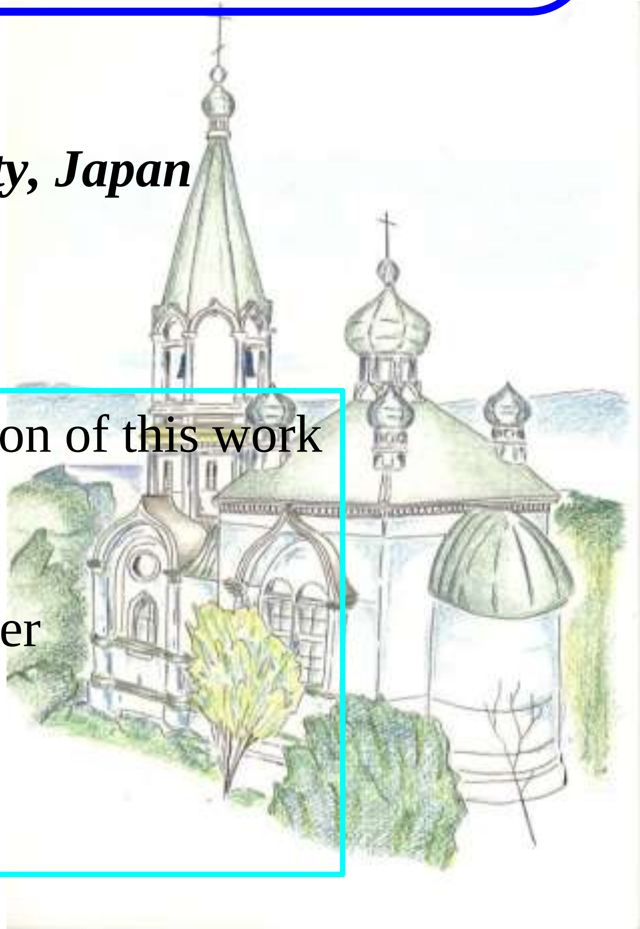
October 26-31, 2014, Workshop on Probing and Understanding Exotic Superconductors and Superfluids,
International Centre for Theoretical Physics, Trieste, Italy

Hetero-pairing and pseudogap phenomena in the BCS-BEC crossover regime of an ultracold Fermi gas with mass imbalance

Yoji Ohashi

Department of Physics, Keio University, Japan

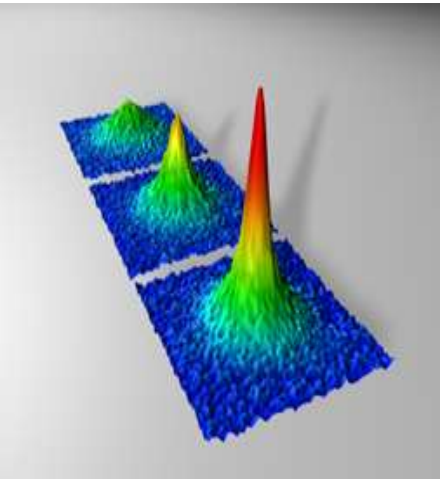


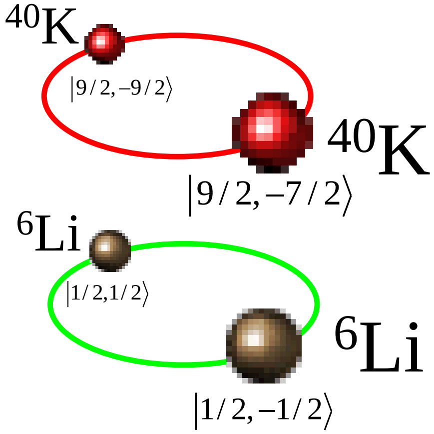
- **Collaborator: R. Hanai (Keio)**

- Introduction: ^{6}Li-^{40}K Fermi gas mixture and motivation of this work
- Formulation: self-consistent T-matrix theory
- Results
  - Tc of ^{6}Li-^{40}K Fermi gas in the BCS-BEC crossover
  - pseudogap and effects of mass imbalance
- Summary

Ultracold superfluid Fermi gas (2014 ~)

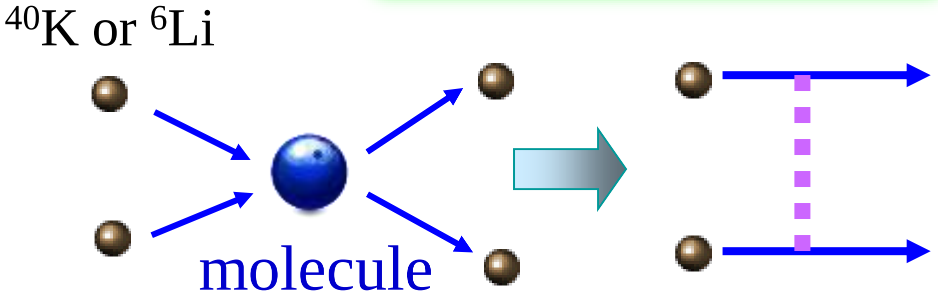
Fermi atom gas





atomic hyperfine state

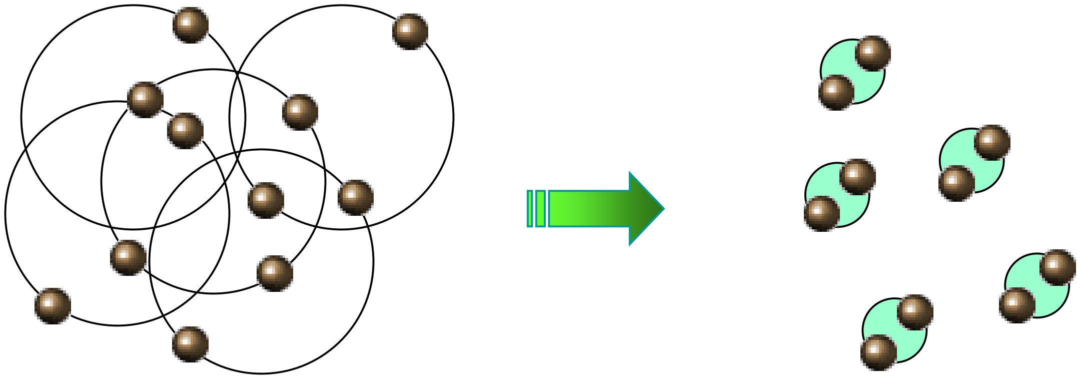
Feshbach resonance



$$V_{eff} = -g^2 \frac{1}{2\nu}$$

tunable **by magnetic field**

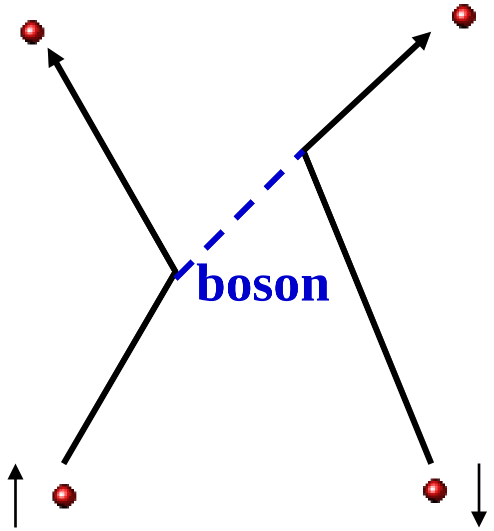
BCS-BEC crossover



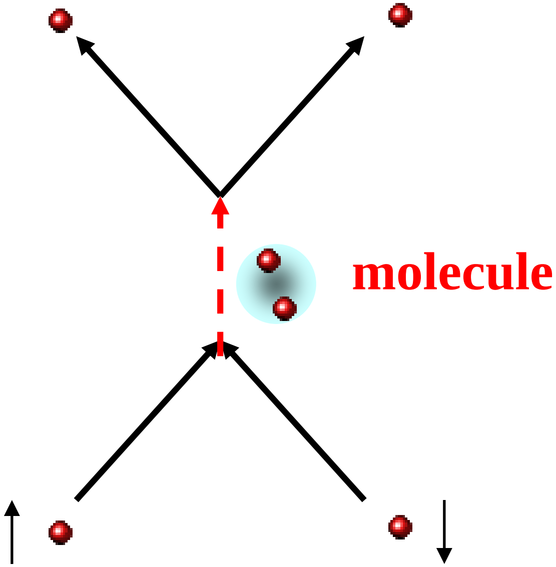
weak-coupling (BCS)

strong-coupling (BEC)

pairing interaction



Boson exchange mechanism



Feshbach mechanism

- superconductivity

  Phonon, AF spin fluctuations…..

- superfluid ^{3}He

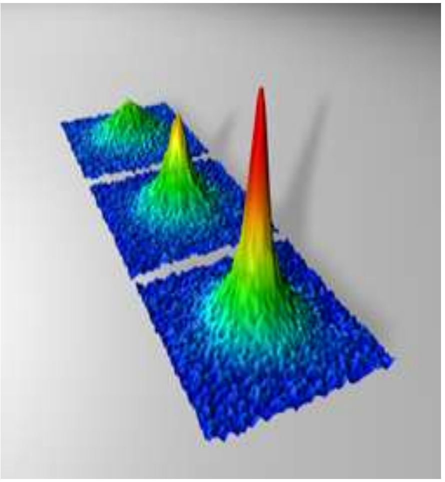
  Ferromagnetic spin fluctuations

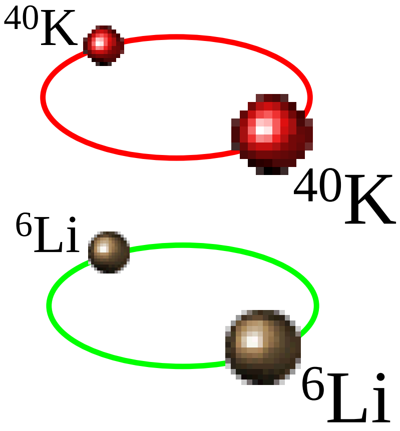
- superfluid Fermi gas

  ^{40}K, ^{6}Li

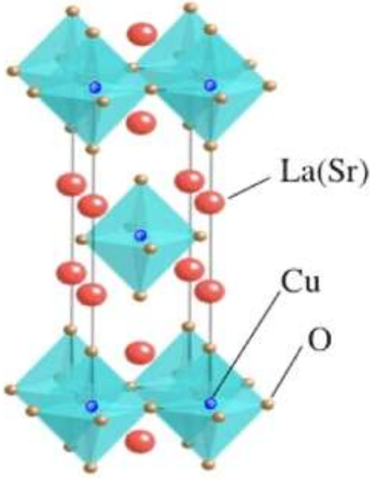
“Conventional” Fermi superfluids

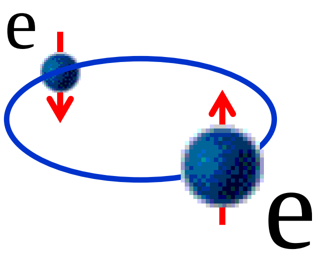
Fermi atom gas



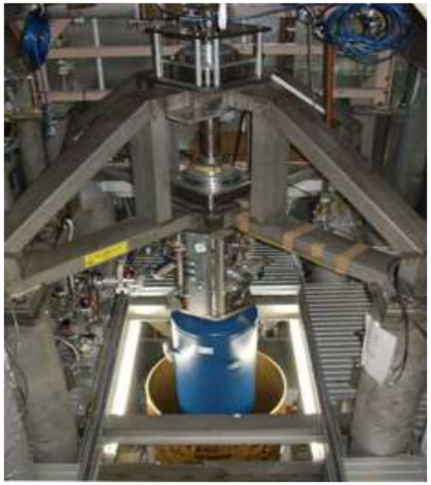


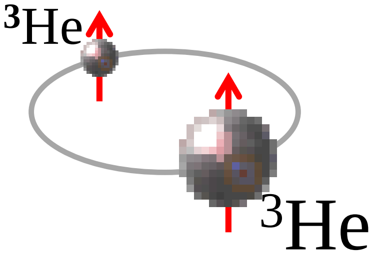
superconductivity



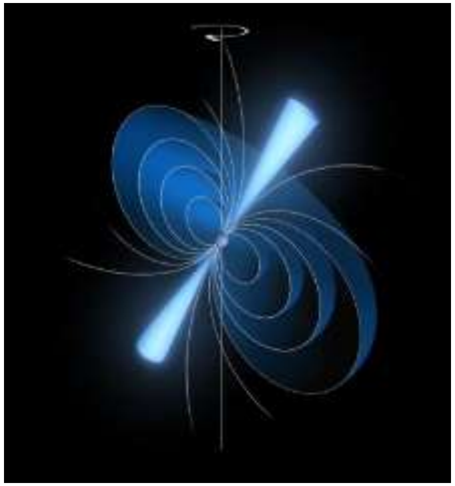


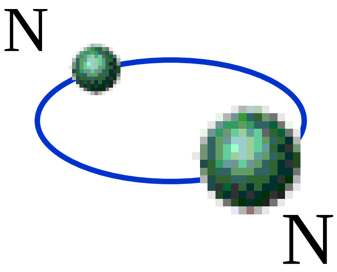
liquid 3He





neutron star

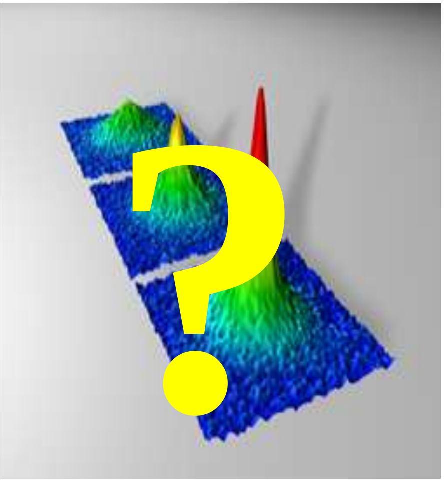


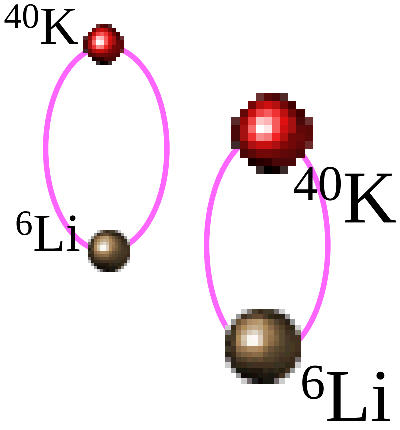


Cooper pairs are formed between the same kind of fermions.

