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Nonlinear instability and plasma dynamics

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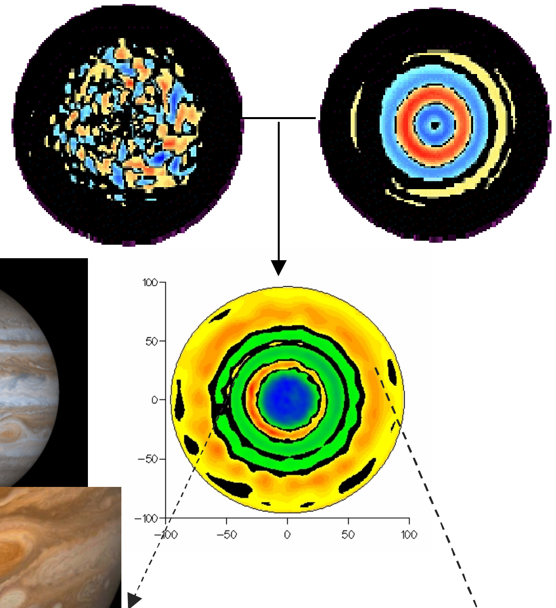
- August 19 : Structure/dynamics of fluctuations in fusion plasmas
- August 19 : Nonlinear instability and plasma dynamics
- August 21 : Interaction between different scales

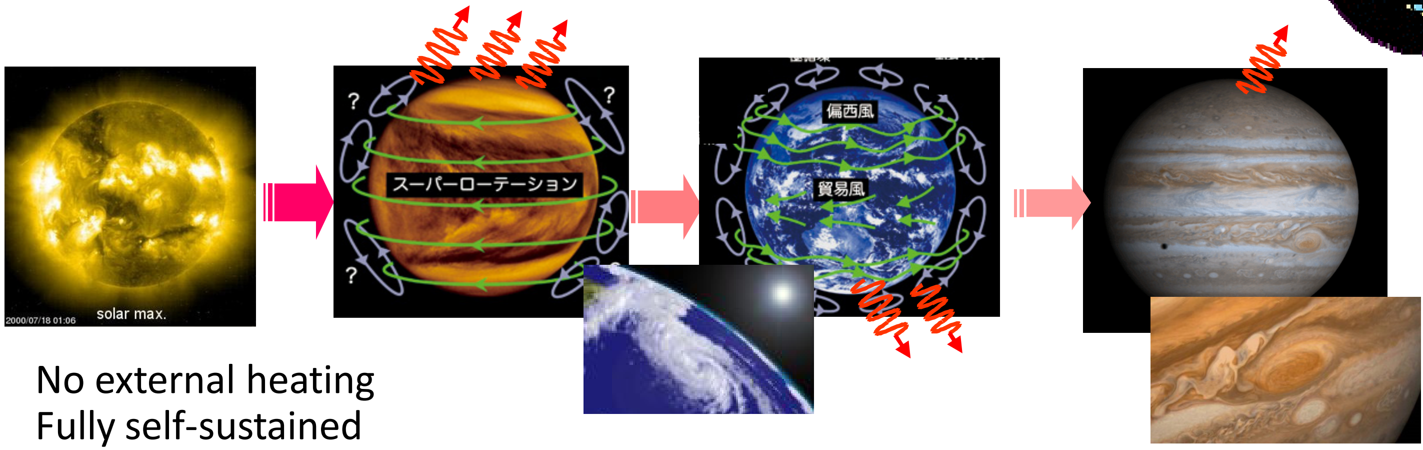
Planetary environment and fusion plasmas

World dominated by

- Micro-turbulences $E^{(turb)}$
- Macro-flows $E^{(ZF)}$
  (sometime with low mode)
- Large scale structure $E^{(LSS)}$

Micro-scale vortices are embedded in macro-scale flows



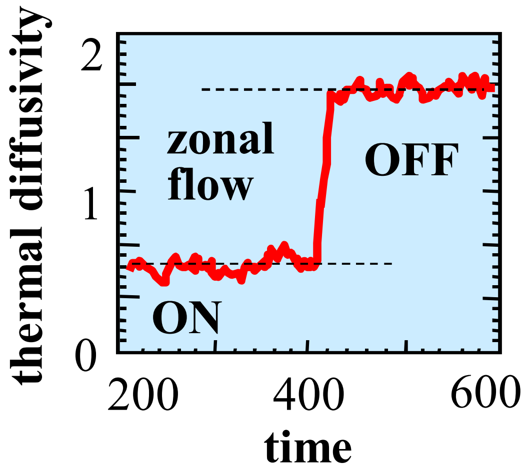


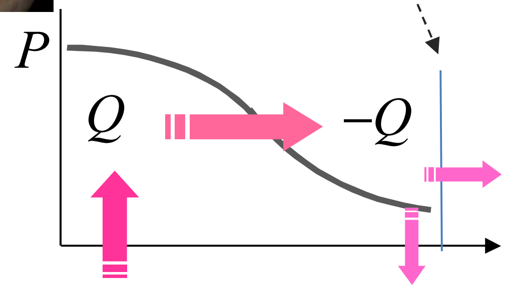
No external heating
Fully self-sustained

What is the role of ZF and LSS on transport ?

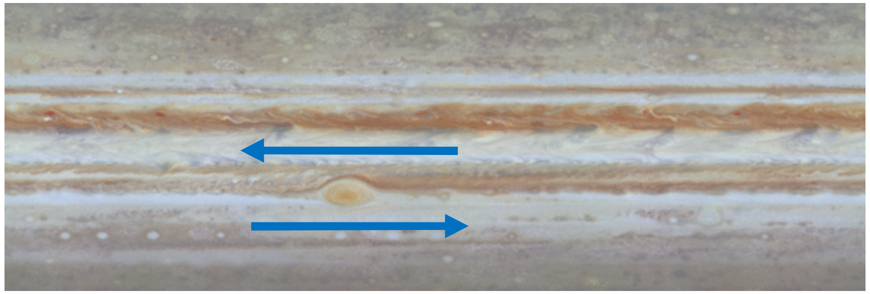
$$\eta_{ZF} \equiv \frac{E^{(ZF)}}{E^{(tot)} + E^{(ZF)} + E^{(LSS)}}$$

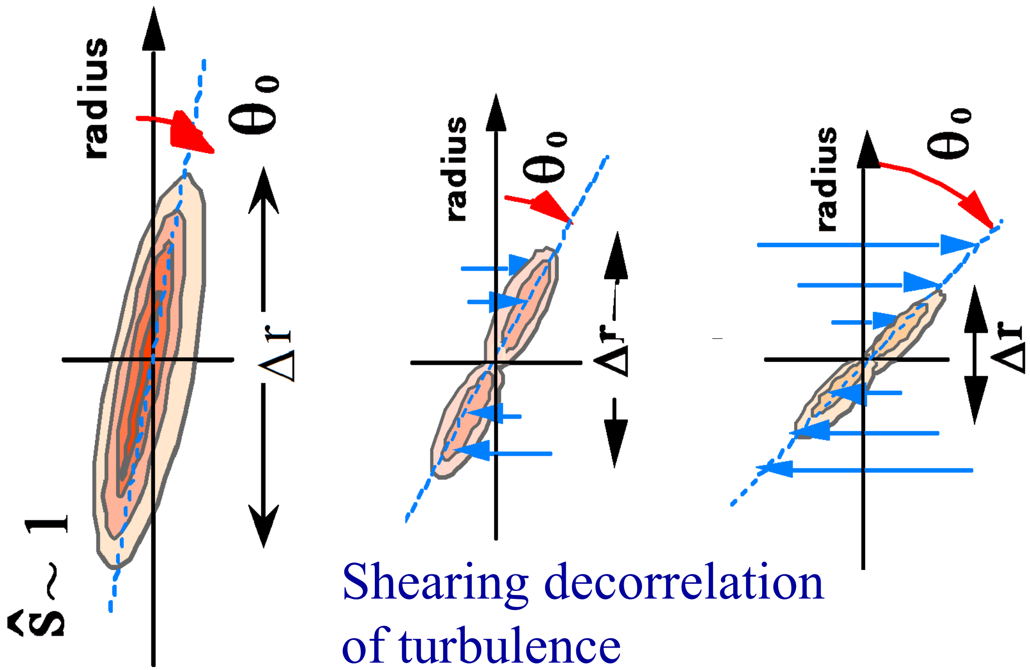
$$\eta_{ZF} \equiv \frac{E^{(ZF)}\tau_{ZF}}{E^{(turb)}\tau_t + E^{(ZF)}\tau_{ZF} + E^{(LSS)}\tau_{LSS}}$$



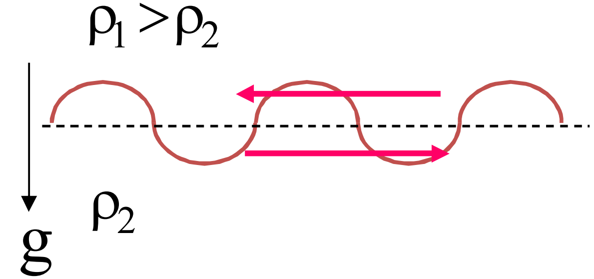


Interaction between flows & fluctuations





Shearing decorrelation
of turbulence



$$\gamma = \sqrt{k_y g - \alpha\left(\frac{\partial V_y}{\partial x}\right)}$$

$$\left(1+k_\perp^2\right)\frac{\partial}{\partial t}\phi_{k_x} = -ik_y\phi_{k_x} + \boxed{\upsilon'_{sf}\left(1+k_\perp^2\right)k_y\frac{d\phi_{k_x}}{dk_x}} + \left[\phi,\nabla_\perp^2\phi\right]_{k_x}$$

