

# Second sound and the superfluid fraction in a resonantly interacting Fermi gas

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清华大学

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Superconductors and Superfluids

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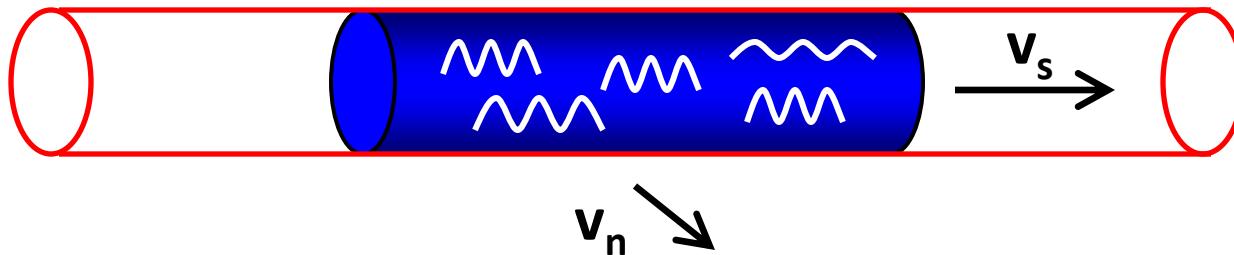


# Outline

- **Second sound**
- **Experimental results**
- **Obtaining temperature dependence  
of the superfluid fraction**

# Two-fluid model of superfluid helium

$$T \neq 0$$



a superfluid at finite temperature

Lev D. Landau

= a superfluid component + a normal component

zero viscosity  
+  
zero entropy

finite viscosity  
+  
carries entropy

# Two-fluid model of superfluid helium

$$n = n_n + n_s$$

normal part      superfluid part

TWO sound modes are possible:

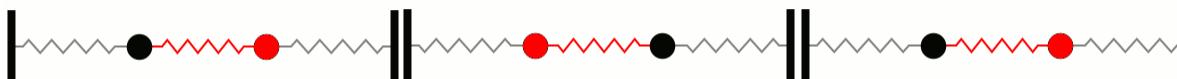
$$u_1 = \sqrt{\frac{1}{m} \left( \frac{\partial P}{\partial n} \right)_{\bar{s}}}$$

