Flows and Physics in Stellar Atmospheres

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Based On:

1. S.M. Mahajan, R.Miklaszevski, K.I. Nikol'skaya & N.L. Shatashvili. ***Phys. Plasmas*. 8,** 1340 (2001);
***Adv. Space Res*., 30,** 345 (2002).
2-3. S. Ohsaki, N.L.Shatashvili, Z. Yoshida and S.M. Mahajan. ***The Astrophys. J***. **559**, L61 (2001); **570**, 395 (2002)
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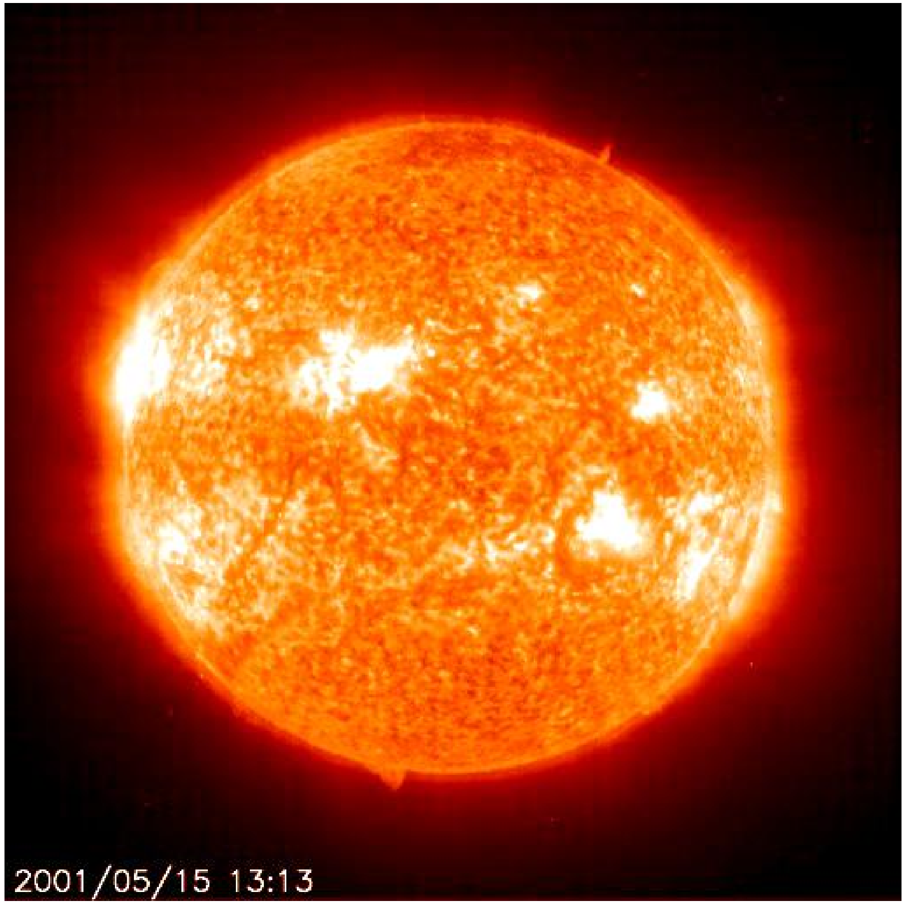
6. S.M. Mahajan, N.L. Shatashvili, S.V. Mikeladze & K.I. Sigua. ***Phys. Plasmas***. **13**, 062902 (2006)
7. S.M. Mahajan & N.L. Shatashvili. arXiv:0709.1535 (astro-ph), (2007).

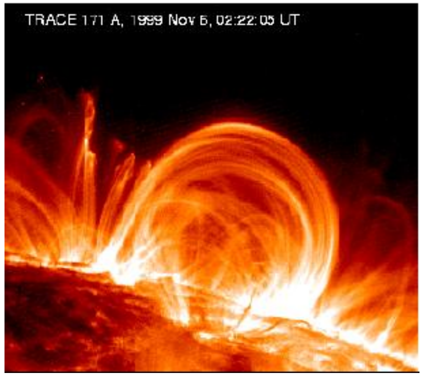
Outline

- Solar atmosphere - fields and flows; ***Dynamic finely structured Solar Atmosphere***
- Corona - observations and inferences. ***Heating of the Solar Corona***
- Towards a General unifying model $\Rightarrow$ ***Magnetofluid coupling***
- **Simultaneous Formation and primary heating** of the coronal structure.
- **Acceleration / Generation of flows - incompressible plasma case –** ***Catastrophe***
- **Acceleration / Generation of flows - incompressible plasma case –** ***Reverse Dynamo***
- **Acceleration / Generation of flows - compressible plasma case**
- **Beltrami-Bernoulli Equilibria in White Dwarfs –** ***indications & plans***
- Summary and Conclusions

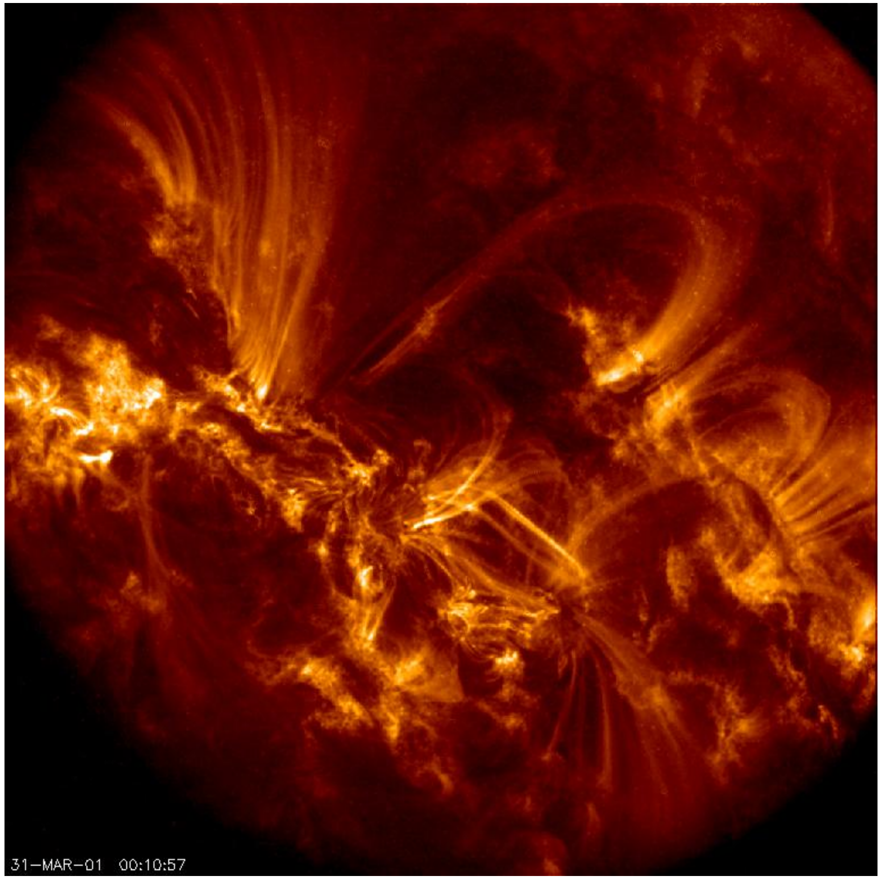
Dynamic multi-scale Solar Corona



- **The solar corona – a highly dynamic arena replete with multi-species multiple–scale spatiotemporal structures.**
- **Magnetic field was always known to be a controlling player.**
- **Strong flows are found everywhere in the low Solar atmosphere — in the sub-coronal (*chromosphere*) as well as in coronal regions (loops)** – *recent observations from HINODE (De Pontieu et al. 2011-2014).*



Active region of the corona with:



Co-existing dynamic structures:

- Flares
- Spicules
- Different-scale dynamic closed/open structures

Message:

- Different temperatures
- Different life-times

Indication:

- Any particular mechanism may be dominant in a specific region of parameter space.

Equally important: ***the plasma flows may complement the abilities of the magnetic field in the creation of the amazing richness observed in the Atmosphere***

Loops at different temperatures exist in the same general region and may be co–located to within their measured diameters.

Flows are found within loops.

Challenge – to develop a theory of energy transformations for understanding the **quiescent and eruptive/explosive events in solar atmosphere.**

Recently developed **theory that the formation and heating of coronal structures may be simultaneous**

and

directed flows may be the carriers of energy within a broad uniform physics framework opens a new channel for exploration.

How does the Solar Corona gets to be so hot ($\geq 10^6$K)?

Still an unsolved problem?!

- Is it the ohmic or the viscous dissipation?
- Or is it the shocks or the waves that impart energy to the particles
- And do the observations support the ”other consequences” of a given model?

Heating due to the viscous dissipation of the flow vorticity:

$$\left[\frac{d}{dt}\left(\frac{m_i \mathbf{V}^2}{2}\right)\right]_{\text{visc}} = -m_i n \nu_i \left(\frac{1}{2}(\nabla \times \mathbf{V})^2 + \frac{2}{3}(\nabla \cdot \mathbf{V})^2\right). \qquad (1)$$

Towards a General Unifying Model:

Conjecture:

Formation and primary heating of coronal structures as well as the more violent events (flares, erupting prominences and CMEs) **are the expressions of different aspects of the same general global dynamics that operates in a given coronal region.**

- The **plasma flows**, the source of both the particles and energy (part of which is converted to heat), ***interacting with the magnetic field, become dynamic determinants of a wide variety of plasma states*** $\Longrightarrow$

- **the immense diversity of the observed coronal structures.**

Magneto-fluid Coupling

$\boldsymbol{V}$ — the flow velocity field of the plasma

Total current $\boldsymbol{j} = \boldsymbol{j_0} + \boldsymbol{j_s}$. $\boldsymbol{j_s}$ – self-current (generates $\boldsymbol{B_s}$).

Total (observed) magnetic field — $\boldsymbol{B} = \boldsymbol{B_0} + \boldsymbol{B_s}$.

The stellar atmosphere is finely structured. Multi-species, multi-scales.

Simplest – two-fluid approach.

Quasineutrality condition: $n_e \approx n_i = n$

The kinetic pressure: $p = p_i + p_e \approx 2\, nT;$ $T = T_i \approx T_e$

Electron and proton flow velocities are different:

$$\boldsymbol{V_i} = \boldsymbol{V} \;; \qquad \boldsymbol{V_e} = (\boldsymbol{V} - \boldsymbol{j} / en)$$

Nondissipative limit: field frozen in electron fluid; ion fluid (finite inertia) moves distinctly.

