Current patterns and optical conductivity in disordered superconductors:

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Collaborations
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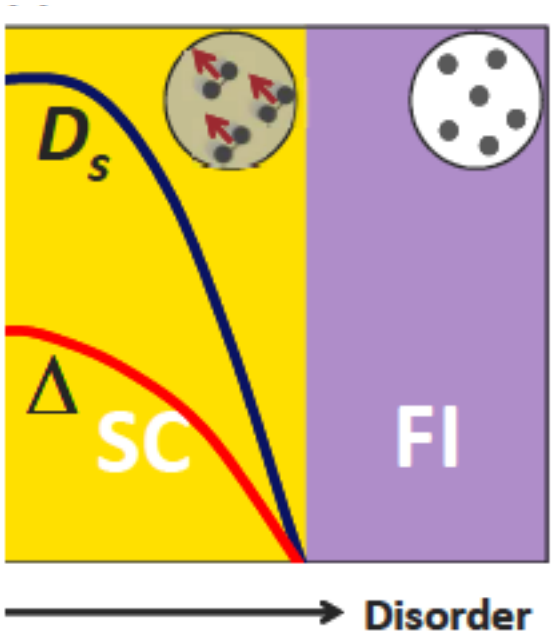
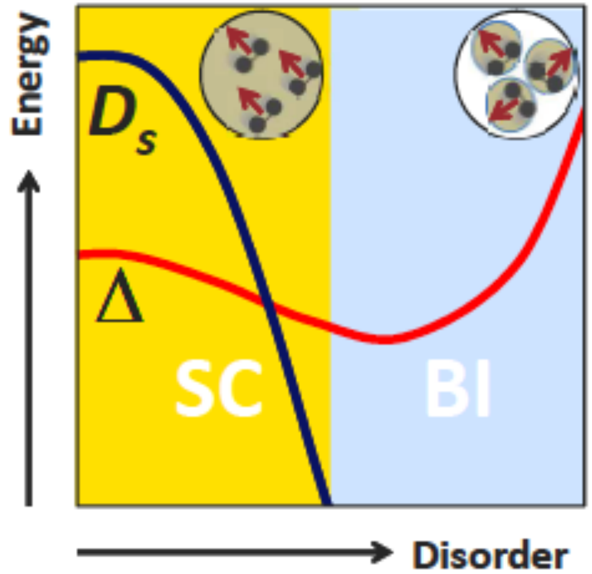
References:
Seibold et al. PRL 108, 207004 (2012)
G. Lemarié et al. PhysRevB 87, 184509 (2013)
Cea, Bucheli, Seibold, Benfatto, Lorenzana,
and Castellani, PhysRevB 89,174506 (2014).

Disappearing of SC by increasing disorder

- “Fermionic” vs “bosonic” mechanism



Superconductor-Insulator Transition (SIT)



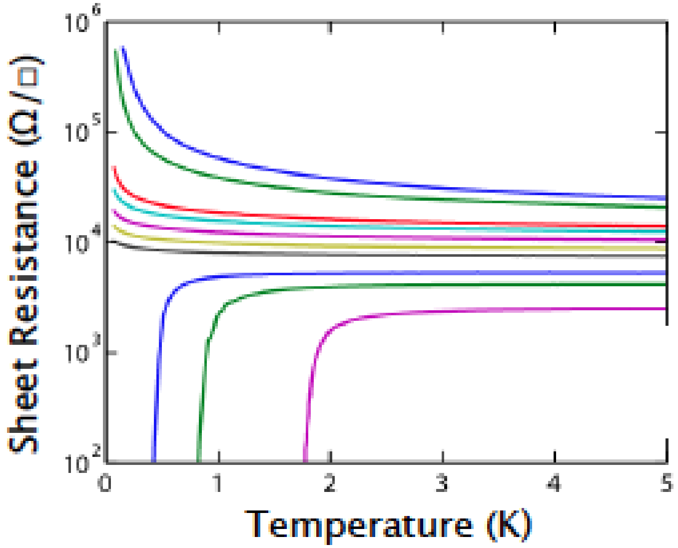
“Fermionic” mechanism (Finkelstein):
disorder enhances Coulomb repulsion,
pairing strength decreases,
both Tc and Δ go to zero ⇒ FM or FI

“Bosonic” mechanism (Fisher, Ma & Lee, etc.)
direct localization of Cooper pairs,
finite pairing in the insulating phase

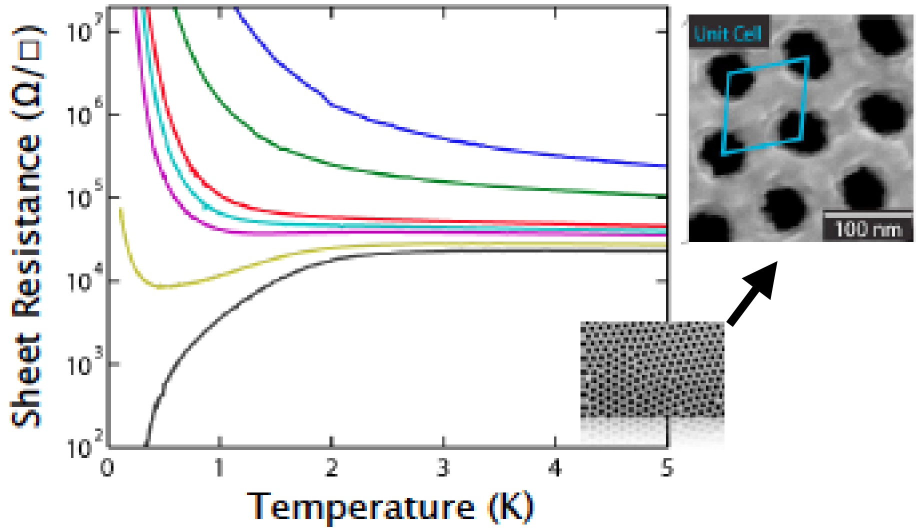
Jim Valles , Leiden 2011

Thickness tuned SIT: amorphous Bi films

Uniform



Nano-honeycomb (50nm)



- $\Delta_{gap} \Rightarrow 0$ as $T \Rightarrow Tc$
- Insulator : Weakly localized electrons
- Small positive MR

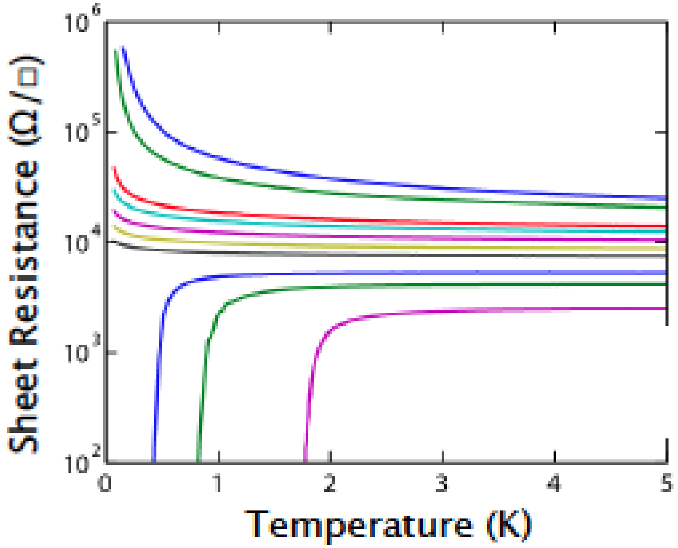
- $\Delta_{gap} \neq 0$ at $T > T_c$
- Localized Cooper Pairs with activated transport $R = R_0 e^{To/T}$ $T_0 \Rightarrow 0$ at SIT
- Giant positive MR

Bosonic mechanism at work ≠from granular

Jim Valles , Leiden 2011

Thickness tuned SIT: amorphous Bi films

Uniform



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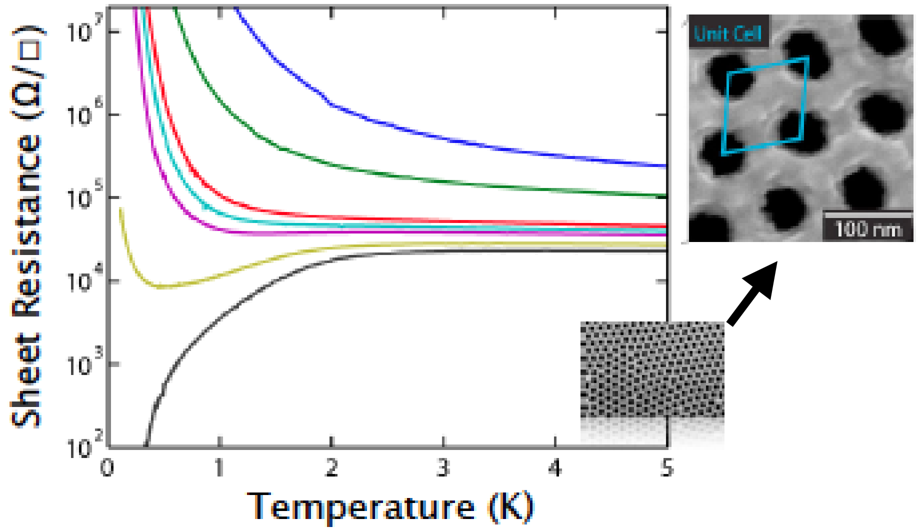

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- Localized Cooper Pairs with activated transport $R = R_0 e^{To/T}$ $T_0 \Rightarrow 0$ at SIT
- Giant positive MR

Trivial consequence of artificial nanostructure

Superconductor to insulator transition: bosonic mechanism at work

- Homogeneous InO, TiN, NbN films share the same features : Δ_{gap} stays $\neq 0$ above T_c activated transport $R=R_0 e^{To/T}$, giant positive MR
- Are they inhomogeneous? Effectively inhomogeneous?
- General mechanism for inhomogeneity?

Ioffe and Mezard proposal for disordered superconductors

- low temperature glassy phase (with one-step replica symmetry breaking):

  self-generated inhomogeneity on a mesoscopic scale (much) larger than the scale of disorder and of pairing (ξ_{sc})

Ioffe and Mezard, PRL 105, 037001 (2010);
Feigelman, Ioffe and Mezard, PRB 82, 18534 (2010)

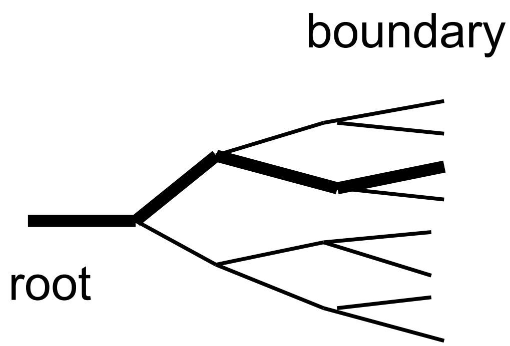
Ioffe and Mezard proposal for disordered superconductors

- Ising model in a transverse random field (strong coupling pairing with $<\sigma^x_i>$ = superconducting order parameter) on a Cayley tree with K-branching

$$H = -\sum_i \xi_i \sigma_i^z - g/K \sum_{<ij>} \sigma_i^x \sigma_j^x$$

- Cavity methods: recursion formula for the Weiss field acting on σ^x_i (and mapping to directed polymer problem)

At low T only few paths contribute to pairing susceptibility.
Self-generated quasi-1d inhomogeneity, non self-averaging properties, anomalous distribution of the local order parameter $s=<\sigma^x_i>$



Ioffe and Mezard proposal for disordered superconductors

Various questions:

- Quasi-1d, non self-averaging and anomalous distribution will survive from Cayley tree to real lattices (2d)?
  (The replica symmetry breaking issue in finite d)
- Intrinsically strong coupling (bosonic) model: what for a fermionic model of superconductivity from weak to strong coupling? ⇒ **attractive Hubbard model**
- Price to pay: from cavity methods to mean field, however MF can be a reasonable description of the ordered phase
- Study of **distribution** (static) and **optical conductivity** (dynamics)

The attractive Hubbard model

- we consider the attractive Hubbard model with on site disorder

$$H = -t\sum_{<ij>,\sigma} a^{+}_{i\sigma} a_{j\sigma} - |U| \sum_{i} n_{i\uparrow} n_{i\downarrow} - \sum_{i\sigma} \xi_i n_{i\sigma}$$

- and solve the mean field Bogoliubov-de Gennes eqs on a 2d finite cluster at T=0 with site dependent SC order parameter $\Delta_i = |U| < a^{+}_{\uparrow i} a^{+}_{i\downarrow} >$, and $-V_0 < \xi_i < V_0$

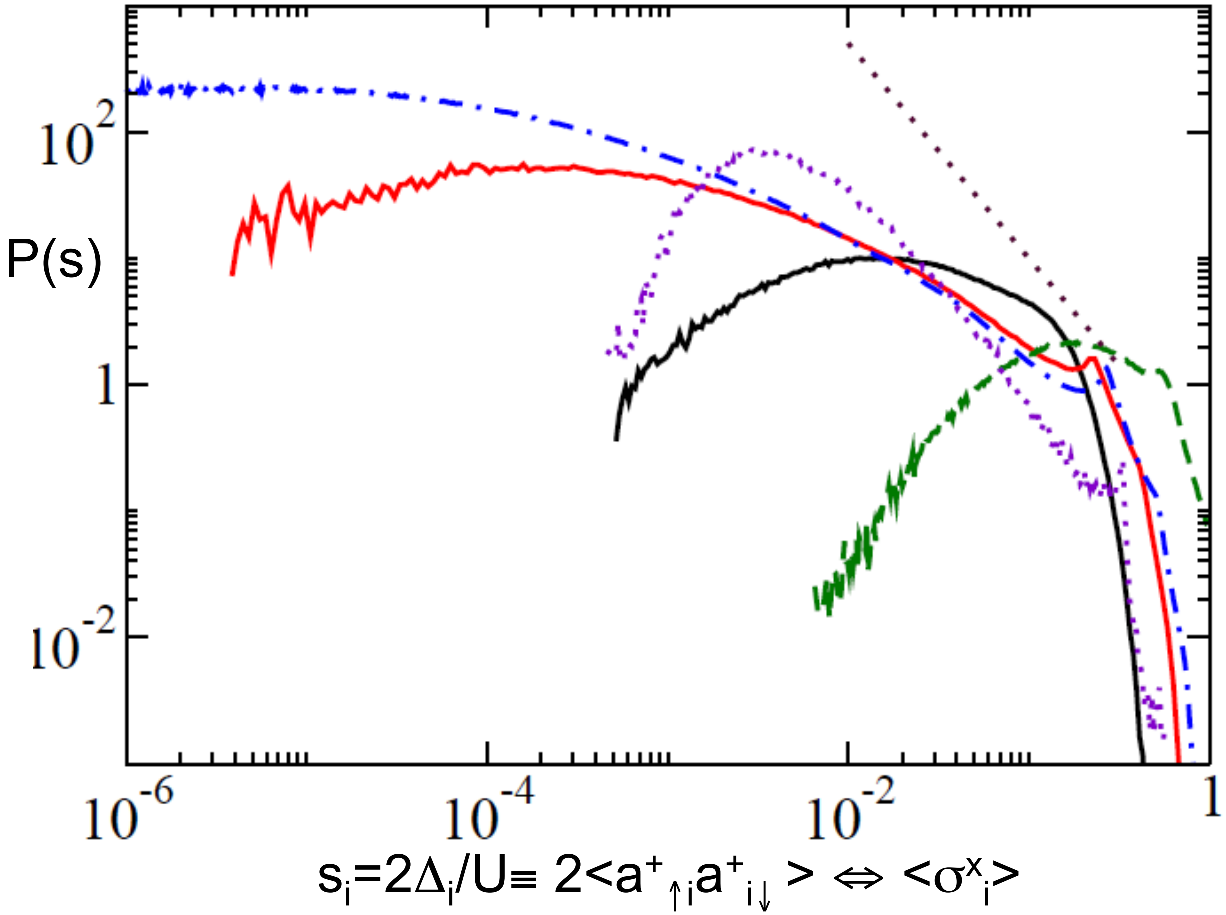
The attractive Hubbard model

- Parameters: $t=1$, $U=1.5\div10$, $V_0=0.1\div3$,$<n>=0.1 \div1$
- Various known results from previous BdG and Monte Carlo. Huge literature. See: Ghosal, Randeria and Trivedi, PRL(1998) and PRB (2001)
- Even for not too large U (with $V_0\sim1$) superconductivity is established by coherence of local pairs. Cfr: Feigelman et al Ann.Phys (2010)
- Strong variations of local SC order parameter Δ_i ($\leq\Delta_{gap}$ spectral gap from local Density of States) "superconducting islands" (inhomogeneity)