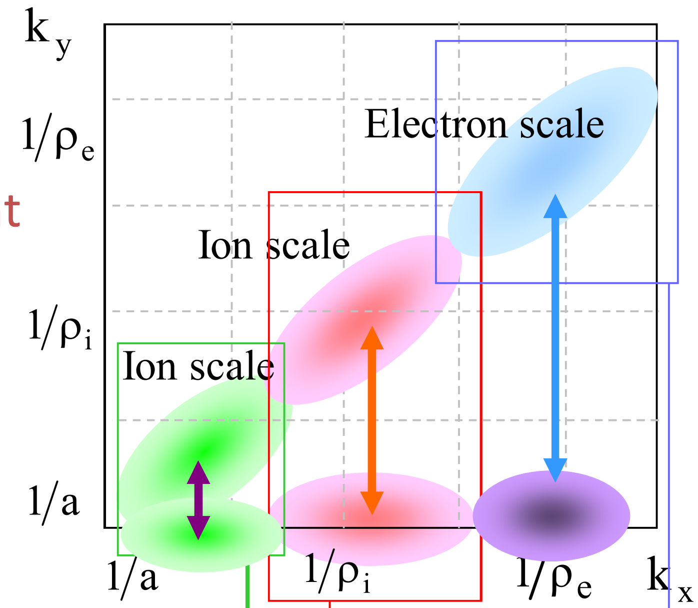
$$\left\langle \delta k^2 \right\rangle = t^2 k^2 V_E'^2$$

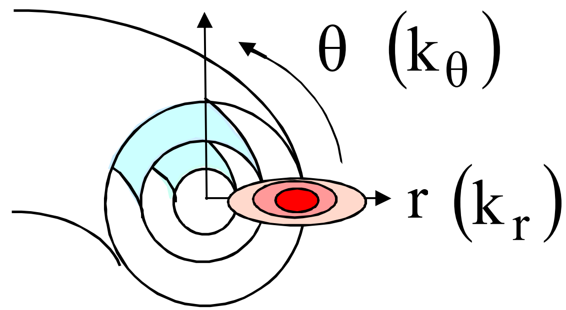
Shearing decorrelation is a local interaction in k space

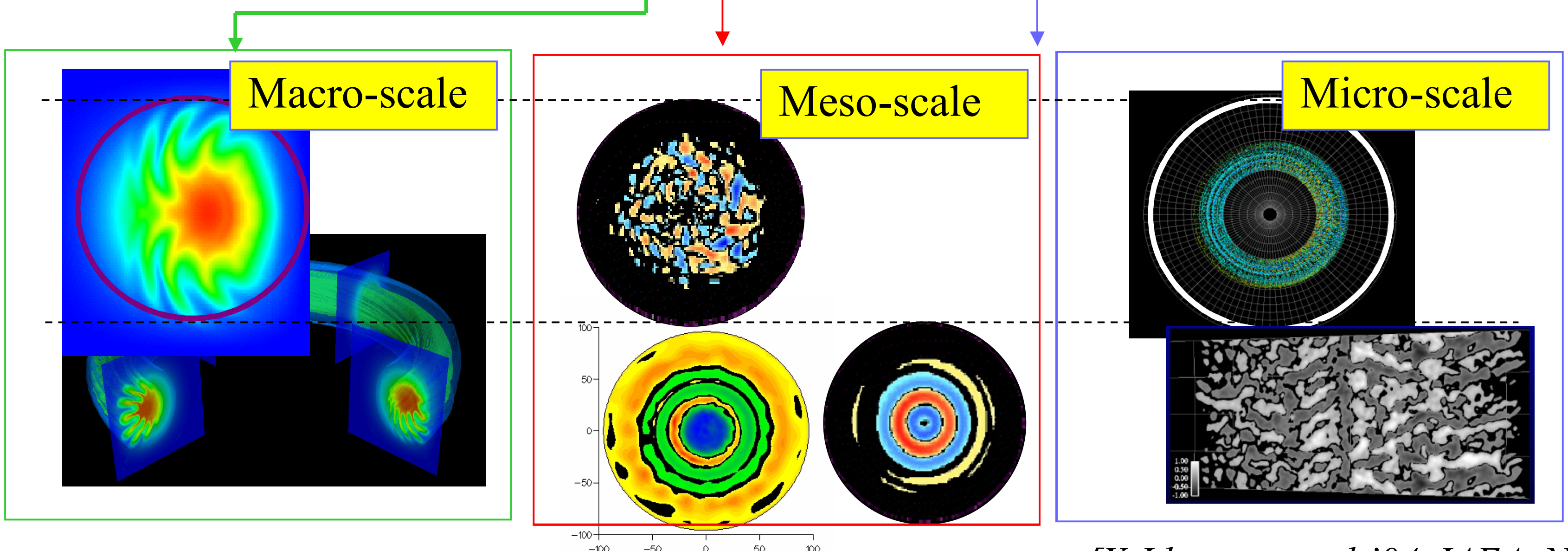
Fluctuation with different time and spatial scales

- Coexistence of different scale fluctuations :

$$\Delta k \sim 10^5 \quad \Delta\omega \sim 10^8$$







[*courtesy of Kagei (JAEA)*]

[Y. Kishimoto et al. '96, PoP]

[Y. Idomura et al. '04, IAEA, NF]

Quasi-linear transport due to drift wave (1)

$$n(x,t) = n_0(x) + \tilde{n}(x,t)$$

$$\boldsymbol{u}(x,t) = \boldsymbol{u}_0(x) + \tilde{\boldsymbol{u}}(x,t)$$

$$\frac{\partial \mathrm{n}_0}{\partial \mathrm{t}} + \frac{\partial \tilde{\mathrm{n}}}{\partial \mathrm{t}} + \nabla\cdot\left[\mathrm{n}_0 \tilde{\mathbf{v}} + \tilde{\mathrm{n}}\tilde{\mathbf{v}}\right] = 0$$

$$\frac{\partial \mathrm{n}_{\mathbf{k}}}{\partial \mathrm{t}} + \nabla\cdot\left(\mathrm{n}_0 \mathbf{v}_{\mathbf{k}}\right) + \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \nabla\cdot\left(\mathrm{n}_{\mathbf{k}'} \mathbf{v}_{\mathbf{k}''}\right) = 0$$

$$\frac{\partial \mathrm{n}_0}{\partial \mathrm{t}} + \nabla\cdot\left[\underbrace{\sum_{\mathbf{k}} \mathrm{Re}\left(\mathrm{n}_{\mathbf{k}} \mathbf{v}_{\mathbf{k}}^{*}\right) \exp\left(2\gamma_{\mathbf{k}} \mathrm{t}\right)}\right] = 0$$

Flux : $\Gamma = -D\nabla n$

Quasi-linear transport due to drift wave (2)

$$\mathbf{k} = k_{\parallel}\hat{\mathbf{b}} + \mathbf{k}_b + k_r\hat{\mathbf{r}}$$

Coupling between ion motion and density gradient (drift wave)

$$\mathbf{E}_k = -\nabla\phi_{\mathbf{k}} = -i\mathrm{k}\,\phi_{\mathbf{k}}$$

$$\mathbf{v}_{\mathbf{k}} = -i\frac{c\phi_{\mathbf{k}}(\mathbf{k}\times\mathbf{b})}{B} = -i\left(\frac{cT_i}{eB}\right)\left(\frac{e\phi_{\mathbf{k}}}{T_i}\right)\left(k_b\hat{\mathbf{r}} + k_r\hat{\mathbf{r}}\times\hat{\mathbf{b}}\right)$$

$$\frac{n_{\mathbf{k}}^{(ExB)}}{n_0} = -i\frac{\nabla n_0\cdot\mathbf{v}_{\mathbf{k}}}{n_0\omega_{\mathbf{k}}} = -\frac{\omega_{*i}}{\omega_{\mathbf{k}}}\left(\frac{e\phi_{\mathbf{k}}}{T_i}\right) = -\frac{\omega_{*i}\omega_{\mathbf{k}r}}{|\omega_{\mathbf{k}}|^2}\left(\frac{e\phi_{\mathbf{k}}}{T_i}\right)\left(1 - i\frac{\gamma_{\mathbf{k}}}{\omega_{\mathbf{k}r}}\right)$$

$$\omega_{*i} = k_b v_{di}$$

: dia-magnetic drift frequency

$$\mathbf{v}_d = -\frac{T_e\mathbf{B}_0\times\nabla n_0}{en_0B_0^{\,2}}$$

$$v_{dj} = -\left(\frac{cT_j}{e_jB}\right)\frac{1}{L_{pj}} = -\frac{\rho_j v_j}{L_{pj}} = -\frac{\rho_j v_j}{L_{nj}}(1+\eta_j)$$

Quasi-linear transport due to drift wave (3)

$$\Gamma = \sum_{\mathbf{k}} n_{\mathbf{k}} v^{*}_{\mathbf{k}r} = -\, n_0 \left(\frac{cT_i}{eB}\right) \sum_{\mathbf{k}} \left[k_b \frac{\omega_{*i}\gamma_{\mathbf{k}}}{|\omega_{\mathbf{k}}|^2} \left|\frac{e\phi_{\mathbf{k}}}{T_i}\right|^2 \exp(2\gamma_{\mathbf{k}} t) \right]$$

