Superfluid and Transport Properties of Disordered Bose Gases in Two Dimensions

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- Correlated disorder in homogeneous 2D:
  - superfluid transition
  - transport (Bose glass)

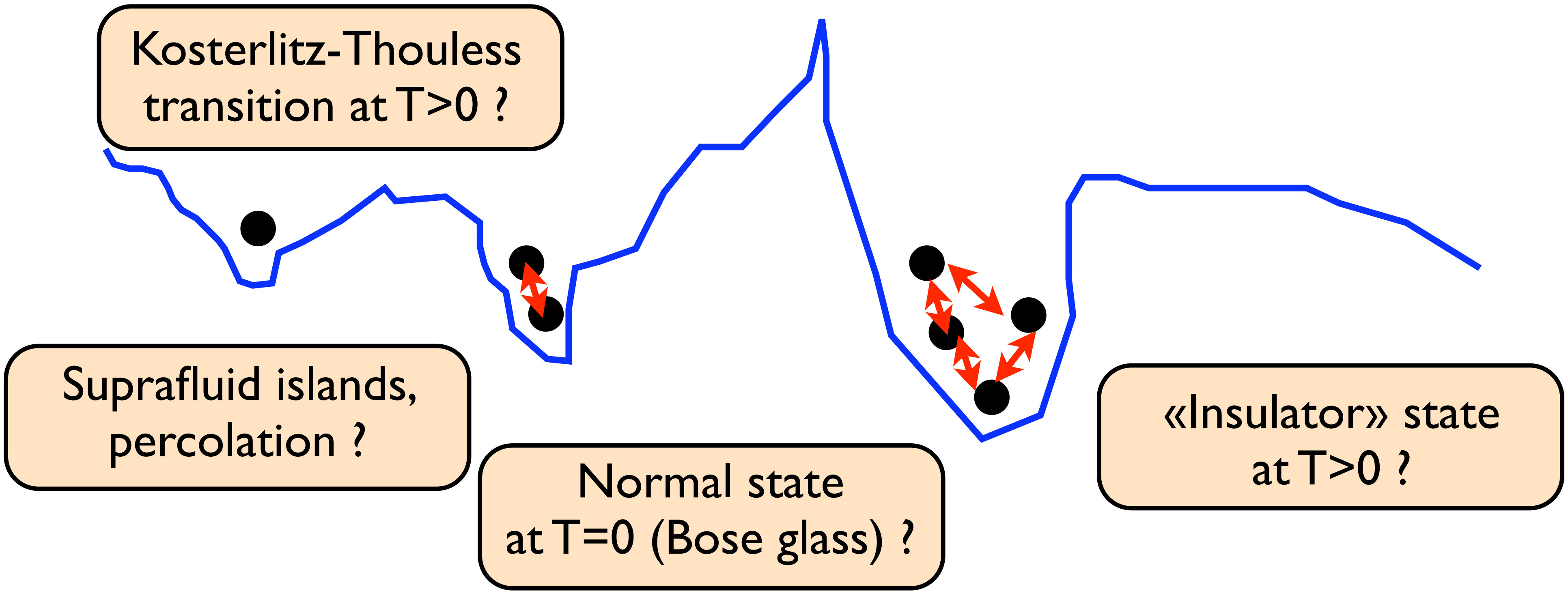
G. Carleo,
G. Boeris,
L.Sanchez-Palencia

- Correlated disorder in 2D traps:
  - experiment

B. Allard, T. Plisson,
G. Salomon, A. Aspect,
P. Bouyer, T. Bourdel

Dirty Boson Problem (2 Dimensions)

Competition between interaction (g) and disorder (V)



Experimental realizations:

- suprafluid transition of ^{4}Helium in vycor,
- superconductor-insulator transition in high Tc
- ✖ trapped ultracold atoms with artificial disorder (2D)

2D Correlated Disorder potential: Speckle

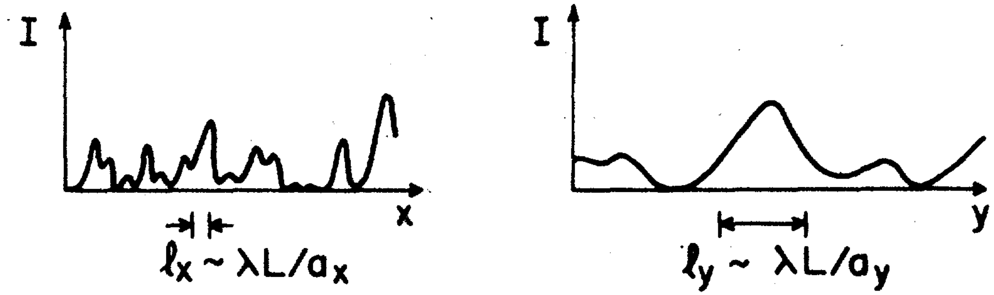
scattered light from diffuse scatterers

$$V(x,y) \propto I(x,y)$$

Intensity I(x,y) pseudo-random

$$\overline{V(x,y)} = V$$

$$\overline{V(x,y)V(0,0)} = V^2 f(x,y)$$



Gaussian correlations with correlation lengths σ_x, σ_y

Classical Percolation transition:

experiment: percolation threshold at area fraction ≈0.41 (V/μ≈1.9)

Smith, Lobb, Phys. Rev. B 20, 3653 (1979).

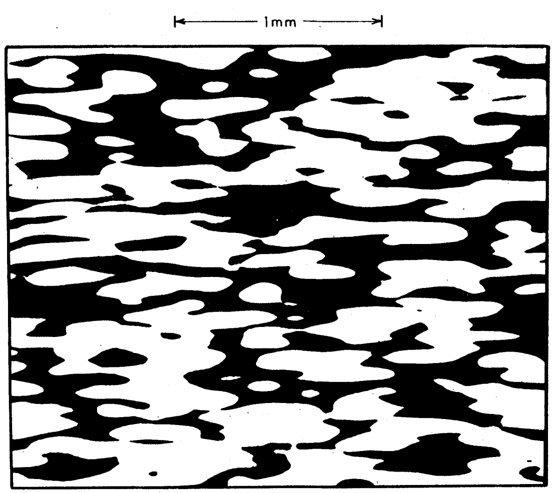


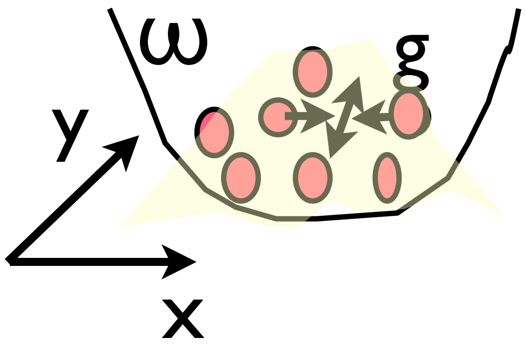
FIG. 4. Photograph of a section from an anisotropic sample: $\alpha=5$, $f=0.407$. The black areas are metal. Note that the magnification is less than that of Fig. 3.

Two-dimensional Ultracold Bosons in a harmonic trap:

(Experiments ↔ theory: Path-Integral Monte-Carlo simulations)

N≈40 000 Bosons

with short-ranged (hard-core) 3D interaction in anisotropic trap



ω: trap frequency

g: interaction constant

N: number of Bosons

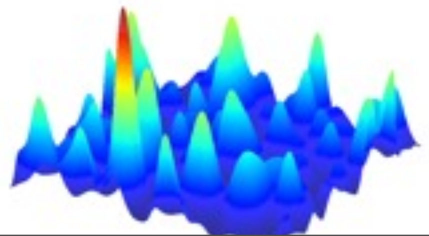
quasi2D: $\omega_z \gg \omega_x \approx \omega_y$

g ~$(\log na^2)^{-1}$ for hard disks of diameter a

- ideal Bosons (g=0): Bose-Einstein transition at T_c^0
- interacting Bosons (g>0): Kosterlitz-Thouless-Berenzinski transition at $T_{KT} < T_c^0$

How does disorder modify the transition ?

speckle potential



correlation length σ

amplitude V

Clean Bosons: Coherence properties (Experiment+QMC)