Normalizations: $n \to n_0$ – the density at some appropriate distance from surface,
$\boldsymbol{B} \to \boldsymbol{B_0}$ – the ambient field strength at the same distance, $|\boldsymbol{V}| \to V_{A0}$ – Alfvén speed

Parameters: $r_{A0} = GM / V_{A0}^2 R_0 = 2\beta_0\, r_{c0}$; $\alpha_0 = \lambda_0 / R_0$; $\beta_0 = c_{s0}^2 / V_{A0}^2$;
c_{s0} — sound speed , R_0 — the characteristic scale length ,
$\lambda_0 = c/\omega_{i\,0}$ — the collisionless ion skin depth **are defined with** n_0 ; T_0 ; $\boldsymbol{B_0}$.

Hall current contributions are significant when $\alpha_0 > \boldsymbol{\eta}$
($\boldsymbol{\eta}$ - inverse Lundquist number)

Important in: interstellar medium, turbulence in the early universe, white dwarfs, neutron stars, stellar atmosphere.

Typical solar plasma: **condition is easily satisfied.**

Construction of a Typical Coronal structure

Solar Corona — $T_c = (1 \div 4) \cdot 10^6\ K$ $n_c \leq 10^{10}\ \mathrm{cm}^{-3}$.

Standard picture – Corona is first formed and then heated.

3 principal heating mechanisms:

- By Waves / Alfven Waves,
- By Magnetic reconnection in current sheets,
- MHD Turbulence.

All of these attempts fall short of providing a continuous energy supply that is required to support the observed coronal structures.

New concept:

Formation and heating are contemporaneous – primary flows are trapped & a part of their kinetic energy dissipates during their trapping period.

It is the Initial & Boundary conditions that define the characteristics of a given structure $T_c >> T_{0f} \sim 1eV$.

Observations → **there are strongly separated scales both in time and space in the solar atmosphere.** *And that is good.*

A closed coronal structure – 2 distinct eras:

1. **A hectic dynamic period when it acquires particles & energy (accumulation + primary heating)**

Full description needed: time dependent dissipative two-fluid equations are used. Heating takes place while particles accumulate (get trapped) in a curved magnetic field (*viscosity is taken local as well as the radiation is local),*

2. **Quasistationary period when it "shines" as a bright, high temperature object — a reduced equilibrium description suffices**

collisional effects and time dependence are ignored.

Equilibrium: each coronal structure has a nearly constant $\boldsymbol{T}$,

but different structures have different characteristic $\boldsymbol{T}$-s,

i.e. **bright corona seen as a single entity will have considerable $\boldsymbol{T}$–variation**

1st Era – Fast dynamic

Energy losses from corona: $F \sim (5 \cdot 10^5 \div 5 \cdot 10^6)$ erg/cm^2 s.

If the conversion of the kinetic energy in the **Primary Flows** were to compensate for these losses, we would require a radial energy flux

$$\frac{1}{2} m_i n_0 V_0^2 \; V_0 \geq F$$

For initial $V_0 \sim (100 \div 900)$ km/s $n \sim 9 \cdot 10^5 \div 10^7$ cm^{-3}

Viscous dissipation of the flow takes place on a time:

$$t_{\text{visc}} \sim \frac{L^2}{\nu_i} \qquad (2)$$

For flow with $T_0 = 3\text{eV} = 3.5 \cdot 10^4$ K , $n_0 = 4 \cdot 10^8$ cm^{-3}

Creating a quiet coronal structure of size $L = (2 \cdot 10^8 \div 10^{10})$ cm

$$t_{\mathrm{visc}} \sim (3.5 \cdot 10^8 \div 10^{10}) \text{ s}$$

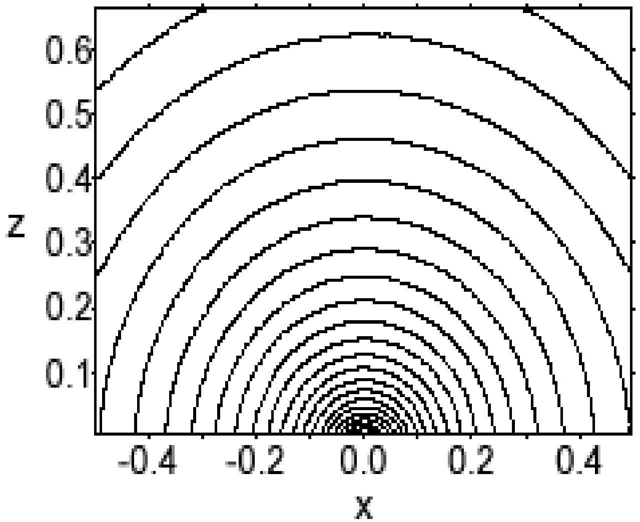
Note: (2) is an overestimate. $t_{\mathrm{real}} \ll t_{\mathrm{visc}}$.

Reasons:

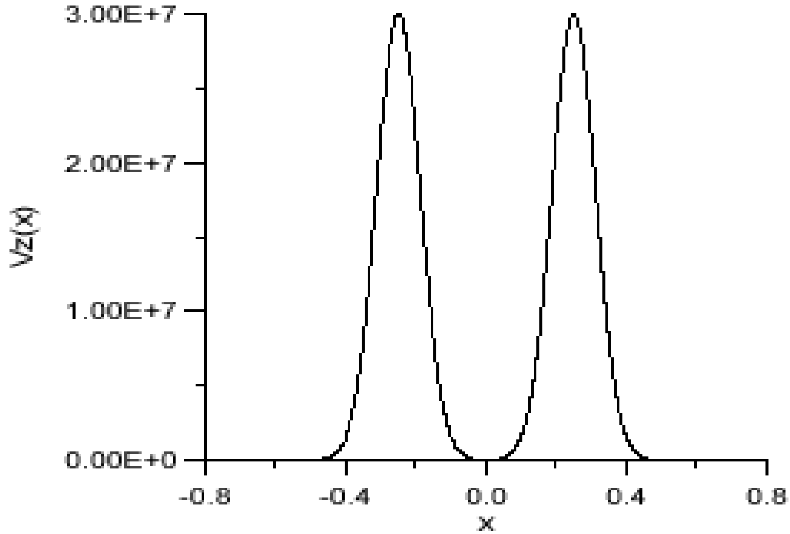
1) $\nu_i = \nu_i(t, \mathbf{r})$ will vary along the structure,

2) the spatial gradients of the ***V***–field can be on a scale much shorter than *L* (defined by the smooth part of ***B***–field).

Initial and Boundary conditions



Contour plots for the vector potential **A** (flux function) in the $x - z$ plane for a **typical arcade- like solar magnetic field**



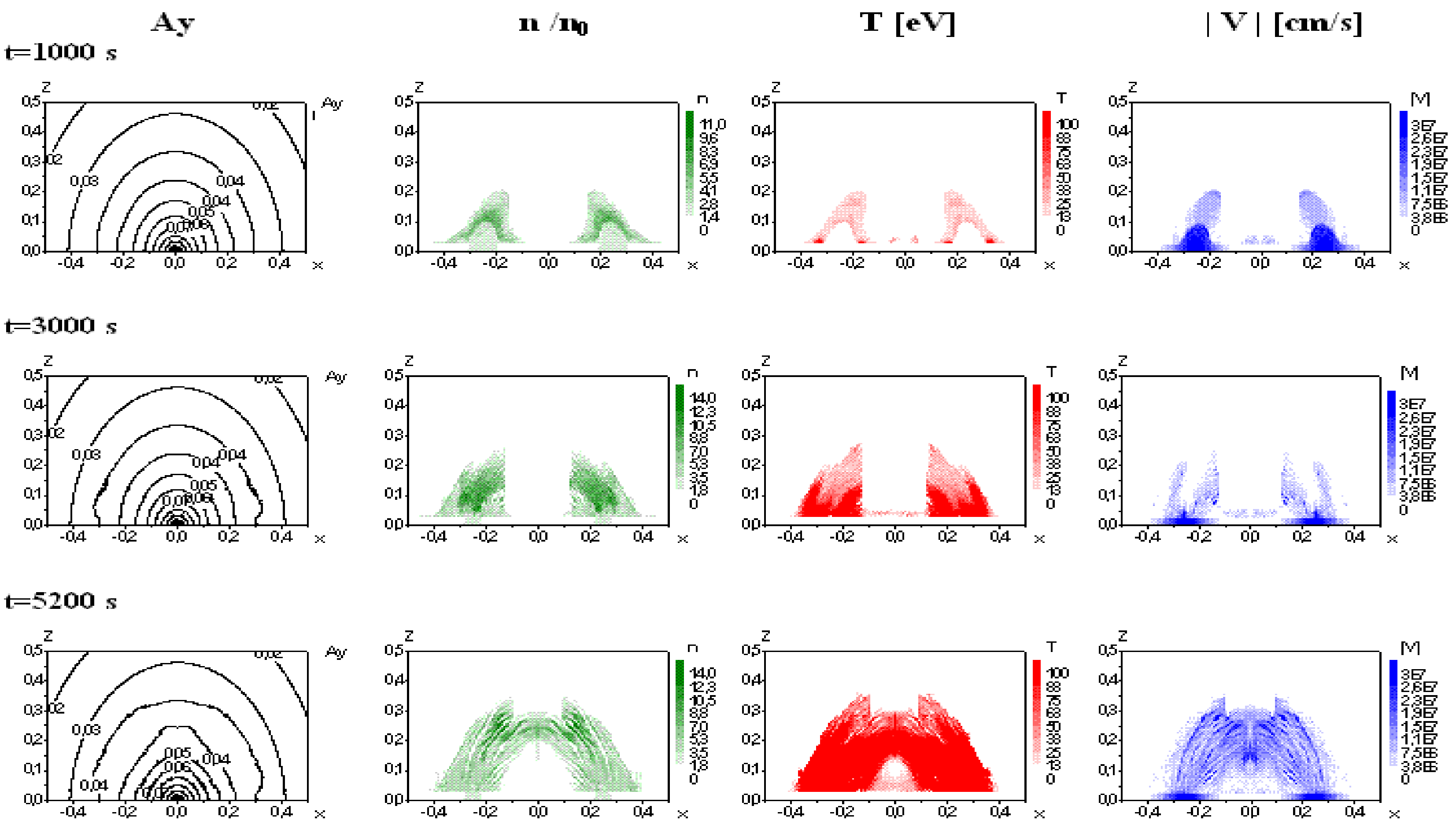
The distribution of the radial component $\mathbf{V_z}$ (with a maximum of **300 km/s at *t*= 0**) for the **symmetric, spatially nonuniform velocity field.**

2.5D numerical simulation of the general two-fluid equations in Cartesian Geometry.
Code: Mahajan et al. PoP (2001), Mahajan et al, ApJ (2005).