The attractive Hubbard model

- Distribution of the local order parameter $s_i=2\Delta_i/U\equiv 2\langle a^+_{\uparrow i}a^+_{i\downarrow}\rangle \Leftrightarrow \langle\sigma^x_i\rangle$. Note: Δ_i is not the DOS gap, it is a measure of coherence
- predictions of FIM in the ordered phase: $P(s) \sim s^m_{typ}/s^{1+m}$ for large s, with m <1. "Unbounded" distribution but for the physical constraint $s \leq 1$. Averaged $\langle s\rangle \gg s_{typ}$ with $s_{typ} = \exp\langle \ln s\rangle$
- We find a qualitative agreement (broad P(s)), but a different distribution

Distribution of the order parameter



Very crowded:

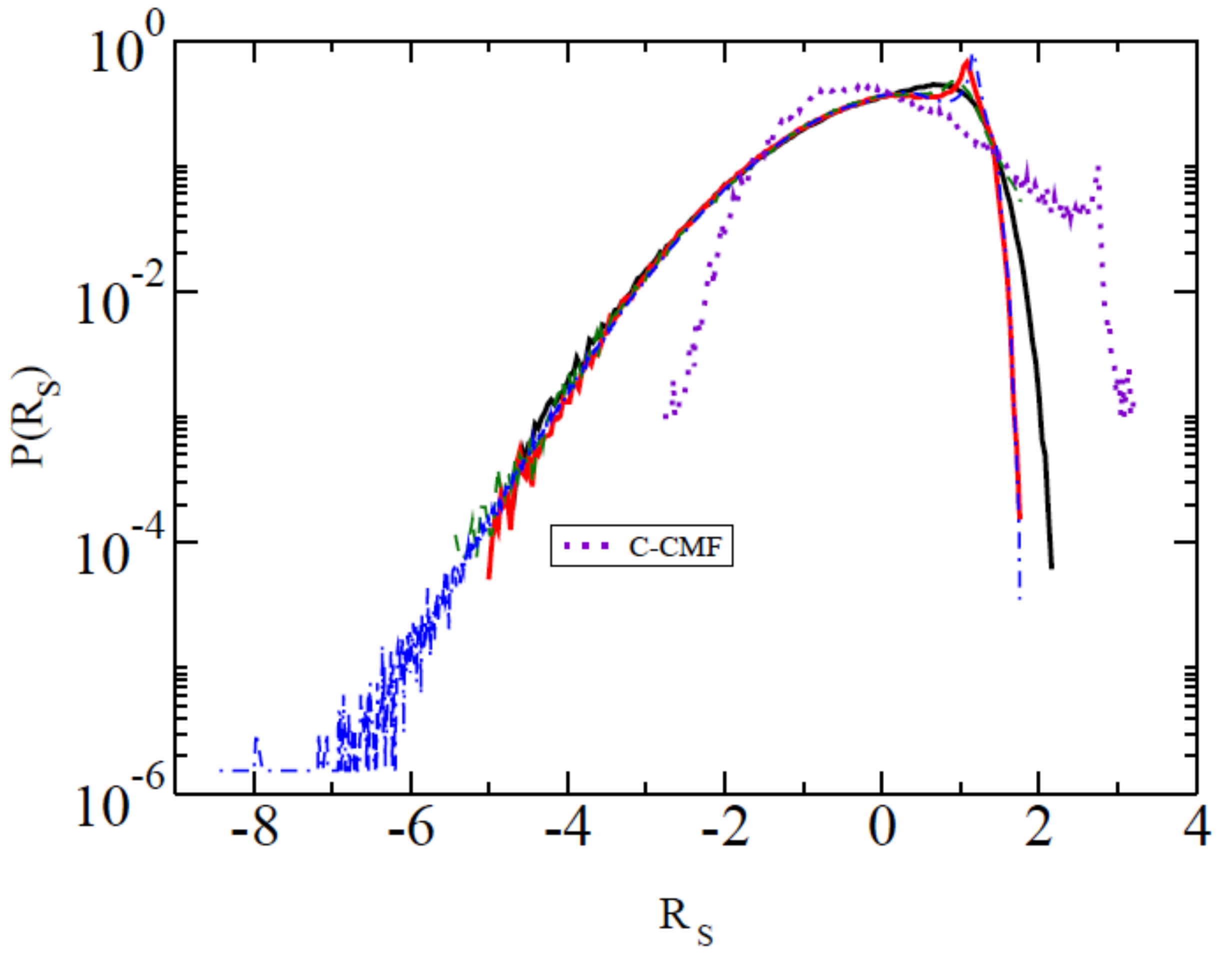
Hubbard
$U=5$, $g=t^2/UV_0=0.08$, $n=0.85$
$U=5$, $g=0.2$, $n=1$
XX-Z
2DCMF (Monthus and Garel 2012), $g=J/V_0=0.4$
MF, $g=0.2$ (dashed-dotted)
FIM: Cayley, $K=3$, $g=0.2$ (dotted)

Distribution of the order parameter

Universal by rescaling $\mathbf{R=(Ins\text{-}Ins_{typ})/\sigma_s}$

$Ins_{typ}=\langle Ins\rangle$

$\sigma_s{}^2=\langle(Ins- Ins_{typ})^2\rangle$



Hubbard

U=5, $g=t^2/UV_0=0.08$, n=0.85

U=5, g=0.2,n=1

XX-Z

2DCMF (Monthus and Garel 2012), $g=J/V_0=0.4$

MF, g=0.2 (dashed-dotted)

FIM: Cayley, K=3, g=0.2 (dotted)

Distribution of the order parameter

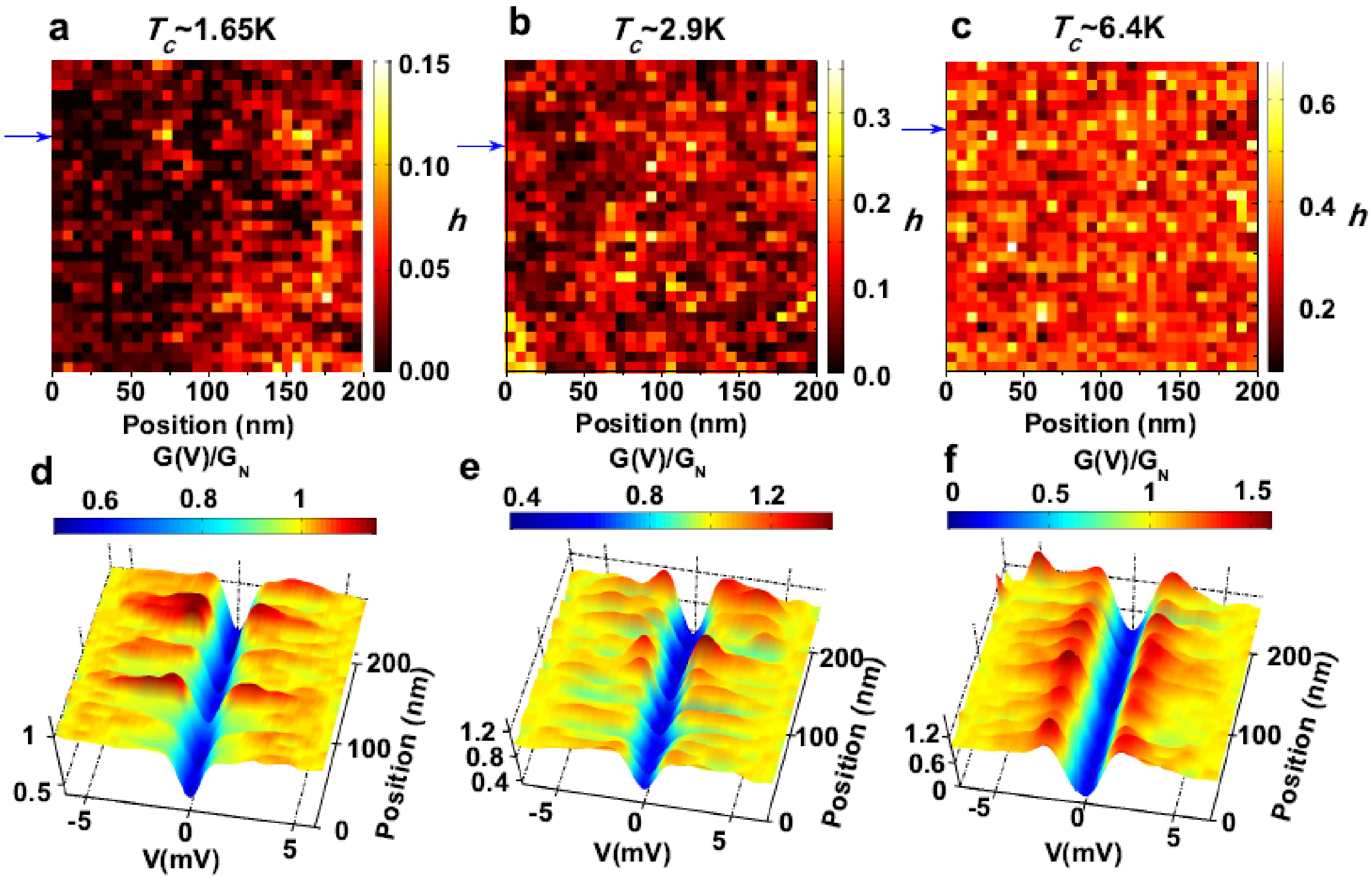
- Universal distribution
- the rescaling seems to work in a wide range of parameters, with the variance $\sigma_s \sim -\ln s_{typ}$ increasing for increasing disorder
- In the disordered phase analogy with directed polymer problem physics in 2D: $\ln s_L = -c\,L + L^{\omega}\,u$ with $\omega \approx 1/3 > 0$ ($\omega = 0$ on the Cayley tree) L=distance from ordered boundary, u=random variable with Tracy-Widom distribution
- Tracy-Widom is reasonably good also for ordered phase

Distribution of the order parameter

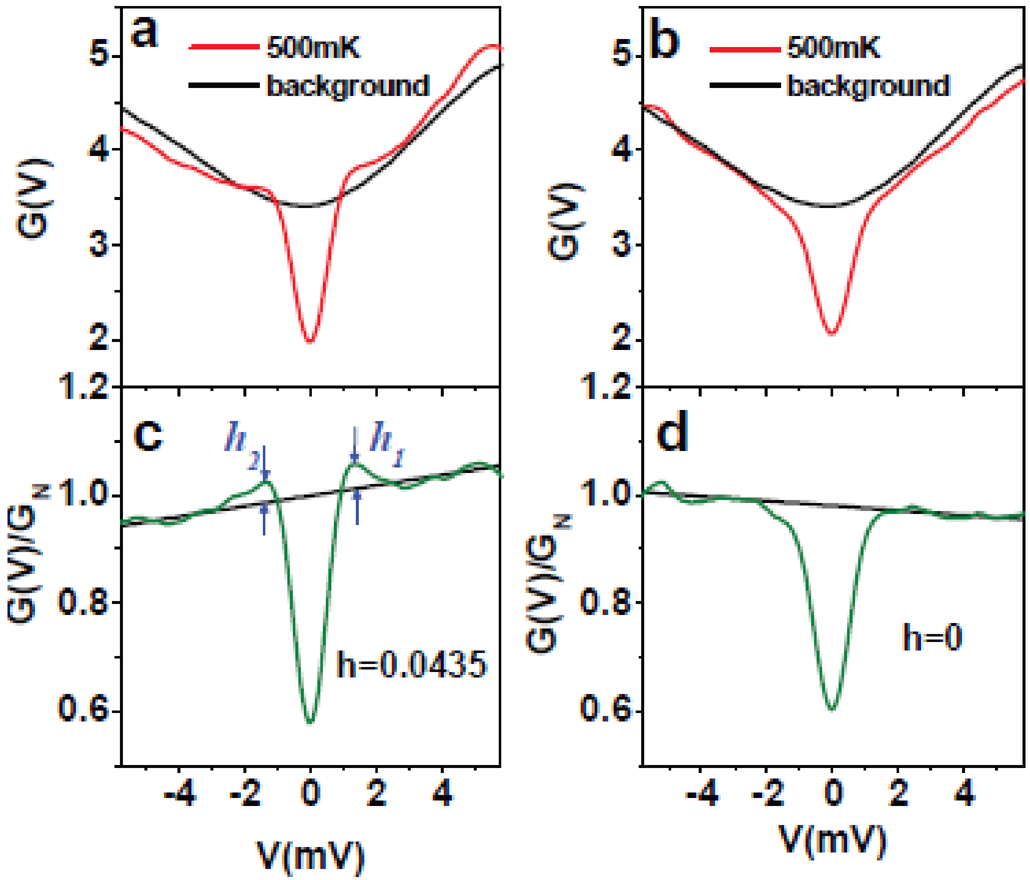
- Comparison with experimental data on NbN films from Tata Institute group (Pratap Raychaudhuri and collaborators)

  Hypothesis: $\Delta(r)$ correlates with relative height of coherent peaks in local density of states in STM (Sacépé et al, Nature Phys. 7, 239, (2011): STM in InO films)

Lemarié et al. PRB 87, 184509 (2013)
STM in NbN films

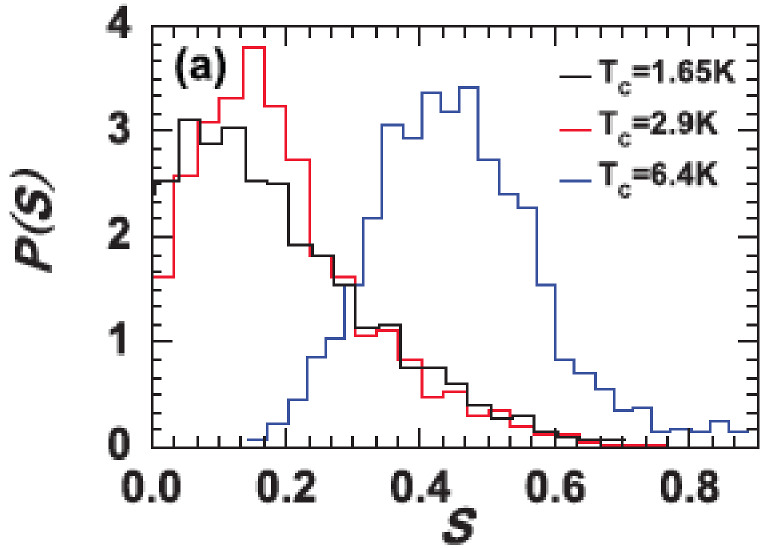


Lemarie et al. PRB 87, 184509 (2013)
STM in NbN films

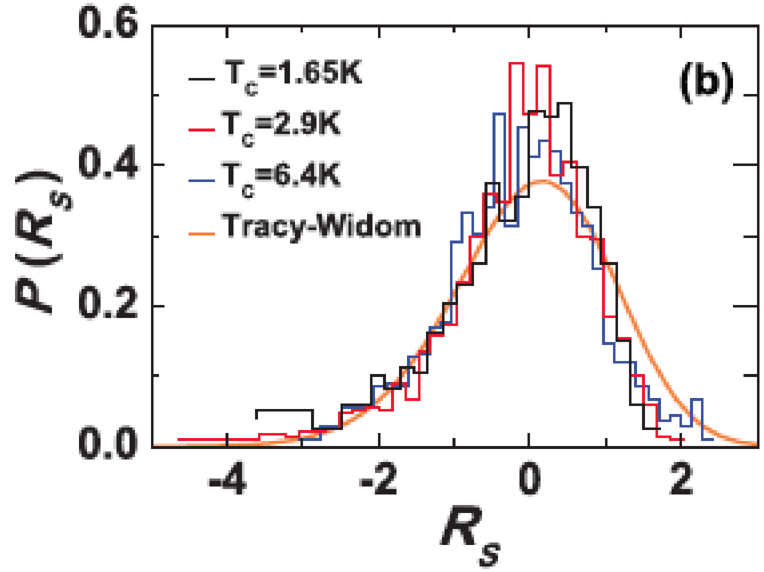


Define peak height $h=(G_{peak}-G_{min})/G_{min} \propto$ SC order parameter **s**

Lemarie et al. PRB 87, 184509 (2013)
STM in NbN films



Distribution of the local peak height h ∝SC order parameter **s**



Rescaled distribution $R_s=(\ln \mathbf{s}-\ln \mathbf{s}_{typ})/\sigma_s$

Inhomogeneity and glassy physics

- The emergent mesoscopic inhomogeneity is a signature of **“glassy”** superconductivity?