FIRST SOUND – isentropic oscillation

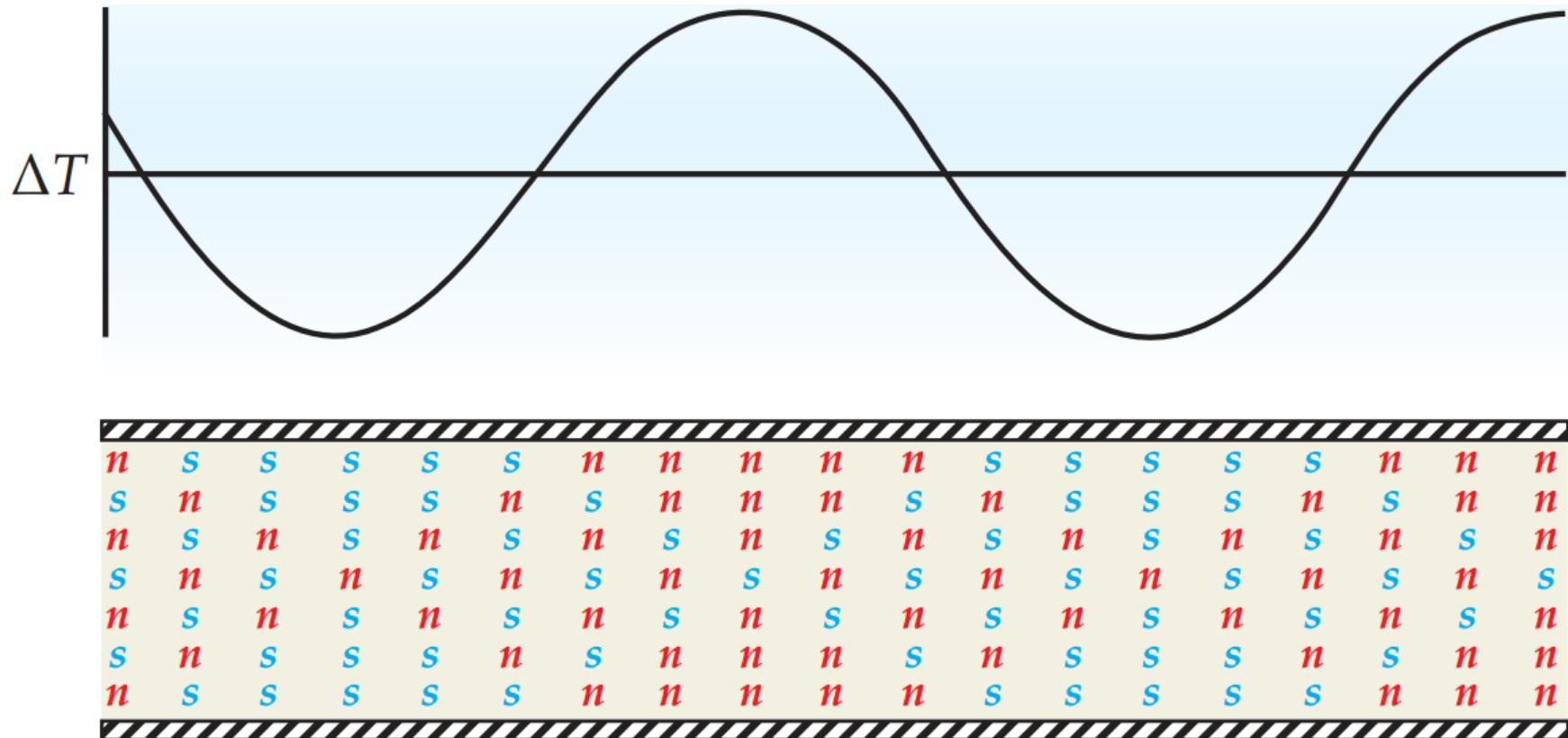


$$u_2 = \sqrt{\frac{1}{m} \frac{T \bar{s}^2}{\bar{c}_p} \frac{n_s}{n_n}}$$

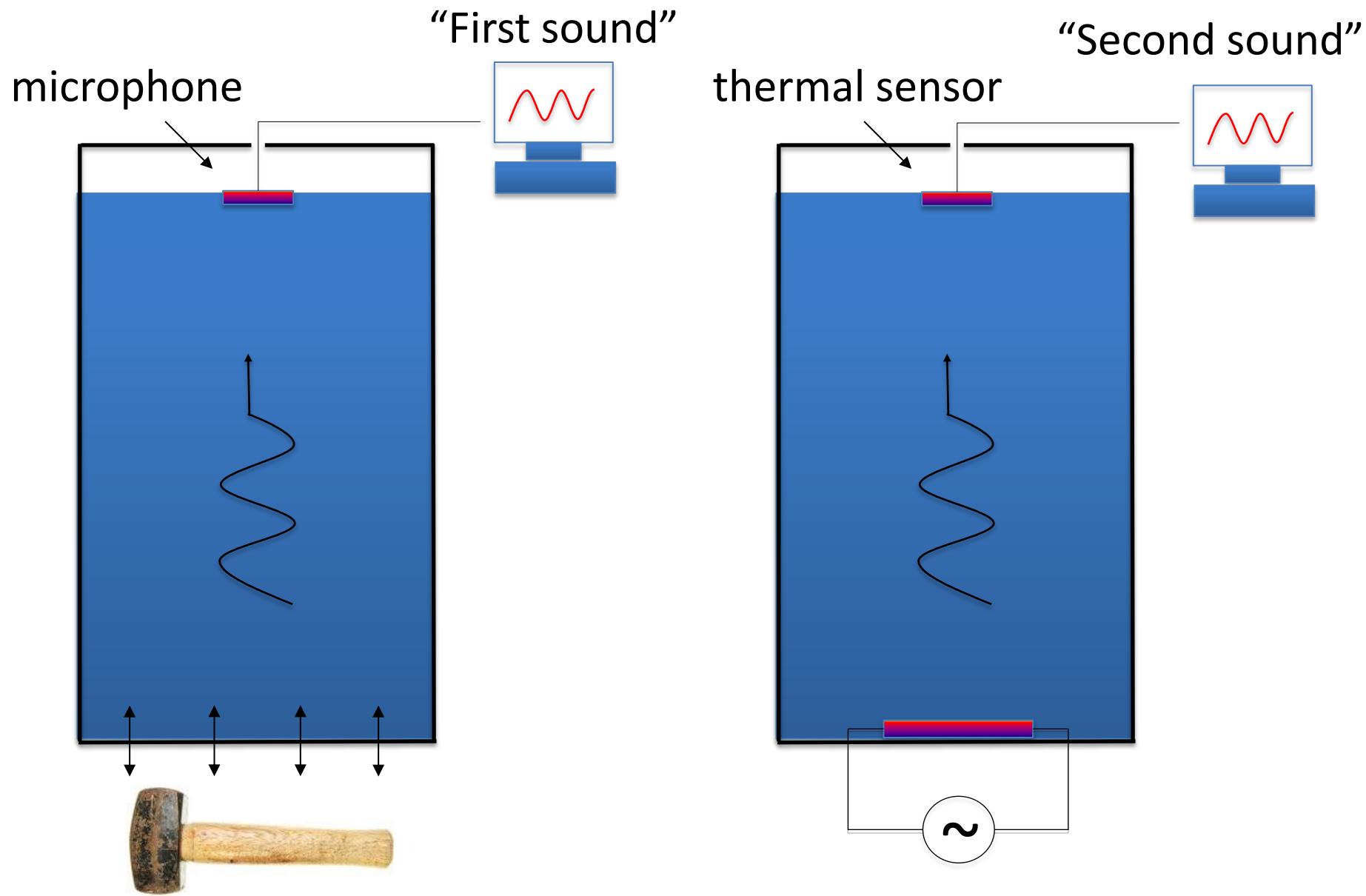
SECOND SOUND – isobaric oscillation



# second sound in superfluid helium

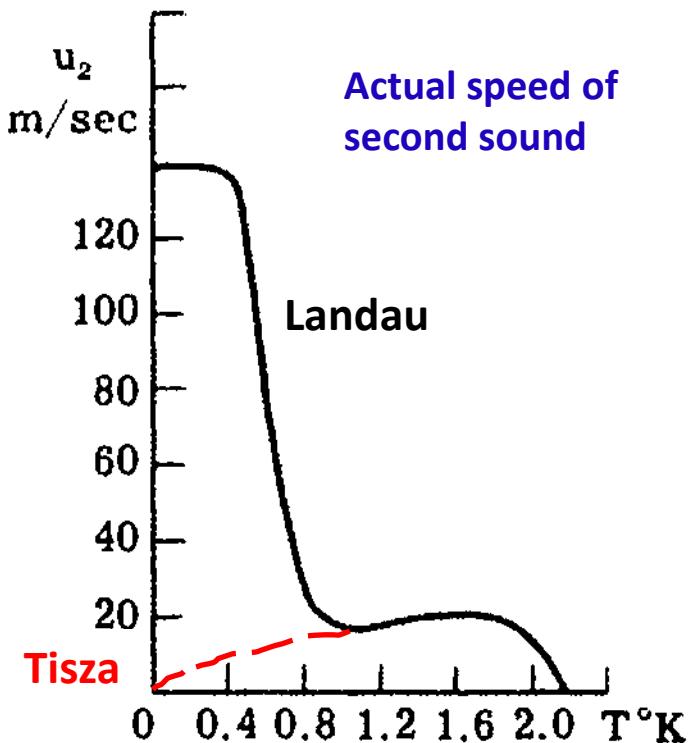


# First and second sound in superfluid helium

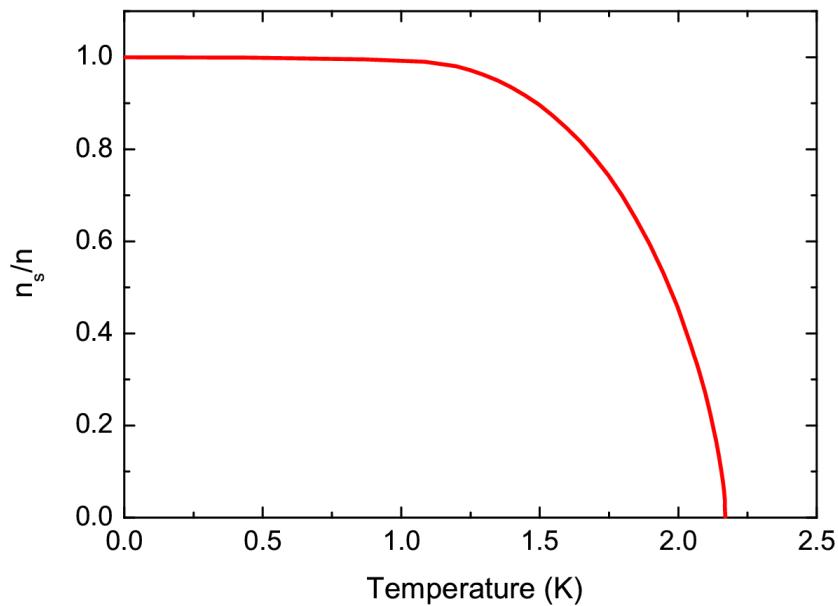


# Significance of second sound

Landau's model vs Tisza's BEC model



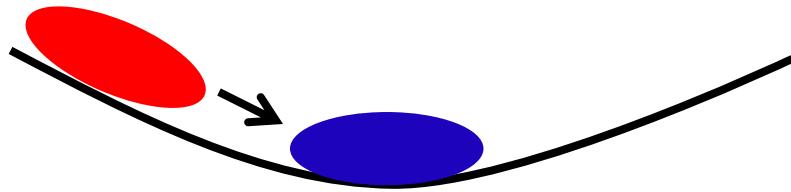
Superfluid fraction in Helium II



# Observing second sound in cold atoms?

## Second sound in BEC

Stamper-Kurn et al, Phys. Rev. Lett. 81, 500–503 (1998),  
Mappelink et al., Phys. Rev. Lett. 103, 265301 (2009),  
Mappelink et al., Phys. Rev. A 80, 043605 (2009).



## Differences from second sound in Helium II:

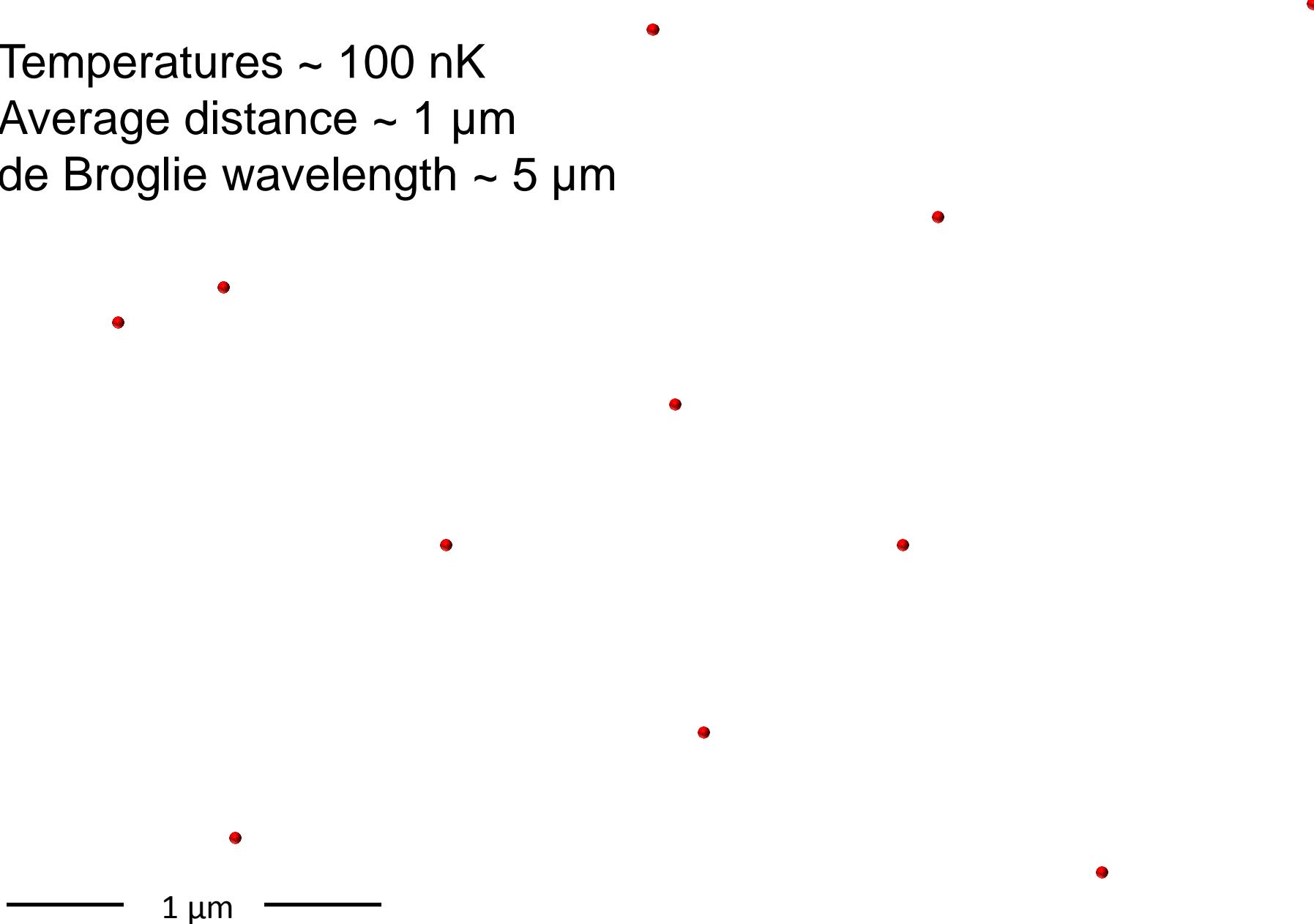
- Second sound in a BEC reduces to the motion of a condensate over a stationary thermal gas.
- hydrodynamic conditions not well satisfied.