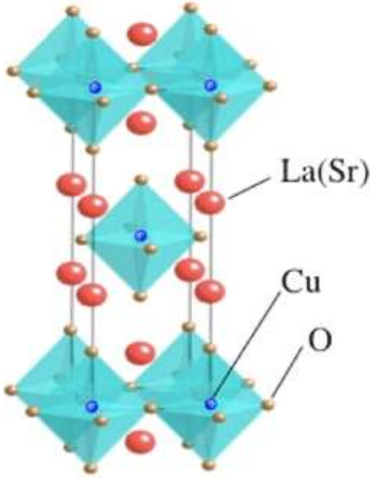
Next challenge in cold Fermi gas physics

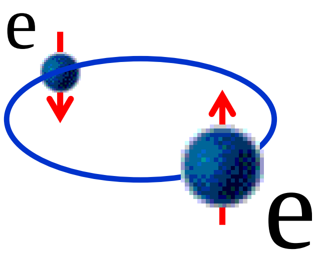
Fermi atom gas



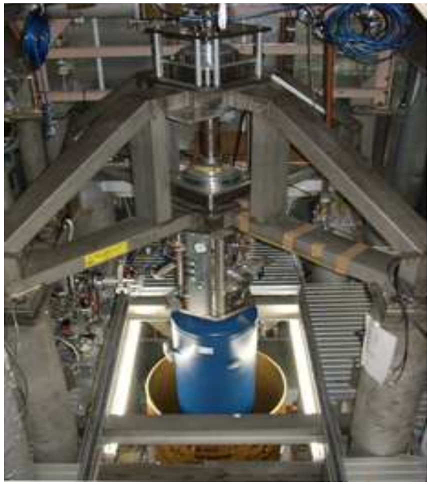


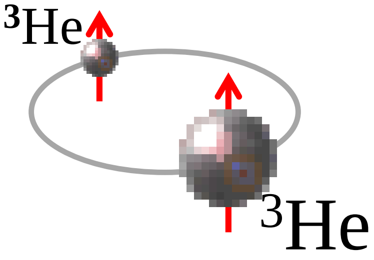
superconductivity



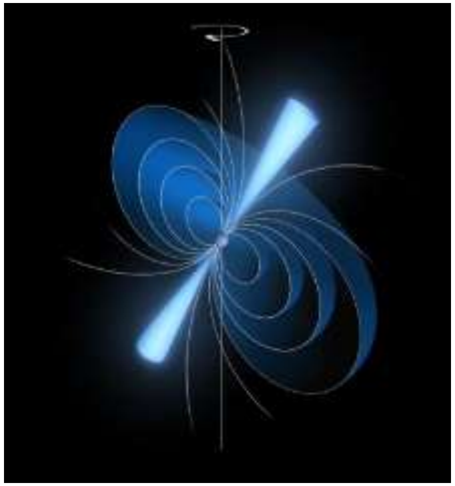


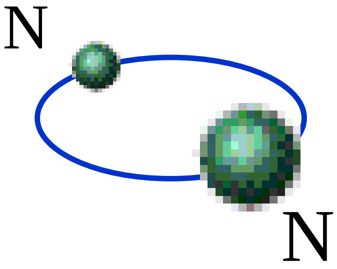
liquid 3He





neutron star

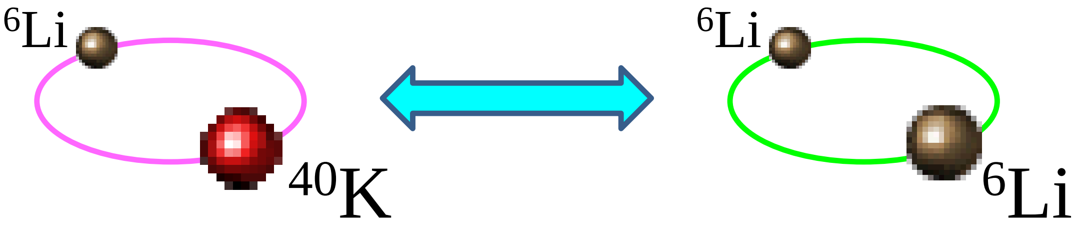




Fermi superfluid with hetero-Cooper pairs

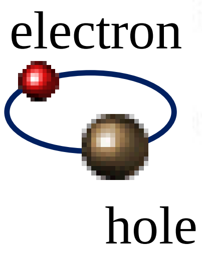
^{6}Li-^{40}K mixture: "mass imbalanced" Fermi gas

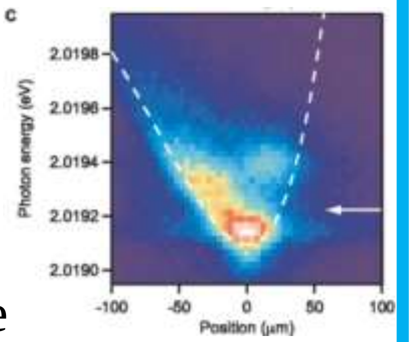
$$\boxed{\frac{m_{\mathrm{Li}}}{m_{\mathrm{K}}}=\frac{6}{40}=0.15<<1}$$



BCS-BEC crossover

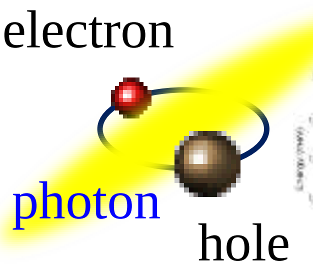
exciton condensate

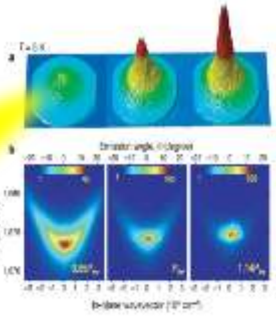




K. Yoshioka, *et al.*,
Nature Com.**2** (2011) 328.

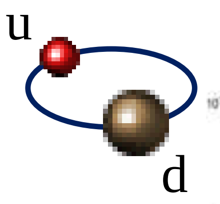
exciton-polariton gas

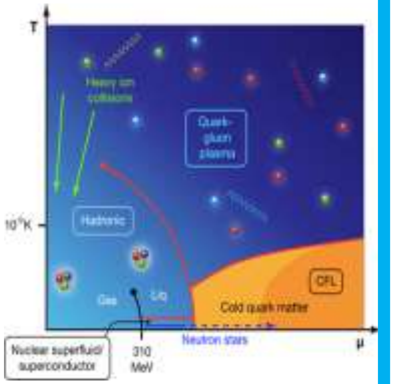




Kasprzak, *et al.*, Nature
443,409(2006).

color superconductivity





http://www.tech.plym.ac.uk/Research/
applied_mathematics/PPP.html

^{6}Li-^{40}K mixture: "mass imbalanced" Fermi gas

$$\boxed{\frac{m_{\mathrm{Li}}}{m_{\mathrm{K}}} = \frac{6}{40} = 0.15 << 1}$$

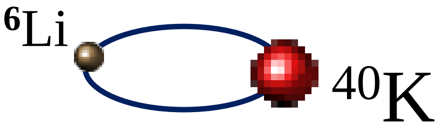
- ✓ **Fermi degeneracy** $N \sim 10^{5}$

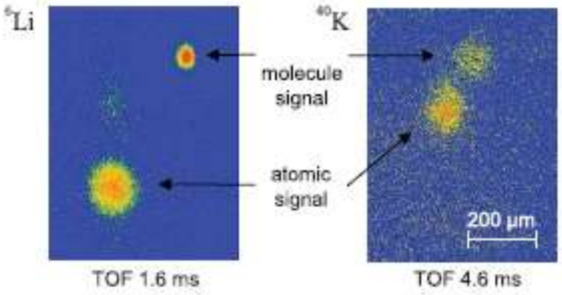
$$T \sim 200\mathrm{nK} < T_{\mathrm{F}}^{\mathrm{K}} = 0.47\mu\mathrm{K},\ T_{\mathrm{F}}^{\mathrm{Li}} = 1.2\mu\mathrm{K}$$

M. Taglieber, *et al.*,
PRL **100**, 010401 (2008).

- ✓ **Feshbach resonance and ^{6}Li-^{40}K pairs**

A. Voigt, *et al.*,
PRL **102**, 020405 (2009).

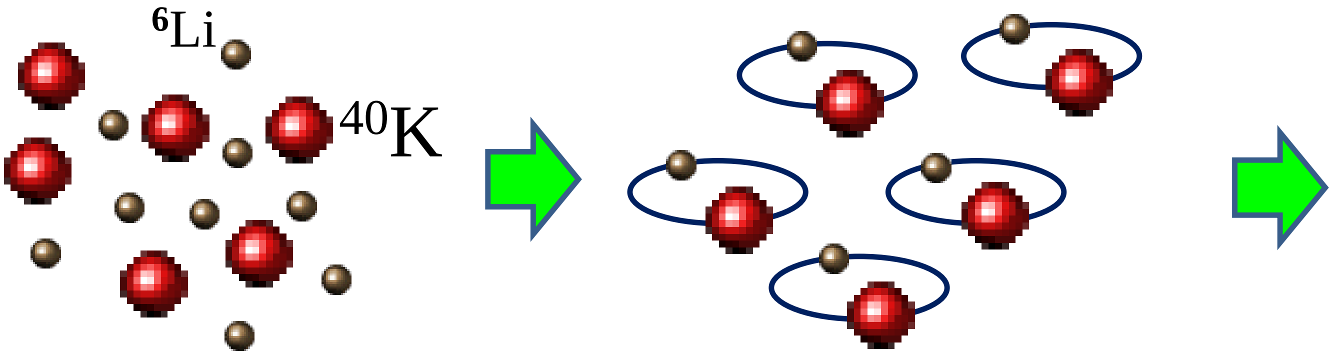




In the current stage of research for ^{6}Li-^{40}K Fermi-Fermi mixture, the quantitative prediction of Tc is a crucial theoretical issue.

Today’s talk

We theoretically investigate an ultracold Fermi gas with mass imbalance, focusing on ^{6}Li-^{40}K mixture. Including strong-coupling effects within the self-consistent T-matrix theory, we clarify how the mass imbalance affects T_c , as well as physical properties of this system, in the BCS-BEC crossover region.

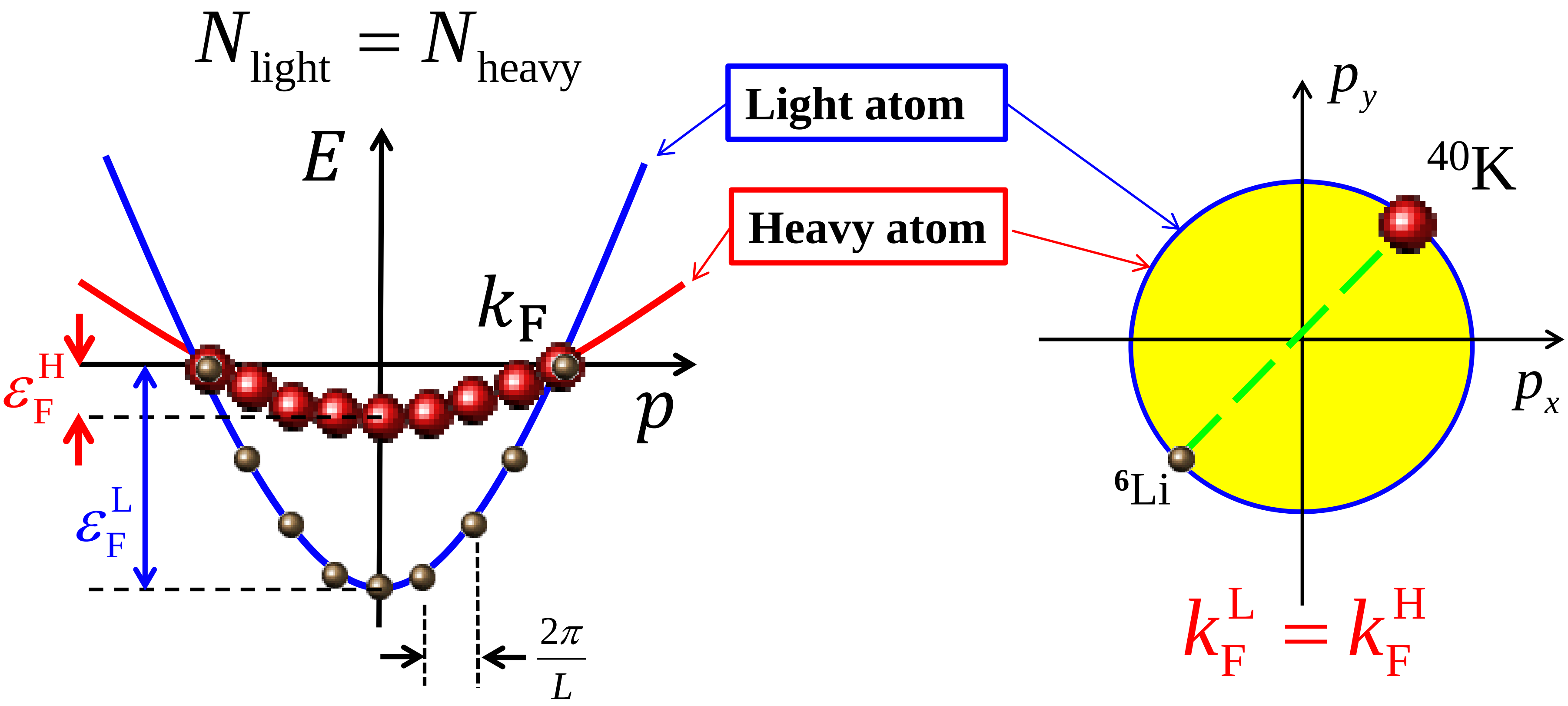


^{6}Li-^{40}K SF?

- **Keys to understand physics of *mass-imbalanced* Fermi gas**
    - **thermal excitations**
    - **Fermi surface size**
    - **scaled temperature (in terms of T_F)**

Key physics in mass-imbalanced Fermi gas

$T = 0$



- <u>“scaled” temperature</u>

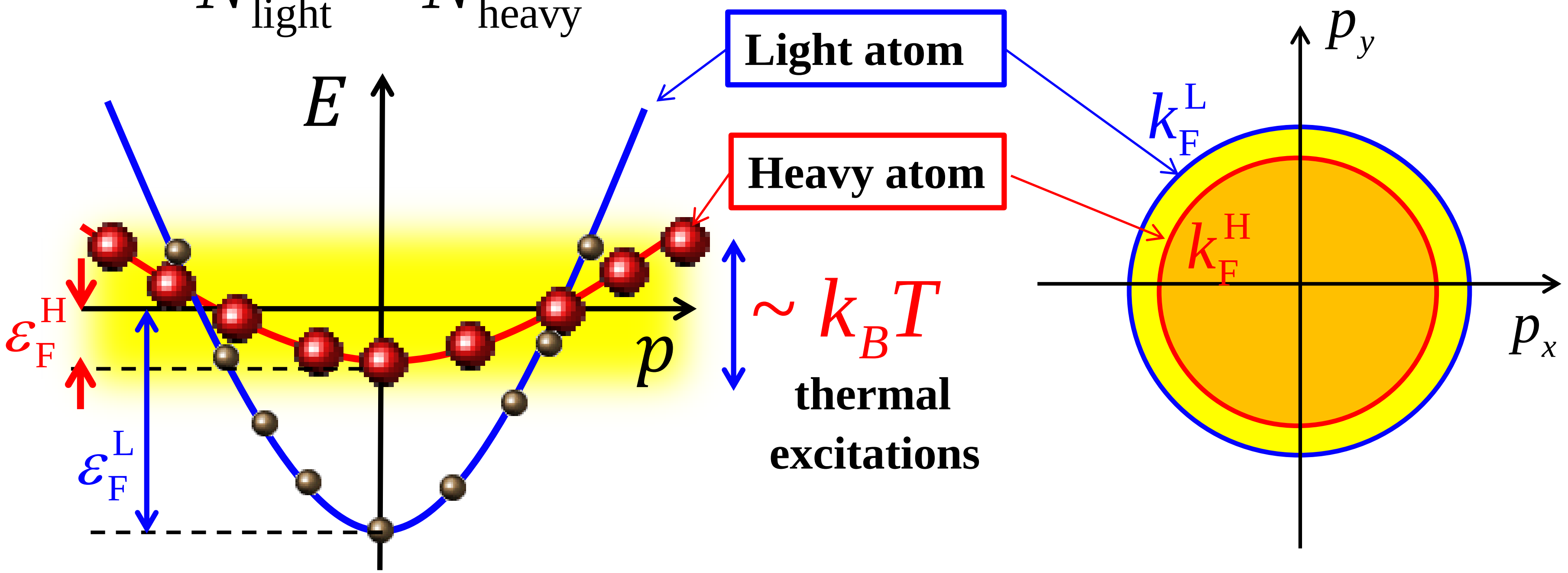
$$\frac{T}{T_F^L} < \frac{T}{T_F^H}$$

: **Heavy atoms feel higher *scaled* temperature.**

Key physics in mass-imbalanced Fermi gas

$T > 0$

$$N_{\text{light}} = N_{\text{heavy}}$$



Light atom

Heavy atom

$\sim k_B T$

thermal excitations

- **“Fermi surface size at T>0”**

$$k_F^{L,H} = \sqrt{2m_{L,H}\mu_{L,H}} = k_F\sqrt{1-\frac{\pi^2}{12}\left(\frac{T}{T_F^{L.H}}\right)^2}$$

⇨ $k_{\mathrm{F}}^{\mathrm{L}} > k_{\mathrm{F}}^{\mathrm{H}}$ **: Temperature works as an effective magnetic field.**