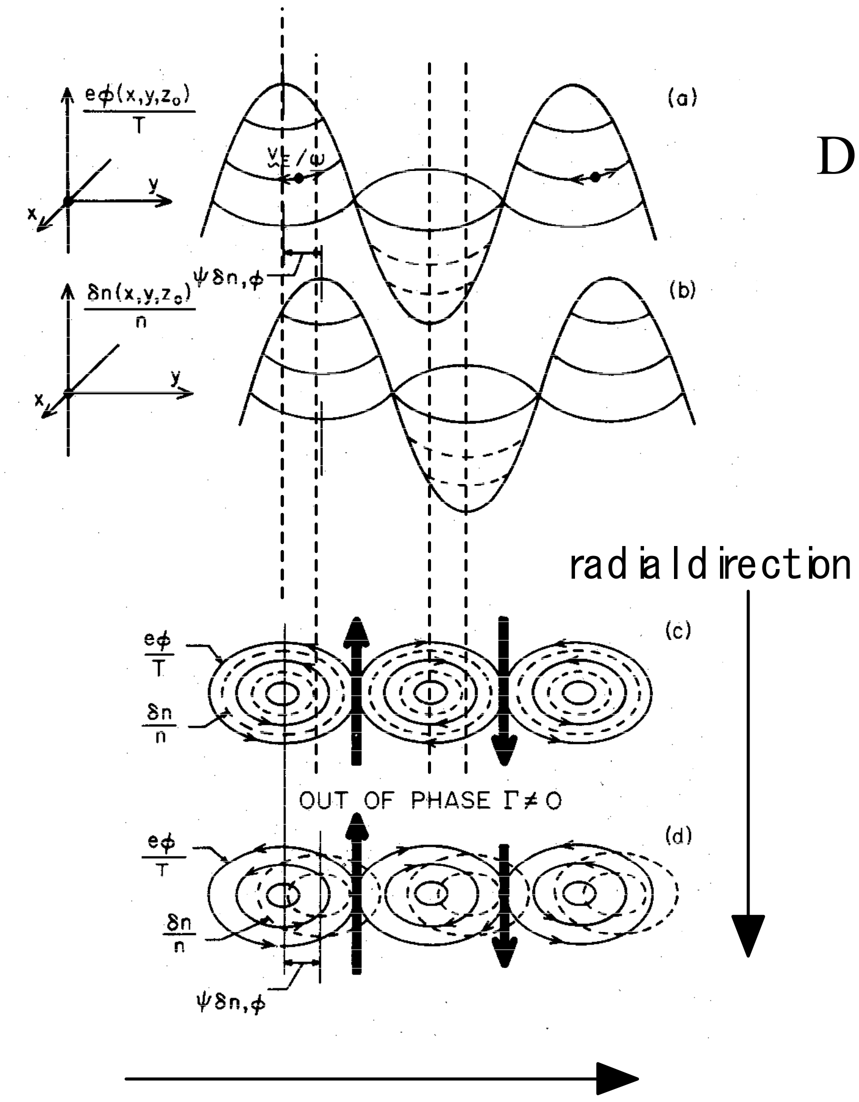
$$D = \left(\frac{cT_i}{eB}\right) \sum_{\mathbf{k}} \left[(k_b L_n) \frac{\gamma_{\mathbf{k}}}{\omega_{*i}} \left|\frac{n_{\mathbf{k}}}{n_0}\right|^2 \exp(2\gamma_{\mathbf{k}} t) \right] \qquad \text{cf. } D = \Gamma\left(\frac{L_n}{n_0}\right)$$

cf. $\left(\dfrac{cT_i}{eB}\right) = \rho_i v_i = \rho_i^{\,2}\omega_{ci}$

Step size : gyro-radius
correlation time : gyration period

$$D_B = \frac{1}{16}\frac{cT}{eB} \qquad \tau_B \propto a^2\left(\frac{B}{T}\right) \qquad \text{cf. } D_B \propto \frac{a^2}{\tau_B}$$

Quasi-linear transport due to drift wave (4)



$$D = -\left(\frac{cT_i}{eB}\right)\sum_{\mathbf{k}}\left[(k_b L_n)\frac{\gamma_{\mathbf{k}}}{\omega_{*i}}\left|\frac{n_{\mathbf{k}}}{n_0}\right|^2 \exp(2\gamma_{\mathbf{k}} t)\right]$$

$$\gamma_k = 0 \rightarrow \text{Zero transport}$$

$$\frac{n_{\mathbf{k}}^{(ExB)}}{n_0} = -i\frac{\nabla n_0 \cdot \mathbf{v}_{\mathbf{k}}}{n_0 \omega_{\mathbf{k}}}$$

$$= -\frac{\omega_{*i}\omega_{\mathbf{k}r}}{|\omega_{\mathbf{k}}|^2}\left(\frac{e\phi_{\mathbf{k}}}{T_i}\right)\left(1 - i\frac{\gamma_{\mathbf{k}}}{\omega_{\mathbf{k}r}}\right)$$

Phase angle between density and potential

$$\frac{n_{\mathbf{k}e}}{n_0} \cong \frac{e\phi_{\mathbf{k}}}{T_e}(1-i\delta_{\mathbf{k}}) \quad \alpha_{\mathbf{k}} = \tan^{-1}\left(\frac{\gamma_{\mathbf{k}}}{\omega_{\mathbf{k}r}}\right)$$

Correlation time

$$\mathbf{v}_E = -\frac{\nabla_\perp \phi \times \mathbf{B}_0}{{B_0}^2} \longrightarrow \frac{d\xi_r}{dt} = \frac{c\widetilde{E}_{\theta k}}{B} \qquad \xi_r = \left(\frac{c\widetilde{E}_{\theta k}}{B}\right)\tau_{ck} = -i\left(\frac{ck_\theta \phi_k}{B}\right)\tau_{ck}$$

Correlation time

Assuming Markov process

$$D \sim \frac{\xi_r^2}{\delta\tau_c} \sim \sum_{\mathbf{k}} \left|\frac{ck_\theta \widetilde{\phi}_{\mathbf{k}}}{B}\right|^2 \delta\tau_{c\mathbf{k}}$$

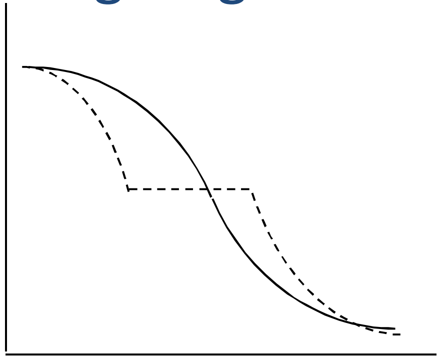
generation → linear growth rate γ_{Lk}

annihilation → scattering of fluctuation due to various origins γ_{Nk}

$$\phi = \mathbf{k}\cdot\mathbf{r} - \omega_{\mathbf{k}r} t + \left(\gamma_{L\mathbf{k}} - \gamma_{N\mathbf{k}}\right)t \qquad \text{steady state}$$

$$\gamma = \gamma_{Lk} - \gamma_{Nk} \cong 0 \qquad \tau_{ck} \sim \frac{1}{\gamma_{Nk}} \sim \frac{1}{\gamma_{Lk}}$$

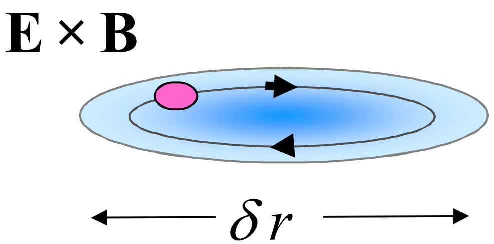
- Mixing length estimate



$$\frac{\partial}{\partial r}\left[\tilde{n}(r,t)+n_0\right] \sim 0 \quad \left|\frac{\tilde{n}}{n_0}\right| \sim \frac{1}{L_n k_r}$$

$$D = \left(\frac{cT}{eB}\right)\sum_{\mathbf{k}}\left[\left(k_b L_n\right)\frac{\gamma_{\mathbf{k}}}{\omega_*}\left|\frac{n_{\mathbf{k}}}{n_0}\right|^2 \exp\left(2\gamma_k t\right)\right] \sim \frac{\gamma_{\mathbf{k}}}{k_r^2}$$

- Wave breaking estimate



$$\mathbf{v}_E = -\frac{\nabla_\perp \phi \times \mathbf{B}_0}{B_0{}^2} \qquad \frac{d\xi_r}{dt} = \frac{c\tilde{E}_{\theta k}}{B}$$

$$\xi_r = \left(\frac{c\tilde{E}_{\theta k}}{B}\right)\tau_{ck} = -i\left(\frac{c k_\theta \phi_k}{B}\right)\tau_{ck}$$ Correlation time

$$\xi_r \sim \frac{c k_b \left|\varphi_k\right|}{\gamma_k B} \sim \frac{1}{k_r}$$

$$\left|\frac{e\phi_k}{T}\right| \sim \frac{1}{k_b \rho_s}\frac{\gamma_k}{k_r v_s} \sim \frac{1}{k_r L_n} \sim \left|\frac{n_k}{n_0}\right|$$

Assuming Markov process

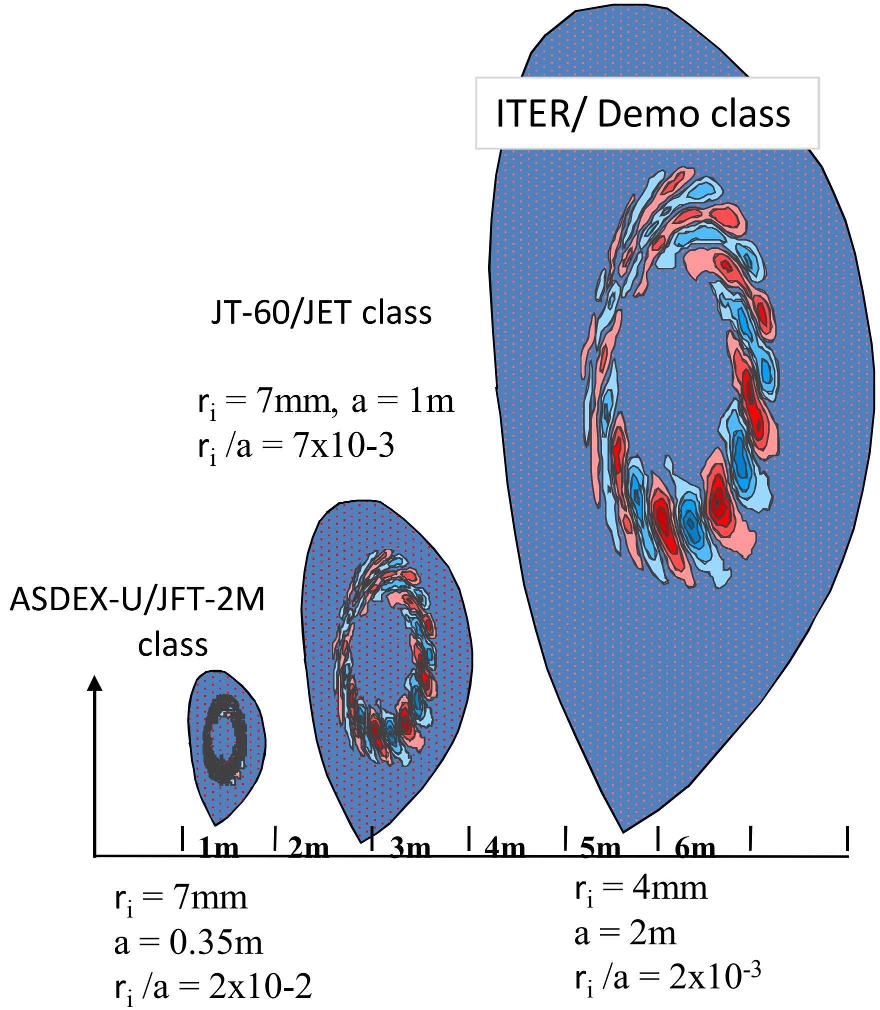
$$D \sim \frac{\xi_r^2}{\delta\tau_c} \sim \sum_{\mathbf{k}}\left|\frac{c k_\theta \tilde{\phi}_{\mathbf{k}}}{B}\right|^2 \delta\tau_{c\mathbf{k}}$$

$$\gamma_k \sim \omega_{kr} \sim \omega_{*i} \qquad \Delta r \sim \rho_i^{1-\alpha} L_T^{\alpha} \sim \frac{1}{k_r}$$

