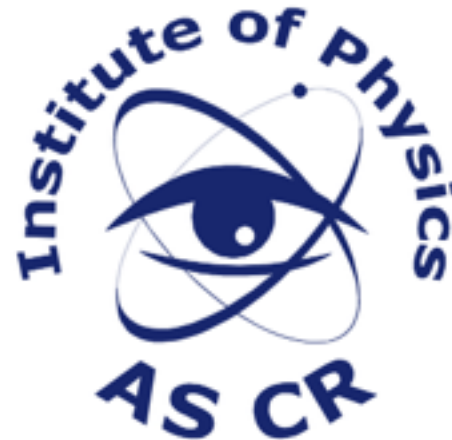
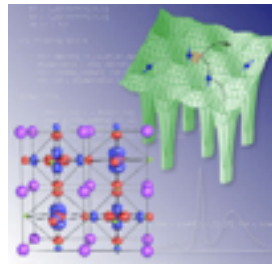


Excitonic Condensation of Strongly Correlated Electrons

Jan Kuneš and Pavel Augustinský



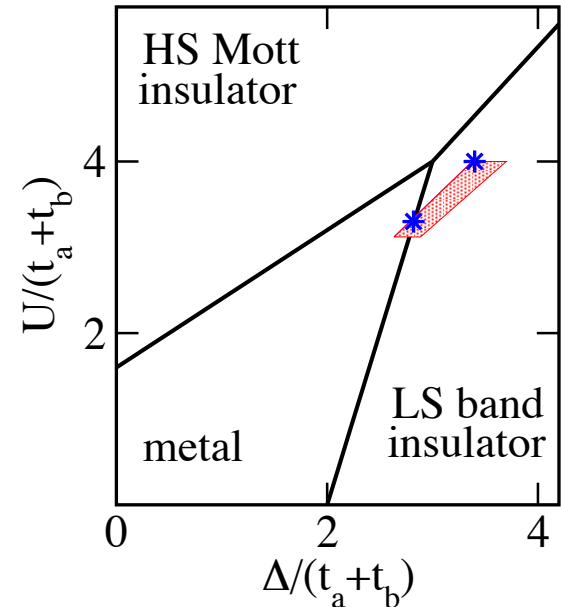
DFG FOR1346



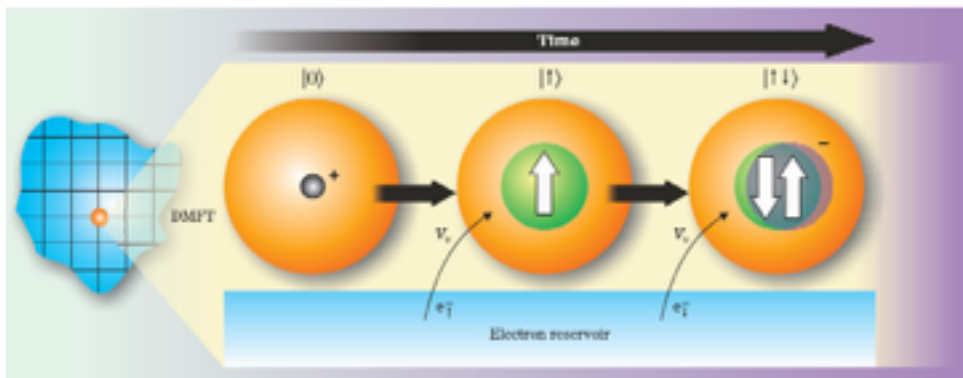
Minimal lattice model with spin-state transition

Two-band Hubbard model at $n=2$ (half filling)

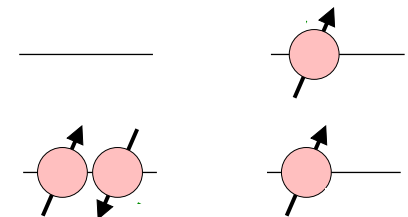
$$\begin{aligned}
 H_t &= \frac{\Delta}{2} \sum_{i,\sigma} (n_{i\sigma}^a - n_{i\sigma}^b) + \sum_{i,j,\sigma} (t_a a_{i\sigma}^\dagger a_{j\sigma} + t_b b_{i\sigma}^\dagger b_{j\sigma}) \\
 &\quad + \sum_{\langle ij \rangle, \sigma} (V_1 a_{i\sigma}^\dagger b_{j\sigma} + V_2 b_{i\sigma}^\dagger a_{j\sigma} + c.c.) \\
 H_{\text{int}}^{\text{dd}} &= U \sum_i (n_{i\uparrow}^a n_{i\downarrow}^a + n_{i\uparrow}^b n_{i\downarrow}^b) + (U - 2J) \sum_{i,\sigma} n_{i\sigma}^a n_{i-\sigma}^b \\
 &\quad + (U - 3J) \sum_{i\sigma} n_{i\sigma}^a n_{i\sigma}^b \\
 H'_{\text{int}} &= J \sum_{i\sigma} a_{i\sigma}^\dagger b_{i-\sigma}^\dagger a_{i-\sigma} b_{i\sigma} + J' \sum_i (a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger b_{i\downarrow} b_{i\uparrow} + c.c.).
 \end{aligned}$$