T. Plisson, B. Allard, M. H., G. Salomon, A. Aspect, P. Bouyer, and T. Bourdel, Phys. Rev. A 84, 061606(R) (2011)

$$g = 0.096, \quad \omega_x \approx \omega_y/2, \quad \omega_z/\omega_x = 100$$

Time-of-Flight ⇒ momentum distribution

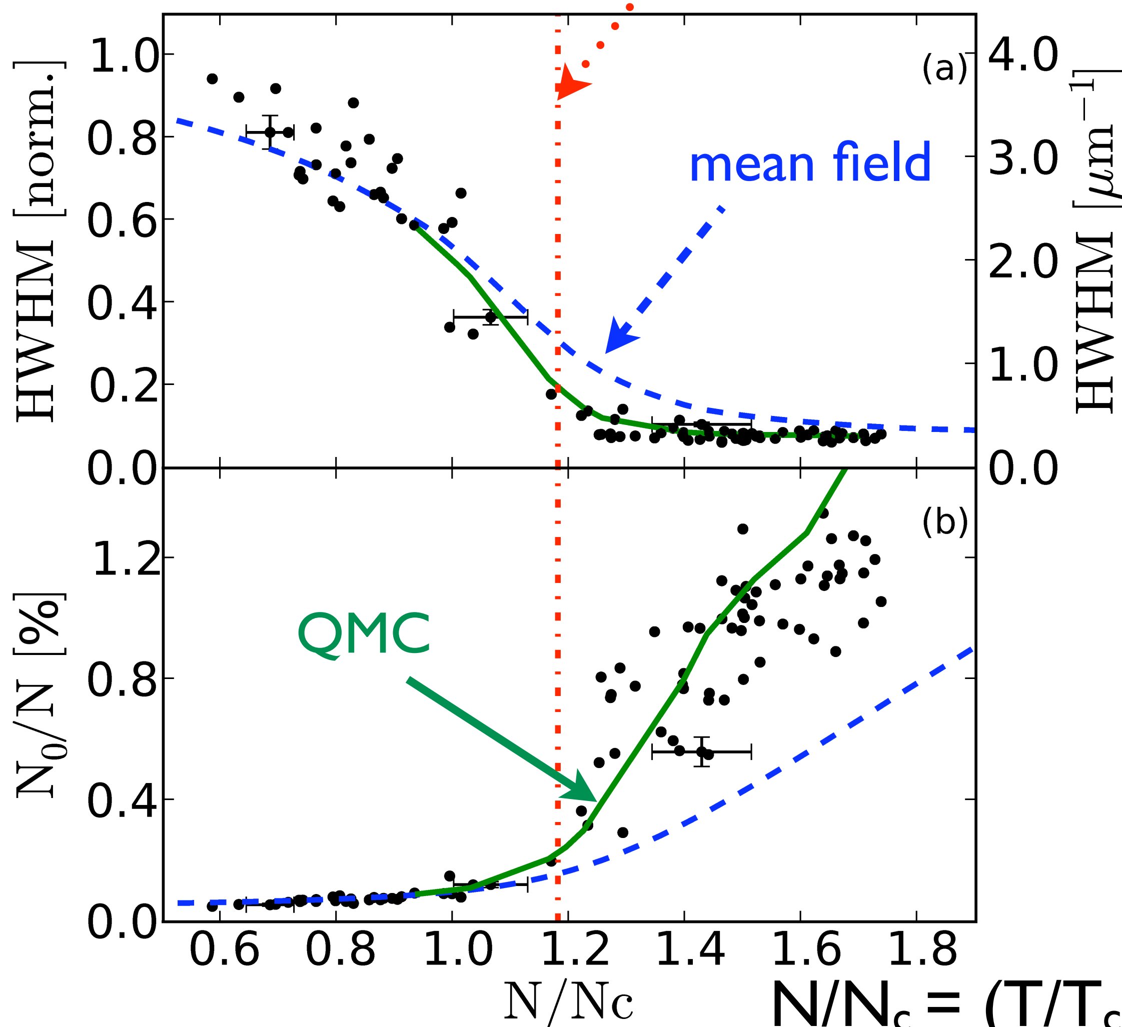
Kosterlitz-Thouless transition: N_{KT}

Characterization of Coherence (Peak around k=0):

Width of the peak:
HWHM

Height of the peak:

Fraction of particles in k=0 peak:
N_0/N



$N/N_c = (T/T_c)^{-2}$

Coherence properties: Where is Kosterlitz-Thouless in trap?

Infinite (homogeneous) system:

Low T-phase with algebraic order: $n_k \sim k^{-[2-\eta(T)]}$ for $k \to 0$

with $1/4 \leq \eta(T) \leq 2$

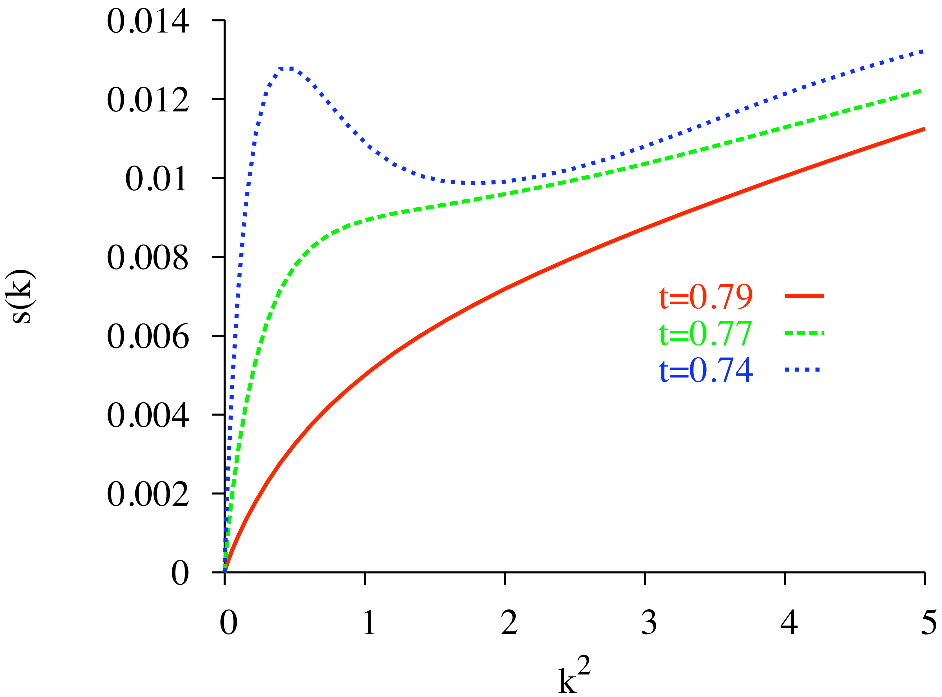
High T-phase normal: $n_k \sim 1$

KT transition at η=1/4

Trapped (finite,inhomogeneous) system:

$\Rightarrow$ plot $s(k) = n_k k^{2-1/4}$

s(k) for finite trapped system (QMC simulation):