Formulation: model mass-imbalanced Fermi gas

$$H=\sum_{\boldsymbol{p},\sigma}\xi_{\boldsymbol{p}\sigma}c^{\dagger}_{\boldsymbol{p}\sigma}c_{\boldsymbol{p}\sigma}-U\sum_{\boldsymbol{q},\boldsymbol{p},\boldsymbol{p}'}c^{\dagger}_{\boldsymbol{p}+\frac{\boldsymbol{q}}{2},L}\,c^{\dagger}_{-\boldsymbol{p}+\frac{\boldsymbol{q}}{2},H}\,c_{-\boldsymbol{p}'+\frac{\boldsymbol{q}}{2},H}\,c_{\boldsymbol{p}'+\frac{\boldsymbol{q}}{2},L}$$

$$\xi_{\boldsymbol{p}\sigma}=\frac{p^2}{2\boldsymbol{m_\sigma}}-\mu_\sigma$$

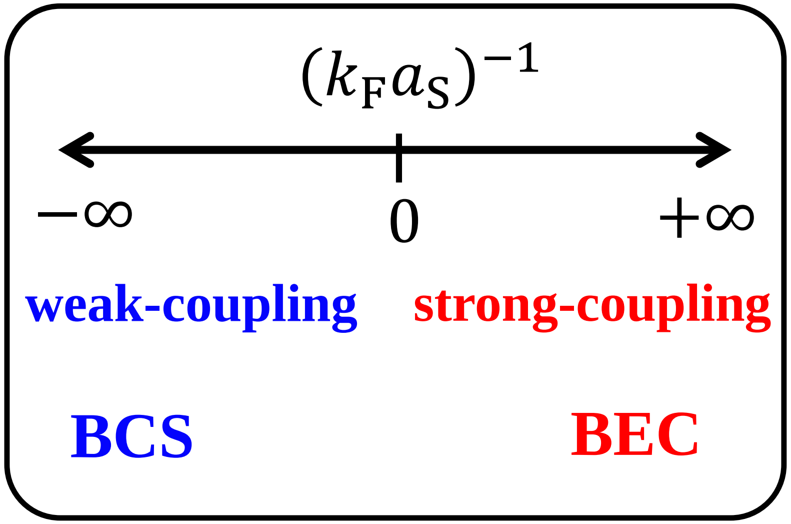
$$\sigma=L,H$$

$N_\mathrm{L}=N_\mathrm{H}$

Effects of mass difference

$$\frac{4\pi a_s}{m}=\frac{-U}{1-U\sum_{\boldsymbol{p}}^{\omega_c}\frac{m}{p^2}}$$

$$m=\frac{2m_L m_\mathrm{H}}{m_L+m_\mathrm{H}}$$

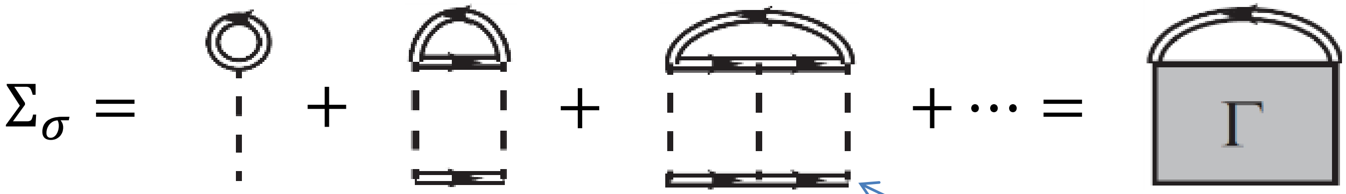


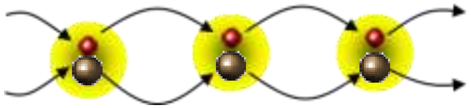
Formulation: self-consistent T-matrix approximation

- **single-particle Green's function**

$$G_{\boldsymbol{p}\sigma}(i\omega_n) = \frac{1}{i\omega_n - \varepsilon_{\boldsymbol{p}} + \mu_\sigma - \Sigma_{\boldsymbol{p}\sigma}(i\omega_n)}$$

self-energy describing fluctuations in the Cooper channel





Dressed G

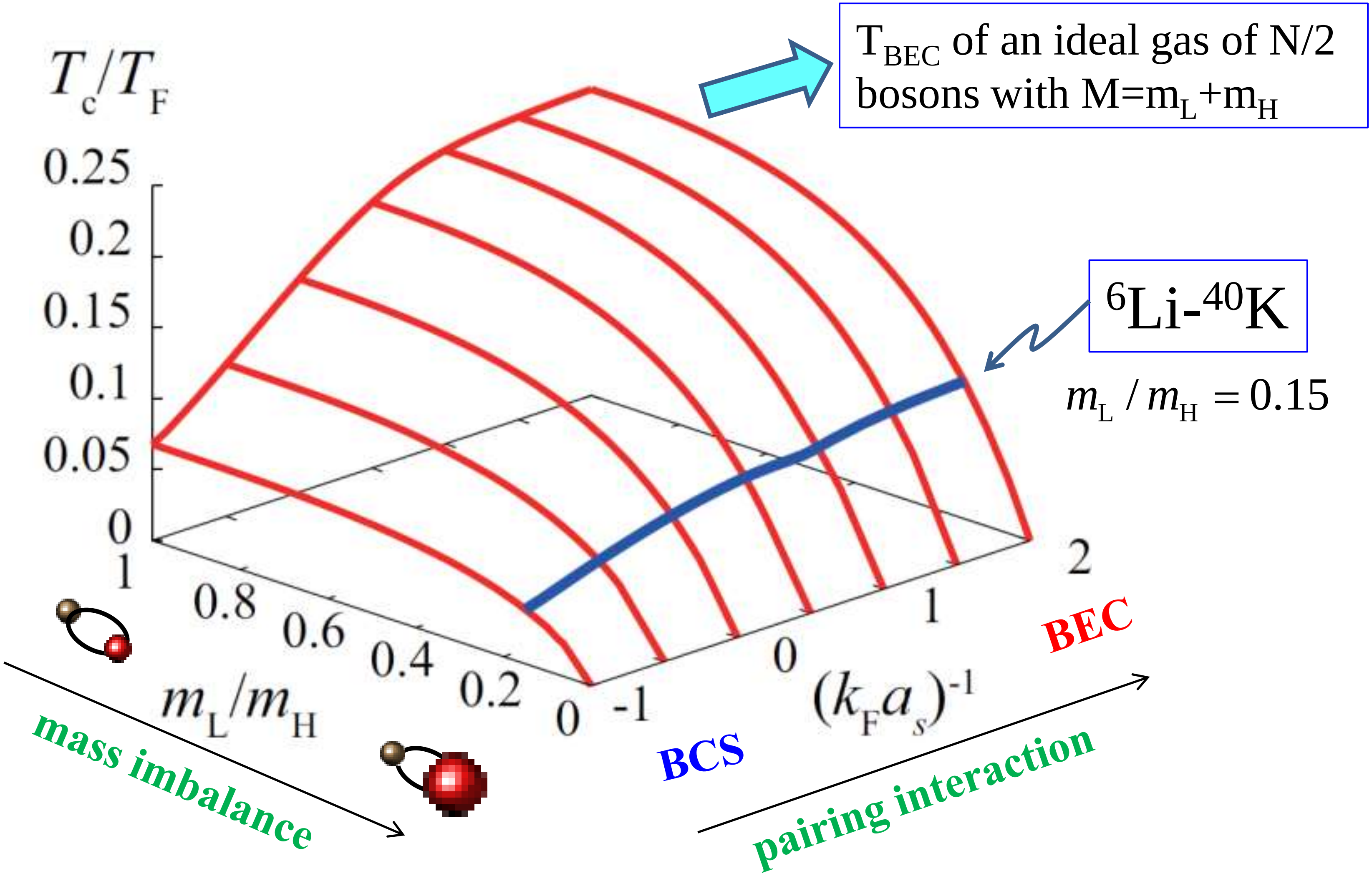
- **Tc: Thouless criterion**

$$\Gamma^{-1}(\mathbf{q}=0, i\nu_n = 0) = 0$$

$$N_\sigma = -T\sum_{\mathbf{p}, i\omega_n} G_\sigma(\mathbf{p}, i\omega_n) \quad (N_L = N_H)$$

We self-consistently determine T_c, μ_L, and μ_H.

BCS-BEC crossover behavior of Tc in the presence of mass imbalance



T_{BEC} of an ideal gas of N/2 bosons with $M=m_L+m_H$

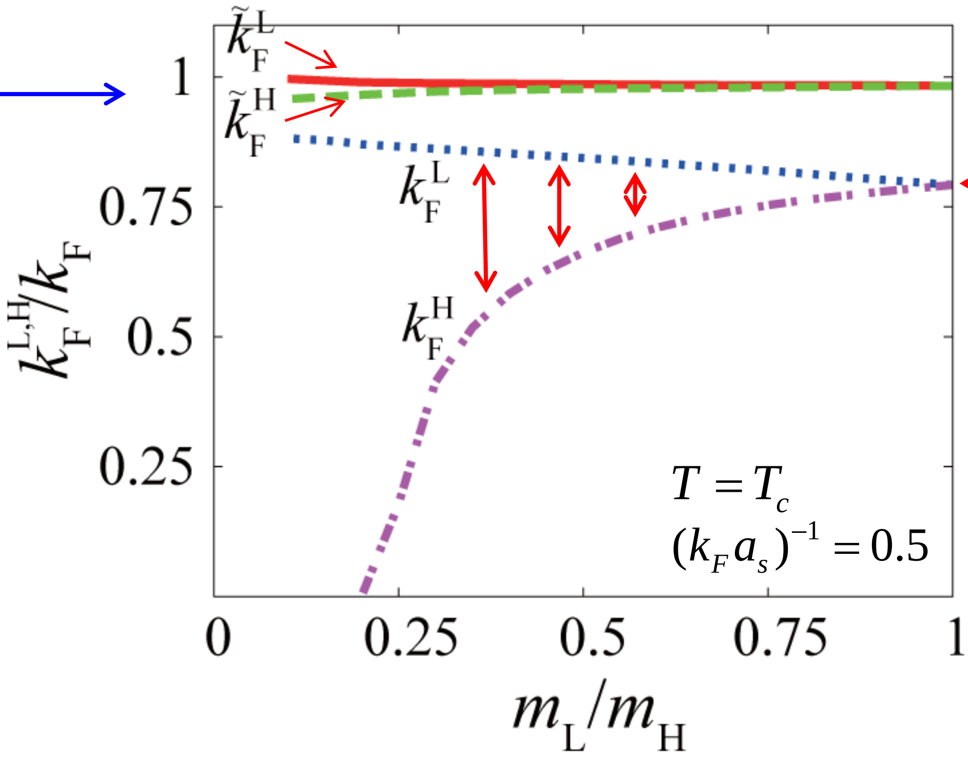
^{6}Li-^{40}K

$m_L / m_H = 0.15$

Hanai, Ohashi, PRA 90 (2014) 043622

NOTE: T_F is the Fermi temperature of an ideal Fermi gas with $m=2m_L m_H/(m_L+m_H)$.

“effective Fermi surface size” at Tc



free Fermi gas

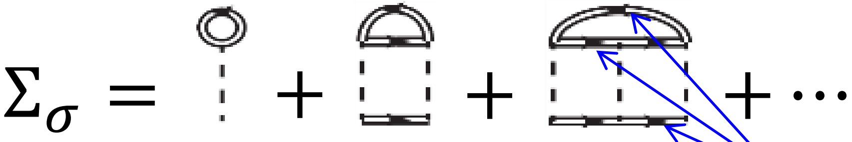
$$k_F^{L,H} = \sqrt{2m_{L,H}\mu_{L,H}}$$

$$\tilde{k}_F^{L,H} = \sqrt{2m_{L,H}\left[\mu_{L,H} - \mathrm{Re}[\Sigma_{L,H}(\tilde{k}_F^{L,H}, \omega = 0)]\right]}$$

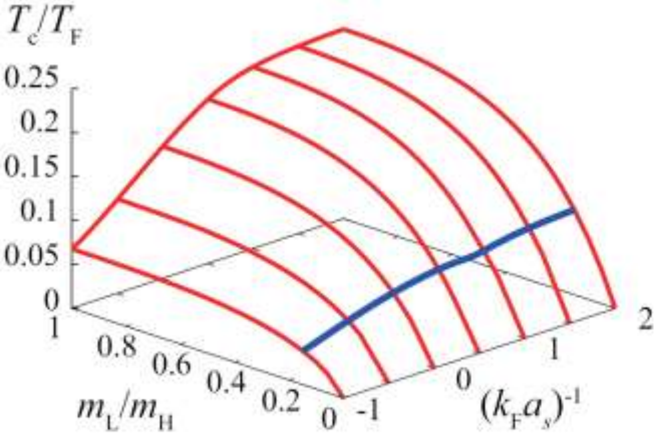
Inclusion of self-energy correction, describing pairing fluctuations

$$G_{\boldsymbol{p}\sigma}(i\omega_n) = \frac{1}{i\omega_n - \varepsilon_{\boldsymbol{p}} + \mu_\sigma - \Sigma_{\boldsymbol{p}\sigma}(i\omega_n)}$$

"effecvtive Fermi surface size" at Tc

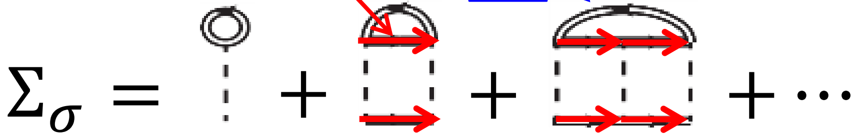


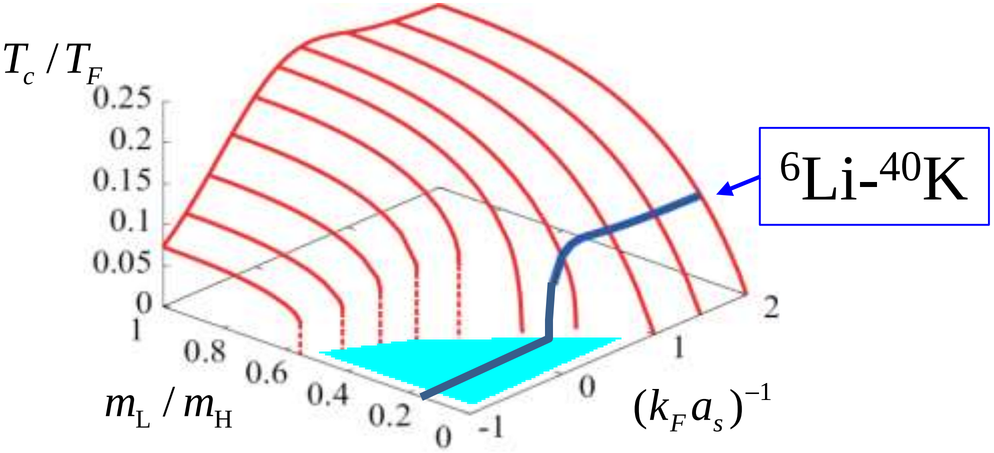
$$G_\sigma = \frac{1}{i\omega - \xi_{\mathbf{p}\sigma} - \Sigma_\sigma}$$



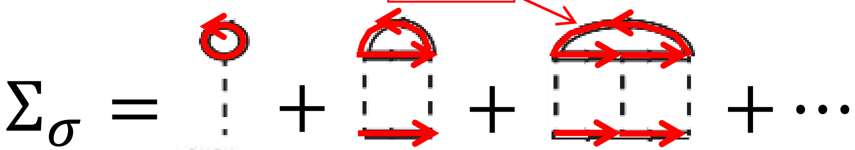
$$G_\sigma^{\text{bare}} = \frac{1}{i\omega - \xi_{\mathbf{p},\sigma}}$$

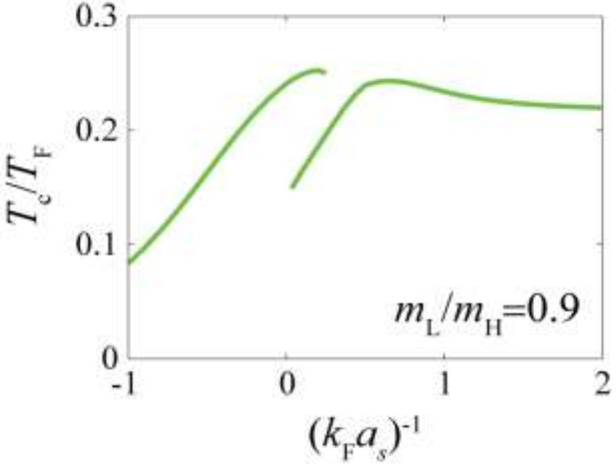
G





G_σ^{bare}

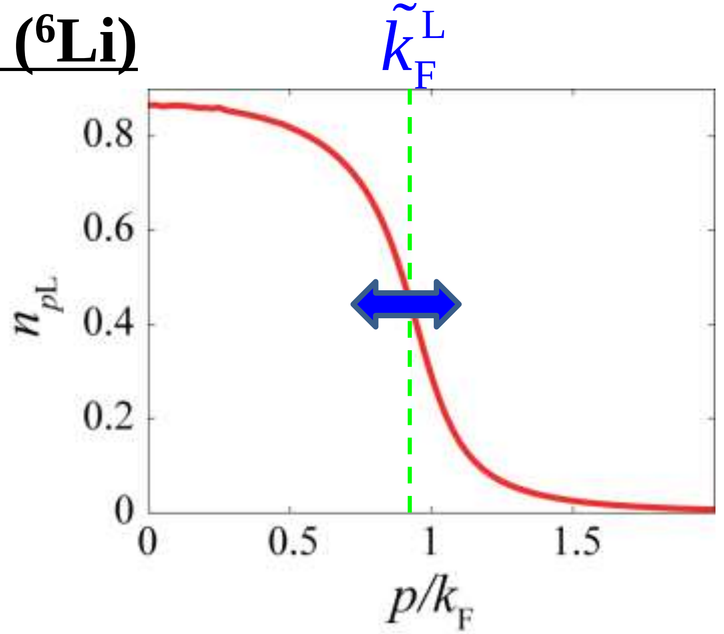


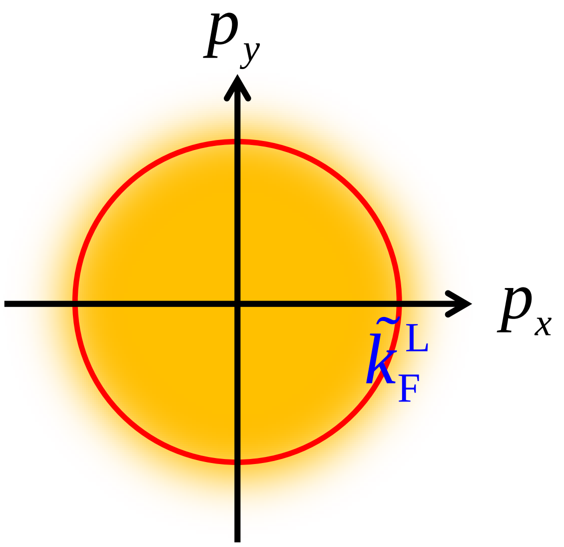


momentum distribution at Tc (m_L/m_H=0.15)

light atom (^{6}Li)

$$\left\langle c^{\dagger}_{\mathbf{p},\mathrm{L}} c_{\mathbf{p},\mathrm{L}} \right\rangle$$