Simulation system contains: 1) dissipation (local) and heat flux; 2) plasma is compressible ; 3) ***Radiation*** is local (modified Bremsstahlung) - extra possibility for micro-scale structure creation.

Transport coefficients are taken from Braginskii and are local. Diffusion time of magnetic field > duration of interaction process (would require $T \leq$ a few eV -s).

Hot coronal structure formation



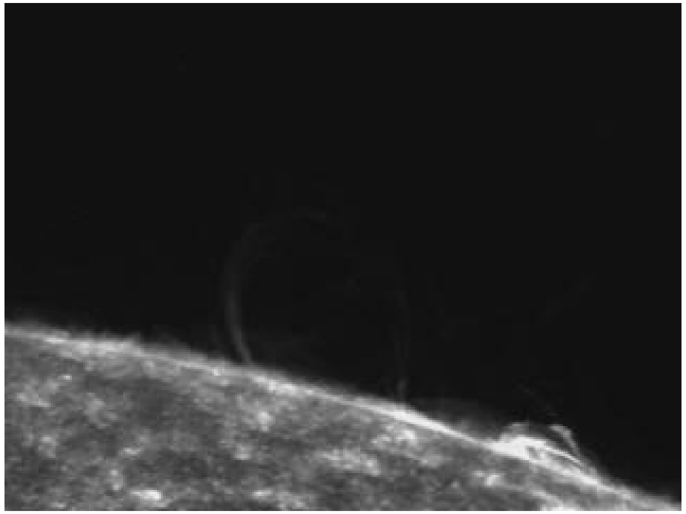
Flow T_0 =3eV, n_0 =4·10^{8}cm^{-3}, initial background density = 2·10^{8}cm^{-3} , B_{max} (x_0 , z_0=0) = 20G.

Much of the primary flow kinetic energy has been converted to heat via shock generation.

Simulations examples –formation & heating of hot structure

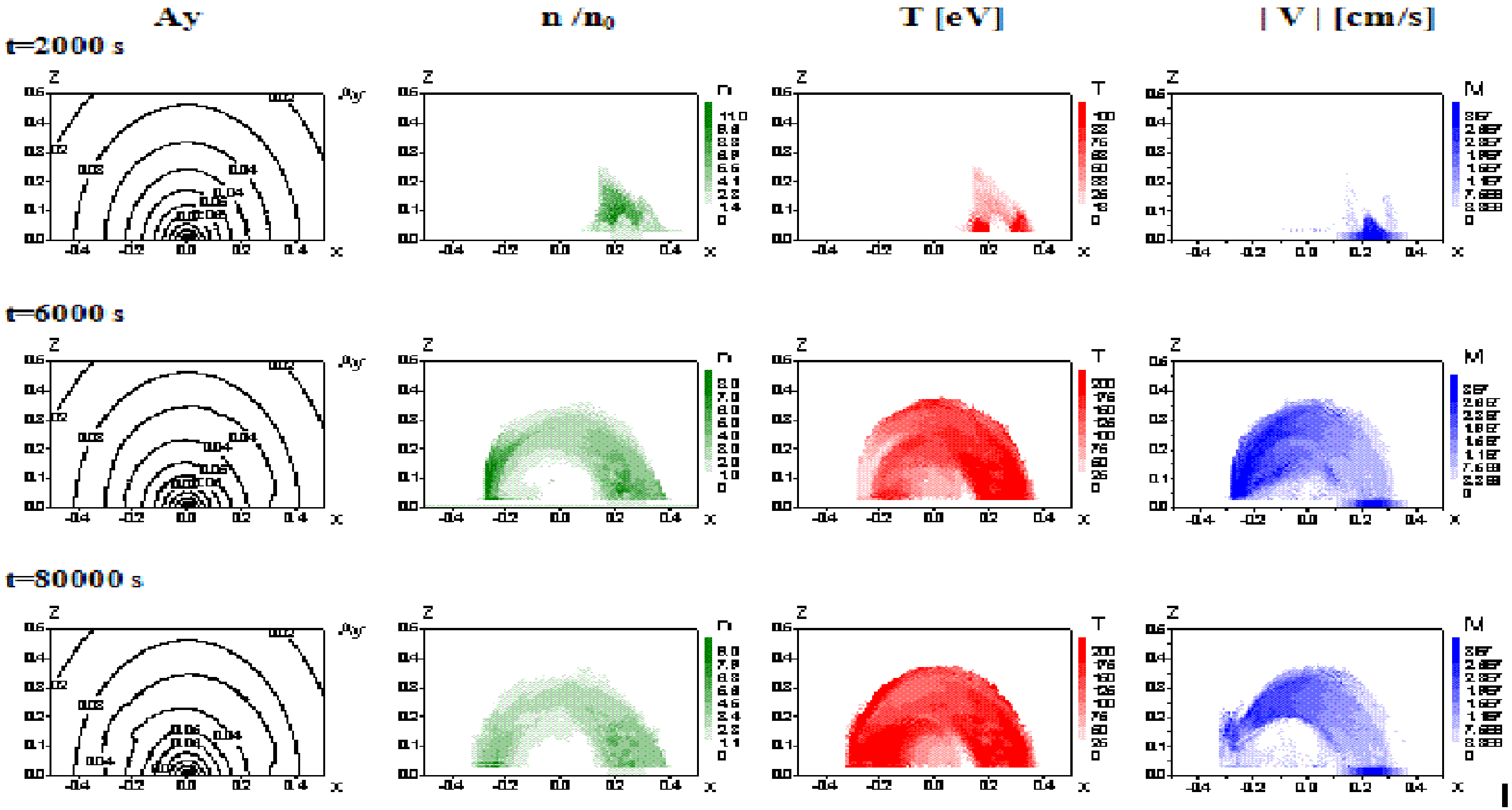
Simulation example 1 – symmetric case:
2 identical constant in time flows interact with closed ***B***-field structure. $B_{0max} = 20G$, $V_{z0max} = 300km/s$, $T_0 = 3eV$.
Primary heating is very fast – hot base is created in few 100s of seconds.

Observations show hot closed structure formation being different for different structures. **In the same region one observes different speeds of formation + heating – we see loop when it is hot.**



Left Column - no resistivity, right column – local resistivity included with coefficient ~ $2 \cdot 10^{-3}$.

Hot coronal structure formation

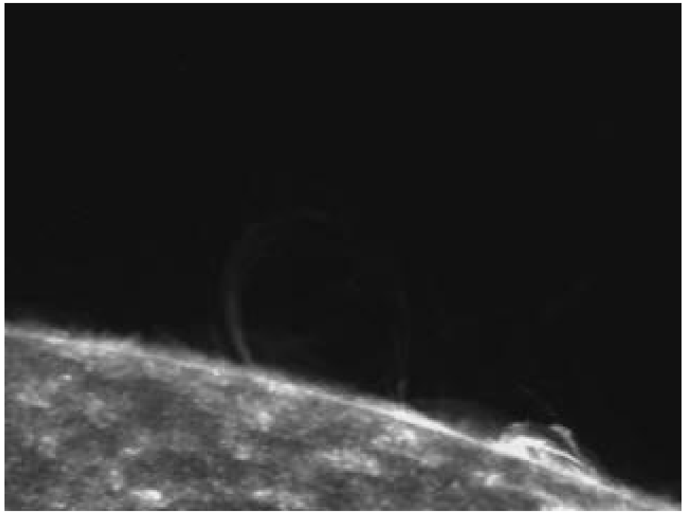


The interaction of an **initially asymmetric, spatially nonuniform primary flow** (*just the right pulse*) with a strong arcade-like magnetic field B_{max} $(x_0\ , z_0 = 0) = 20$ G.

Downflows, and the imbalance in primary heating are revealed

Flows found in the loops

Observations show that coronal structure **formation + heating is never a symmetric process; there are flows inside hot loops.**



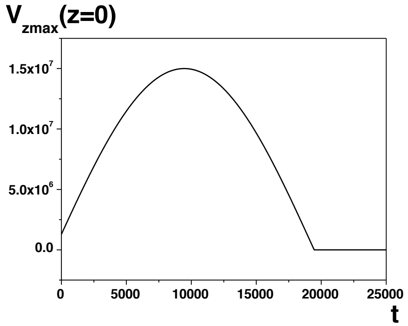
Simulation example 2 – non-symmetric case:
1 flow (constant in time) interacts with closed ***B***-field structure. $B_{0max} = 20G$, $V_{z0max} = 300km/s$, $T_0 = 3eV$. **Process of formation + heating is slower than in symmetric case.**
Flow remains along loop, just slowed down.

Left Column - no resistivity, right column – local resistivity included with coefficient $\sim 2\cdot 10^{-3}$.

Dependence on the initial and boundary conditions

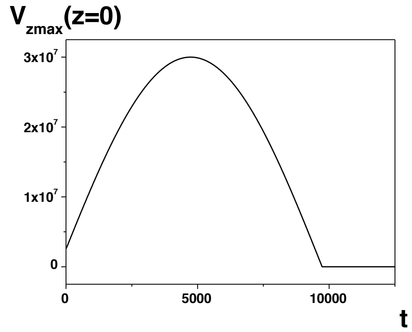
$V_{z0max}(z=0)=150km/s$

Left column - constant in time initial flow, **right column** – initial flow has *Life-time = 20000s*; $B_{0max} = 10G$.



$V_{z0max}(z=0)=300km/s$

Left column - constant in time initial flow & $B_{0max} = 10G$; **right column** – initial flow has *Life-time* = 10000s & $B_{0max} = 20G$.

