

Joint ICTP-IAEA College on Advanced Plasma Physics  
International Centre for Theoretical Physics  
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# Structure /Dynamics of Fluctuation in Fusion Plasmas

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*collaboration with K. Imadera and J.Q. Li*

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Kyoto University, Japan

- August 19 : Structure/dynamics of fluctuations in fusion plasmas
- August 19 : Nonlinear instability and plasma dynamics
- August 21 : Interaction between different scales

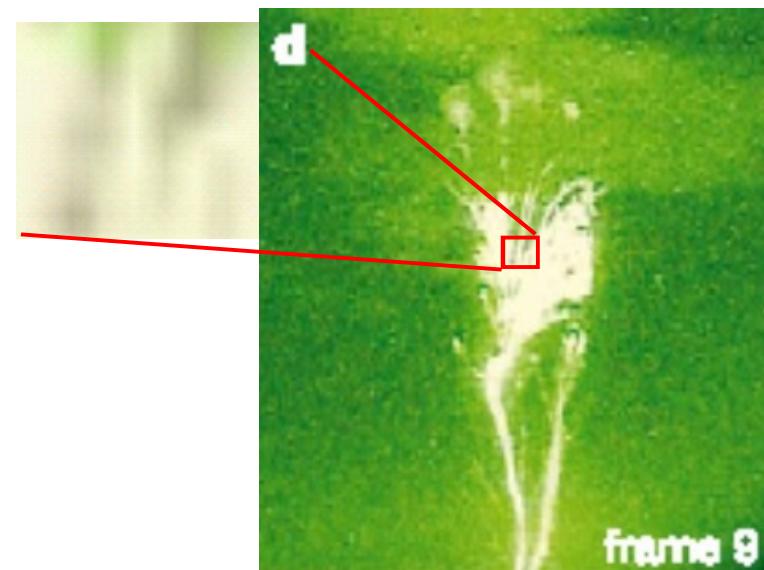
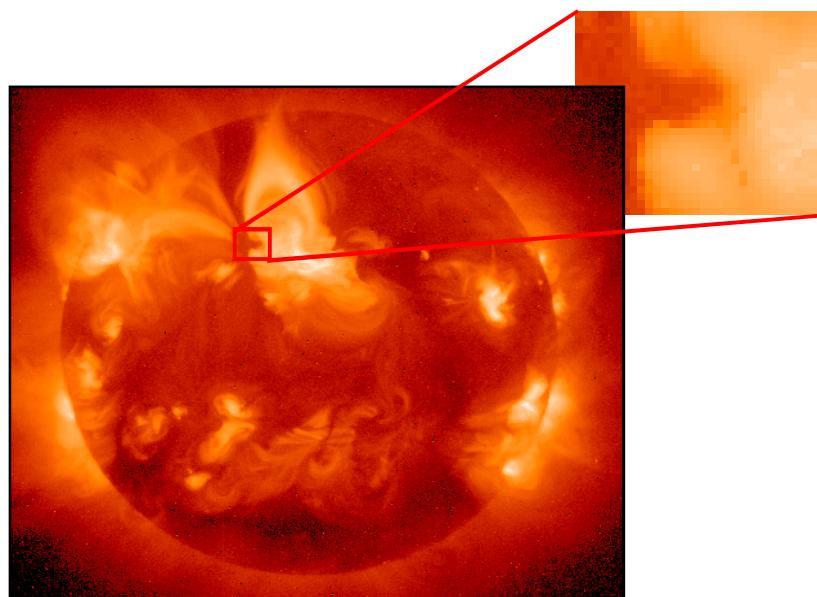
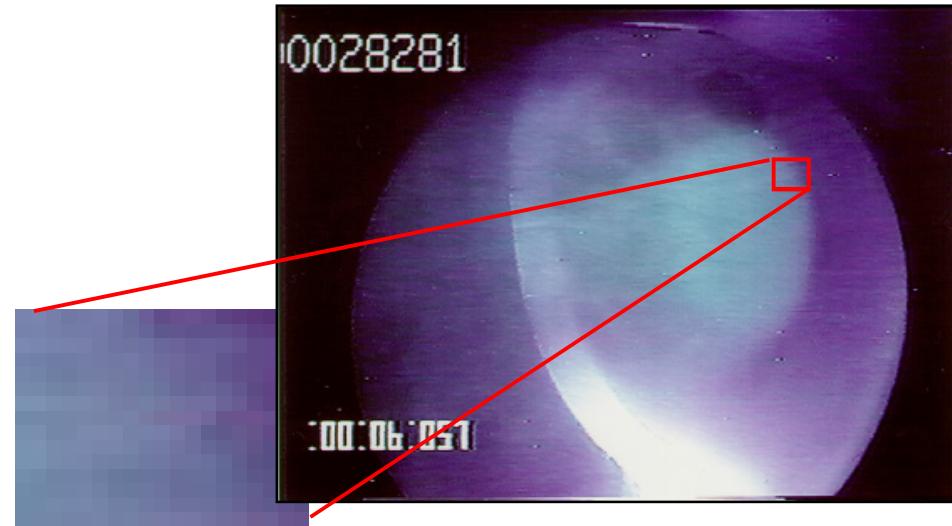
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## Group discussion (fusion) :

- ✓ More information for confinement including experiments, why we need to focus on fluctuation, e.g. what is fluctuation, how micro-turbulence is related to global confinement

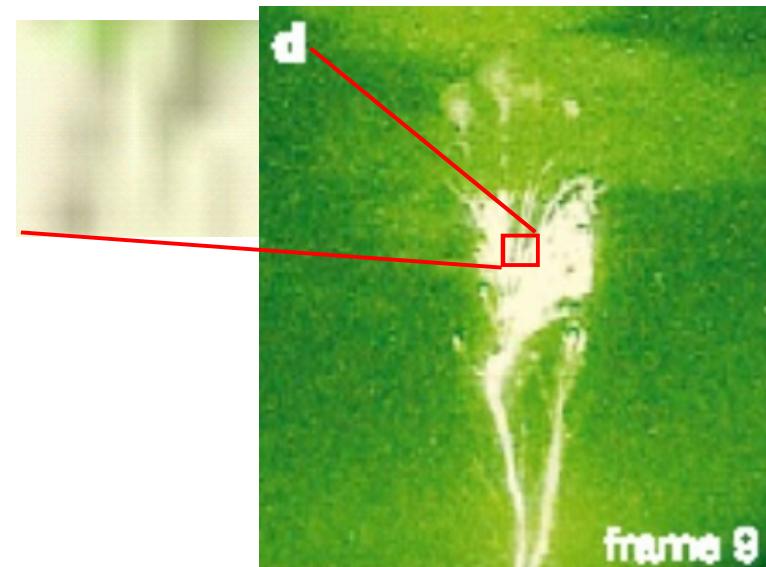
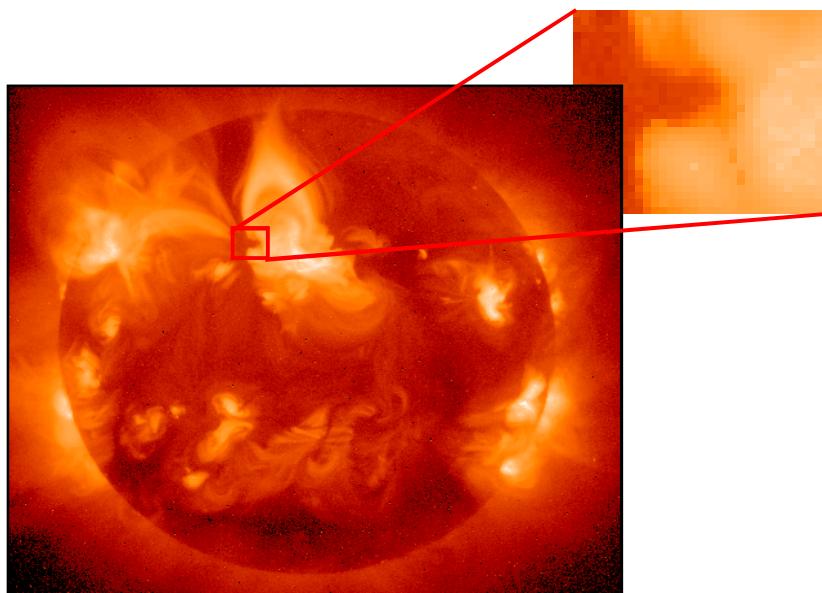
# Understanding plasmas is to understand nature

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# Understanding plasmas is to understand nature

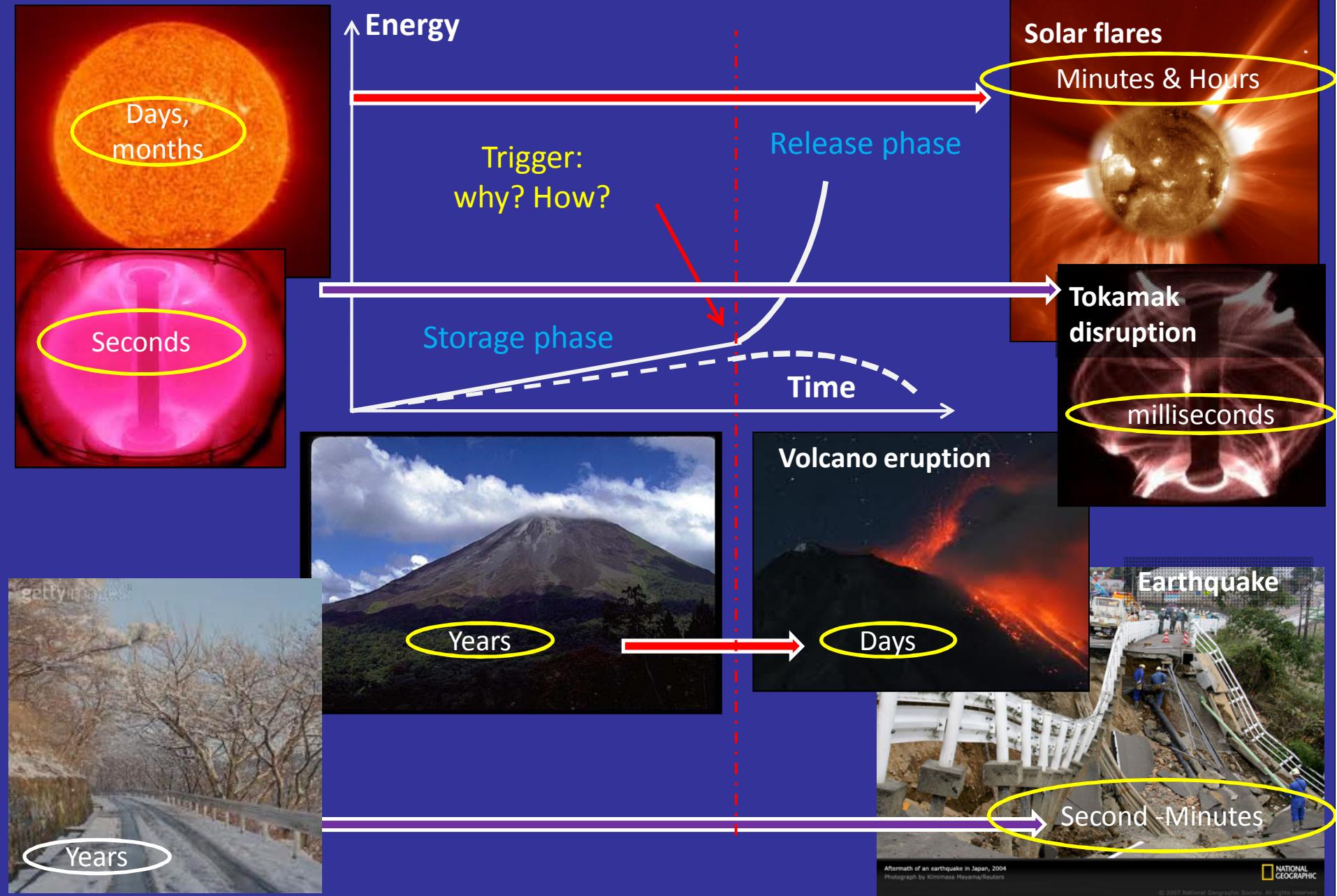
- ✓ Looking at a “**part**”, it is uncertain existence, but looking at a “**whole**” far away from the object, it exhibits clear and beautiful “**structure**” and “**dynamics**” with no specific symmetry.
- ✓ Careful investigation is necessary in choosing proper scales in “**space**” and “**time**”
- ✓ “**Small scale structure**” and “**large scale structure**” are incorporated through “**long range force**” and “**short range force**”.



frame 9

# sudden events in nature

Courtesy of Dr. Miho Janvier

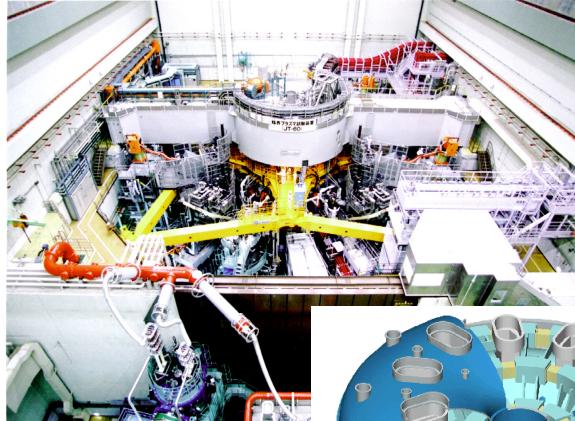


Aftermath of an earthquake in Japan, 2004  
Photograph by Kimimasa Mayama/Reuters

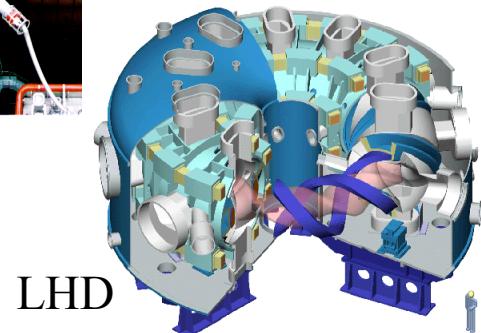
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# Magnetic fields in fusion device



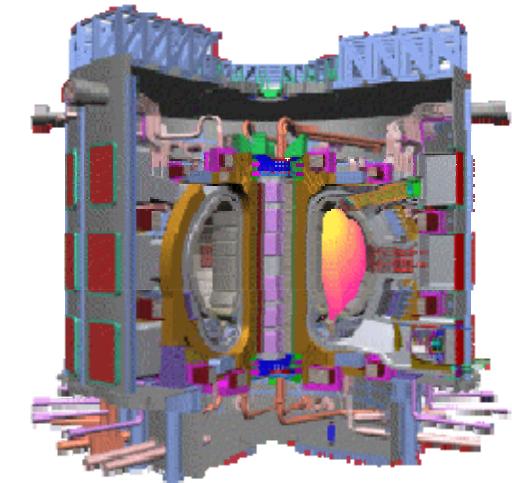
JT-60U



LHD

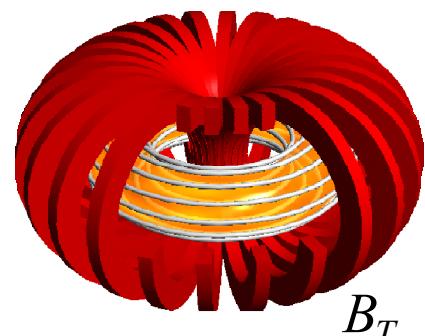


Joint European Torus (JET)

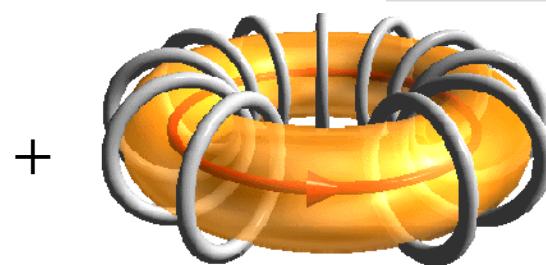


IETR

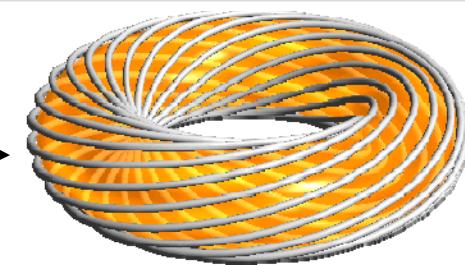
Limited volume covered by a infinite set of nested magnetic surface (magnetic cage)