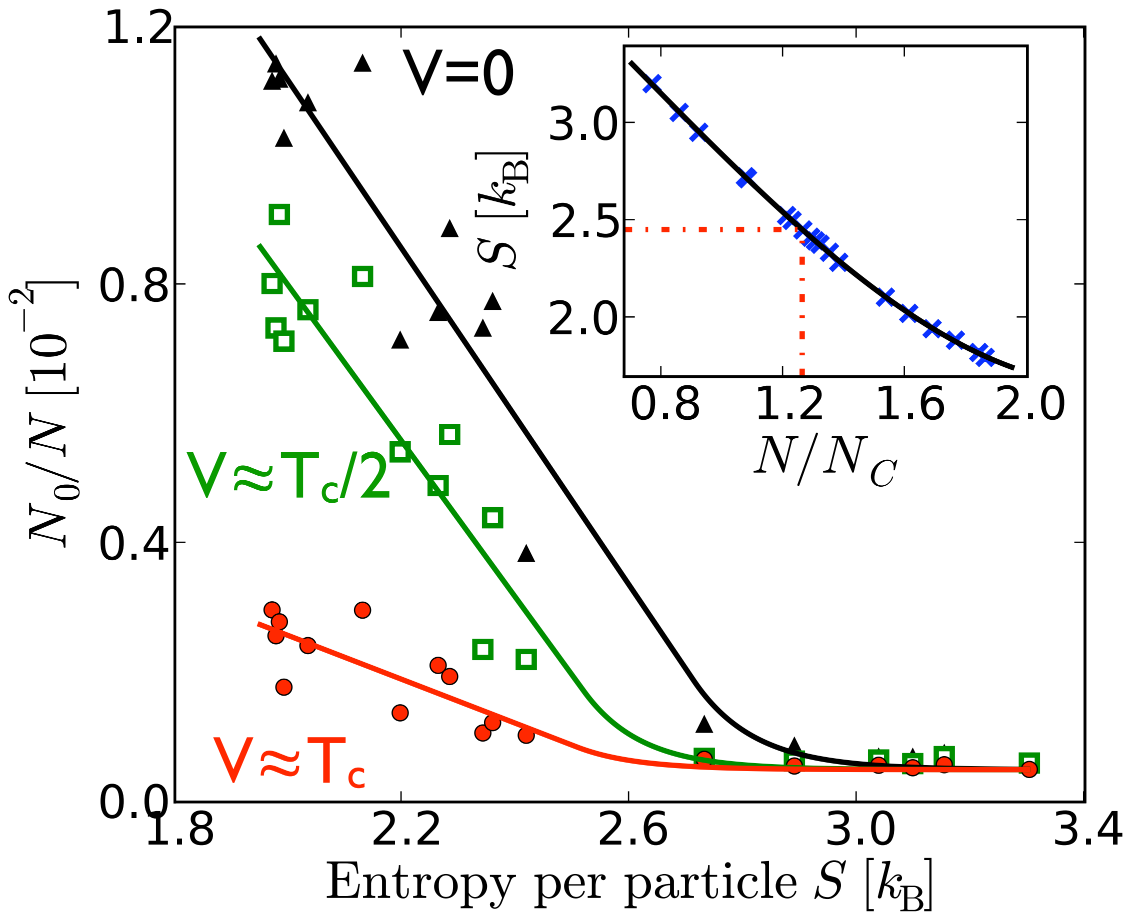
Experiment: Influence of disorder

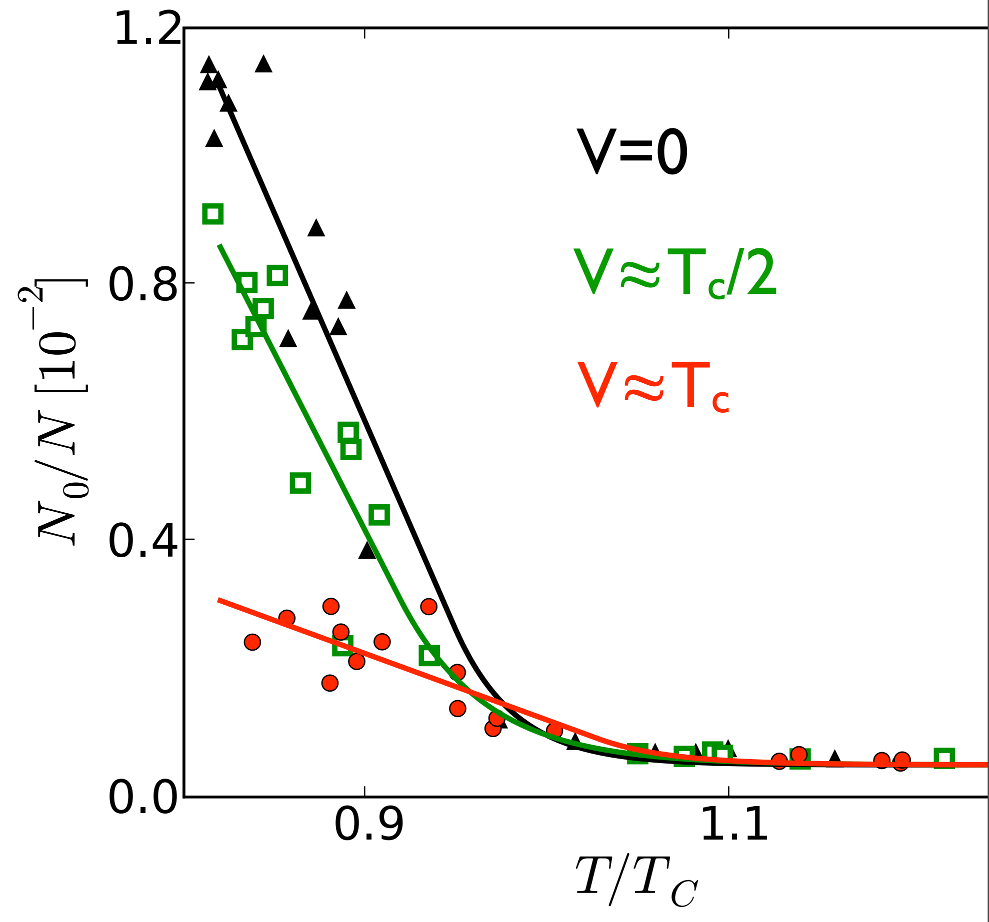
B. Allard, T. Plisson, M. H., G. Salomon, A. Aspect, P. Bouyer, and T. Bourdel, Phys. Rev. A 85, 033602 (2012)

adiabatic ramping of correlated disorder potential (speckle)

$$\sigma_x/2 = \sigma_y \approx \lambda_T = \sqrt{2\pi/mT}$$

supression of peak density N_0 observed





Dirty Bosons: QMC study (homogeneous)

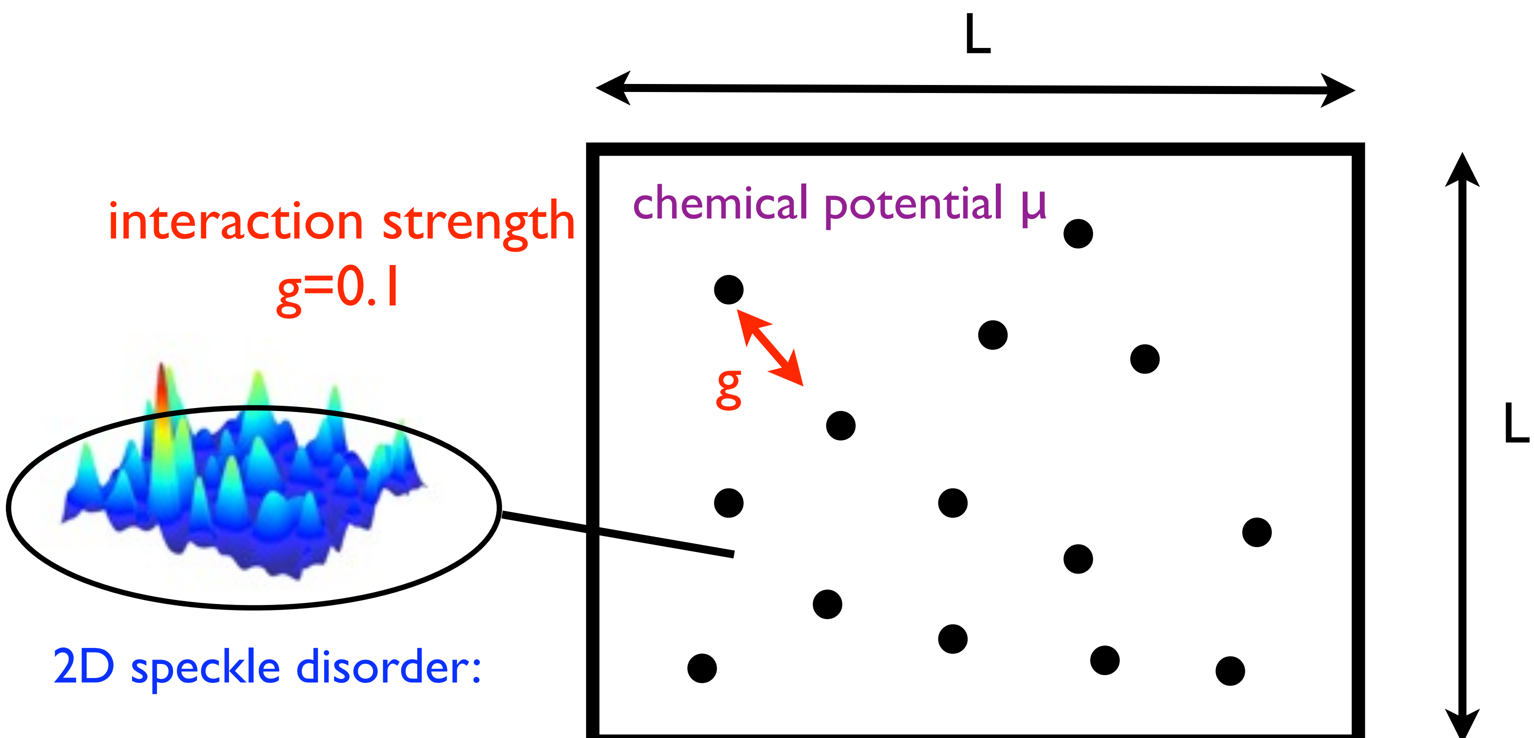
G. Carleo, G. Boéris, M. Holzmann, and L. Sanchez-Palencia, Phys. Rev. Lett. 111, 037203 (2013)

Grand-canonical simulation (worm-algo)

$N \approx 10^5$ Bosons in 2D Box with periodic boundary conditions

Similar study in 3D: S. Pilati, S. Giorgini, N. Prokof'ev, Phys. Rev. Lett. 102, 150402 (2009)
S. Pilati, S. Giorgini, M. Modugno, N. Prokof'ev, New J. Phys. 12, 073003 (2010)

interaction strength
g=0.1



2D speckle disorder:

amplitude V_R

isotropic correlation length σ_R

$$\sigma_R = \sqrt{\frac{5}{2m\mu}}$$

(averaged over ≈40 disorder configurations)