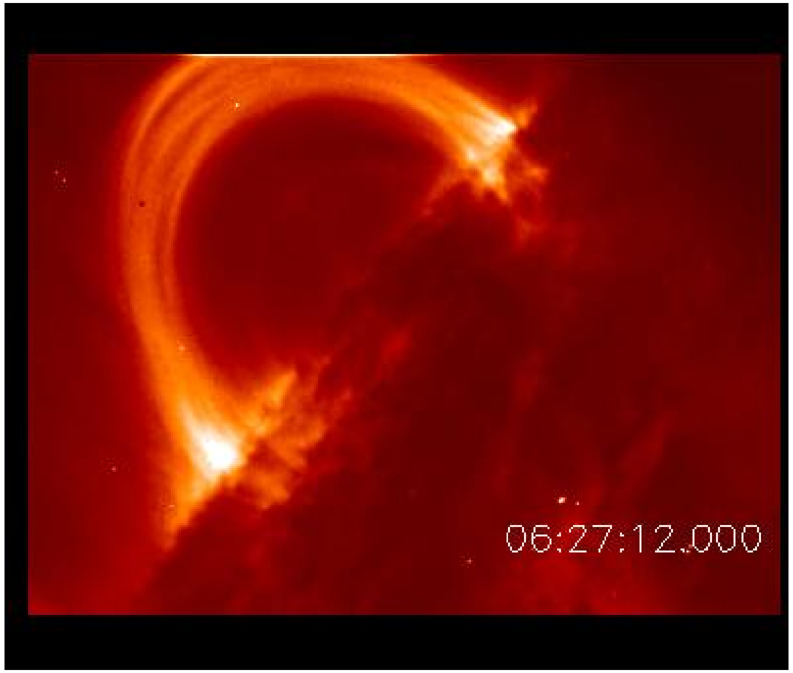


Simulation Results

- **Primary plasma flows are capable of thermalizing during interaction with primary magnetic fields** (that are curved) **to form the hot coronal structure**.
- **Two distinct eras are distinguishable in the life of a hot closed structure** – a fast era of the formation (plus primary heating), and a relatively calm era of in which the hot structure persists in a state of quasi-equilibrium.
- **Parameters of the hot closed structure (in quasi-equilibrium) are fully determined by the characteristics of the primary flow and the ambient magnetic fields**; the greater the primary flow initial velocity and initial magnetic field $\boldsymbol{B_0}$, the hotter is the coronal base.
- **For the same primary flows the maximum heating is achieved at some height independent of $\boldsymbol{B_0}$** (in agreement with observations).
- **The greater the resistivity, the shorter is the life-time of the quasi-equilibrium structure**.
- **The formation time of the hot closed structure is strictly dependent on the magnitudes of primary flow & primary magnetic field, as well as their initial time dependence (life-time).**
- The duration of the primary heating is directly determined by the parameters of primary flow and magnetic fields. **Greater the fields, the faster is the primary heating.**

2nd Era – Quasi-Equilibrium

Quasistationary period when closed coronal structure ”shines” as a bright, high temperature object.



Observations: A loop system may be quiescent for a long time with individual loops living for several hours (2nd era of quasi-equilibrium in the life of the closed coronal structure).

Quiescent periods may be followed by rapid activity (loops are ”turned on”/disappear in ≤ 10 - 40 min).

In equilibrium each coronal structure has a nearly constant T , but **different structures have different characteristic *T*-s,**

i.e.

bright corona seen as a single entity will have considerable T –variation.

- **The familiar magneto hydrodynamics (MHD) theory (*single fluid*) is inadequate** – The fundamental contributions of the velocity field do not come through.

- Equilibrium states (relaxed minimum energy states) encountered in MHD do not have enough structural richness.

In a two-fluid description, the velocity field interacting with the magnetic field provides:

1. **new pressure confining states**
2. **the possibility of heating these equilibrium states by dissipation of short scale kinetic energy.**

Let us now construct a simple equilibrium theory.

A Quasi-equilibrium Structure

Model: **recently developed magnetofluid theory.**

Assumption: at some distance there exist fully ionized and magnetized plasma structures such that the quasi–equilibrium two–fluid model will capture the essential physics of the system.

Simplest two–fluid equilibria: $T = \mathrm{const} \longrightarrow \mathrm{n}^{-1}\nabla \mathrm{p} \rightarrow \mathrm{T}\nabla \ln \mathrm{n}.$

Generalization to homentropic fluid: $p = \mathrm{const}\cdot \mathrm{n}^{\gamma}$ is straightforward.

The **dimensionless equations:**

$$\frac{1}{n}\nabla \times \mathbf{b} \times \mathbf{b} + \nabla\left(\frac{r_{A0}}{r} - \beta_0 \ln\ n - \frac{V^2}{2}\right) + \mathbf{V} \times (\nabla \times \mathbf{V}) = 0, \quad (3)$$

$$\nabla \times \left[\left(\mathbf{V} - \frac{\alpha_0}{n}\nabla \times \mathbf{b}\right) \times \mathbf{b}\right] = 0, \quad (4)$$

$$\nabla \cdot (n\mathbf{V}) = 0, \quad (5)$$

$$\nabla \cdot \mathbf{b} = 0, \quad (6)$$

The system allows the following **relaxed state solution**

$$\mathbf{b} + \alpha_0 \nabla \times \mathbf{V} = d\ n\ \mathbf{V}, \qquad \mathbf{b} = a\ n\ \left[\mathbf{V} - \frac{\alpha_0}{n} \nabla \times \mathbf{b}\right] \tag{7}$$

augmented by the **Bernoulli Condition**

$$\nabla \left(\frac{2\beta_0 r_{c0}}{r} - \beta_0 \ln\, n - \frac{V^2}{2} \right) = 0 \tag{8}$$

a and *d* — dimensionless constants related to **ideal invariants**:
the Magnetic and the Generalized helicities

$$h_1 = \int (\mathbf{A} \cdot \mathbf{b})\ d^3x \tag{9}$$

$$h_2 = \int (\mathbf{A} + \mathbf{V}) \cdot (\mathbf{b} + \nabla \times \mathbf{V}) d^3x \tag{10}$$

The system is obtained by minimizing **the energy** $E = \int (\mathbf{b} \cdot \mathbf{b} + n\mathbf{V} \cdot \mathbf{V})\ d^3x$ keeping h_1 and h_2 invariant.

Equations (7) yield

$$\frac{\alpha_0^2}{n}\nabla\times\nabla\times\mathbf{V}+\alpha_0\,\nabla\times\left(\frac{1}{a}-d\,n\right)\mathbf{V}+\left(1-\frac{d}{a}\right)\mathbf{V}=0 \tag{11}$$

which must be solved with (8) for n and $\mathbf{V}$.

Equation (8) is solved to obtain ($g(r) = r_{c0}/r$) .

$$n=\exp\left(-\left[2g_0-\frac{V_0^2}{2\beta_0(T)}-2g+\frac{V^2}{2\beta_0(T)}\right]\right) \tag{12}$$

The variation in density can be quite large for a low β_0 plasma if the gravity and the flow kinetic energy vary on length scales comparable to the extent of the structure.

Model calculation – temperature varying but density constant ($n = 1$) .

The following still holds (where $\boldsymbol{Q}$ is either $\mathbf{V}$ or $\boldsymbol{b}$):

$$\alpha_0^2\,\nabla\times\nabla\times\mathbf{Q}+\alpha_0\,\left(\frac{1}{a}-d\right)\nabla\times\mathbf{Q}+\left(1-\frac{d}{a}\right)\mathbf{Q}=0 \tag{13}$$

Analysis of the *Curl Curl* Equation, Typical Equilibria

The existence of **two, rather than one** (as in the standard relaxed equilibria) **parameter in this theory is an indication that we may have found an extra clue to answer the extremely important question:**

why do the coronal structures have a variety of length scales, and what are the determinants of these scales?

$\alpha_0 \sim 10^{-7} - 10^{-8}$ for typical densities $(\sim (10^7 - 10^9\,\mathrm{cm}^{-3})$.

Suppose: a structure has a span $\epsilon R_\odot$, where $\epsilon \ll 1$. For a structure of order 1000 km , $\epsilon \sim 10^{-3}$.

The ratio of the orders of various terms in Eq. (13) are $(|\nabla| \sim L^{-1})$

$$\frac{\alpha_0^2}{\epsilon^2} : \quad \frac{\alpha_0}{\epsilon}\left(\frac{1}{a} - d\right) : \quad \left(1 - \frac{d}{a}\right)$$
$$(1) \qquad\qquad (2) \qquad\qquad (3)$$

The following two principle balances are representative:

(a) The last two terms are of the same order, and the first << them:

$$\epsilon \sim \alpha_0 \frac{1/a - d}{1 - d/a} \tag{14}$$

For our desired structure to exist ($\alpha_0 \sim 10^{-8}$ for $n_0 \sim 10^9$ cm^{-3}):

$$\frac{1/a - d}{1 - d/a} \sim 10^5 \tag{15}$$

which is possible if d/a tends to be extremely close to unity.

For the first term to be negligible, we would further need

$$\frac{\alpha_0}{\epsilon} \ll \frac{1}{a} - d \qquad \Rightarrow \qquad \epsilon \gg \frac{10^{-8}}{1/a - d} \tag{16}$$

easy to satisfy as long as neither of $a \approx d$ is close to unity.

Standard relaxed state: flows are not supposed to play an important part.
Extreme sub–Alfvénic flows: a ~ d >> 1.

The new term introduces a qualitatively new phenomenon:

$\nabla \times (\nabla \times \mathbf{b})$ is a singular perturbation of the system; its effect on the standard root (2) ~ (3) >> (1) will be small, but it introduces a new root for which the $|\nabla|$ must be large (short length scale!)

For a and d so chosen to generate a 1000km structure

$$d/a \sim 1 + 10^{-4}, \qquad d \simeq a = -10, \qquad |\nabla|^{-1} \sim 10^2 \text{ cm},$$

an equilibrium root with variation on the scale of 100cm will be automatically introduced by the flows.

Even if flows are weak $(a \simeq d \simeq 10)$, the departure from $\nabla \times \mathbf{B} = \alpha \mathbf{B}$ can be essential: **it introduces a totally different (small!) scale solution** $\Rightarrow$ fundamental importance in understanding the effects of viscosity on the dynamics of structures.

Dissipation of short scale structures $\rightarrow$ primary heating.