Toroidal magnetic field



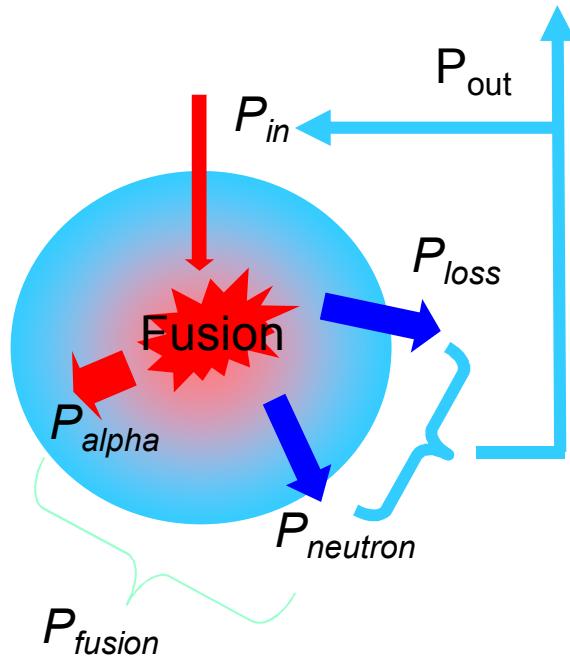
Poloidal magnetic field



$$\mathbf{B} = (B_T \hat{\phi} + B_P \hat{\theta})$$

Tilted Magnetic cage

## Schedule and key aspect



$$Q \equiv \frac{P_{fusion}}{P_{in}} \sim \frac{\langle n^2 \langle \sigma v \rangle E_f \rangle}{\langle nT \rangle / \tau_E} \sim (n\tau_E) \left( \frac{\langle \sigma v \rangle E_f}{T} \right)$$

key aspect :

- high pressure state in limited volume  
(not free space, cf. space/universe)  
→ **high fluctuation level inevitable** as Q increases
- **Open system**, but **high autonomous system**  
as Q increases

How we can understand and explore control methodology of high pressure plasma medium with a **high fluctuation level**, maintaining the **autonomous nature**

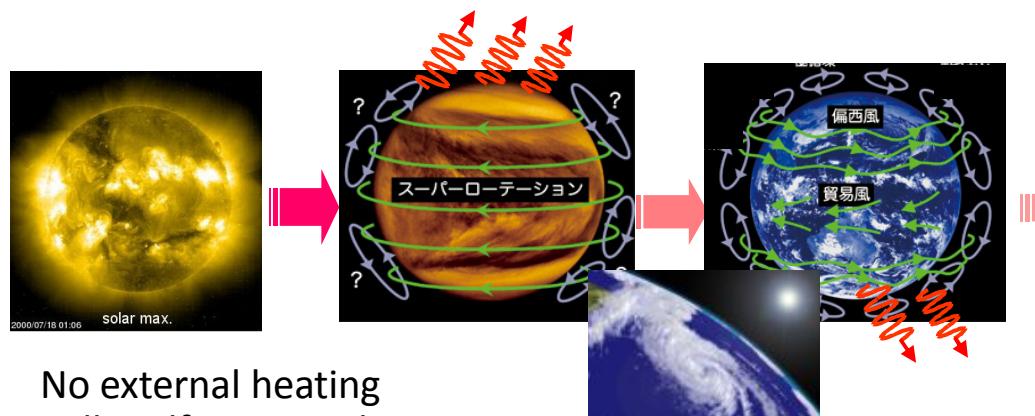
Group discussion (fusion) :

- ✓ Larger devise (lower fluctuation), which promises stable operation
- ✓ Smaller device (higher fluctuation), but, uncertain in stable operation

# Planetary environment and fusion plasmas

World dominated by

- Micro-turbulences
- Macro-flows
- (sometime with low mode)
- Large scale structure



No external heating  
Fully self-sustained

$$\eta_{ZF} \equiv \frac{E^{(ZF)}}{E^{(tot)} + E^{(ZF)} + E^{(LSS)}}$$

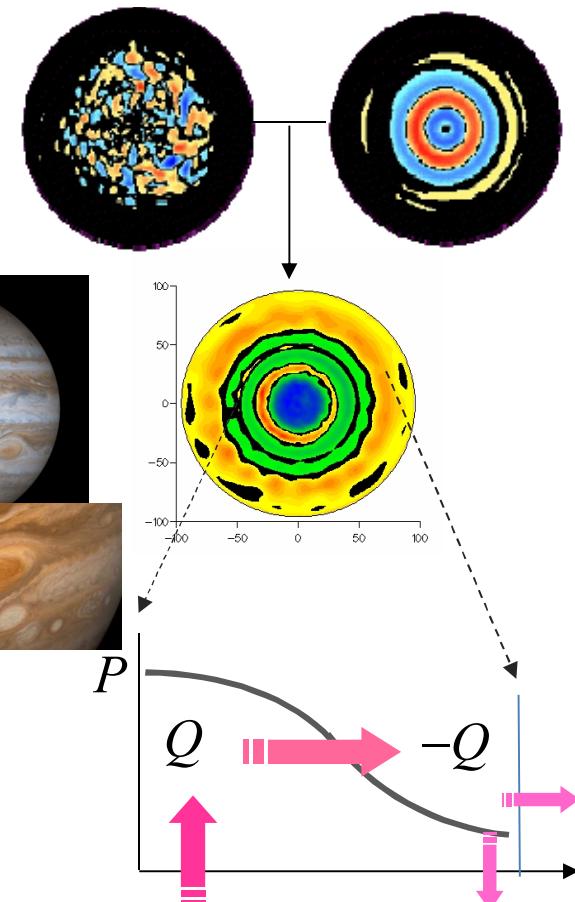
$$\eta_{ZF} \equiv \frac{E^{(ZF)} \tau_{ZF}}{E^{(turb)} \tau_t + E^{(ZF)} \tau_{ZF} + E^{(LSS)} \tau_{LSS}}$$

$$E^{(turb)}$$

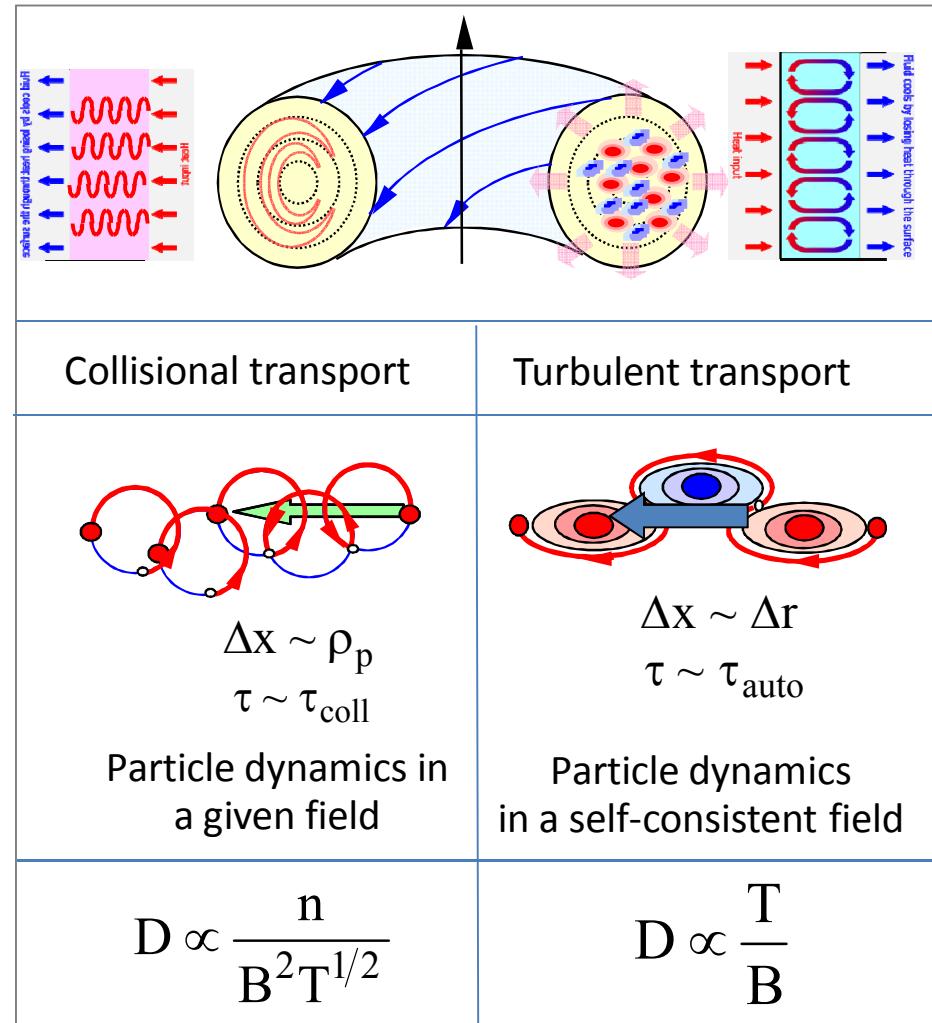
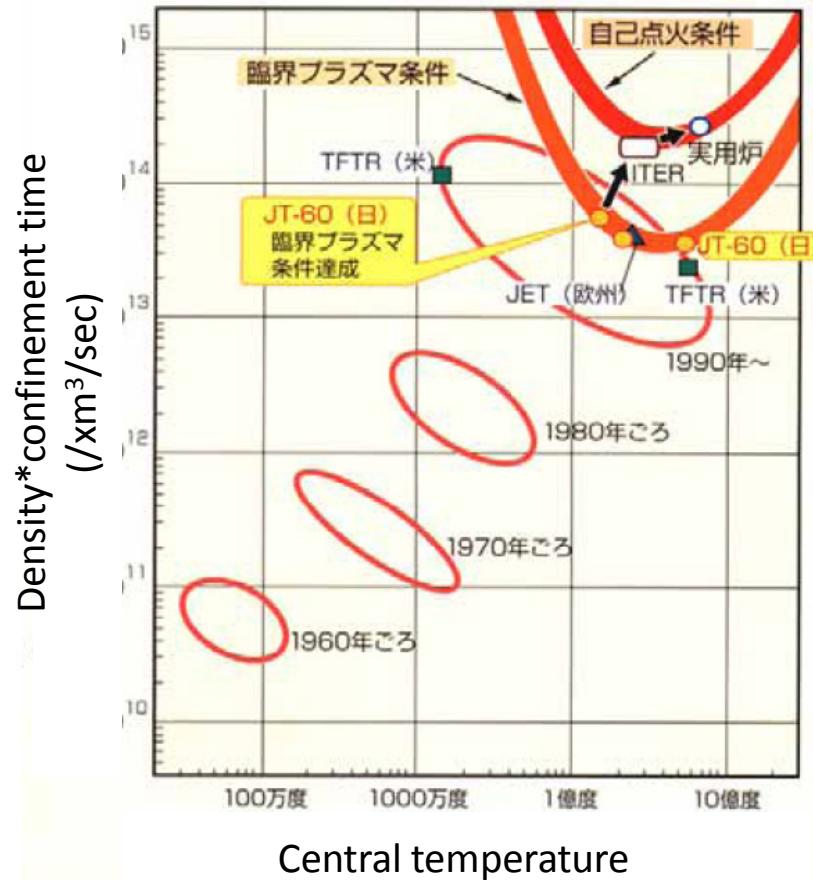
$$E^{(ZF)}$$

$$E^{(LSS)}$$

Micro-scale vortices are embedded in macro-scale flows

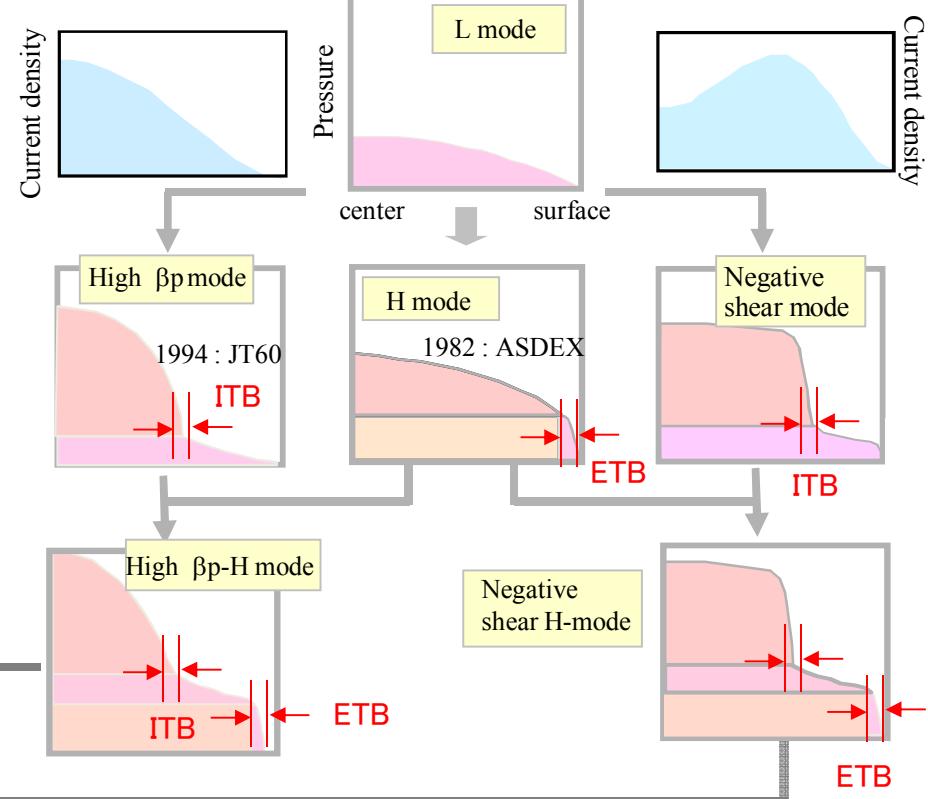
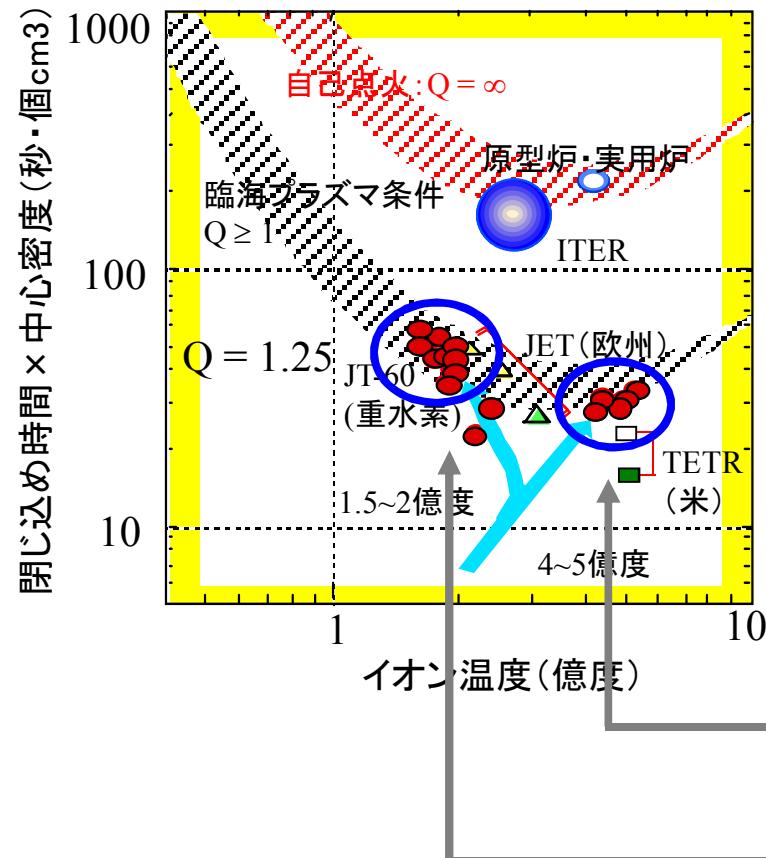


What is the role of ZF and LSS on transport ?