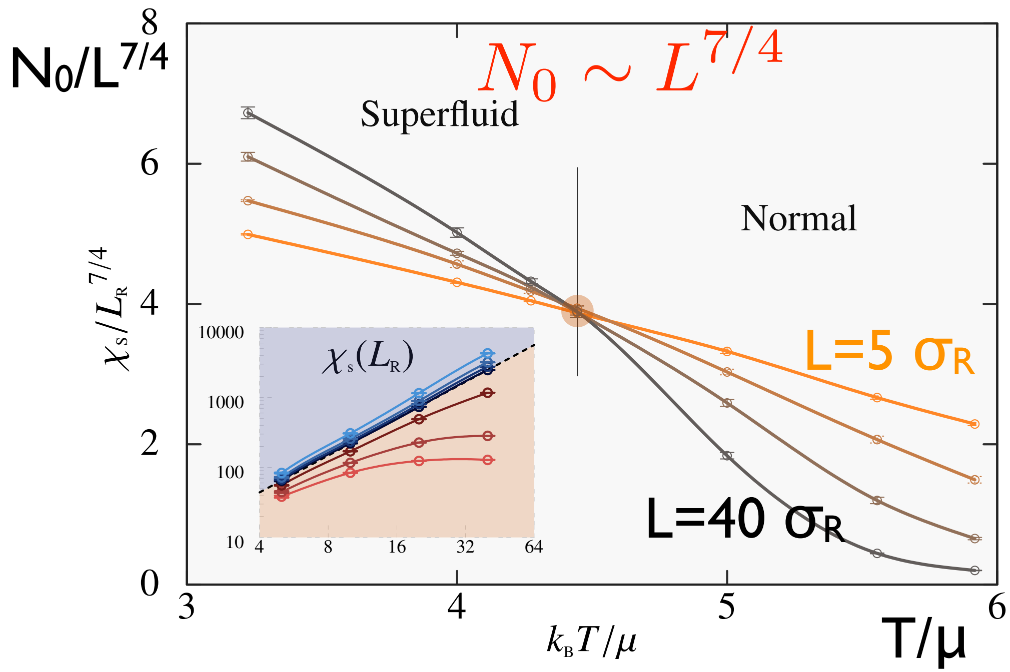
Dirty Bosons: Suprafluid transition

Number of condensed particles N_0 at fixed disorder amplitude V_R

at T_{KT}

superfluid phase:

$$N_0 \sim L^{2-\eta(T)}$$



normal phase:

$$N_0 = \mathcal{O}(1)$$

Suprafluid transition remains in Kosterlitz-Thouless universality class
up to high disorder amplitude
(Harris criterium: small amplitude)

Dirty Bosons in two dimensions: Phase diagram

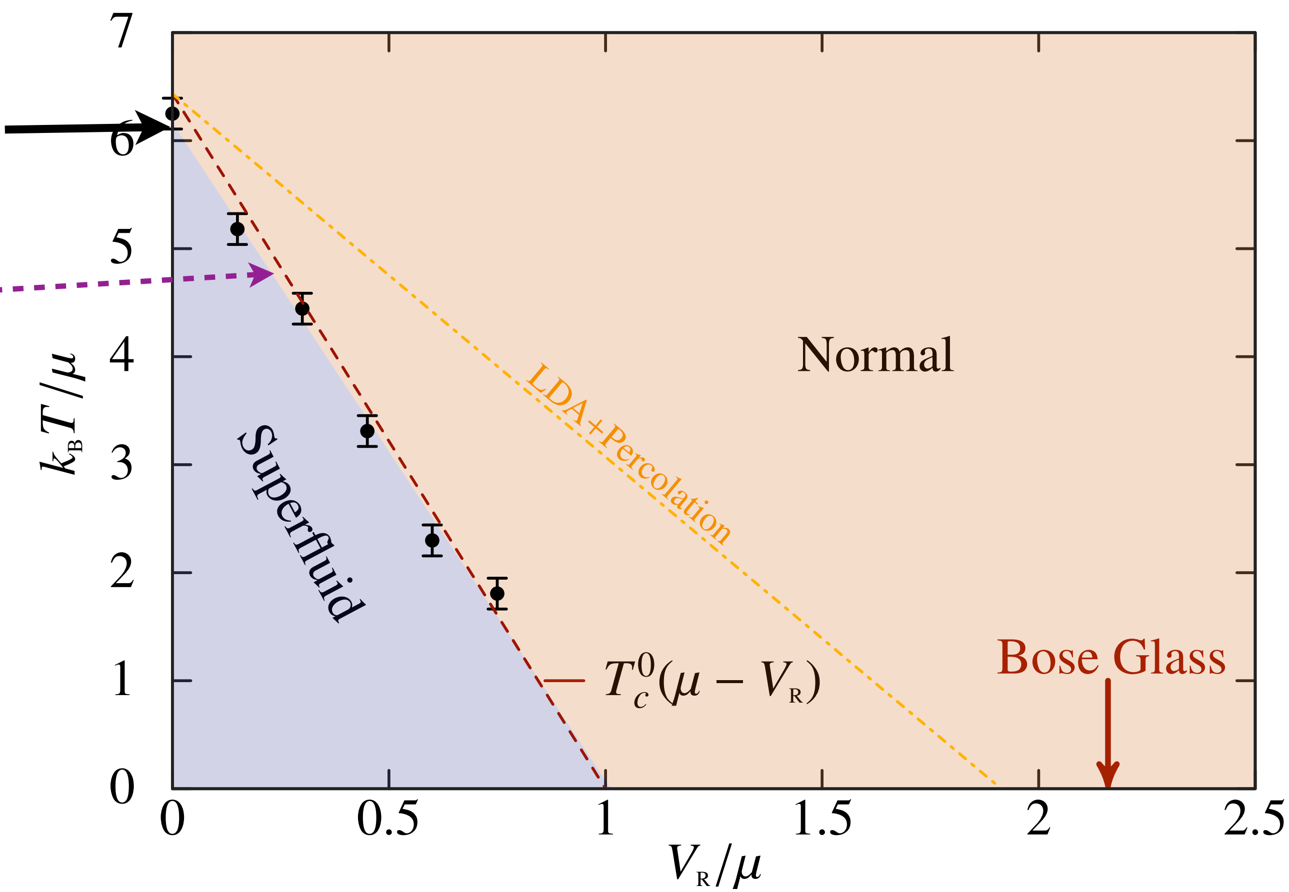
Phase diagram as a function of temperature T and disorder amplitude V_R

$2m\sigma^2\mu=5$ fixed

$T_c^0(\mu)$

Kosterlitz-Thouless universality class

(different from Percolation transition!)