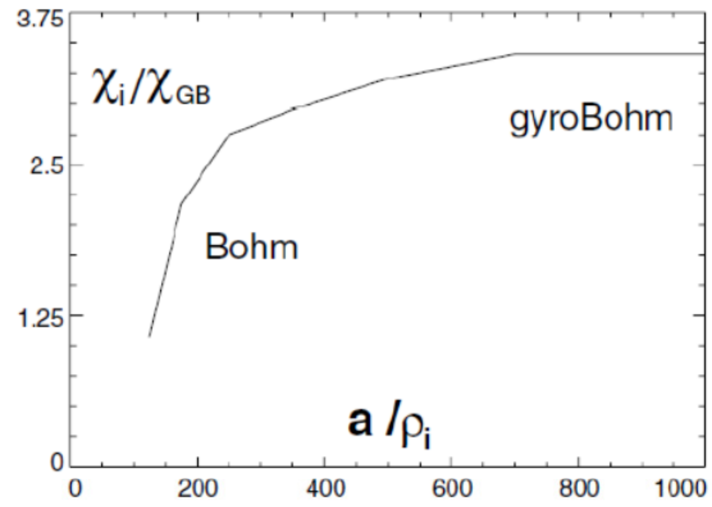
$$D \sim \frac{\gamma_k}{k_r^2} \sim \frac{\omega_{*i}}{k_r^2} \sim (k_\theta \rho_i)\left(\frac{v_i}{L_T}\right)\left(\rho_i^{1-\alpha} L_T^{\alpha}\right)^2 \sim (\rho_i v_i)\left(\frac{\rho_i}{L_T}\right)^{1-2\alpha} \sim \left(\frac{cT_i}{eB}\right)\left(\frac{\rho_i}{L_T}\right)^{1-2\alpha}$$

$$W = \left|\frac{e\phi}{T}\right|^2 \sim \left(\frac{1}{k_r L_n}\right)^2 \sim \left(\frac{\rho_s}{L_n}\right)^{2(1-\alpha)}$$

$$\left\{\begin{array}{l}
\alpha = 0 \ \left(\Delta r \sim \rho_i\right) \quad : \ \text{gyro-Bohm} \\[1em]
D \sim \dfrac{\gamma_k}{k_r^2} \sim \left(\dfrac{cT_i}{eB}\right)\left(\dfrac{\rho_i}{L_T}\right) \qquad W \sim \left(\dfrac{\rho_s}{L_n}\right)^2 \\[1em]
\alpha = 0 \ \left(\Delta r \sim \sqrt{\rho_i L_T}\right) \ : \ \text{Bohm} \\[1em]
D \sim \dfrac{\gamma_k}{k_r^2} \sim \left(\dfrac{cT_i}{eB}\right) \qquad W \sim \dfrac{\rho_s}{L_n}
\end{array}\right.$$



An interesting observation in global gyrokinetic simulations of toroidal ITG turbulence is that for small device size plasma, the fluctuations are microscopic and local, that transport is diffusive, while the resulting turbulent transport is not gyro-Bohm, shown in Fig.1.[1] The local transport coefficient exhibits a gradual transition from a Bohm-like scaling for small device sizes to a gyro-Bohm scaling for future larger devices in the presence of self-consistent zonal flows.

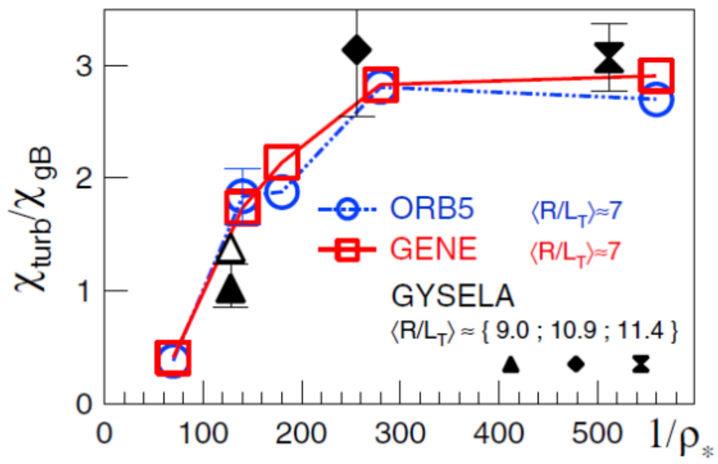


[1] Z. Lin et al., PRL **88**, 195004 (2002).

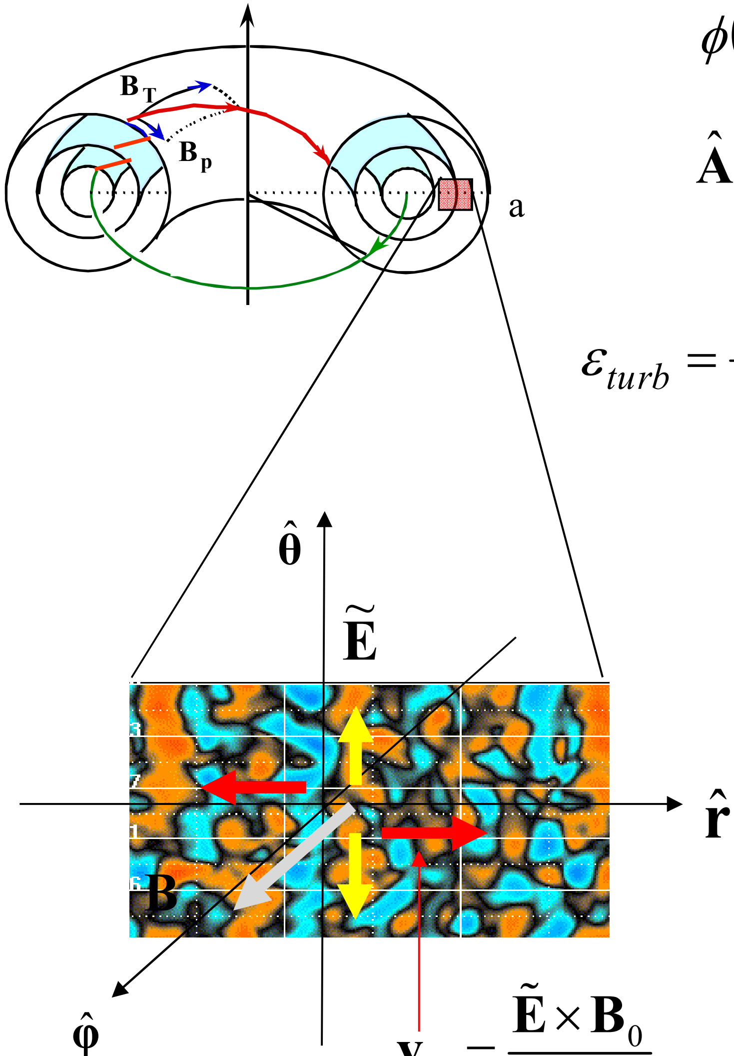
Fig.1. Ion heat conductivity vs tokamak minor radius. [Copied from Fig.3 in Ref. [1].

Fig. 4. Effective heat diffusivity of three global gyrokinetic codes as a function of $\rho*$. (copied from Fig.5 in Ref. 3)

[3] Sarazin, *et. al*, Nucl. Fusion **51** (2011)



Fluctuating filed in confinement device



$$\mathbf{v}_E = \frac{\tilde{\mathbf{E}} \times \mathbf{B}_0}{B_0^{\ 2}}$$

$$\phi(\boldsymbol{r},t) = \sum_k \phi(\boldsymbol{k},\omega_k)\exp[i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_k t)]$$

$$\hat{\mathbf{A}}(\mathbf{r},t) = \sum_k \mathbf{A}(\mathbf{k},\omega_k)\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega_k t)]$$

$$\varepsilon_{turb} = \frac{1}{8\pi}\left(E^2 + B^2\right)$$

- Weak turbulence : $\dfrac{\varepsilon_{turb}}{nT} << 1$
- Strong turbulence : $\dfrac{\varepsilon_{turb}}{nT} \sim 1$

$$\Gamma_x = < \tilde{n}\tilde{v}_x^* > = \sum_k \tilde{n}_k \tilde{v}_k^*$$

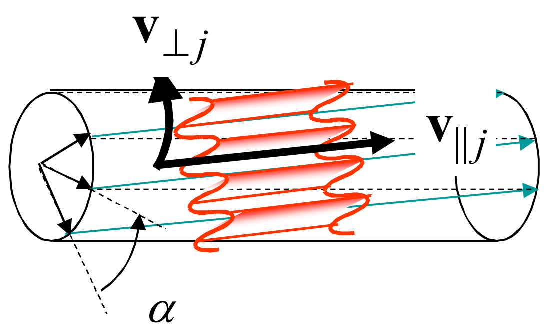
$$Q_x = \frac{3}{2} < \tilde{p}\tilde{v}_x^* > = \frac{3}{2}\sum_k \tilde{p}_k \tilde{v}_k^*$$