# Observing second sound in cold atoms?

Temperatures  $\sim 100$  nK

Average distance  $\sim 1$   $\mu\text{m}$

de Broglie wavelength  $\sim 5$   $\mu\text{m}$

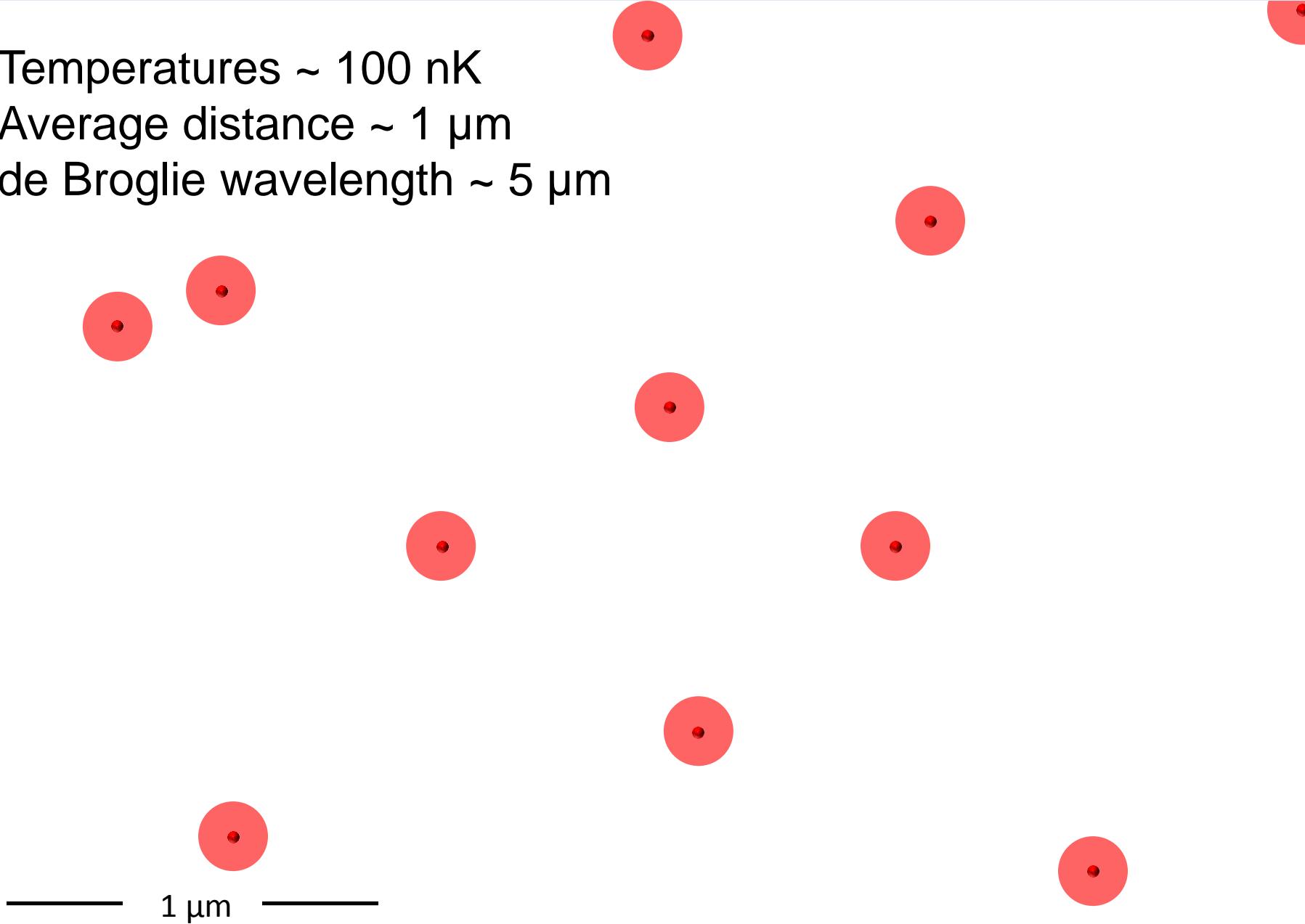


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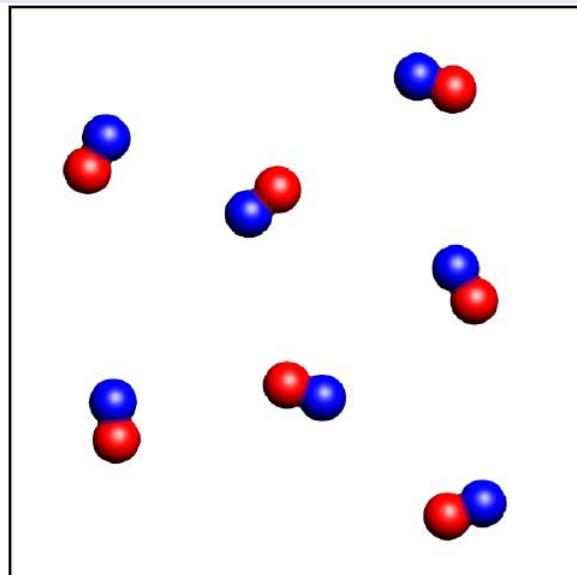
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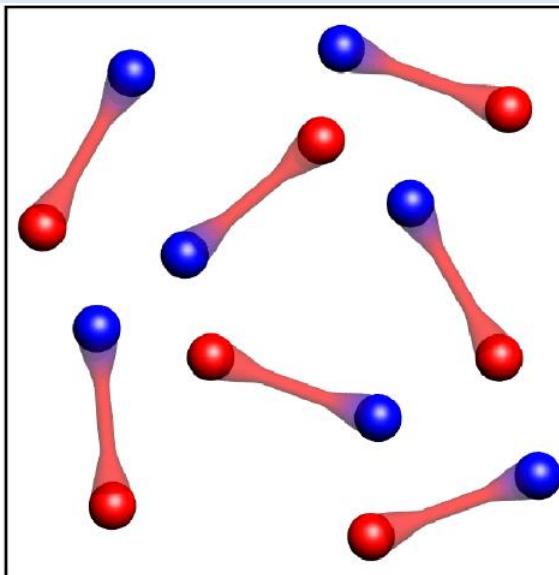
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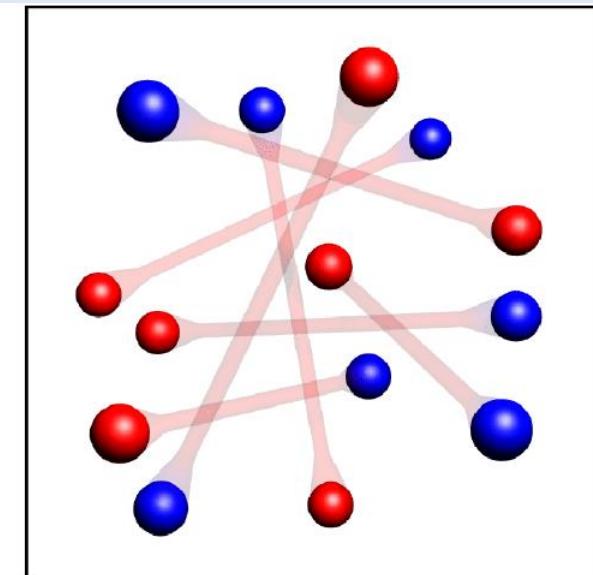
# Observing second sound in cold atoms?



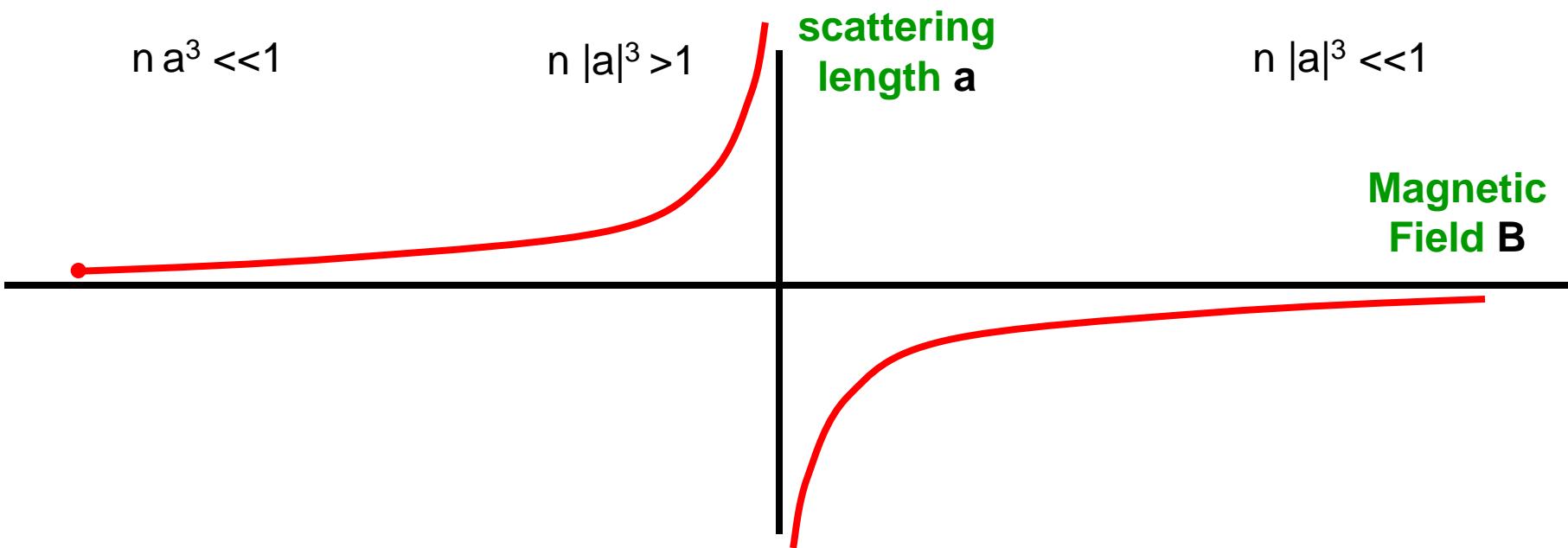
BEC of Molecules



Crossover Superfluid



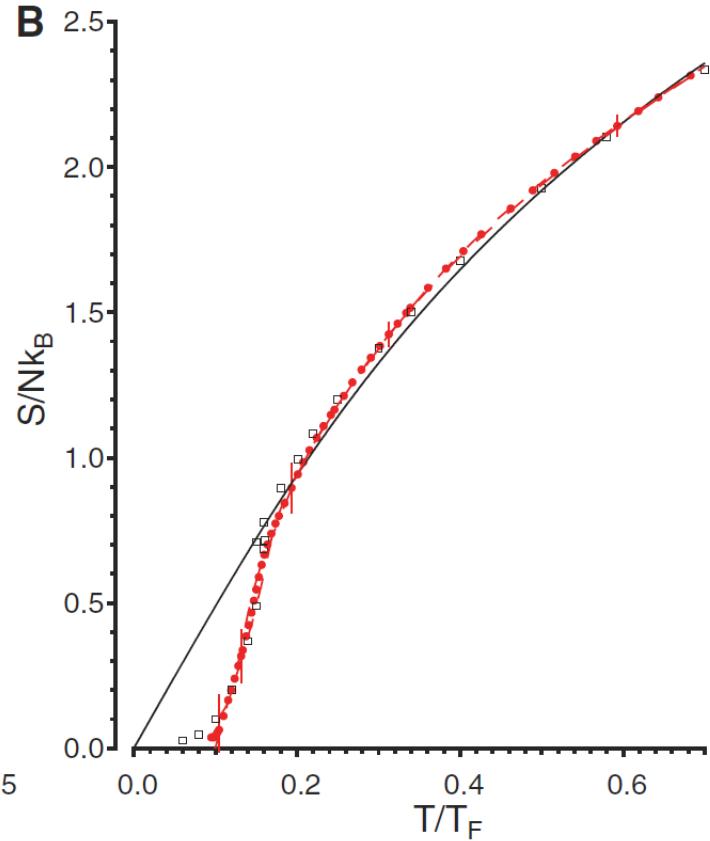
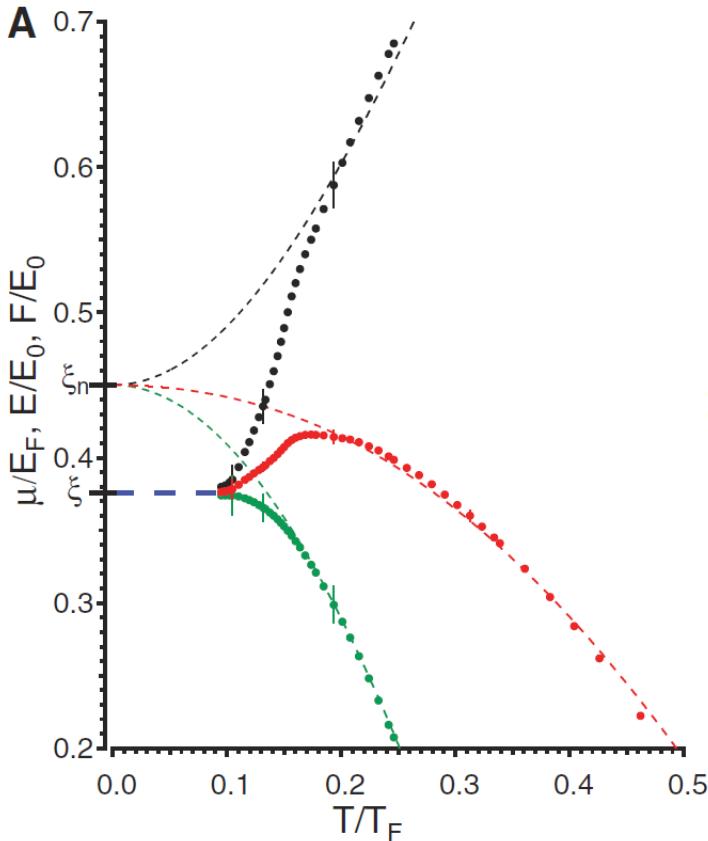
BCS state