Serious challenge  
 $Q \sim n\tau_E T$    Opposite dependence to the goal  
 (Bohm scaling)

# “Selection of pass” and structure formation in high pressure state leading to burn, i.e. $Q \sim 1$



# JT-60 High Performance Shot

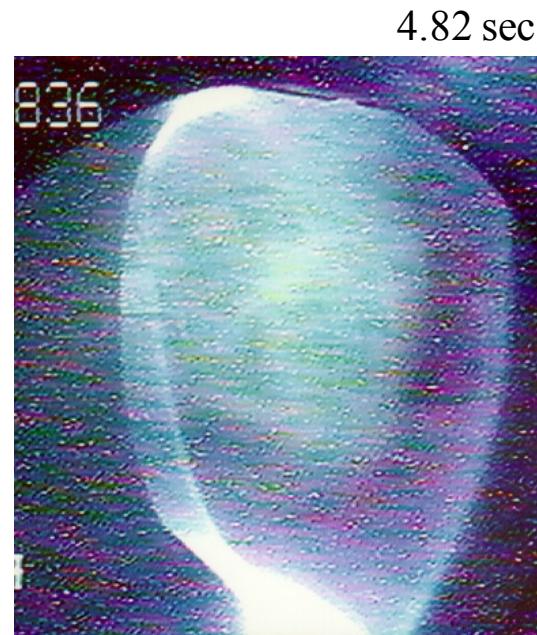
June 11, (June 25 :press) 1998  
Shot number E31872 ,  $Q_{DT} \sim 1.25$

- Reversed magnetic shear configuration
- Internal transport barrier formation

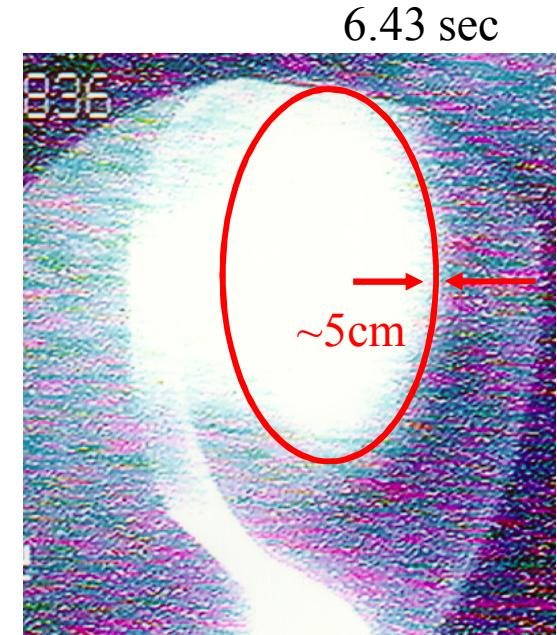
[*Ishida, et al., IAEA, '98, NF, '99*]



Tem. : 7 keV  
Den. :  $1.5 \times 10^{13} / \text{cm}^3$

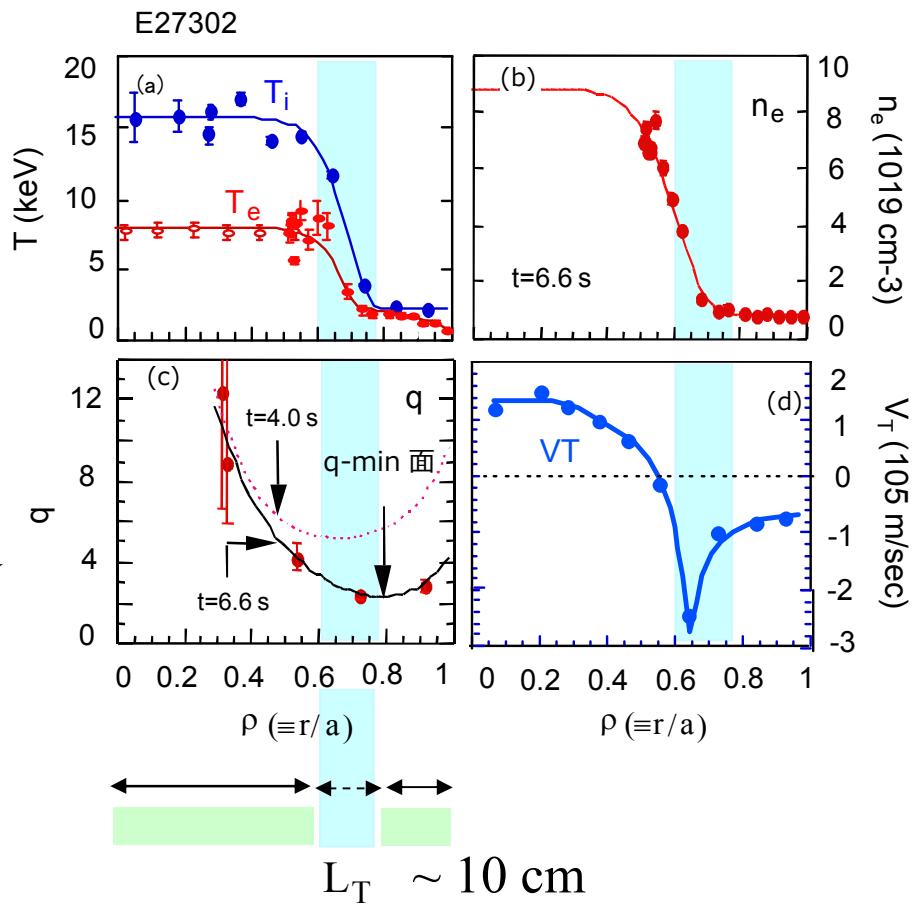
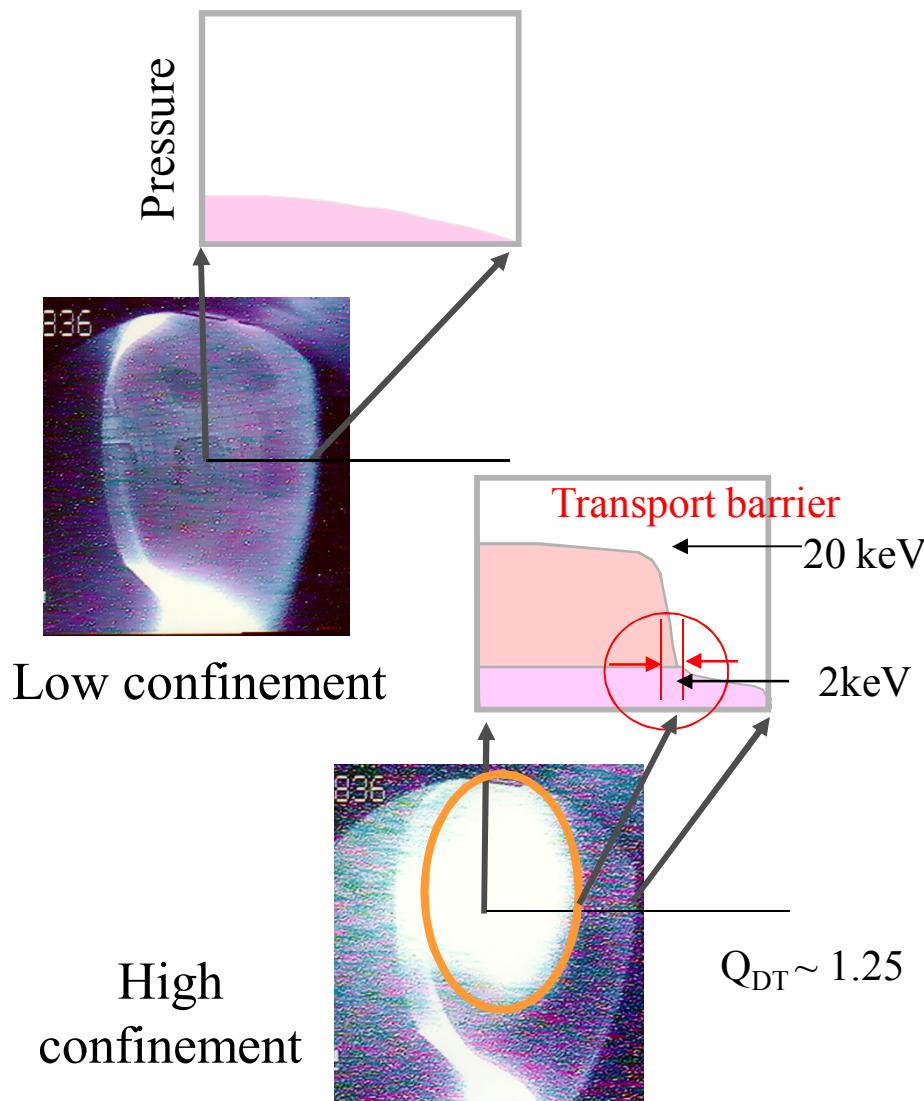


Tem. : 14 keV  
Den. :  $3.1 \times 10^{13} / \text{cm}^3$

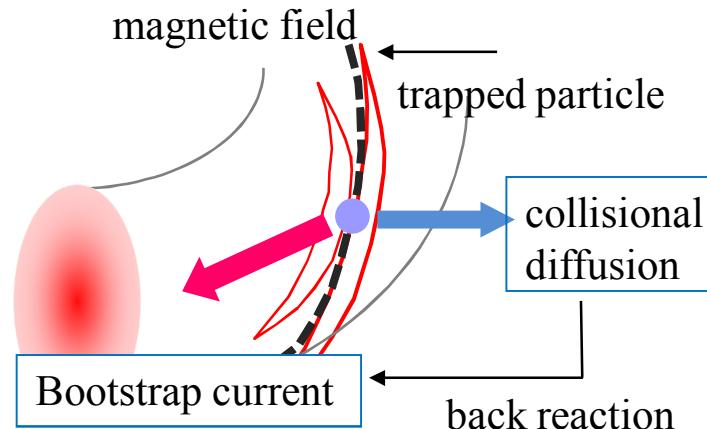


Tem. : 19 keV  
Den. :  $4.8 \times 10^{13} / \text{cm}^3$

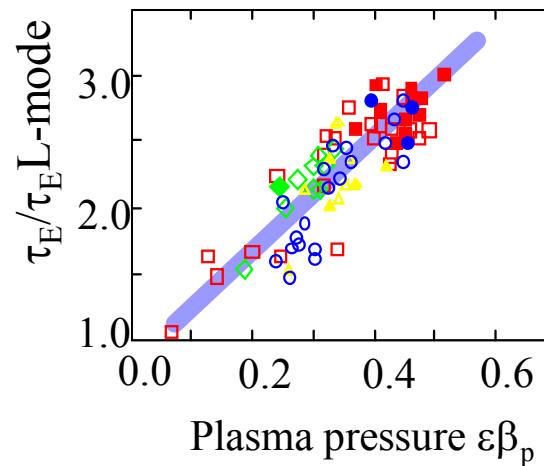
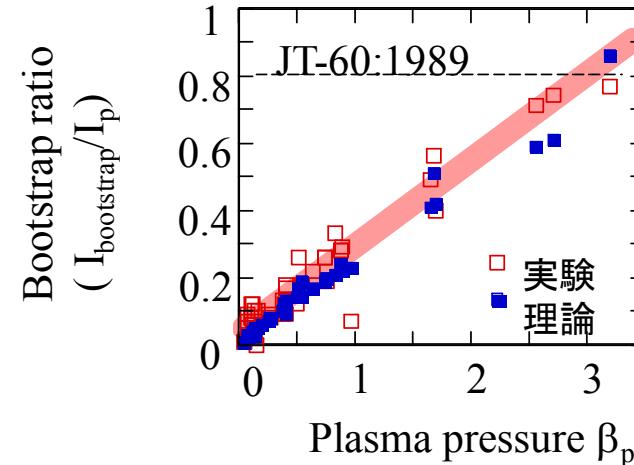
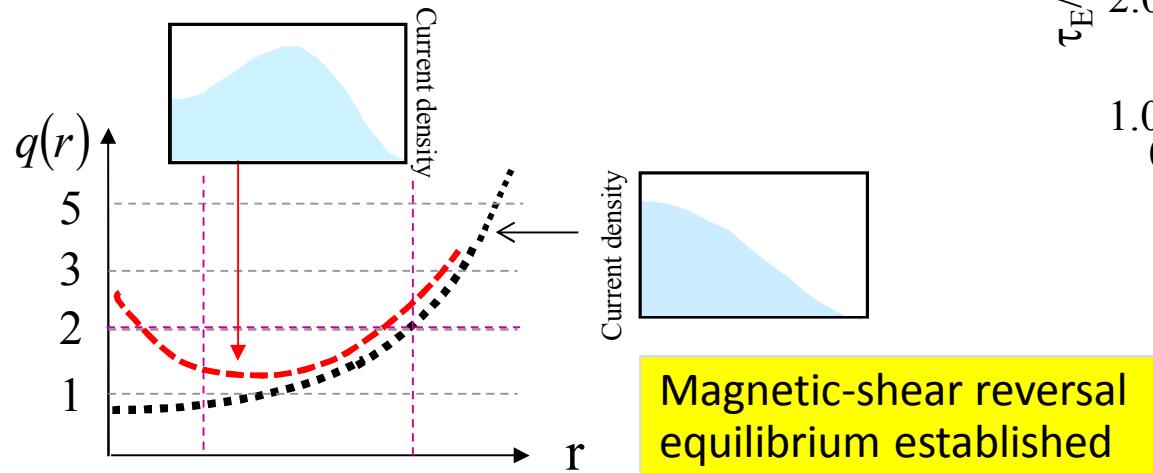
# High performance plasma (internal transport barrier : ITB ) achieved by having a structure