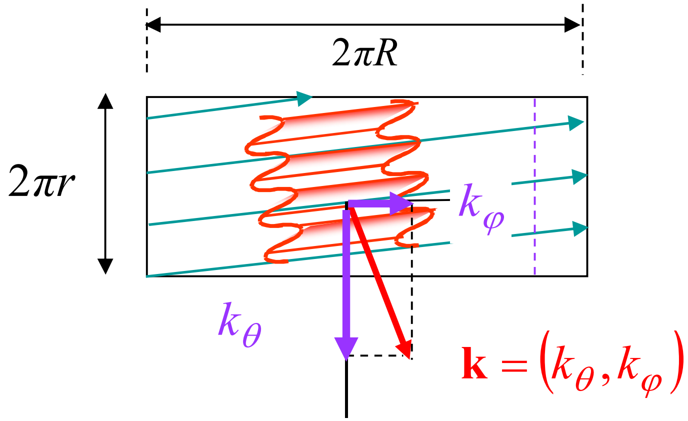
Phase difference causes flux :

$$\tilde{n} = |\tilde{n}|\exp(i\delta_n), \quad \tilde{v} = |\tilde{v}|\exp(i\delta_\varphi)$$

$$\Gamma_x \sim \tilde{n}\tilde{u}^* \sim |\tilde{n}||\tilde{v}|\sin(\delta_n - \delta_\varphi)$$

Particle motion :





$$\phi(\mathbf{r},t) = \sum_k \phi(\mathbf{k},\omega_k)\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega_k t)]$$

$$= \sum_{m,n} \phi_{m,n}(r,t)\exp[i(m\theta - n\varphi)]$$

$$k_\theta = \frac{2\pi m}{L_\theta} = \frac{m}{r} \qquad k_\varphi = \frac{2\pi m}{L_\varphi} = \frac{n}{R}$$

Electron motion (parallel to the magnetic field

$$n_e m \frac{dv_\parallel}{dt} = -en_e \hat{\boldsymbol{b}}\cdot\boldsymbol{E} - \hat{\boldsymbol{b}}\cdot\nabla p_e$$

→ Adiabatic response (simplest case)

$$\tilde{n}_{m,n} = n_0(x)\frac{e(\phi_{m,n} - \langle\phi\rangle)}{T_e}$$

Ion motion (perpendicular to the magnetic field

$$n_i m_i \frac{d\boldsymbol{v}_i}{dt} = en_i(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B}_0) - \nabla p_i$$

1th order drift

$$\mathbf{v}_E = -\frac{\nabla_\perp\phi\times\mathbf{B}_0}{B_0^{\ 2}} \qquad \mathbf{v}_d = -\frac{T_e\mathbf{B}_0\times\nabla n_0}{en_0 B_0^{\ 2}}$$

$$\vec{\upsilon}_D = -\frac{\vec{\boldsymbol{B}}_0\times\nabla B_0}{B_0^{\ 2}}\frac{\upsilon_{//}^2 + \upsilon_\perp^2/2}{\omega_c}$$

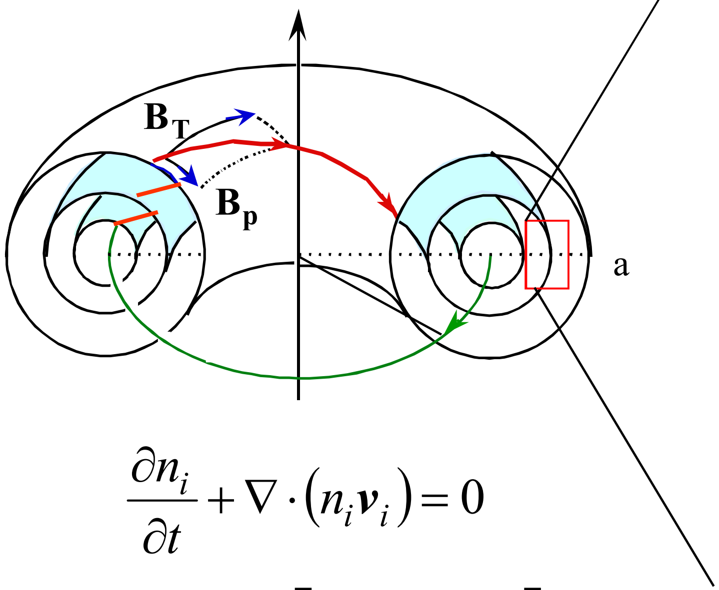
2th order, polarization drift

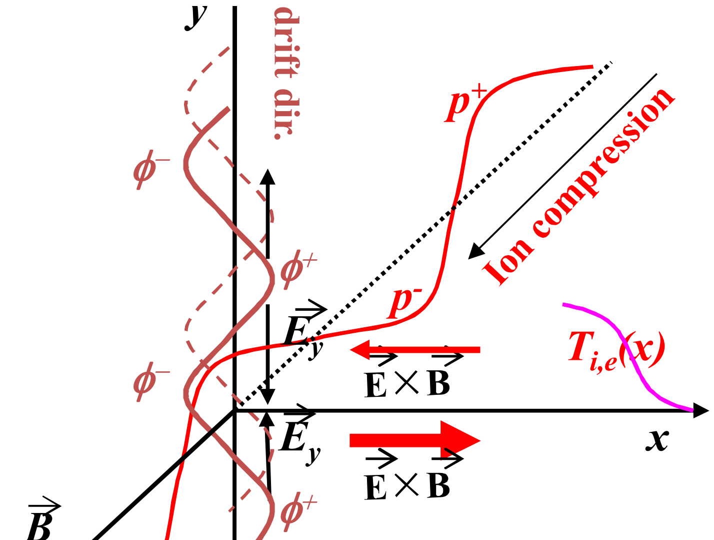
$$\boldsymbol{v}_p = \frac{1}{\omega_c B_0}\left[-\frac{\partial}{\partial t}\nabla_\perp\phi - (\boldsymbol{v}_E\cdot\nabla_\perp)\nabla_\perp\phi\right]$$

Drift wave caused by ∇p drift

$$\mathbf{v}_d = -\frac{\mathbf{B}_0 \times \nabla n}{e n_0 {B_0}^2} = \frac{cT_i}{eB}\left(\frac{1}{n}\frac{\partial n}{\partial x}\right)\hat{\mathbf{y}}$$

Low frequency ion motion





Physical picture of slab ITG mode

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \boldsymbol{v}_i) = 0$$

$$\frac{\partial \tilde{n}_i}{\partial t} - \nabla \cdot \left[n_0 \frac{c \nabla \phi \times \hat{\boldsymbol{b}}}{B} \right] = 0$$

$$\tilde{n}_e = n_0 \frac{e\phi}{T_e} = \tilde{n}_i$$

Slab ITG: coupling between ∇T_i and ion sound wave

$$\frac{\partial \phi}{\partial t} + v_{di} \frac{\partial \phi}{\partial y} = 0$$

$$\omega_* = k_y v_d = k_y \frac{cT_e}{eB}\left(\frac{1}{n}\frac{\partial n}{\partial x}\right) = -k_y \frac{cT_e}{eB}\frac{1}{L_n} = -(k_y \rho_s)\frac{v_s}{L_n}$$

$$\omega = v_d k_\theta = \omega_{*e}$$

$$\longrightarrow \quad \phi = \phi(y - v_d t)$$

Key point: Density perturbation is in phase with potential, just drift wave

Drift wave becomes can be unstable, leading instability

Drift wave : a plasma oscillation propagating
along diamagnetic drift direction

If some mechanism may pressure perturbation is out of phase with the potential perturbation, the drift wave becomes "unstable"

Electron drift wave

$$m_e n_0 \frac{\partial \upsilon_{//}}{\partial t} = -e n_0 \nabla_{//} \varphi - \nabla_{//} p - \frac{\hat{b} \cdot (\nabla \cdot \vec{\pi})}{e n_e} - m_e n_0 \gamma_{ei} \upsilon_{//} \approx 0$$

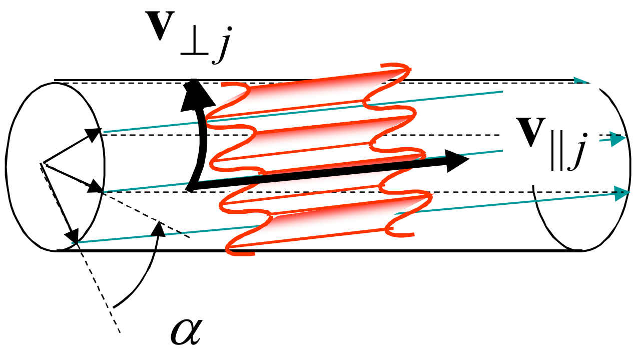
iδ model : $\tilde{n}_e = \frac{e\varphi}{T_e}(1 - i\delta) \qquad \tilde{n}_i = \tilde{n}_e$

$$\omega = k_y \upsilon_d = k_y \frac{T_e}{eB} \frac{\partial n_0}{n_0 \partial x} \approx \omega_* (1 + i\delta)$$

Collisional electron drift wave:

$$\omega \approx \omega_{*e} + i\gamma_{ei} \left(\frac{k_y}{k_{//}} \right)^2 \left(\frac{\omega_*^2}{\omega_{ce}\omega} \right)$$

Hasegawa-Mima equation



$$\mathbf{v}_E = -\frac{\nabla_\perp \phi \times \mathbf{B}_0}{B_0{}^2} \qquad \varepsilon = \frac{\omega}{\omega_{cj}} << 1$$

$$\mathbf{v}_p = \frac{1}{\omega_{ce} B_0}\left[-\frac{\partial}{\partial t}\nabla_\perp \phi - (\mathbf{v}_E \cdot \nabla_\perp)\nabla_\perp \phi \right]$$

$$\frac{\partial n_i}{\partial t} - \nabla \cdot \left[n_i \left(\mathbf{v}_E + \mathbf{v}_p \right) \right] = 0 \qquad \tilde{n}_e = n_0(x)\frac{e\left(\tilde{\phi} - \langle \phi \rangle\right)}{T_e} \qquad \tilde{n}_i = \tilde{n}_e$$

$$(1 - \nabla^2)\frac{\partial}{\partial t}\phi + \upsilon_d \frac{\partial}{\partial y}\phi - \underbrace{\left(\nabla \phi \times \boldsymbol{b} \cdot \nabla\right)\left(\nabla^2 \phi\right)} = 0$$

$$[\phi, \nabla^2 \phi] = \frac{\partial \phi}{\partial x}\frac{\partial \nabla^2 \phi}{\partial y} - \frac{\partial \phi}{\partial y}\frac{\partial \nabla^2 \phi}{\partial x}$$