(b) **The other balance**: we have a complete departure from conventional relaxed state: **all three terms are of the same order**

$$\epsilon \sim \alpha_0 \frac{1}{1/a - d} \sim \alpha_0 \frac{1/a - d}{1 - d/a} \tag{17}$$

which translates as:

$$\left(\frac{1}{a} - d\right)^2 \sim 1 - \frac{d}{a}, \qquad \frac{1}{a} - d \sim \alpha_0 \frac{1}{\epsilon} \tag{18}$$

For a **1000km structure**, $\alpha_0 \cdot 1/\epsilon \sim 10^{-5}$ and ***a ~ d ~ 1***

we would need the flows to be almost perfectly Alfvénic!

Such flow conditions are in the weak magnetic field regions.

(1) Alfvénic flows are capable of creating entirely new kinds of structures – quite different from the ones that we normally deal with.

(2) Though they also have two length scales, these length scales are quite comparable to one another.

(3) Two length scales can become complex conjugate giving rise to fundamentally different structures in ***b*** and **V** .

Curl Curl Equation – Double-Beltrami states

With $p = (1/a - d)$ and $q = (1 - d/a)$, Eq. (13) $\Longrightarrow$

$$(\alpha_0 \nabla \times - \lambda)(\alpha_0 \nabla \times - \mu)\mathbf{b} = 0 \tag{19}$$

where λ (λ_+) and μ(λ_-) are the solutions of the quadratic equation

$$\alpha_0 \lambda_\pm = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}. \tag{20}$$

If G_λ is the solution of the **Beltrami Equation** (a_λ and a_μ are constants)

$$\nabla \times \mathbf{G}(\lambda) = \lambda \mathbf{G}(\lambda), \qquad then \tag{21}$$

$$\mathbf{b} = a_\lambda \mathbf{G}(\lambda) + a_\mu \mathbf{G}(\mu) \tag{22}$$

is the general solution of the double *curl* equation. Velocity field is:

$$\mathbf{V} = \frac{\mathbf{b}}{a} + \alpha_0 \boldsymbol{\nabla} \times \mathbf{b} = \left(\frac{1}{a} + \alpha_0 \lambda\right) a_\lambda \mathbf{G}(\lambda) + \left(\frac{1}{a} + \alpha_0 \mu\right) a_\mu \mathbf{G}(\mu) \tag{23}$$

Double curl equation is fully solved in terms of the solutions of Eq. (21).

Double Beltrami States

- **There are two scales in equilibrium** unlike the standard case.

- **A possible clue for answering the extremely important question: why do the coronal structures have a variety of length scales, and what are the determinants of these scales?**

- **The scales could be vastly separated** – are determined by the constants of the motion – the original preparation of the system.
  These constants also determine the relative kinetic & magnetic energy in quasi-equilibrium.

- **These vastly richer structures can & do model the quiescent solar phenomena** rather well – construction of coronal arcades fields, slow acceleration, spatial rearrangement of energy etc.

An Example of structural richness

Closed Coronal structure: the magnetic field is relatively smooth but the velocity field must have a considerable short–scale component if its dissipation were to heat the plasma. Can a DB state provide that?

Sub–Alfvénic Flow: $a \sim d >> 1 \Longrightarrow \lambda \sim (d - a) / \alpha_0 d\, a\, ; \quad \mu = d / \alpha_0$.

$$\mathbf{V} = \frac{1}{a} a_\lambda \mathbf{G}_\lambda + d a_\mu \mathbf{G}(\mu) \tag{24}$$

$$\mathbf{b} = a_\lambda \mathbf{G}_\lambda + a_\mu \mathbf{G}(\mu) \tag{25}$$

while, the slowly varying component of velocity is smaller by a factor ($a^{-1} \approx d^{-1}$) compared to similar part of ***b***-field, the fast varying component is a factor of d larger than the fast varying component of ***b***-field!

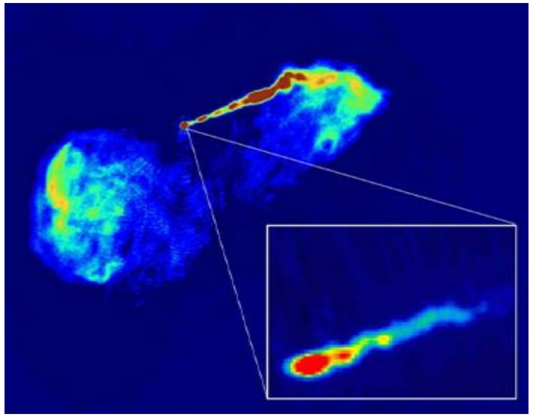
Result: for an extreme sub–Alfvénic flow (e.g. $|\mathbf{V}| \sim d^{-1} \sim 0.1$) ,

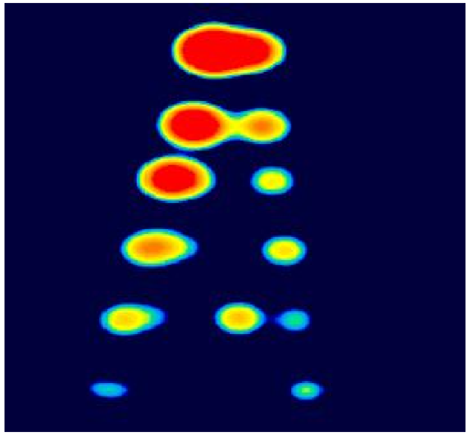
$$\frac{|\mathbf{V}(\mu)|}{|\mathbf{V}(\lambda)|} \simeq 1 \tag{26}$$

the velocity field is equally divided between slow and fast scales.

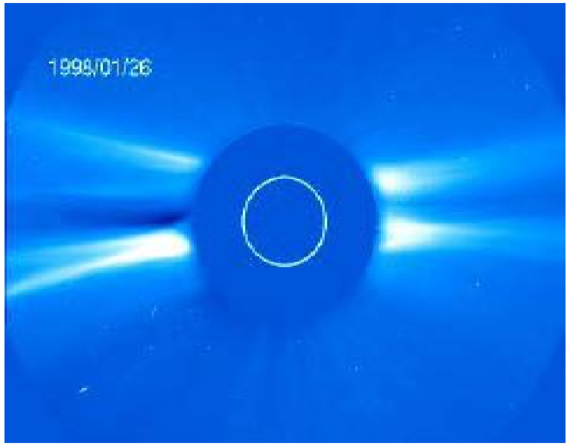
Acceleration / Generation of *Large Scale Flows*

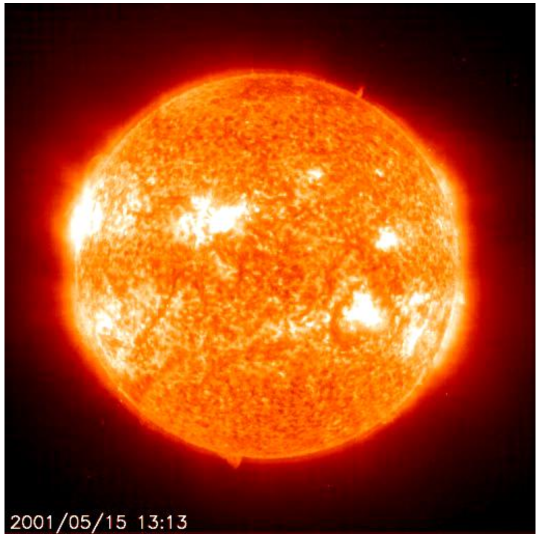
Acceleration of large-scale flows - creation of stellar winds, variety of outflows, jets, & etc. – often observed **in astrophysical objects.**





Recent observations - **solar corona is a highly dynamic arena replete with multi-species multiple–scale spatiotemporal structures.**





Sources of Energy for *Large-Scale Plasma Flow* Acceleration

The most obvious process for acceleration (***rotation is ignored***):

the **conversion of**

- **magnetic**
- **and/or the thermal energy**
- **turbulence energy**

===> to plasma kinetic energy

- **Magnetically driven transient but sudden flow–generation models:**
  - **Catastrophic models**
  - **Magnetic reconnection models**
  - **Models based on instabilities**

Quiescent pathway:

- ***Bernoulli mechanism converting thermal energy into kinetic***
- **General magneto-fluid rearrangement of a relatively constant kinetic energy:** going from an initial *high density–low velocity* to *a low density–high velocity state*.

Magneto-Fluid coupling – model equations

Minimal two-fluid model – **incompressible, constant density Hall MHD** – ***gravity & rotation are ignored.***

Dimensionless system in standard Alfvenic units.

Velocities - normalized to the Alfven speed with appropriate magnetic field.

Times **-** measured in terms of the (cyclotron time)$^{-1}$, **Lengths** - to collisionless skin depth λ_{i0}.

Defining equations are:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \Big[[\boldsymbol{V} - \boldsymbol{\nabla} \times \boldsymbol{B}] \times \boldsymbol{B} \Big], \qquad \boldsymbol{V}_e = \boldsymbol{V} - \boldsymbol{\nabla} \times \boldsymbol{B} \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial t} = \boldsymbol{V} \times (\boldsymbol{\nabla} \times \boldsymbol{V}) + (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} \left(P + \frac{V^2}{2} \right) \tag{2}$$

The red terms are due to Hall current and the **blue terms are vorticity forces**.

Quasi-equilibrium Approach

Assumption **(gravity included):** there exists fully ionized & magnetized plasma structures ==> the quasi–equilibrium two–fluid model will capture the essential physics of flow acceleration

Simplest two–fluid equilibria: $T = \mathrm{const} \longrightarrow \mathrm{n}^{-1}\nabla \mathrm{p} \rightarrow \mathrm{T}\nabla \ln \mathrm{n}.$ **Generalization to homentropic fluid is straightforward.**

The dimensionless equations for compressible case: *Mahajan et al. 2001, PoP*

$$\frac{1}{n}\nabla \times \mathbf{b} \times \mathbf{b} + \nabla\left(\frac{r_{A0}}{r} - \beta_0 \ln\, n - \frac{V^2}{2}\right) + \mathbf{V} \times (\nabla \times \mathbf{V}) = 0, \tag{3}$$

$$\nabla \times \left[\left(\mathbf{V} - \frac{\alpha_0}{n}\nabla \times \mathbf{b}\right) \times \mathbf{b}\right] = 0 \tag{4}$$

$$\nabla \cdot (n\mathbf{V}) = 0 \quad (5) \qquad \nabla \cdot \mathbf{b} = 0 \quad (6)$$

Parameters: $r_{A0} = GM/V_{A0}^2 R_0 = 2\beta_0\, r_{c0},\quad \alpha_0 = \lambda_{i0}/R_0,\quad \beta_0 = c_{s0}^2/V_{A0}^2,$

$\lambda_{i0} = c/\omega_{i0}$ - the collisionless ion-skin depth, **are defined by** $n_0,\ T_0,\ B_0$

Hall current contributions are significant when $\alpha_0 > \eta$, (η - inverse Lundquist number)

Important in: interstellar medium, early universe, white dwarfs, neutron stars, stellar atmosphere.