# Resonantly-interacting Fermi gases

## Equation of State

$P = P(n, T)$



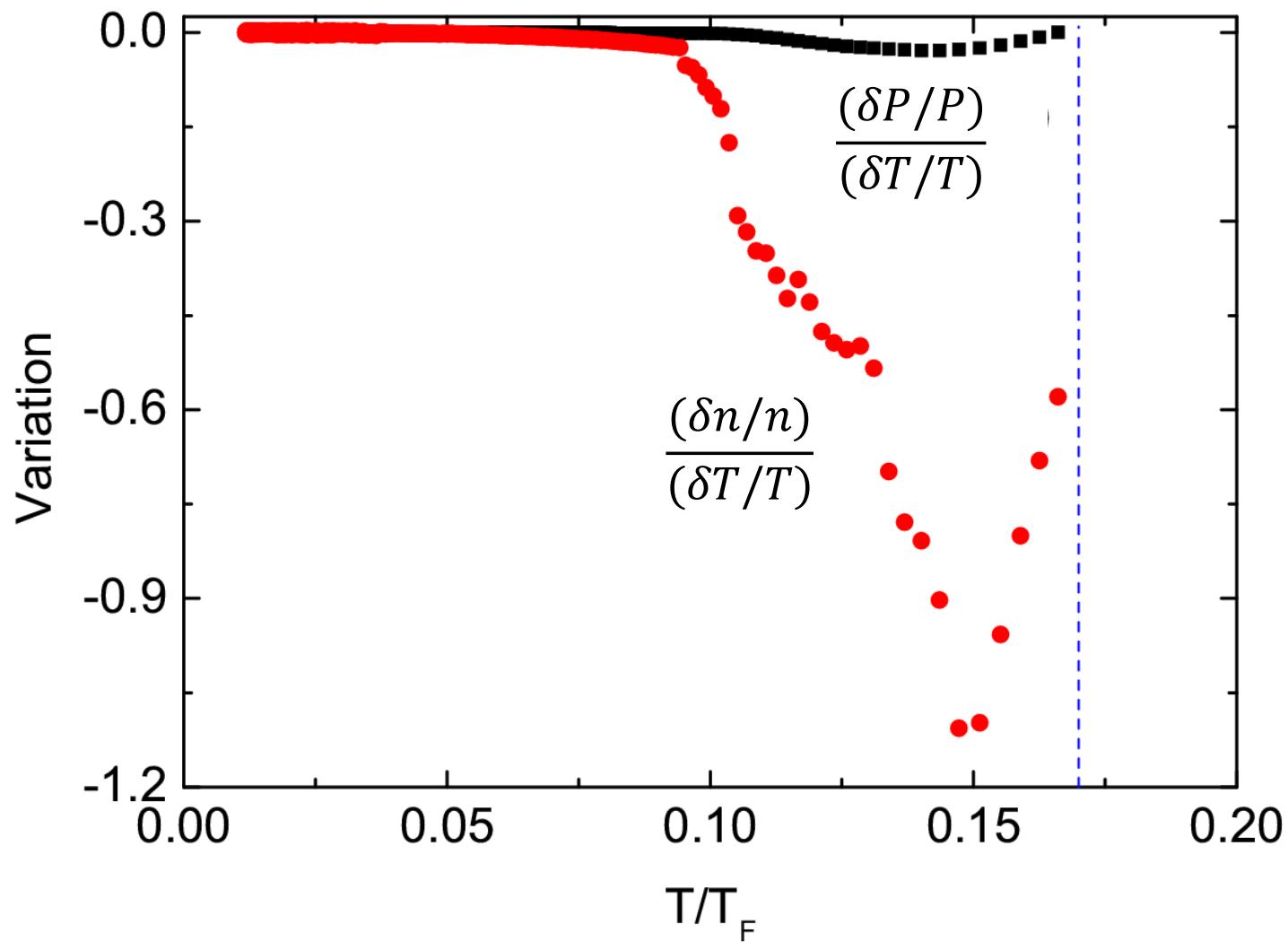
Ku et al., Science 335, pg 563-567 (2012) (MIT)

Nascimbène et al., Nature 463, 1057–1060 (2010) (ENS)

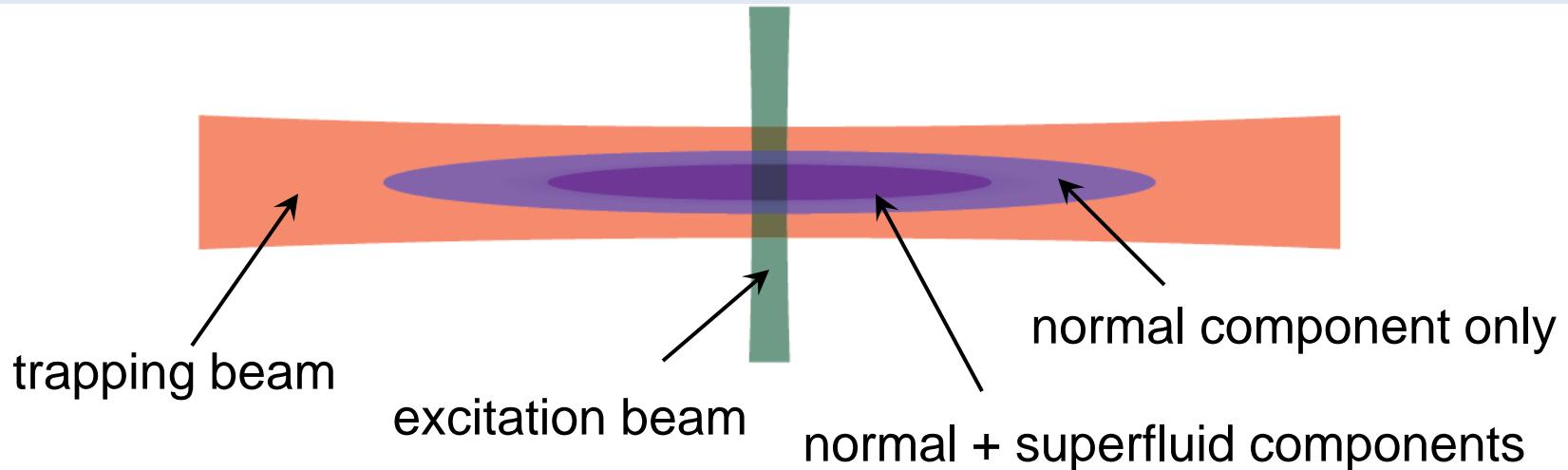
Horikoshi et al., Science 327, 442–445 (2010) (Tokyo)

Kinast et al., Science 307, 1296–1299 (2005) (Duke)

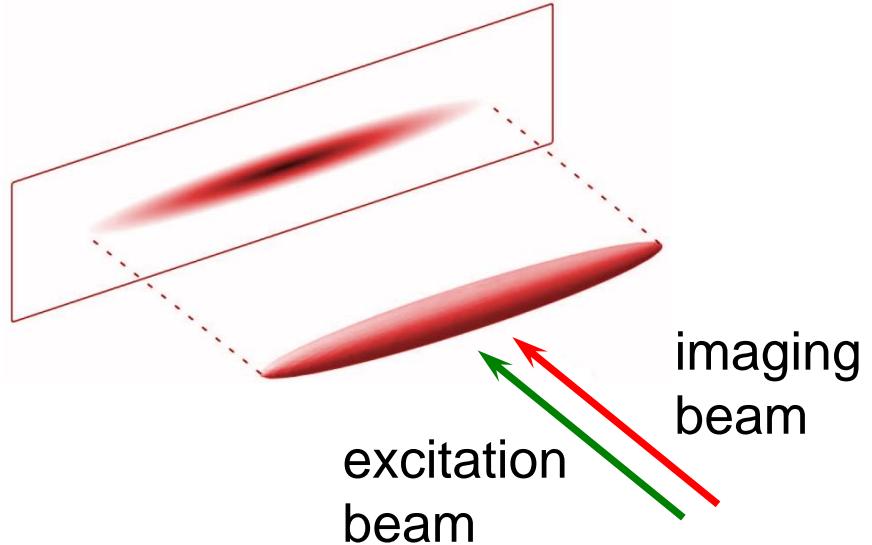
# Coupling between temperature and density variations in second sound for a resonantly interacting Fermi gas