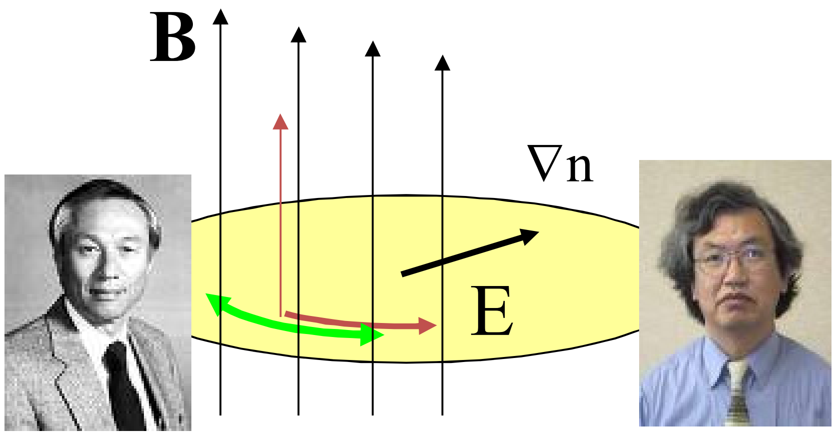
$$(1 - \delta - \nabla^2)\frac{\partial}{\partial t}\varphi + \upsilon_d \frac{\partial}{\partial y}\varphi - [\varphi, \nabla^2 \varphi] = 0$$

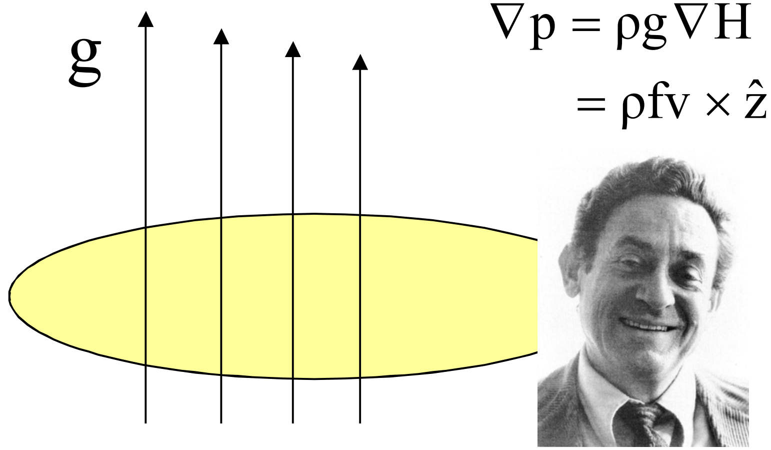
for zonal flows : δ=1

for others : δ=0

Turbulence in quasi-two dimensional bounded system

Drift wave : Hasegawa-Mima eq.	Rossby wave : Charney equation
$\left(1-\nabla^2\right)\frac{\partial\varphi}{\partial t}+v_d\frac{\partial\varphi}{\partial y}-\left[\left(b\times\nabla\varphi\right)\cdot\nabla\right]\nabla^2\varphi=0$	$\left(1-\nabla^2\right)\frac{\partial h}{\partial t}+v_R\frac{\partial h}{\partial y}-\left[\left(\hat{z}\times\nabla h\right)\cdot\nabla\right]\nabla^2 h=0$





$$\nabla p = \rho g \nabla H$$
$$= \rho f v \times \hat{z}$$

Drift wave	Rossby wave
$v_E=\frac{c}{B}\hat{z}\times\nabla\varphi \qquad v_p=-\frac{c}{\omega_{ci}B}\frac{d}{dt}\nabla_\perp\varphi$	$\boldsymbol{v}_c=\frac{g}{f}\hat{z}\times\nabla H \qquad \boldsymbol{v}_p=-\frac{gH_0}{f^2}\frac{d}{dt}\nabla h$

Conserving quantities

Energy : $W=(\nabla\phi)^2+\phi^2$ Enstrophy : $U=(\nabla\phi)^2+\left(\nabla^2\phi\right)^2$

Fluid equation of ITG in tokamak

With adiabatic election, simplified 3-field equations for ITG mode

$$\left(1-\nabla_\perp^2\right)\frac{\partial\varphi}{\partial t}=-\frac{a}{L_n}\nabla_\theta\varphi-(1+\eta_i)\frac{a}{L_n}\nabla_\theta\nabla_\perp^2\varphi$$

$$\boxed{-2\Gamma\omega_D(\varphi+p_i)-\frac{1}{2}\omega_D\nabla_\perp^2\varphi}\boxed{-\nabla_{//}\upsilon_{//}}$$

Toroidal curvature for toroidal ITG mode

$$\frac{\partial\upsilon_{//}}{\partial t}=\boxed{-\nabla_{//}\varphi-\nabla_{//}p_i}$$

Parallel compression for slab ITG mode

$$\frac{\partial p_i}{\partial t}=-(1+\eta_i)\frac{a}{L_n}\nabla_\theta\varphi-\left(\Gamma-\frac{1}{3}\right)\Gamma(1+\eta_i)\frac{a}{L_n}\nabla_\theta\nabla_\perp^2\varphi$$

$$\boxed{+4\Gamma\omega_D p_i+\left(\Gamma-\frac{1}{2}\right)\omega_D\nabla_\perp^2\varphi}\boxed{-\Gamma\nabla_{//}\upsilon_{//}}-\sqrt{\frac{8}{\pi}}\left|k_{//}\right|(p_i-\varphi)$$

with $\omega_D f=2\varepsilon\left(\cos\theta\nabla_\theta-\sin\theta\nabla_r\right)f$

Discussion:

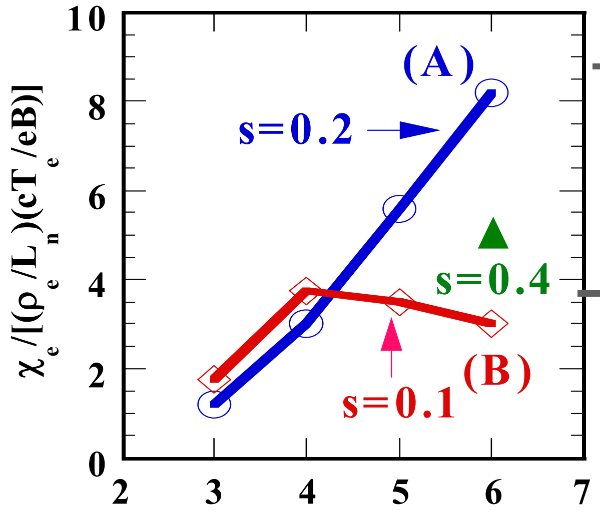
✓Ignoring toroidal curvature and parallel compression term, reduced to just drift wave;

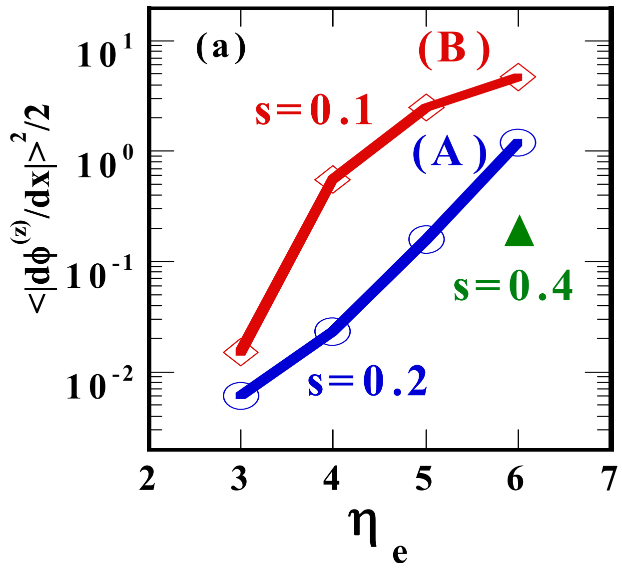
✓ Ignoring toroidal curvature effects, reduced to slab ITG

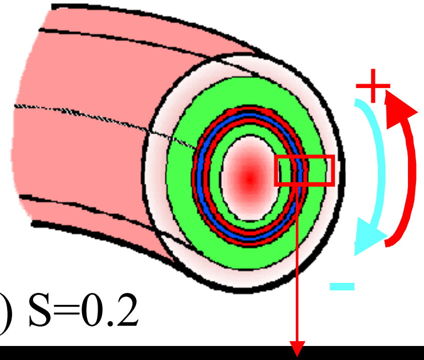
✓ Ignoring parallel compression, reduced to toroidal ITG;

Conditional flow generation in high pressure region

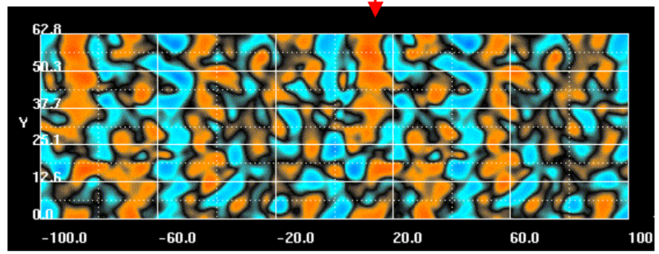
[*Kishimoto,Li, et al., IAEA '02*]