# Synergetic relation between self-sustainment of current and increase of confinement (self-organization realized)

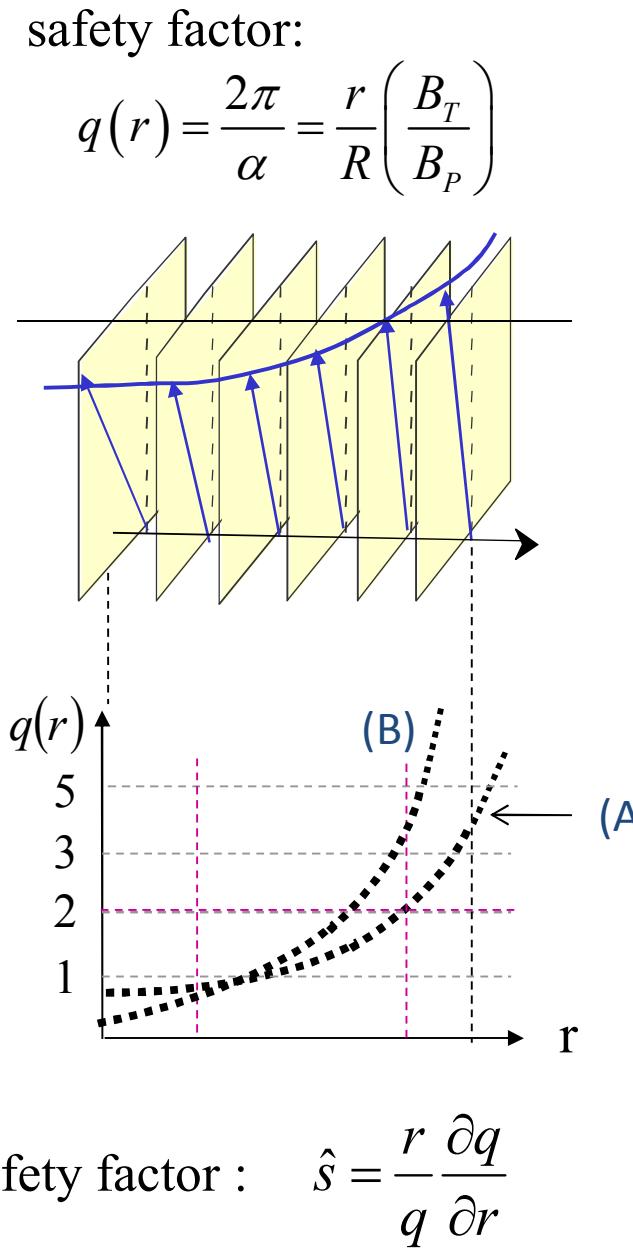
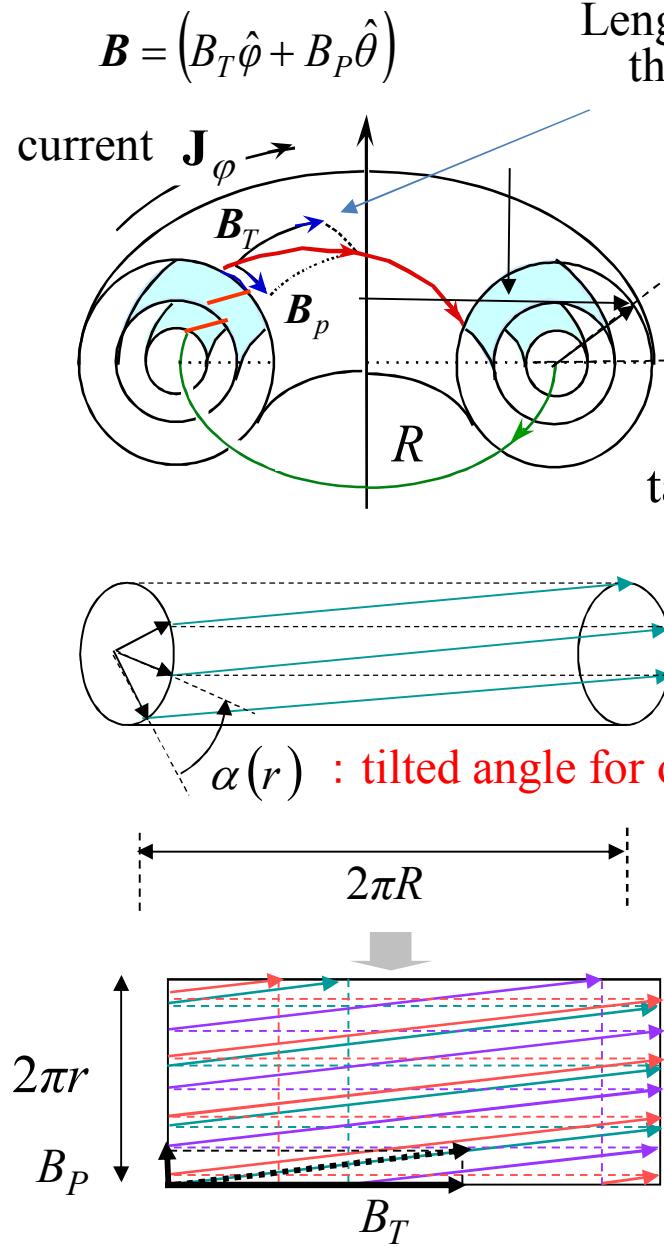


$$J_b = \frac{\sqrt{\varepsilon}}{B_p} \frac{dp}{dp} \quad \frac{I_{BS}}{I_{total}} \propto \sqrt{\varepsilon} \beta_p \quad \left( \beta_p \equiv \frac{p}{B_p^2/2\mu_0} \right)$$



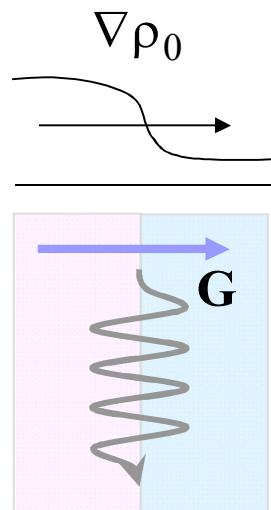
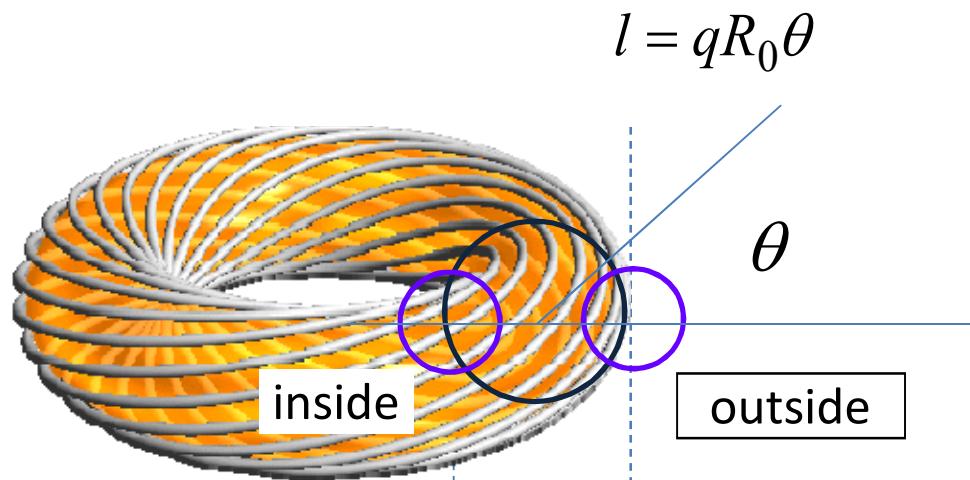
Should be unstable from the linear aspect

# Magnetic fields in fusion device

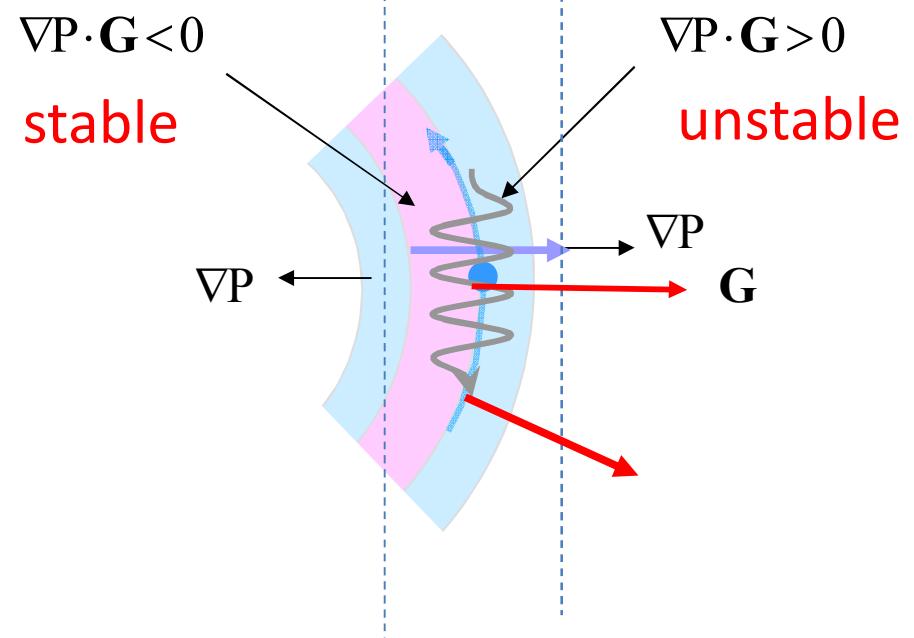


Length from  $\theta=0$  (outside)  
to  $\theta=\pi$  (inside)

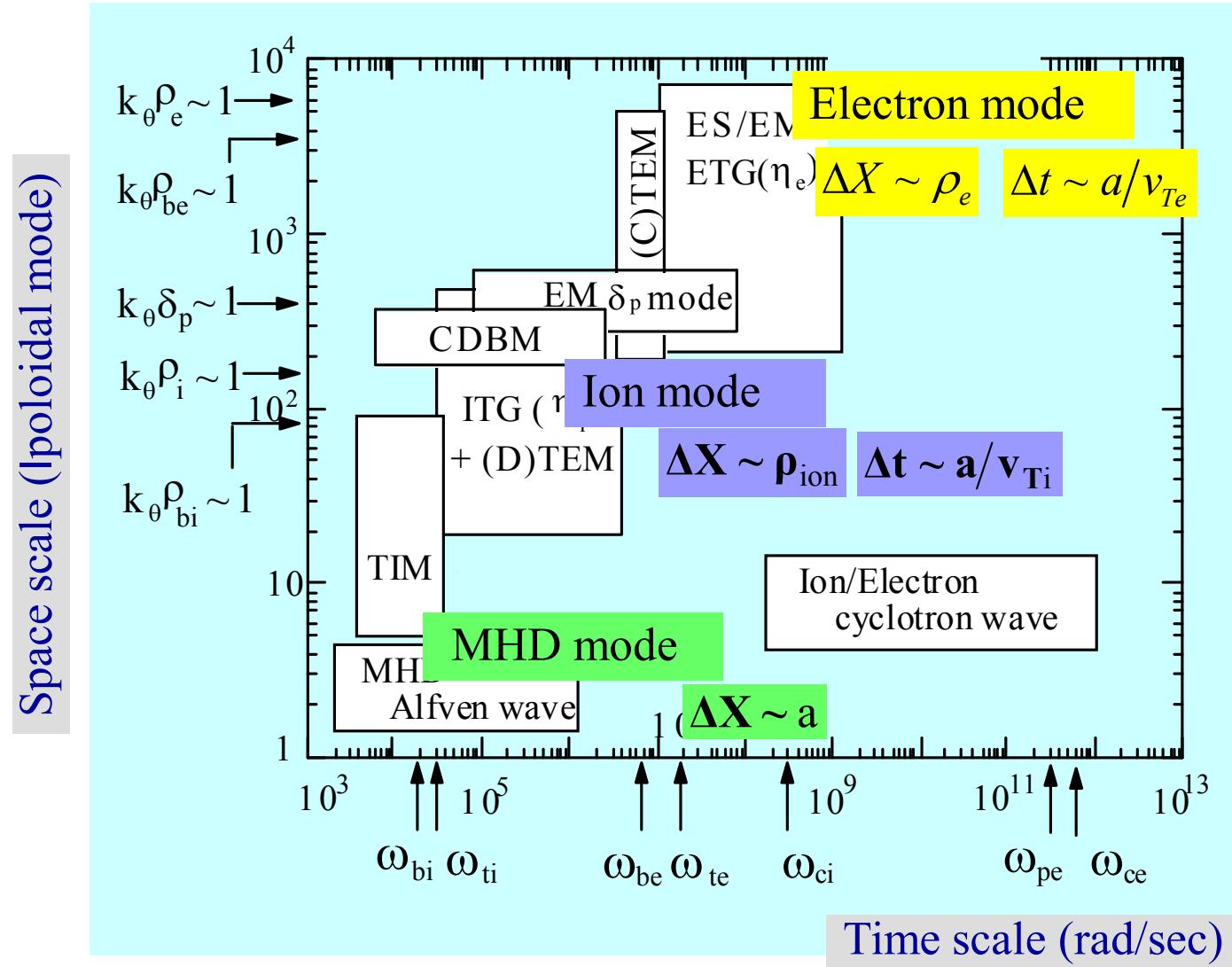
$$l = \pi q R_0$$



$\nabla\rho_0 \cdot \mathbf{G} > 0$   
Rayleigh-Taylor  
Instability



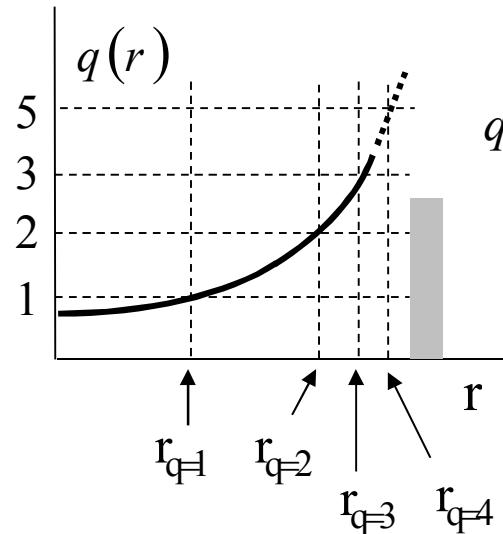
# Fluctuations in magnetically confined plasma



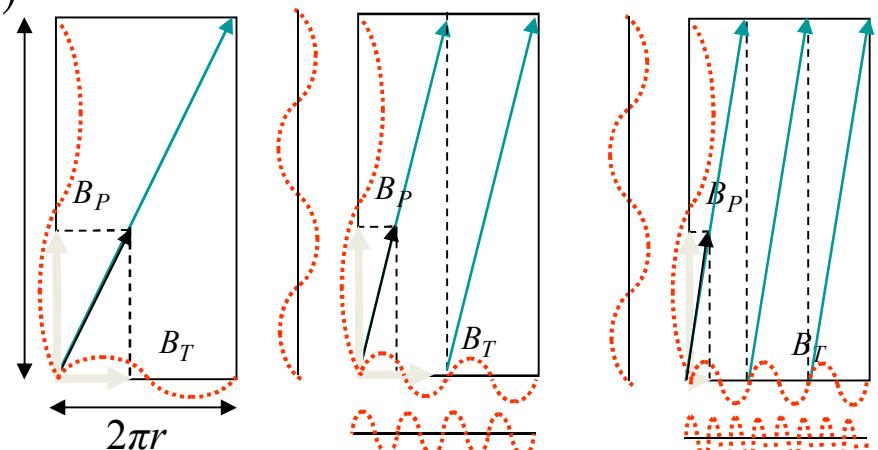
# Coupling between magnetic structure and wave : resonance

perturbation  $A(\mathbf{x}, t) = A_0 \exp(-in\phi + im\theta - i\omega t)$

non-rational and non-rational surface :



$$q(r) \equiv \frac{2\pi}{\alpha} = \frac{m \text{ (poloidal mode)}}{n \text{ (toroidal mode)}}$$



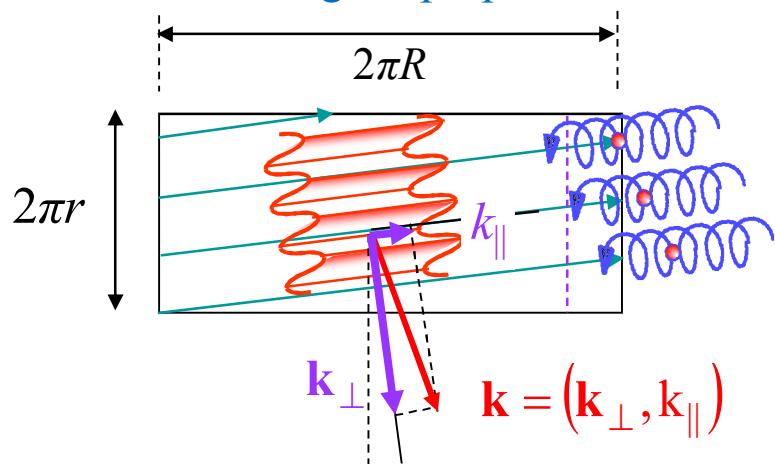
$$\begin{aligned} m/n &= 1/1 \\ &= 2/2 \\ &= 3/3 \end{aligned}$$

$$\begin{aligned} m/n &= 2/1 \\ &= 4/2 \\ &= 6/3 \end{aligned}$$

$$\begin{aligned} m/n &= 3/1 \\ &= 6/2 \\ &= 9/3 \end{aligned}$$

Long wavelength : along field line

Short wavelength : perpendicular to field line



$$\mathbf{k} = k_{\parallel} \mathbf{b} + \mathbf{k}_{\perp}$$

$$k_{\parallel} = \hat{\mathbf{b}} \cdot \mathbf{k} = i \hat{\mathbf{b}} \cdot \nabla = i \left( \hat{\boldsymbol{\phi}} + \frac{r}{qR} \hat{\boldsymbol{\theta}} \right) \cdot \left( \frac{\hat{\boldsymbol{\phi}}}{R} \frac{\partial}{\partial \phi} + \frac{\hat{\boldsymbol{\theta}}}{r} \frac{\partial}{\partial \theta} \right)$$

