The Superfluid Mass Density and the Landau Criterion for Superfluidity

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Multi-Condensate Workshop on Probing and Understanding Exotic Superconductors and Superfluids

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CERN Courier Archive: 1968

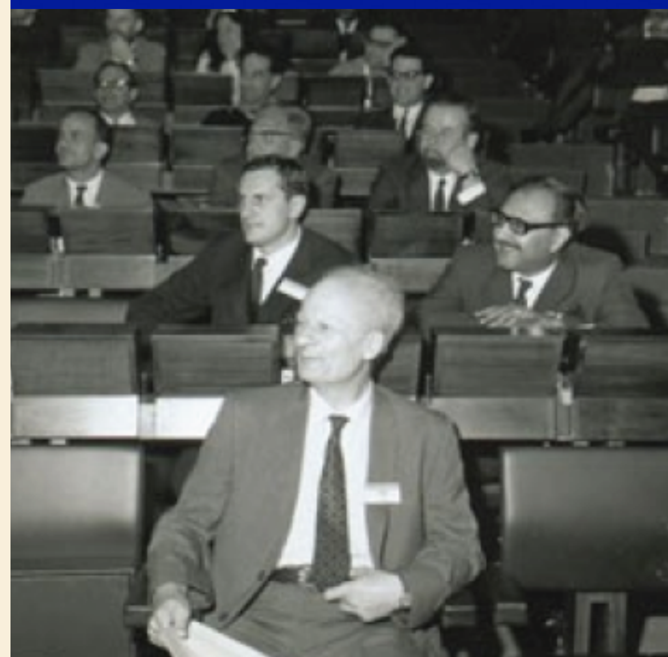
A look back to CERN Courier vol. 8, July/August 1968, compiled by Peggie Rimmer

NEWS FROM ABROAD

International Centre for Theoretical Physics

During the three weeks of 7–29 June, about 350 leading physicists from countries throughout the world gathered at the International Centre for Theoretical Physics (ICTP) in Trieste, Italy, for a major symposium to inaugurate the fine new building which has just become its permanent home.

Professor Abdus Salam, director of ICTP, described the aim of the meeting: "to review the whole spectrum of modern theoretical physics, to share the insights of different disciplines and to acquire, if possible, a deep sense of the scope and unifying nature of the subject." Among the participants were Nobel prize winners M A Bethe, F C Crick, P A M Dirac, W Heisenberg, C H Townes, T D Lee, J Schwinger and E P Wigner.



Non-zero superfluid mass density is defining characteristic of superfluids:

from liquid helium to superconductors (penetration depth) to nuclei and nuclear matter to atomic condensates

Microscopic definition

Landau criterion for superfluidity is neither necessary nor sufficient:

when violated have superfluid mass density less than total density.
Possible new inhomogeneous superfluid states.

Landau Two-Fluid Model

Can picture superfluids as containing two interpenetrating fluids:

Normal: density $\rho_n(T)$, velocity v_n
Superfluid: density $\rho_s(T)$, velocity v_s

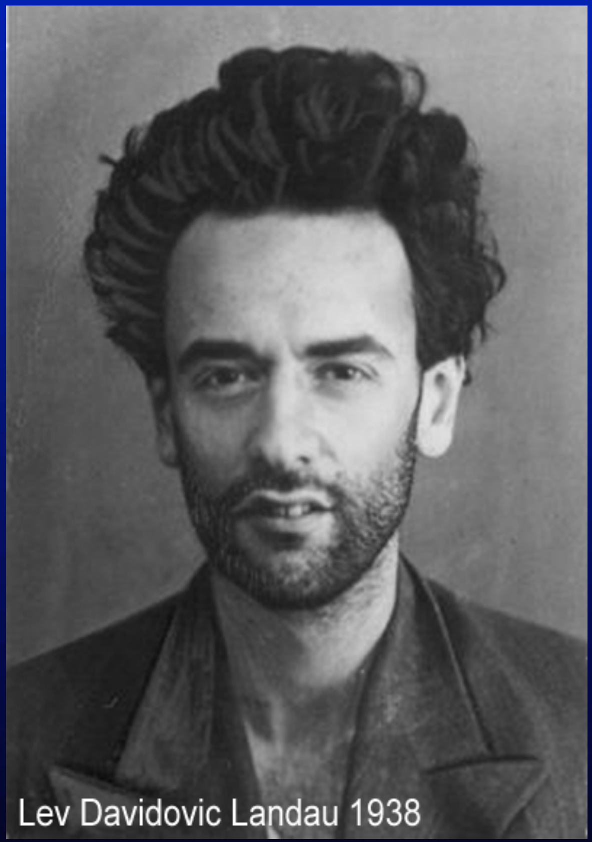
T = temperature

$$\rho = \rho_n(T) + \rho_s(T)$$

Mass current = $\rho_s v_s + \rho_n v_n$

Entropy current = $s v_n$
:carried by normal fluid only

Second sound (collective mode) =
counter-oscillating normal and superfluids

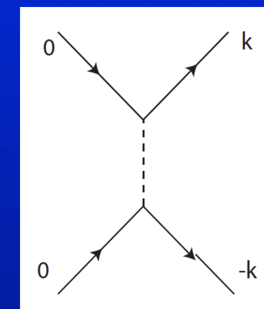


Superfluid density and condensate density are <u>different</u>

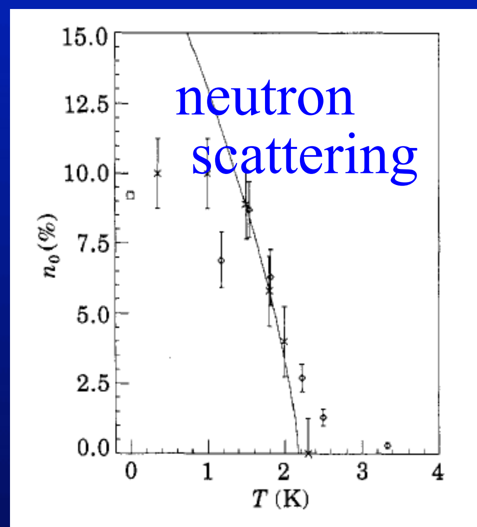
$\psi(\mathbf{x})$ = order parameter; $\mathbf{n_0} = |\psi|^2$ = condensate density

$$\boxed{\rho_\mathbf{s} \neq \mathbf{mn_0}}$$

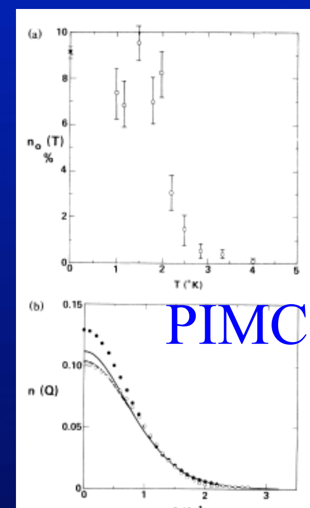
In ground state, interactions drive particles into non-zero momentum single particle states:



In ^{4}He at T=0, $\rho_s/\rho = 1$, while <10% of particles are in condensate



Snow, Wang, & Sokol, Europhys. Lett. 19 (1992)

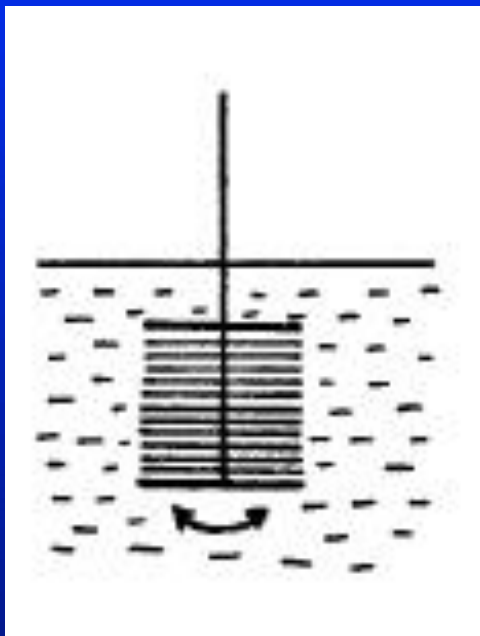


Ceperley & Pollock, PRL 56 (1986)

In superfluid Berezinskii-Kosterlitz-Thouless systems, $n_0 = 0$, and at T_c :

$$\rho_s = \frac{m^2 T_c}{2\pi\eta} \qquad \eta = 1/4$$

Try to rotate superfluid slowly. Normal fluid component rotates, but superfluid component stays put. Reduced moment of inertia.



E.L. Andronikashvili, J. Physics, USSR, 1946

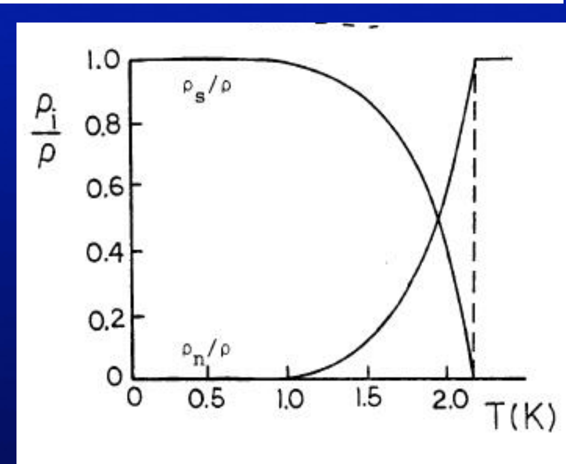
Androniskashvili experiment in He-II – with stack of closely spaced disks oscillating back and forth – measure how much fluid rotates

Moment of inertia

$$I = I_{\text{disk}} + I_{\text{fluid}}$$

Measure resonant frequency, and deduce I_{fluid} from

$$I\frac{d^2\theta}{dt^2} = -k\theta$$



Penetration depth in superconductors: Meissner effect analog of reduced moment of inertia:

$$\frac{1}{\Lambda^2} = \frac{4\pi n e^2}{mc^2}\frac{\rho_s}{\rho}$$

Accurate measurements in He-II up to the Lambda point

J.R. Clow and J.D. Reppy, PRL 1964

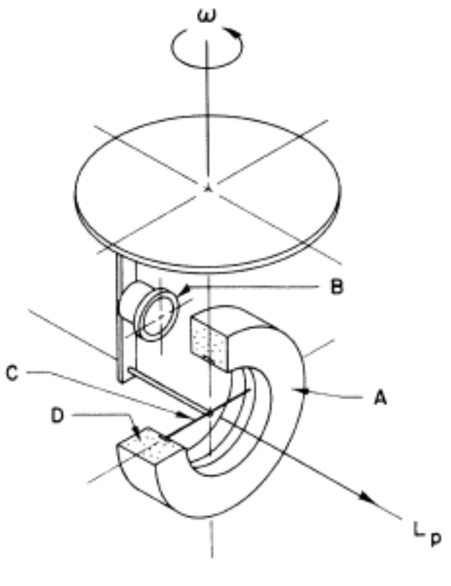
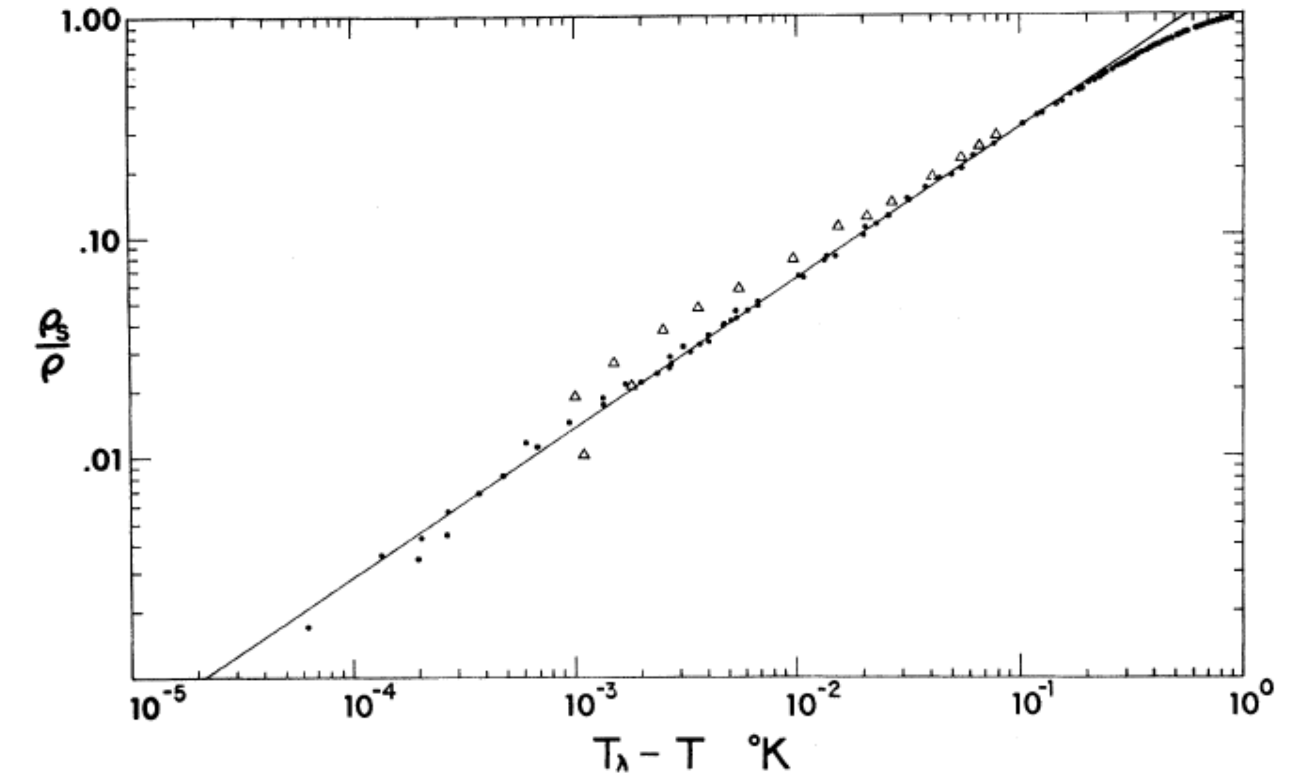


FIG. 1. Schematic of apparatus. The persistent current with angular momentum $\vec{L}_p$ is formed in the annular container *A*. The container dimensions are 5.0 cm o.d., 3.0 cm i.d., and 1.5 cm long. It is filled with fibrous foam, *D*. During rotation $\vec{\omega}$, about the vertical axis, the container deflects against a 2-mil tungsten fiber, *C*. This deflection is sensed by an oscillator tank coil, *B*.



Near T_λ have critical scaling behavior:

$$\frac{\rho_s}{\rho} \sim (T_\lambda - T)^{\alpha}$$

$$\alpha = 0.67 \pm 0.03$$

Determination of ρ_s from penetration depth in (non-conventional) superconductors

$$\frac{1}{\Lambda^2} = \frac{4\pi n e^2}{mc^2}\frac{\rho_s}{\rho}$$

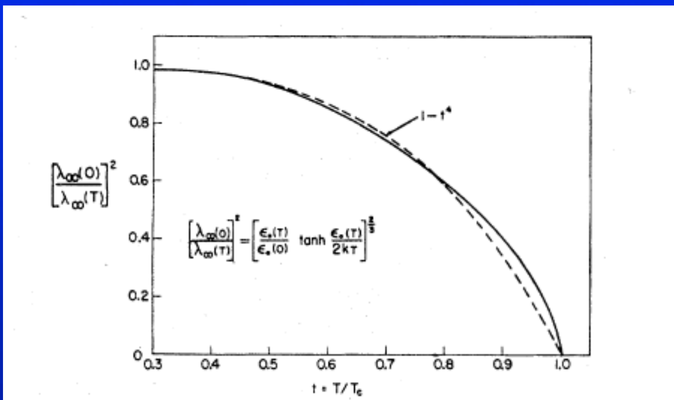
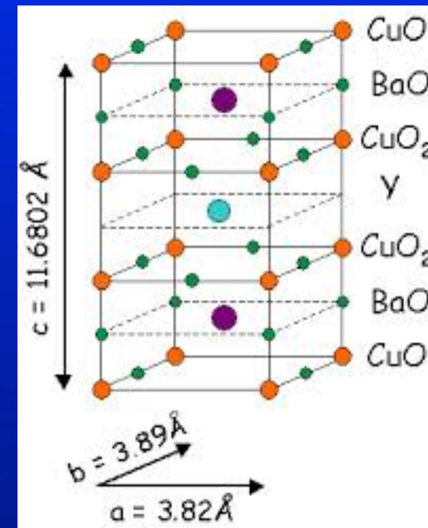


FIG. 6. The temperature variation of the penetration depth, λ_∞, in the infinite coherence distance limit, $(\xi_0/\lambda)\to\infty$, compared with the empirical law, $[\lambda(0)/\lambda(T)]^2 = 1-t^4$.

ρ_s in original BCS paper

R. Prozorov & R.W. Gianetta, Supercond. Sci. Technol. 19, R41 (2006).



YBCO

ρ_s has tensor structure, diagonalized along a, b, c axes.

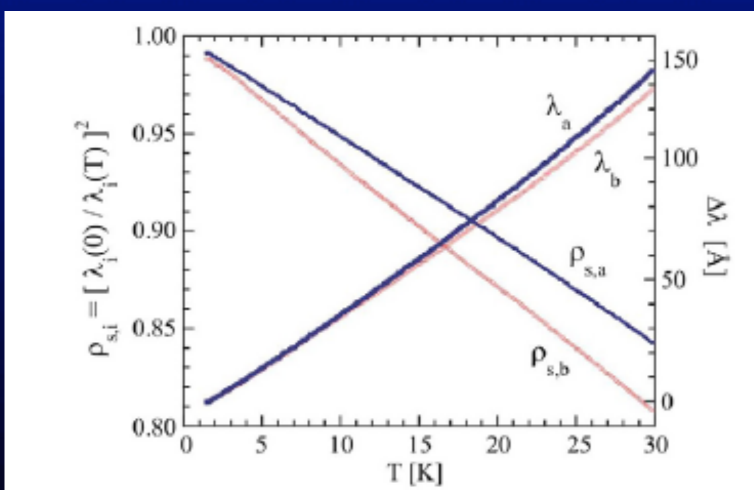


Figure 5. Superfluid density and penetration depth versus temperature in optimally doped YBCO crystal. Data are shown for both *a* and *b* directions within the conducting plane [46].

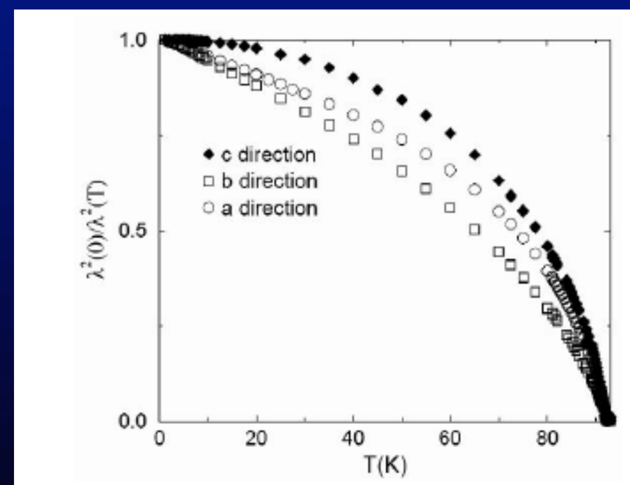


Figure 23. Comparison of *c*-axis superfluid density and *a*- and *b*-axis response in YBCO crystal clearly showing the change from linear for *a*- and *b*-axis to quadratic variation the for *c*-axis. Figure reprinted with permission from [94]. Copyright 1998 by the American Physical Society.

Moment of inertia of superfluid

Reduction of moment of inertia due to condensation

= analog of Meissner effect.

$$I = \frac{\rho_n}{\rho} I_{classical}$$

Rotational spectra of nuclei:

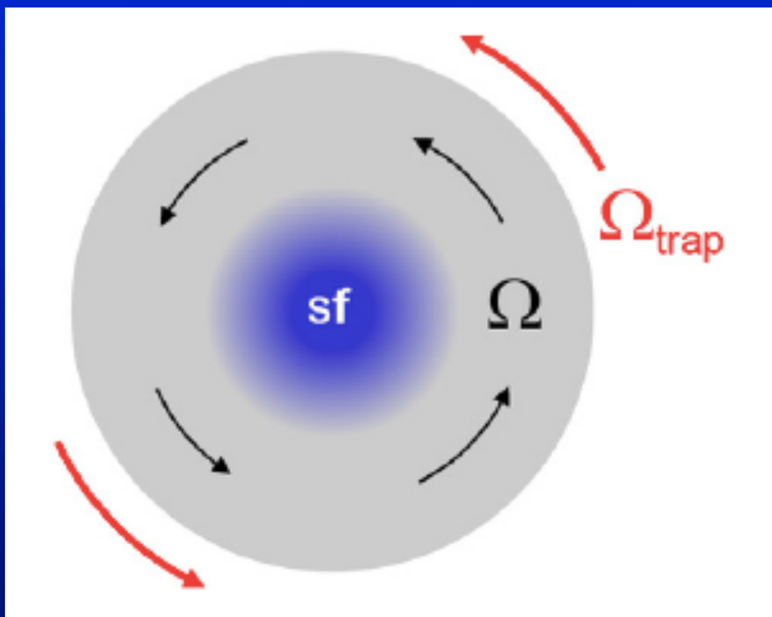
E = J(J+1)/ 2I,

(J=angular momentum) indicate moment of inertia, I, reduced from rigid body value, I_{cl} . Migdal (1959). BCS pairing.

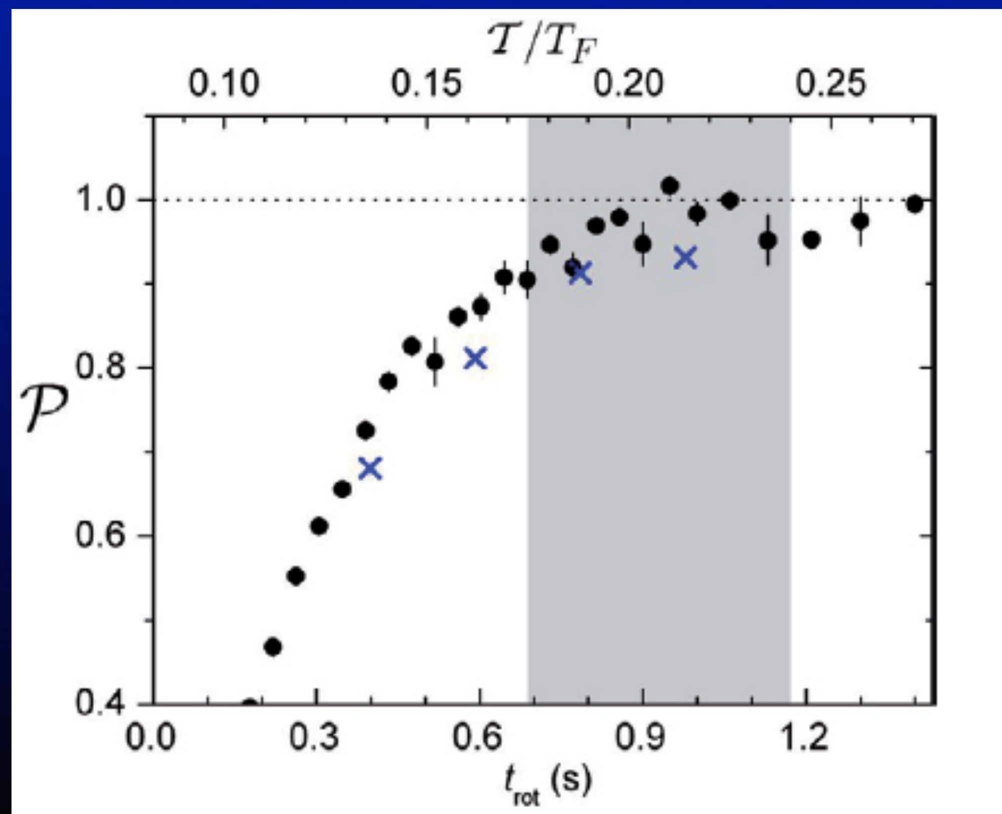
Element	β [7]	x_p	x_n	$\left(\frac{\mathscr{I}}{\mathscr{I}_0}\right)_{rect.}$	$\left(\frac{\mathscr{I}}{\mathscr{I}_0}\right)_{osc.}$	$\left(\frac{\mathscr{I}}{\mathscr{I}_0}\right)_{exper.}$ [7]
Nd^{150}	0.26	0.54	0.94	0.15	0.38	0.35
Sm^{152}	0.24	0.65	1.02	0.17	0.43	0.38
Gd^{154}	0.26	0.52	0.88	0.13	0.35	0.36
Gd^{156}	0.33	0.87	1.37	0.22	0.57	0.48
Gd^{157}	0.29	0.93	1.60	0.22	0.64	0.60
Dy^{162}	0.30	0.84	1.43	0.23	0.57	0.50
Hf^{179}	0.20	0.99	1.75	0.27	0.66	0.52
Os^{186}	0.18	0.44	0.69	0.09	0.26	0.28
Th^{230}	0.22	0.63	0.95	0.15	0.40	0.43
Th^{232}	0.22	0.84	1.42	0.24	0.60	0.44
U^{238}	0.24	0.83	1.29	0.22	0.54	0.43

