Many-body Anderson localization of strongly interacting Bose-Einstein condensates

Mott Lobes, Superfluidity and Bose Glass

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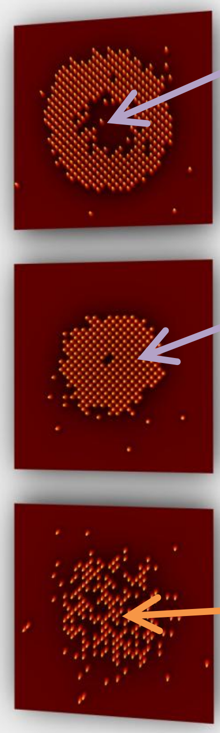
BEC with disorder W and interaction U

Strong correlations
Mott regime

U

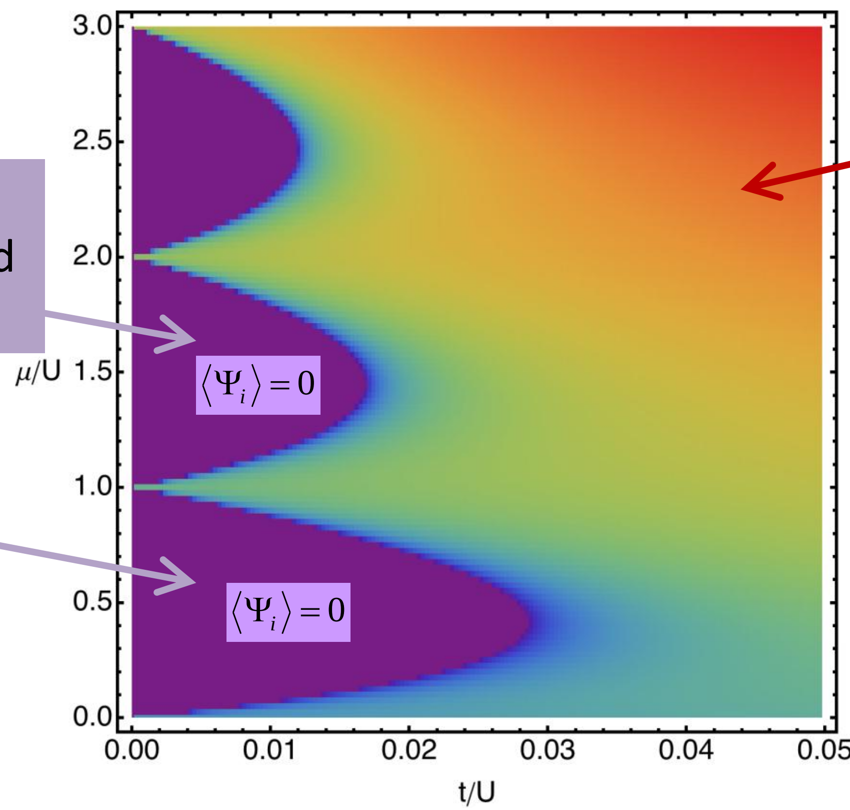
Weak interactions
Bogoliubov regime

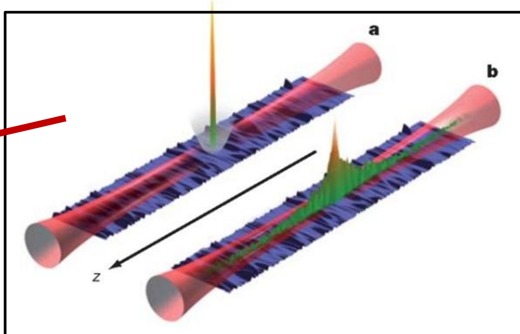


Mott localized n=2

n=1

mobile





Anderson localization of weakly interacting BEC wave function in a random potential

Mott-Hubbard BEC insulator
Bloch (2002)

Experiment: Aspect, inguscio (2008)

Theory:
Gross-Pitaevskii + disorder
interference $\leftrightarrow$ dephasing
Basko, Aleiner, Altshuler, Flach…

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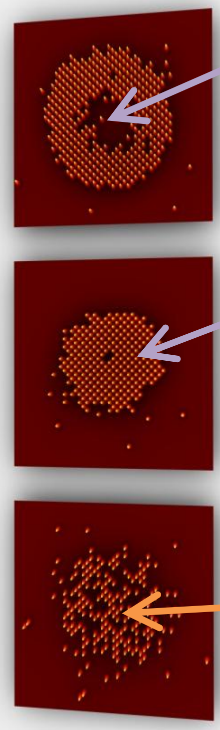