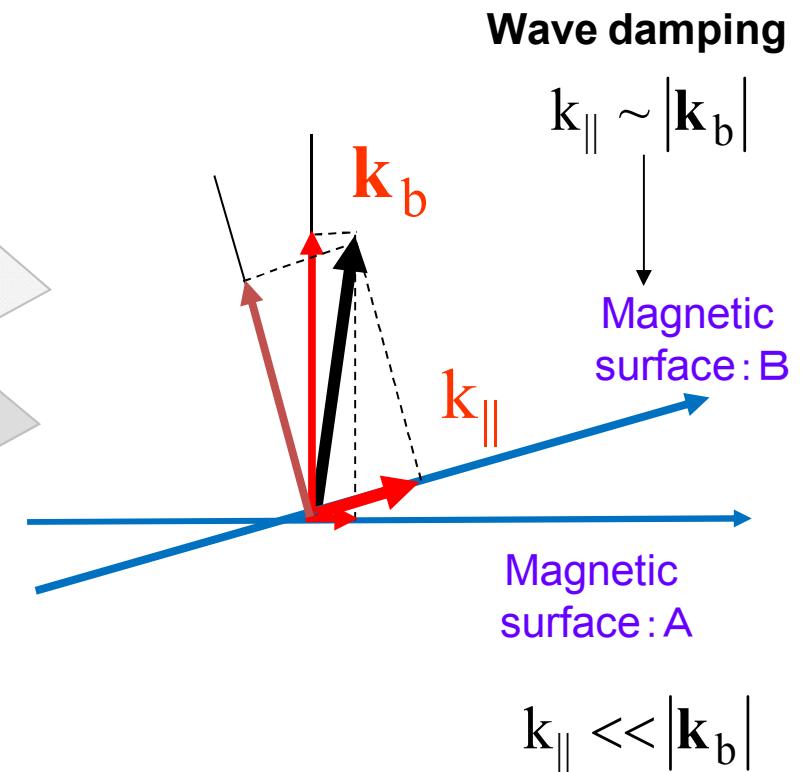
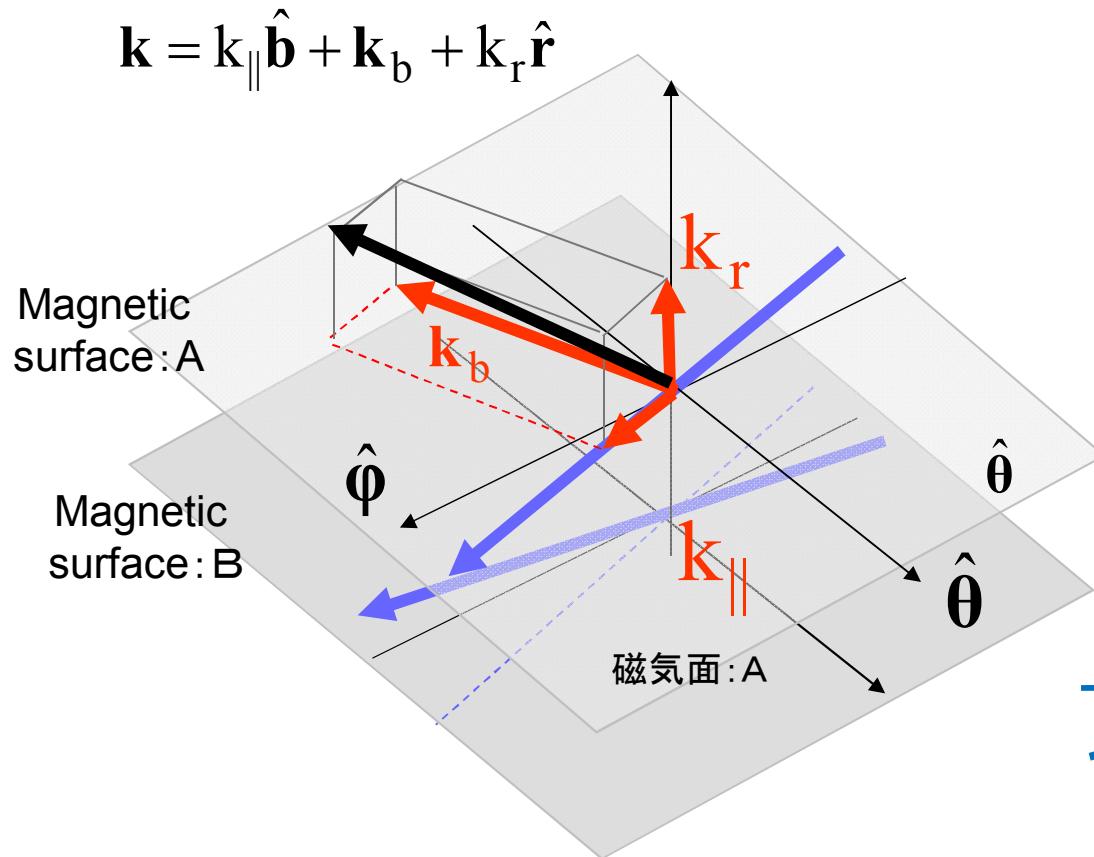
$$= \frac{1}{R} \left( n - \frac{m}{q(r)} \right) \quad k_{\parallel} = 0 \rightarrow q(r) = \frac{m}{n}$$

$$|\mathbf{k}_{\perp}| < \frac{1}{\rho_j} \quad k_{\parallel} \ll |\mathbf{k}_b| \quad \left( k_{\parallel} \sim \frac{1}{qR} \right)$$

# Radial localization of wave by having “magnetic shear”

$$q(r) = \frac{2\pi}{\alpha} = \frac{r}{R} \left( \frac{B_T}{B_P} \right) \quad \hat{s} = \frac{r}{q} \frac{\partial q}{\partial r}$$

$$\begin{aligned} (\hat{r}, \hat{\theta}, \hat{\phi}) &\rightarrow (\hat{r}, \hat{r} \times \hat{\mathbf{b}}, \hat{\mathbf{b}}) \\ (k_r, k_\theta, k_\phi) &\rightarrow (k_r, |k_b|, k_{||}) \end{aligned}$$



# Typical radial mode width of the fluctuation

$$k_{\parallel} = \hat{\mathbf{b}} \cdot \mathbf{k} = \left( \hat{\phi} + \frac{r}{qR} \hat{\theta} \right) \cdot \mathbf{k}$$

$$= \frac{1}{qR} [nq(r) - m]$$

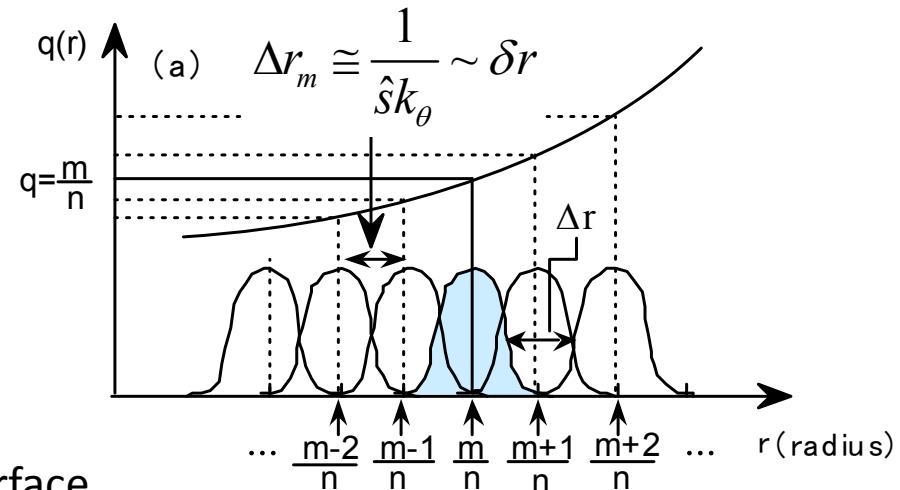
$$q(r) = q(r = r_m) + \frac{\partial q}{\partial r} (r - r_m) + \dots$$

Distance between two different rational surface

$$\Delta r_m \cong \frac{1}{\hat{s} k_\theta}$$

$$k_{\parallel} = \frac{\hat{s}}{qR} k_\theta (r - r_m) \sim \frac{1}{qR}$$

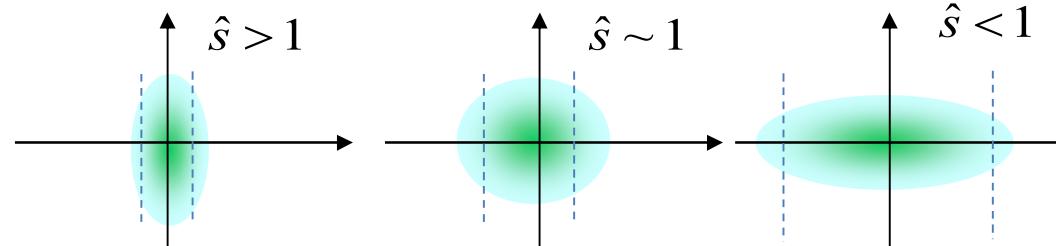
$$\delta r \sim |r - r_m| \sim \frac{1}{\hat{s} k_\theta} \sim \frac{\rho_j}{\hat{s}} \quad |k_\theta| \sim \frac{1}{\rho_j}$$



Distance of poloidal wave number for adjacent two rational surface

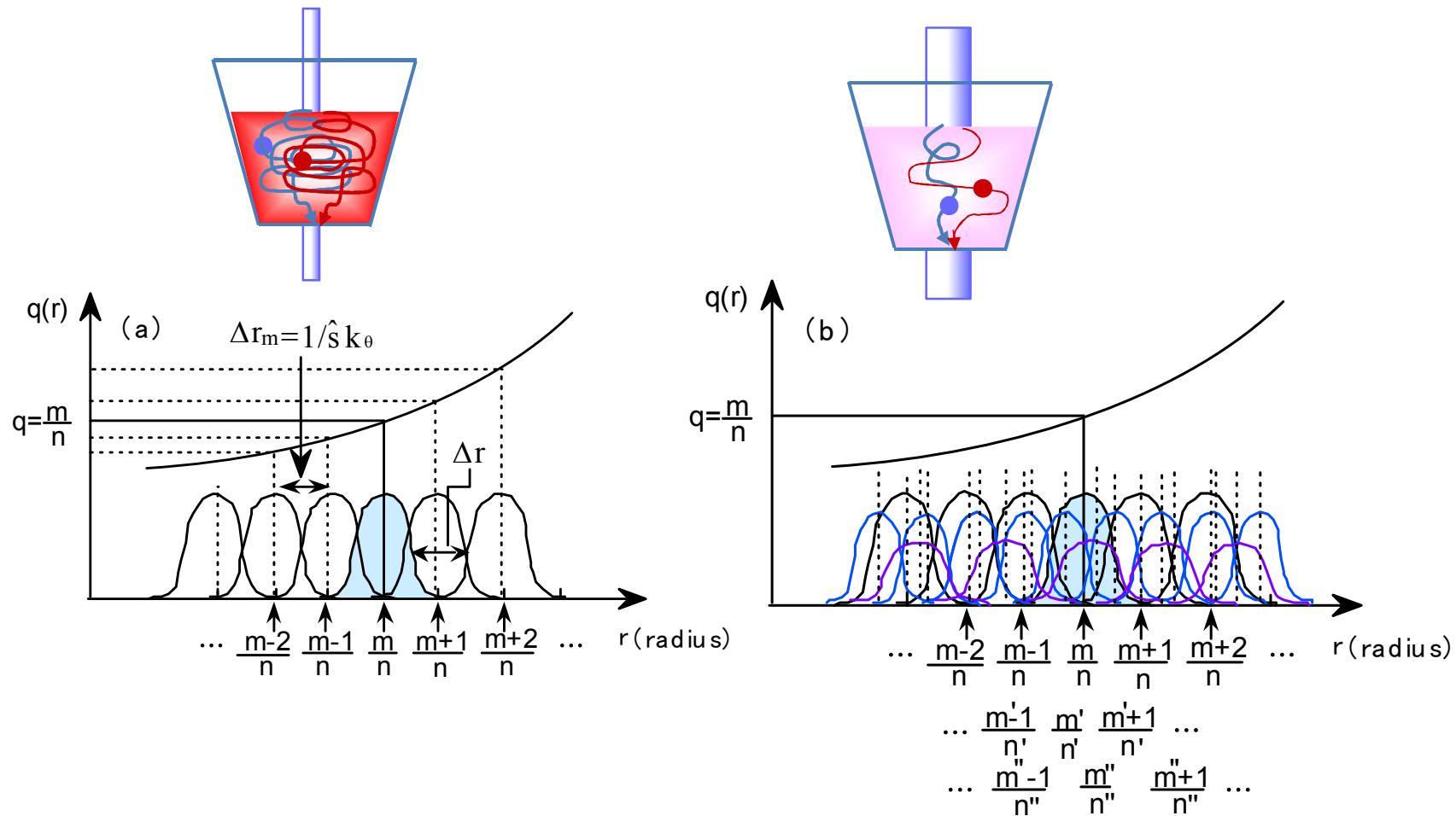
$$\Delta k_\theta = \left| \frac{m}{r_m} - \frac{m+1}{r_m + \Delta r_m} \right| \cong \frac{k_\theta}{m} \left| 1 - \frac{1}{\hat{s}} \right| < \frac{k_\theta}{m}$$

Wave excitation across the different rational surface with same toroidal number



# Distribution of rational surface : incomplete cage

Origin of leaking of across the magnetic surface



$$\Delta r_m \cong \frac{1}{\hat{s} k_\theta} \cong \delta r$$

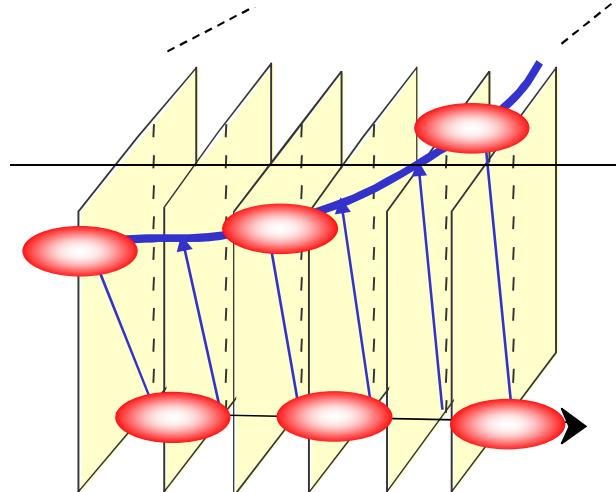
$$\Delta r_{m,n} \cong \frac{1}{n^2 |\partial q / \partial r|} \cong \frac{\Delta r_m}{n}$$