Typical solar plasma: condition is easily satisfied.

Hall currents modifying the dynamics of the microscopic flows/fields - have a profound impact on the generation of macroscopic magnetic fields & macroscopic flows

The double Beltrami solutions are

$$\mathbf{b} + \alpha_0 \nabla \times \mathbf{V} = d\ n\ \mathbf{V}, \qquad \mathbf{b} = a\ n\ \left[\mathbf{V} - \frac{\alpha_0}{n} \nabla \times \mathbf{b}\right], \tag{7}$$

a and ***d*** — **dimensionless constants related to ideal invariants:**

the Magnetic helicity

$$h_1 = \int (\mathbf{A} \cdot \mathbf{b})\ d^3x, \tag{8}$$

& the Generalized helicity

$$h_2 = \int (\mathbf{A} + \mathbf{V}) \cdot (\mathbf{b} + \nabla \times \mathbf{V}) d^3x. \tag{9}$$

obeying the ***Bernoulli Condition***

$$\nabla \left(\frac{2\beta_0 r_{c0}}{r} - \beta_0 \ln n - \frac{V^2}{2} \right) = 0, \tag{10}$$

relating the density with the flow kinetic energy & gravity.

Quasi-equilibrium → Eruptive and Explosive events, Flaring

Incompressible case

The parameters of the DB field change *– assumption*

- **the parameter change is sufficiently slow / adiabatic.**
- **at each stage, the system can find its local DB equilibrium.**
- **in slow evolution the dynamical invariants: h_1 , h_2, & the total (magnetic + fluid) energy E are conserved.**

The General equilibrium solution ***for incompressible*** **case is shown to be**

(*G_λ , G_μ - solutions of Beltrami equation*)

$$\boldsymbol{b} = C_\mu \boldsymbol{G}_\mu(\mu) + C_\lambda \boldsymbol{G}_\lambda(\lambda), \tag{11}$$

$$\boldsymbol{V} = \left(\frac{1}{a} + \mu\right) C_\mu \boldsymbol{G}_\mu(\mu) + C_\lambda \left(\frac{1}{a} + \lambda\right) \boldsymbol{G}_\lambda(\lambda). \tag{12}$$

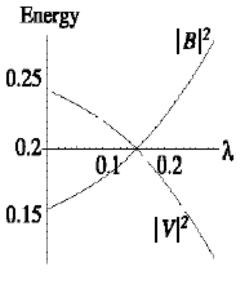
The catastrophic loss of equilibrium may occur in one of the following two ways:

1. The ***determining length scales*** (λ - large-scale, μ - short-scale) ***for the field variation*, go from being real to complex**.

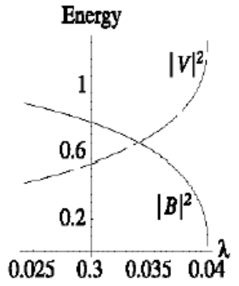
2. amplitude of either of the 2 states (G_λ , G_μ) ceases to be real.

- **Large scale λ – control parameter — observationally motivated choice.**
- ***Example*: structure–structure interactions (2D Beltrami ABC field with periodic boundary conditions). Choosing real λ , μ for quasi–equilibrium.**
- **Conditions for catastrophic changes in Slowly Evolving Solar Structures (sequence of DB states) leading to a fundamental transformation of the initial state are derived as:**
- **For $E > E_c = 2\left(h_1 \pm \sqrt{h_1 h_2}\ \right)$ the DB equilibrium suddenly relaxes to a SB state corresponding to the large macroscopic size.**
- **All of the short–scale magnetic energy is catastrophically transformed to the flow kinetic energy. *Seeds of destruction lie in the conditions of birth.***
- **The proposed mechanism for the energy transformation work in all regions of Solar Atmosphere with different dynamical evolution depending on the *Initial & Boundary Conditions* for a given region.**

Large Scale Plasma flow acceleration – catastrophe ($n = const$)



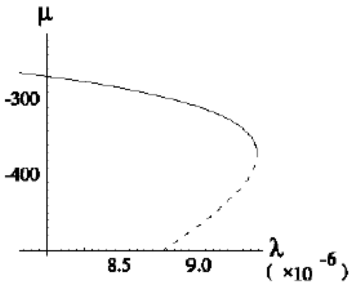
(a)

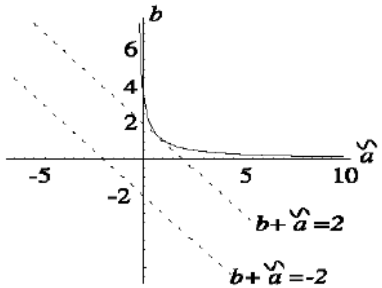


(b)

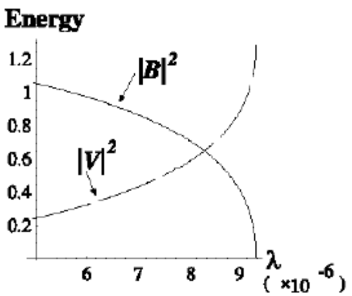
a) No catastrophe initial conditions

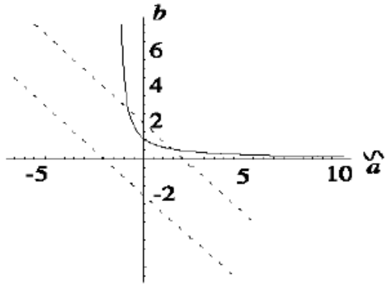
b) Catastrophe initial conditions





(a)



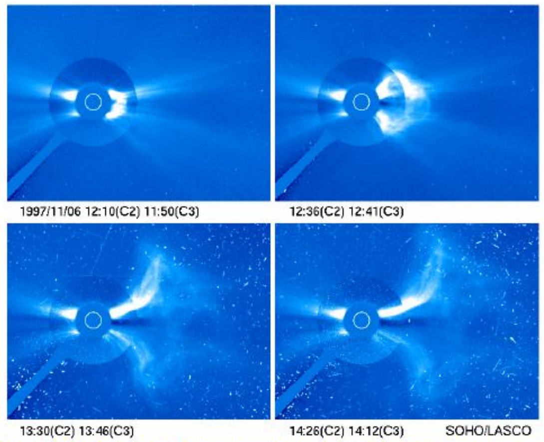


(b)

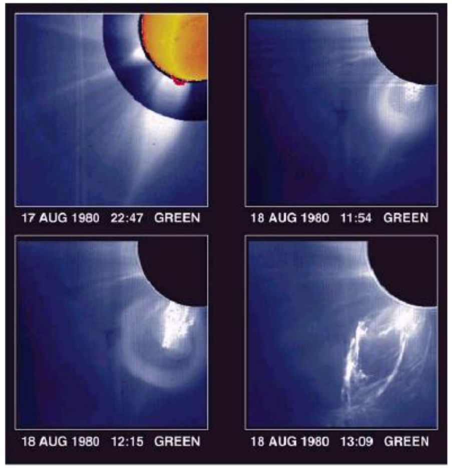
Solar Atmosphere:
Almost all initial magnetic energy (short scale) is transferred to flow

Root coalescence:
No separation between roots at the transition!

Coronal Mass Ejection



SOHO/LASCO images of a coronal mass ejection on 6 November 1997



A time sequence of *Solar Maximum Mission* coronagraph images showing a CME on August 18, 1980 (from Hundhausen (1999))

Quasi-equilibrium → Eruptive and Explosive events, Flaring

Compressible case

Closed HMHD system (3-6) of equilibrium equations ($g(r) = r_{c0} / r$) ===>

1D simulation - a variety of boundary conditions: Flow with 3.3 km/s ends up with ~ 100 km/s .

For small α_0 there exists some height where density drops sharply with a corresponding sharp rise in the flow speed ===>

- **There is a catastrophe in the system.**
- **The distance over which it appears is determined by the strength of gravity *g(z)* .**
- **Amplification of flow is determined by local β_0 .**

If density fall is at a much slower rate than the slow scale 1D problem solution gives:

(if ***a ~ d >>1***; or ***a ~ d << 1*** , or *an equation of state is assumed*, ***T≠const***):

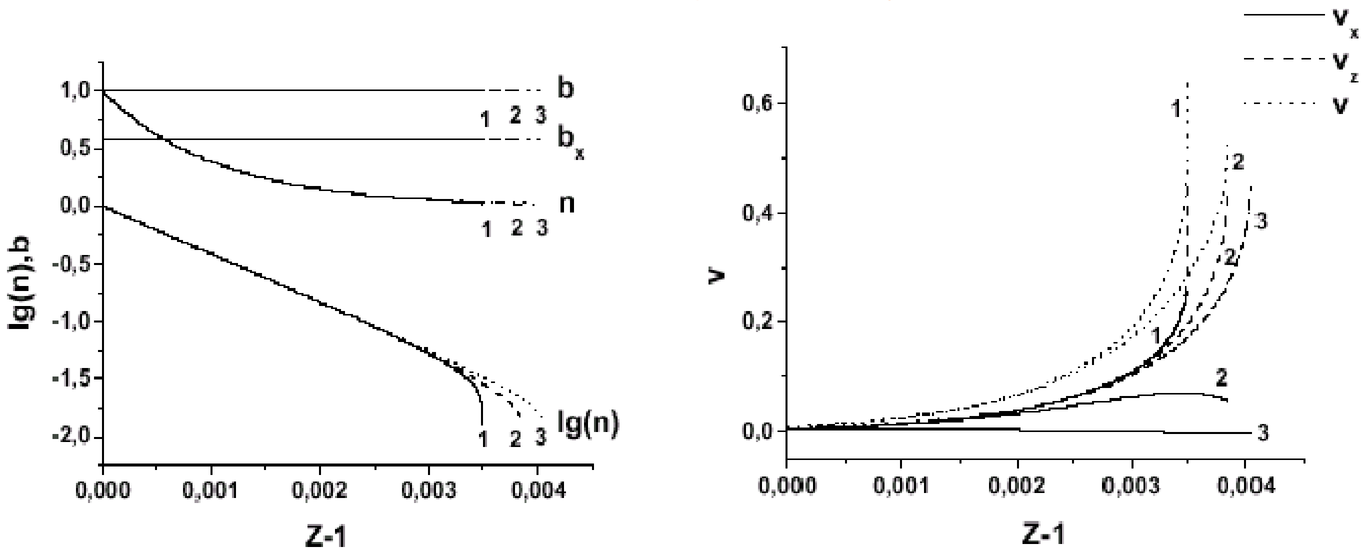
$$|V_{max}| = \frac{1}{d\, n_{min}}$$

$$|V|^2 = 1/d^2 n^2 \qquad |\mathbf{b}|^2 = const \qquad (13)$$

The Bernoulli condition transforms to the defining differential equation for density ==>

$$n_{min} = (2\beta_0)^{-1/2} d^{-1} \qquad (14)$$

Plasma flow acceleration – catastrophe (*n ≠ const*)