S Riedl, E R Sánchez Guajardo, C Kohstall, J Hecker Denschlag, and R Grimm, New J. Physics 13, 035003 (2011)

^{6}Li at unitarity

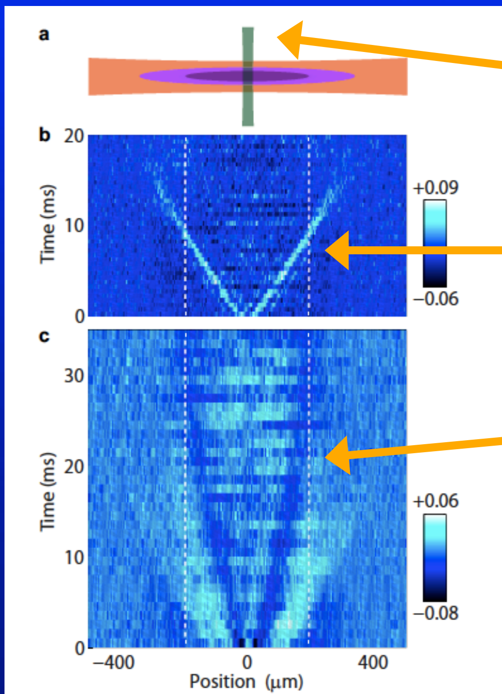


$$\mathcal{P} = \frac{\mathbf{I}}{\mathbf{I_{cl}}} \frac{\mathbf{\Omega}}{\mathbf{\Omega_{trap}}} \simeq \frac{\rho_{\mathbf{n}}}{\rho}$$



Second sound & superfluid mass density in a unitary Fermi gas

Sidorenkov, Tey, Grimm, Hou, Pitaevskii, & Stringari, Nature 498, 78 (2013)

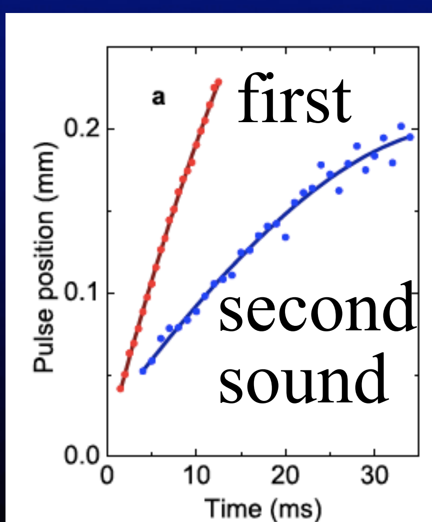


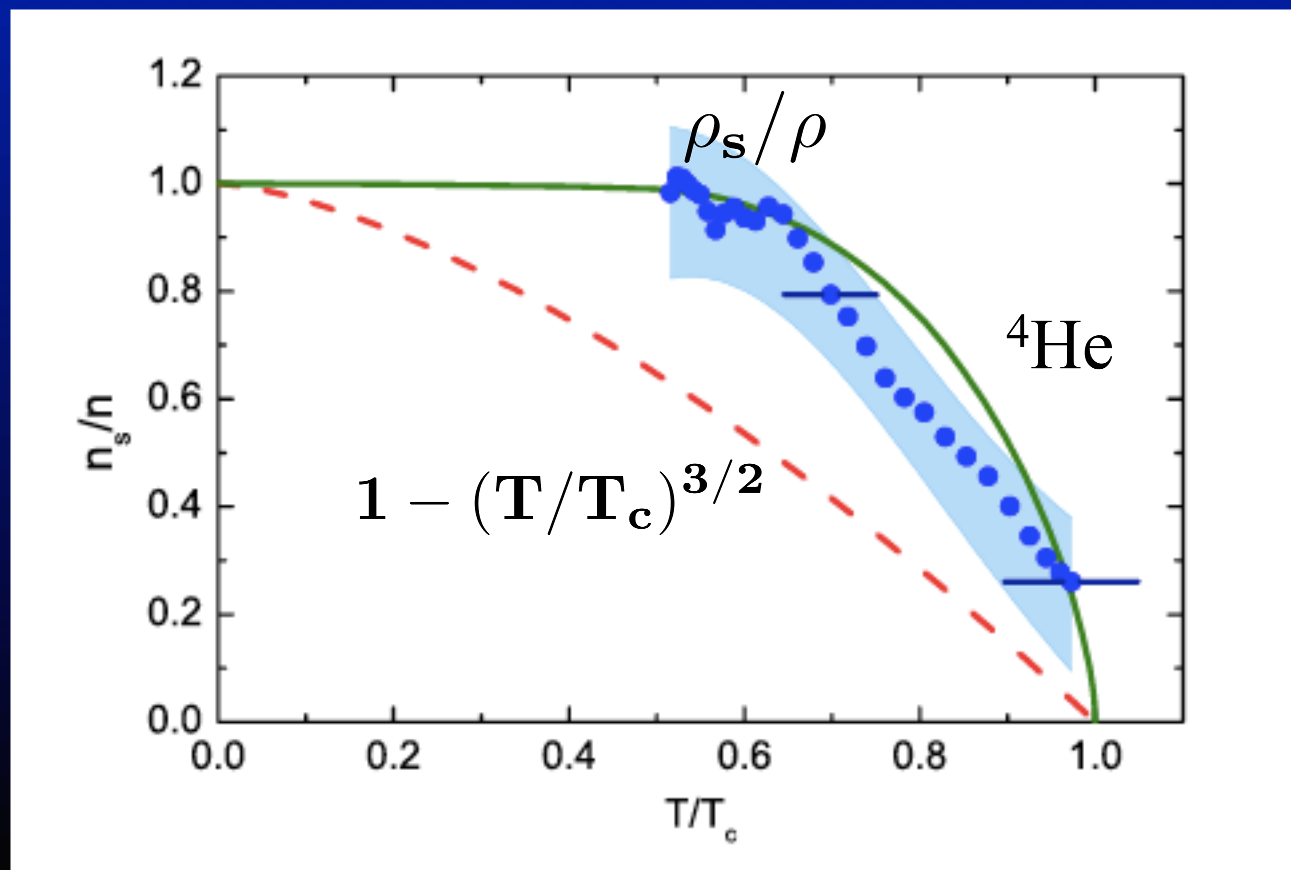
Blue shifted laser

^{6}Li

first sound

second sound -- velocity $\mathbf{s_2} \sim (\rho_\mathbf{s}/\rho_\mathbf{n})^{1/2}$





Microscopic definition of ρ_s

Superfluid flowing down a pipe (at rest) with superfluid velocity v_s in z direction has free energy density:

$$F(v_s, T, \mu) = F(0, T, \mu) + \frac{1}{2}\rho_s v_s^2$$

Differentiate partition function: $\mathbf{Z} = \mathrm{Tr}e^{-\beta(\mathbf{H} + \tilde{\mathbf{P}}\cdot\tilde{\mathbf{v}}_s + \frac{1}{2}\mathbf{M}\mathbf{v}_s^2)}$

$\partial \mathbf{F}/\partial \mathbf{v_s} = \langle \mathbf{P_z} + \mathbf{M}\mathbf{v_s}\rangle/\mathbf{V}$ where P_z is the total momentum in the frame in which the superfluid is at rest.

$\partial^2 \mathbf{F}/\partial \mathbf{v_s^2} = -\beta\langle \mathbf{P_z^2}\rangle/\mathbf{V} + \rho$ so that $\boxed{\rho_{\mathbf{n}} = \beta\langle \mathbf{P_z^2}\rangle/\mathbf{V}}$

In normal state, total momentum is Gaussianly distributed:
$\propto \mathbf{e}^{-\beta\mathbf{P_z^2}/2\mathbf{M}}$ so that $\beta\langle \mathbf{P_z^2}\rangle/\mathbf{V} = \mathbf{M}/\mathbf{V}$ and $\rho_n = \rho$.
In superfluid phase, the total momentum and v_s are entangled and total momentum distribution is not classical.

Two realizations: Landau's quasiparticle calculation, and in terms of transverse current autocorrelation functions.

Landau calculation of ρ_s for system w. quasiparticles

$$\langle \boldsymbol{P} \rangle = \sum \boldsymbol{p} \langle N_{\boldsymbol{p}} \rangle$$

qp carry the momentum

$$\langle N_{\boldsymbol{p}} \rangle = [e^{\beta\{\varepsilon_{\boldsymbol{p}} + \boldsymbol{p}\cdot(\boldsymbol{v}_s - \boldsymbol{v}_n)\}} - 1]^{-1}$$

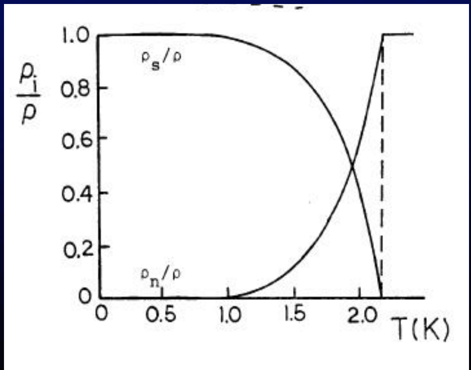
using Galilean invariance

$$\langle \boldsymbol{P} \rangle = -\sum_{\boldsymbol{p}} \boldsymbol{p}(\boldsymbol{p}\cdot\boldsymbol{v}_n) \frac{\partial}{\partial \varepsilon_{\boldsymbol{p}}} \frac{1}{e^{\beta\varepsilon_{\boldsymbol{p}}} - 1} = \rho_{\mathbf{n}} \tilde{\mathbf{v}}_{\mathbf{n}}$$

$$\rho_n = -\int \frac{d\boldsymbol{p}}{(2\pi)^3} \frac{p^2}{3} \frac{\partial}{\partial \varepsilon_{\boldsymbol{p}}} \frac{1}{e^{\beta\varepsilon_{\boldsymbol{p}}} - 1}$$

for phonons (He-II, BEC, ...)

$$\rho_{\mathbf{n}}(\mathrm{T}) = \frac{2\pi^2}{45\hbar^3 \mathbf{s}^5} \mathrm{T}^4$$



Exact representation of ρ_s in terms of transverse current-current correlation functions

$$\Upsilon_{ij}(\mathbf{r},\mathbf{r}',\omega) = \int dt e^{i\omega(t-t')}\langle[j_i(\mathbf{r},t), j_j(\mathbf{r}',t')]\rangle$$

$$\mathbf{j}(\mathbf{r}) = \frac{1}{2im}[\psi^\dagger(\mathbf{r})\nabla\psi(\mathbf{r}) - \nabla\psi^\dagger(\mathbf{r})\psi(\mathbf{r})]$$