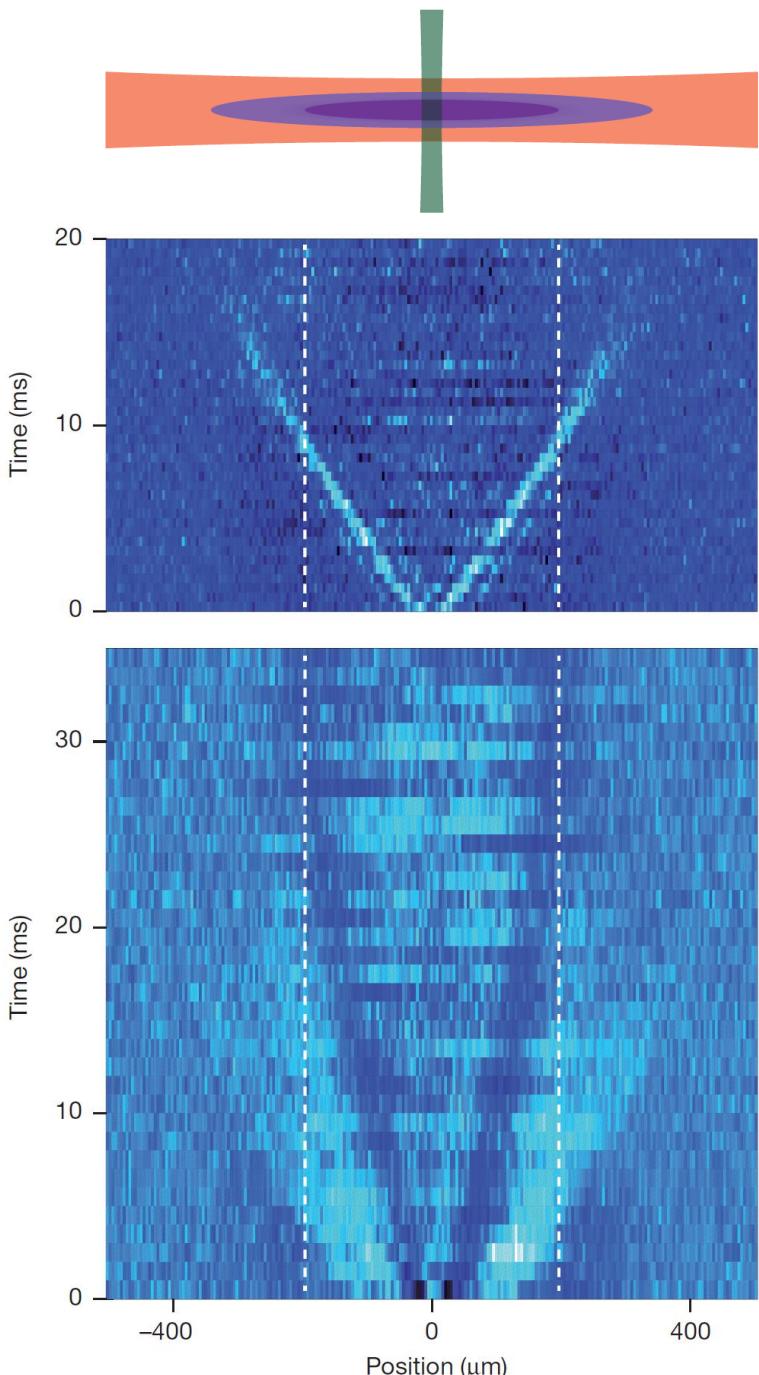
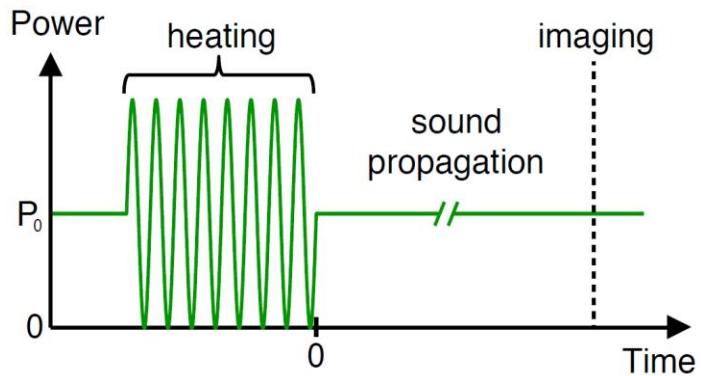
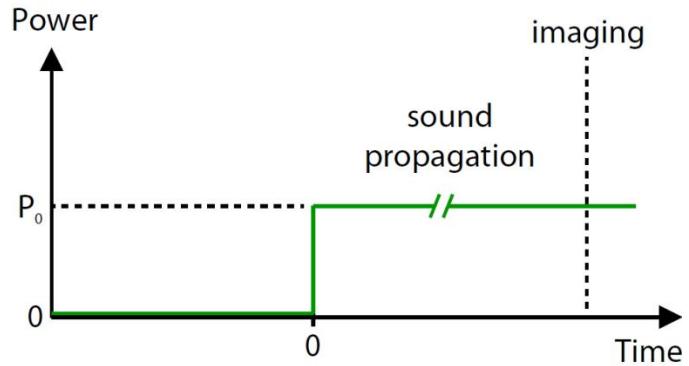
# Experiment



- ${}^6\text{Li}$  atoms, 50/50 spin mixture
- $B = 834 \text{ G}$
- 150000 atoms per spin state
- $T = 0.135(10) T_F^{\text{trap}}$



# Second sound excitation



# '1D' Landau two-fluid model



Strongly-interacting Fermi gas in an elongated trap

Assumptions: 1. Thermal equilibrium along transverse direction,  
2. flow fields independent of radial position.

Making use of local density approximation, Landau's two-fluid hydrodynamic equations become:

$$\partial_t s_1 + \partial_z(s_1 v_n^z) = 0$$

1D thermodynamic quantities

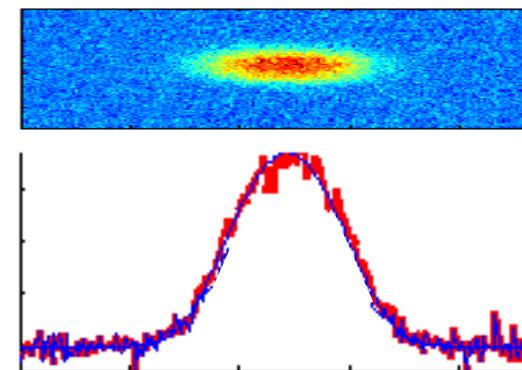
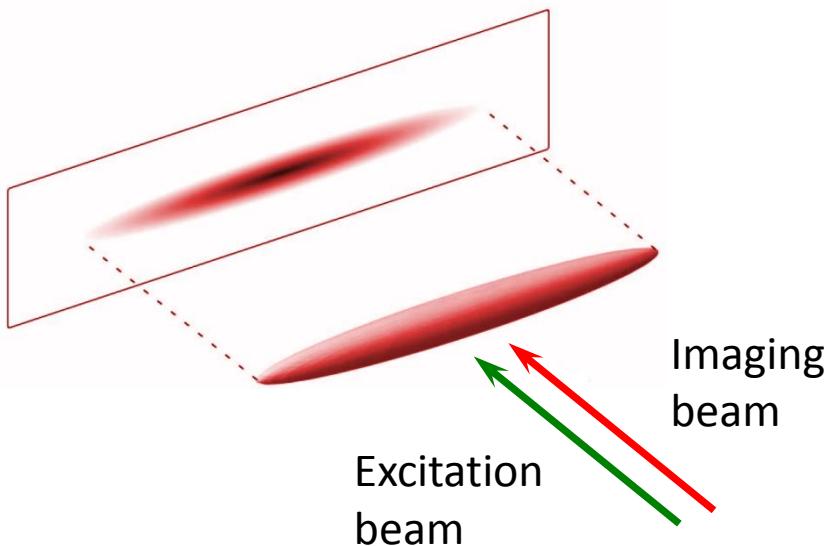
$$m\partial_t n_1 + \partial_z j_z = 0$$

$$X_1 = \int_0^\infty X 2\pi r dr$$

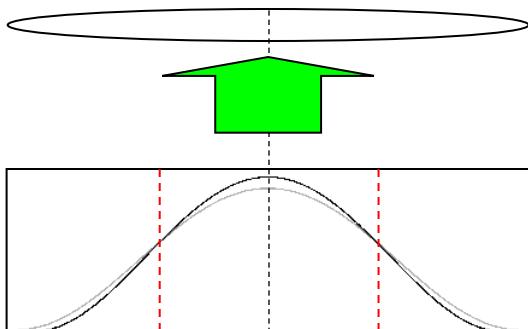
$$m\partial_t v_s^z = -\partial_z(\mu_1(z) + V_{ext}(z))$$