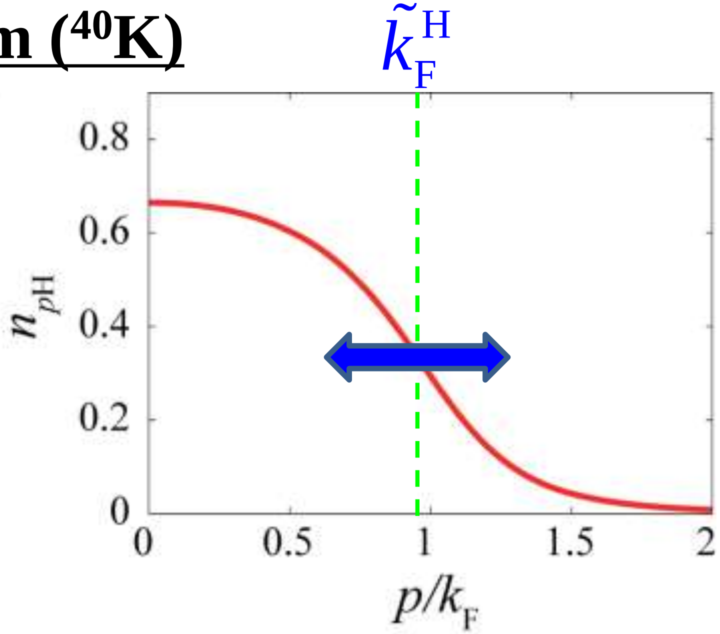


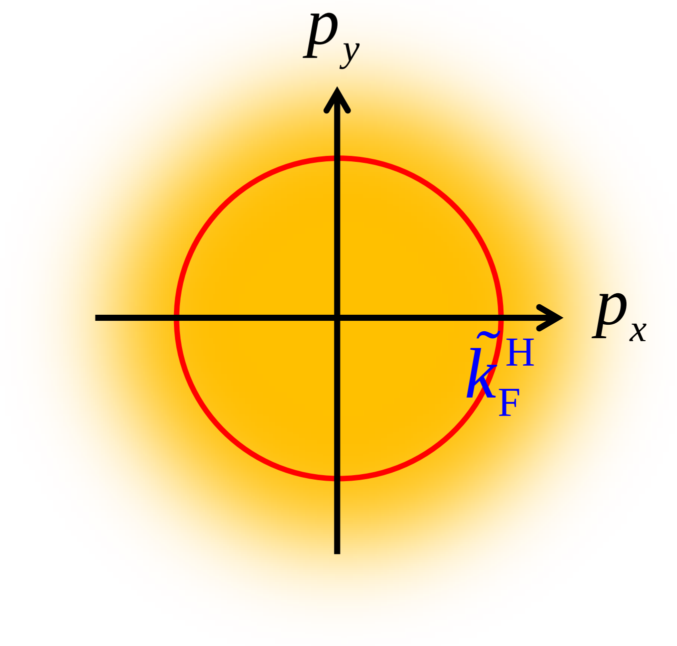
$$\frac{T}{T_F^L}$$

$$\wedge$$

heavy atom (^{40}K)

$$\left\langle c^{\dagger}_{\mathbf{p},\mathrm{H}} c_{\mathbf{p},\mathrm{H}} \right\rangle$$

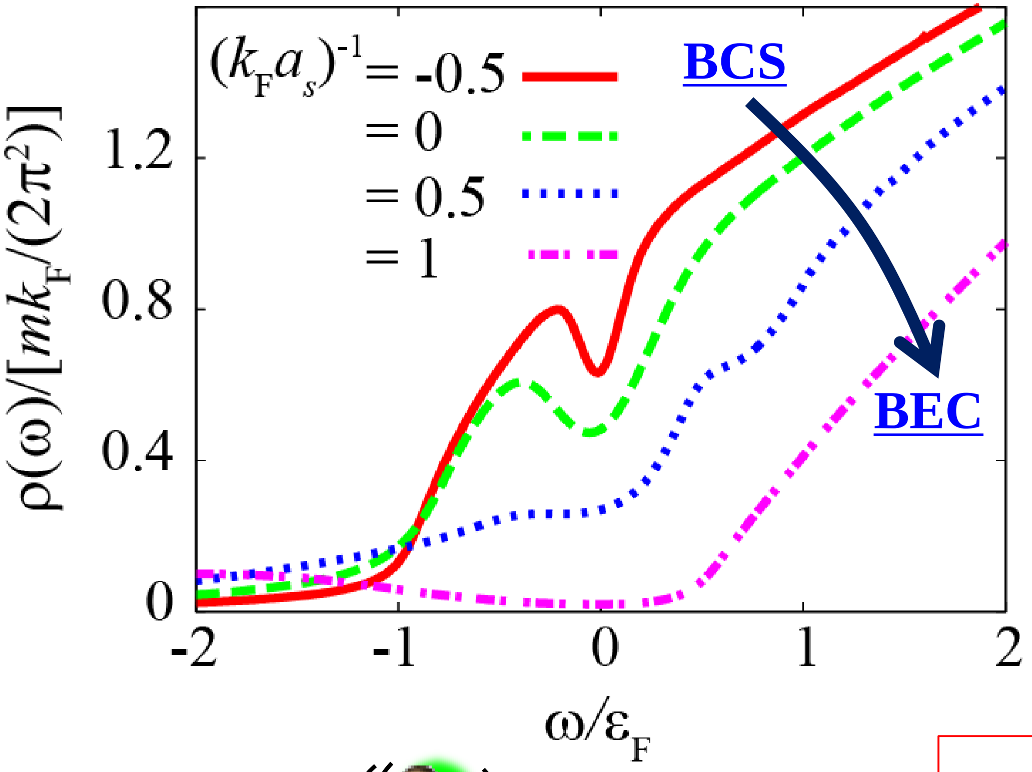


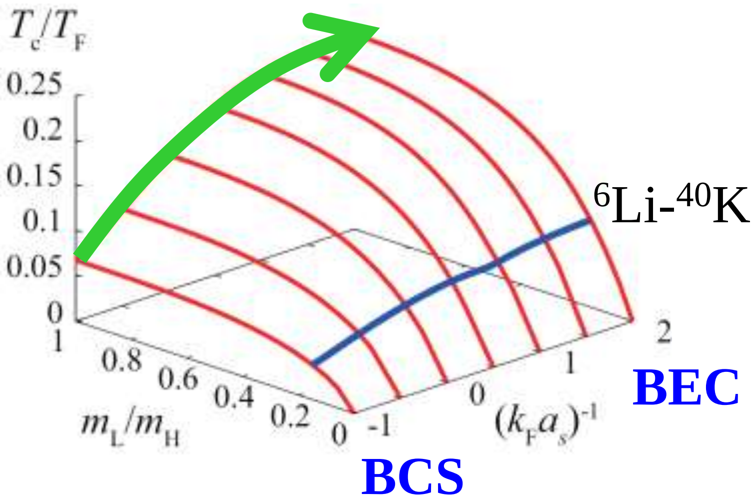


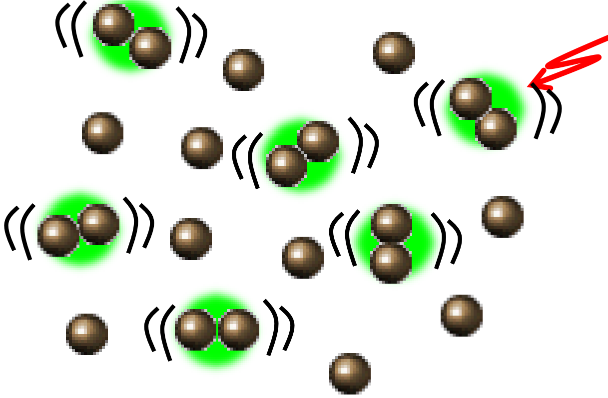
$$\frac{T}{T_F^H}$$

Pseudogap phenomenon (***mass balanced*** case: $m_L/m_H=1$)

$T = T_c \ (\Delta = 0!)$







preformed Cooper pair

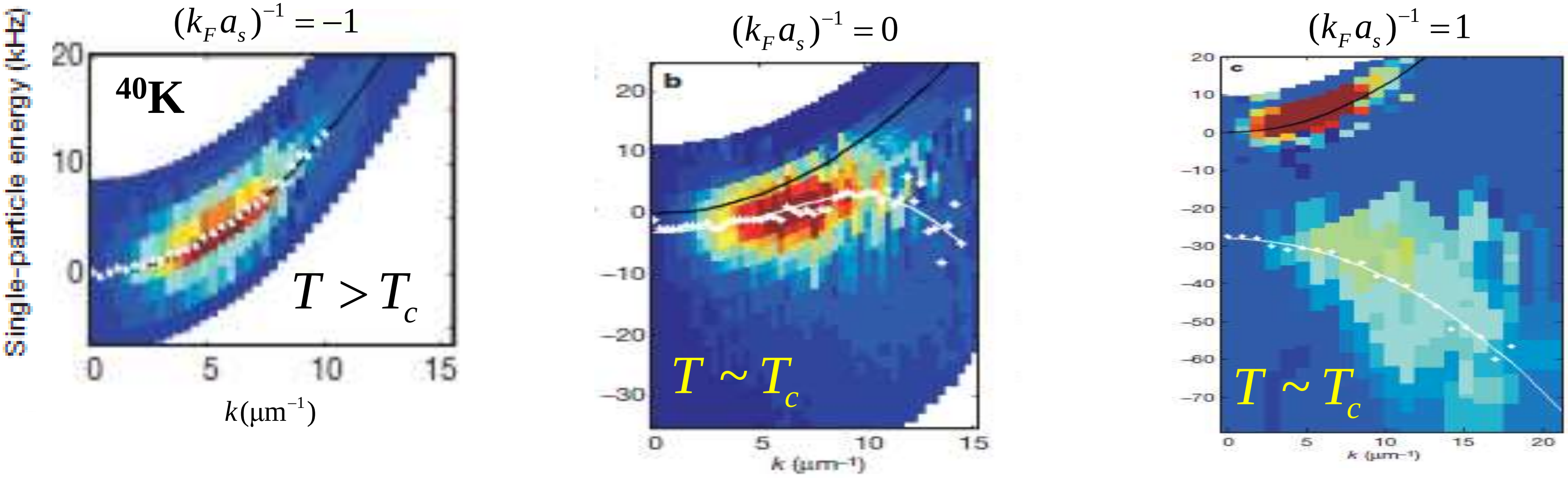
binding energy ↔ pseudogap

Pseudogap effect on photoemission spectra (^{40}K)

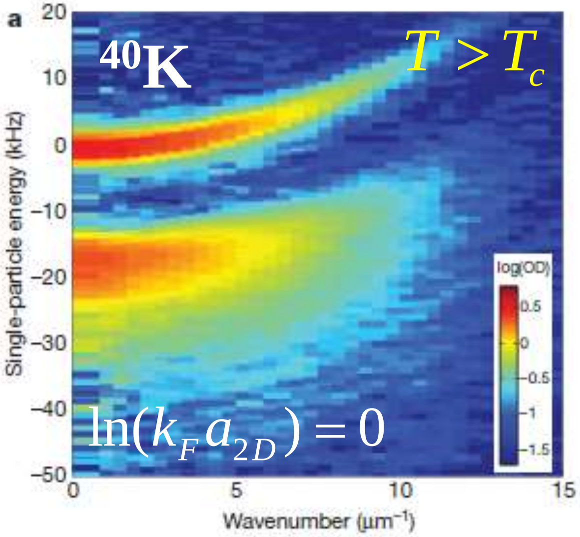
BCS $(k_F a_s)^{-1} = -1$

unitairty $(k_F a_s)^{-1} = 0$

BEC $(k_F a_s)^{-1} = 1$



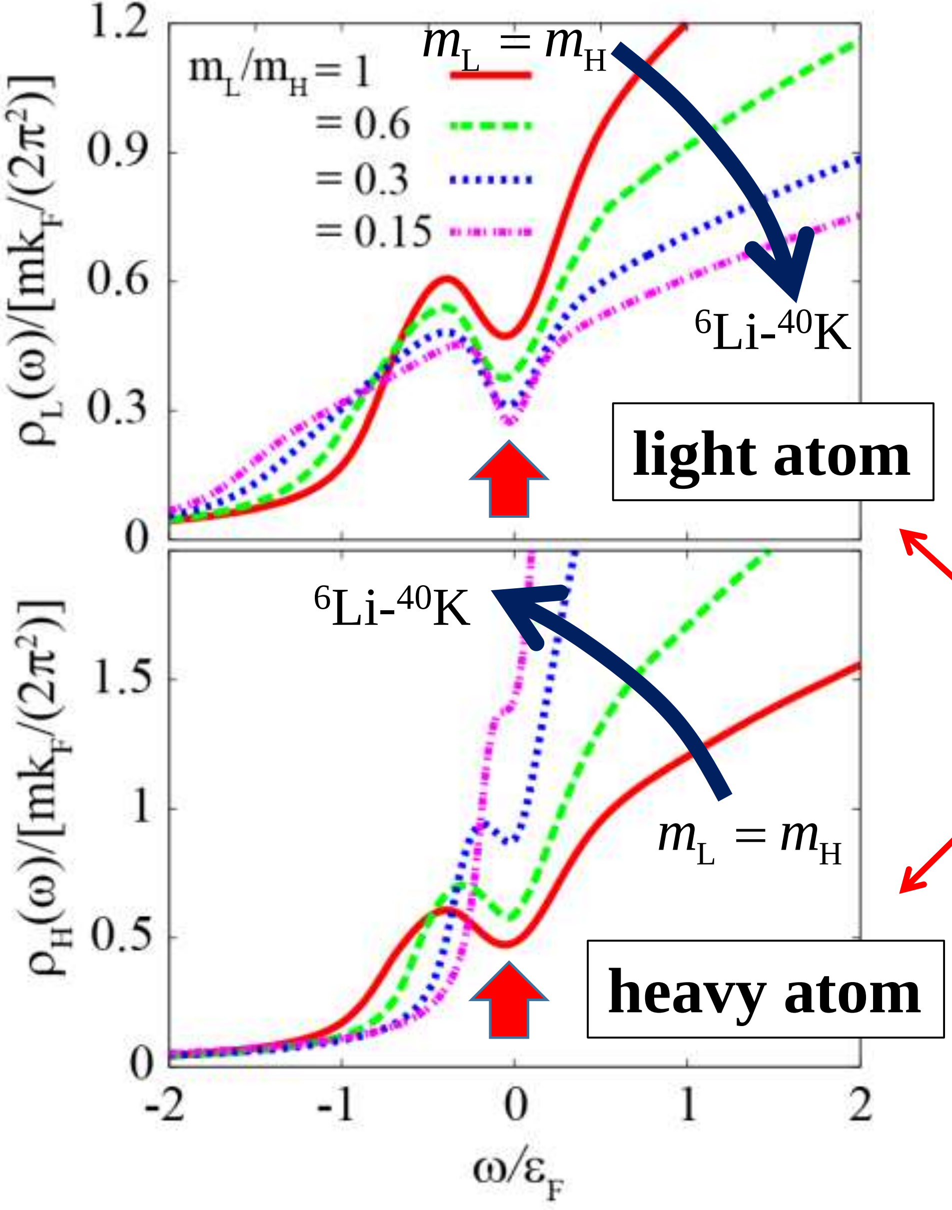
3D: Stewart, Gaebler, Jin, Nature **454** (2008) 744