Decompose into longitudinal and transverse components:

$$\Upsilon_{ij}(\mathbf{k},\omega) = \frac{k_i k_j}{k^2}\Upsilon_L(k,\omega) + \left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)\Upsilon_T(k,\omega).$$

f-sum rule =>

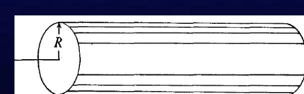
$$\rho = \lim_{k\to 0} m^2 \int \frac{d\omega}{2\pi}\frac{\Upsilon_L(k,\omega)}{\omega}$$

motion in tube with closed ends

Define normal mass density

$$\rho_n = \lim_{k\to 0} m^2 \int \frac{d\omega}{2\pi}\frac{\Upsilon_T(k,\omega)}{\omega}$$

motion in tube with open ends



$\rho_s = \rho - \rho_n$

$\rho_n < \rho$ is basic microscopic characterization of superfluid

Exact relation between ρ_s and the condensate density, via the single particle Green's function

P.C. Hohenberg & P.C. Martin, PRL 22 (1963); B.D. Josephson, PL21 (1966); GB, St. Andrews lectures (1967); A. Griffin PR B30 (1984); M. Holzmann & GB, Phys. Rev. B 76, 092502 (2007)

$$G(k,z) = -i\langle T(\psi^\dagger\psi)\rangle(k,z)$$

z = complex frequency

$$\boxed{\mathbf{G}(\mathbf{k}\to 0, \mathbf{z}=0) \to -\frac{\mathbf{n_0 m^2}}{\rho_\mathbf{s}\mathbf{k^2}}}$$

Ex., in Bogoliubov mean field (n_0=n),

$$\mathbf{G}(\mathbf{k},\mathbf{z}) = \frac{\mathbf{z}+\mathbf{gn}+\mathbf{k^2}/2\mathbf{m}}{\mathbf{z^2}-\mathbf{gnk^2}/\mathbf{m}-\mathbf{k^4}/4\mathbf{m^2}}$$

$$\rho_s = -\lim_{k\to 0}\frac{n_0 m^2}{k^2 G(k,0)} \qquad \Rightarrow \qquad \rho_s = nm$$

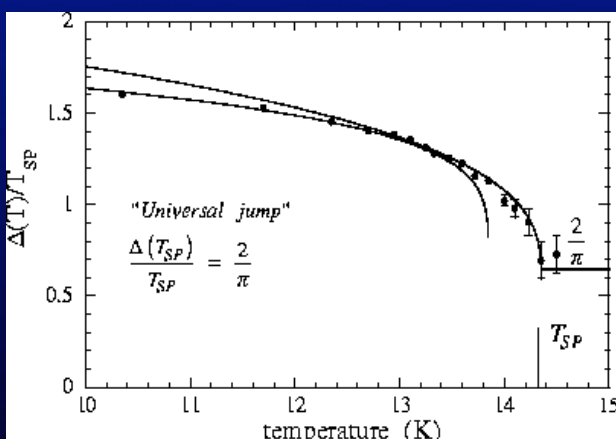
M. Holzmann & GB, PR B 76 (2007);
M. Holzmann, GB, J.-P. Blaizot,
& F. Laloë, PNAS 104 (2007)

Valid in 2D as well as 3D:

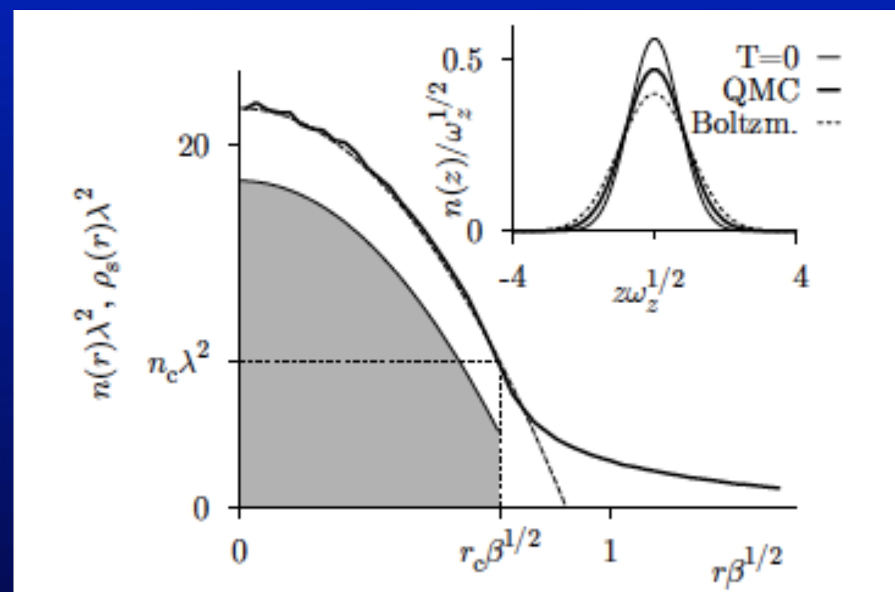
In 2D finite size Berestetskii-Kosterlitz-Thouless system,

$$\mathbf{n_0} \sim \mathbf{1}/(\text{size})^{\eta} \qquad \boldsymbol{G(k,0) \sim 1/k^{2-\eta}}$$

At T_c $\boxed{\boldsymbol{\rho_s = \frac{m^2 T}{2\pi\eta}}}$ $\eta = 1/4$



Order parameter in $CuGeO_3$
Lorenzo et al., EPL. 45 (1999)



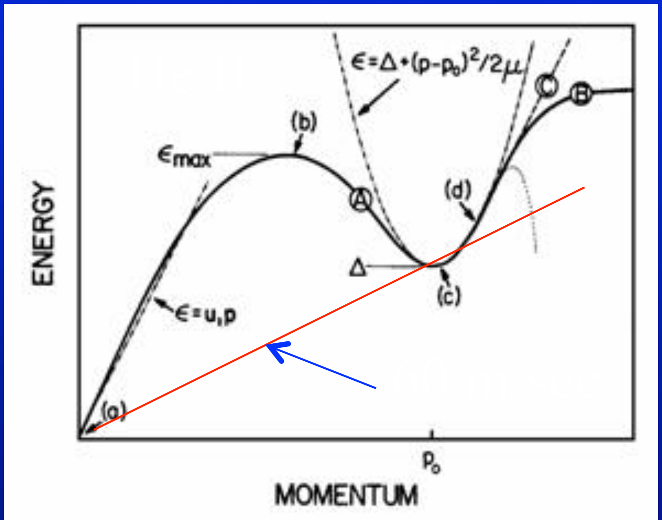
Density profile in 2D trap.
Shaded region ⇔ ρ_s

Holzmann & Krauth, EPL 82 (2008)

GB & Holzmann (2014)

The Landau criterion for superfluidity

Superfluid with elementary excitation spectrum $\varepsilon(q)$



Fluid flowing in pipe in x direction, velocity v with respect to walls. In wall frame excitation energy is

$$\varepsilon_v(q) = \varepsilon(q) + vq_x$$

According to Landau:

For $v < \varepsilon(q)/q$ cannot make spontaneous excitations (which would decay superflow) and *flow is superfluid*.

For v opposite to q_x and $v > \varepsilon(q)/q$ have $\varepsilon_v(q) < 0$
Can then make excitations spontaneously, and *superfluidity ceases*. v_{crit} = 60 m/sec in superfluid He.

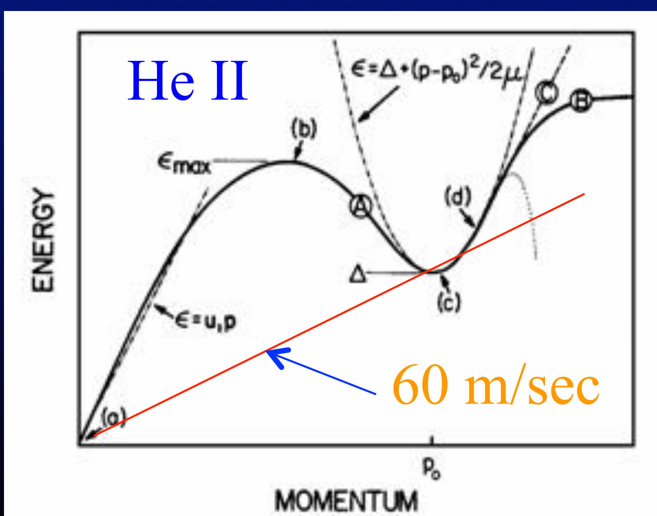
In this way we see that neither phonons nor rotons can be excited if the velocity of flow in helium II is not too large. This means that the flow of the liquid does not slow down, i.e. helium II discloses the phenomenon of superfluidity †.

SUFFICIENT

It must be remarked that already the reasons given above are enough to make the superfluidity vanish at sufficiently large velocities. We leave aside the question as to whether superfluidity disappears at smaller velocities for some other reason (the velocity limit obtained from (4.2) is large—the velocity of sound in helium equals 250 m/sec; (4.4) gives a value only several times lower).

NECESSARY

L.D. Landau, J. Phys. USSR 5, 71 (1941)



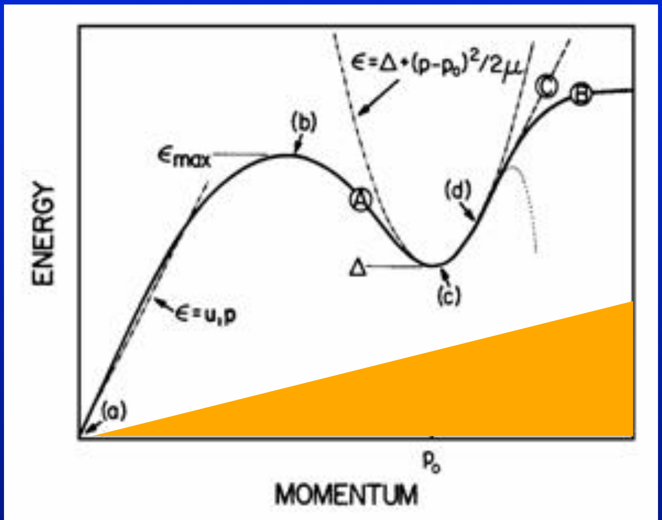
At Landau critical velocity, group and phase velocity of excitations are equal:

$$\frac{\partial \varepsilon}{\partial q} = \frac{\varepsilon}{q}$$

The Landau criterion is neither necessary nor sufficient

Superfluid systems with no "gap":

1) Dilute solutions of degenerate ^{3}He in superfluid ^{4}He:



Particle-hole spectrum

$$\omega = (\vec{p} + \vec{q}\,)^2/2m - \vec{p}^{\,2}/2m$$

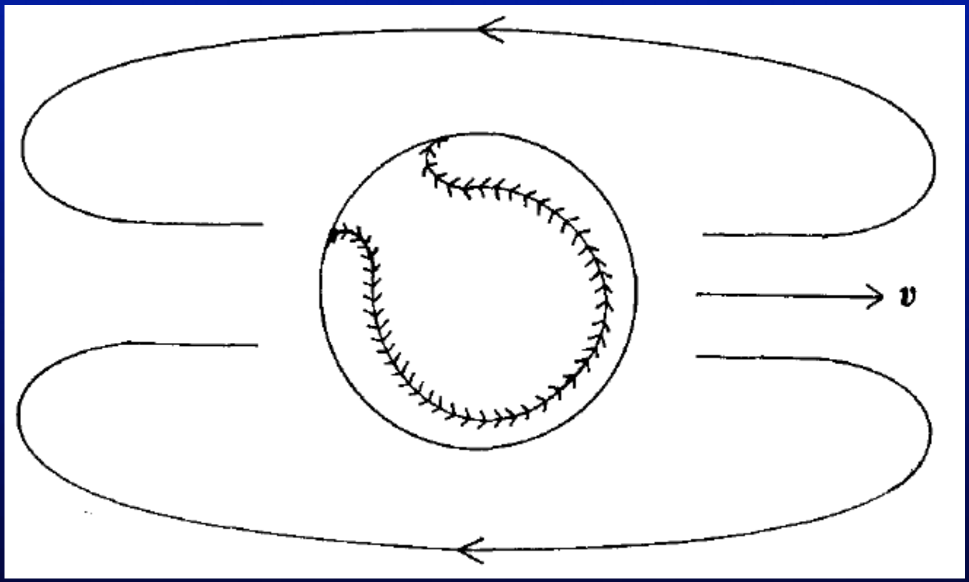
reaches down to $\omega = 0$ at $q \neq 0$.