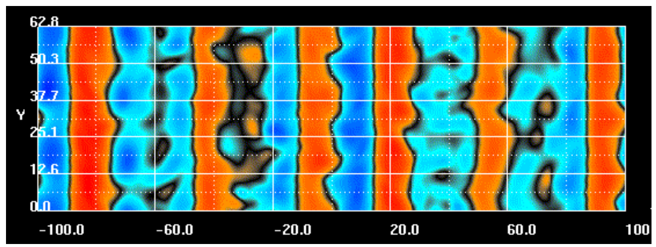




(A) S=0.2



(B) S=0.1 : weak magnetic shear



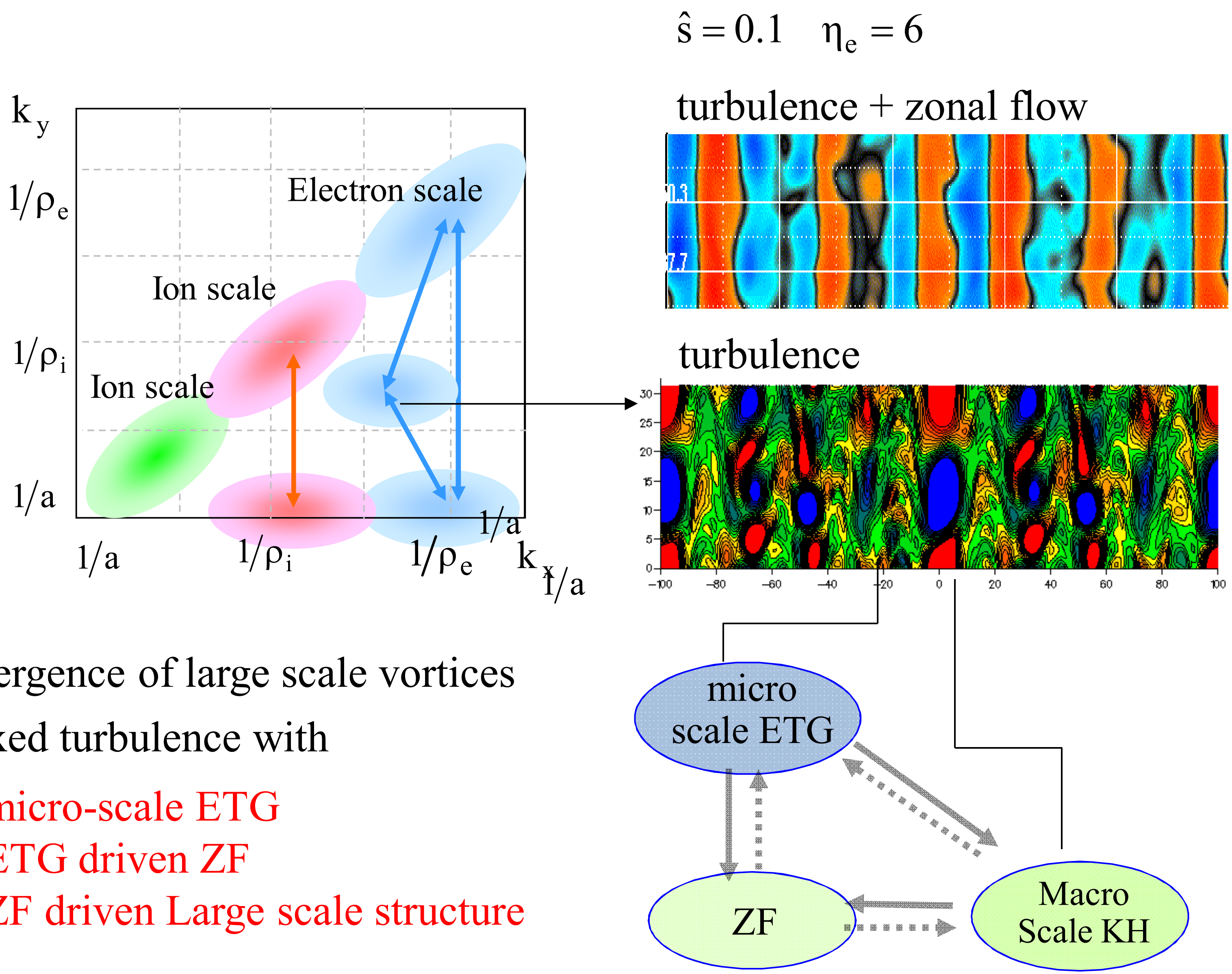
Energy partition low

$$\eta_Z \equiv \frac{E^{(ZF)}}{E^{(turb)} + E^{(ZF)}}$$

fluctuation energy

= turbulent part + laminar part

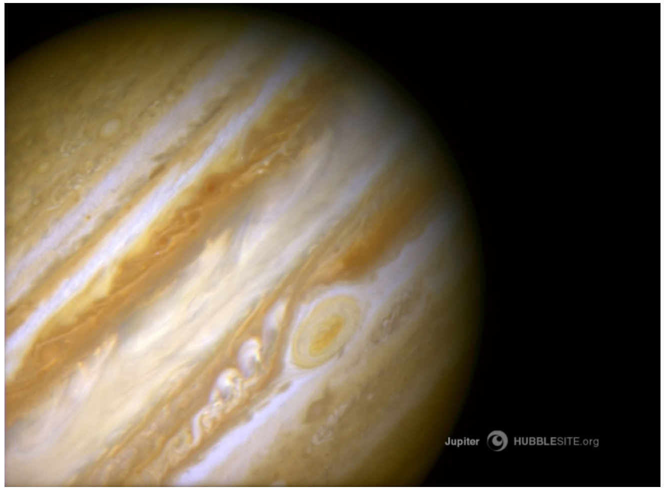
Turbulence dominated by large scale structure



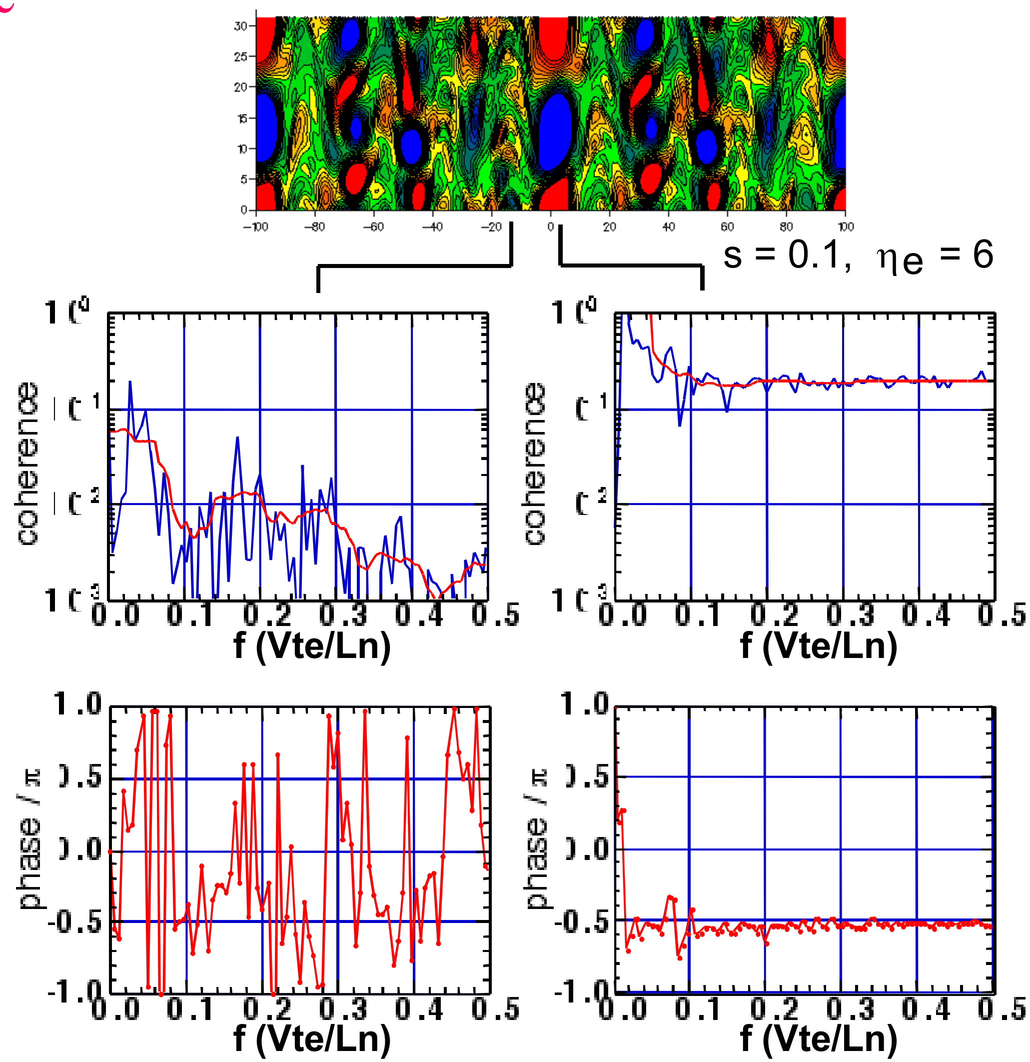
- Emergence of large scale vortices
- Mixed turbulence with
  - micro-scale ETG
  - ETG driven ZF
  - ZF driven Large scale structure