Inhomogeneity and glassy physics

- What about **currents**, **superfluid density** and **optical conductivity** in the strong disordered regime?

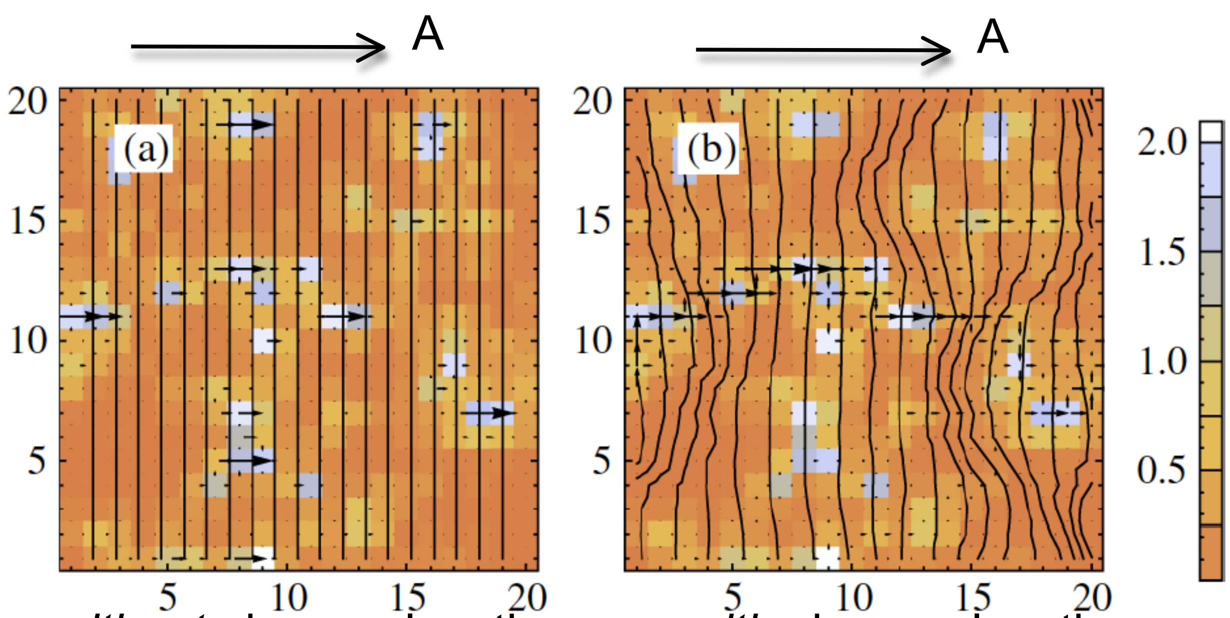
Current response in a disordered SC

- It is a tricky job, even within MF: in a clean system $\boldsymbol{J}_{\mathbf{q}}=\chi^{BCS}(\mathbf{q})\mathbf{A}_{\mathbf{q}}$ and $\chi^{BCS}(\mathbf{q}\to\mathbf{0})$ gives the superfluid density D_S with χ^{BCS} given by the bubble expression with no vertex corrections.
- χ^{BCS} is not gauge invariant, but it is enough since a transverse **A** does not change the phase of the SC order parameter Δ
- In the presence of disorder this is wrong! We solve the BdG eqs in the presence of **A** and evaluate the related local current density $\boldsymbol{J}(\mathbf{r})$

Current patterns

$$H = -t \sum_{<ij>,\sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c.) + \sum_{i,\sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_i n_{i\uparrow} n_{i\downarrow}. \qquad \Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle = |\Delta_i| e^{i\theta_i}$$

- Current in the presence of a finite transverse A, by allowing for the local phases θ_i of the BdG solutions to relax to the applied field A



without phase relaxation *with* phase relaxation

$|U|$=5t, V_0=2t, n=0.85
size=20x20

map: local Δ

lines: constant phase

$\Rightarrow \nabla\theta_i - 2A$

arrows: local current

G. Seibold, L.Benfatto, J. Lorenzana and C. Castellani
PRL 108, 207004 (2012)

Unidimensional patterns for the current:
glassy-like behavior

Current patterns

$$H = -t \sum_{<ij>,\sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c.) + \sum_{i,\sigma} (V_i - \mu) n_{i\sigma} - |U| \sum_{i} n_{i\uparrow} n_{i\downarrow}. \qquad \Delta_i \equiv |U| \langle c_{i\downarrow} c_{i\uparrow} \rangle = |\Delta_i| e^{i\theta_i}$$

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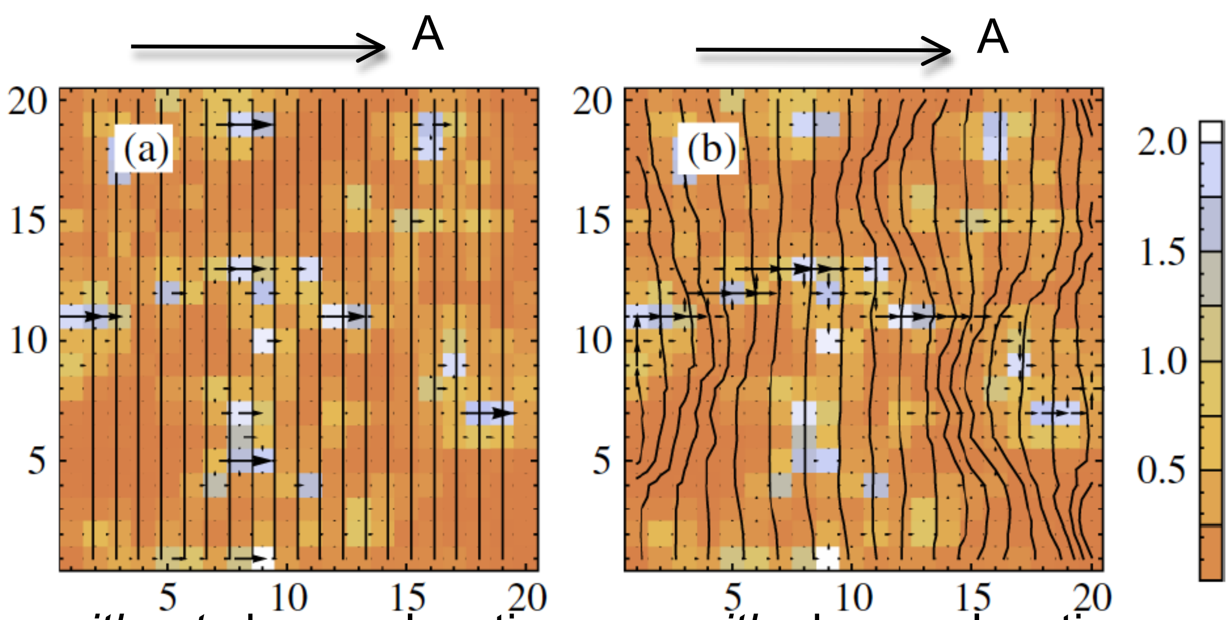


without phase relaxation *with* phase relaxation

Superfluid density

$D_s = 1/L^2 \partial^2 E(A)/\partial A^2$

$D_s < D_s^{BCS}$

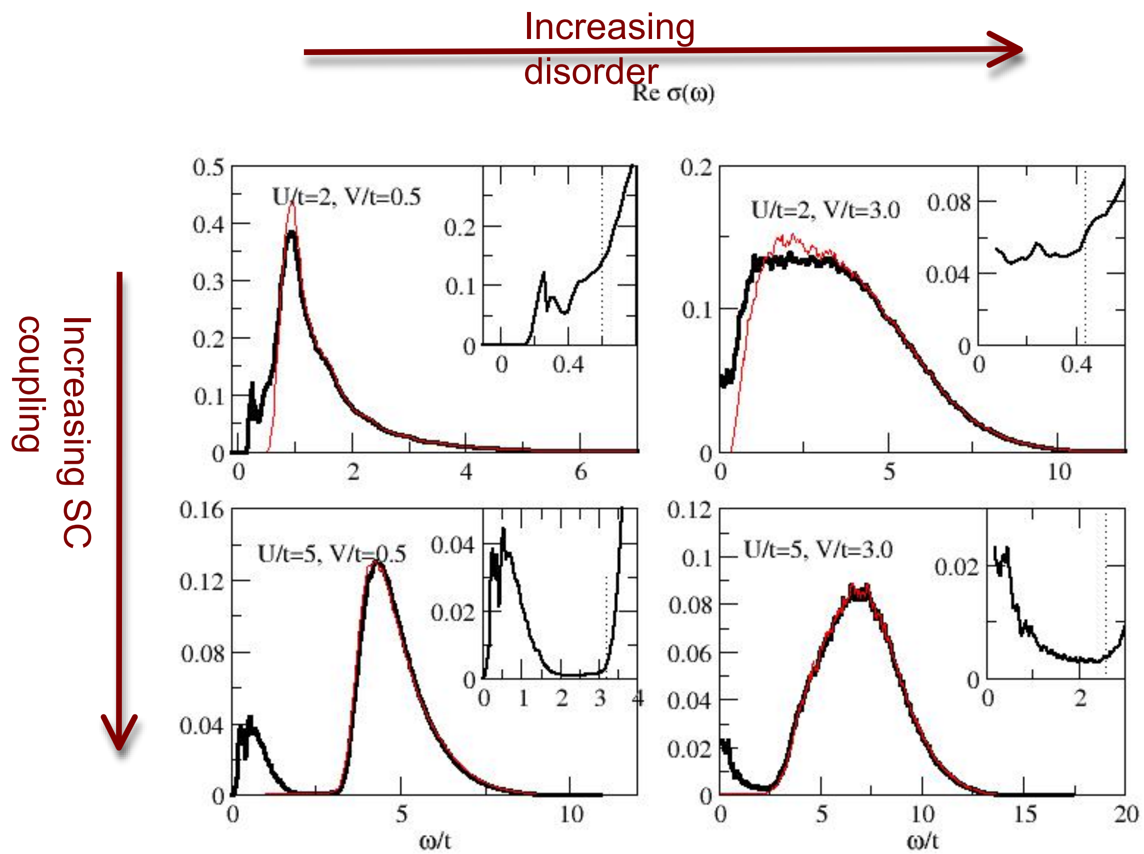
Missing superfluid density $D_s^{BCS} - D_s$ is tranferred to spectral weigth of intragap $\sigma(\omega)$ from collective modes

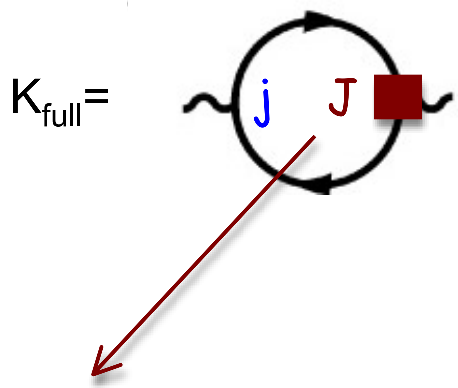
G. Seibold, L.Benfatto, J. Lorenzana and C. Castellani PRL 108, 207004 (2012)

Unidimensional patterns for the current: glassy-like behavior

Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: ***in-gap*** spectral weight due to phase fluctuations





J: collective modes
(**phase**, amplitude, charge)

Sub-gap contribution of phase fluctuations

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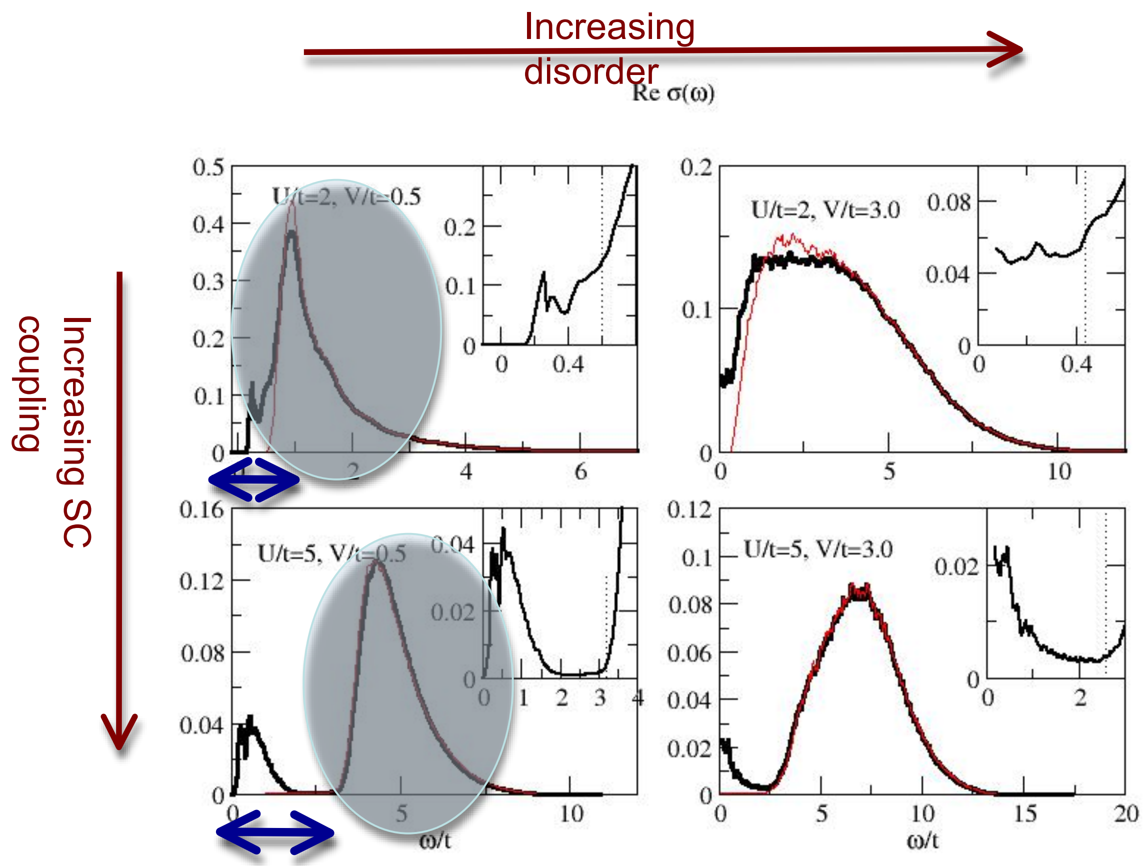
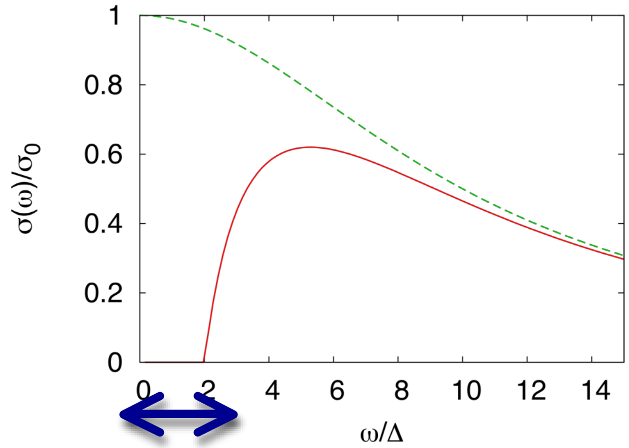


