow J \parallel B \Rightarrow$$
$$\nabla p = 0$$

What we are seeKing is a region in which $P(x)$ is a smooth function

We need isobaric surfaces that are nested



they can neither intersect nor go to infinity

MHD provides such states

$$B \cdot \nabla p = 0 \qquad J \cdot \nabla p = 0$$

Isobaric Surfaces are both Magnetic Surfaces and the Current Surfaces.

Nested Surfaces

Magnetic Confinement ≡ Existence of a set of isobaric nested surfaces.

The particles are constrained on the surfaces

Movement across the surface spells trouble!

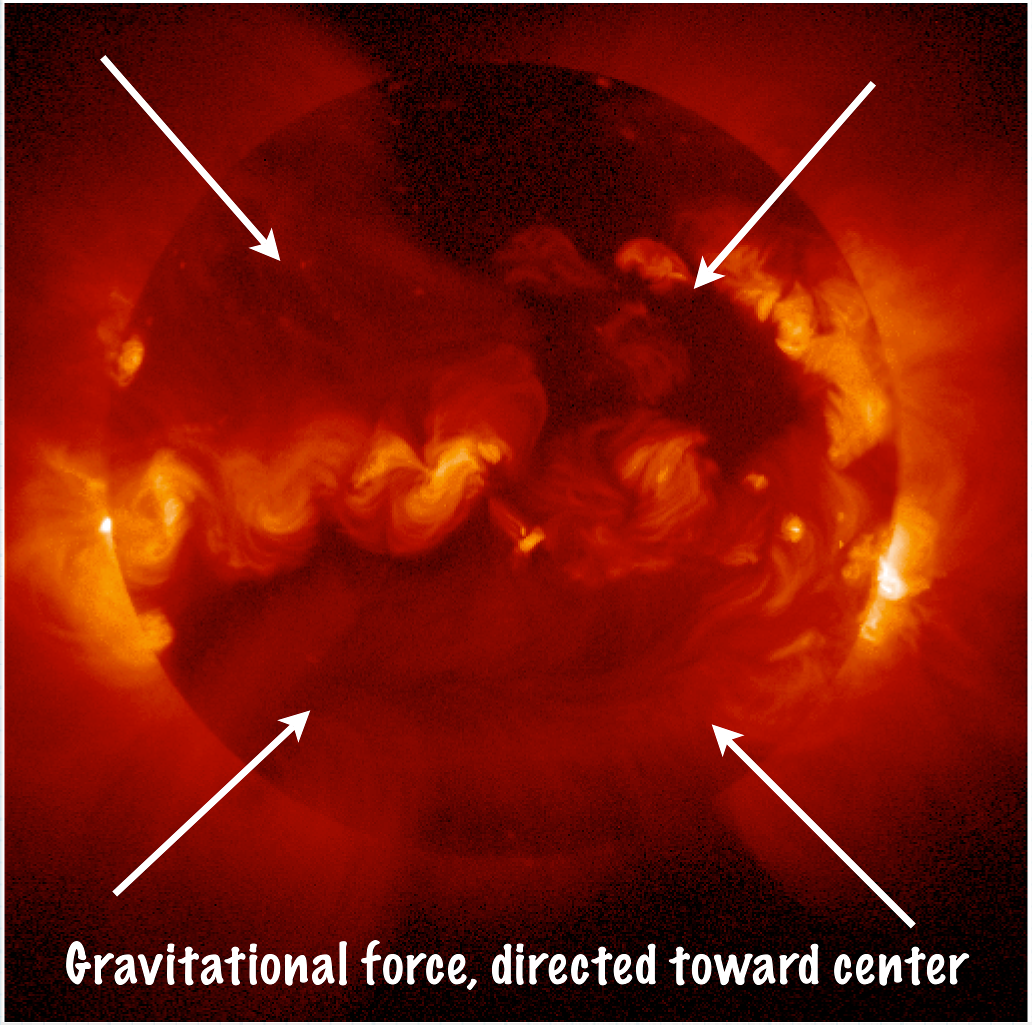
What are the Constraints on the class of desirable surfaces that magnetic fields can generate?