$$T_c(V_R) \approx T_c^0(V_R - \mu)$$

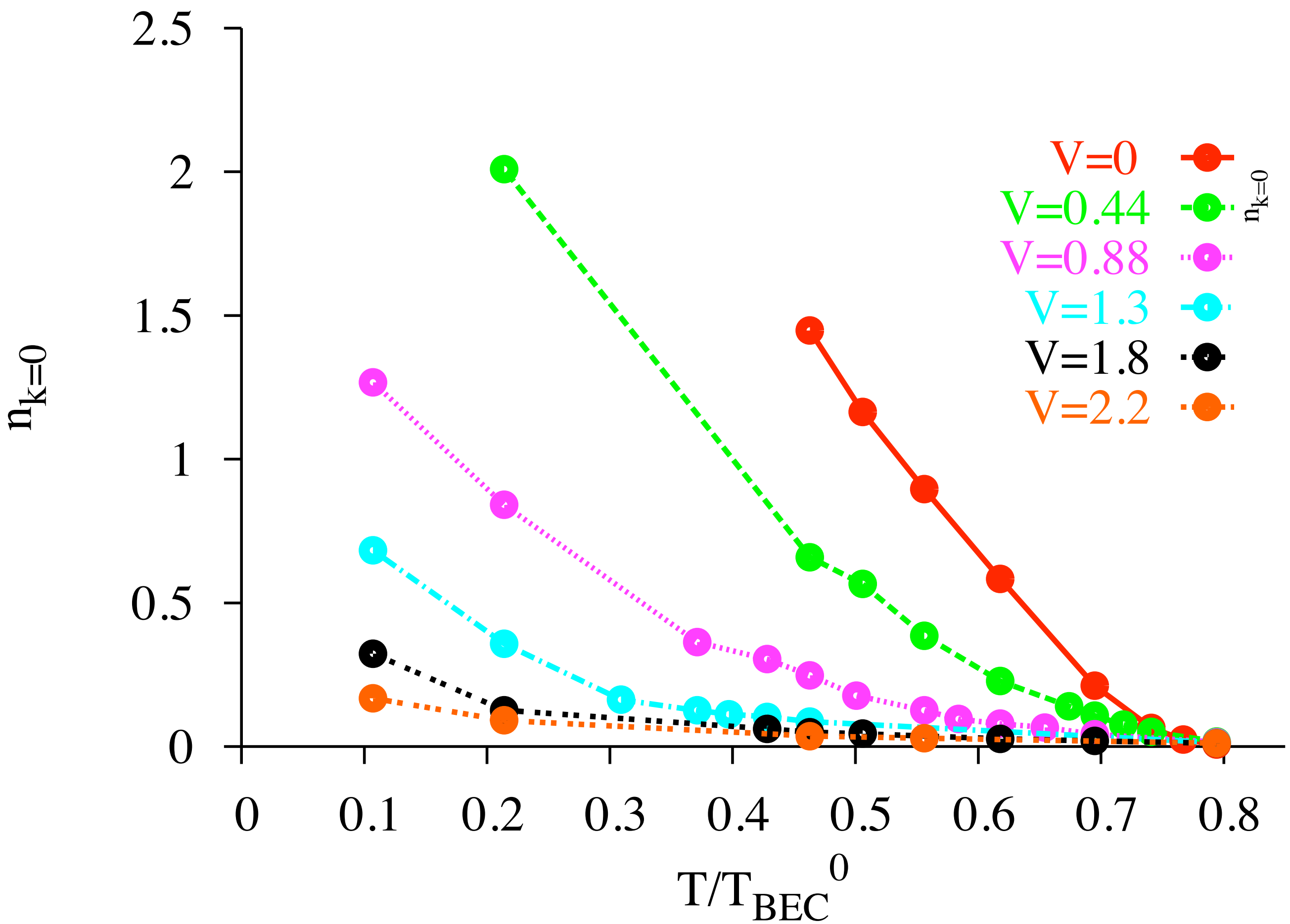
renormalization of μ compared to clean system

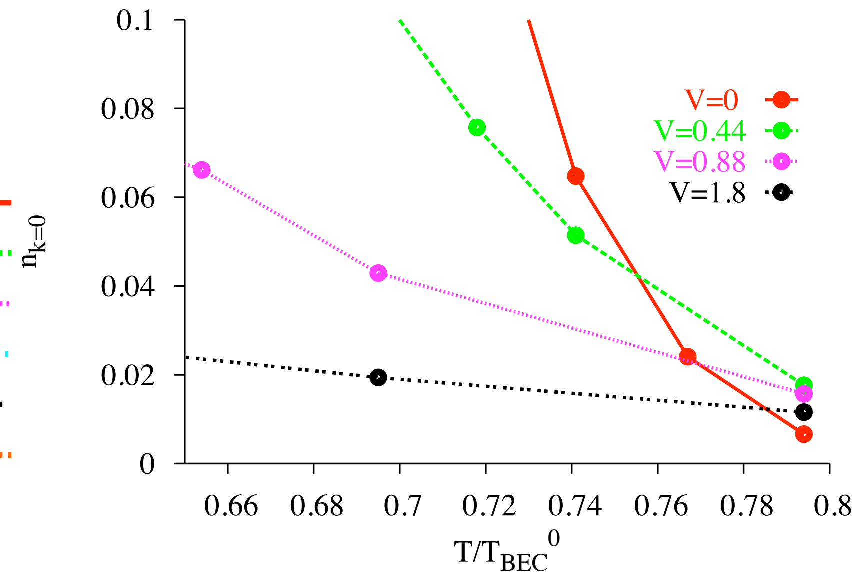
QMC in trap: Momentum distribution n_k

N=38 000, g=0.096, quasi2D, single disorder configuration

Peak density of (normalized) momentum distribution $n_{k=0}$

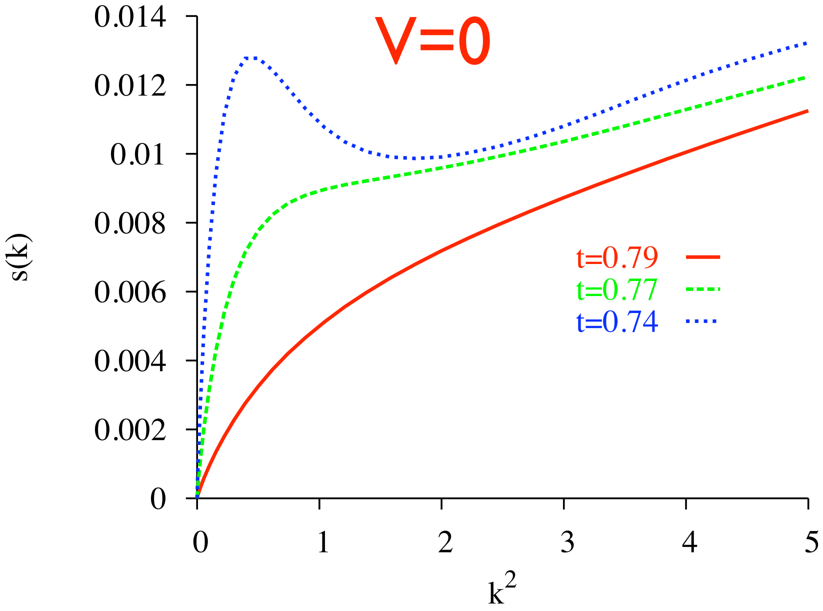
$$\int d\mathbf{k}\, n_k = 1$$

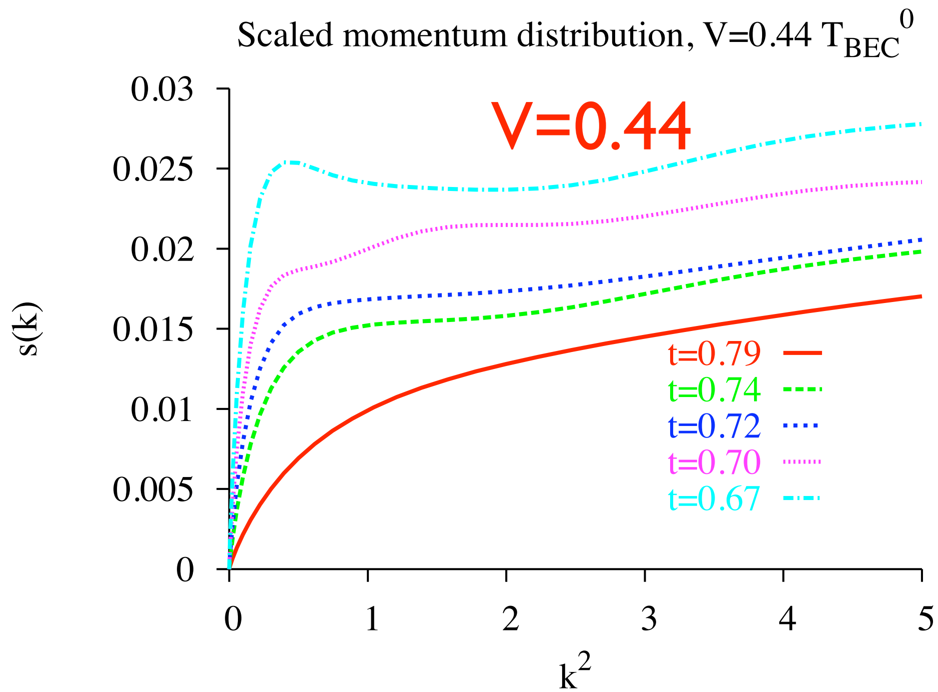


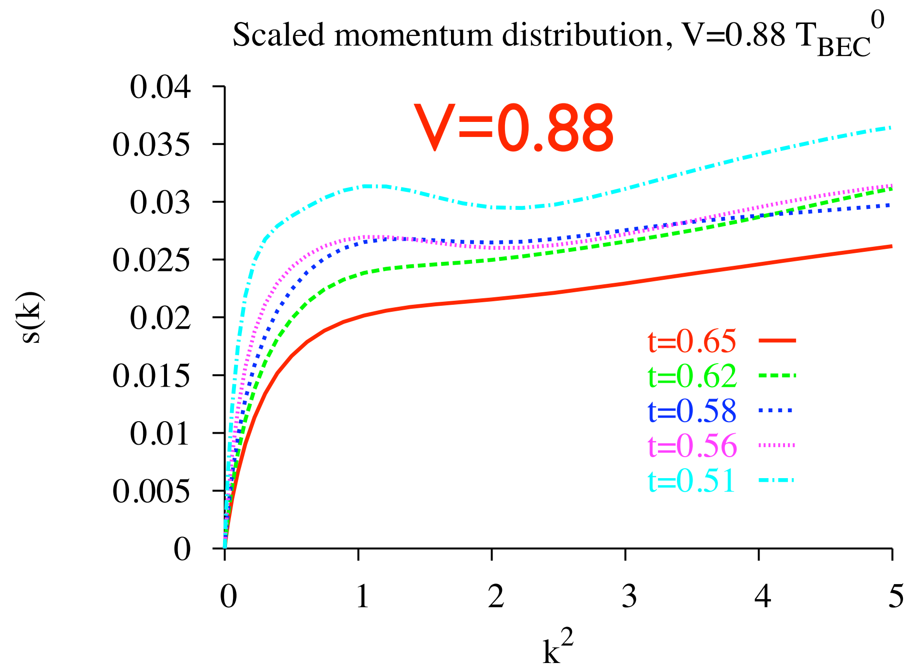


Trap, finite system, disorder: Where is the Kosterlitz-Thouless?