Coherency and phase between E_y and n

High coherency, but keeping phase relation that produces no transport



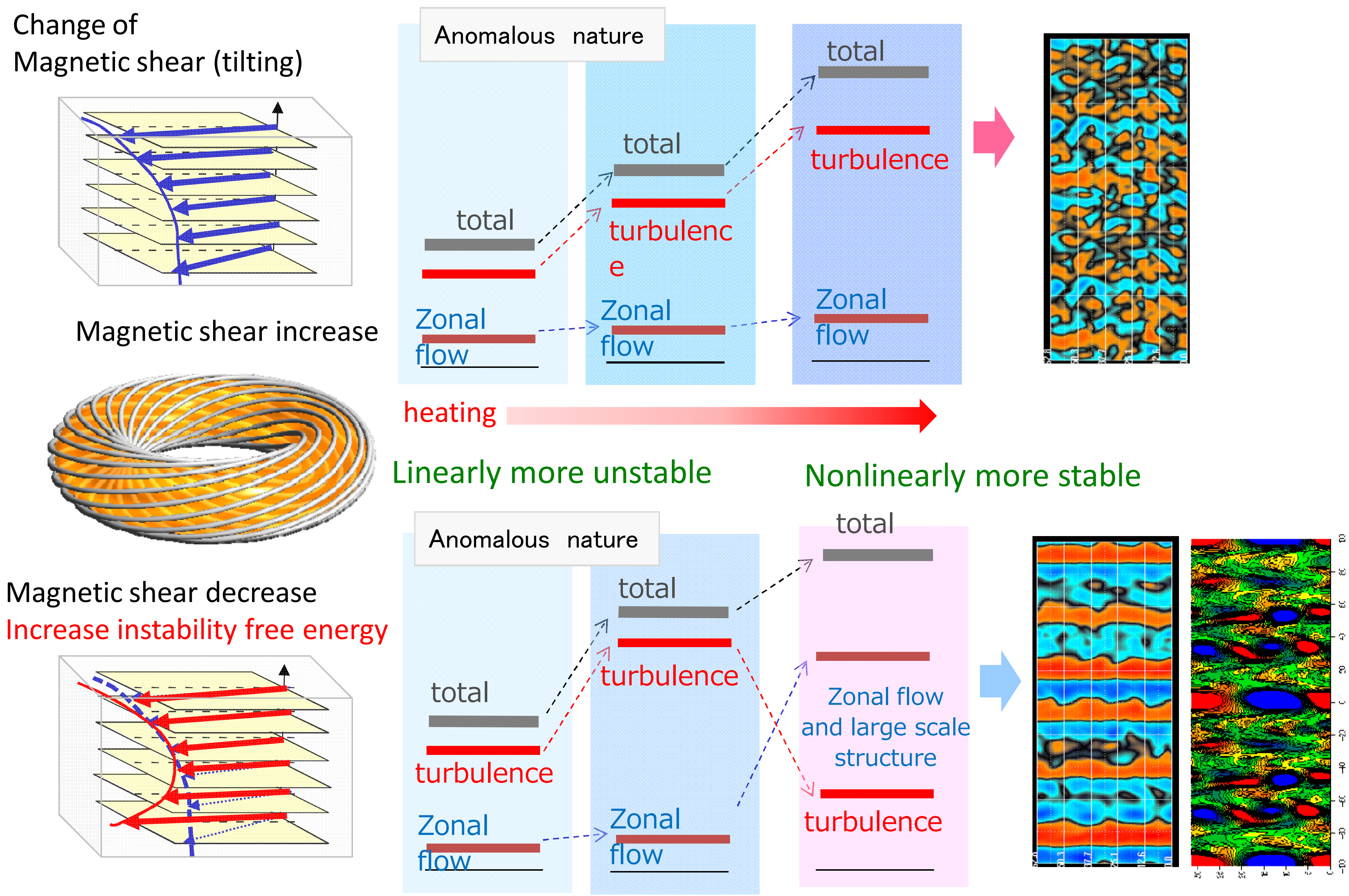
Zonal flow dominated plasma



Micro-scale region

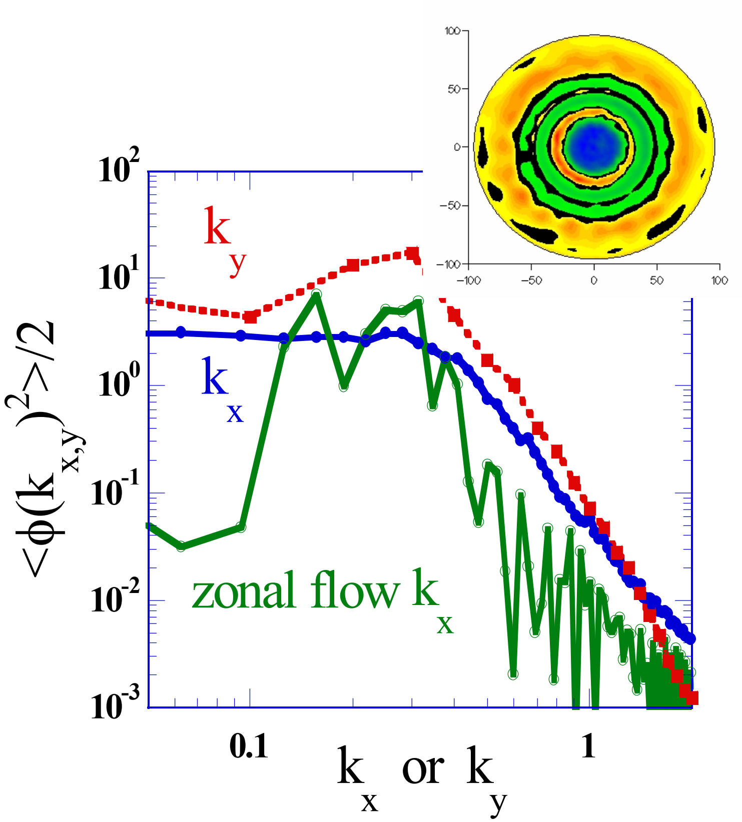
Macro-scale region

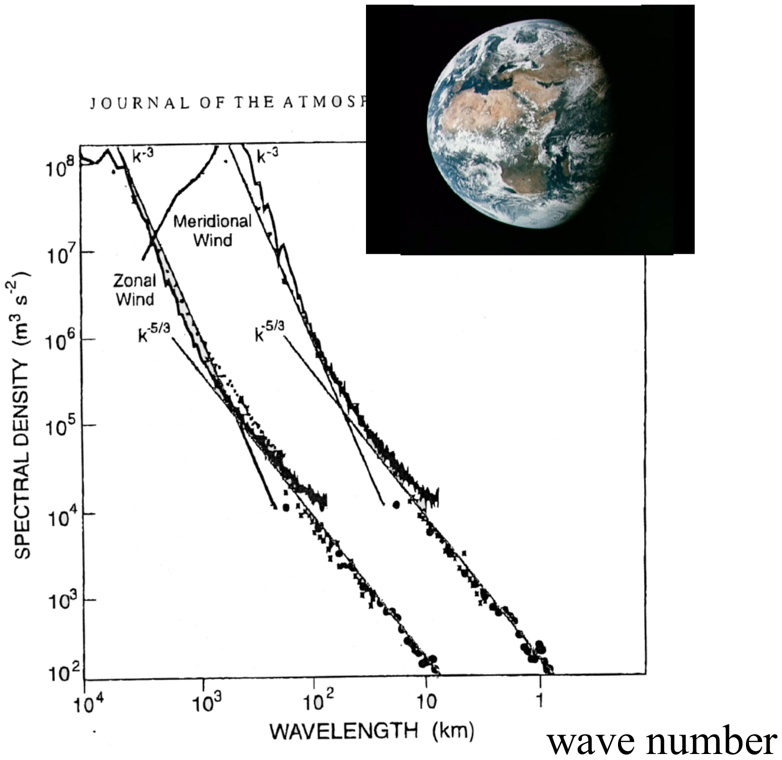
Self-organization in high pressure state



Comparison of atmospheric zonal flows

[Koshyk-Hamilton, JAS, 01]





Hasegawa-Mima Equation

$$\left(1-\nabla^2\right)\frac{\partial \varphi}{\partial t} + v_d \frac{\partial \varphi}{\partial y} - \left[\left(\mathbf{b}\times\nabla\varphi\right)\cdot\nabla\right]\nabla^2\varphi = 0$$

Charney Equation

$$\left(1-\nabla^2\right)\frac{\partial \mathbf{h}}{\partial t} + \mathbf{v}_R \frac{\partial \mathbf{h}}{\partial y} - \left[\left(\hat{\mathbf{z}}\times\nabla\mathbf{h}\right)\cdot\nabla\right]\nabla^2\mathbf{h} = 0$$

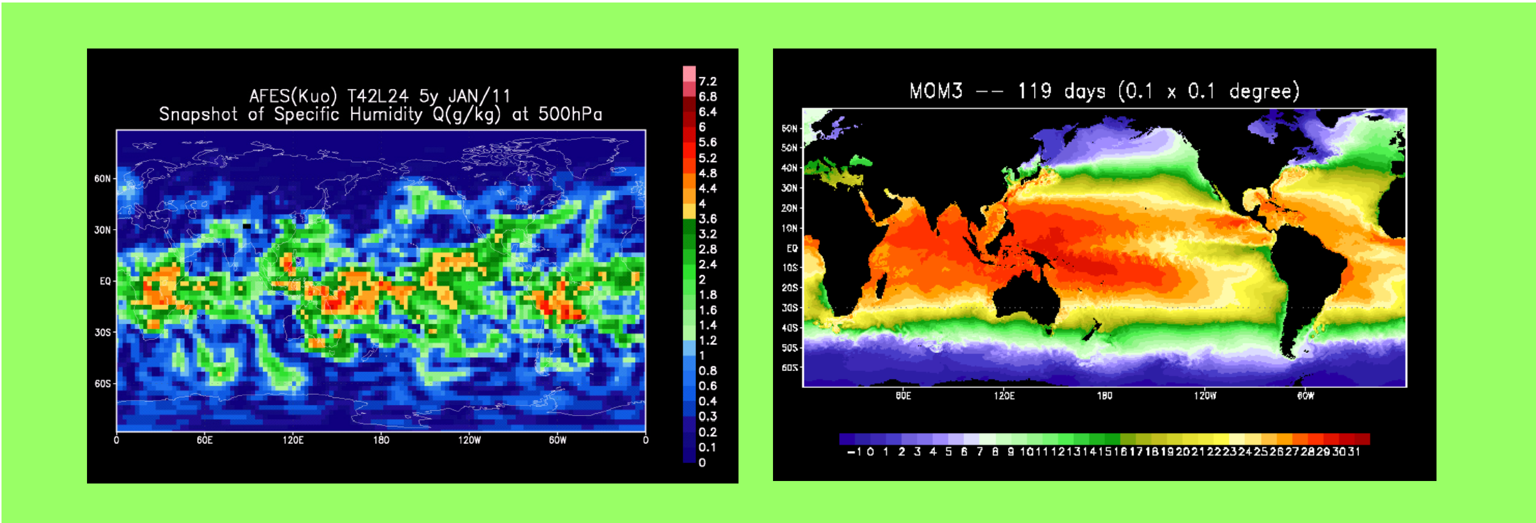
Self-organization in high pressure state

Total fluctuation

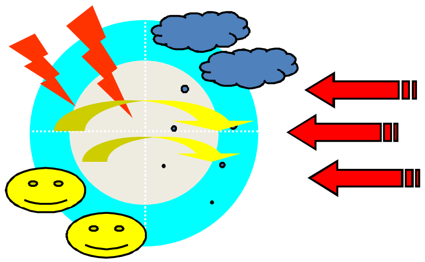
= turbulent fluctuation + zonal fluctuation

(→ induce transport)

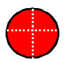
$$\eta_{zF} \equiv \frac{E^{(ZF)}}{E^{(tot)}} = \frac{E^{(ZF)}}{E^{(turb)} + E^{(ZF)}}$$



Earth environment



sun



Earth environment ???

sun