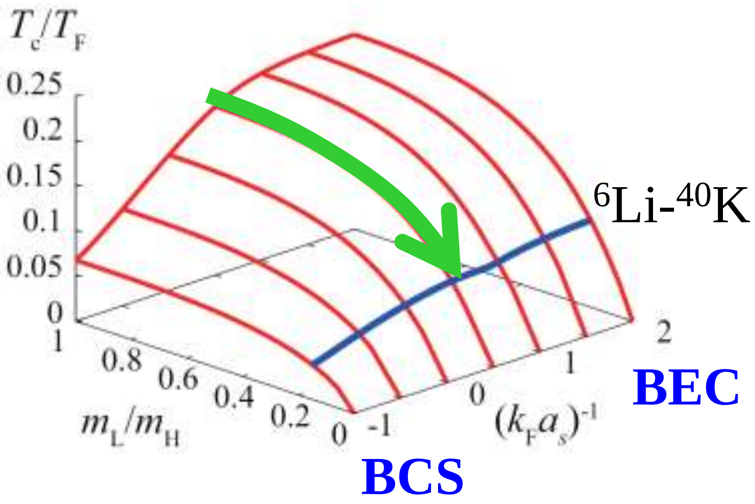


Quasi-2D: M.Feld et. al., Nature 480 (2011) 75

pseudogap in a mass-imbalanced Fermi gas (unitarity)

$T = T_c \ (\Delta = 0!)$



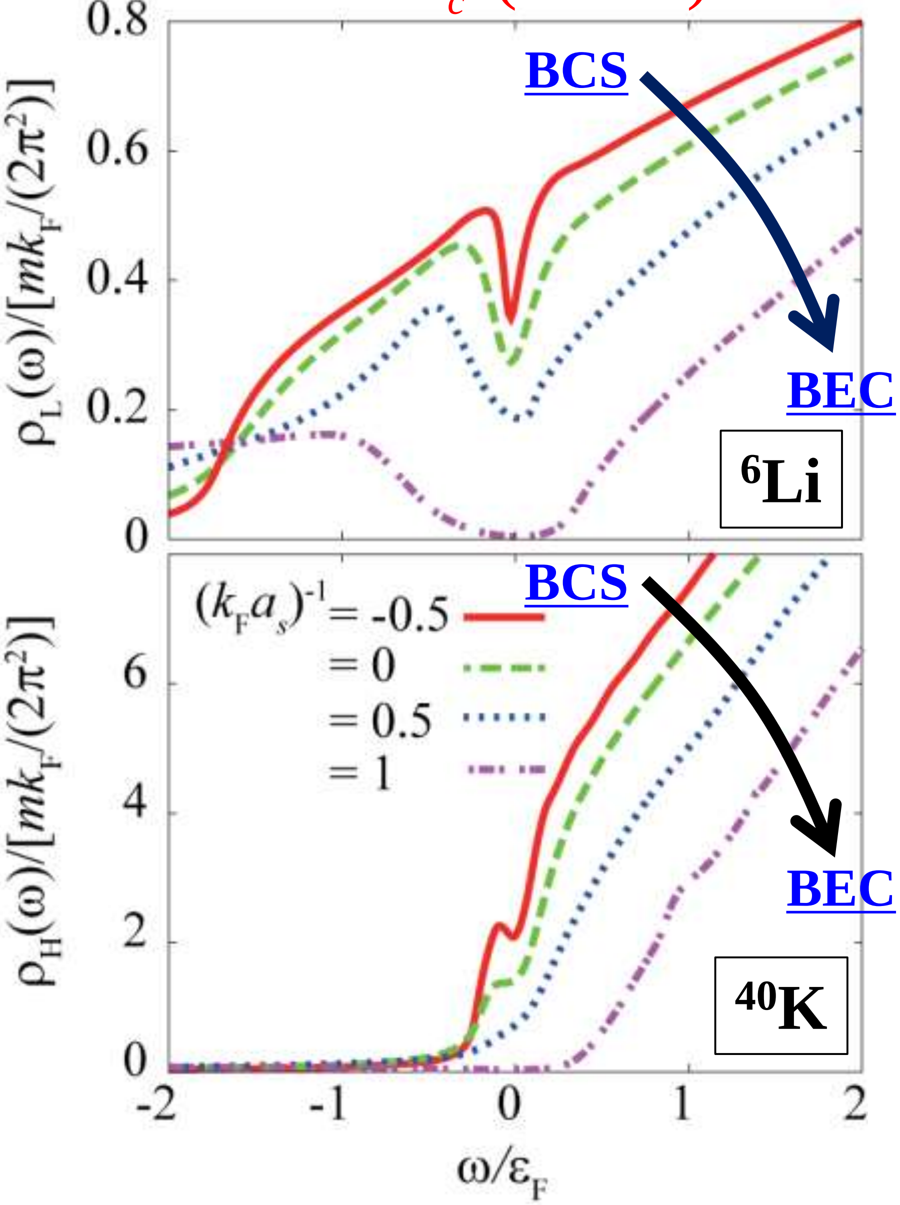


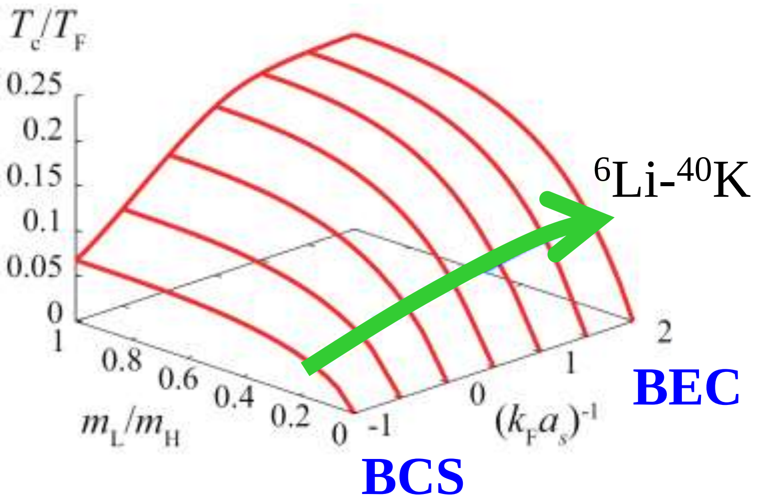
Component-dependent pseudogap phenomenon

While the pseudgap is still seen in the light component, the dip structure gradually disappears in the heavy component.

pseudogap phenomenon in ^{6}Li-^{40}K mixture (m_L/m_H=0.15 <<1)

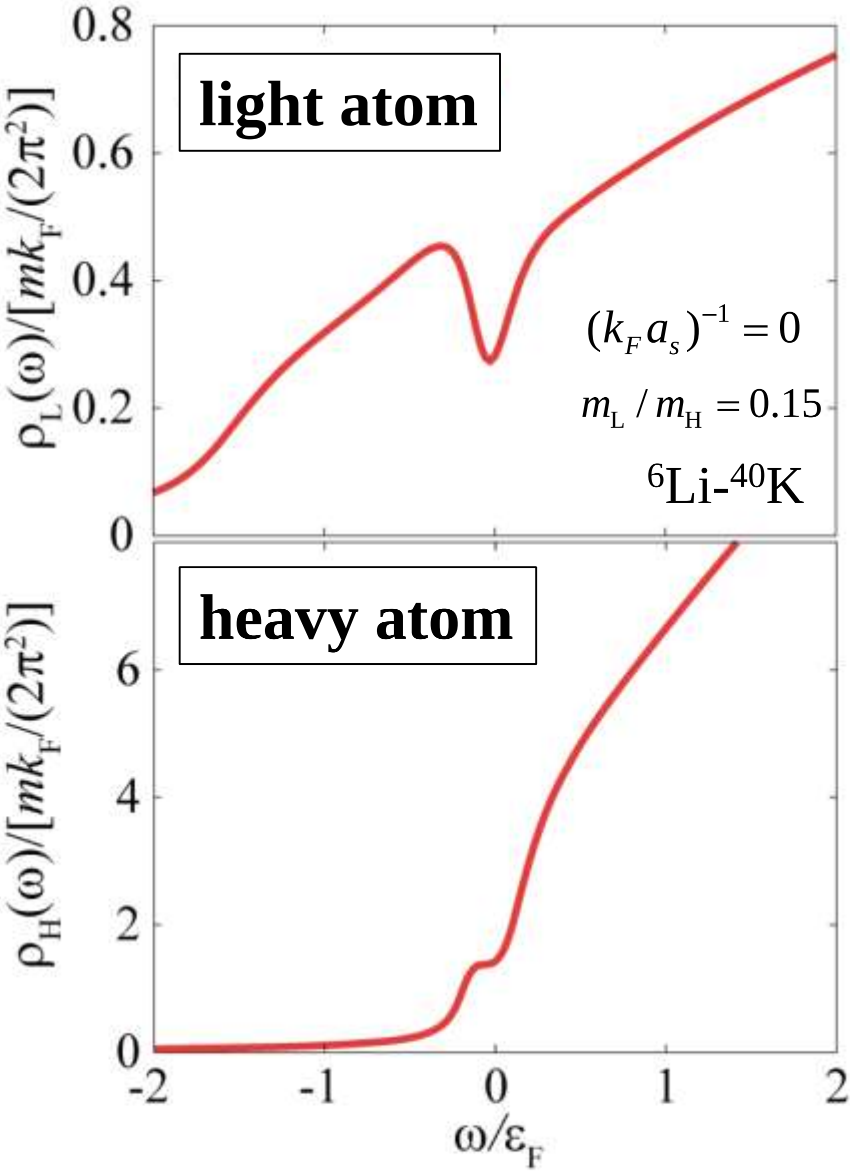
$T = T_c \ (\Delta = 0!)$



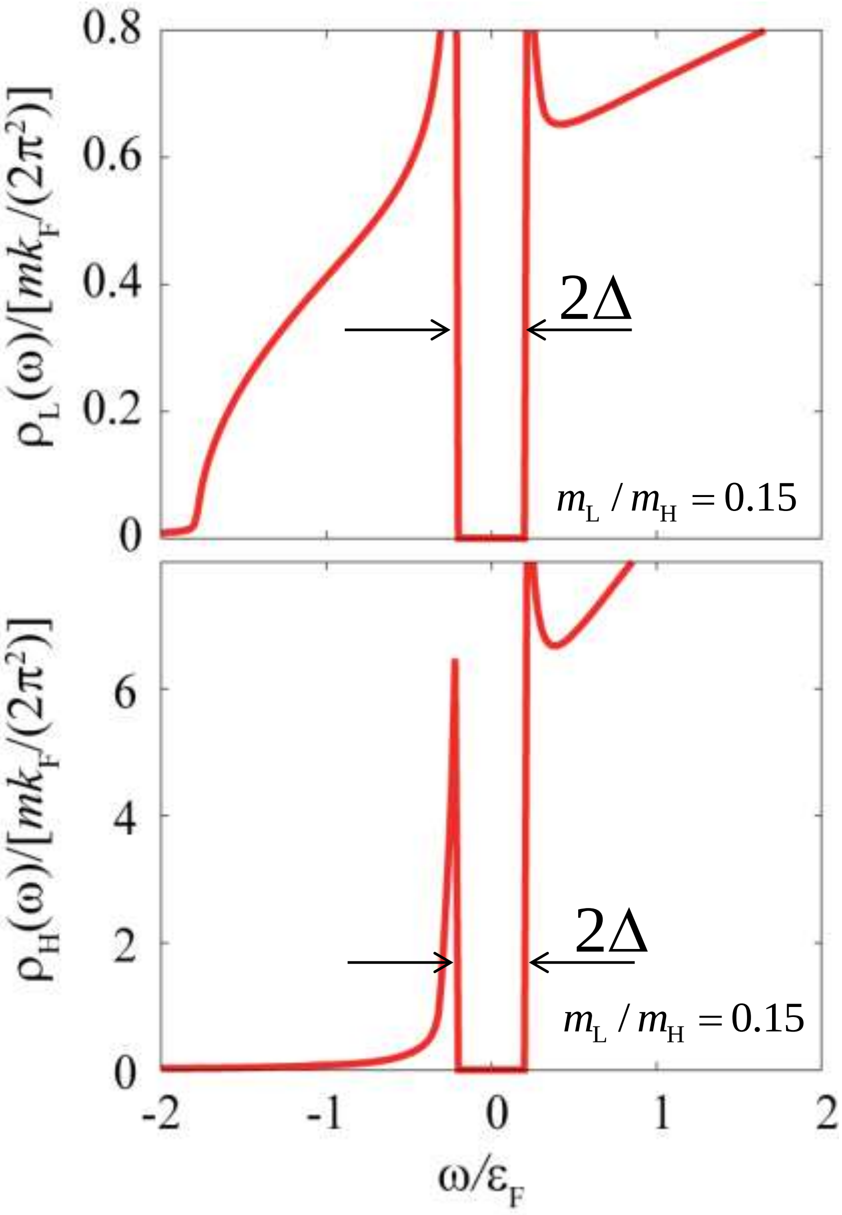


component dependent pseudogap phenomenon ?

$T = T_c \ (\Delta = 0!)$



mass-imbalanced BCS state $(\Delta > 0!)$

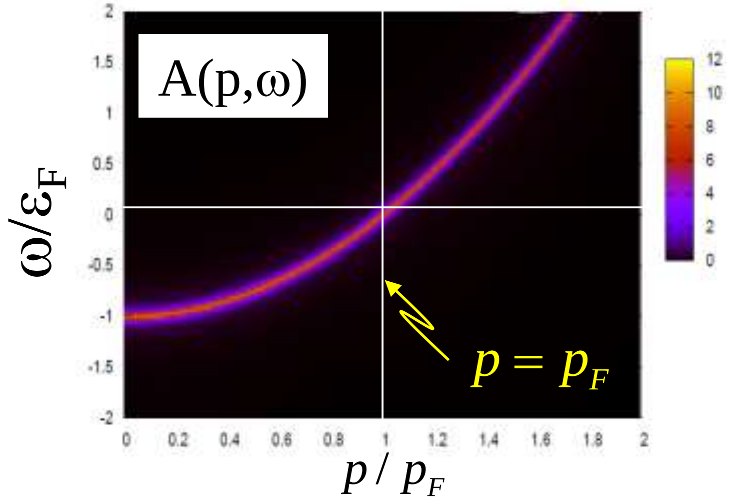


Single-particle spectral weight A(p,ω)

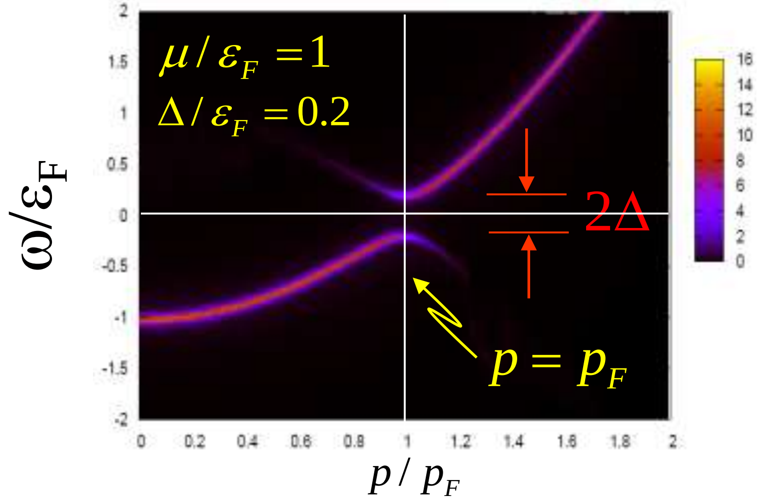
$$\boxed{A(\mathbf{p},\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G(\mathbf{p}, i\omega_n \to \omega_+)}$$

$$\mathrm{DOS} = \sum_{\mathbf{p}} A(\mathbf{p},\omega)$$

free Fermi gas



weak-coupling BCS state



Particle-hole coupling in the BCS state

$$G_{11}^{BCS}(p,i\omega) = -\frac{i\omega+\xi_{\mathbf{p}}}{\omega^2+\xi_{\mathbf{p}}^2+\Delta^2} = \frac{1}{i\omega-\xi_{\mathbf{p}}-\frac{\Delta^2}{i\omega+\xi_{\mathbf{p}}}} \qquad (\xi_{\mathbf{p}}=\varepsilon_{\mathbf{p}}-\mu)$$