⇒ plot $s(k) = n_k k^{2-1/4}$

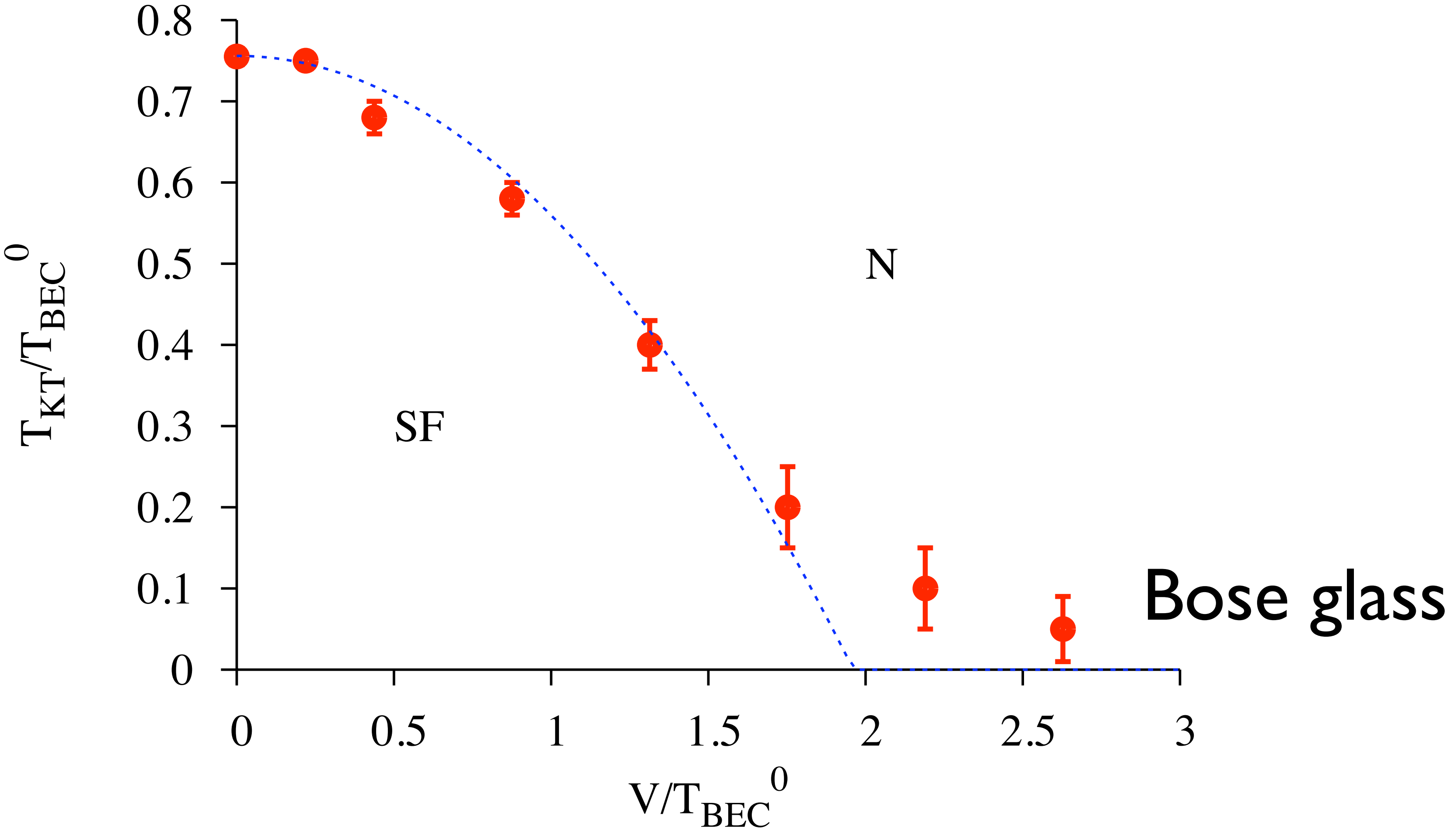






QMC: Phase Diagram in Trap

N=38 000, g=0.096, quasi2D, single disorder configuration, anisotropic speckle

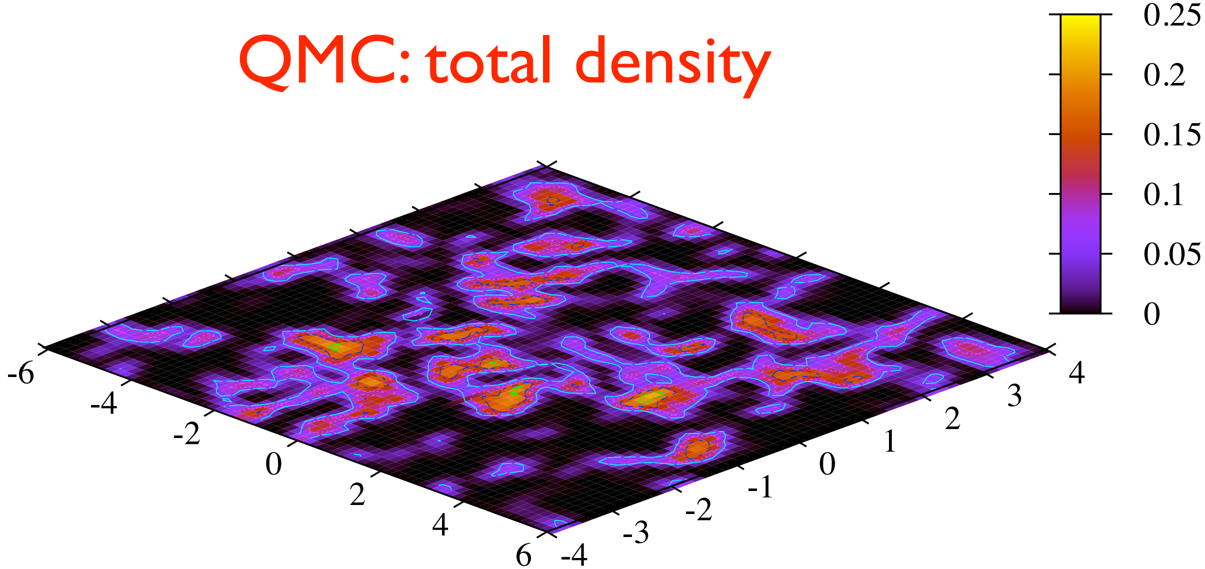


total density

QMC Densities

V=0.88 t=0.46

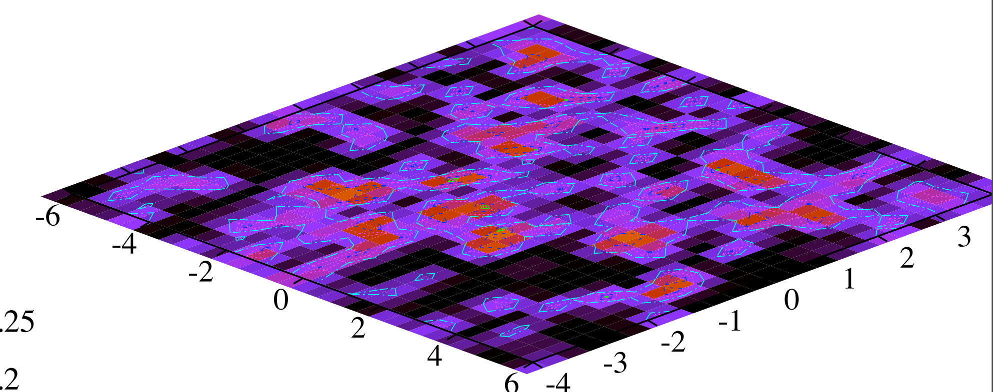
QMC: total density



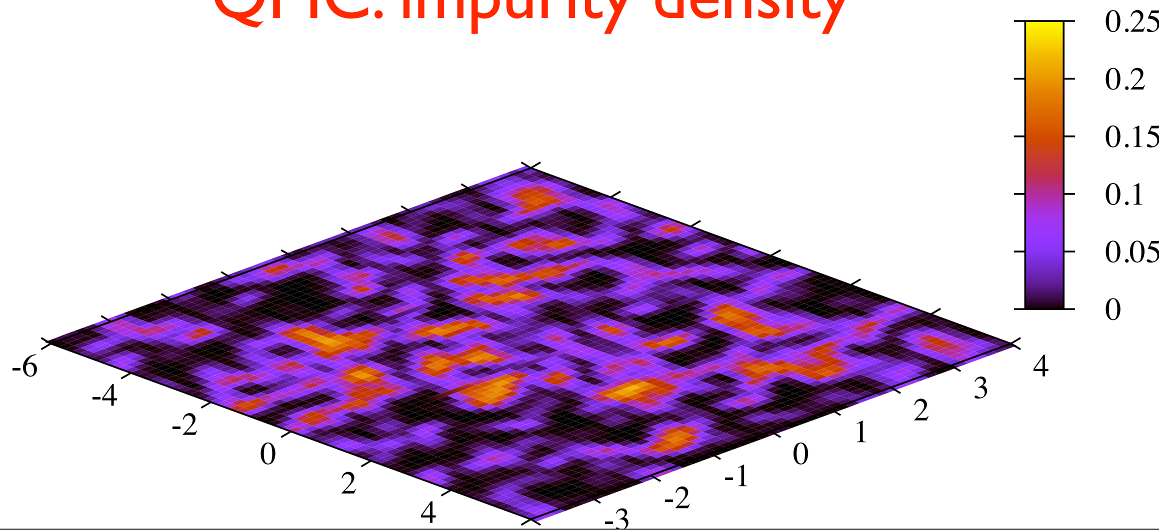
impurity density (exactly 1 cycle)

GPE V=0.88 T_{BEC}^{0}

Gross-Pitaevskii



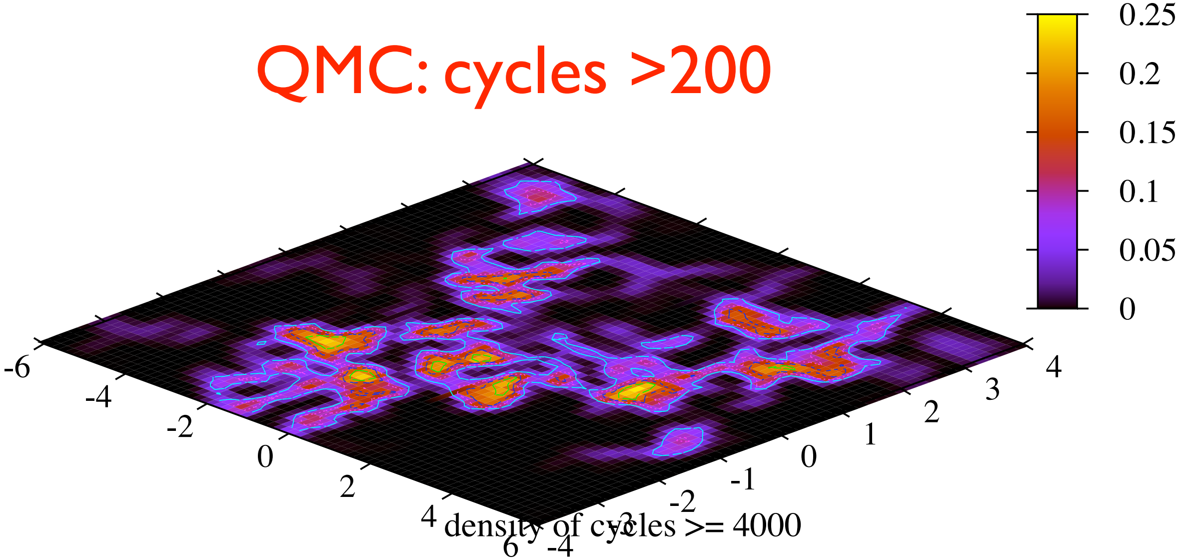
QMC: impurity density



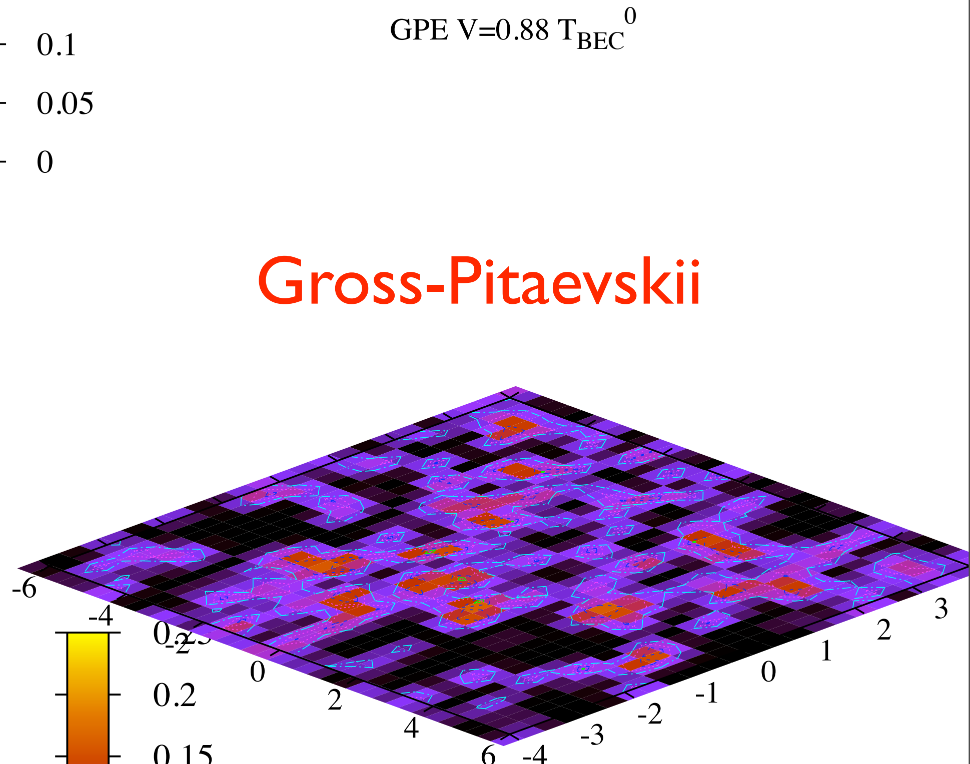
QMC Densities

V=0.88 t=0.46

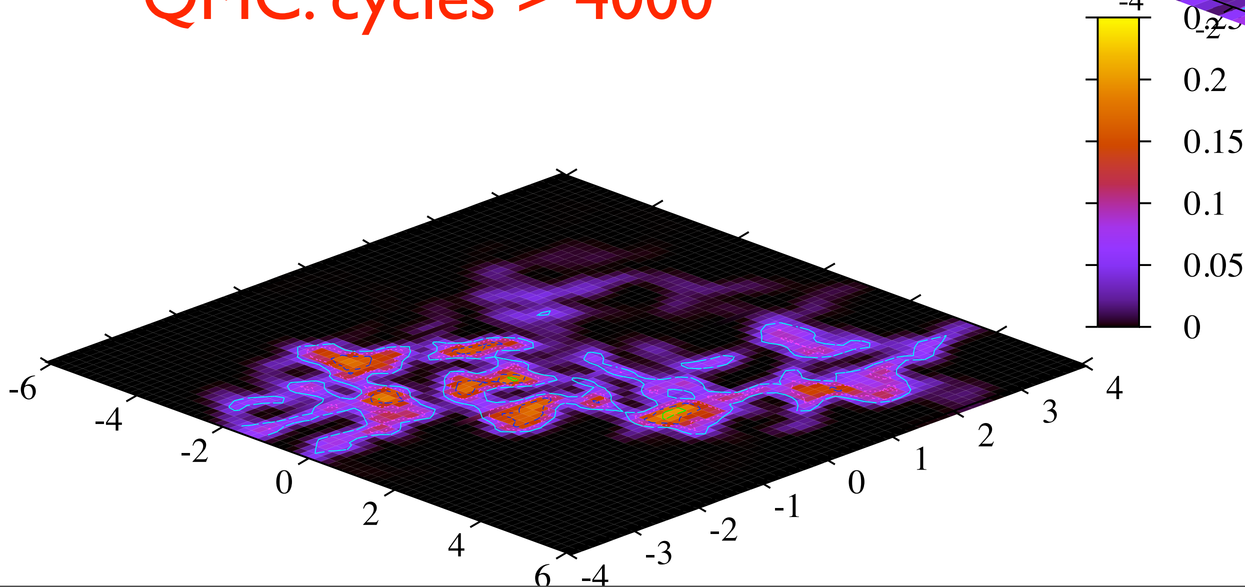
QMC: cycles >200



Gross-Pitaevskii



QMC: cycles > 4000



Transport : Conductance from QMC (homogenous)

G. Carleo, G. Boéris, M. Holzmann, and L. Sanchez-Palencia, Phys. Rev. Lett. 111, 037203 (2013)

current-correlations in imaginary time:

$$\langle J_\alpha(\mathbf{q},\tau) J_\alpha(-\mathbf{q},0)\rangle \equiv \frac{1}{Z}\mathrm{Tr}\left[e^{-(\beta-\tau)H} J_\alpha(\mathbf{q}) e^{-\tau H} J_\alpha(-\mathbf{q})\right]$$

conductance G(0) from inverse Laplace transform:

$$\lim_{q\to 0}\langle J_\alpha(\mathbf{q},\tau) J_\alpha(-\mathbf{q},0)\rangle = 2\int_{-\infty}^{\infty} d\omega \frac{\omega \exp(-\tau\omega)}{1-\exp(-\beta\omega)} G(\omega)$$

direct computations of current-correlations: large errors!

improved QMC estimator: large reduction of variance!