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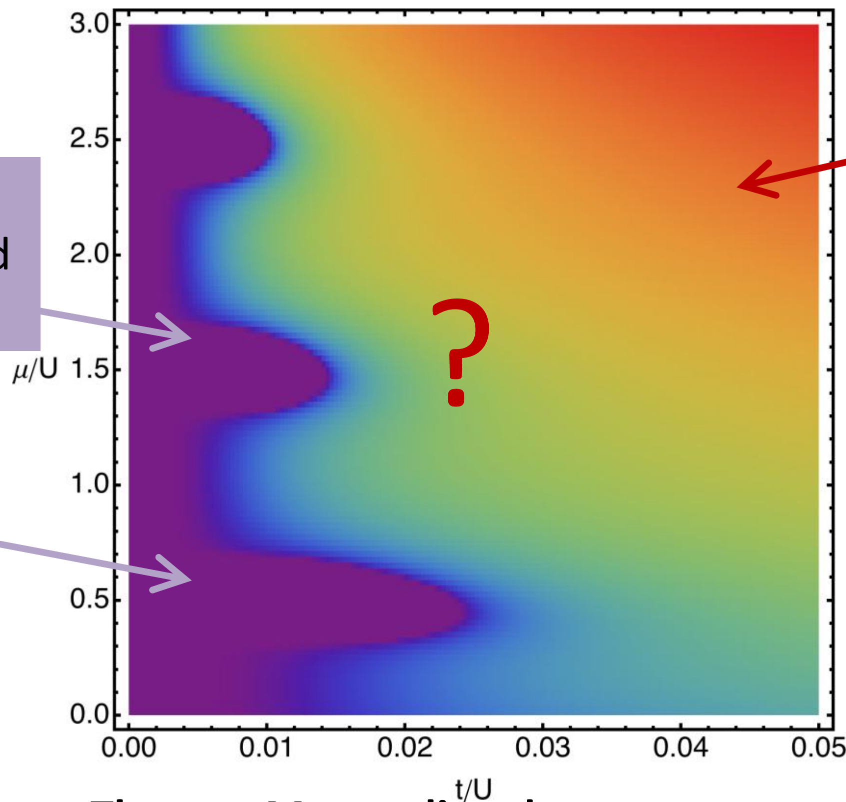
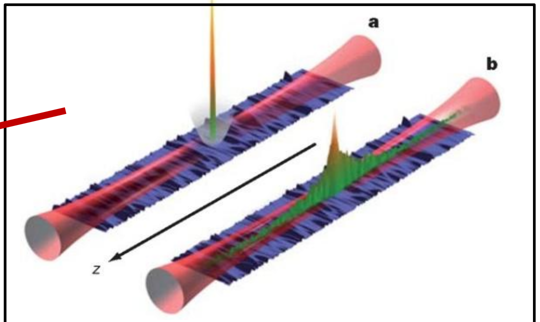




Anderson localization of weakly interacting BEC wave function in a random potential

Experiment: Aspect, inguscio (2008)

Theory: Mott + disorder

„Bose glass“: $\kappa \neq 0$ and $\langle \Psi_i \rangle = 0$

Fisher et al., PRB **40**, 546 (1989), Giamarchi
Bisport, Hofstetter, EPL **86**, 50007 (2009)

Theory:

Gross-Pitaevskii + disorder
interference <-> dephasing
Basko, Aleiner, Altshuler, Flach…

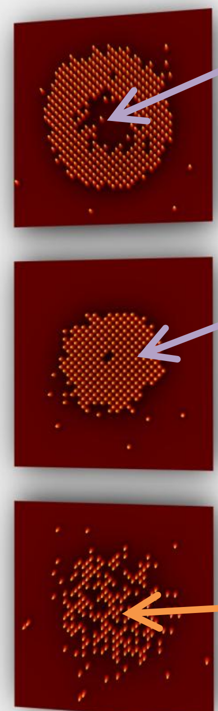
Anderson vs Mott localization of BECs

BEC with disorder W and interaction U

Strong correlations
Mott regime

U

Weak interactions
Bogoliubov regime



Mott localized n=2

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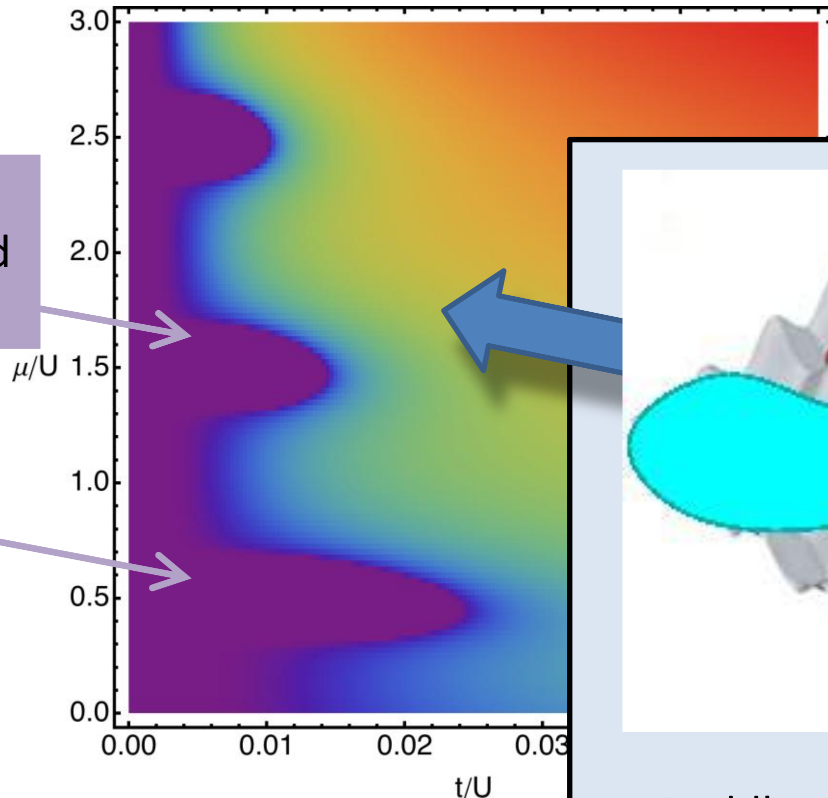
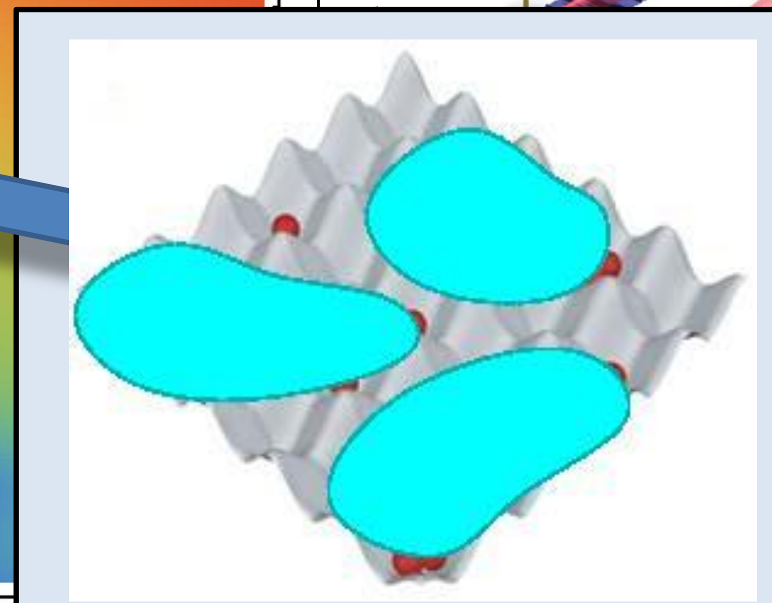