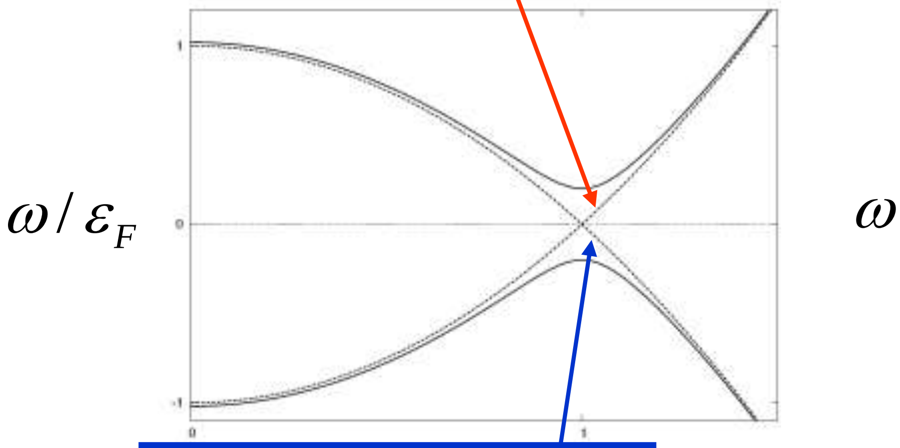
particle Green's function

$$G(p) = \frac{1}{i\omega-\xi_{\mathbf{p}}}$$

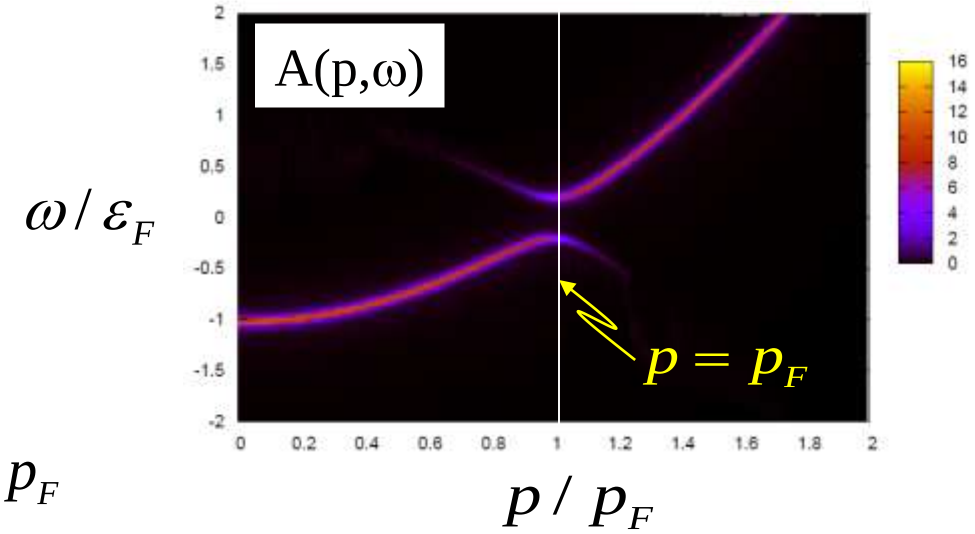
hole Green's function

$$G(p) = \frac{1}{i\omega+\xi_{\mathbf{p}}}$$

$\varepsilon_p-\mu$: **particle band**



$-(\varepsilon_p-\mu)$: **hole band**

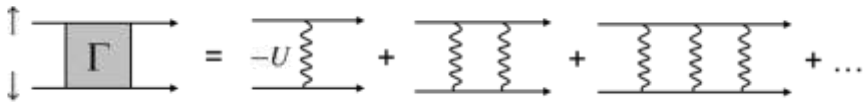


particle-hole coupling by pairing fluctuations

$$G_L(\mathbf{p}, i\omega_n) = \frac{1}{i\omega_n - \xi_{\mathbf{p}} - \Sigma_L(\mathbf{p}, i\omega_n)}$$

$$\Sigma_L(\mathbf{p}, i\omega_n) = -\sum_{\mathbf{q}, i\nu_n} \Gamma(\mathbf{q}, i\nu_n) G_H(\mathbf{q}-\mathbf{p}, i\nu_n - i\omega_n)$$

pairing fluctuations



$$\Sigma_L(\mathbf{p}, i\omega_n) \cong -G_H(-\mathbf{p}, -i\omega_n) T \sum_{\mathbf{q}, i\nu_n} \Gamma(\mathbf{q}, i\nu_n)$$

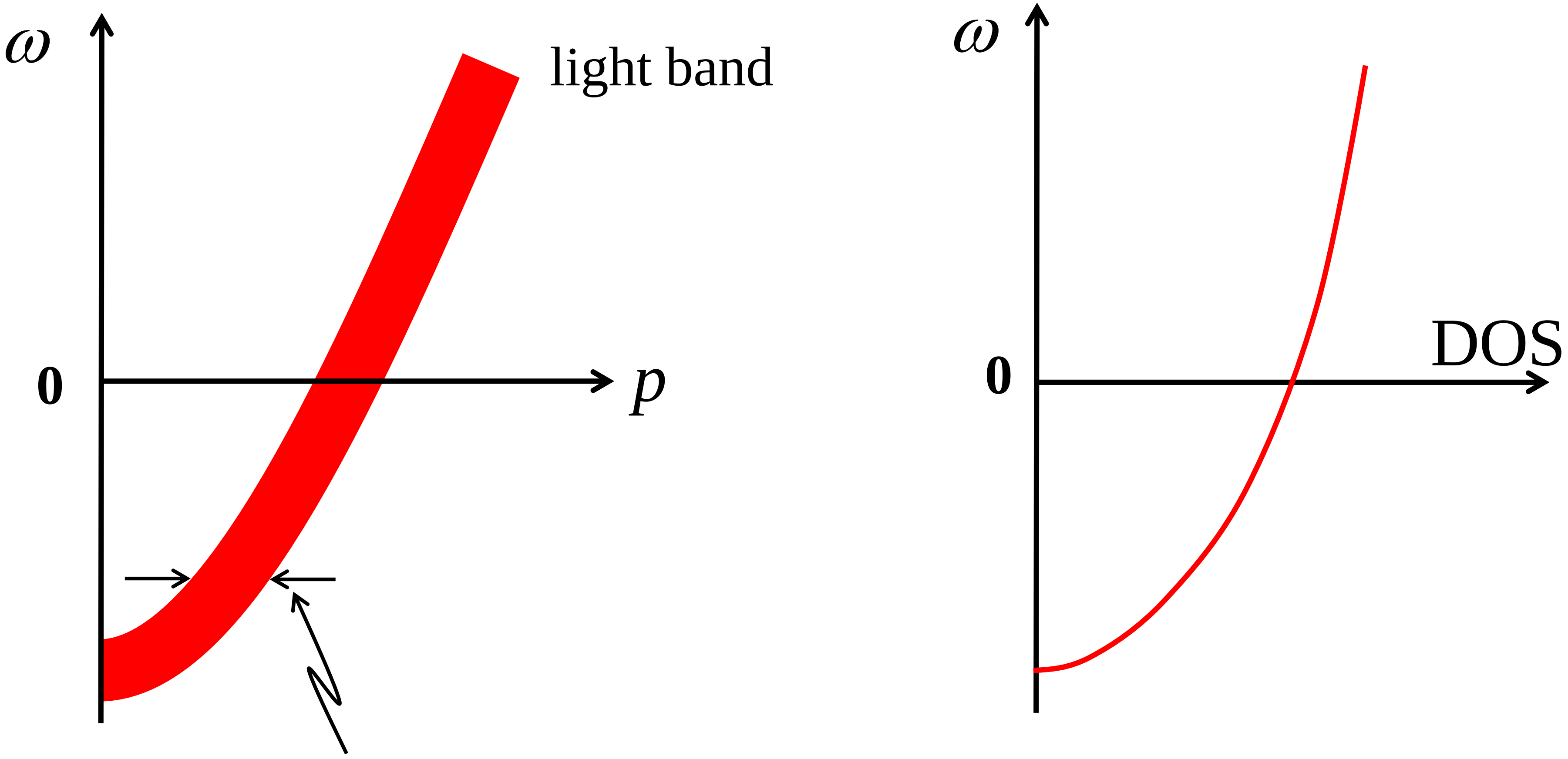
$$G_L(\mathbf{p}, i\omega_n) = \frac{1}{\underbrace{i\omega_n - \xi_{\mathbf{p}}}_{\text{particle}} - \dfrac{\sum_{\mathbf{q}, i\nu_n} \Gamma(\mathbf{q}, i\nu_n)}{\underbrace{i\omega_n + \xi_{\mathbf{p}} + \Sigma_H}_{\text{hole}}}}$$

particle-hole coupling
by pairing fluctuations

$$G_{11}^{BCS}(p, i\omega) = \frac{1}{i\omega - \xi_{\mathbf{p}} - \dfrac{\Delta^2}{i\omega + \xi_{\mathbf{p}}}}$$