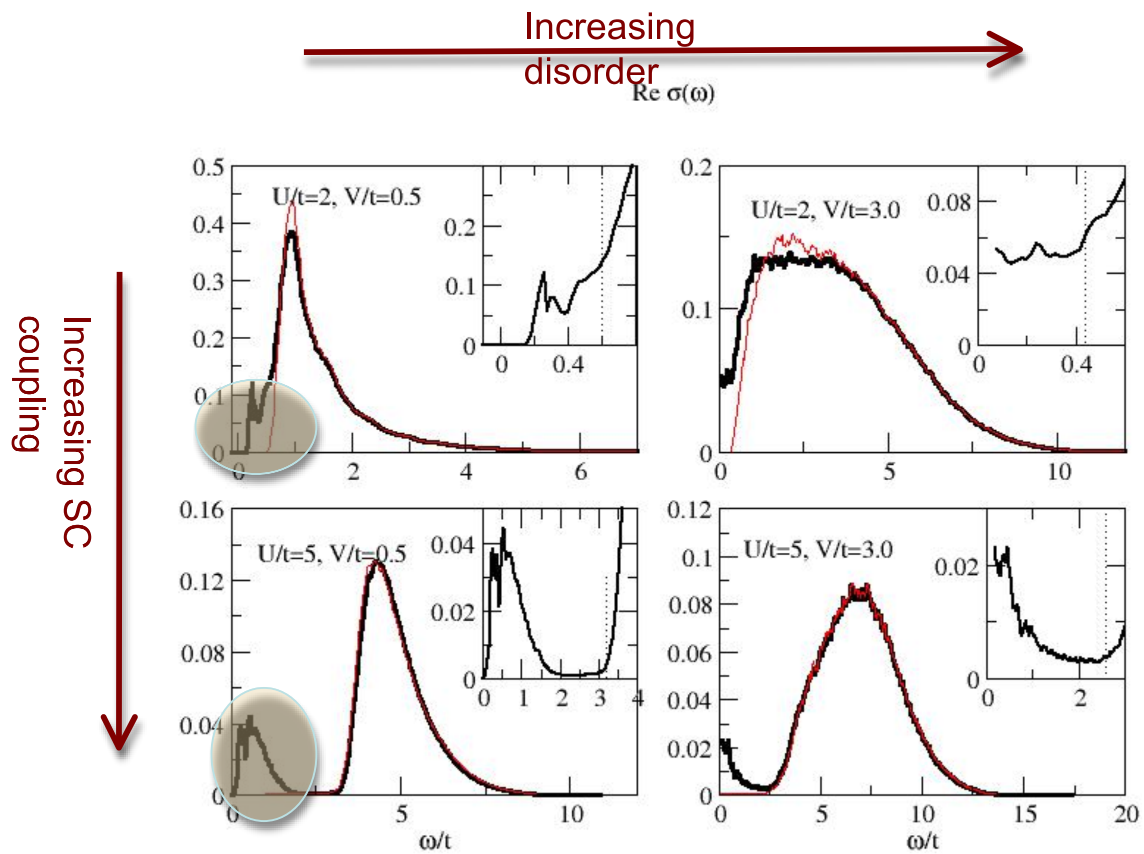


Standard Mattis-Bardeen

Spectral gap Δ, i.e. the same found in the DOS and measured by STM

Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: ***in-gap*** spectral weight due to phase fluctuations



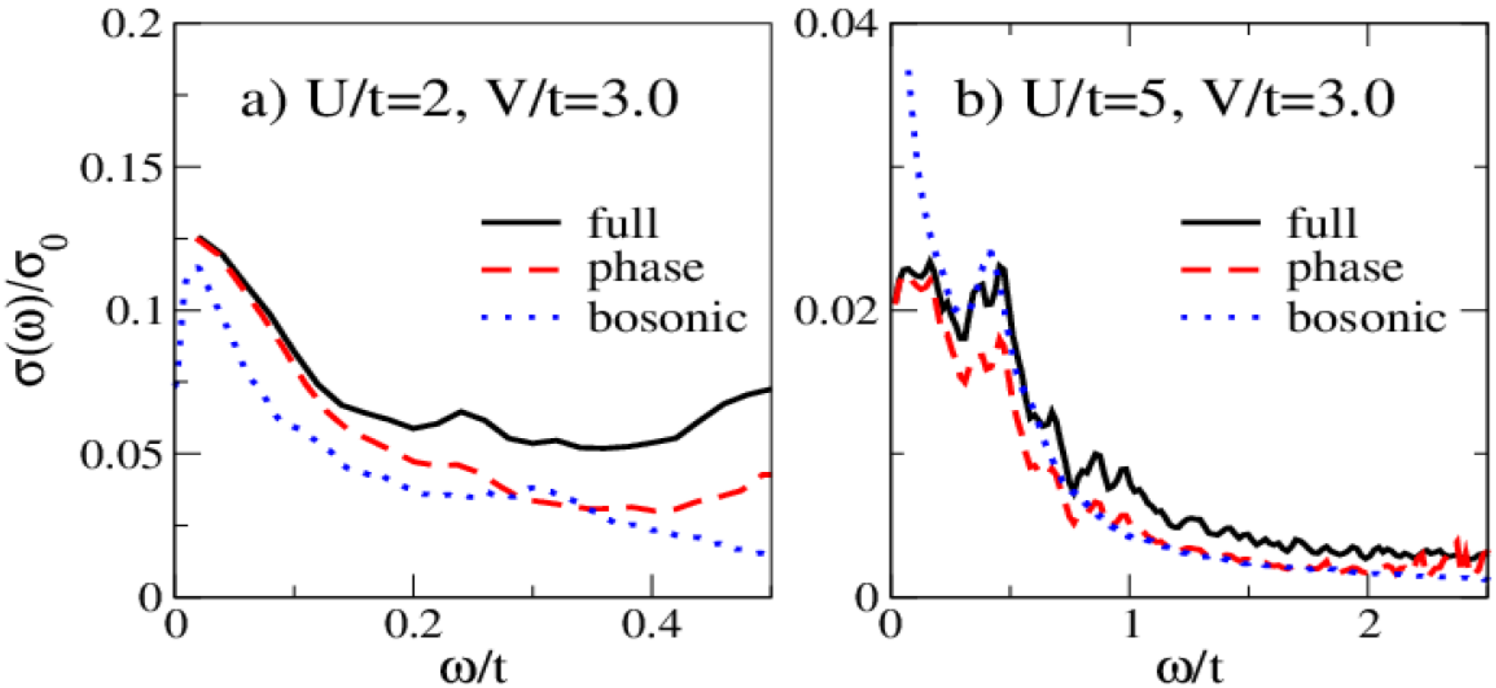
Collective-modes contribution at low energies

"Smaller" optical gap than STM gap? Anomalies in G-THz Spectroscopy?

Sub-gap contribution of phase fluctuations

- Optical conductivity with vertex corrections: ***in-gap*** spectral weight due to phase fluctuations
- Can be derived also from the spin/hard-core boson model

i.e. $\mathcal{H}_{PS} \equiv -2\sum_i \xi_i S_i^z - 2J\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + h.c.\right)$ (cfr Swanson 2013)



boson parameters: $D_{mf}^{B}=D^{BCS}$, $D_s^{B}/D_{mf}=D_s^{F}/D^{BCS}$

Sub-gap contribution of phase fluctuations

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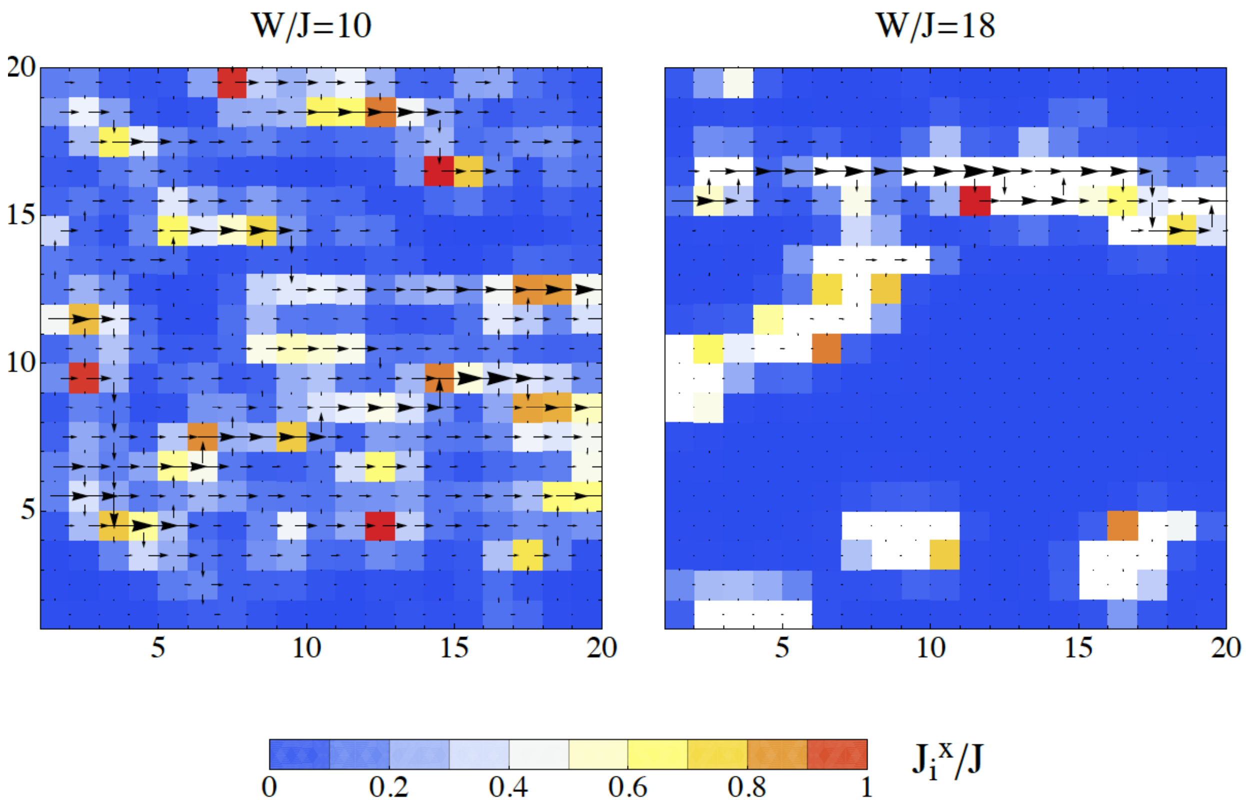

$J_i^x = J \sin\theta_i \sin\theta_{i+x} =$ local diamagnetic term

Optical absorption from "isolated"superconducting islands : "missing" superfluid current transferred in $\sigma(\omega)$, "superfluid" peak moves to finite frequency and dissipates

Sub-gap contribution of phase fluctuations

- A main objection:

  Since intragap contribution to $\sigma(\omega)$ comes from phase modes (sound modes in clean systems) long range force spoils this effect by changing sound mode in high energy plasmons

Optical conductivity for charged systems

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  Since intragap contribution to $\sigma(\omega)$ comes from phase modes (sound modes in clean systems) long range force spoils this effect by changing sound mode in high energy plasmons

Optical conductivity for charged systems

- $\mathbf{J}=\sigma\mathbf{E}$, **E** is the internal field which includes induced charges: σ is evaluated by "irreducible" diagrams (like in a fictitious neutral system)
- Dielectric function $\varepsilon(\omega)=1+4\pi i\sigma(\omega)/\omega$

  ε $(=1+4\pi e^2/q^2 K^{screen})$ is the response to **E**

  $1/\varepsilon$ $(=1-4\pi e^2/q^2 K^{LR})$ is the response to $\mathbf{E_0}$.
  Average $<\varepsilon> \neq <\varepsilon^{-1}>^{-1}$, which is relevant?
- In 2d we could take $V_c \propto 1/q$, and plasmon is $\sqrt{q}$. With disorder: low energy excitations are from local (high q) modes not strongly affected by long range force. We expect both averages to be qualitatively similar

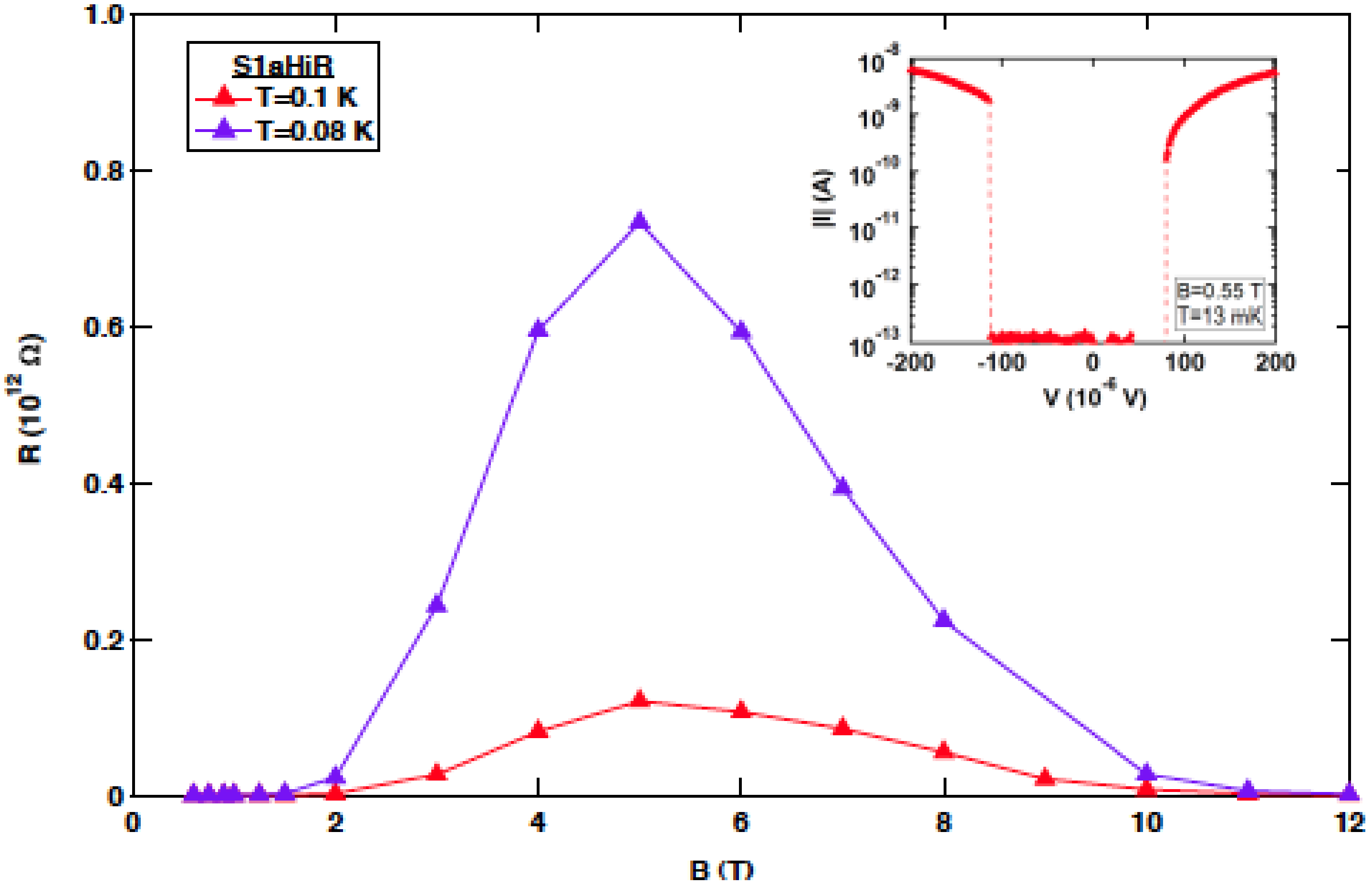
Conclusions

- Disordered superconductors (near SIT): glassy physics?
- Anomalous P(Δ) distribution
- Quasi-1D current paths