Puddles of Anderson-localized,
Strongly interacting BECs

of

scio

pt.

er

interference <-> dephasing

Basko, Aleiner, Altshuler, Flach...

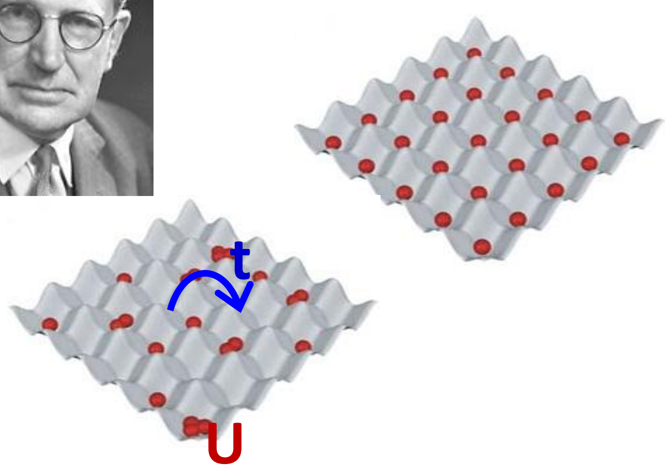
Describing

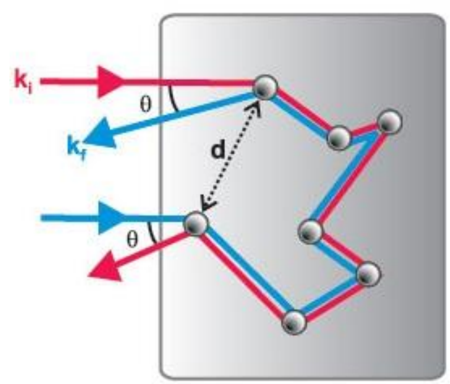
Strong, local Interaction
⇩
``Mottness´´

and

Nonlocal quantum interference
⇩
BEC Anderson localization

in a unified theory.





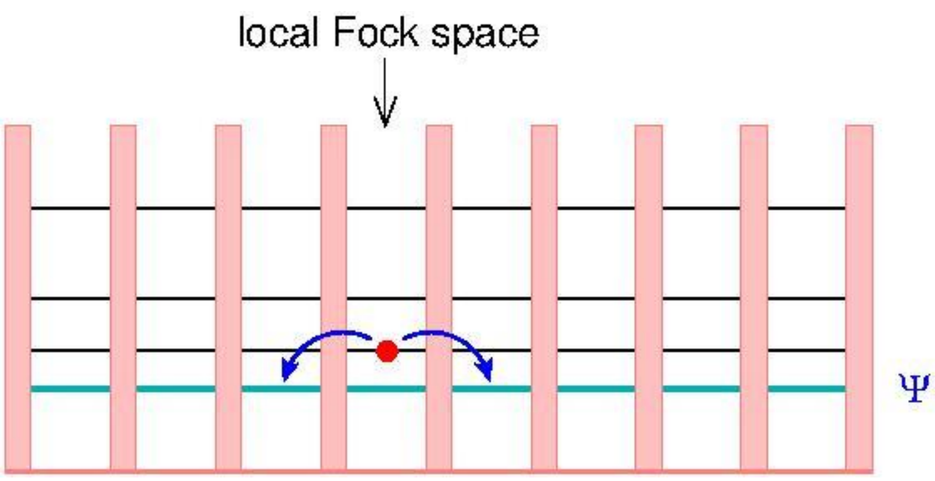
Bose-Hubbard: stochastic condensate DMFT

$$H = \sum_i \left(\varepsilon_i - \mu + \frac{U}{2}(\mathrm{b}_i^\dagger \mathrm{b}_i - 1)\right)\mathrm{b}_i^\dagger \mathrm{b}_i \; - \; t\sum_{\langle ij\rangle} \mathrm{b}_i^\dagger \mathrm{b}_j$$

$$\boxed{\begin{aligned} \mathrm{b}_i &= \langle 0|\mathrm{b}_i|0\rangle + \delta_i \\ \psi_i &= \langle 0|\mathrm{b}_i|0\rangle \end{aligned}}$$

$$\Downarrow$$

$$H = \sum_i \left[\left(\varepsilon_i - \mu + \frac{U}{2}(\mathrm{b}_i^\dagger \mathrm{b}_i - 1)\right)\mathrm{b}_i^\dagger \mathrm{b}_i - t\sum_{j.nn.i}(\psi_j^* \mathrm{b}_i + \psi_j \mathrm{b}_i^\dagger - \psi_j^*\psi_i)\right] \; - \; t\sum_{\langle ij\rangle} \delta_i^\dagger \delta_j$$