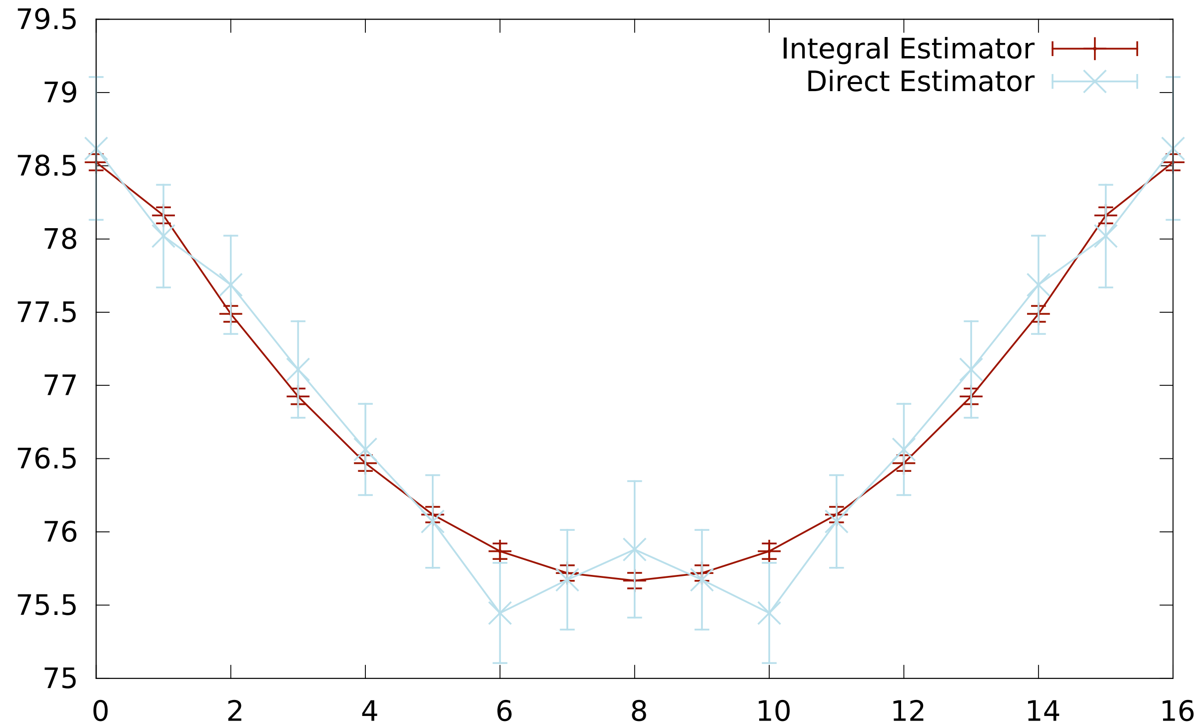


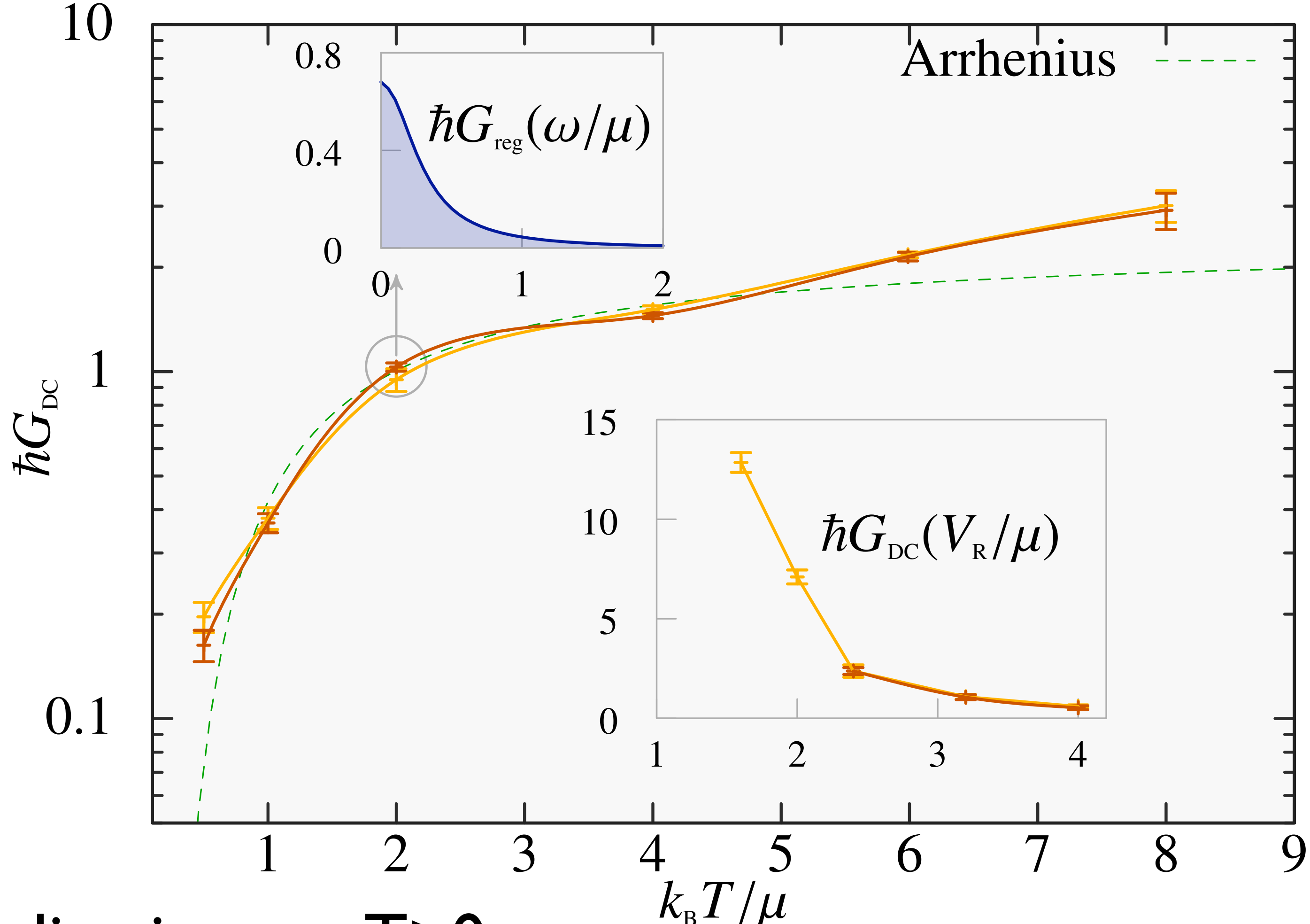
Figure 1: Current-current correlations deep in the disordered phase

Dirty Bosons: transport properties

«Conductance» G_{DC} at $V_R=2.4\mu$

$$G_{DC} \sim e^{-\Delta/T}$$

no indication of localized phase at T>0



Many-Body «delocalization» at T>0:

Perturbation theory around non-interacting localized phase diverges for continuum systems + correlated disorder

R. Nandkishore, cond-mat 1408.6235 (2014).

Summary-Outlook

- Disorder: Kosterlitz-Thouless vs Bose glass

  ✓ Kosterlitz-Thouless transition for disorder strength V smaller than chem. pot. μ

  ✓ Cross-over from normal phase to Bose glass at $T=0$ and strong disorder

∈ full phase-diagram: interaction g, correlation length σ ?

∈ Possibility of many-body (de-)localization at $T>0$?