Sub-Alfvénic flows: Boundary conditions at:

$Z_0 > (1+2.8\bullet10^{-3})R_s$ – the influence of ionization can be neglected

$|b_0| = 1$, $V_0 = 0.01V_{A0}$ (with $V_{x0} = V_{y0} = V_{z0}$)

DB parameters:
d ~a ~100, $(a-d)/a^2$ ~10^{-6}

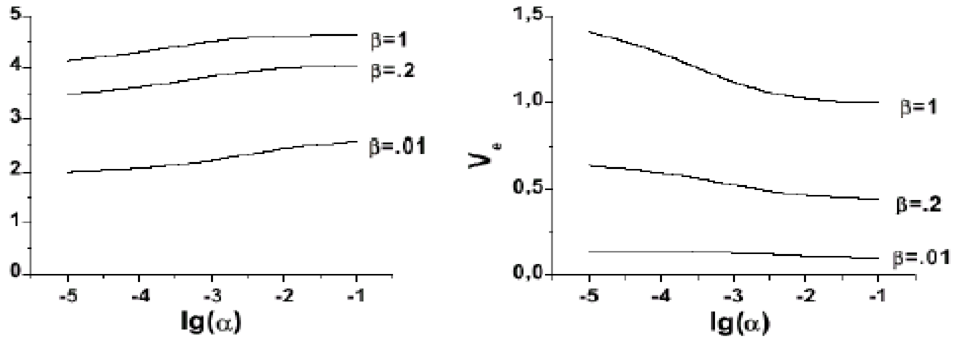
3 sets of curves labeled by α_0 for parameters versus height (Z- 1).
1- 2- 3 correspond to: α_0 = 0.000013; 0.005; 0.1

(a) (b)

(a) Blowup distance

(b) velocity

vs.. α_0



Following are the (n_0 ; B_0 ; T_0 ; V_{A0}):

10^{11}cm^{-3}; 100G; 5eV; 600km/s; β_0~0.007<<1

$|b|^2$~const; Density fall → Velocity increase

Catastrophe!

Acceleration is determined by local β_0

Steady flow generation /acceleration – *Reverse Dynamo*

The Dynamo mechanism - generic process of generating macroscopic magnetic fields from an initially turbulent system

Standard Dynamo – generation of macro-fields from *(primarily microscopic)* velocity (*Flow Dominated Dynamo - FDD*) & magnetic (*Magnetically Dominated Dynamo - MDD*) fields.

Latest understanding - coupling of FDD & MDD at different heights (going from lower scale structures to larger scale structures).

Kinematic dynamo – the velocity field is externally specified & is not a dynamical variable!

"Higher" theories – MHD, Hall MHD, two fluid etc - the velocity field evolves just as the magnetic field does – the fields are in mutual interaction.

A question – A possible inference:

If short–scale turbulence can generate large–scale magnetic fields,
then short-scale turbulence should also be able to generate large-scale velocity fields.

Process of conversion of short–scale kinetic energy to large–scale magnetic → "Dynamo" (D)

The mirror image process - conversion of short–scale magnetic energy to large–scale kinetic energy → "Reverse Dynamo" (RD)

Extending the definitions:

- **Dynamo (D) process -** Generation of large-scale magnetic field from **any mix** of short–scale energy (magnetic & kinetic).
- **Reverse Dynamo (RD) process -** Generation of large-scale flow from **any mix** of short–scale energy (magnetic & kinetic).

Theory and simulation show →

(1) D & RD processes operate simultaneously
(2) The composition of the turbulent energy determines the ratio of the large-scale flow / large-scale magnetic field

Micro (*short-scale*) and Macro (*large-scale*) Fields

The total fields in Eq.-s (1), (2) are broken into ambient & generated.

The **generated fields -** further split into **macro** & **micro** fields**:**

$$\boldsymbol{B} = \boldsymbol{b}_0 + \boldsymbol{H} + \boldsymbol{b}$$
$$\boldsymbol{V} = \boldsymbol{v}_0 + \boldsymbol{U} + \boldsymbol{v}$$

$\boldsymbol{b}_0$, $\boldsymbol{v}_0$ **- equilibrium,** $\boldsymbol{H}$, $\boldsymbol{U}$ **- macroscopic,** $\boldsymbol{b}$, $\boldsymbol{v}$ **- microscopic fields.**

Traditional dynamo theories - the short scale velocity field $\boldsymbol{v}_0$ **is dominant.**

We shall not introduce any initial hierarchy between $\boldsymbol{v}_0$ **&** $\boldsymbol{b}_0$ **.**

We shall develop the natural unified Flow–Field theory.

Equilibrium – Initial State

Departure from the standard dynamo approach - our choice of the initial plasma state.

$$\nabla(p_0 + \boldsymbol{v}_0{}^2/2) = const$$

Equilibrium fields - the DB pair obeying Bernoulli condition

which may be solved in terms of the Single Beltrami (SB) states given by (11) & (12) for equilibrium fields.

Below: λ **- micro-scale,** μ **- macro-scale;** $|b| \ll b_0, \quad |v| < v_0$

Primary interest – to create macro fields from the ambient microfields.

Constructing the closure model of the Hall MHD eq-s & assuming that the original equilibrium is predominantly short-scale (from the DB fields we keep only the λ - part)

$$\boldsymbol{v}_0 = \boldsymbol{b}_0 \left(\lambda + a^{-1}\right) \qquad \boldsymbol{v}_{e0} = \boldsymbol{v}_0 - \nabla \times \boldsymbol{b}_0 = \boldsymbol{b}_0 \, a^{-1} \qquad (15)$$

Straightforward algebra for isotropic ABC initial flow ===>

H **evolves independently of** ***U*** **but evolution of** ***U*** **does require knowledge of** ***H*****.**

Working out the nonlinear solution in linear clothing (*neglecting NLN terms*) we find:

$$\boldsymbol{U} = \frac{q}{(s+r)} \boldsymbol{H} \tag{16}$$

q, s, r **– fully defined by DB parameters & initial turbulent energy** b_0^2 **.**

A few remarkable features of linear solution:

- A choice of ***a; d*** (& hence of ***λ***) **fixes relative amounts of microscopic energy in ambient fields**
  ==> also **fixes the relative amount of energy in the generated macroscopic fields *U* & *H*.**

- **The linear solution makes NLN terms strictly zero – it is an exact (a special class) solution of the NLN system ==> remains valid even as *U* & *H* grow to larger amplitudes**
  (*appears in Alfvénic systems: MHD - nonlinear Alfvén wave:* Walen 1944,1945; in HMHD - Mahajan & Krishan, 2005)

(i) Analytical Results — *An Almost Straight Dynamo*

$a \sim d \gg 1$, **inverse micro scale micro-scales fields are** $\lambda \sim a \gg 1$ **===>**

$\boldsymbol{v}_0 \sim a\,\boldsymbol{b}_0 \gg \boldsymbol{b}_0$ **the ambient micro-scales fields are primarily kinetic.**

Generated macro-fields have opposite ordering - $\boldsymbol{U} \sim a^{-1}\boldsymbol{H} \ll \boldsymbol{H}$

super-Alfvénic ”turbulent flows” lead to steady flows (equally sub–Alfvénic)

(ii) Analytical Results — *An Almost straight Reverse Dynamo*

$a \sim d \ll 1$ $\lambda \sim a - a^{-1} \gg 1$ $\boldsymbol{v}_0 \sim a\,\boldsymbol{b}_0 \ll \boldsymbol{b}_0$

The ambient energy is mostly magnetic; from a strongly sub-Alfvénic turbulent flow

the system generates a strongly super-Alfvénic macro–scale flow

$\boldsymbol{U} \sim a^{-1}\boldsymbol{H} \gg \boldsymbol{H}$

D, RD Summary:

- **Dynamo and "Reverse Dynamo" mechanisms have the same origin – are manifestation of the magneto-fluid coupling**

- ***U*** and ***H*** are generated simultaneously and proportionately. **Greater the macro-scale magnetic field (generated locally), greater the macro-scale velocity field (generated locally)**

- **Growth rate of macro-fields is defined by DB parameters (by the ambient magnetic & generalized helicities) and scales directly with ambient turbulent energy** $\sim b_0^2 \ (v_0^2)$.

- **The composition of the ambient turbulent energy determines the ratio of the large-scale flow / large-scale magnetic field.**

- **Impacts:** on the evolution of large-scale magnetic fields and their opening up with respect to fast particle escape from stellar coronae; on the dynamical and continuous kinetic energy supply of plasma flows observed in astrophysical systems.

A simulation Example for Dynamical Acceleration

Caution: Initial and final states have finite helicities (magnetic and kinetic).

The helicity densities are dynamical parameters that evolve self-consistently during the flow acceleration.

Rotation, dissipation & heat flux as well as compressibility effects were neglected in analysis!