Diagonalizing local Hamiltonian exactly in Fock space:

$E_{i\alpha}$ many-body eigenenergy

$\Psi_i = \Psi(\varepsilon, \mu, t/U)$

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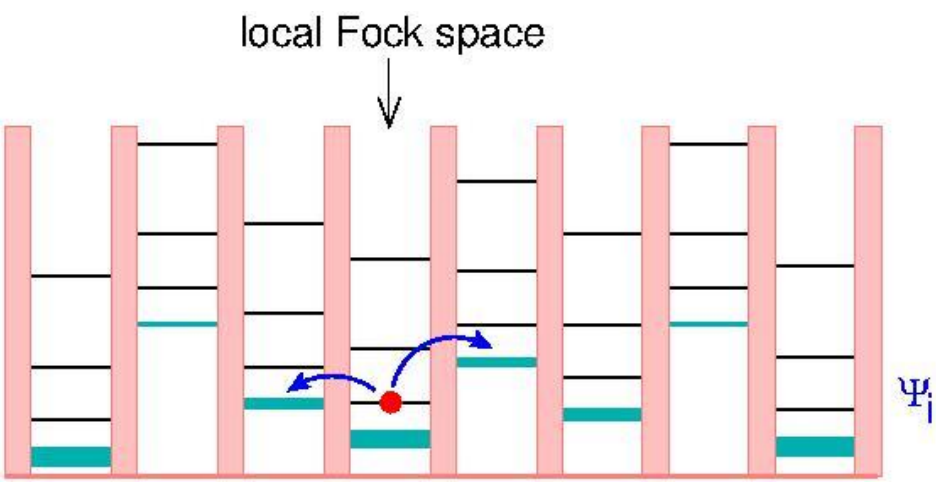
Fisher et al, PRB 40, 546 (1989)
Bisbort, Hofstetter, EPL 86, 50007 (2009)

Disorder: random ε_i

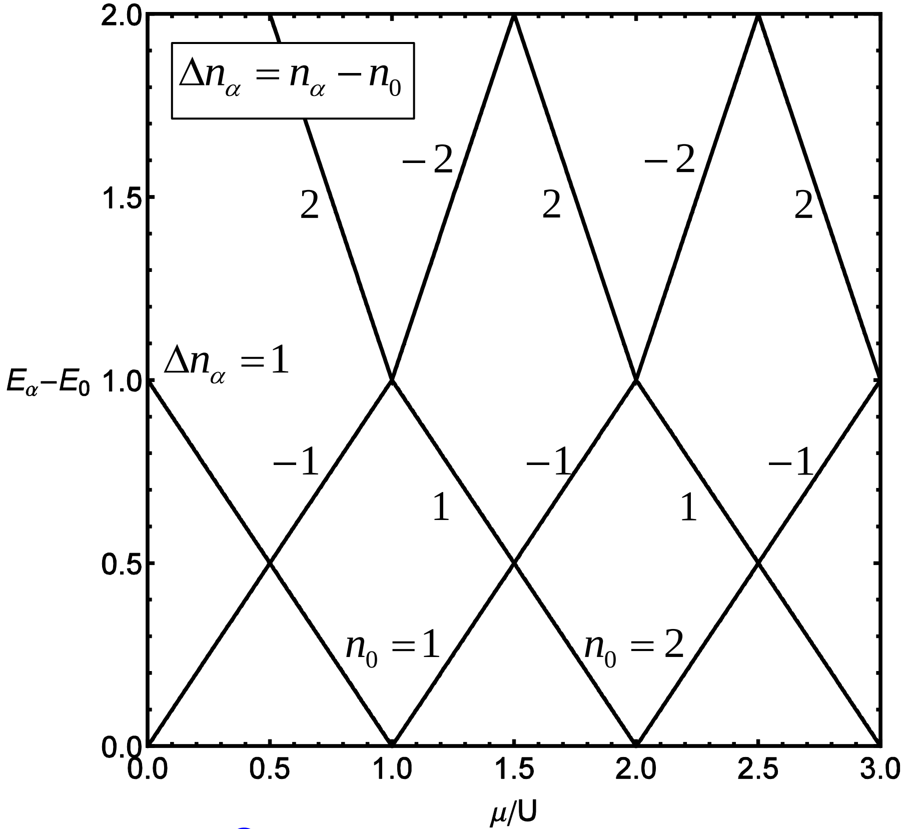


Diagonalizing
local Hamiltonian
exactly in Fock space:

$E_{i\alpha}$ many-body eigenenergy

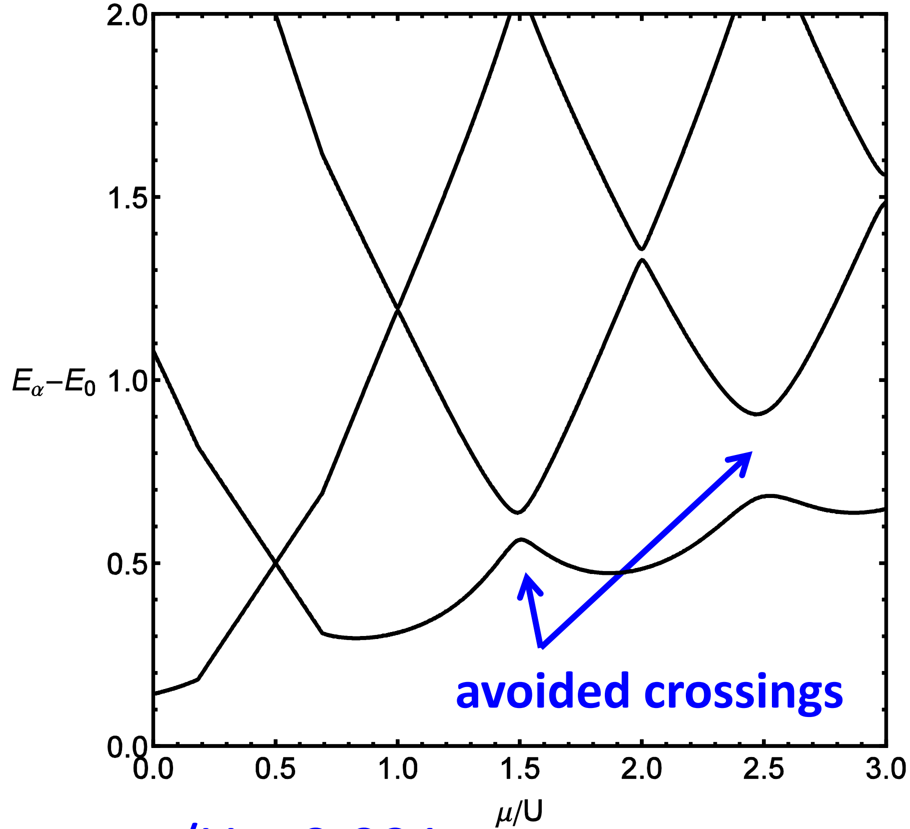
$\Psi_i = \Psi(\varepsilon_i, \mu, t/U, \Psi_j)$

Local many-body bands:



t = 0

no disorder: **W/U = 0.0**



t/U = 0.021

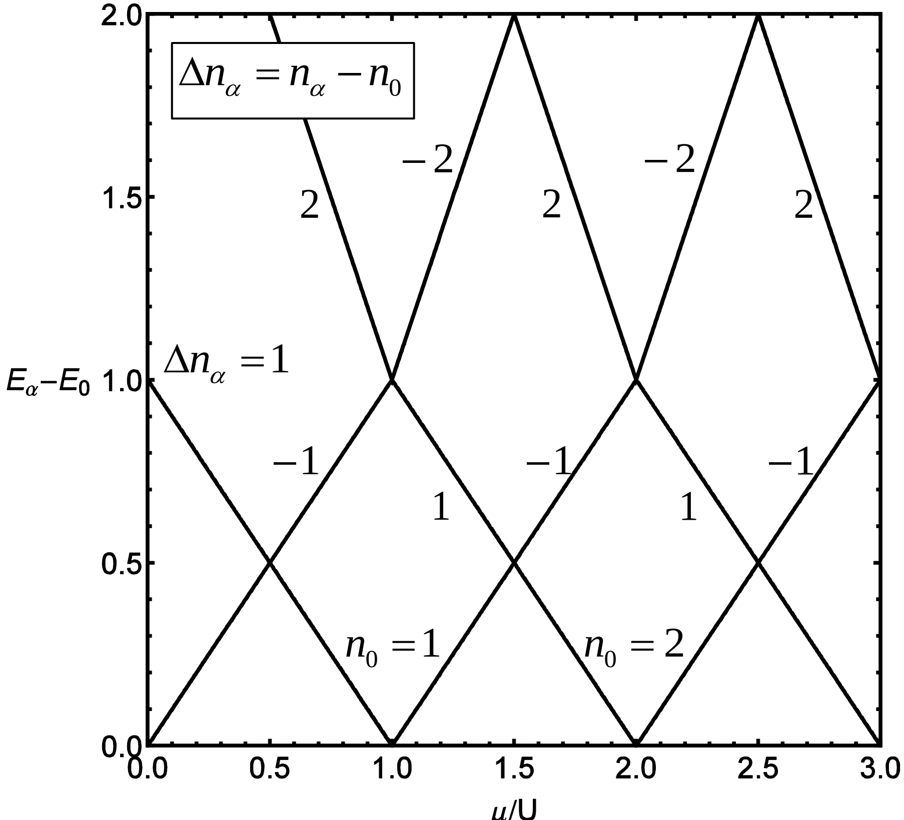
The spectra result from a combination of

- local charging energy:
- discreteness of particle number n_α

$$E_\alpha^{(0)} = (\varepsilon_i - \mu) n_\alpha + \frac{U}{2}(n_\alpha - 1) n_\alpha$$