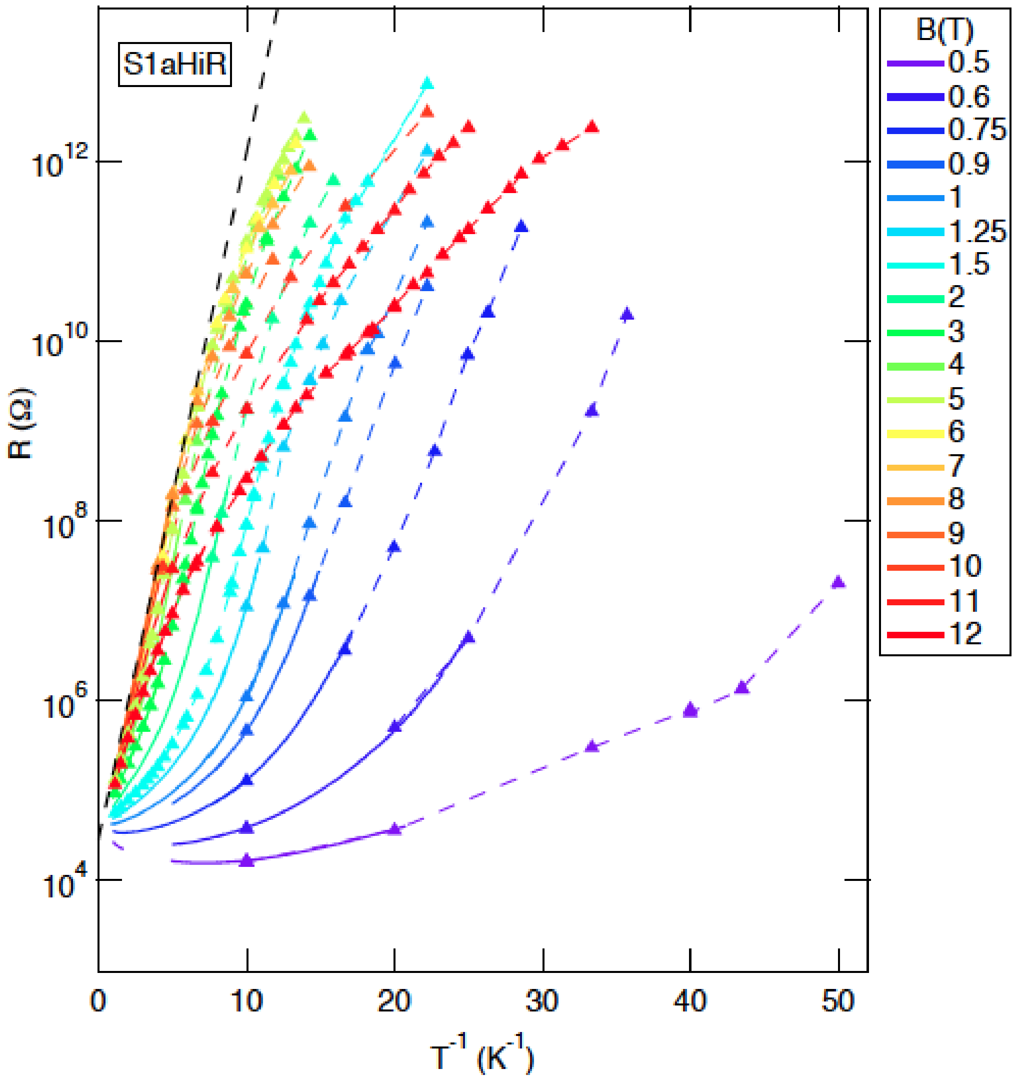
  Physical view: coupled SC islands with large variation of Josephson couplings
- Intra-gap optical absorption (anomalies at G-THz?)
- **Open problems**: dynamics (long range forces), **critical behavior** at SIT and **insulating phase** (many body localization? True glass phase?)

Insulating peak

B-induced SIT in thin film of InO: Ovadia et al. arXiv 1406.7510



T dependence of R

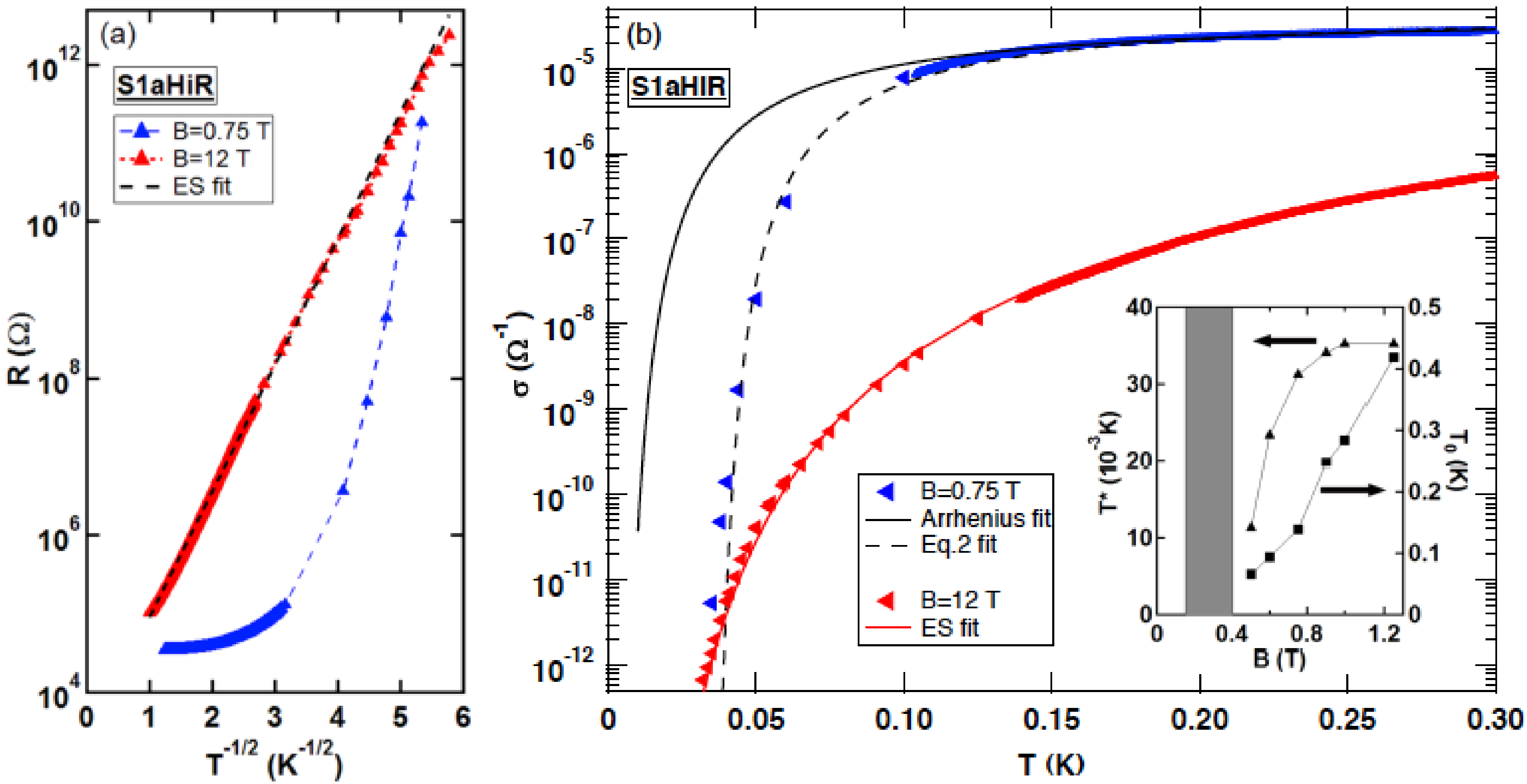


Ovadia et al. arXiv 1406.7510

B-induced SIT in thin film of InO: Insulating phase

$\sigma = \sigma_o \exp[-T_o/(T-T^*)]$

B-induced SIT in thin film of InO: Ovadia et al. arXiv 1406.7510



Optical conductivity for charged systems

- Work in progress: define transport coefficients from average e.m. propagator

$$D = (D_0^{-1} + K)^{-1} \quad \Rightarrow \quad \overline{D} = (D_0^{-1} + \tilde{K})^{-1}$$

$$D_0^{-1} = \left[(q^2 - \omega^2)\delta_{\alpha\beta} - q_\alpha q_\beta \right] \qquad \boxed{\overline{A} = \overline{D} J_{ext}}$$

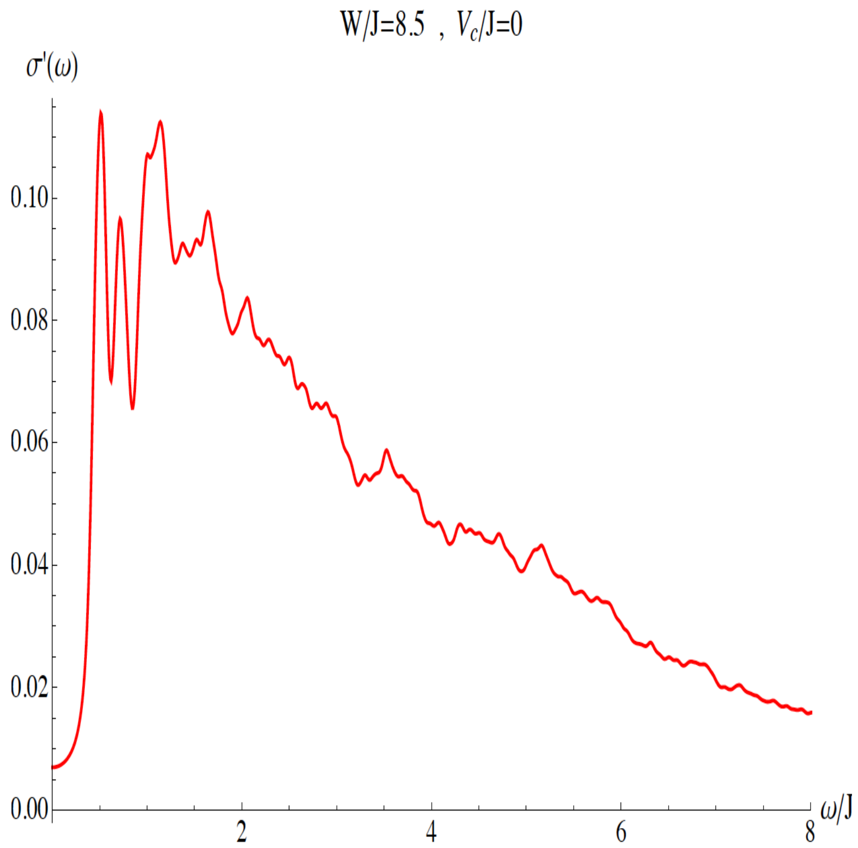
$$\boxed{\overline{J}(q,\omega) = -\tilde{K}(q,\omega)\overline{A}}$$

$$\boxed{1 + 4\pi i \tilde{\sigma}_L / i\omega = <(1/\varepsilon_L)>^{-1}}$$

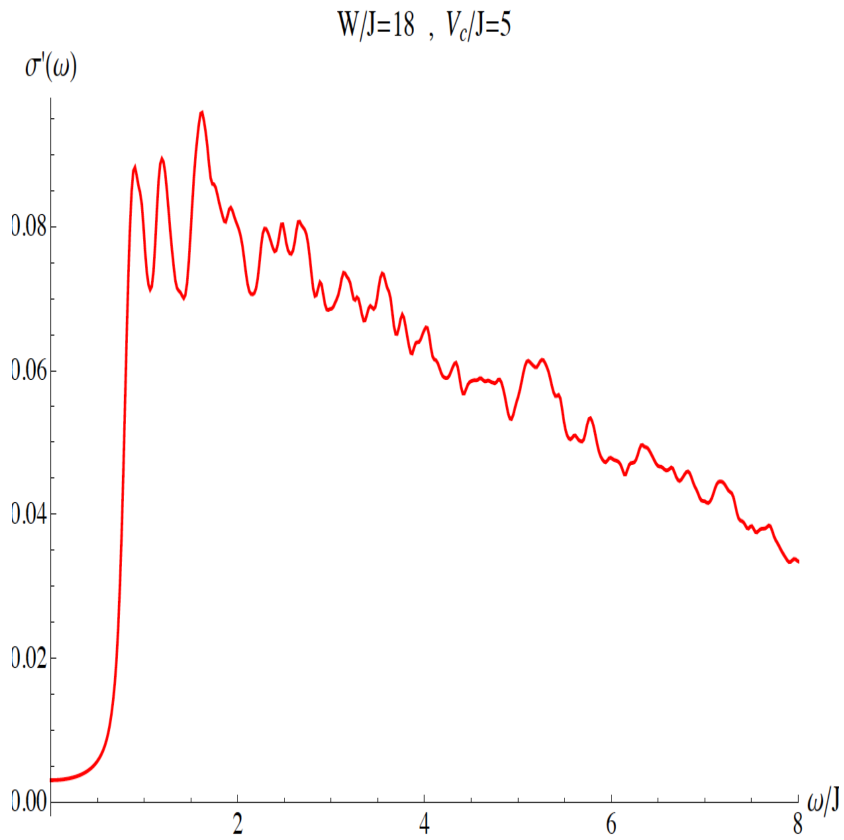
- Preliminary results: by introducing long range forces depletion of low energy absorption towards higher energies, but still in the superconducting gap