## Fluctuation and magnetic shear

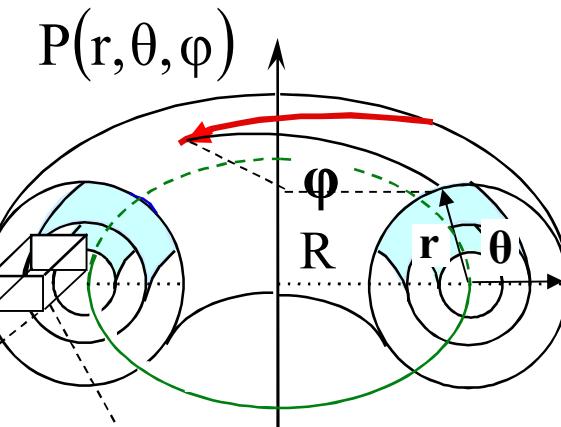
Magnetic fields are designed to minimize plasma fluctuations based on **linear theory**

$$\delta r \sim \frac{1}{\hat{s} k_\theta}$$

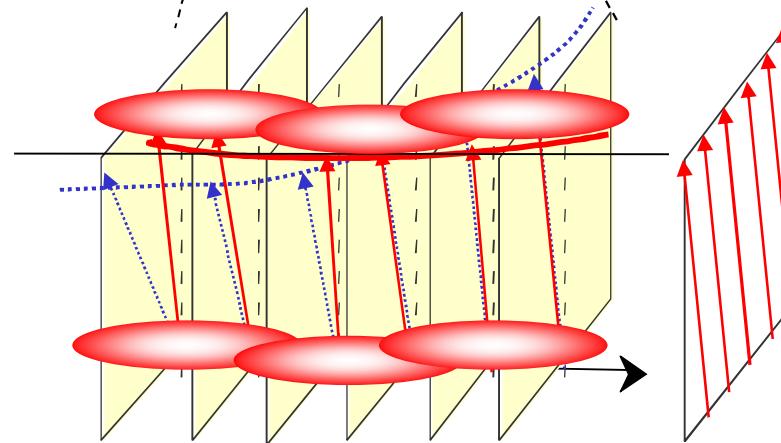
$$\hat{s} = \frac{r}{q} \frac{\partial q}{\partial r} \sim 1$$



disintegration  
along magnetic field line



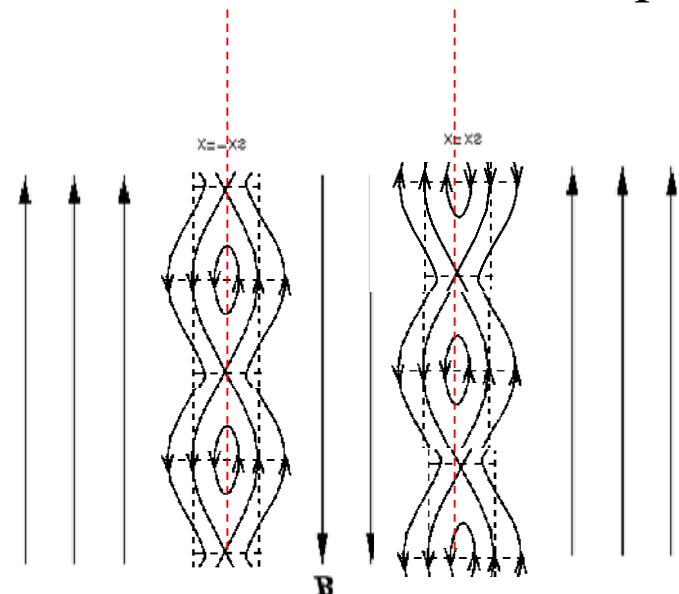
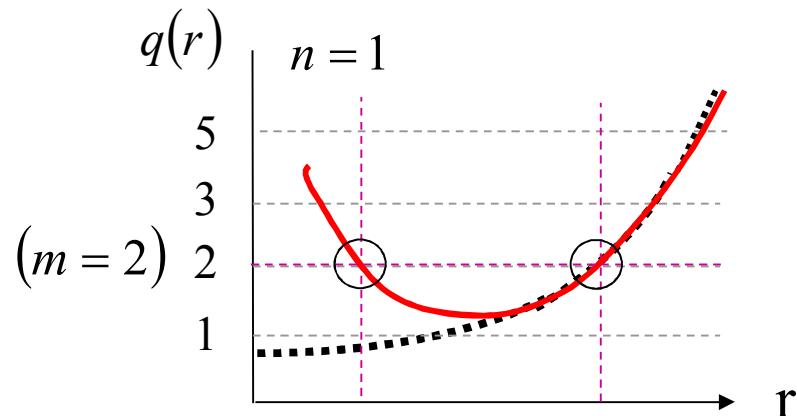
$$\hat{s} = \frac{r}{q} \frac{\partial q}{\partial r} \sim 0$$



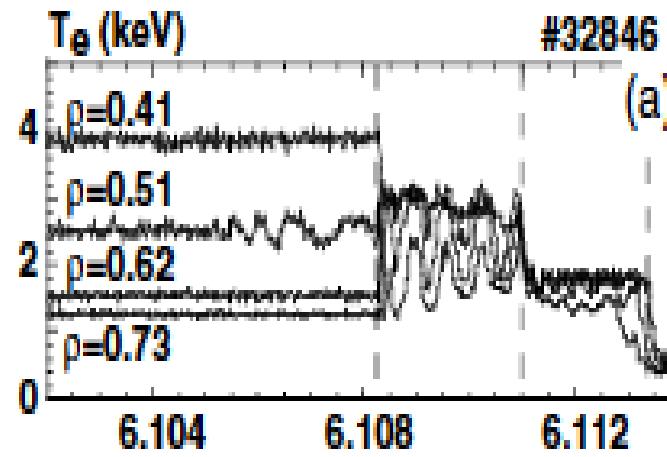
Survive along magnetic field line  
and establish “standing wave”

# Reversed magnetic shear plasma and double tearing mode (DTM)

$$q(r) \equiv \frac{r}{R} \frac{B_T}{B_P} \sim \frac{m}{n}$$



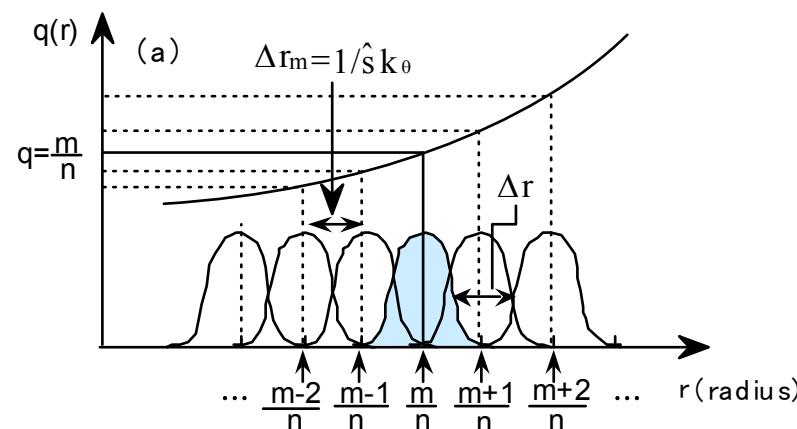
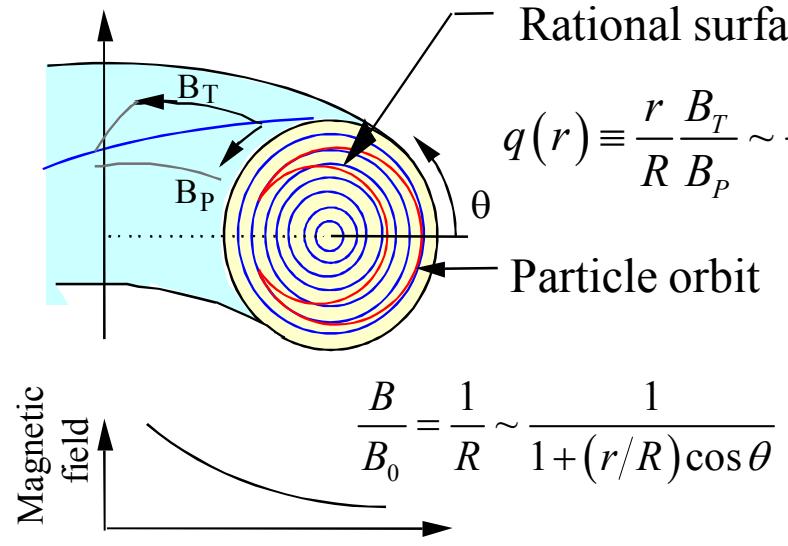
S. Takechi et al., Nucl. Fusion 42, 5 (2002)



- ✓ Disruptive event terminating high performance plasma
- ✓ Explosive (sudden) nature with pre-cursor
- ✓ Double tearing mode (DTM) with multiple current sheet

# Toroidal mode coupling of wave due to poloidal asymmetry

## 0-th order Ballooning mode local theory



- Linear eigen-mode analysis

$$\Phi(\mathbf{x}, t) = \phi(r, \theta) \exp(-in\varphi - i\omega t)$$

$$\phi(r, \theta) = \exp(im_0\theta) \sum_j \phi_j(r) \exp(ij\theta_j)$$

- Bloch theory (translational symmetry)

$$\begin{aligned}\phi_m(x) &= \phi_{m-1}(x-1) \exp(i\theta_0) = \dots \\ &= \phi_0(x-m) \exp(i\mathbf{m}\theta_0)\end{aligned}$$

$\theta_0$  : Bloch angle

- Eigen mode analysis :

$$\omega = \omega_r(\hat{s}, x) + i\gamma_0(\hat{s}, x) \cos \theta_0$$

$$\omega_r(\hat{s}, x) \sim \omega_D \sim 2(k_\theta \rho_i) \frac{v_i}{R}$$

Radial mode width and Bloch angle undetermined

# Global toroidal mode structure

- Mode-structure in non-uniform medium :  
(Extension to non-local ballooning theory)

$$\phi_j(r, x) = A(r) \underbrace{\phi_0(x - j)}_{\rightarrow \text{0th order eigen-}} \exp(i\theta_0 j)$$

$$A(r) = \exp \left[ -\frac{k_\theta \hat{s}^{\text{function}}}{2\gamma_0 \sin \theta_0} \left( \frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r} \right) r^2 \right]$$

## Spatial variation of real frequency (diamagnetic shear )

Spatial variation of plasma rotation

- Representation of 2d-structure :  $(\Delta r, \theta_0)$

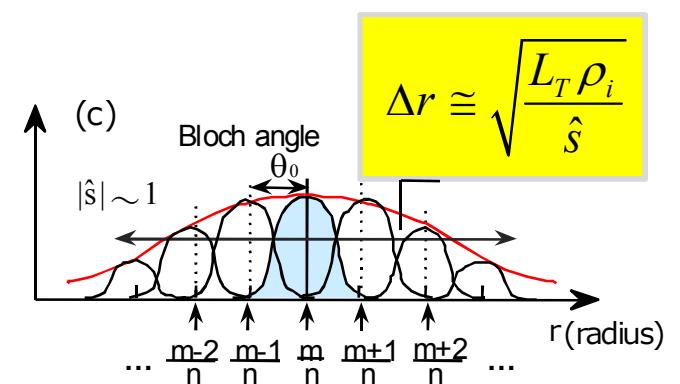
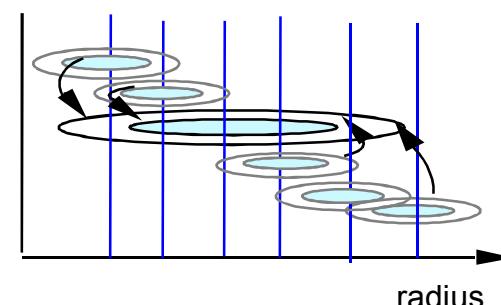
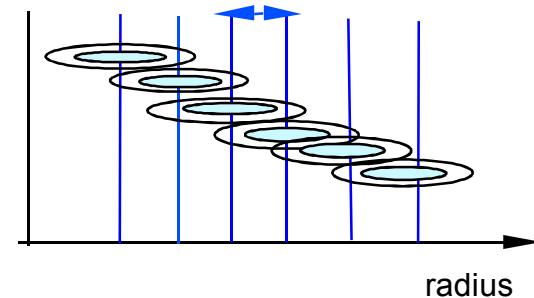
$$\Delta r \quad \approx \quad \left| \frac{2\gamma_0 \sin \theta_0 k_\theta \hat{s}}{k_\theta \hat{s} \left( \frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r} \right)} \right|^{\frac{1}{2}}$$

$$(\Delta\theta_0)_{\max} \cong \mp \left| \frac{\left( \frac{\partial\omega_r}{\partial r} + \frac{\partial\omega_f}{\partial r} \right)}{2k_\theta\gamma_0\hat{s}} \right|^{\frac{1}{3}}$$

$$\gamma(\theta_0) = \gamma_0 \cos \theta_0$$

## Growth rate reduction from mid-plane $\theta_0 = 0$

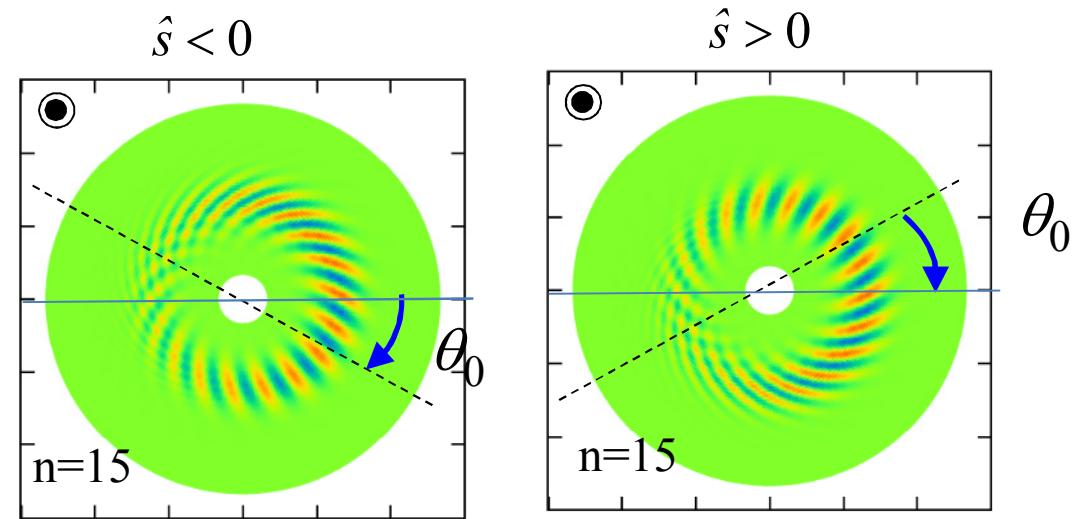
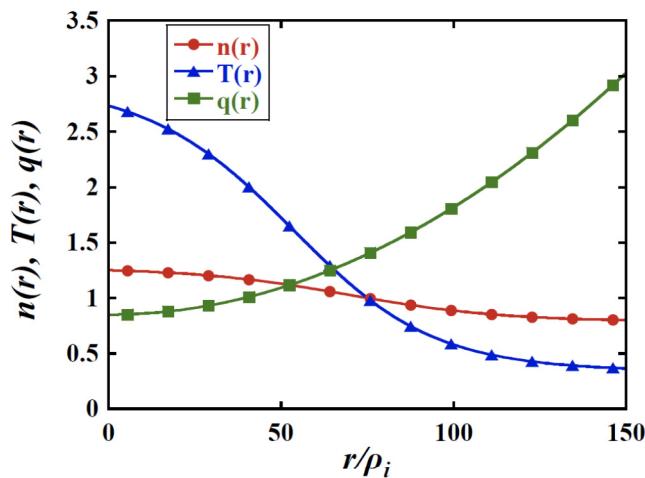
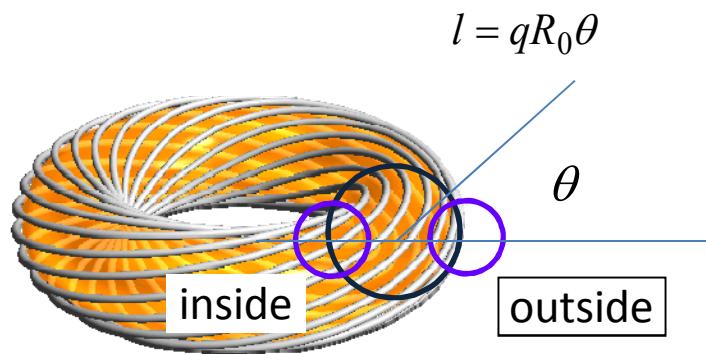
$$\Delta r \equiv \sqrt{\frac{L_T \rho_i}{\hat{s}}}$$