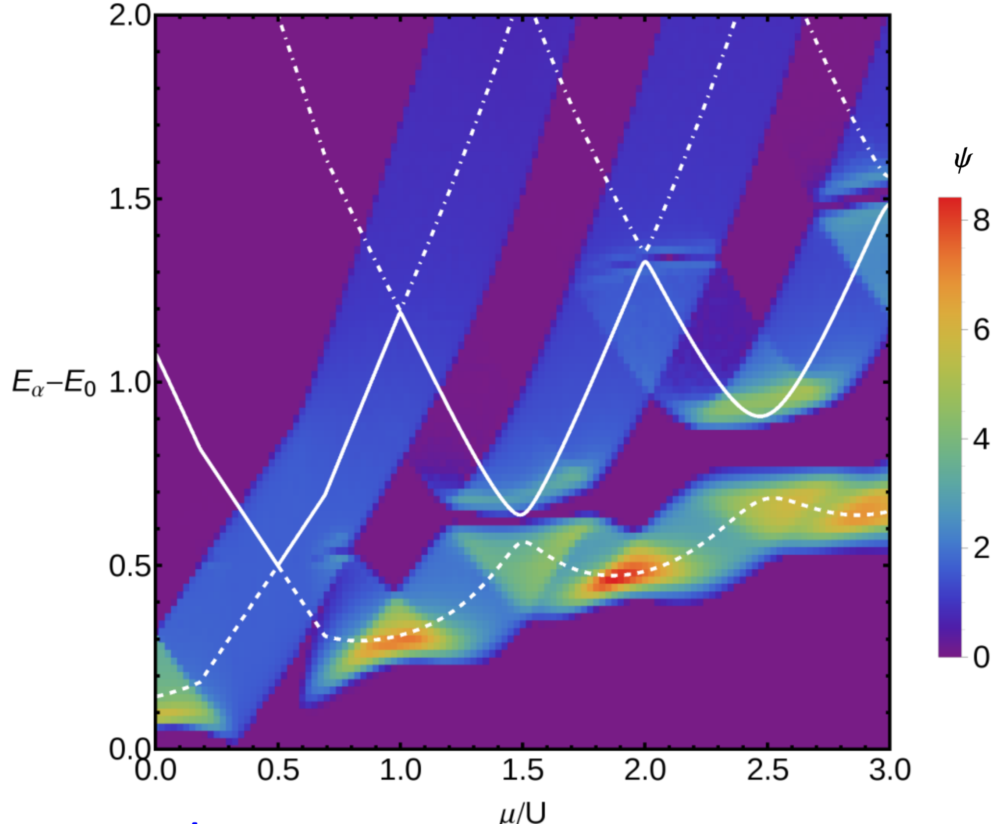
Bose-Hubbard: stochastic condensate DMFT

Local many-body bands:



t = 0

adding disorder: W/U = 0.6



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The spectra result from a combination of

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$$E_\alpha^{(0)} = (\varepsilon_i - \mu) n_\alpha + \frac{U}{2}(n_\alpha - 1) n_\alpha$$

$$H = \sum_i \left[\left(\varepsilon_i - \mu + \frac{U}{2}(b_i^\dagger b_i - 1) \right) b_i^\dagger b_i - t \sum_{j.nn.i} (\psi_j^* b_i + \psi_j b_i^\dagger - \psi_j^* \psi_i) \right] - t \sum_{\langle ij \rangle} \delta_i^\dagger \delta_j$$

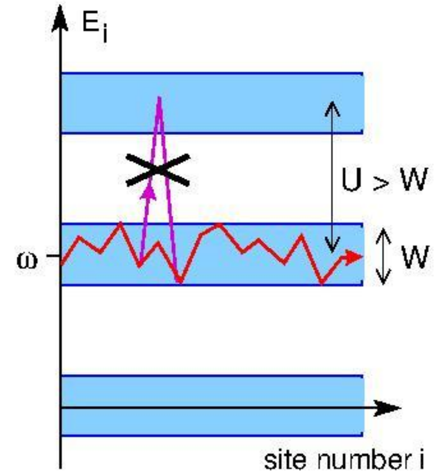
no transport in the local many-body ground states: $\delta_i = b_i - \Psi_i$ → $\boxed{\langle 0_i | \delta_i | 0_i \rangle = 0}$

→ Hopping transport of local many-body excitations

Propagators for local Fock space eigenstates:

$$G_{jj}^R(\omega) = -i\langle b_j b_j^\dagger \rangle = \sum_\alpha \left[\frac{\psi_{j\,0\alpha}\ \psi_{j\,\alpha 0}^*}{\omega - (E_{j\alpha} - E_{j0}) + i\eta} - \frac{\psi_{j\,0\alpha}^*\ \psi_{j\,\alpha 0}}{\omega + (E_{j\alpha} - E_{j0}) + i\eta} \right]$$

$$F_{jj}^R(\omega) = -i\langle b_j b_j \rangle = \sum_\alpha \left[\frac{\psi_{j\,0\alpha}\ \psi_{j\,\alpha 0}}{\omega - (E_{j\alpha} - E_{j0}) + i\eta} - \frac{\psi_{j\,0\alpha}\ \psi_{j\,\alpha 0}}{\omega + (E_{j\alpha} - E_{j0}) + i\eta} \right]$$



→ For strong interaction, U >> t, W:

confinement to one Fock band |ia>

Wick‘s theorem preserved!

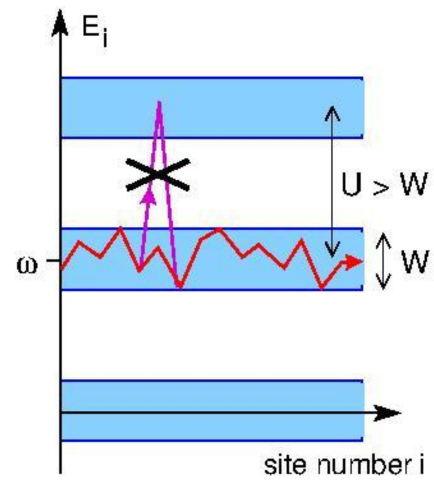
Effective single-particle Hamiltonian

for hopping of local, many-body excitations:

$$H = \sum_i \overline{B_{i\alpha}}^{+} (E_{i\alpha} - E_{i0}) \underline{\underline{\tau}}\, \overline{B_{i\alpha}} - \sum_{\langle ij \rangle; \alpha,\beta\neq 0} \overline{B_{i\alpha}}^{+} \underline{\underline{T_{ij\alpha\beta}}} B_{i\beta}$$

$$\overline{B_{i\alpha}} = \begin{pmatrix} B_{i\alpha} \\ B_{i\alpha}^{+} \end{pmatrix}$$