Optical conductivity for charged systems

Short range conductivity

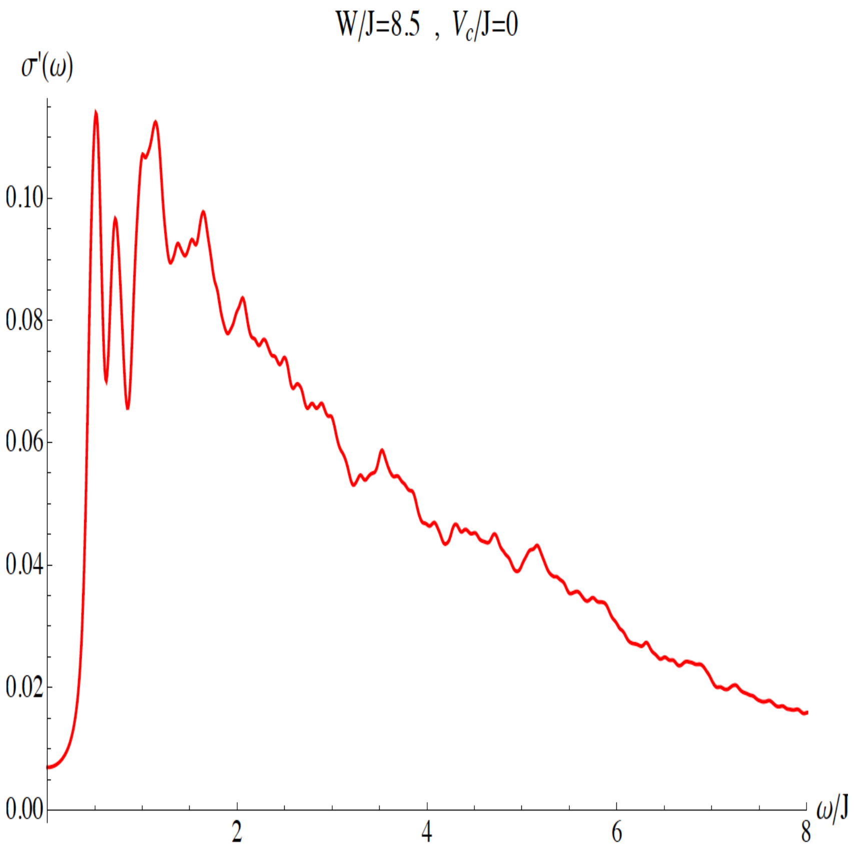


Long range conductivity

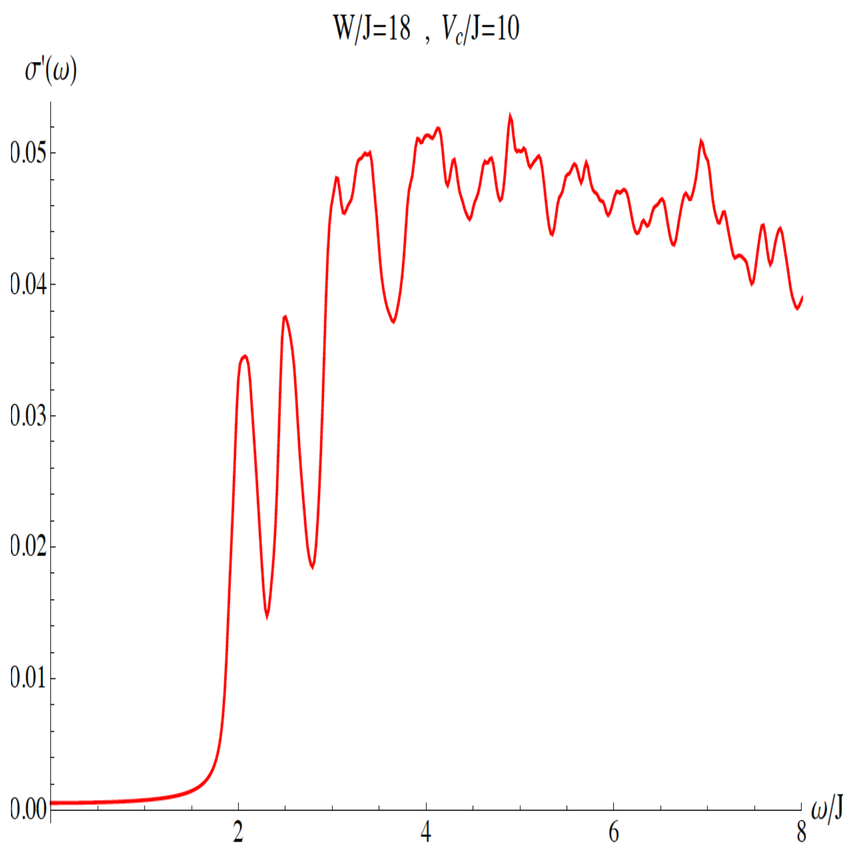


Optical conductivity for charged systems

Short range conductivity



Long range conductivity



Optical conductivity for charged systems

A simple bosonic model in the clean limit:

$S=\int drdt\,\{1/2D_s(\nabla\theta-2e/c\mathbf{A})^2-1/2\chi_0\,(d\theta/dt)^2\}$

D_s superfluid density, χ_0 charghe compressibility

Long range $\chi_0^{-1}\Rightarrow \chi_0^{-1}+V_c$, $\quad V_c \propto 1/q,\ 1/q^2$

Response to a longitudinal external field $J_L=-K_L^{red}A^0_L$,($\sigma=K/i\omega$)

$$K_L^{red}(q,\omega)=D_s\frac{\omega^2}{\omega^2-\omega_P^2}$$

ω_P plasmon frequencies

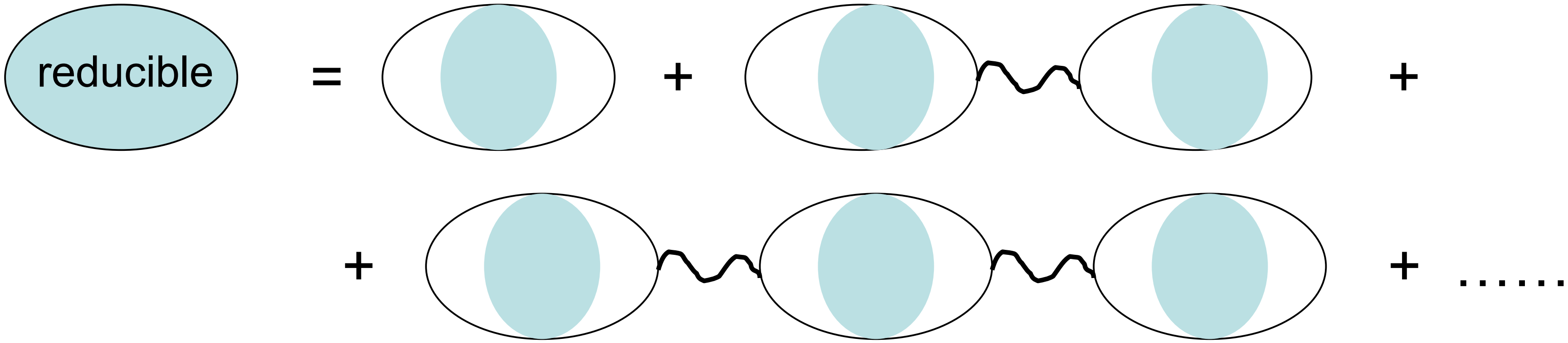
However $J_L=-K_LA_L$

$$K_L(q,\omega)=K_L^{red}(q,\omega)\varepsilon(q,\omega)=D_s\frac{\omega^2}{\omega^2-\omega_s^2}$$

ω_s sound frequencies

$\Rightarrow$Screening of long range, what in the presence of disorder?

Graphics



Spin/bosonic model and H.P. appr.

$$\mathcal{H}_{PS} \equiv -2\sum_i \xi_i S_i^z - 2J\sum_{\langle i,j\rangle}\left(S_i^+ S_j^- + h.c.\right).$$

$$\widetilde{S_i^z} = 1/2 - a_i^+ a_i,\ \widetilde{S_i^+} \simeq a_i \text{ and } \widetilde{S_i^-} \simeq a_i^+$$

$$\mathcal{H}'_{PS} = \sum_{ij}\left[A_{ij}(a_i^\dagger a_j + h.c.) + \frac{1}{2}B_{ij}\left(a_i a_j + h.c.\right)\right]$$
$$= \sum_\alpha E_\alpha \gamma_\alpha^\dagger \gamma_\alpha + const.$$

$$\Phi_i = -2\frac{S_i^y}{\sin\theta_i} = \sum_\alpha i\frac{\phi_{\alpha i}}{\sqrt{2}}(\gamma_\alpha^\dagger - \gamma_\alpha), \qquad a_i = \textstyle\sum_\alpha \left(u_{\alpha i}\gamma_\alpha + v_{\alpha i}\gamma_\alpha^\dagger\right):$$

$$L_i = S_i^\perp \sin\theta_i = \sum_\alpha \frac{\ell_{\alpha i}}{\sqrt{2}}(\gamma_\alpha^\dagger + \gamma_\alpha), \qquad \phi_{\alpha i} = \sqrt{2}\left(v_{\alpha i} - u_{\alpha i}\right)/\sin\theta_i$$

Quantum phase model and σ(ω)

Mean field for <S_i^x>=1/2sinθ_i and Holstein- Primakov transformation to a bilinear boson problem or equivalently to a quadratic phase hamiltonian

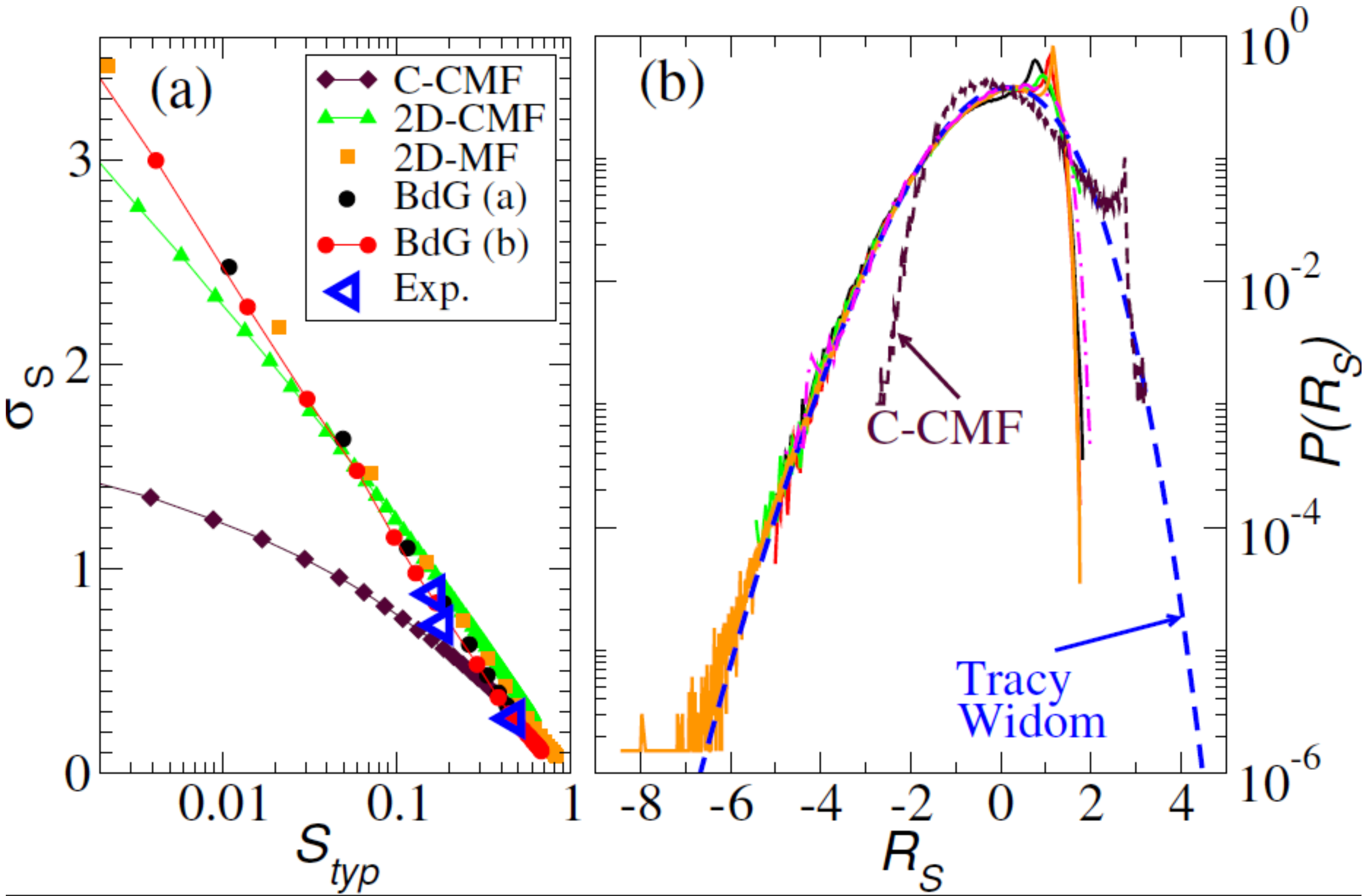
$$\mathcal{H}'_{PS} = \frac{1}{2} \sum_{i,\mu=x,y} J_i^\mu \left[\Delta_\mu \Phi_i\right]^2 + \frac{1}{2} \sum_{ij} \mathcal{X}_{ij}^{-1} L_i L_j$$

$$J_i^\mu \equiv J \sin\theta_i \sin\theta_{i+\hat{\mu}} \qquad \Delta_\mu \Phi_i \to \Delta_\mu \Phi_i - 2eA_\mu$$

$$\sigma_{reg}^B(\omega) = \frac{e^2\pi}{2N} \sum_\alpha Z_\alpha \left[\delta(\omega + E_\alpha) + \delta(\omega - E_\alpha)\right]$$

$$Z_\alpha = \frac{1}{E_\alpha} \left[\sum_i 2J_i^\mu \Delta_\mu \phi_{\alpha i}\right]^2 .$$

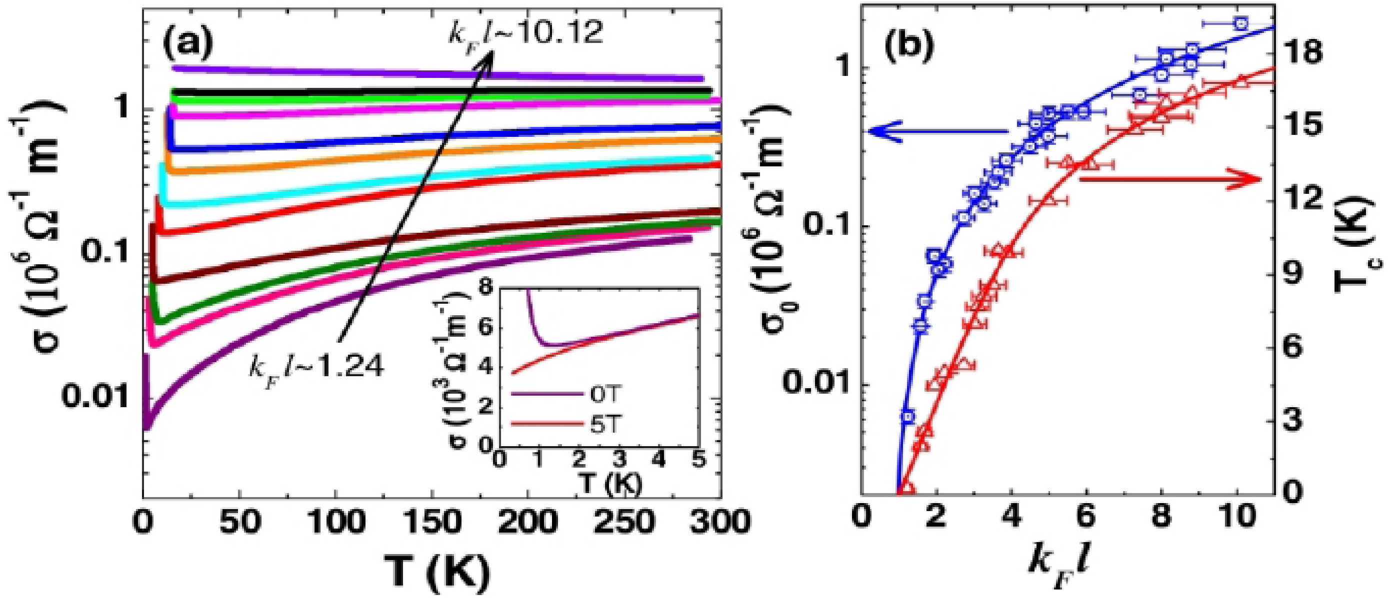
$$D^B = (1/N) \sum_i 4J_i^\mu \qquad D_s^B = D^B - \frac{1}{N} \sum_\alpha Z_\alpha$$

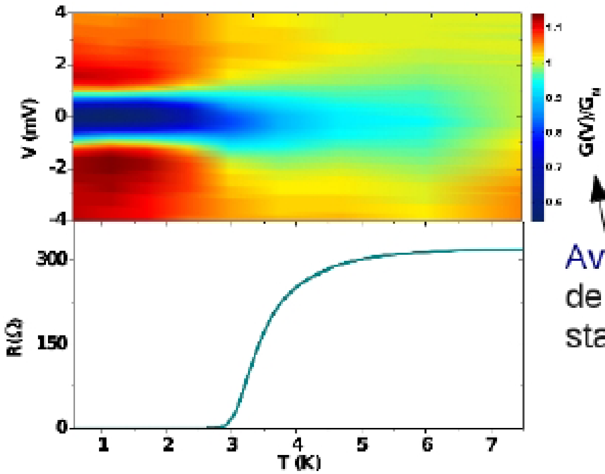


magenta dashed dotted line, BdG with $|U| = 1.5$,
$\langle n \rangle = 0.875$, $L = 25$, $g = 0.2$, i.e., $V_0 = 3.33$.