Origin of the component dependent pseudogap phenomenon

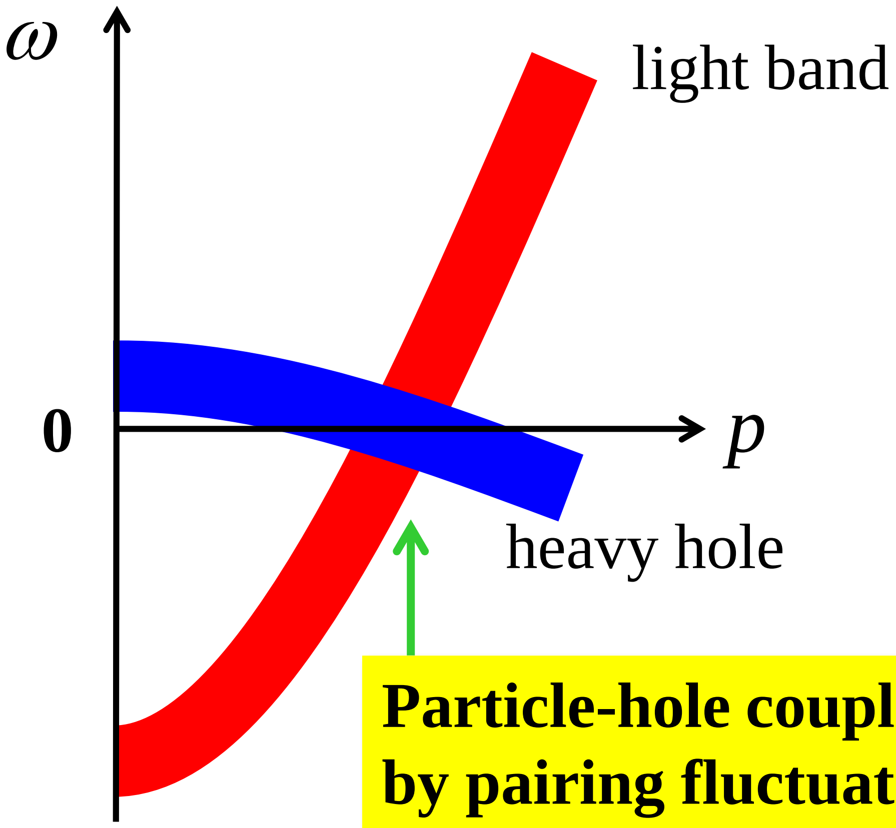
light component



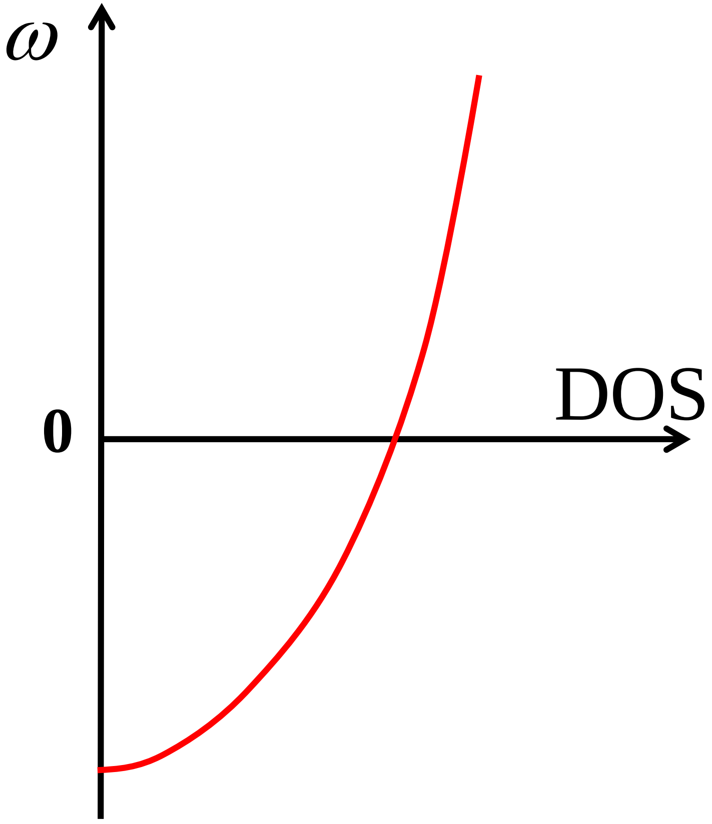
Finite width due to scattering effect

Origin of the component dependent pseudogap phenomenon

light component

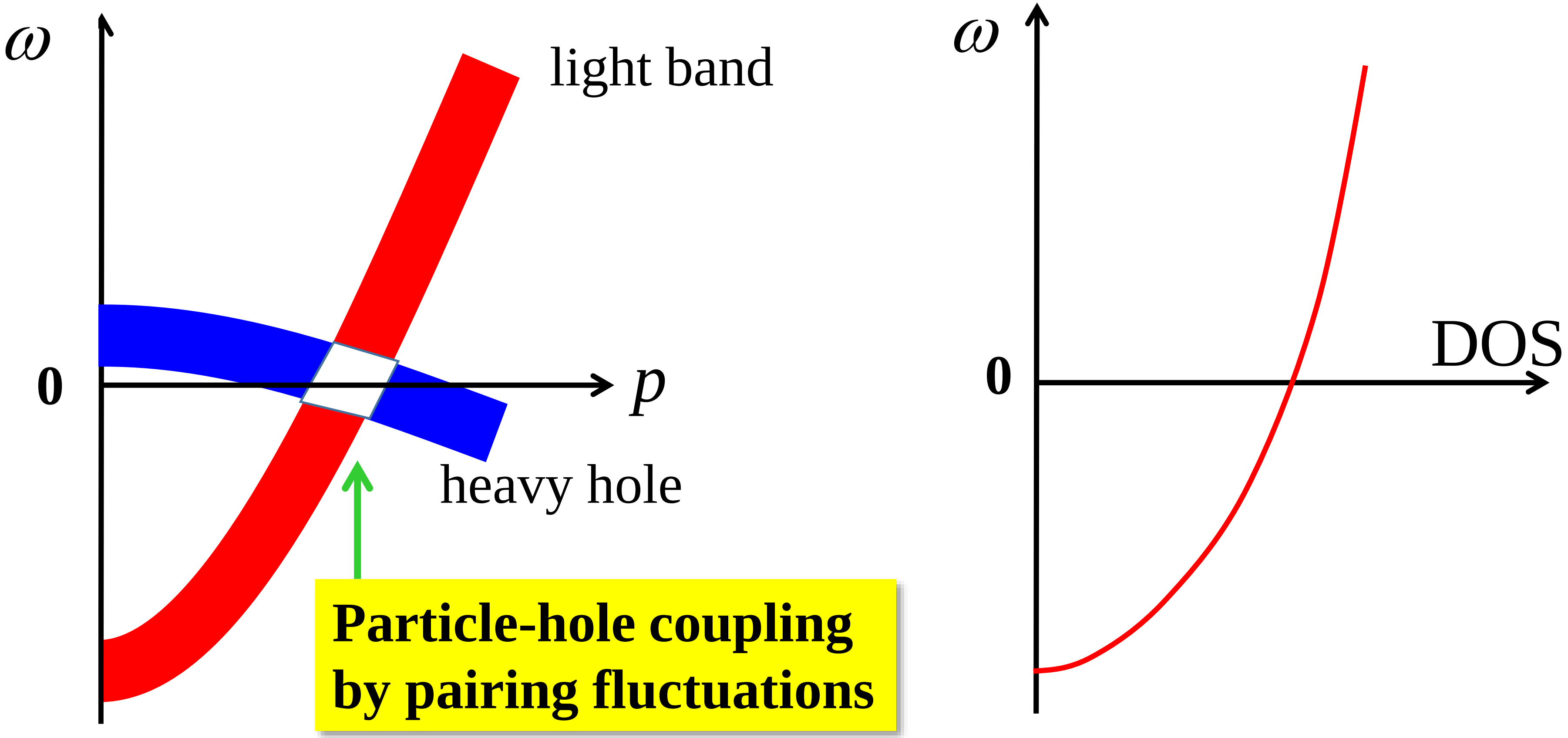


Particle-hole coupling by pairing fluctuations



Origin of the component dependent pseudogap phenomenon

light component

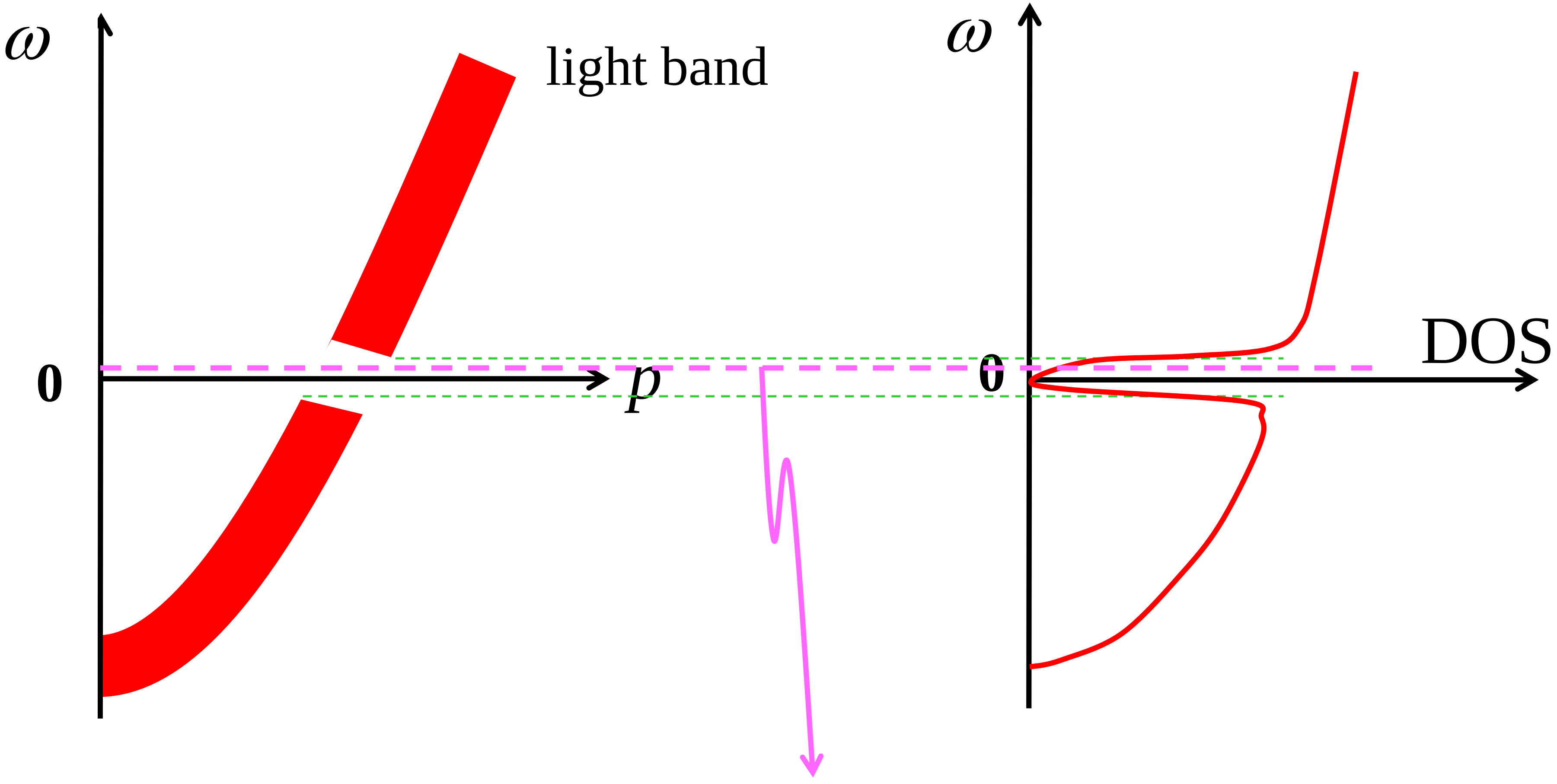


Particle-hole coupling by pairing fluctuations

$$\mathrm{DOS} = \sum_{\mathbf{p}} A(\mathbf{p}, \omega)$$

Origin of the component dependent pseudogap phenomenon

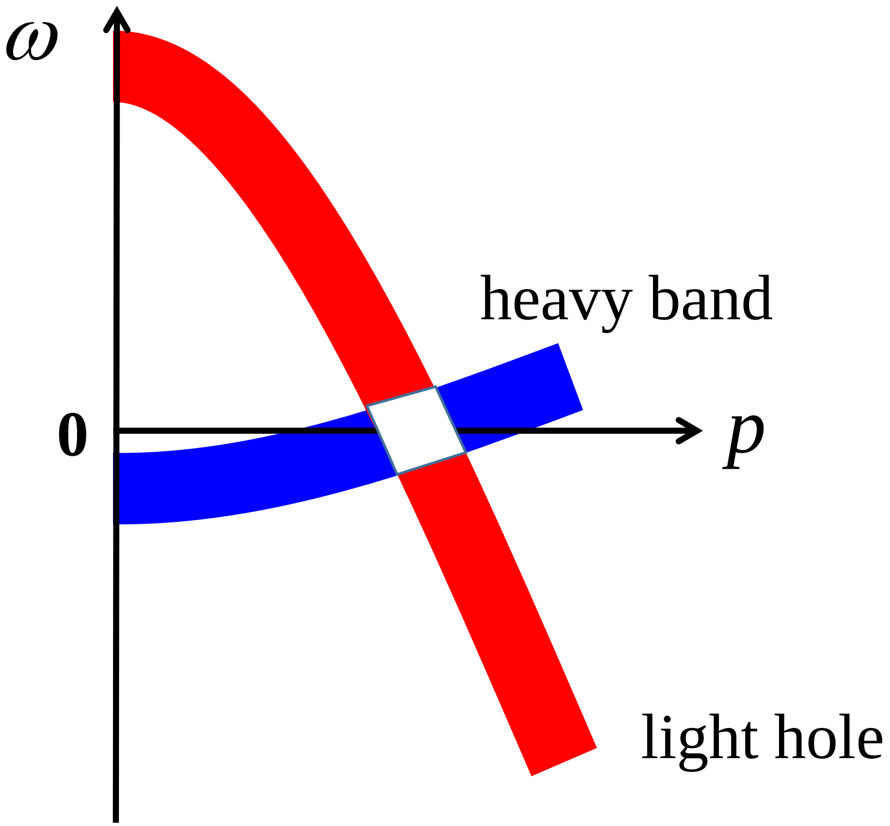
light component

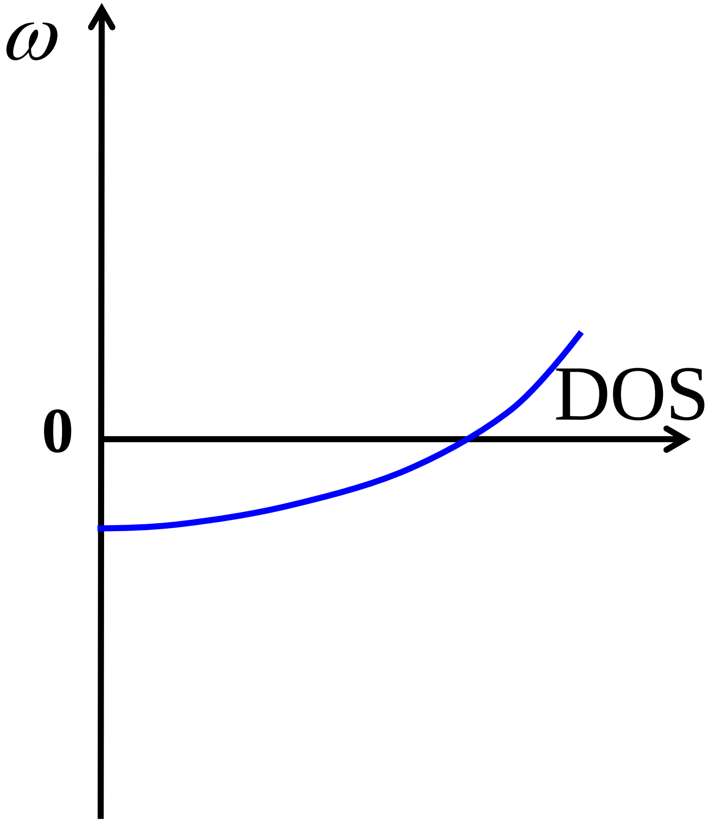


$$\mathrm{DOS} = \sum_{\mathbf{p}} A(\mathbf{p}, \omega)$$

Origin of the component dependent pseudogap phenomenon

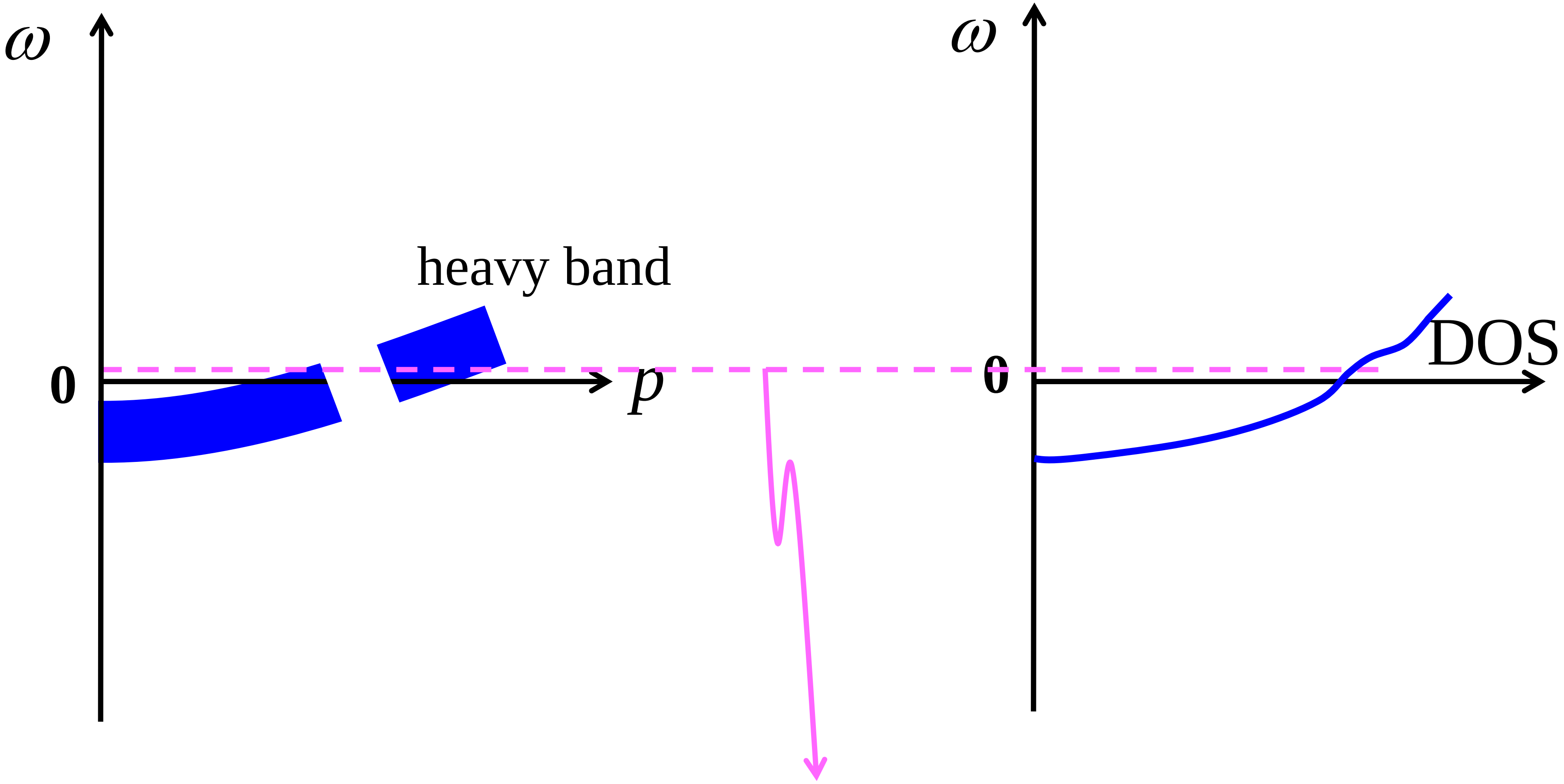
heavy component





Origin of the component dependent pseudogap phenomenon

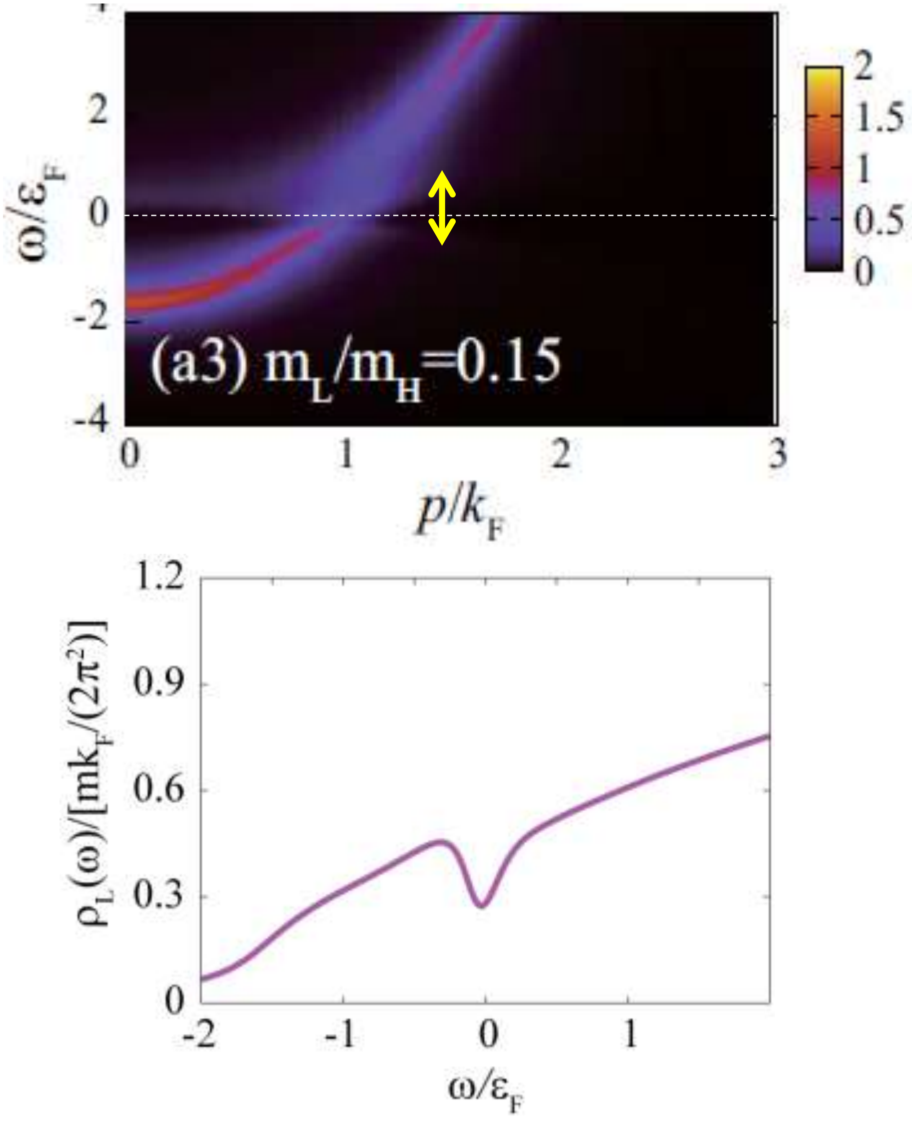
heavy component



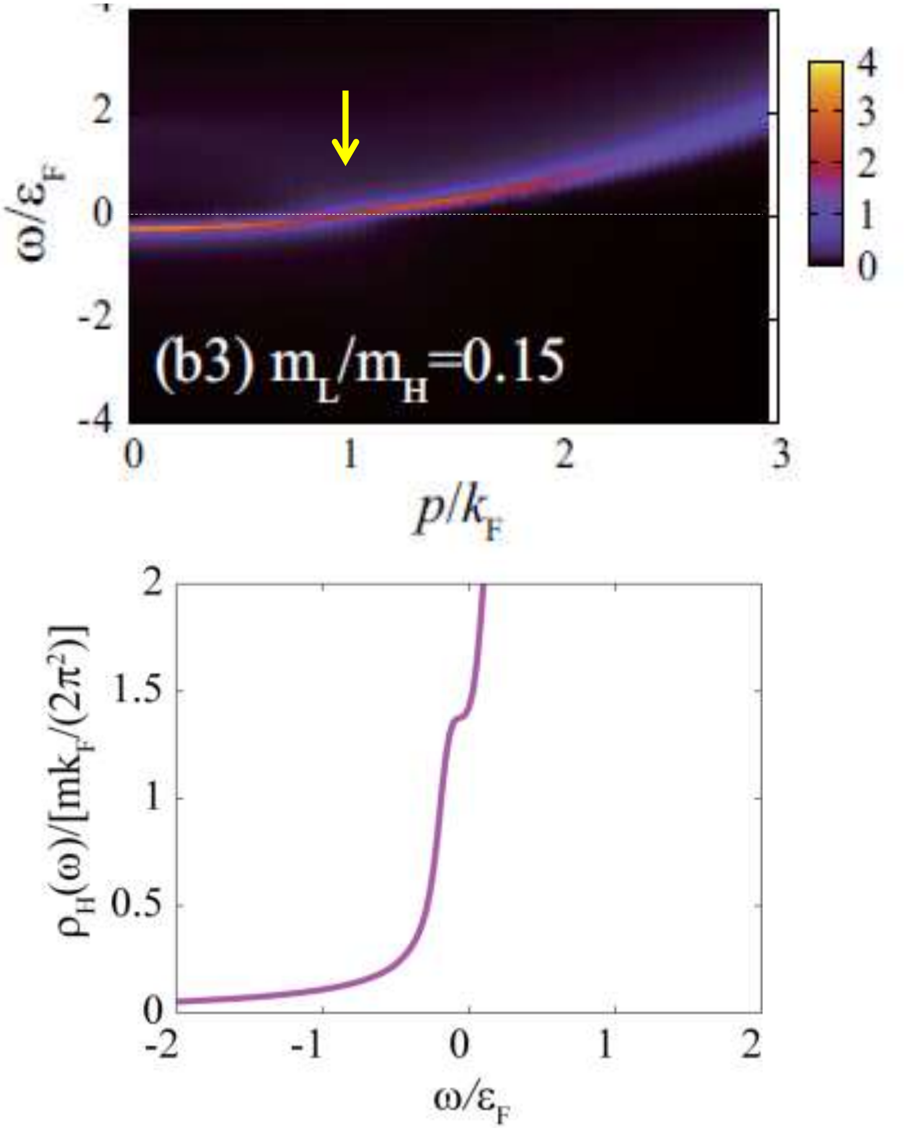
$$\mathrm{DOS} = \sum_{\mathbf{p}} A(\mathbf{p}, \omega)$$