Anomalous, renormalized hopping amplitudes:



$$\underline{\underline{T_{ij\alpha\beta}}} = \begin{pmatrix} T_{ij\alpha\beta} & S_{ij\alpha\beta} \\ S^{*}_{ij\alpha\beta} & T^{*}_{ij\alpha\beta} \end{pmatrix}$$

$$T_{ij\alpha\beta} = t\, \psi^{*}_{i\alpha 0} \psi_{j0\beta}$$

$$S_{ij\alpha\beta} = t\, \psi_{i\alpha 0} \psi_{j0\beta}$$

$$\psi_{i\,\alpha 0} = \langle i\,\alpha | \mathrm{b}_i | i\,0 \rangle$$

$$\psi^{*}_{i\,\alpha 0} = \langle i\,0 | \mathrm{b}_i^{\dagger} | i\,\alpha \rangle$$

Criterion for AL of BEC: vanishing of total superfluid transport

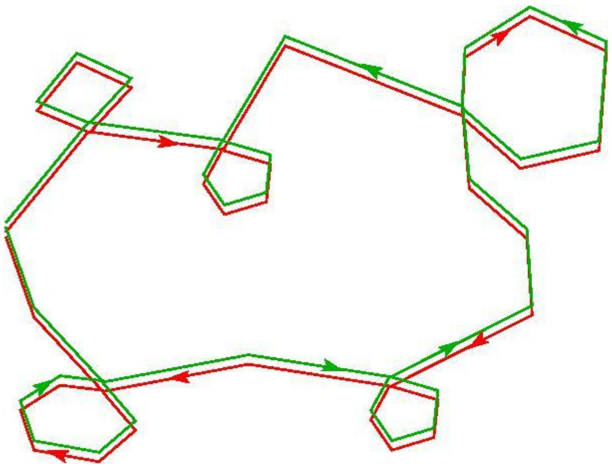
The superfluid current is carried by all $\omega < 0$ (hole-like) many-body excitations:

$$J_{ij} = 2\frac{t}{\hbar}\,\mathrm{Re}\int \frac{\mathrm{d}\omega}{2\pi}\, b(\omega)\ \left[G^{A}_{ij}(\omega) - G^{R}_{ij}(\omega)\right] \qquad b(\omega) = \theta(-\omega)$$

analogous to SC junctions!

SF current is driven by a phase difference between distant sites i and j:

$$J_{ij} = 2\frac{t}{\hbar}\,\sin(\phi_j - \phi_i)\ \mathrm{Im}\int \frac{\mathrm{d}\omega}{2\pi}\, b(\omega)\ \left[G^{(n)\,A}_{ij}(\omega) - G^{(n)\,R}_{ij}(\omega)\right]$$

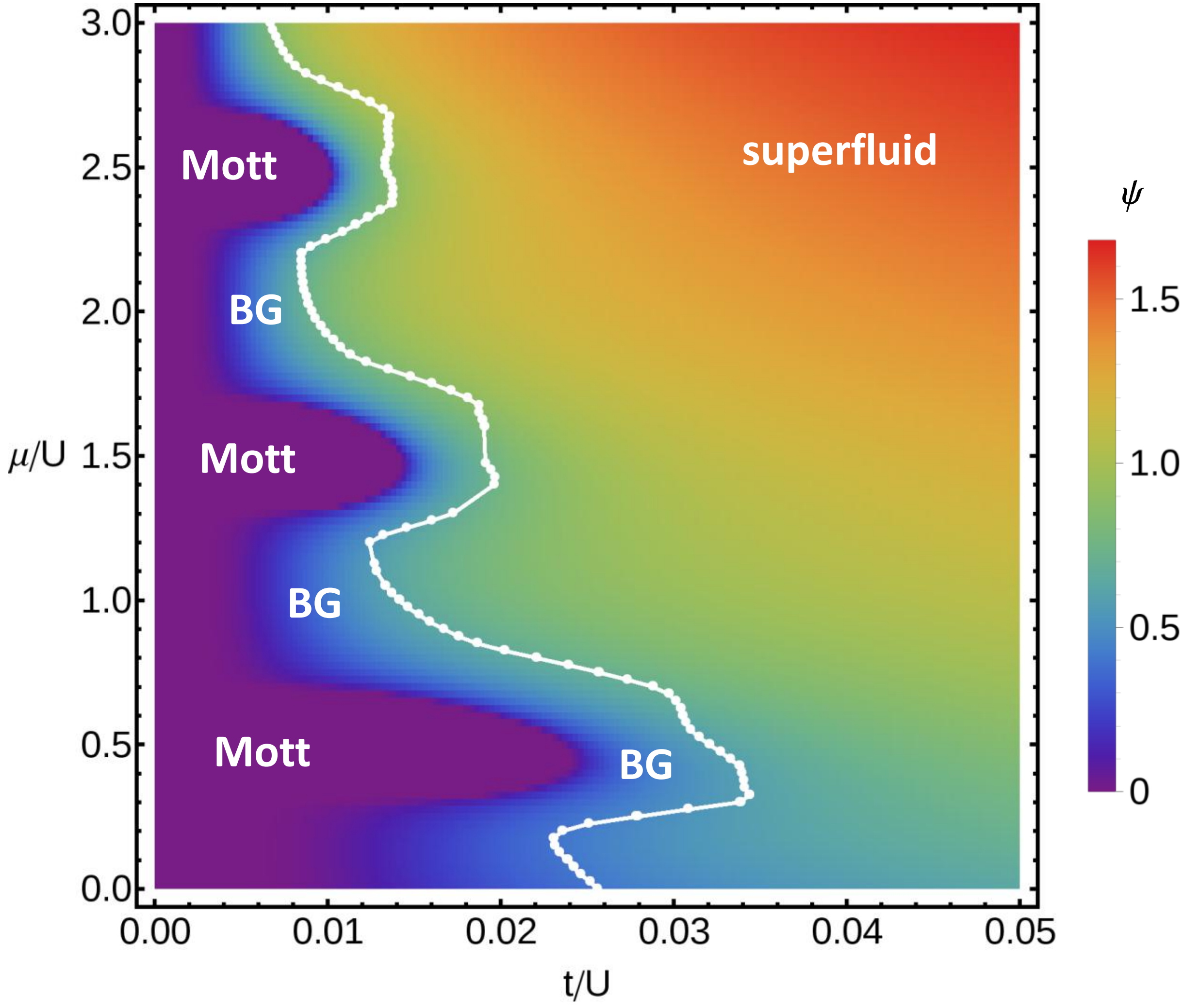


AL of BECs amounts to the
AL of **all** hole-like many-body excitations!