Let us have some fun investigating!

Plasma confinement

Cool plasma is easy to confine

But fusion plasma cannot survive contact with any wall: heat loss quenches plasma (only minor damage to wall).



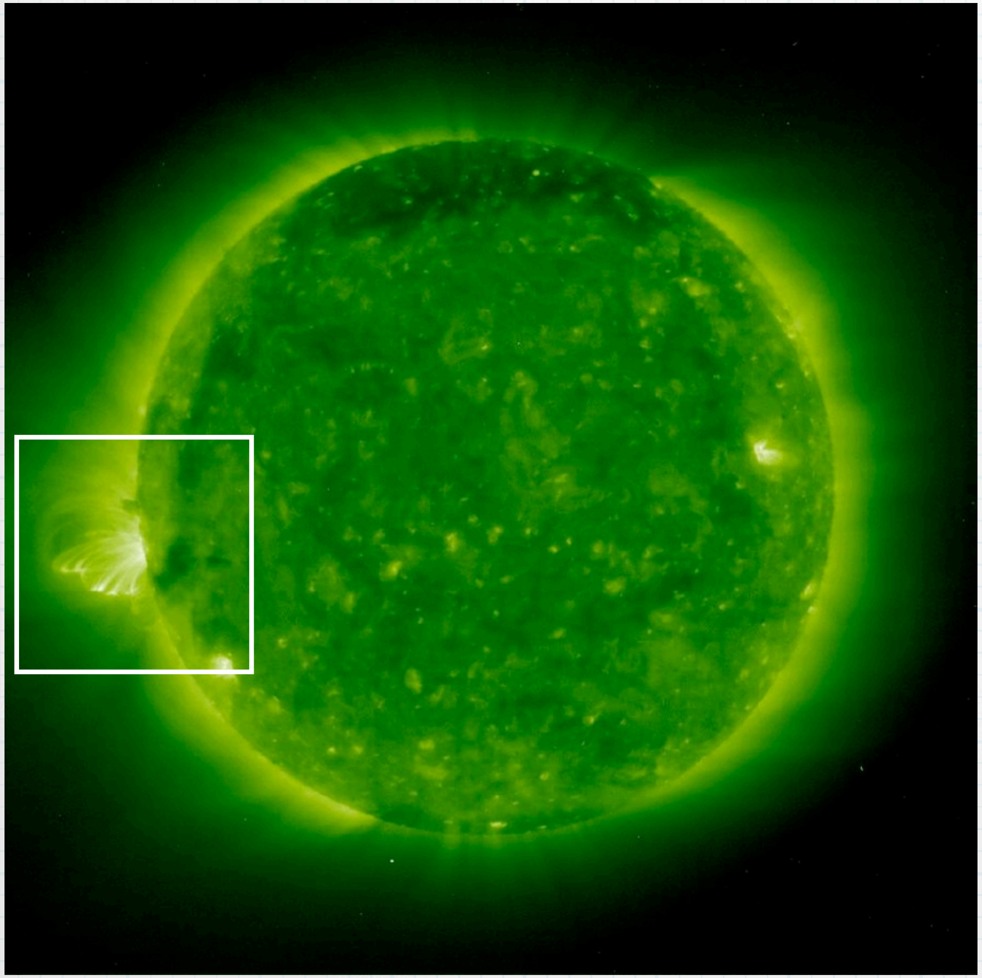
Solar plasma confinement:

Gravity holds plasma together, allowing fusion

But gravitational force is proportional to mass:

Solar confinement works because sun is large and massive

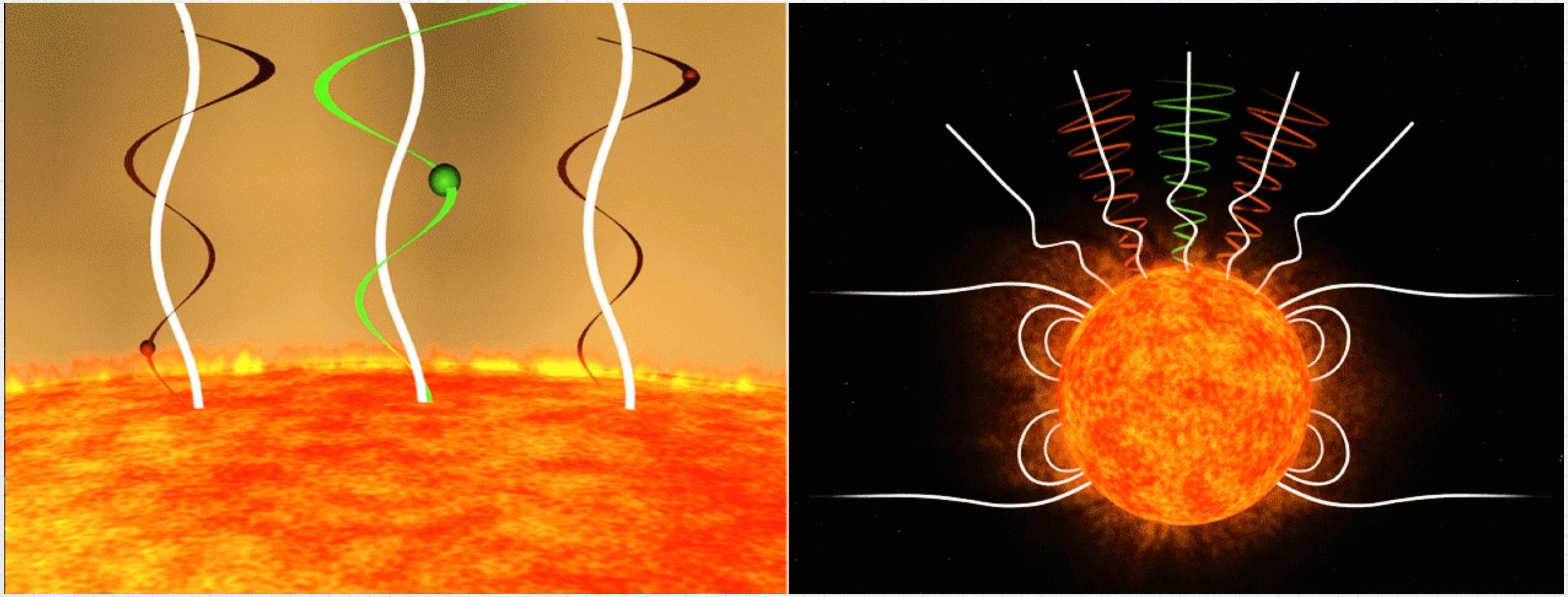
Solar corona: a different sort of confinement



Filaments and loops reveal charged particles trapped on magnetic field lines

Magnetic force is independent of mass: acts equally on large and small scales

Magnetic force links plasma (charged particles) to "field lines"



Motion across field lines is tightly constrained; but motion along field lines is not affected. ("2-D confinement.")

Key to magnetic confinement

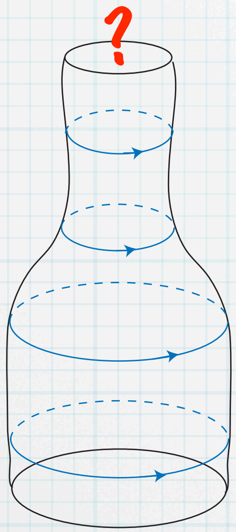
Suppose magnetic field lines lie on a surface, rather than wandering through some 3D volume.

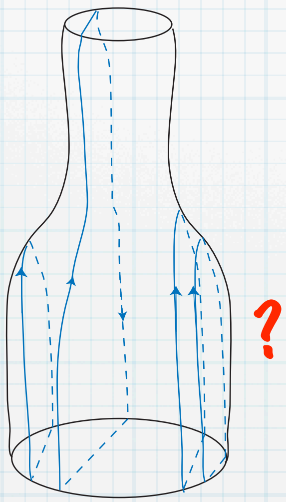
A surface covered by magnetic field lines is called a magnetic surface.