Landau critical velocity vanishes, but system is superfluid.

2) Superfluid ^{4}He at non-zero temperature: Can scatter a phonon of momentum k to –k with zero energy change. Again Landau critical velocity vanishes, but system remains a perfectly good superfluid.

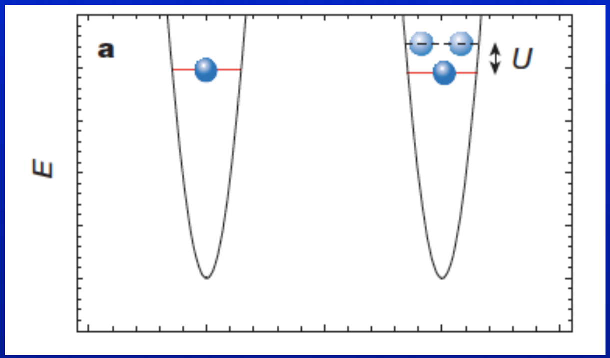
Landau criterion well describes an object travelling through a superfluid (e.g, a neutron).

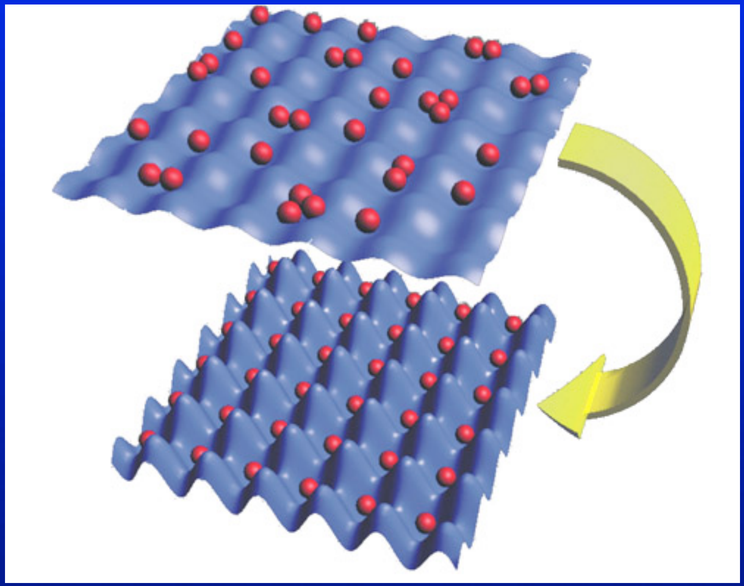
The object will not excite excitations and nor experience drag until it reaches the Landau critical velocity.



Gap also not sufficient to guarantee superfluidity:

ex. bosons in optical lattice:

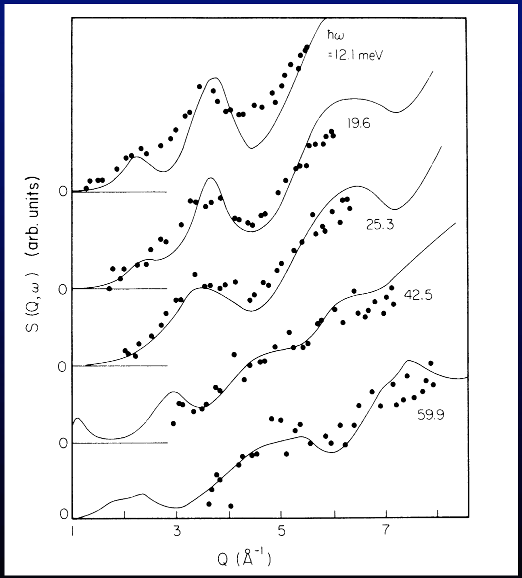




superfluid

Mott insulator

Amorphous solids, e.g., Si doped with H, not superfluid.



What happens when the Landau criterion is violated?

The superfluid mass density becomes less than the total mass density. It does not necessarily vanish!

In dilute solutions of ^{3}He in superfluid ^{4}He,

m^* = ^{3}He effective mass, m_3 = bare mass

$$\rho_s = \rho - (m^* - m_3)n_3$$

In superfluid ^{4}He at nonzero temperature,

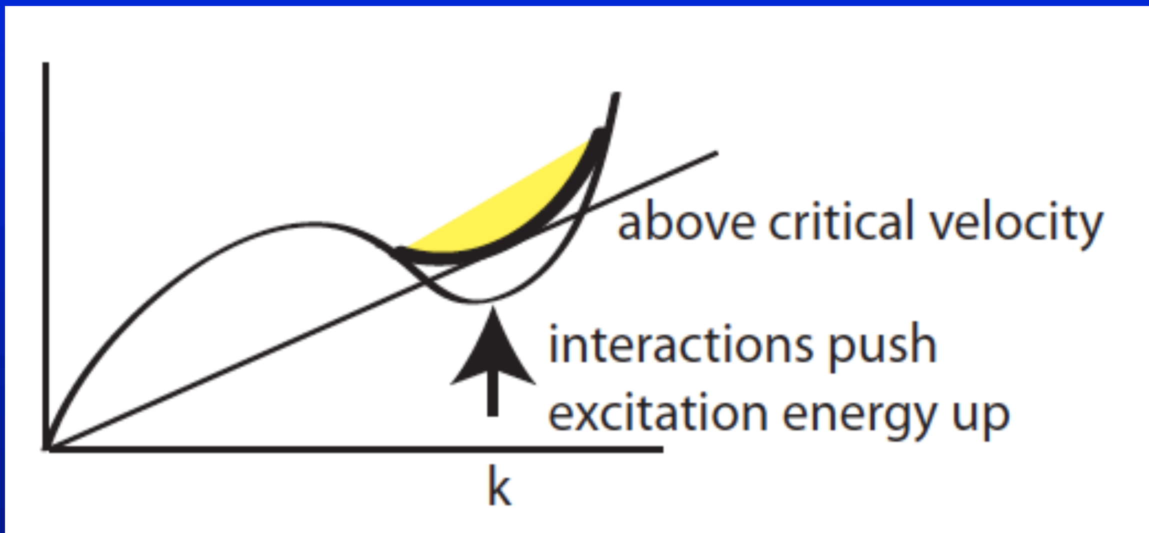
$$\rho_s = \rho - aT^4 - \cdots$$

Formation of non-uniform states, which remain superfluid

Formation of non-uniform states

L. Pitaevskii, JLTP 87, 127 (1992), GB & CJ Pethick, PRA 86 023602 (2012)

L. P. Pitaevskii, Pis'ma Zh. Eksp. Teor. Fiz. 39, 423 (1984);
GB & CJ Pethick, PRA86, 023602 (2012)



Beyond the critical velocity spontaneously form excitations of finite momentum k.

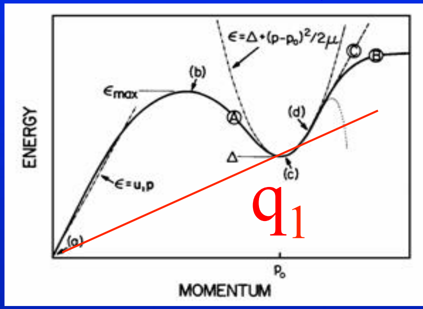
Excitations near the critical momenta are "levons."
Pitaevskii => levon "condensate" in He-II.

Interactions of levons, when repulsive, raises energy of unstable mode k, to make velocity just critical.

Mixing in of modes of momentum k in condensate wave function causes condensate to become non-uniform.

Simple model when Landau critical velocity is exceeded

GB & CJ Pethick, PRA86, 023602 (2012) *(GB and CJ Pethick)*



Weakly interacting Bose gas with finite range interaction g(r), and thus g(q) (> 0) , produces v_{crit} at non-zero $q = q_1$

In Bogoliubov approx:

$$\varepsilon(q) = \left[\frac{ng(q)q^2}{m} + \left(\frac{q^2}{2m}\right)^2\right]^{1/2}$$

Critical point: where group velocity = phase velocity,

$$\frac{d(gn)}{d(q^2/2m)} = -1,$$

Study stability of uniform condensate $\psi(x) = \sqrt{n}e^{iqx}$ for $q>q_1$

Since excitations at q_1 can form spontaneously, generate new condensate of form

$$\boxed{\psi(x) = e^{iqx}\left[\sqrt{n_0} + ue^{-iq_1x} + ve^{iq_1x}\right]}$$

(like periodic array of solitons; here of lower energy!)

Let $u = \zeta\cosh(\phi/2), \qquad v = \zeta\sinh(\phi/2)$

Stable solution $\zeta^2 = (v - v_{crit})\dfrac{q_1}{2G}$ above critical velocity

$$G = \frac{g_1}{4\varepsilon_1^2}\left[g_1 g_2 n^2 - \frac{q_1^2}{m}\left(\frac{q_1^2}{m} + 2g_1 n\right)\right]$$

= effective repulsion of excitations near q_1

$$g_\kappa \equiv g(\kappa q_1)$$

System remains superfluid above the critical velocity but with reduced superfluid mass density:

$$\boxed{\rho_s = mn - mq_1\frac{\partial\zeta^2}{\partial q} = mn - \frac{q_1^2}{2G}}$$

Non-uniform density:

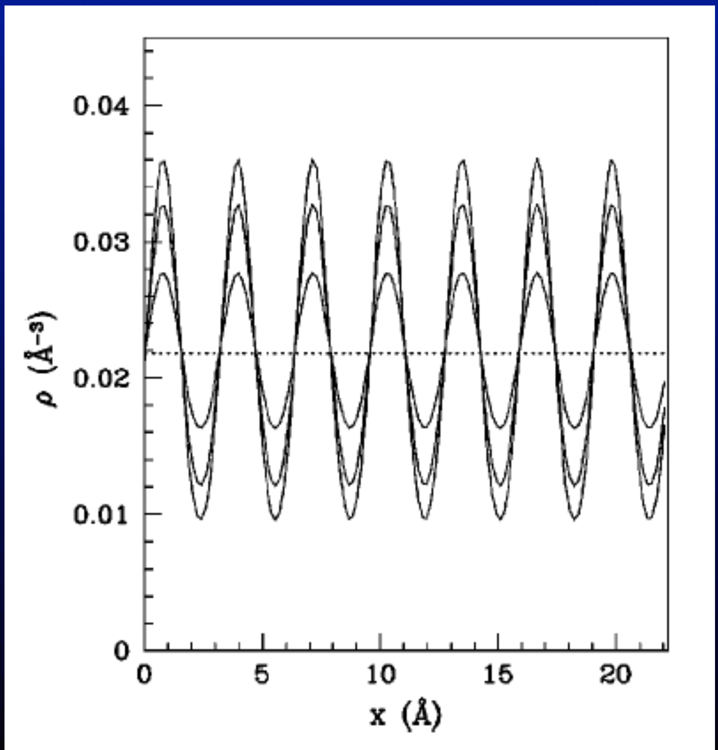
$$\psi(x) = e^{iqx}\left[\sqrt{n_0} + ue^{-iq_1x} + ve^{iq_1x}\right]$$

$$u = \zeta\cosh(\phi/2), \qquad v = \zeta\sinh(\phi/2)$$

$$n(x) = n + 2\sqrt{n_0}\zeta\cosh\phi\cos(q_1x)$$

Density functional simulation of supercritical flow in He-II

Ancilotto, Dalfovo, Pitaevskii, & Toigo, PR B 71, 104530 (2005)



Amplitude of density modulation

$\sim (v - v_c)^{1/2}$

in agreement with model calculation:

$$\zeta^2 = (v - v_{crit})\frac{q_1}{2G}$$

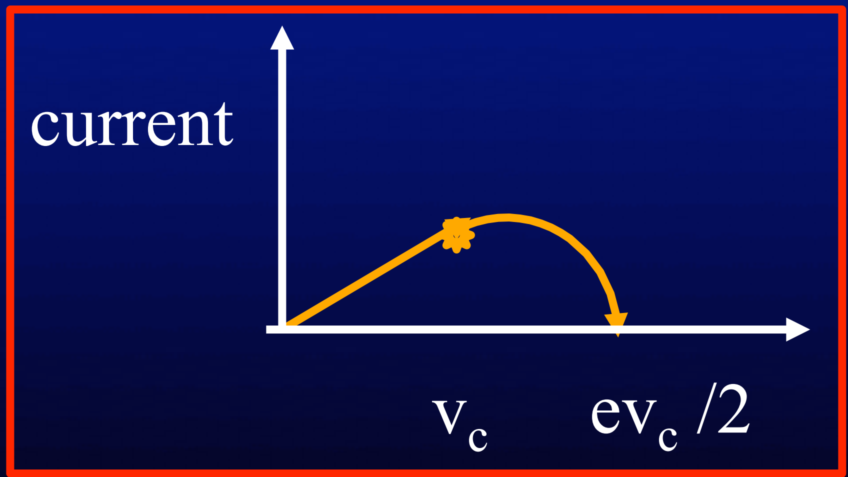
Comparison with BCS superconductors

K.T. Rogers, Univ. Ill. Ph.D. thesis (1960)
J. Bardeen, RMP 34, 667 (1962)

Critical flow velocity = $v_c = \Delta / p_f$

For $v > v_c$ generate quasiparticles. Pauli prevents runaway. No density modulation.