Analysis with the selfconsistent AL theory
Vollhardt, Wölfle (1980)

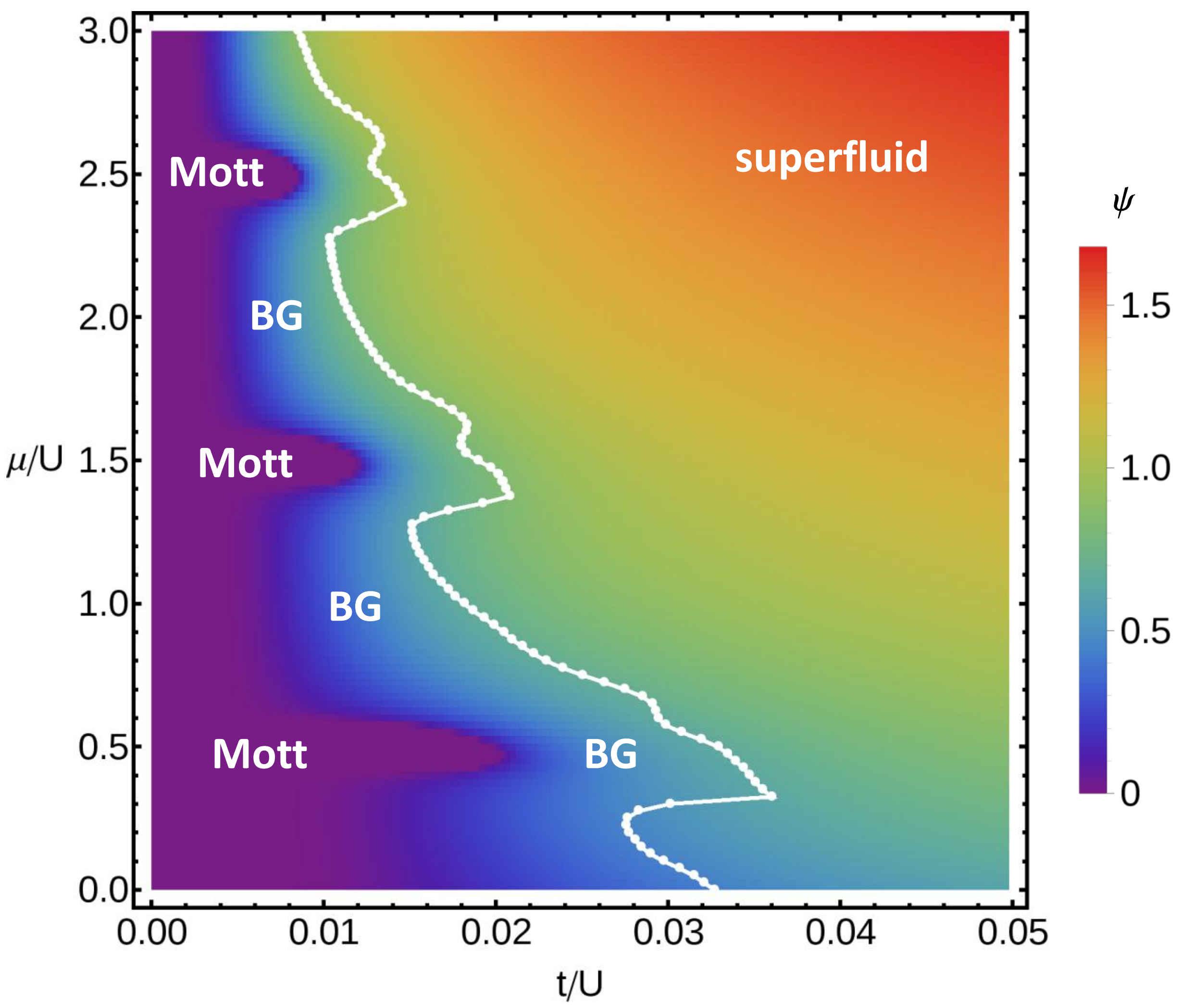
Bose glass phase diagram

Phase diagram **disorder: W/U = 0.6**



- The theorem of inclusions is respected:
  no direct transition
  Mott ↔ superfluid
  Polet, Prokof'ef, Troyer, PRL 103, 104402 (2009)
- The phase boundary reflects the charging spectrum of the on-site many-body excitations

Phase diagram **disorder: W/U = 0.8**



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  Polet, Prokof'ef, Troyer, PRL 103, 104402 (2009)
- The phase boundary reflects the charging spectrum of the on-site many-body excitations

Conclusion

- For strong interaction: $U \gg t, W$:
  Mapping of full many-body problem onto
  Transport theory of hopping many-body excitations

- Simultaneous description of
  Mott phase and Anderson localization (q.interference)
  Stochastic dynamical mean-field theory for the avg local OP
  local many-body spectrum in Fock space

- The phase diagram
  - respects the theorem of inclusions
  - reflects the features of the local many-body excitation spectrum