$$\partial_t j_z = -\partial_z P_1 - n_1 \partial_z V_{ext}(z)$$

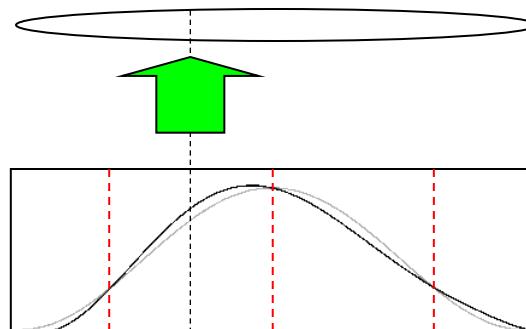
# Higher order collective oscillations



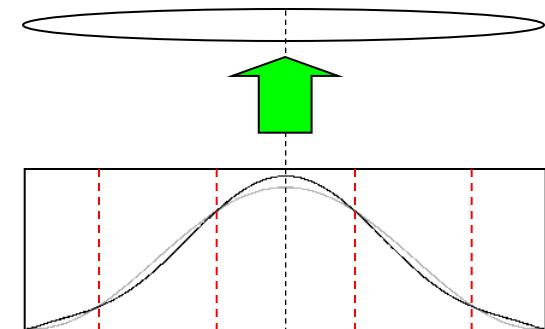
$k=1$



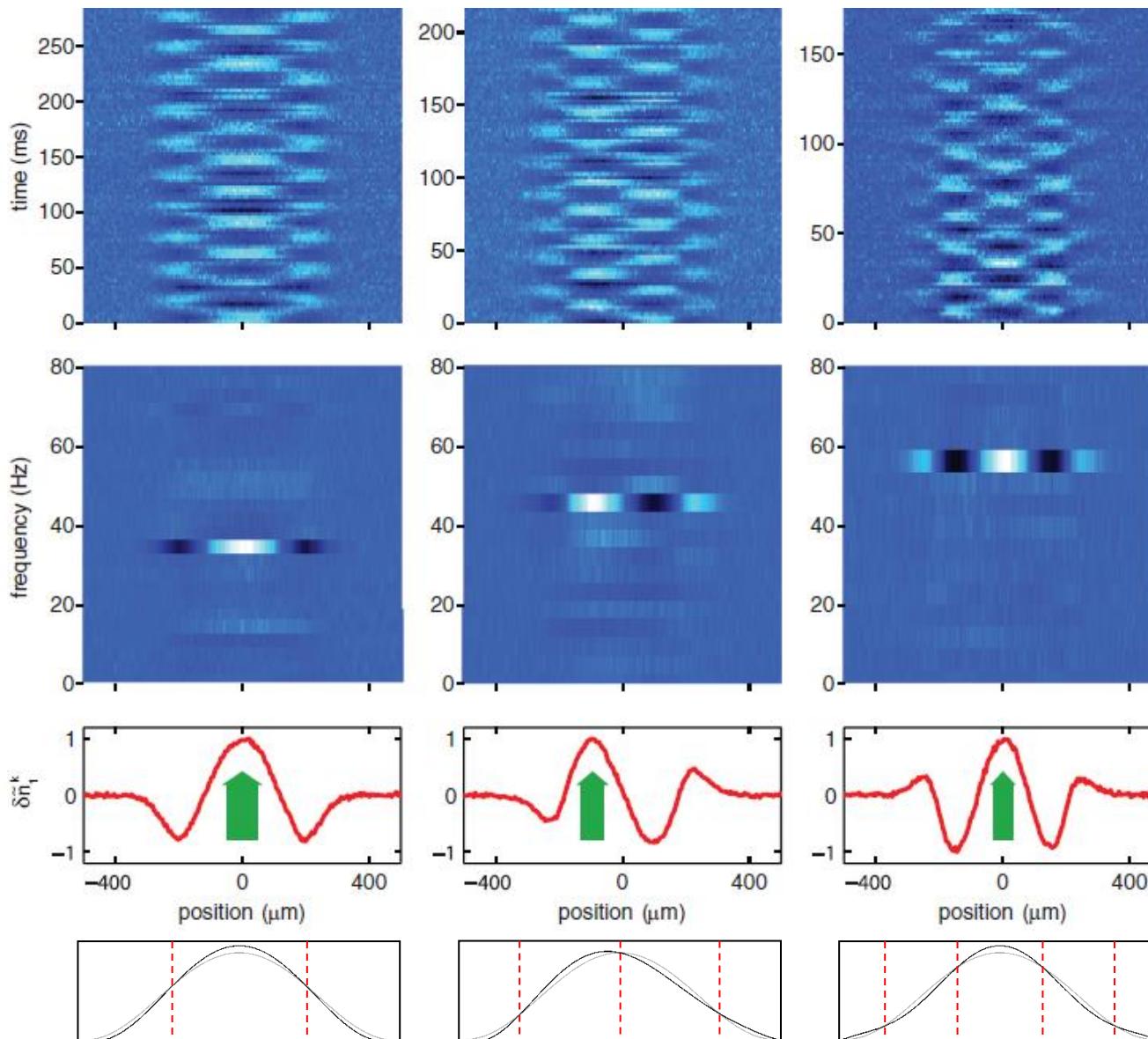
$k=2$



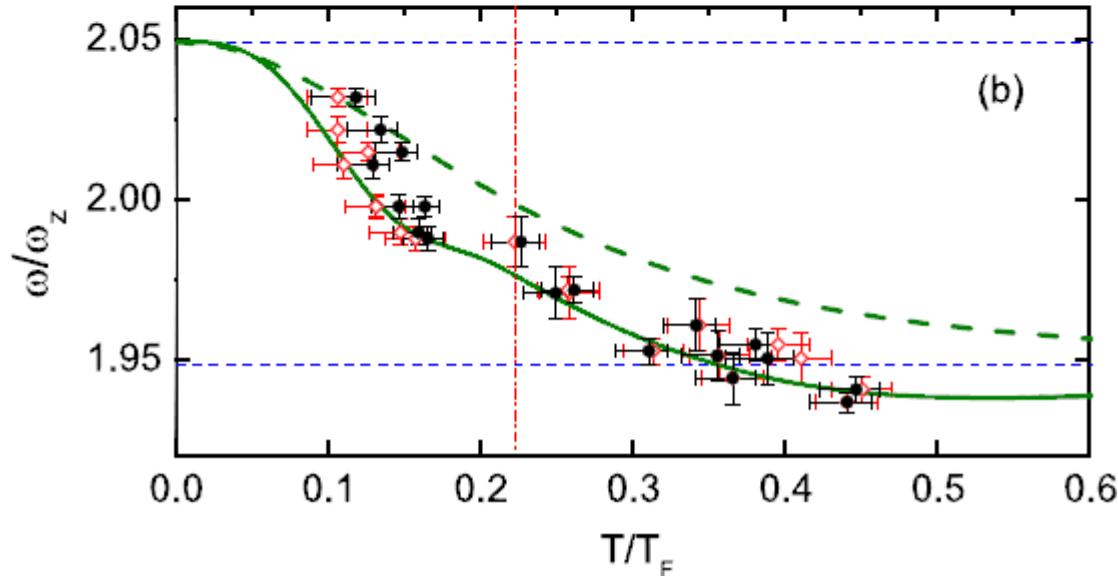
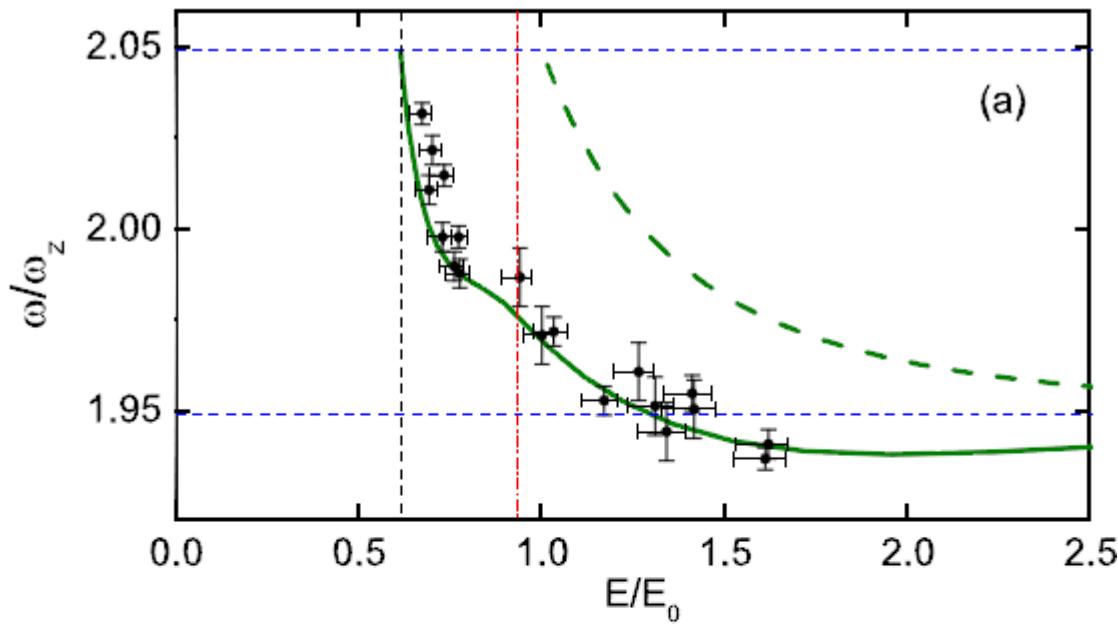
$k=3$