A closed magnetic surface will confine plasma.

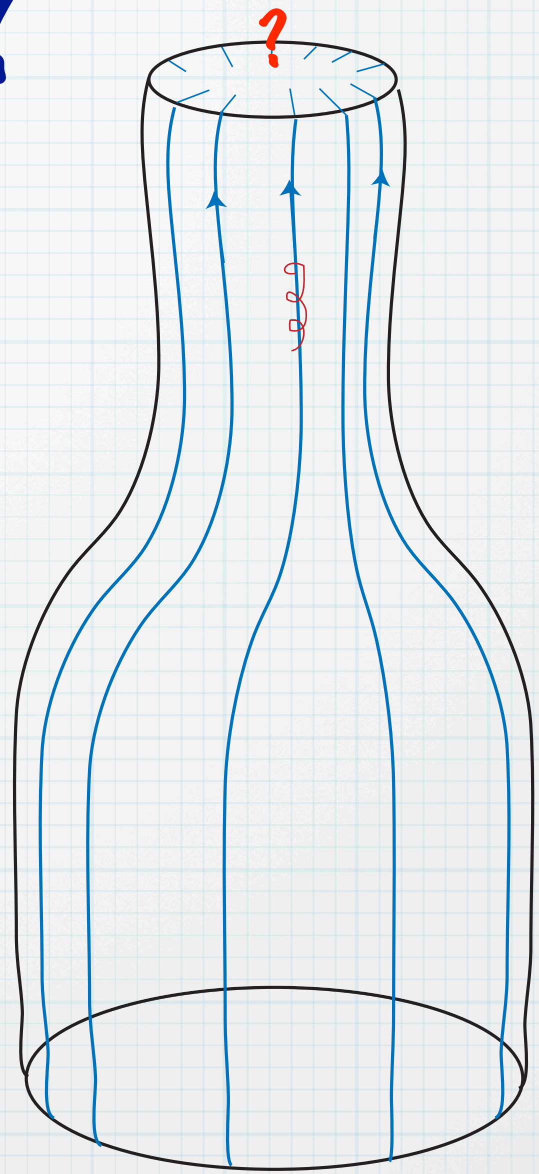
Magnetic bottle?