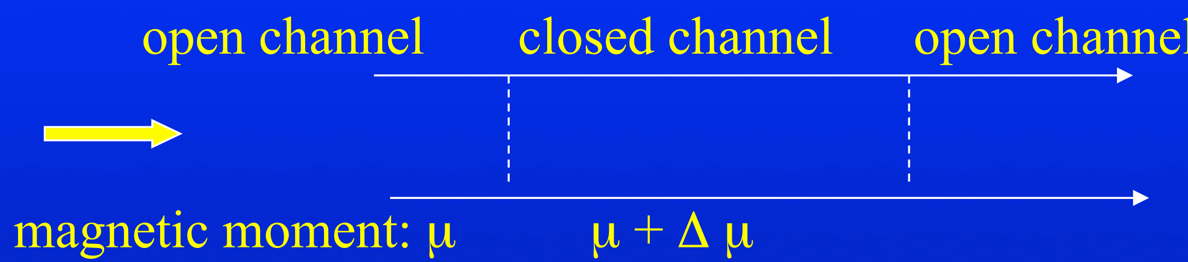
Current increases with flow velocity. Vanishes at $v = (e/2)v_c$ (e=2.718...)



QP creation => normal fluid, even at T=0.

Experimental ways to produce spectrum with a critical velocity

1) with a Feshbach resonance



Effective interaction between particles of momentum p and –p near resonance:

$$g(p)_{\rm res} \simeq \frac{|M|^2}{\Delta E + p^2/4m}$$

ΔE = initial – intermediate energies

$$n\frac{dg(p)}{d(p^2/2m)} \simeq -\frac{n}{2}\frac{|M|^2}{(\Delta E + p^2/4m)^2}$$

can be driven to -1 close to the resonance (ΔE =0)

$$\frac{d(gn)}{d(q^2/2m)} = -1,$$

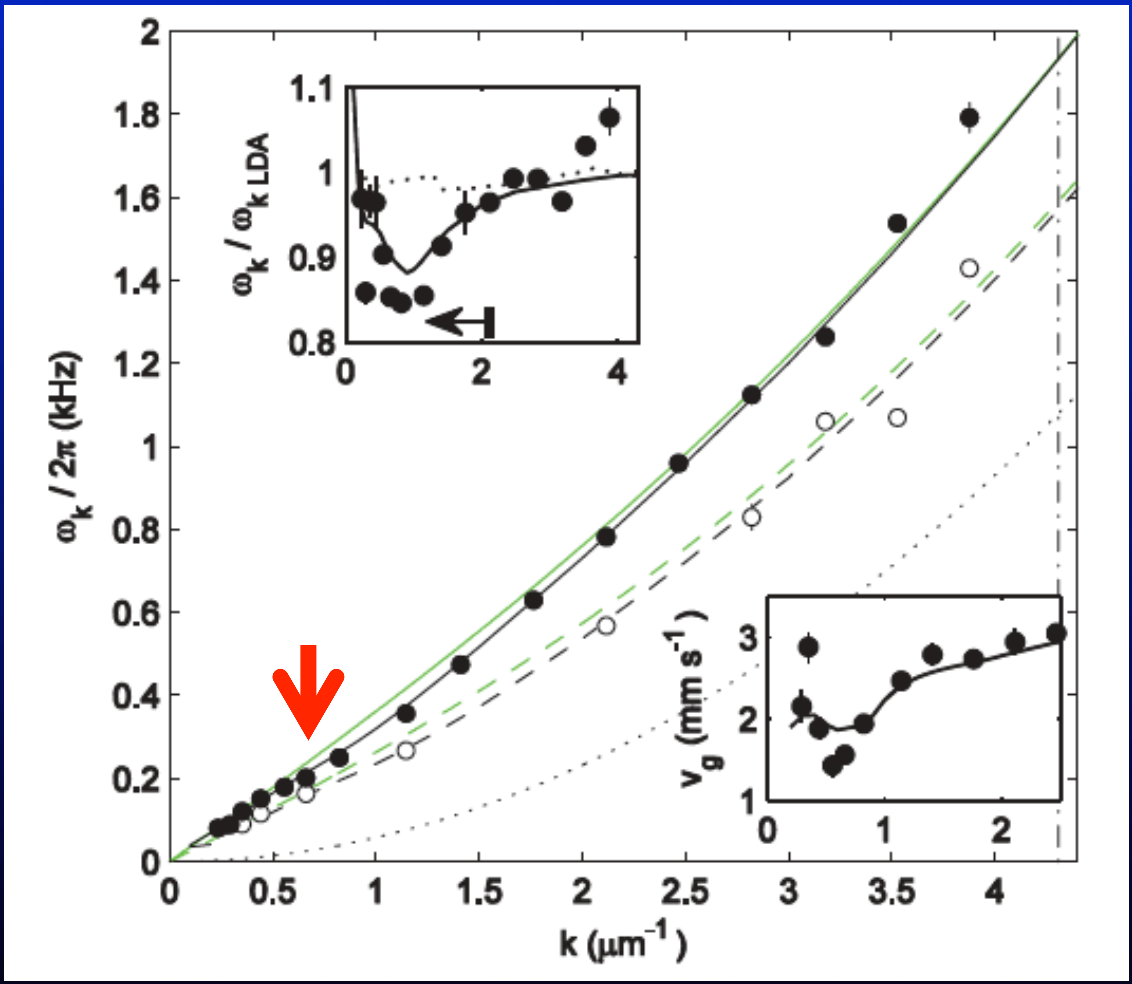
2) short range correlations: $E_p = p^2/(2m\,S(p))$ (à la Feynman)

3) bosons:
in elongated (prolate) traps
with spin-orbit coupling,
or with
shaking of optical lattice

Experimental realization of levons, I

I. Shammass, S. Rinott, A. Berkovitz, R. Schley, and J. Steinhauer, PRL 109, 195301 (2012)

^{87}Rb in prolate trap. $\omega_{\mathbf{c}}/\mathbf{2}\pi = \mathbf{26}$Hz, $\omega_{\perp}/\mathbf{2}\pi = \mathbf{22}$Hz



Short Bragg pulse creates standing wave excitations.

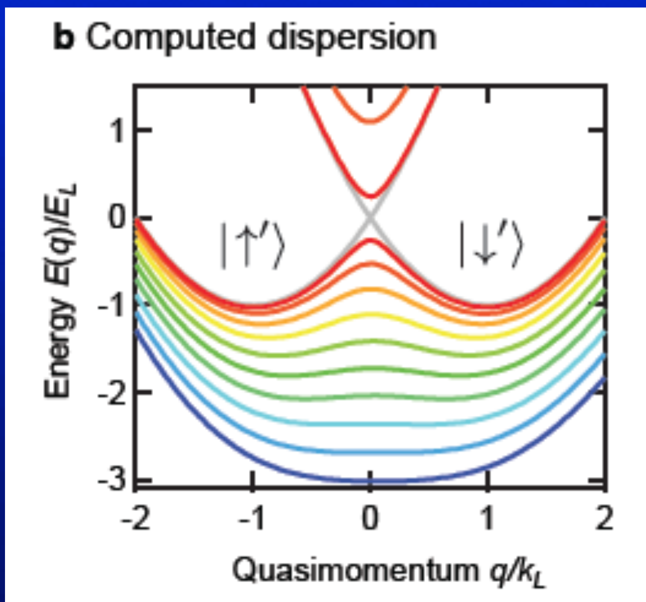
Inflection pt. in excitation spectrum ~ crossover from 1D to 3D behavior.

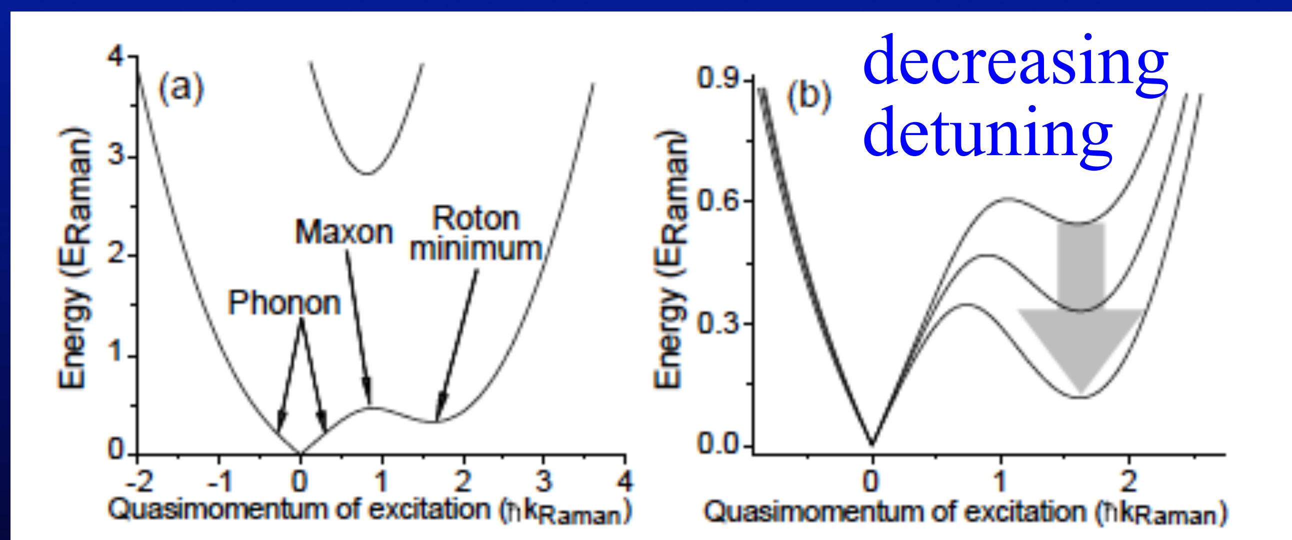
$$\omega/\mathbf{k} = \mathbf{v_c} = \mathbf{1.91mm/sec}$$

Experimental realization of levons, II

M. A. Khamehchi, Y. Zhang, C. Hamner, T. Busch, & P. Engels, arXiv: 1409.5387

^{87}Rb Bose gas with simulated spin-orbit coupling, $\sim p_z \sigma_z$, develops roton minimum in excitation spectrum