# Global toroidal mode structure

$$\Delta r \cong \left| \frac{2\gamma_0 \sin \theta_0 \hat{s}}{k_\theta \hat{s} \left( \frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r} \right)} \right|^{\frac{1}{2}} \cong \sqrt{\frac{L_T \rho_i}{\hat{s}}}$$

$$(\Delta \theta_0)_{\max} \cong \mp \left| \frac{\left( \frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r} \right)}{2 k_\theta \gamma_0 \hat{s}} \right|^{\frac{1}{3}} \cong \mp \left| \frac{1}{\hat{s} k_\theta L_T} \right|^{\frac{1}{3}}$$



$$a / R_0 = 0.36$$

$$R_0 / L_n = 2.22$$

$$R_0 / L_T = 10$$

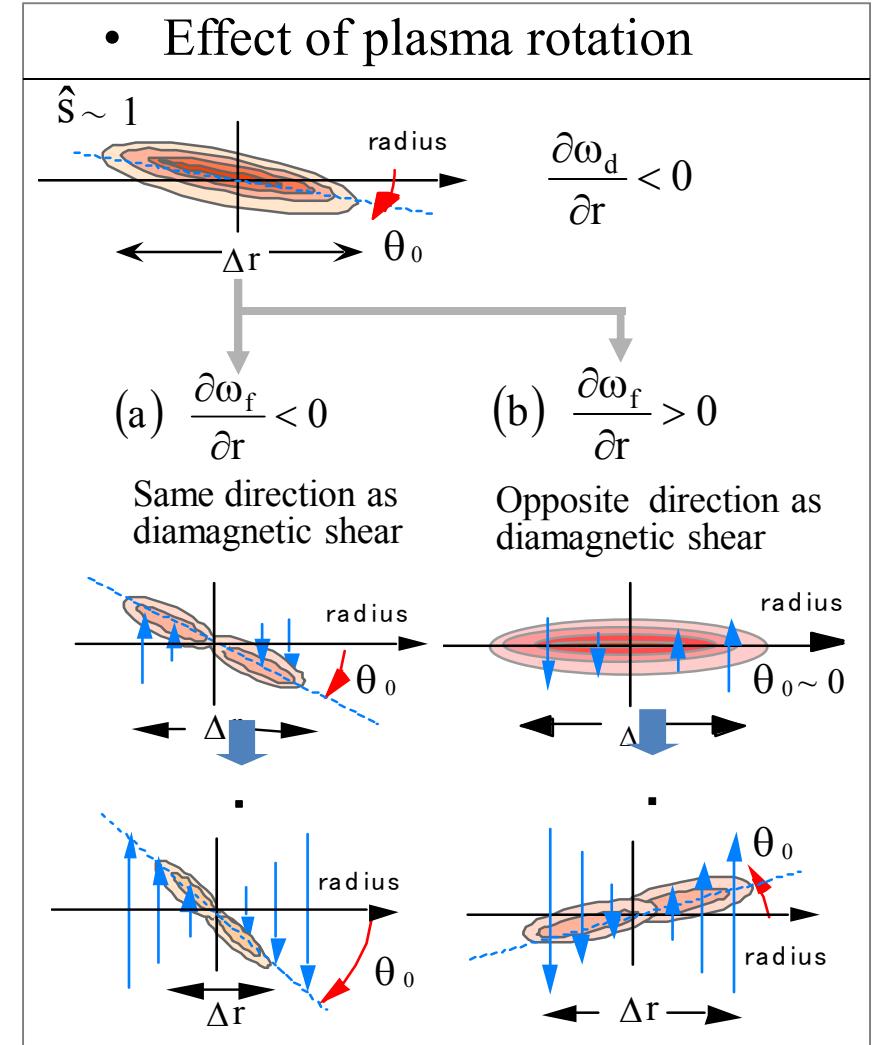
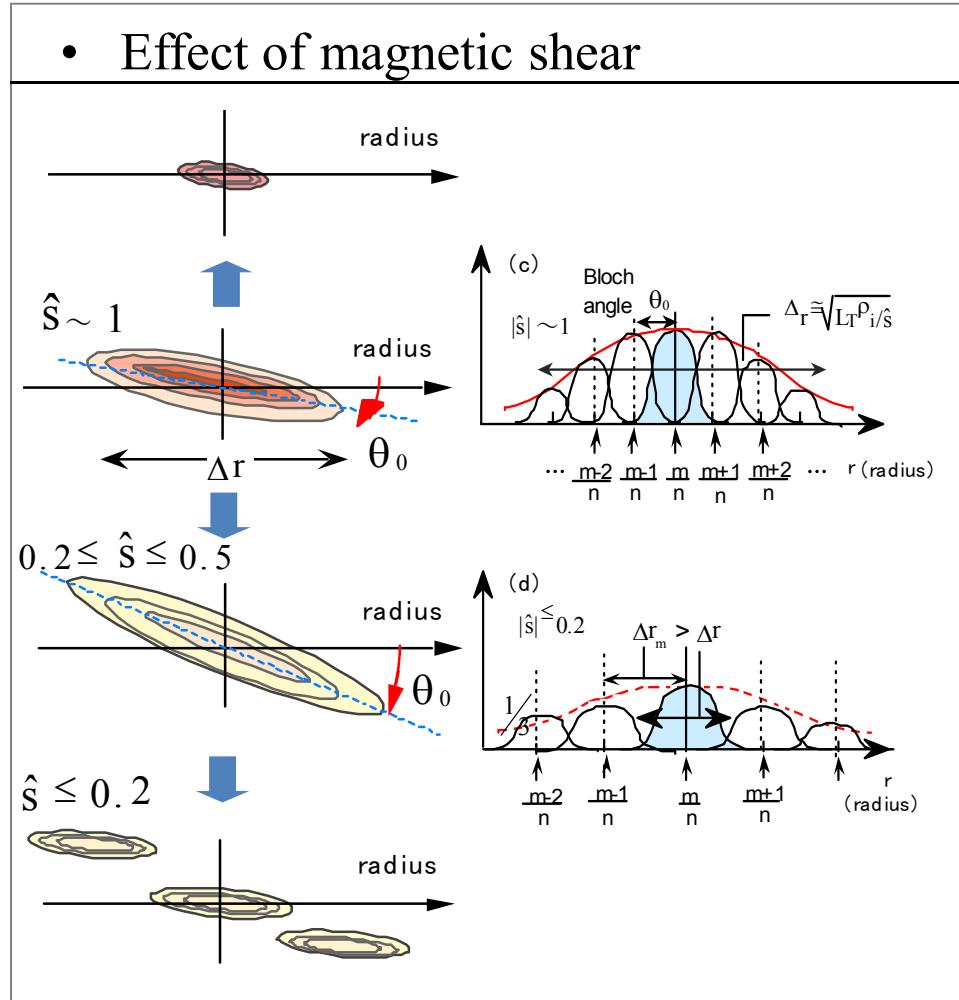
$$L_T = 41.7 \rho_i$$

$$\gamma(\theta_0) \cong \gamma_0 \cos \theta_0$$

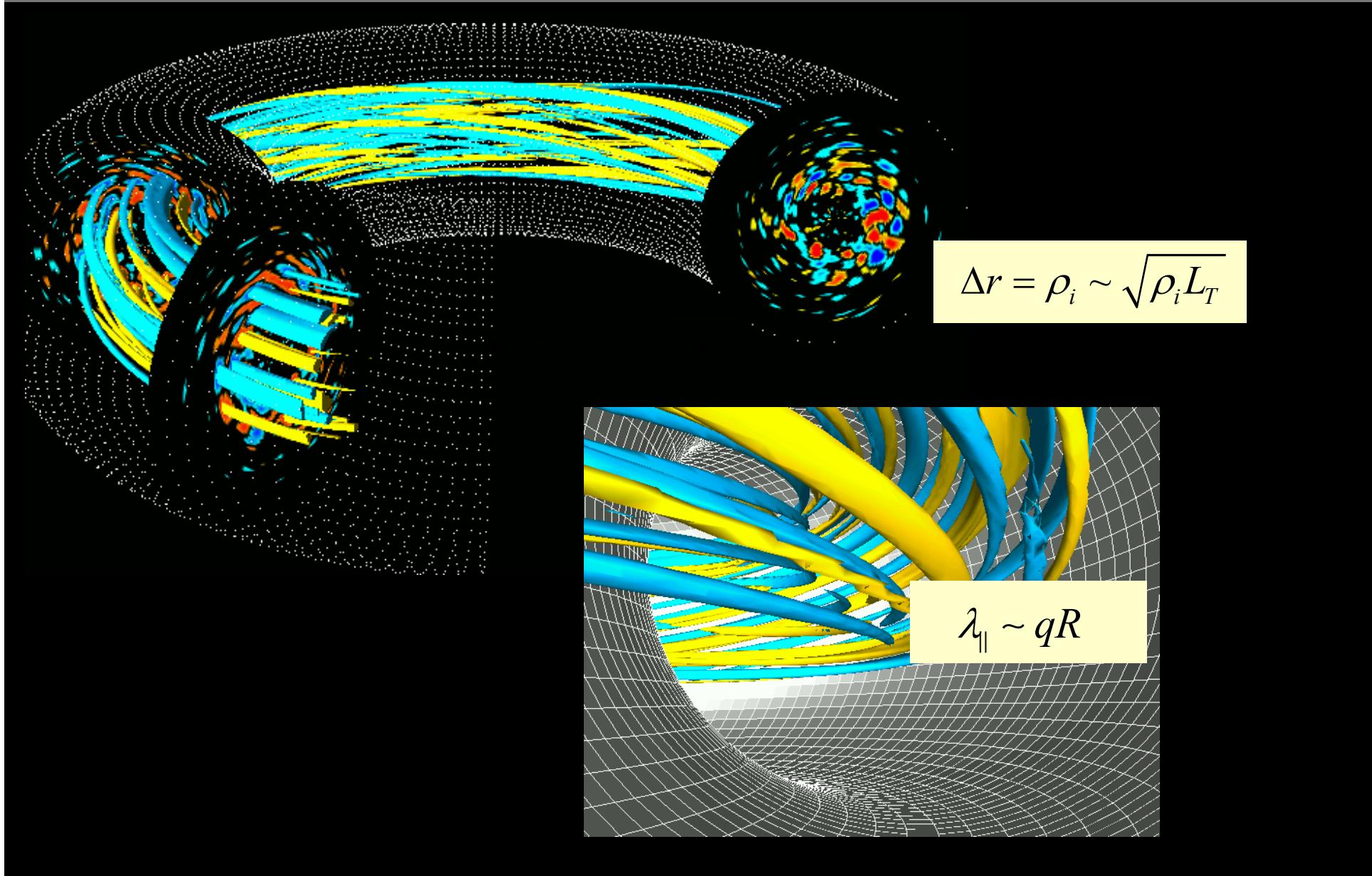
Growth rate reduction  
as the tilting angle increase

# Global toroidal ITG mode structure including profile effect

$$\Delta r \approx \frac{2\gamma_0 \sin \theta_0}{k_\theta \hat{s} \left( \frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r} \right)} \quad (\Delta \theta_0)_{max} \approx \mp \sqrt[3]{\frac{\left( \frac{\partial \omega_r}{\partial r} + \frac{\partial \omega_f}{\partial r} \right)}{2k_\theta \gamma_0 \hat{s}}} \quad \gamma(\theta_0) \approx \gamma_0 \cos \theta_0$$

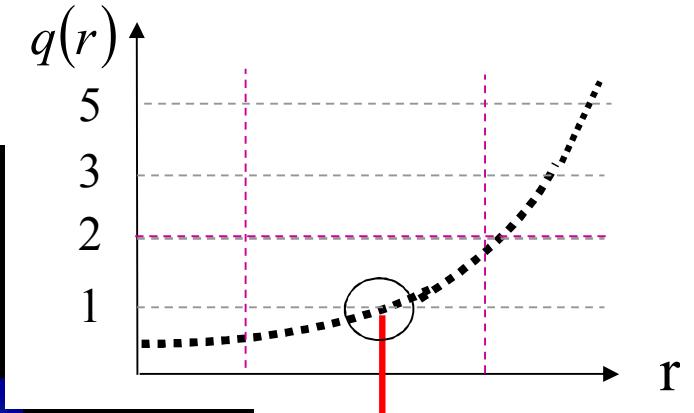
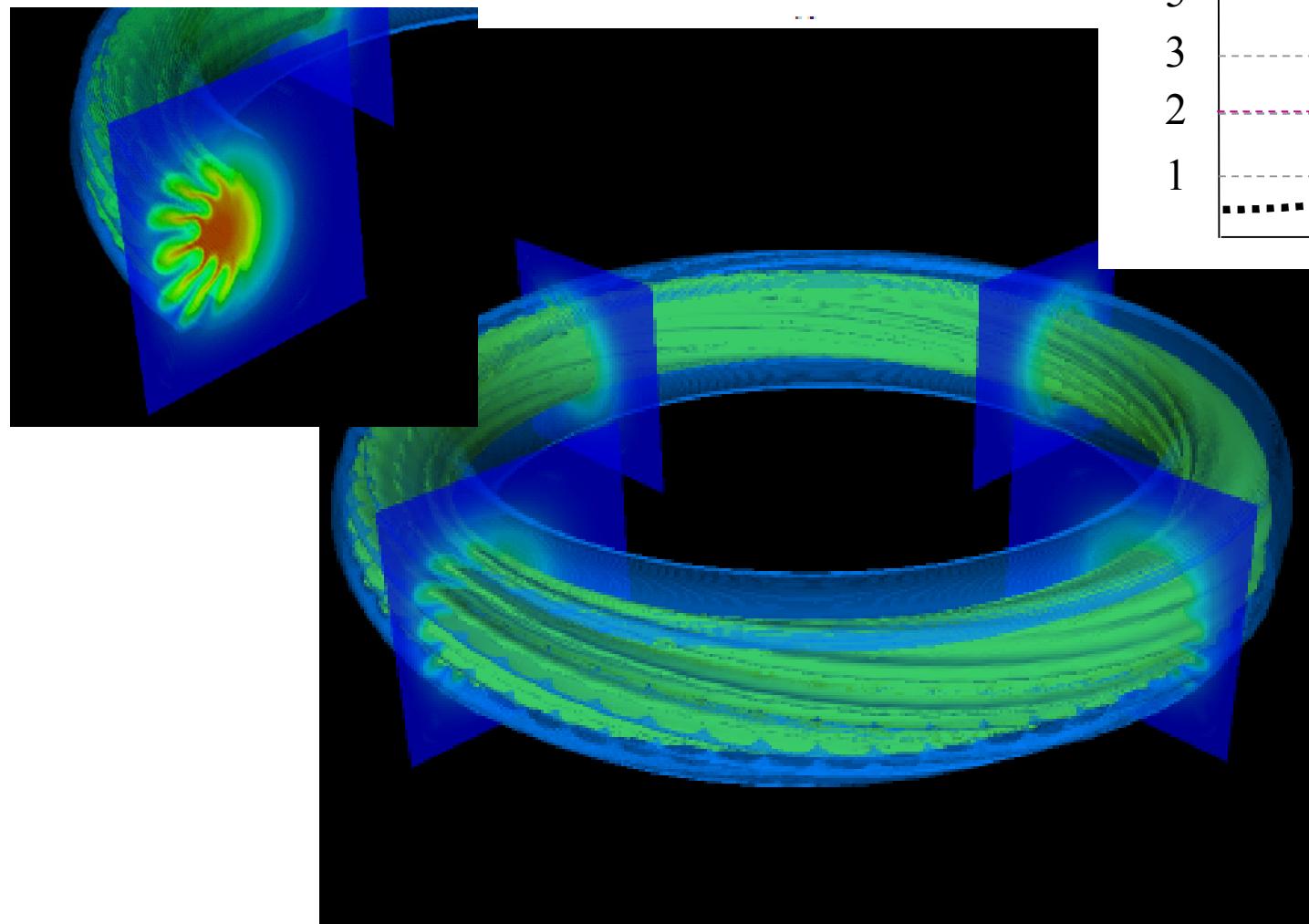


# Global turbulent simulation of ion temperature gradient mode (ITG)



# Interaction between 1/1 internal kink and high-m/n ballooning mode

*Y. Kagei and Y. Kishimoto:  
Full set non-linear MHD simulation 2006, JPS*



$$q(r) \equiv \frac{r}{R} \frac{B_T}{B_P} \sim \frac{m}{n}$$

$$\frac{m}{n} = \frac{1}{1}$$

*and*

$$\frac{m}{n} \sim \frac{30}{30}$$

# Development of toroidal full-f GK code

Developed by Imadera/Kishimoto : Kyoto University

$$\frac{\partial f}{\partial t} + \frac{dr}{dt} \frac{\partial f}{\partial r} + \frac{1}{r} \frac{d\theta}{dt} \frac{\partial f}{\partial \theta} + \frac{1}{R} \frac{d\varphi}{dt} \frac{\partial f}{\partial \varphi} + \frac{dv_{||}}{dt} \frac{\partial f}{\partial v_{||}} = C_{coll}$$

GK

Vlasov Eq.