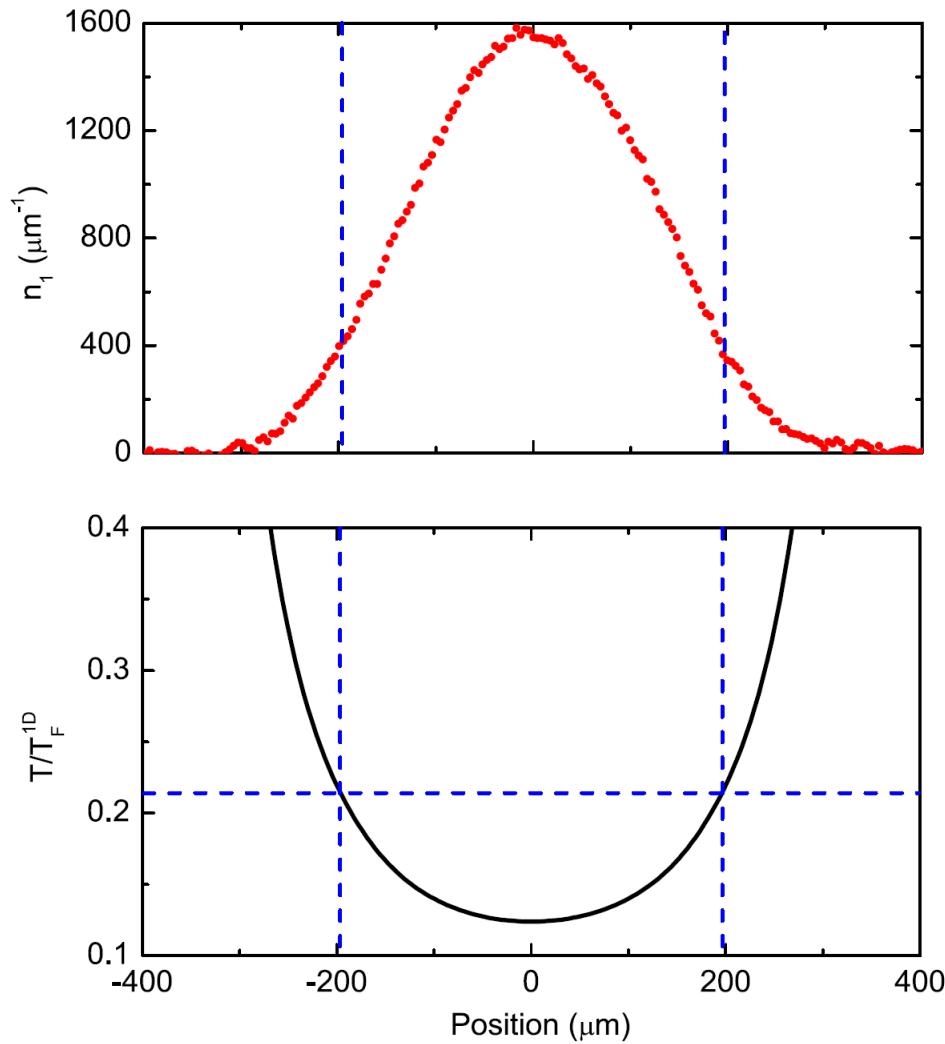
# Higher order collective oscillations



# Higher order collective oscillations



# Advantages of trap inhomogeneity

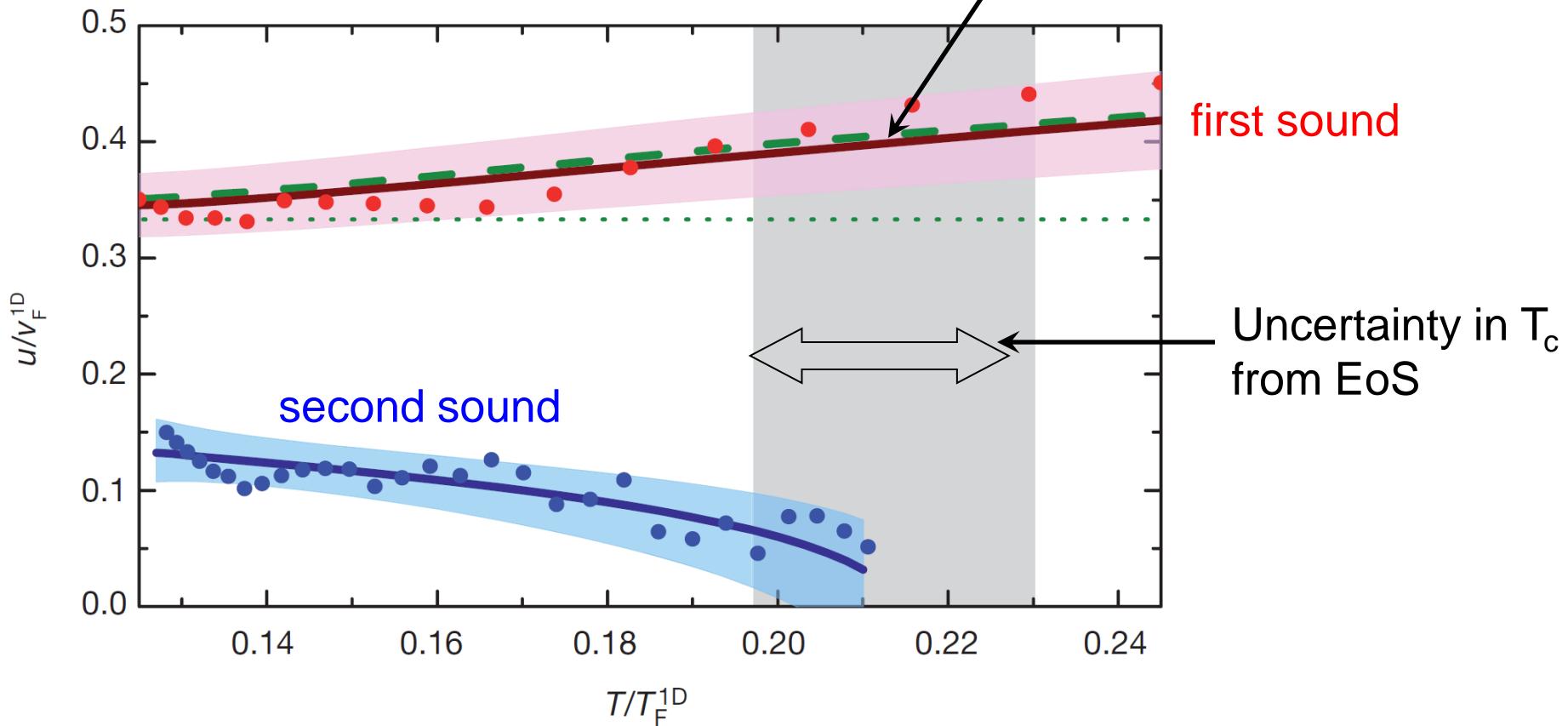


Temperature dependence for free!

# Normalized speeds of the first and second sound

Calculation based on 1D hydrodynamics model for the first sound

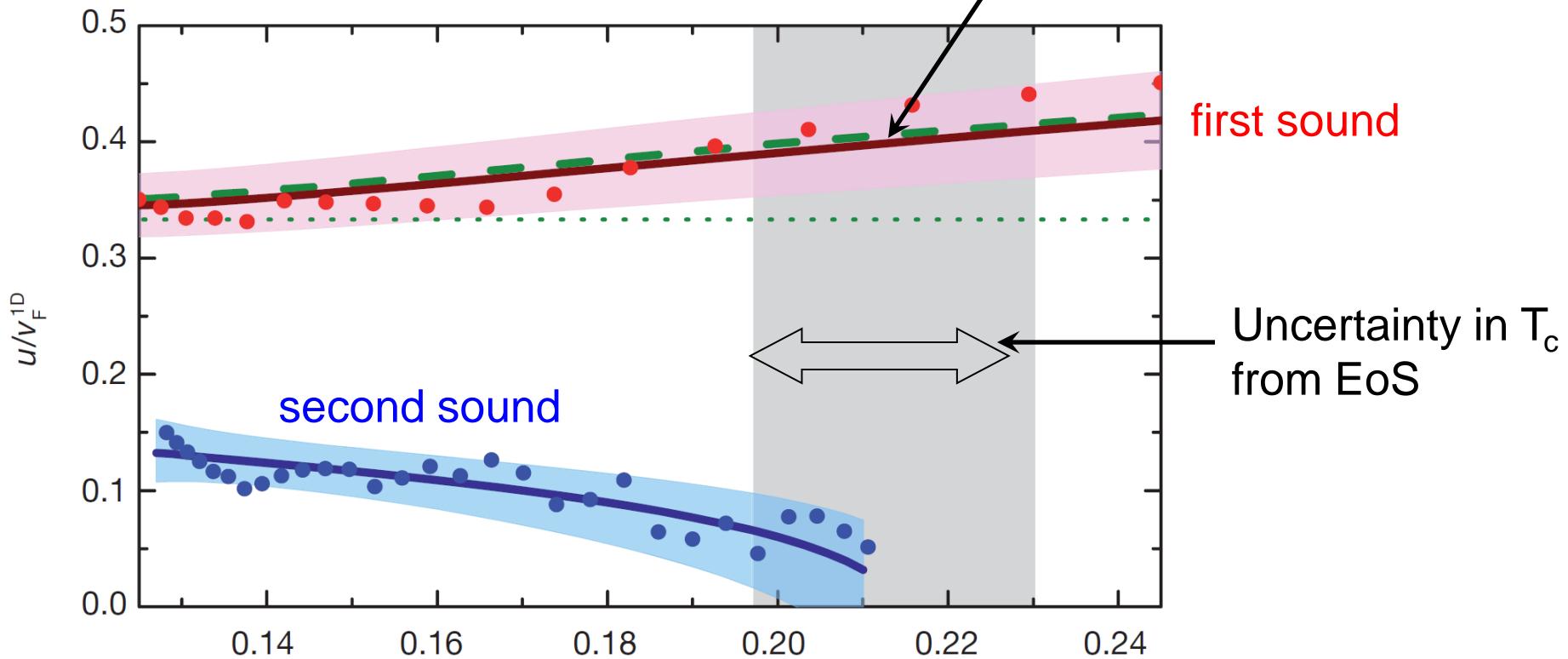
$$\frac{u_1}{v_F^{1D}} = \sqrt{\frac{7}{10} \frac{P_1}{n_1 k_B T_F^{1D}}}$$



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Calculation based on 1D hydrodynamics model for the first sound

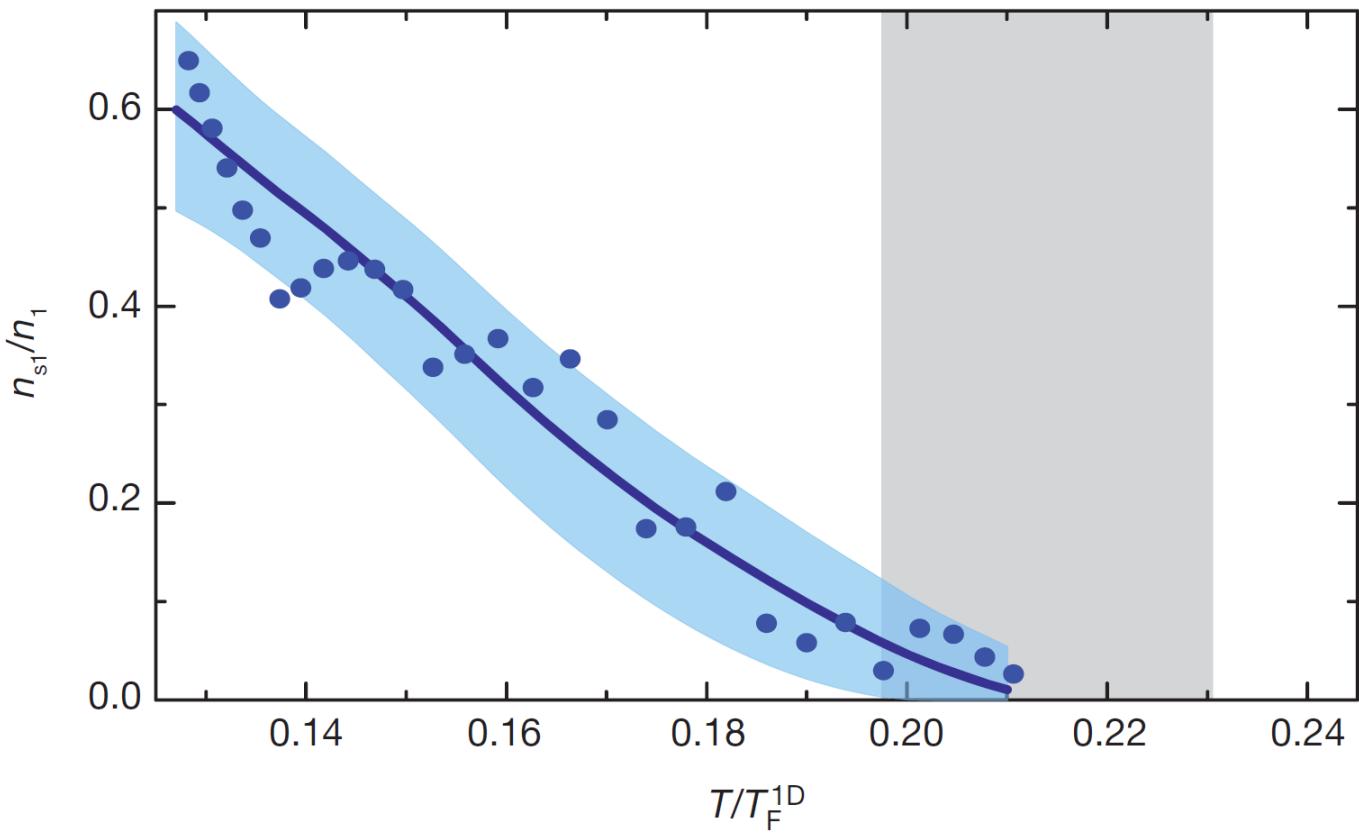
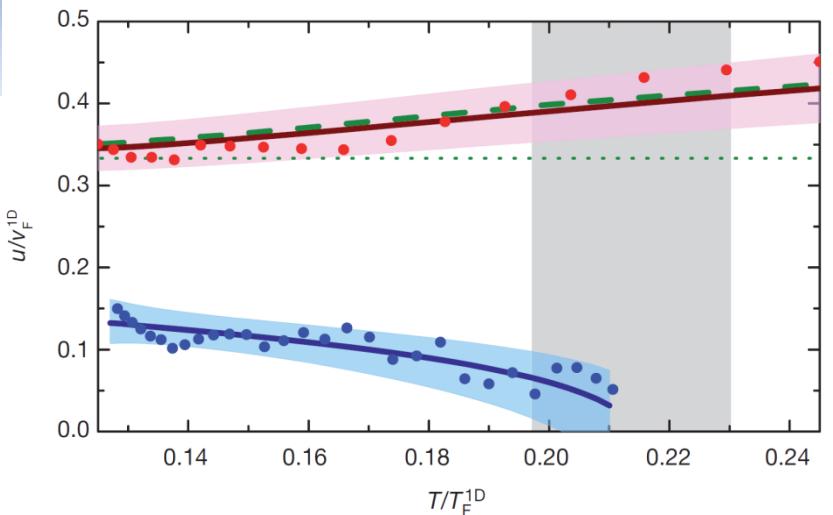
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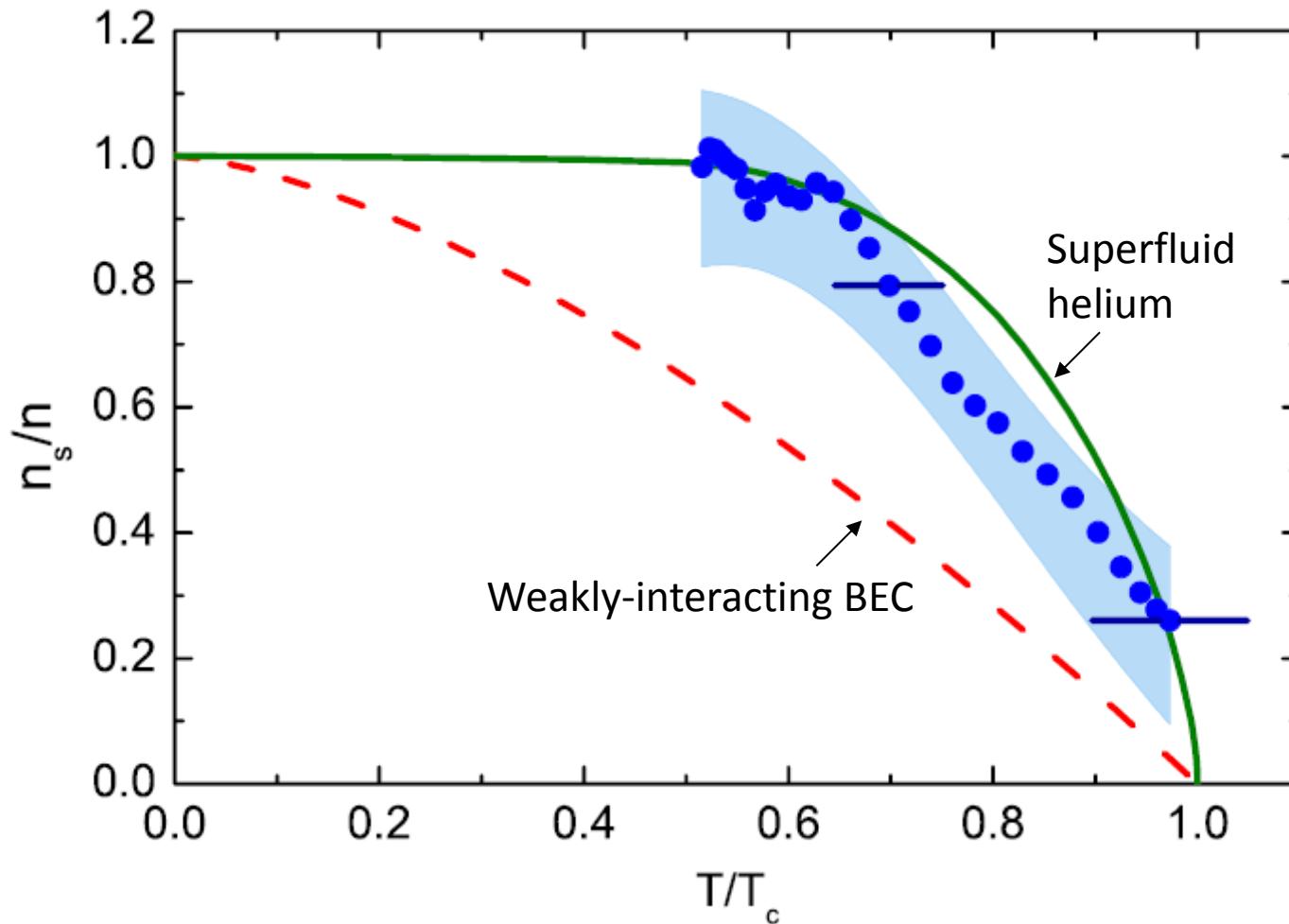
$$\frac{u_2}{v_F^{1D}} = \sqrt{\frac{T}{2k_B T_F^{1D}} \frac{\bar{s}_1^2}{\bar{c}_{p1}} \frac{n_{s1}}{n_{n1}}} \quad T/T_F^{1D}$$