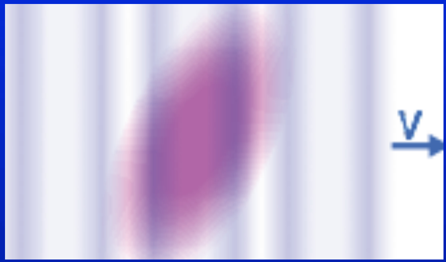


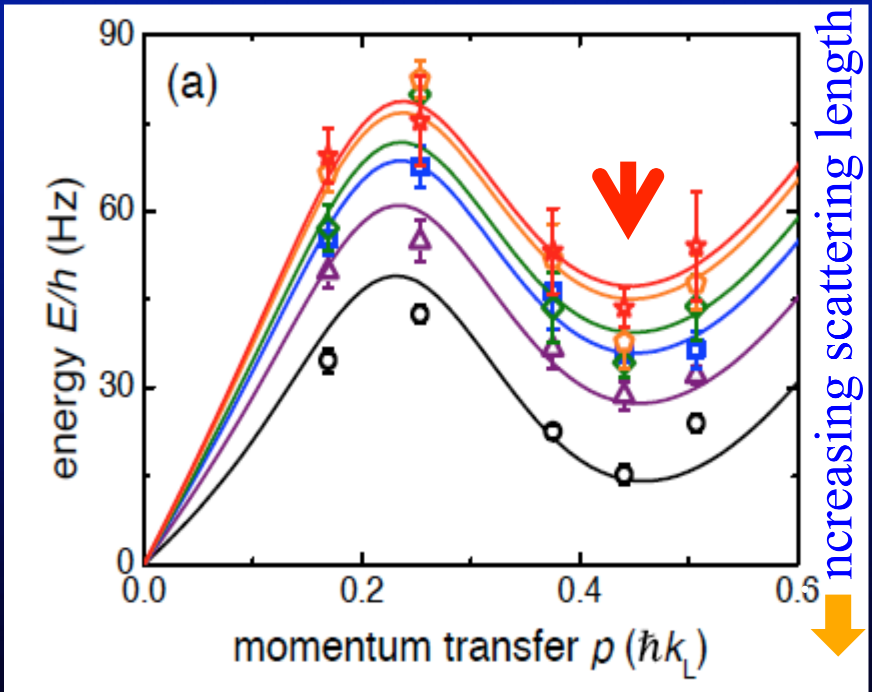
populate right minimum

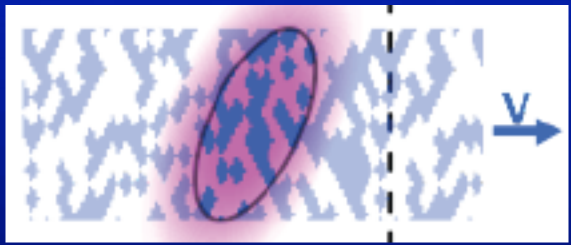
Experimental realization of levons, III

L.-C. Ha, L.W. Clark, C.V. Parker, B. M. Anderson ,& C. Chin, arXiv:1407.7157

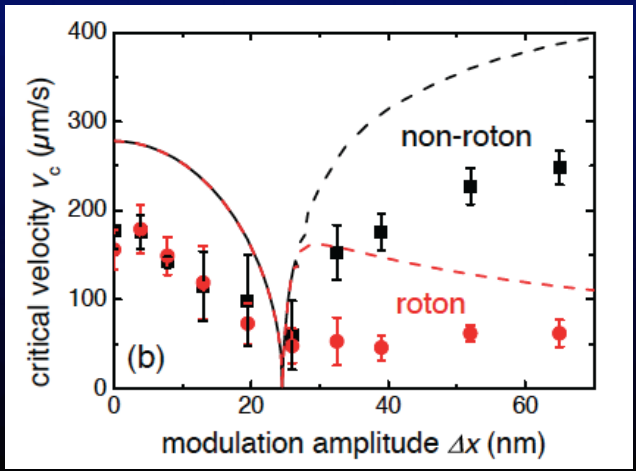
^{133}Cs Bose gas (3D) in shaken 1D optical lattice;
also develops roton minimum in excitation spectrum.
(Shaking couples in higher bands.)





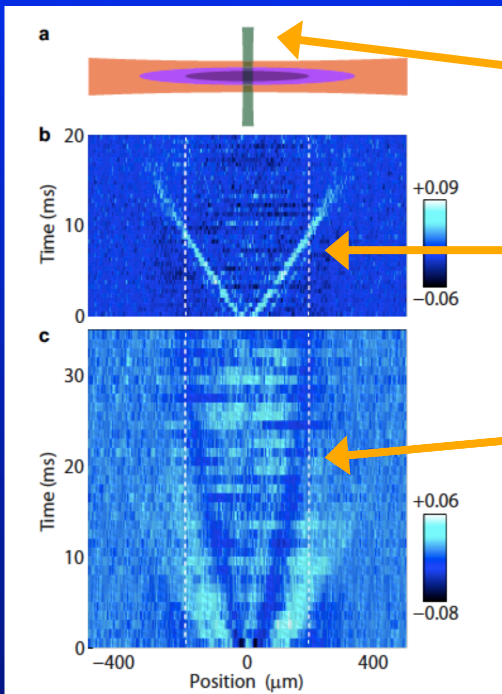


measure critical velocity of speckle potential moving through system



Second sound & superfluid mass density in a unitary Fermi gas

Sidorenkov, Tey, Grimm, Hou, Pitaevskii, & Stringari, Nature 498, 78 (2013)

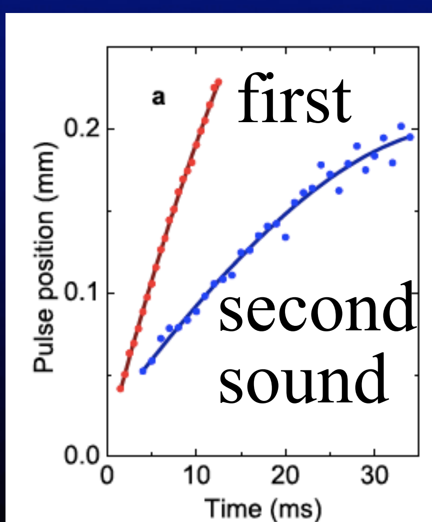


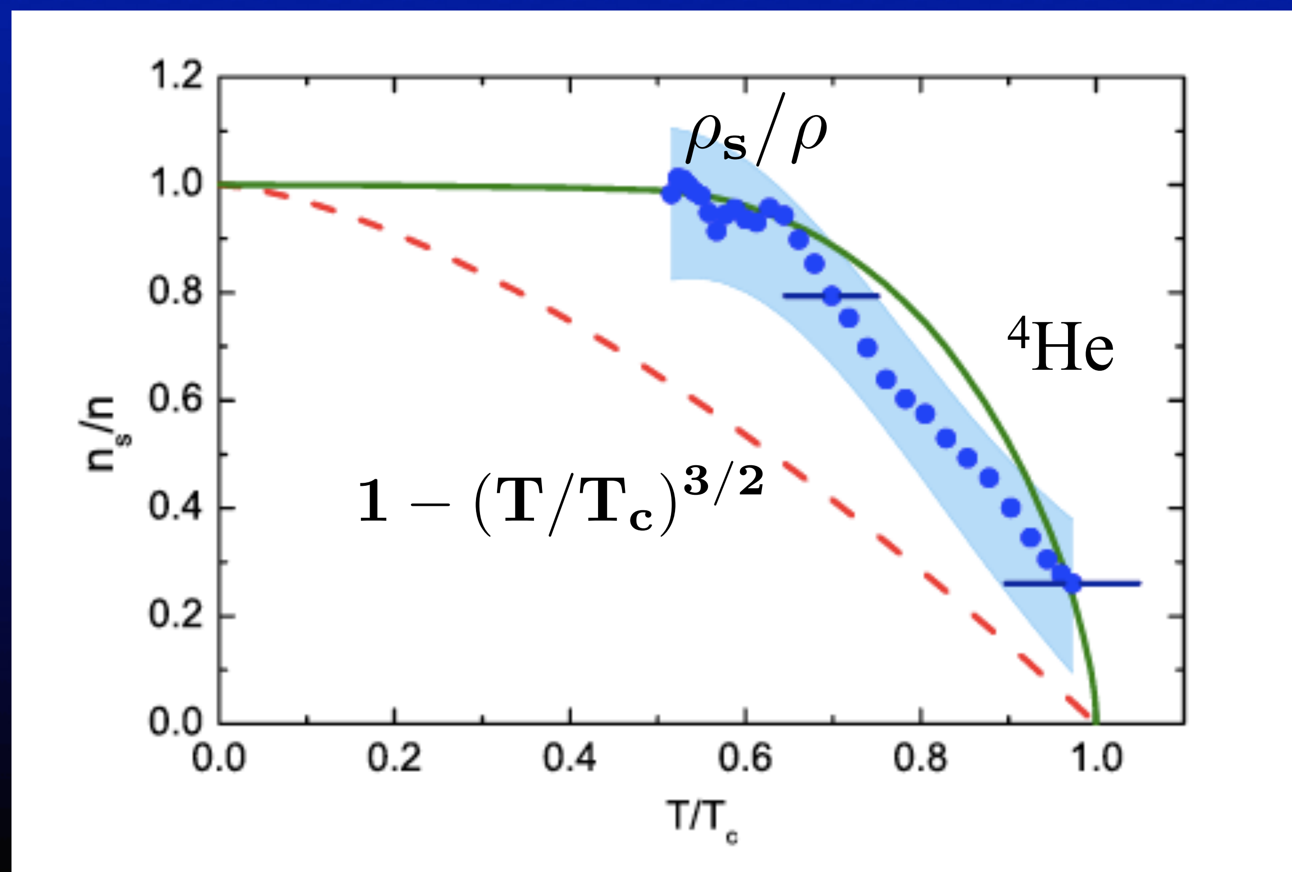
Blue shifted laser

^{6}Li

first sound

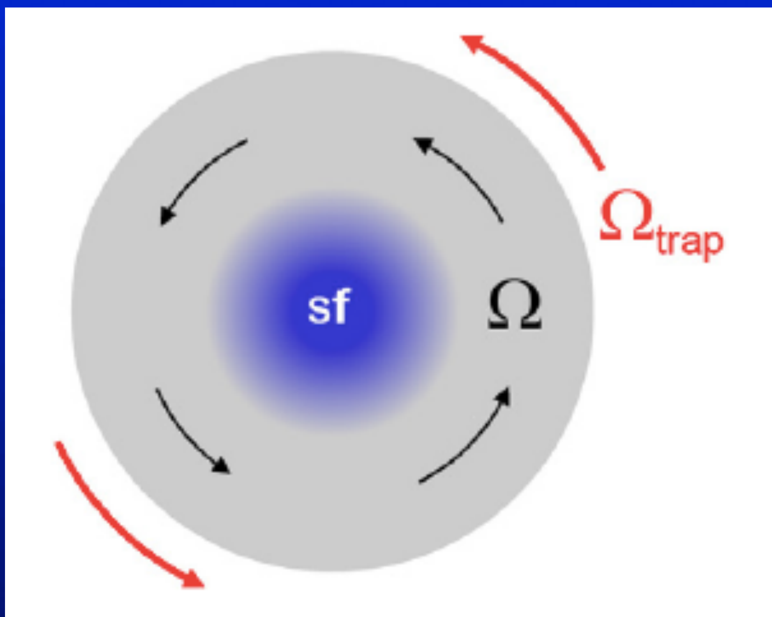
second sound -- velocity $\mathbf{s_2} \sim (\rho_\mathbf{s}/\rho_\mathbf{n})^{1/2}$