Disorder driven Superconductor-Insulator transition in 3D NbN

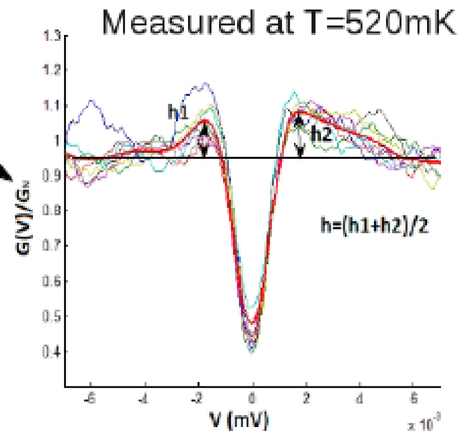
Mondal et. al (2011); Chand et al. (2012)





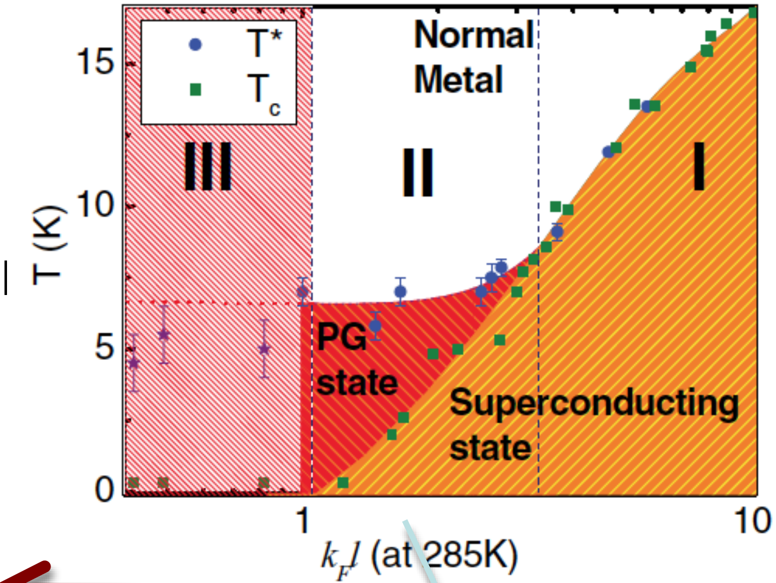
Averaged density of states

Local density of states

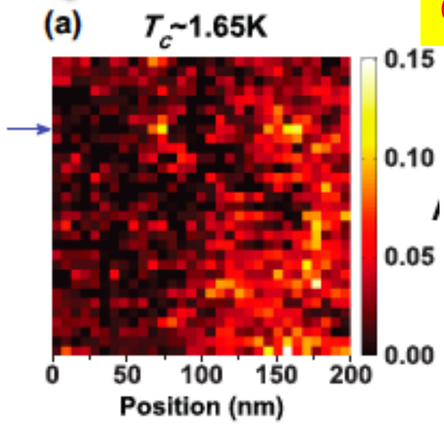


Inhomogeneity and glassy physics

NbN films
M. Chand et al
PRB 2012



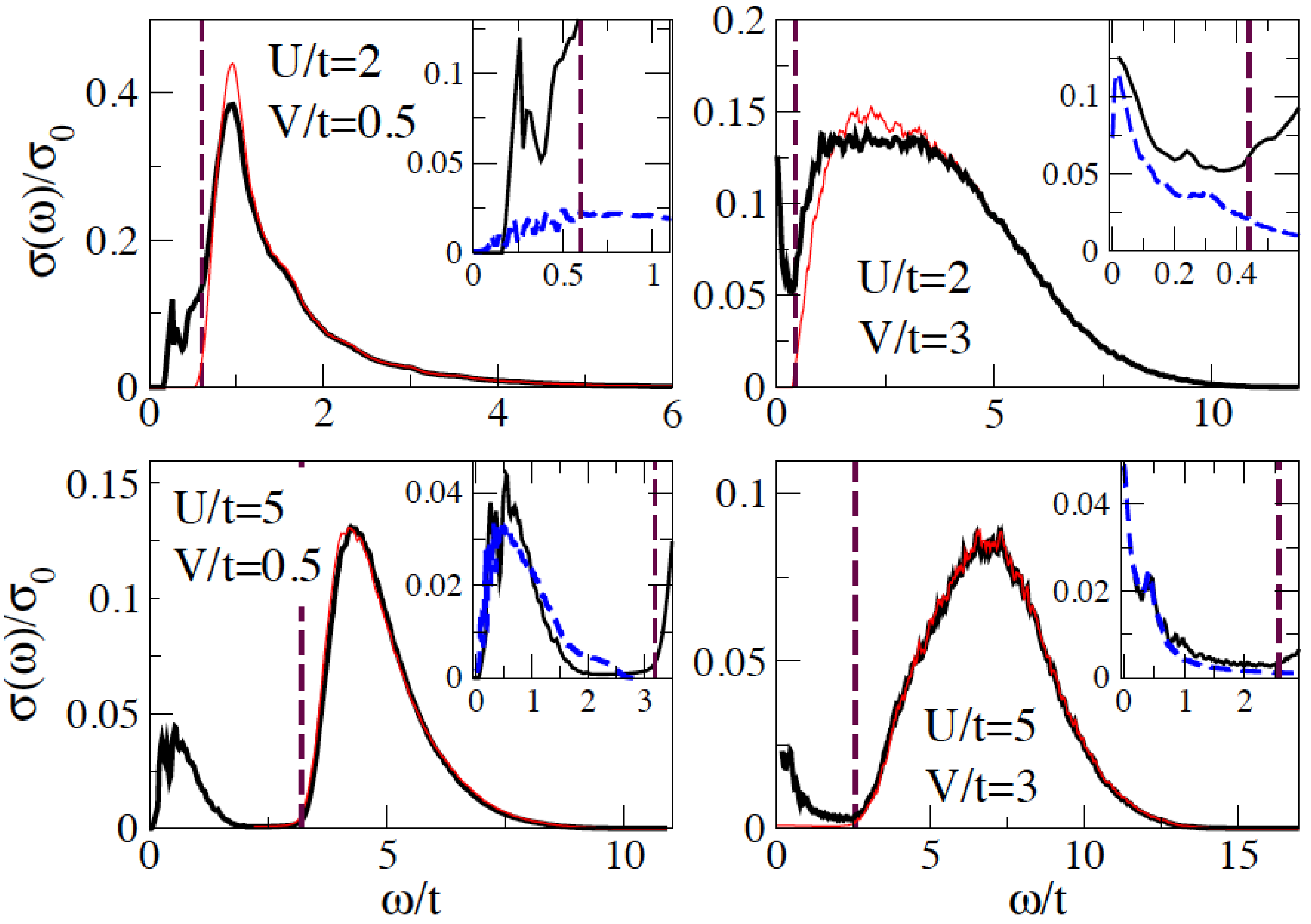
Increasing disorder



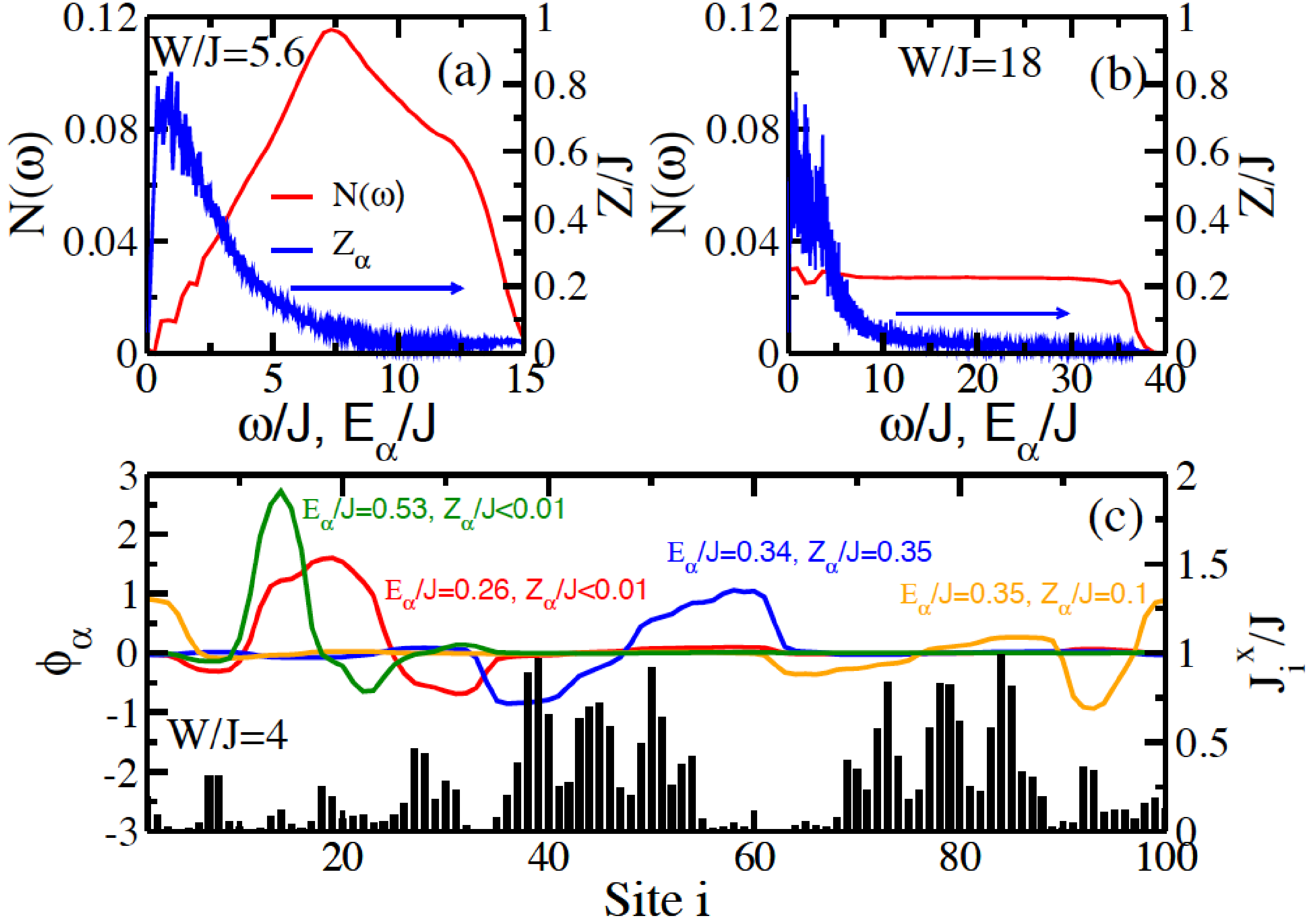
The emergent mesoscopic inhomogeneity is a signature of **"glassy"** superconductivity?

Make predictions on currents, superfluid density and optical conductivity

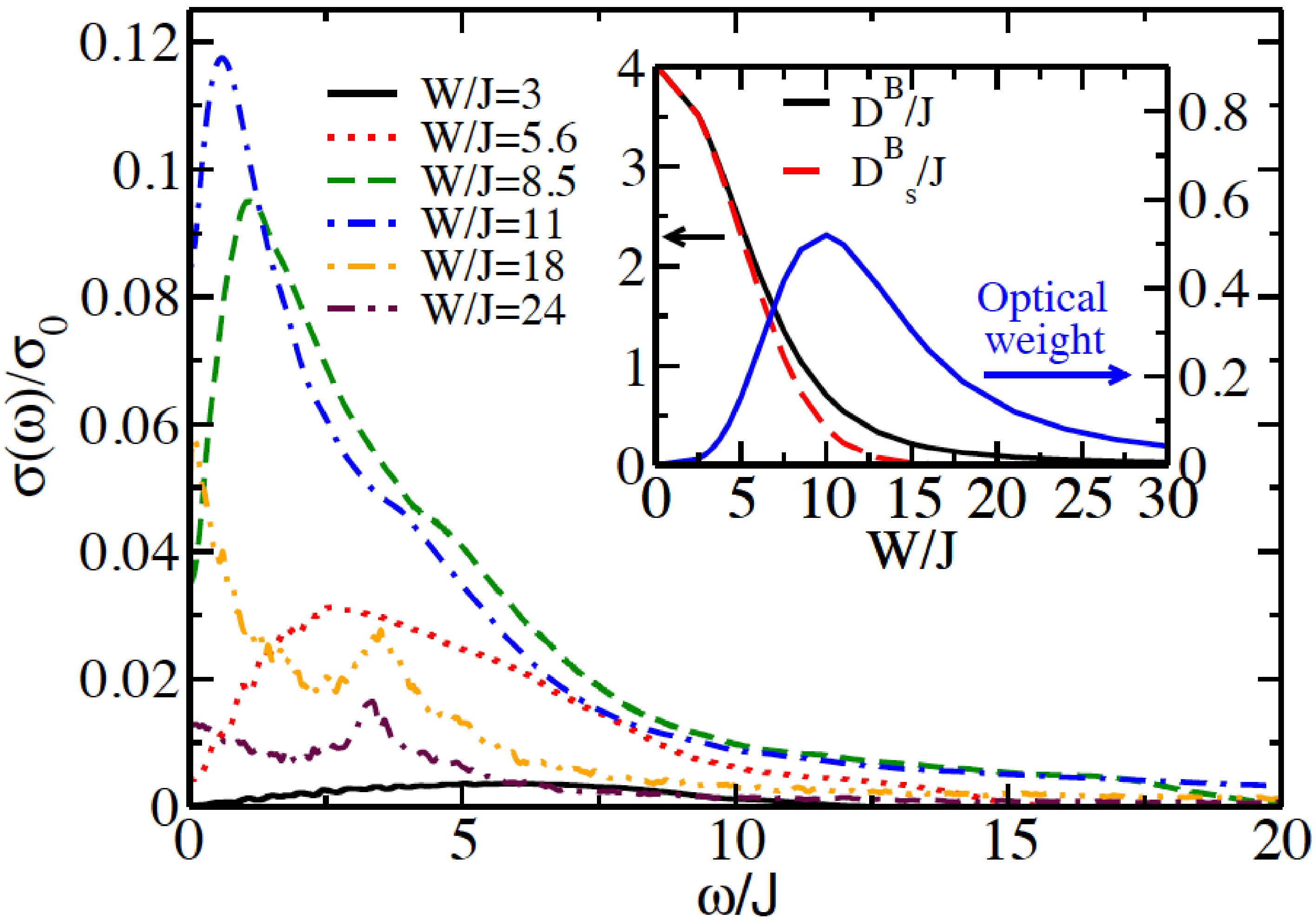
Optical conductivity in disordered attractive Hubbard model