# 1D superfluid fraction

$$\frac{u_2}{v_F^{1D}} = \sqrt{\frac{T}{2k_B T_F^{1D}} \frac{\bar{s}_1^2}{\bar{c}_{p1}} \frac{n_{s1}}{n_{n1}}}$$

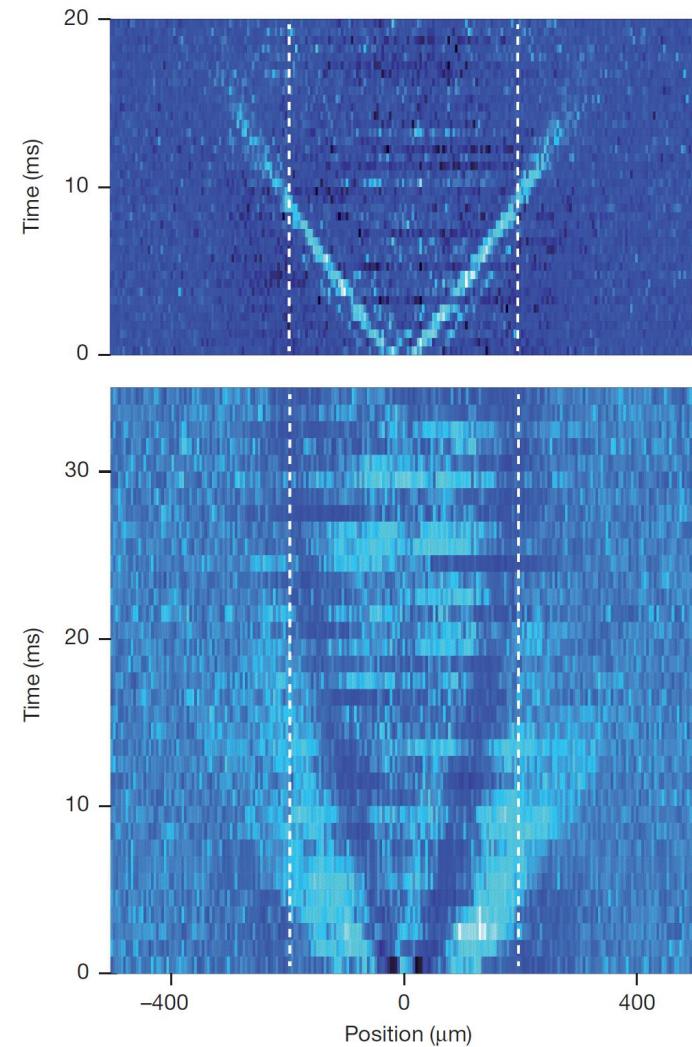
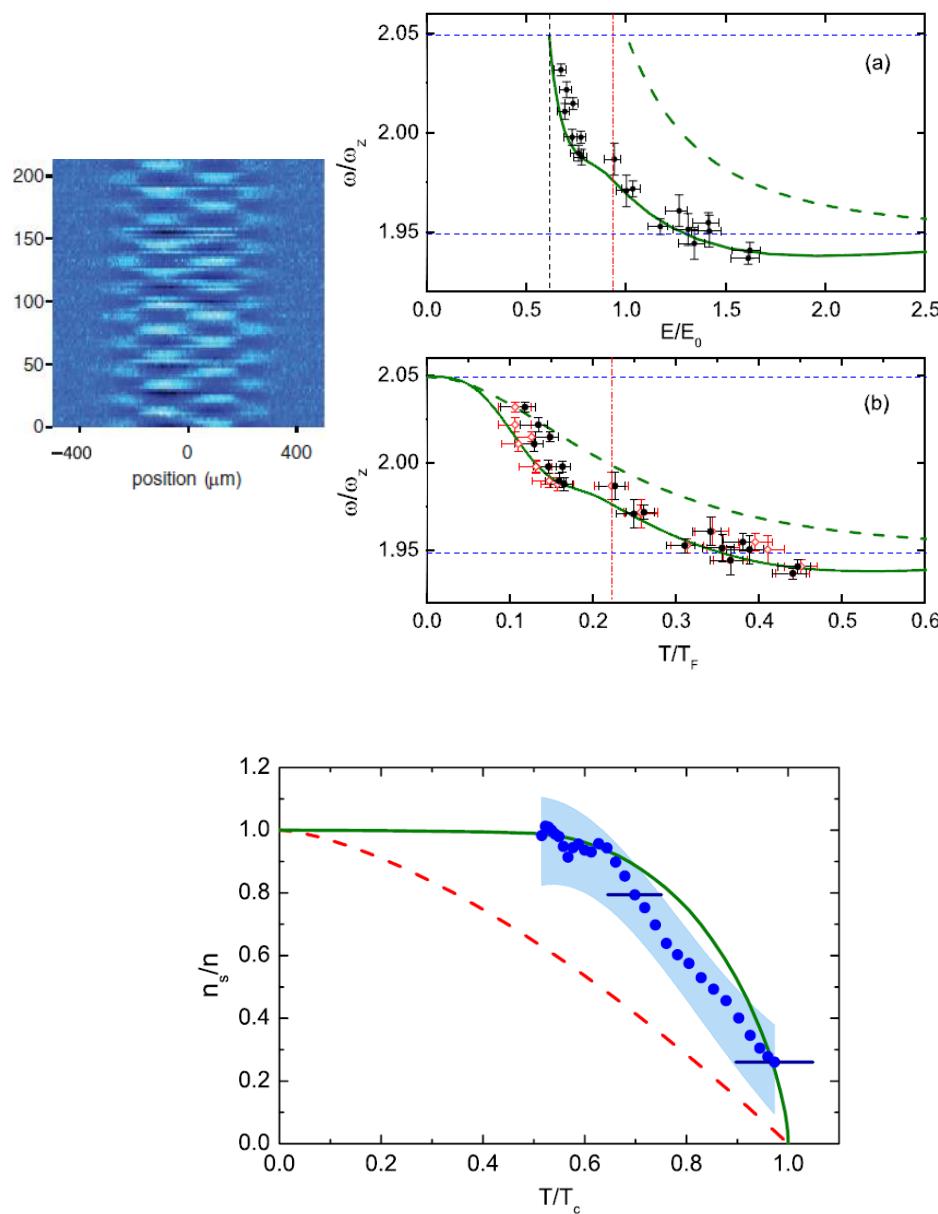


# Superfluid fraction in the UNIFORM system



$$\frac{n_s}{n} = \frac{f_{ns}(x_0)}{f_n(x_0)} = \frac{1}{f_n(x_0)} \frac{d}{dx_0} \left[ \frac{n_{s1}}{n_1} \int_{-\infty}^{x_0} f_n(x) dx \right]$$

# Conclusions



- Sánchez Guajardo et al., PRA 87, 063601 (2013)  
Yan-hua Hou et al., Phys. Rev. A 88, 043630 (2014)  
M.K. Tey et al., PRL 110, 055303 (2013)  
Sidorenkov et al., Nature 498, 78 (2013)

**Thank you  
for your attention.**