2.5D numerical simulation of the general two-fluid equations in Cartesian Geometry.
Code: Mahajan et al. PoP 2001, Mahajan et al, 2005

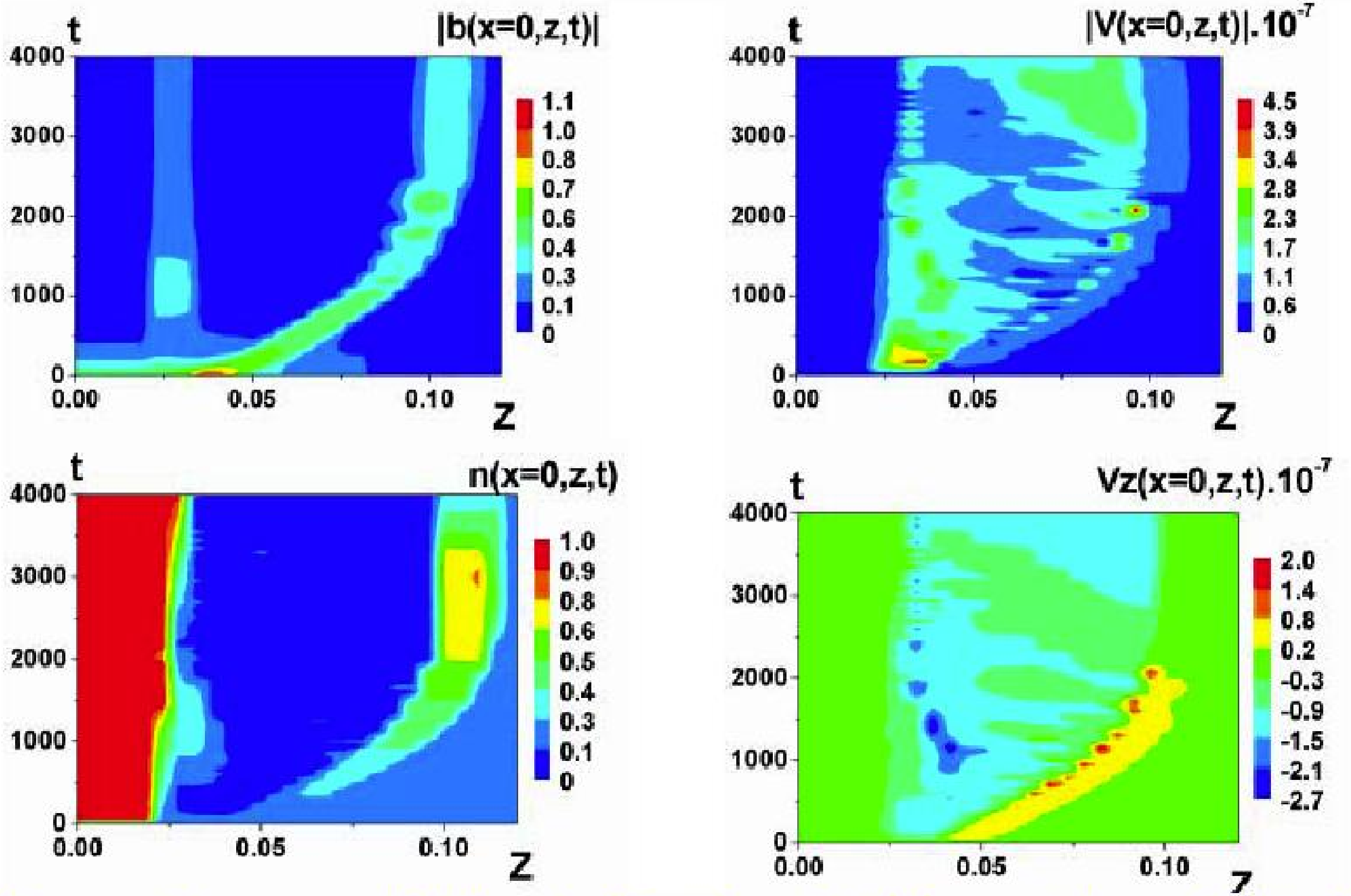
Simulation system contains:

- **an ambient macroscopic field**
- **effects not included in the analysis**:
  1. dissipation and heat flux
  2. plasma is compressible embedded in a gravitational field →
     extra possibility for micro-scale structure creation.

Transport coefficients are taken from Braginskii and are local.
Diffusion time of magnetic field > duration of interaction process (would require T ≤· a few eV -s).

Study of trapping and amplification of a weak flow impinging on a single closed-line magnetic structure ===>

Dynamo and Reverce Dynamo Phenomena
In the center of the original closed magnetic field structure



Dynamical Emergence of the new magnetic field in region different from original; flux moves to the upper heights with time!

Accelerated flow follows the maximum field localization area – RD! D & RD phenomena have oscillating/pulsating character.

Generated field maximum ~$0.5b_0$; accelerated flow max. radial speed ~200km/s ; at ~2000sec time flow converts to down-flow!

Simulation Summary:

- **Dissipation present: Hall term** ***(through the mediation of micro-scale physics)*** **plays a crucial role in acceleration / heating processes.**

- **Initial fast acceleration in the region of maximum original magnetic field + the creation of new areas of macro-scale magnetic field localization with simultaneous transfer of the micro-scale magnetic energy to flow kinetic energy = manifestations of the combined effects of the D and RD phenomena**

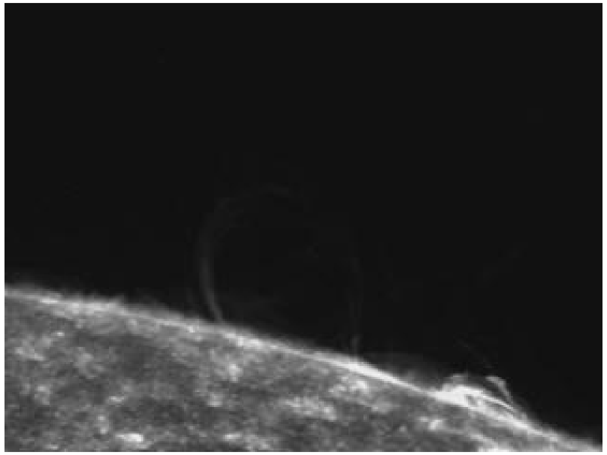
- **Continuous energy supply from fluctuations (dissipative, Hall, vorticity) ===> maintenance of quasi-steady flows for significant period**

- **Simulation: actual** h_1 **,** h_2 **are dynamical.**

  Even if they are not in the required range initially, their evolution could bring them in the range where they could satisfy conditions needed to efficiently generate flows ===>

  several phases of acceleration

- **In the presence of dissipation, these up-flows play a fundamental role in the heating of the finely structured stellar atmospheres; their relevance to the solar wind is also obvious.**



Formation / primary heating of Solar coronal loopby up-flows; flow remains along loop, just slowed down – Mahajan at al (2001)

Beltrami-Bernoulli Equilibria in White Dwarfs – indications & plans

- **Star collapses & cools down** - the density of lighter elements increases affecting the total pressure/ enthalpy of unit fluid element – **beyond the hot, pre-white dwarf stage, photon cooling dominates and gravitational contraction is dramatically reduced → a strongly degenerate electron gas.**
- Mechanical & thermal properties separate; **the degenerate electrons provide the dominant pressure**; the thermal motions of ions make a negligible contribution to the mechanical support.
- **The roles of electrons & ions are reversed in their contribution to energy** - the only significant source of energy is the reservoir of thermal energy in the nearly classical ideal gas of the ions.
- Compact astro-objects: white & brown dwarfs, neutron stars, magnetars $n_e \sim 10^{26} - 10^{34}\ cm^{-3}$

$$\frac{\partial}{\partial t}\left(\sqrt{1+R^2}\,\mathbf{p}_e\right) + m_e c^2 \nabla\left(\sqrt{1+R^2}\,\gamma_e\right) = -e\mathbf{E} + \mathbf{V}_e \times \mathbf{\Omega}_e$$

$$\nabla\left(\beta_0 \ln N + \mu_0\sqrt{(1+R^2)}\,\gamma + \frac{V^2}{2}\right) = 0$$

$$R = p_F / m_e c$$

- For **large R, degeneracy pressure >> the thermal pressure → a nontrivial Beltrami–Bernoulli equilibrium** (*Double-Beltrami*) **state is possible even for a zero temperature plasma** (classical neutral zero-beta plasmas - only the trivial, single Beltrami states are accessible).

Degeneracy of electrons automatically introduces the richness in the structures available in compact objects with degenerate electrons <=> **when degeneracy becomes significant while star contraction its outer layers keep the multi-structure character with density defined by electron degeneracy pressure.**

Summary and Conclusions

- The **structures which comprise the solar corona can be created by particle (plasma) flows observed near the Sun's surface**
- **The primary heating of these structures is caused by the viscous dissipation of the flow kinetic energy.**
- It is during trapping and accumulation in closed field regions, that the relatively cold and fast flows thermalize (*due to the dissipation of the short scale flow energy*) leading to a bright and hot coronal structure.
- **The formation and primary heating of a closed coronal structure (*loop at the end*) are simultaneous.**
- The **heating** caused by the dissipation of flow energy **may, in addition, be augmented by one or several modes of secondary heating**. In our model, the **"secondary heating" may occur to simply sustain** (against, say, radiation losses) **the hot bright loop**.
- **The emerging scenario, then, is not the filling of some hypothetical virtual loop with hot gas.** The loop, in fact, is created by the interaction of the flow and the ambient field; its **formation and heating are simultaneous & "loop" has no ontological priority to the flow**.

Dynamic processes in Solar Atmosphere involving flows

At any quasi-equilibrium stage of the accelerating plasma flow [*acceleration scenario could be one of many*], the **nascent intermittent flows will blend & interact with pre–existing closed field structures on varying scales**.

”New” **flows** could be trapped by other structures with strong / weak magnetic fields and **participate in creating different dynamical scenarios** (when dissipation is present) **leading to**:

1) **Formation & heating of a new structure of finely structured atmosphere** [see (Mahajan et al. 1999; Mahajan et al. 2001)].
2) **Explosive events/prominences/CME eruption** [see (Ohsaki et al. 2001; Ohsaki et al. 2002; Mahajan et al. 2002a)].
3) **Creation of a dynamic escape channel** *(providing important clues toward the creation of the solar wind* [see (Mahajan et al. 2002b; Mahajan et al. 2003)]).
4) **Instabilities, and wave-generation could also be triggered.**