$$\begin{cases} \frac{dr}{dt} = \frac{1}{B_{||}^*} \left( -\frac{v_{||}^2 + \mu B}{R} \sin \theta - \frac{1}{r} \frac{\partial \langle \Phi \rangle_\phi}{\partial \theta} + \frac{r}{qR_0 R} \frac{\partial \langle \Phi \rangle_\phi}{\partial \varphi} \right) \\ \frac{1}{r} \frac{d\theta}{dt} = \frac{1}{B_{||}^*} \left( \frac{v_{||}}{qR} - \frac{v_{||}^2 + \mu B}{rR} \cos \theta + \frac{1}{r} \frac{\partial \langle \Phi \rangle_\phi}{\partial r} \right) \\ \frac{1}{R} \frac{d\varphi}{dt} = \frac{1}{B_{||}^*} \left[ \left( B + \frac{2-\hat{s}}{qR_0} v_{||} \right) \frac{v_{||}}{R} + \mu B \frac{r \cos \theta}{qR_0 R^2} - \frac{r}{qR_0 R} \frac{\partial \langle \Phi \rangle_\phi}{\partial r} \right] \\ \frac{dv_{||}}{dt} = \frac{1}{B_{||}^*} \left[ -\mu B \frac{r \sin \theta}{qR^2} - \frac{1}{qR} \frac{\partial \langle \Phi \rangle_\phi}{\partial \theta} - \left( B + \frac{2-\hat{s}}{qR_0} v_{||} \right) \frac{1}{R} \frac{\partial \langle \Phi \rangle_\phi}{\partial \varphi} + \frac{v_{||}}{R} \left( \frac{\partial \langle \Phi \rangle_\phi}{\partial r} \sin \theta + \frac{\partial \langle \Phi \rangle_\phi}{\partial \theta} \frac{\cos \theta}{r} \right) \right] \end{cases} \quad \left( B_{||}^* = B + \frac{R_0 + (1-\hat{s})R}{qR_0 R} v_{||} \right)$$

GK Q.N. Eq.

$$\Phi - \langle \langle \Phi \rangle \rangle_\phi + \frac{1}{T_{e0}(r)} (\Phi - \langle \Phi \rangle_f) = \frac{1}{n_{i0}(r)} \iint \langle f_1 \rangle_\phi B_{||}^* dv_{||} d\mu$$

DK

Collision

$$C_{coll}(f) = \frac{\partial}{\partial u} \left[ \frac{3\sqrt{\pi}}{2} \frac{n}{v_{th}} \frac{\Phi(v) - \Psi(v)}{2v} \frac{\varepsilon^{3/2}}{q_0 R_0} v_* \left( \frac{\partial f}{\partial u} + \frac{u}{v_{th}^2} f \right) \right] - \frac{1}{n} [aF(x) + bG(x)\xi + cH(x)] f_M$$

$$\Phi(v) = \frac{2}{\sqrt{\pi}} \int_0^v e^{-x^2} dx, \quad \Psi(v) = \frac{\Phi(v) - v\Phi'(v)}{2v^2} = \frac{1}{2v^2} \left( \frac{2}{\sqrt{\pi}} \int_0^v e^{-x^2} dx - \frac{2v}{\sqrt{\pi}} e^{-v^2} \right)$$

# Development of toroidal full-f GK code

Developed by Imadera/Kishimoto : Kyoto University

- Vlasov solver : 4th-order Morinishi scheme
- Field solver : FDM(LU decomposition,  $r$ )  
+ Fourier expansion( $\theta$ - $\varphi$ )
- Time integration : 4th-order RKG scheme
- Parallelization: 3D( $r$ - $\theta$ - $\mu$ ) MPI decomposition
- Heat source and sink

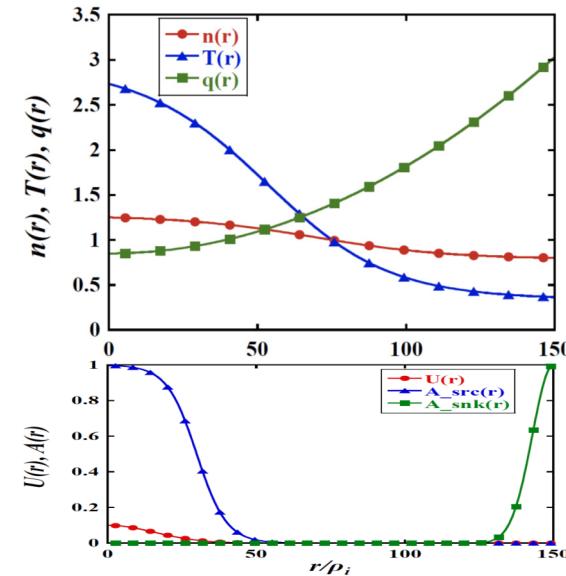
$$S_{src} = A_{src}(x) \tau_{src}^{-1} \left\{ f_M(2\bar{T}_0) - f_M(\bar{T}_0) \right\}$$

$$S_{snk} = -A_{snk}(x) \tau_{snk}^{-1} \left\{ f(t) - f(t=0) \right\}$$

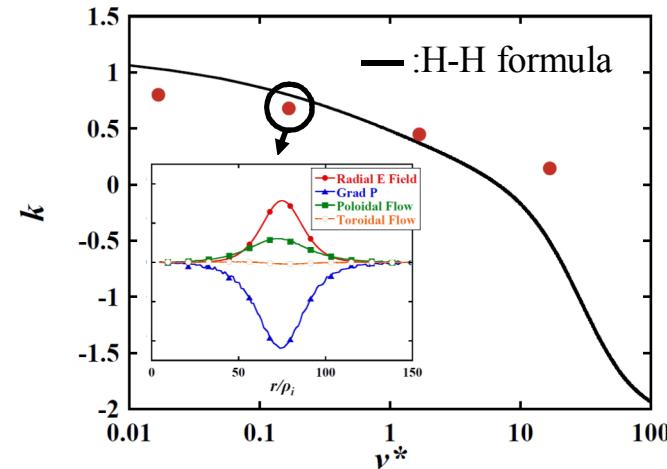
[Y. Idomura, et. al., Nucl. Fusion 49, 065029 (2009).]

- Typical simulation parameter

$$\begin{cases} \rho_* = \frac{\rho_{ti}}{a} = \frac{1}{150} & (N_r, N_\theta, N_\varphi, N_{v_\parallel}, N_\mu) \\ & = (128, 128, 64, 32, 16) \\ \varepsilon = \frac{a}{R_0} = 0.36 & (L_r, L_\theta, L_\varphi, L_{v_\parallel}, L_\mu) \\ & = (150\rho_{ti}, 2\pi, \pi, 12v_{ti}, 18v_{ti}^2 / B_0) \\ \frac{R_0}{L_T} = 10 & P_{in} = 4[MW], 16[MW] \\ \frac{R_0}{L_n} = 2.22 & \tau_{snk}^{-1} \frac{R_0}{v_{ti}} = 0.25, v_* \frac{R_0}{v_{ti}} = 0.5 \end{cases}$$



$$E_r + v_\theta B_\varphi - v_\varphi B_\theta = \frac{1}{n} \frac{dP}{dr}, \quad v_\theta = \frac{k}{B} \frac{dT}{dr}$$

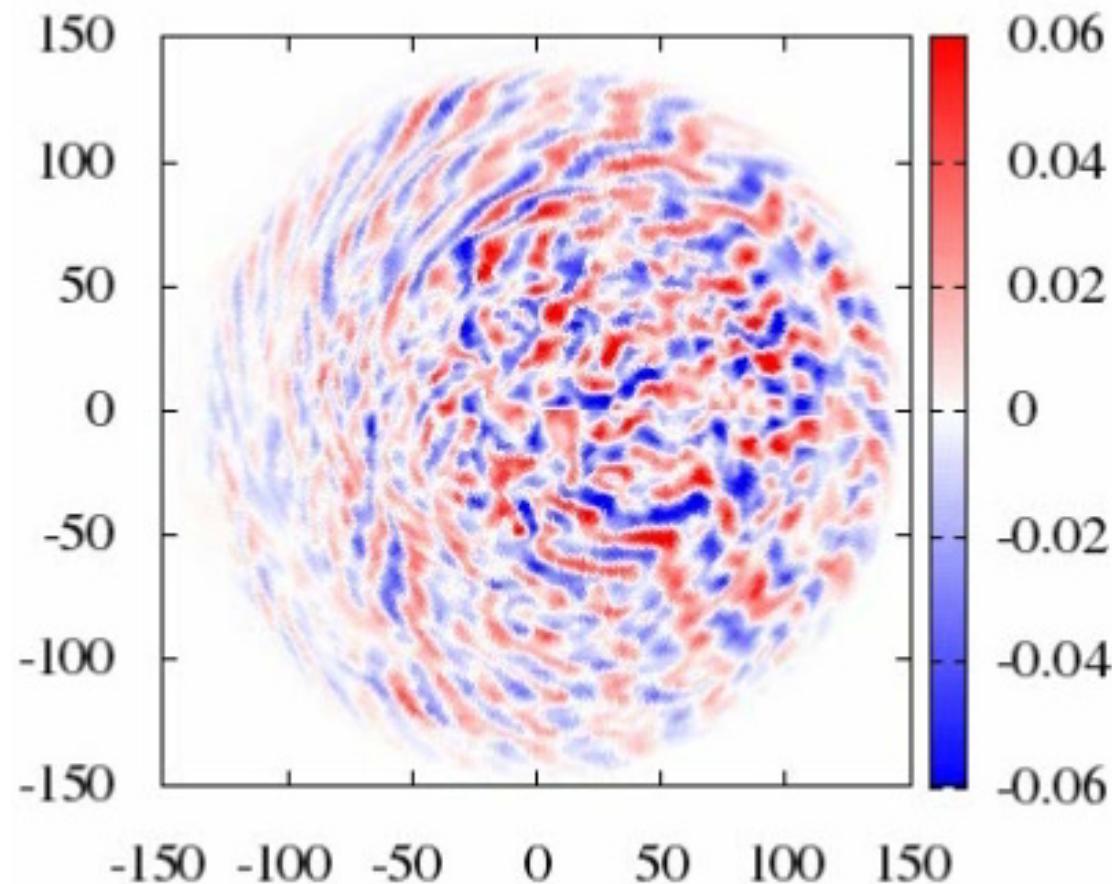


$t = 534$

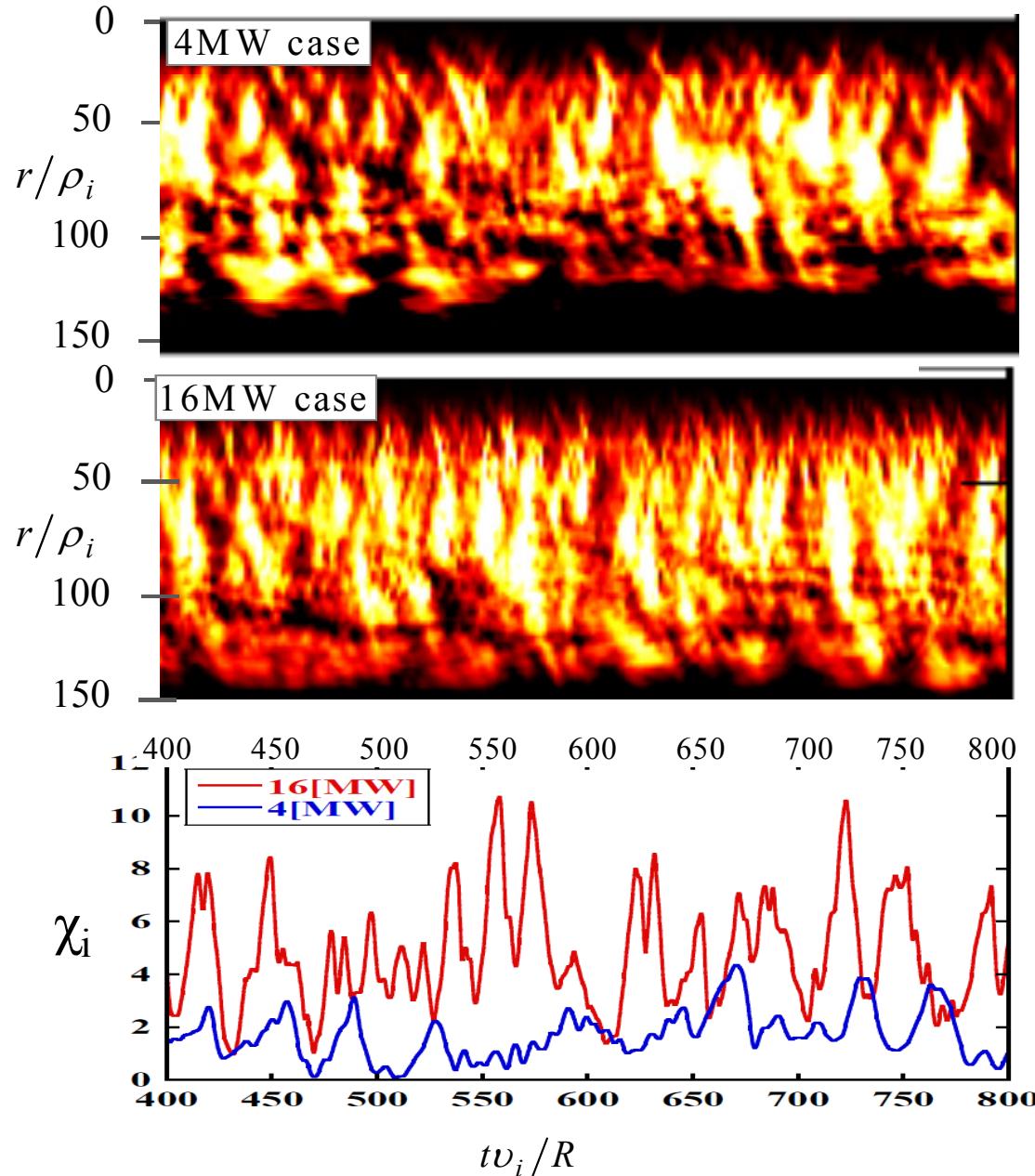
$t = 556$

$t = 574$

$t = 500.00000000000000$



# ITG turbulence and profile formation



- Various types of heat event
  - ① Heat avalanches radially propagates both down an up-words

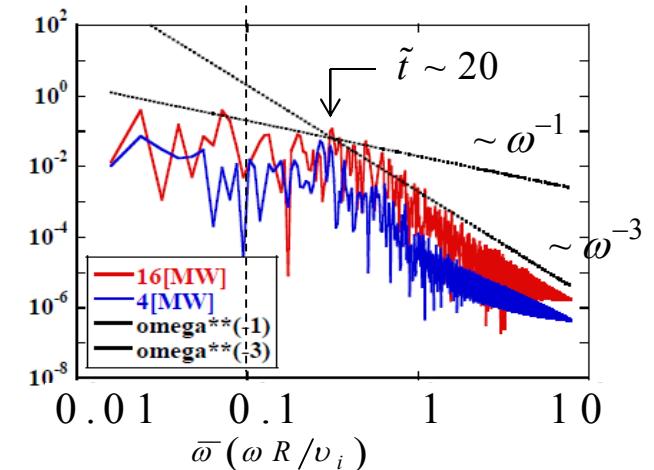
$$v_{av} \leq 2.5 v_b \sim 2.5 C_s \left( \rho_i^*/A \right) \sim C_s \rho_i^*$$

- ② Radially extended bursts (meso-macro scales)

$$\ell_c \sim (L_T \rho_i)^{1/2} - L_T$$

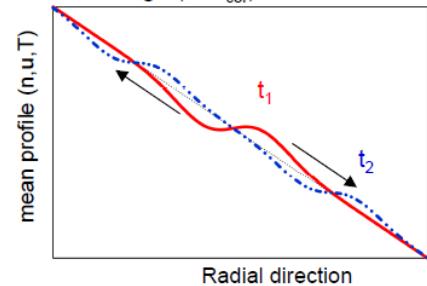
$$\Delta\tau_c \sim \tau_{rec} \sim 20 - 100 \tau_d$$

- Frequency spectrum of heat



# Avalanche transport

## Ballistic propagation of localized heat event



Diamond-Hahm  
PoP (1995)



[Y. Idomura, *et al.* Nucl. Fusion, 49, 065029 (2009).]

[Y. Sarazin, *et.al.* Nucl. Fusion, 50, 054004 (2010).]

[B. F. McMillan, Phys. Plasmas, 16, 022310 (2009)]

- Propagating velocity  $\sim \pm \rho_{ti} v_{ti} / R_0$
- Long positive PDF tails
- The direction depends on the sign of the  $E \times B$  shear determined by the neo-classical  $E_r$  shear.