An arbitrary surface **cannot** be covered with smooth field lines



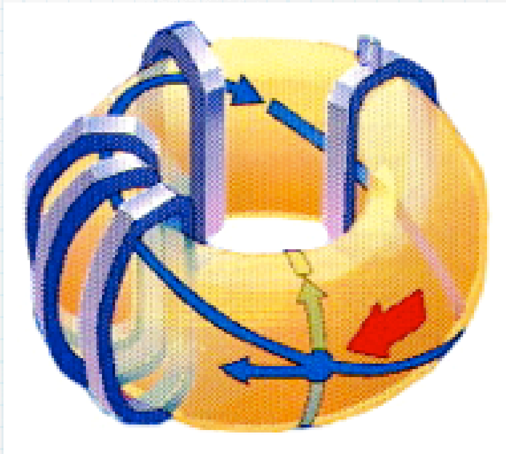


Either singular point, or null point, somewhere on surface



Closed magnetic surface must be toroidal

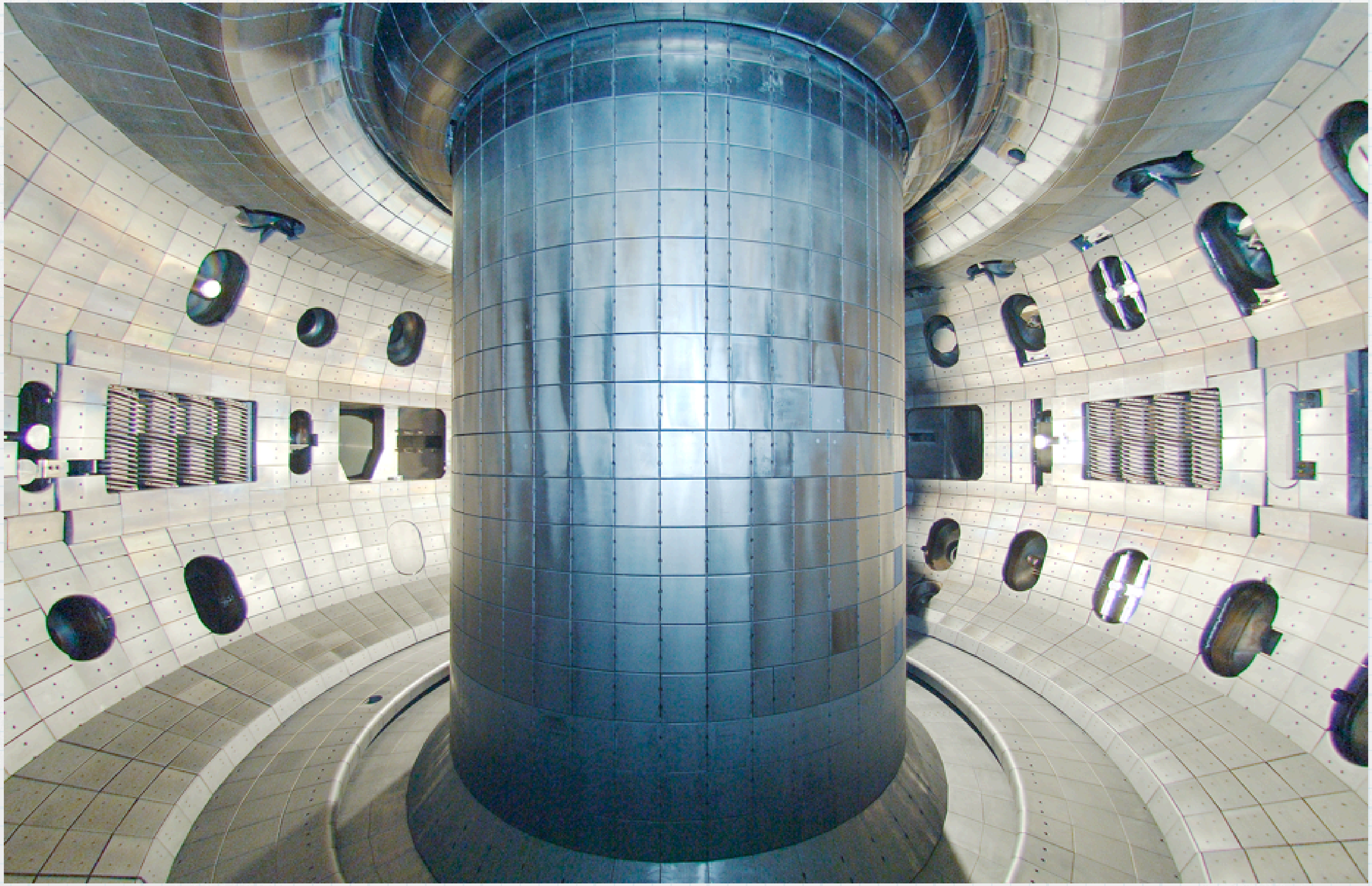
Krispy Kreme



Tokamak

No ends to cap: field lines cover surface

Tokamak interior



Few Comments on Stability

Confined Equilibria are, by def, not thermal Equilibria $\nabla p \neq 0$

There is always free energy

$\Rightarrow$ drive the system to a thermal state

Instability is a mechanism by which a system seeks a lower energy state

All the thermonuclear fusion attempts can state access states with a finite

Life Time τ = Confinement time

The larger the τ, the better the chances of eventual fusion.

$n T \tau \rightarrow$ triple product - Metric

(In) Stability

Instability, when not virulent, could be a mechanism for quiescent

Energy Transfer

thermal - kinetic - electromagnetic

heating, turbulent transfer, dynamo ....

When Virulent, it is a mechanism for violent, explosive transfer

Solar Flares, Coronal Mass Ejection, Tokamak Disruptions .....

For Thermonuclear Fusion to light our bulbs one day, we vote for complete stability!