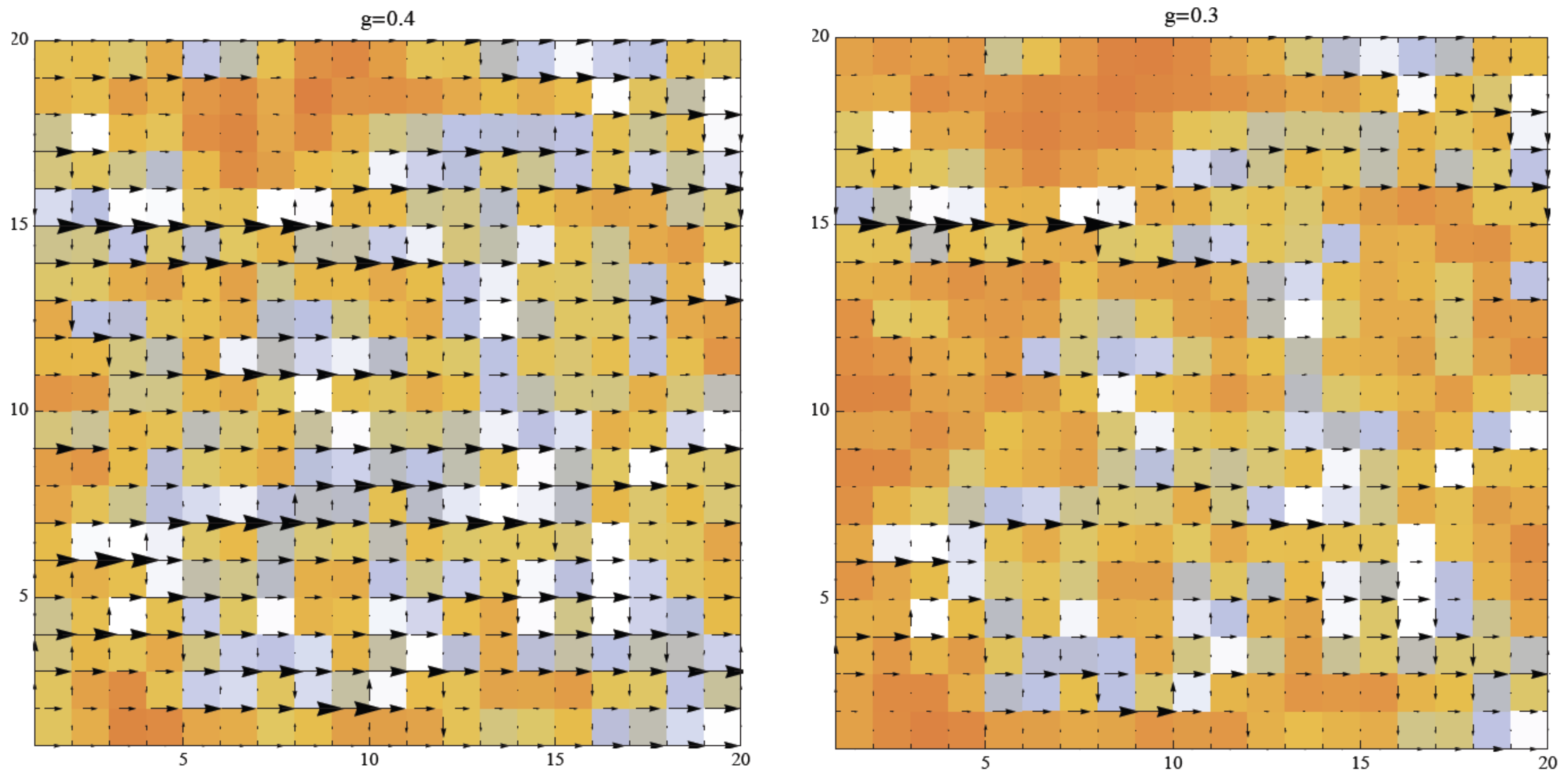
Optics in bosonic model



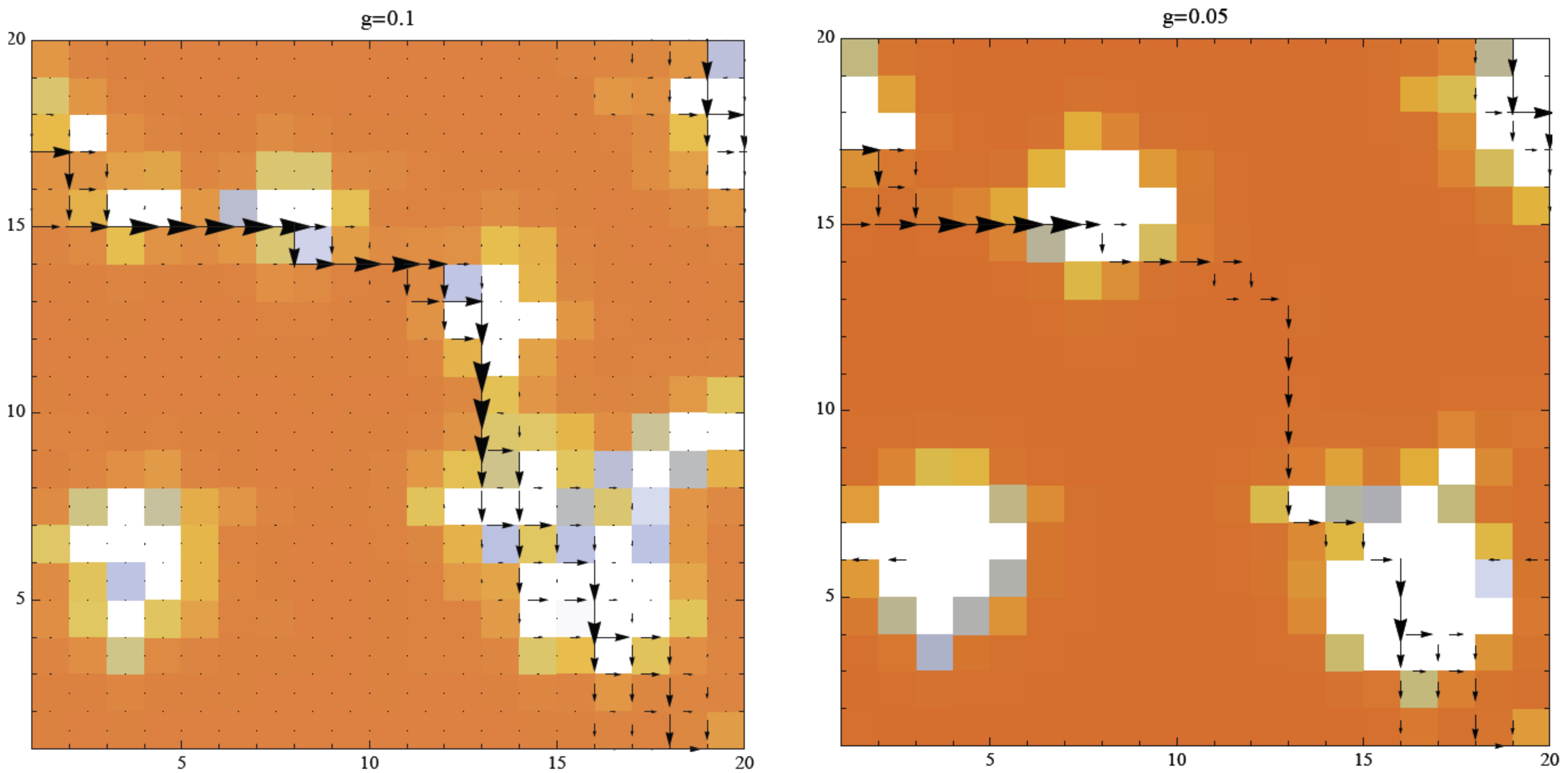
Optical conductivity in the bosonic model



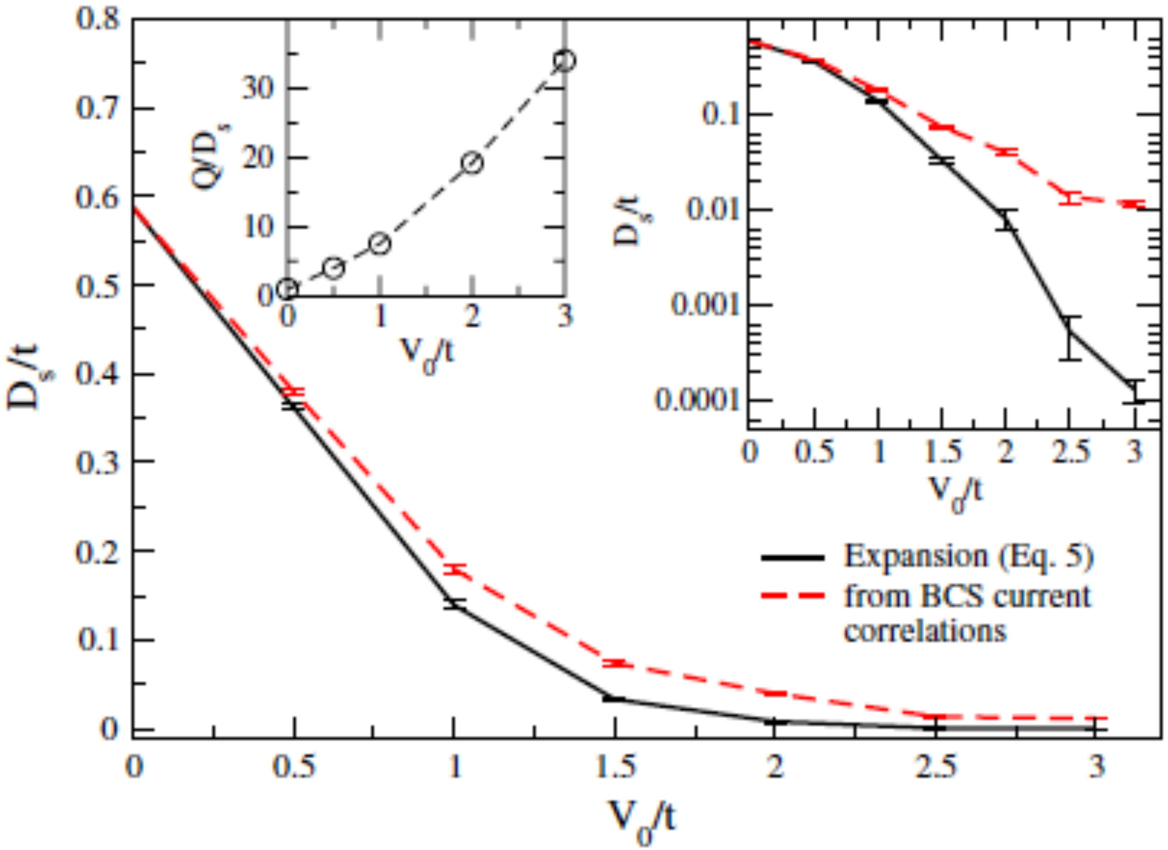
Current response in a disordered SC



Current response in a disordered SC



Current response in a disordered SC



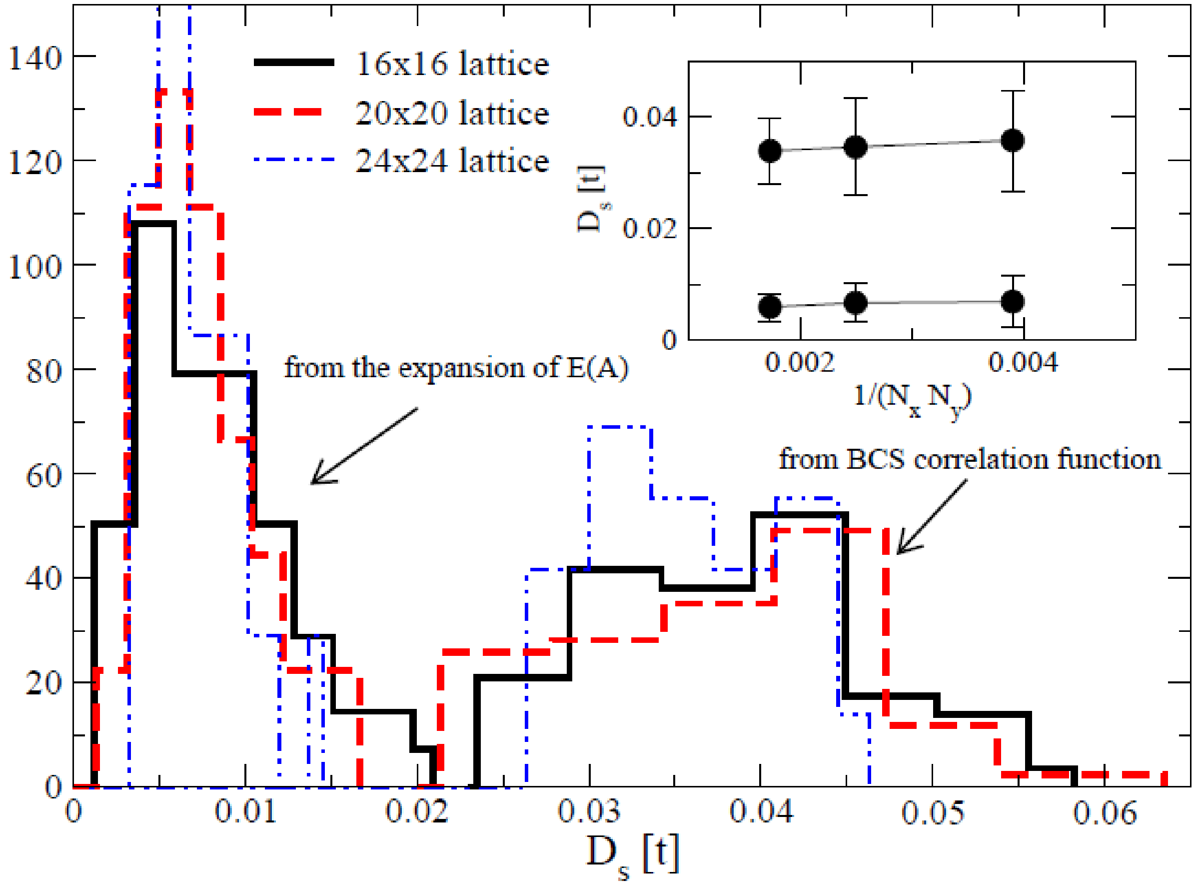
Superfluid density from **$D_s=1/L^2\partial^2E(A)/\partial A^2$**

$|U|=5t$, $n=0.85$ vs V_0

Average over 20 ÷40 disorder configurations

Q/D_s measure of anharmonic terms

Current response in a disordered SC



Distributions of the superfluid density with increasing lattice size

$|U|=5t$, $V_0=2t$
$n=0.85$

Current-current correlations