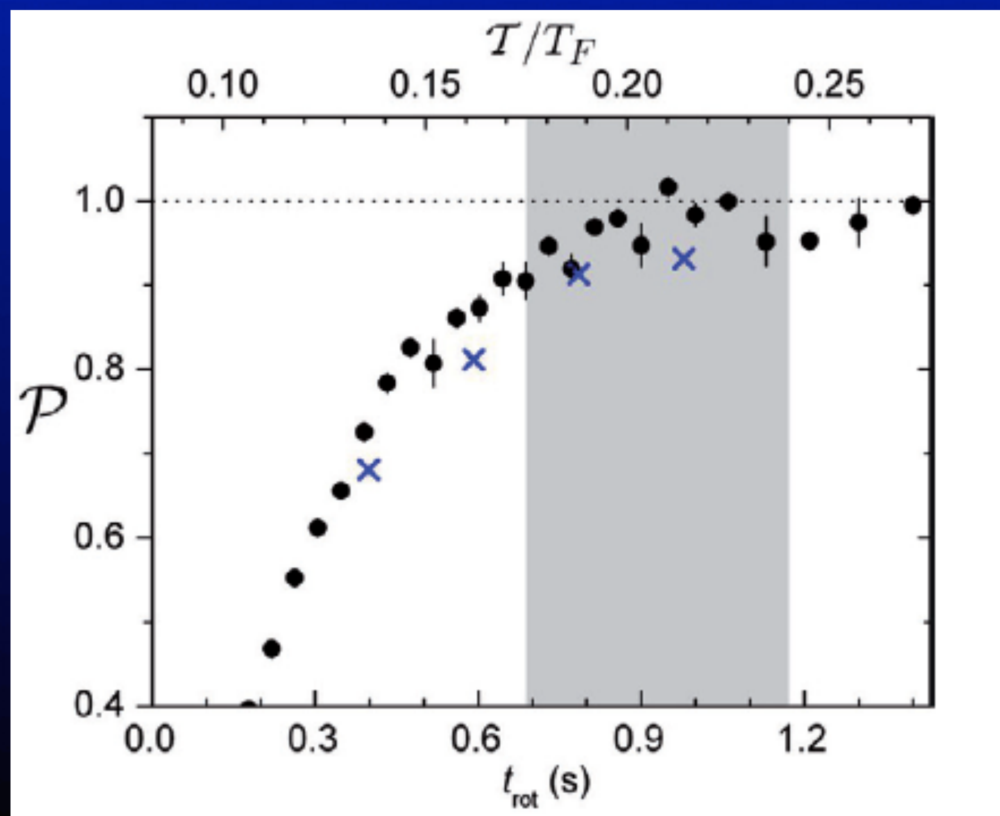
Moment of inertia of a strongly interacting Fermi gas

S Riedl, E R Sánchez Guajardo, C Kohstall, J Hecker Denschlag, and R Grimm, New J. Physics 13, 035003 (2011)



^{6}Li at unitarity

$$\mathcal{P} = \frac{\mathbf{I}}{\mathbf{I_{cl}}} \frac{\mathbf{\Omega}}{\mathbf{\Omega_{trap}}} \simeq \frac{\rho_{\mathbf{n}}}{\rho}$$



Normal mass density in paired gas at unitarity

GB and CJ Pethick ,PRA 88, 043631 (2013)

Basic physics of paired superfluid is BCS, with excitations:

1) quasiparticles: $\mathbf{E_p \simeq \left((\epsilon_p - \epsilon_F)^2 + \Delta(T)^2\right)^{1/2}}$

2) first sound: $\omega^{(1)} = \mathbf{sk}$, 3) second sound: $\omega^{(2)} = \mathbf{s_2 k}$

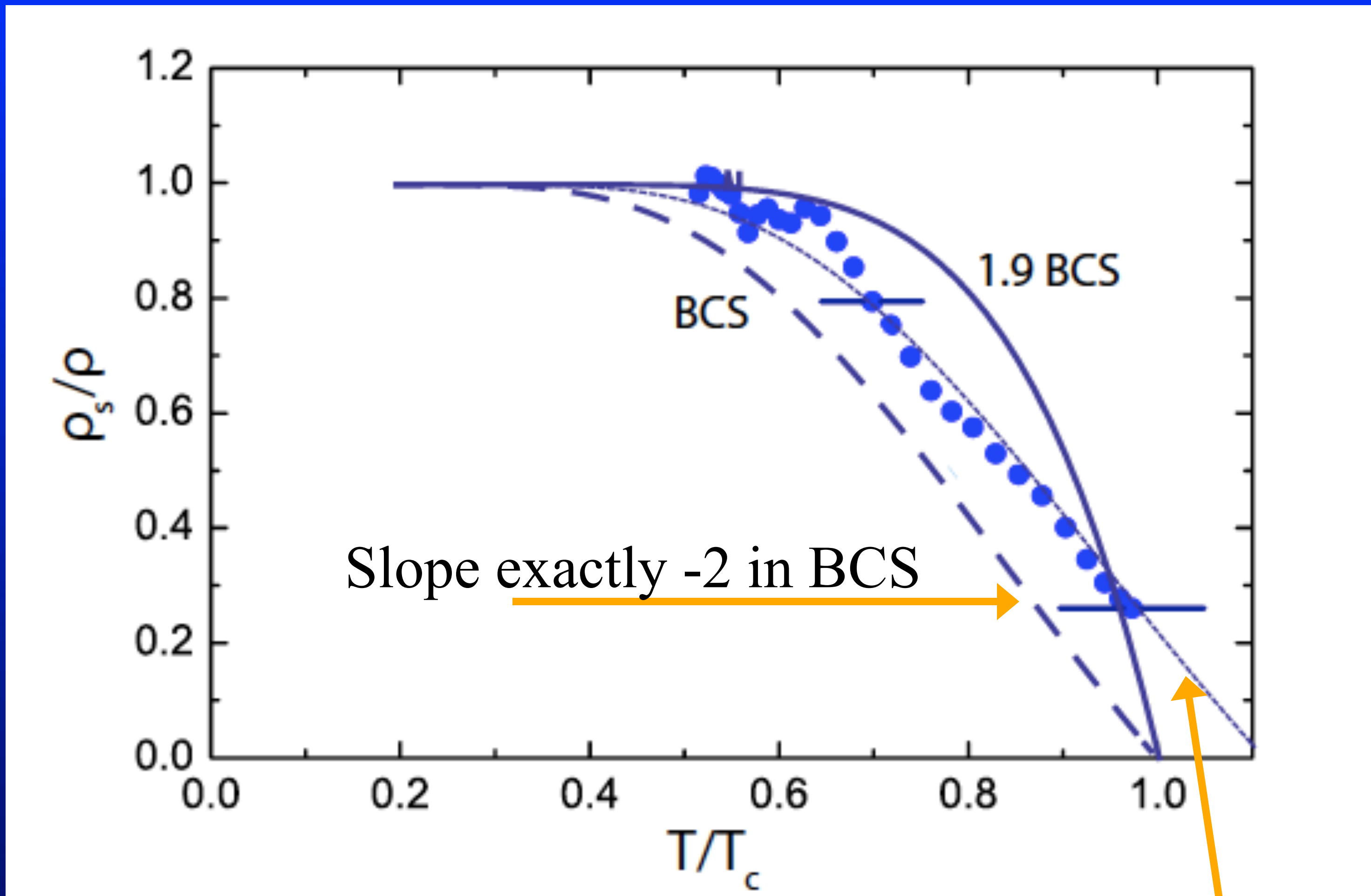
Each branch of well-defined excitations (i) contributes

$$\rho_{\mathbf{n}}^{(\mathbf{i})} = -\int \frac{\mathbf{d^3k}}{(\mathbf{2}\pi)^{\mathbf{3}}} \frac{\mathbf{k^2}}{\mathbf{3}} \frac{\partial \mathbf{f_i(k)}}{\partial \epsilon_{\mathbf{i}}(\mathbf{k})}$$

to the normal mass density (except for small double counting since sound modes are collective modes of quasiparticle excitations)

Near T_c, $\mathbf{\Delta(T) \simeq \Delta_c \sqrt{1 - T/T_c}}$, with Δ_c a parameter.

In BCS, $\Delta_c = 3.06\, T_c$



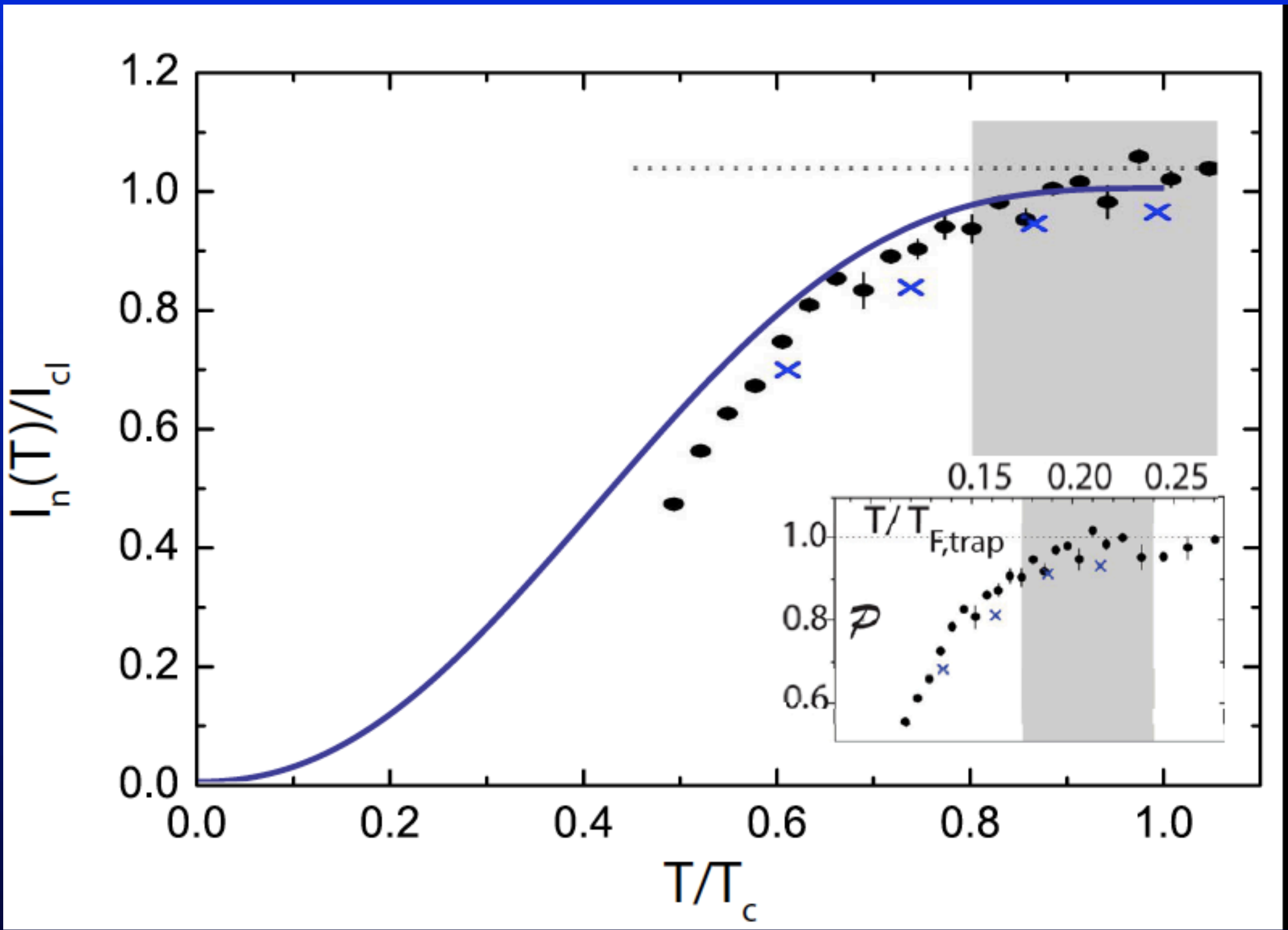
BCS result with revised temperature scale, with T_c fixed.

$$\Delta_{\rm MC}(T=0)/\Delta_{\rm BCS}(T=0) \simeq 1.9$$

Magierski et al. (2009)

Moment of inertia of a strongly interacting Fermi gas

S Riedl, E R Sánchez Guajardo, C Kohstall, J Hecker Denschlag, and R Grimm, New J. Physics 13, 035003, (2011)



Trap => slow falloff as T → T_c

GB & C.J. Pethick, PRA 88, 043631 (2013)

Measurement of normal mass density sensitive probe of gap near T_c

Summary and conclusions

Non-zero superfluid mass density is defining characteristic of superfluids:

from liquid helium to superconductors (penetration depth) to nuclei and nuclear matter to atomic condensates

Microscopic definition

Landau criterion for superfluidity is neither necessary nor sufficient:

when violated have superfluid mass density less than total density. Possible new inhomogeneous superfluid states.