Origin of the *component dependent* pseudogap phenomenon

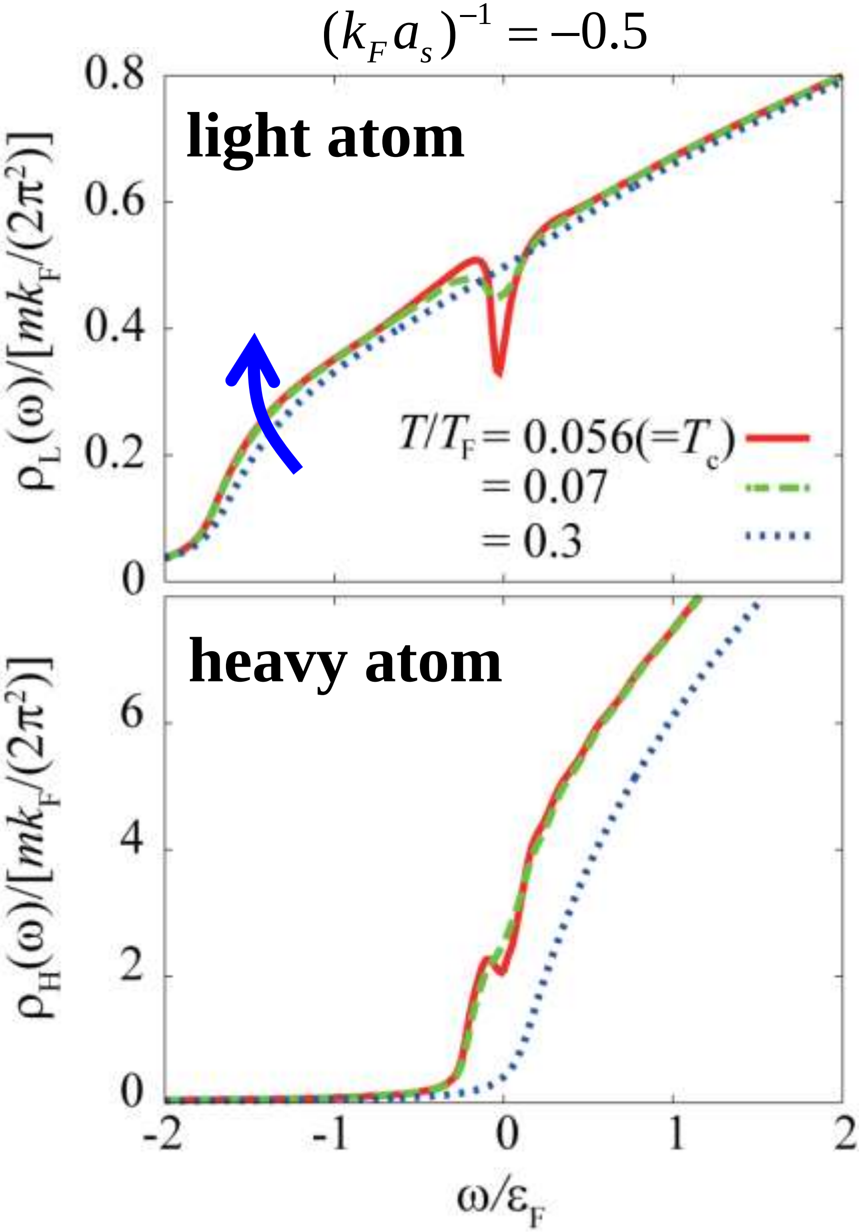
light component



heavy component



pseudogap above Tc: ^{6}Li-^{40}K mixture (m_L/m_H=0.15 <<1)

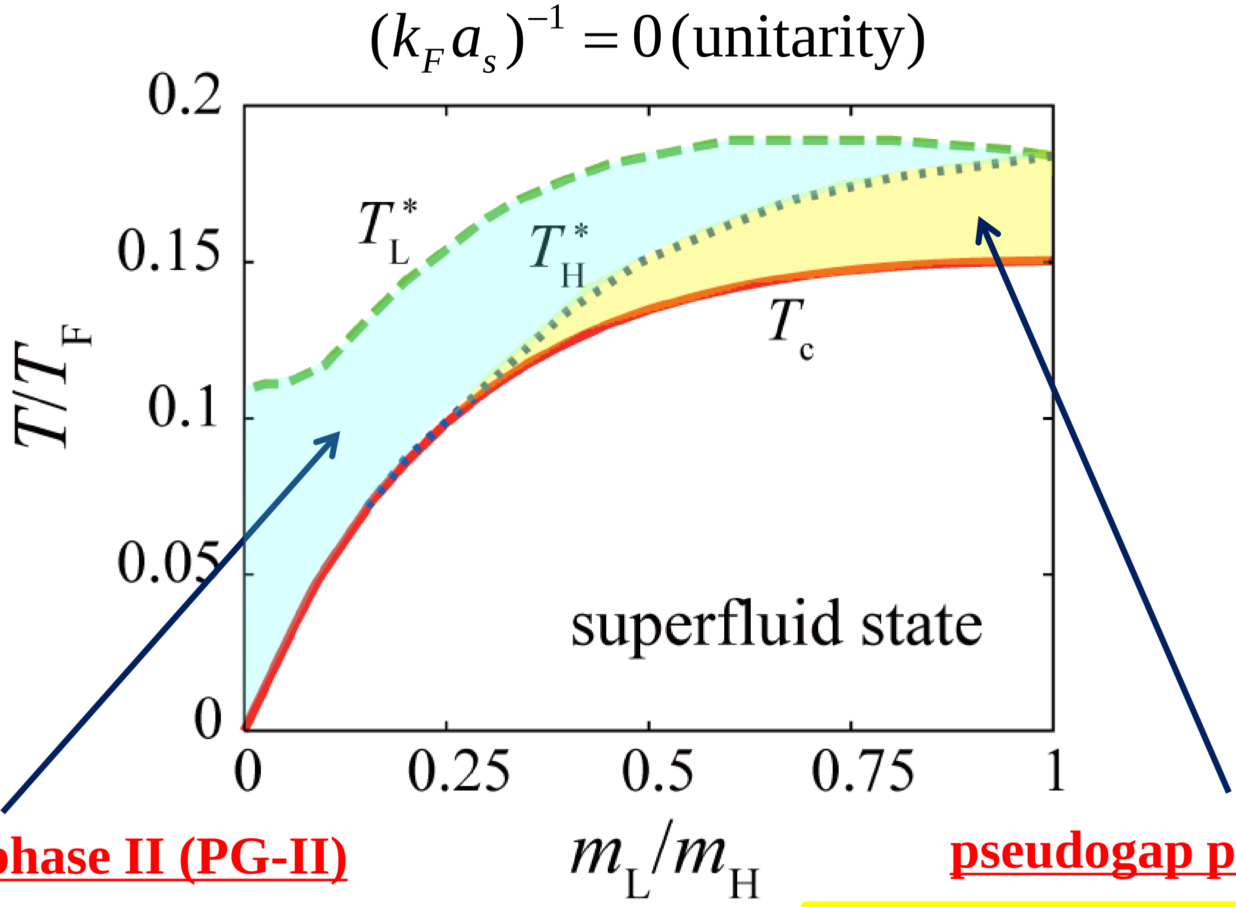


pseudogap temperature

T_L^* T_H^*

temperature at which the dip structure disappears in DOS of the light/heavy component

Two pseudogap phases in a mass imbalanced Fermi gas



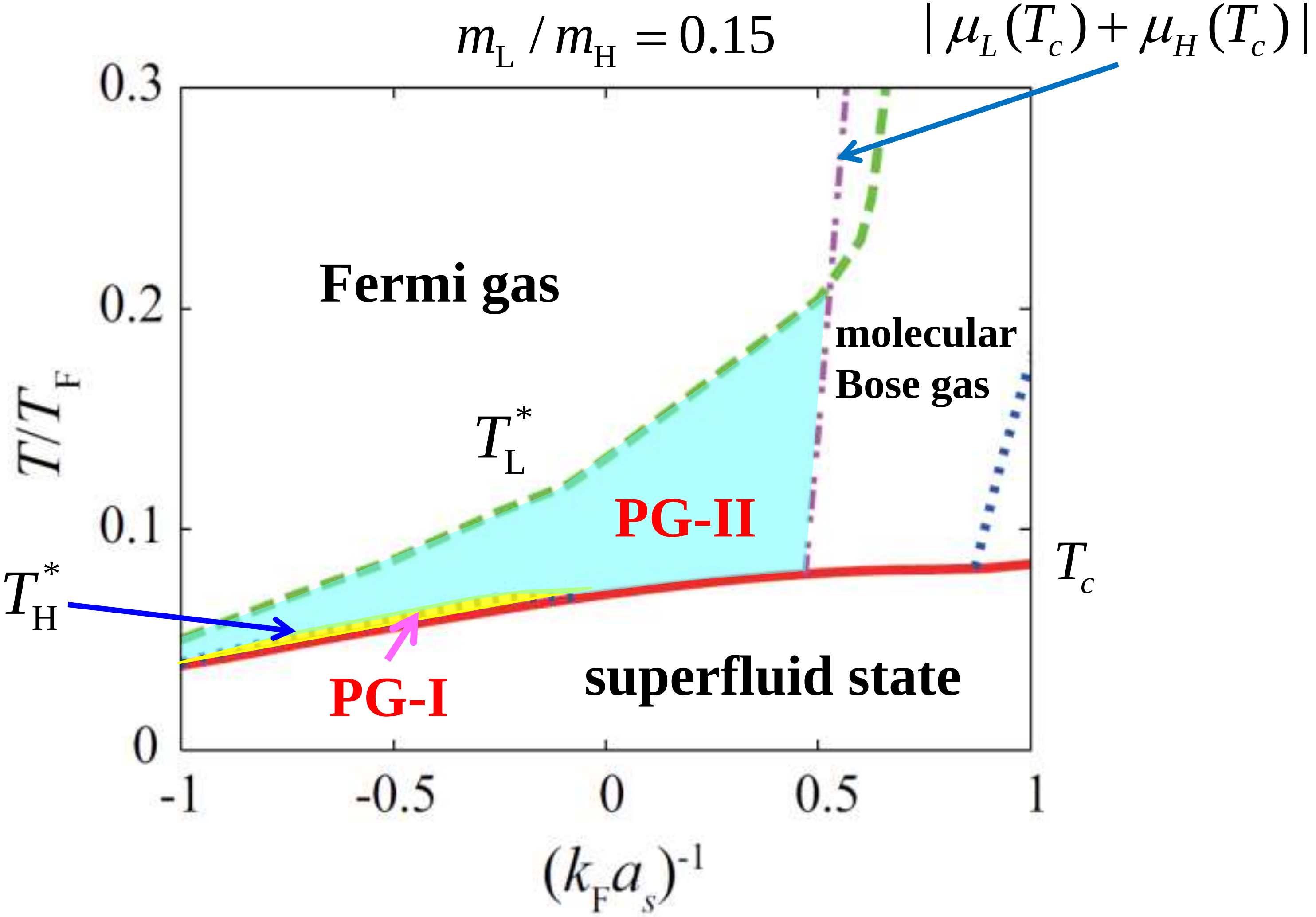
pseudogap phase II (PG-II)

The pseudogap is seen only in the light component.

pseudogap phase I (PG-I)

The pseudogap appears in both the light and heavy components.

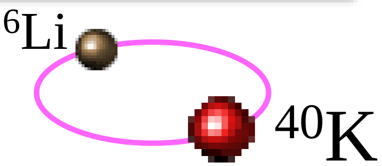
Phase diagram of a ^{6}Li-^{40}K Fermi gas



PG-I : Pseudogap appears in both the components.

PG-II: Pseudogap only appears in the light component.

Summary

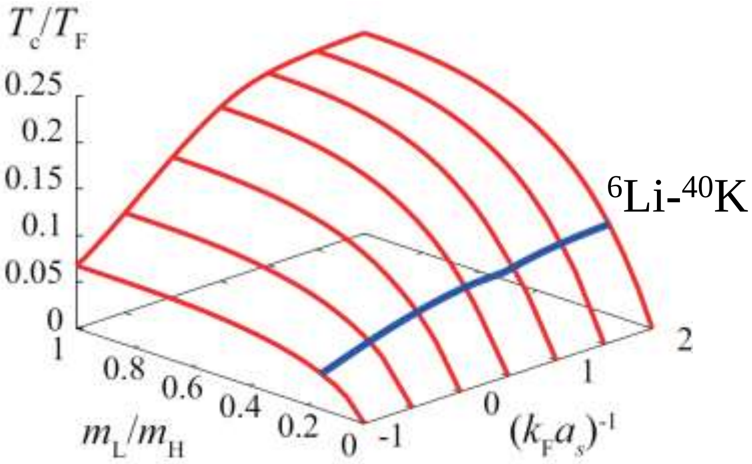


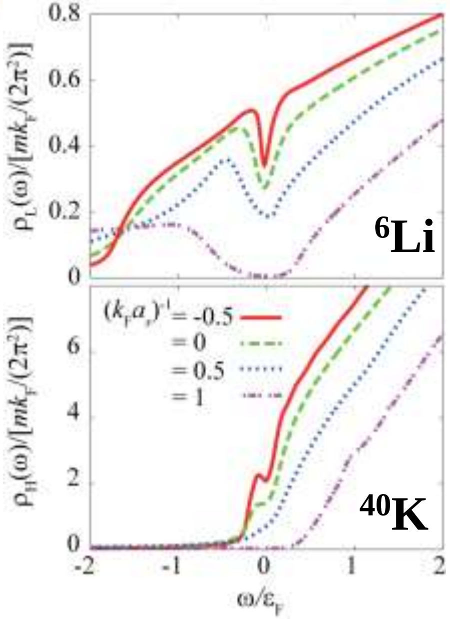
We have discussed strong-coupling properties of a mass-imbalanced Fermi gas. Within the framework of the self-consistent T-matrix theory, we determined Tc and effects of mass imbalance in the BCS-BEC crossover region. We have also examined the pseudogap phenomenon in the presence of mass imbalance.

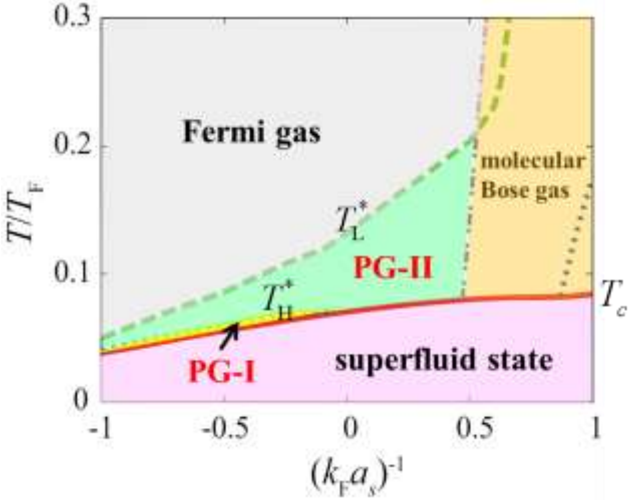
superfluid phase transition temperature Tc

component-dependent pseudogap phenomenon

phase diagram of ^{6}Li-^{40}K mixture







Hanai, Ohashi, PRA 90 (2014) 043622