Dynamical Mean-Field Theory



low spin $S=0$ high spin $S=1$



Excitonic instability - DMFT linear response

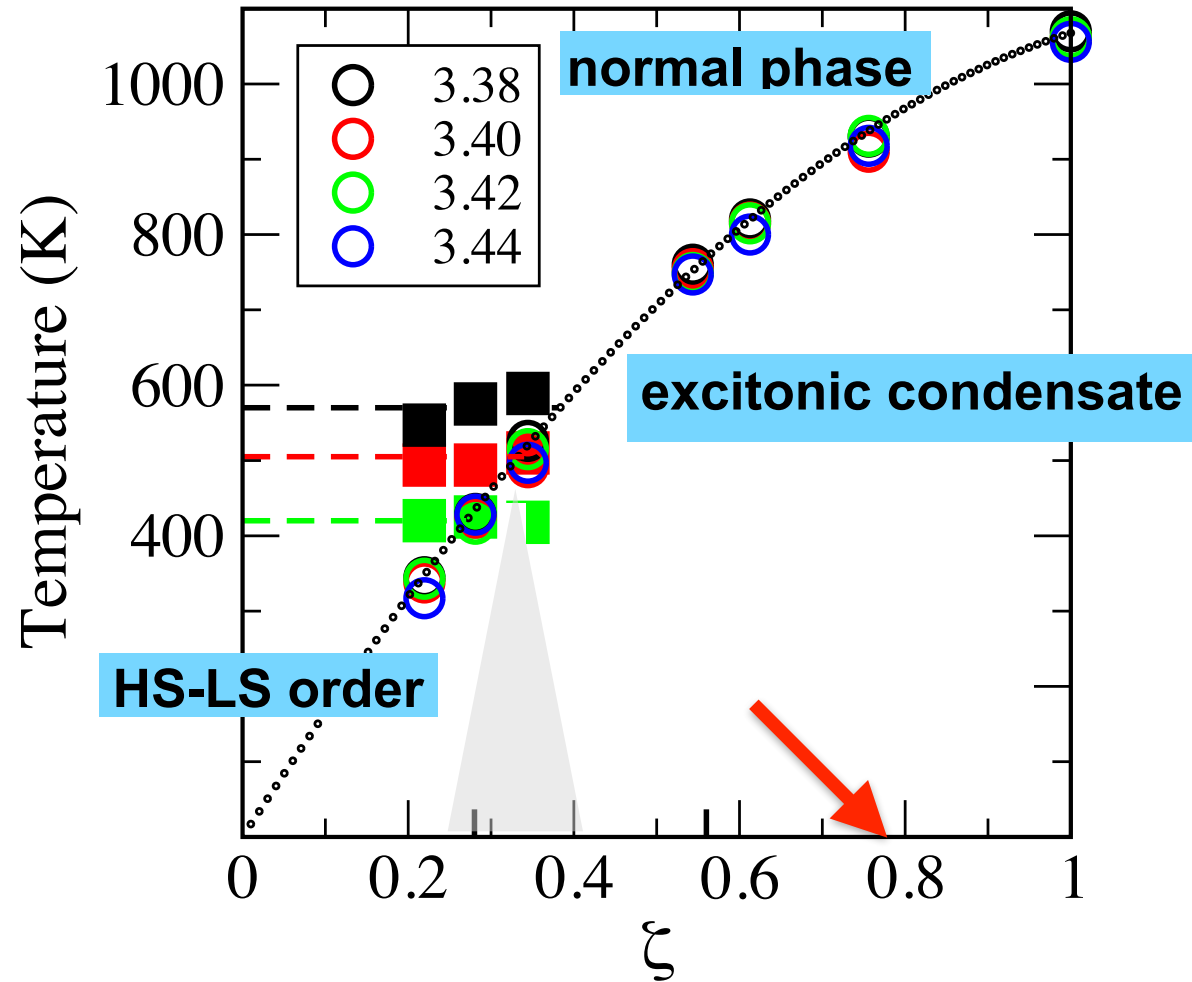
$$U=4, J=1$$

$$t_a^2 + t_b^2 = \text{const}$$

$$\Delta = 3.40 + \delta$$

$$V=0$$

$$\zeta = \frac{2t_a t_b}{t_a^2 + t_b^2}$$



Excitonic condensate - order parameter

Order parameter is a complex vector: $\phi^\gamma = \sum_{\alpha\beta} \sigma_{\alpha\beta}^\gamma \langle a_\alpha^\dagger b_\beta \rangle$

z-axis parallel to: $i(\bar{\phi} \wedge \phi)$

No cross hopping ($V=0$) \rightarrow overall phase of ϕ does not matter
 \rightarrow phases are distinguished by $|\phi^+|$ and $|\phi^-|$

3 possible phases:

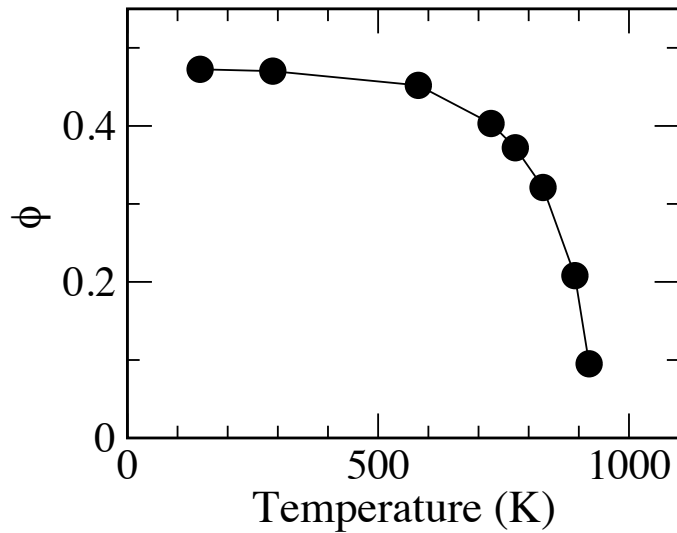
linear (L) $|\phi^+| = |\phi^-| \neq 0$

circular (C) $|\phi^+| = 0, |\phi^-| \neq 0$

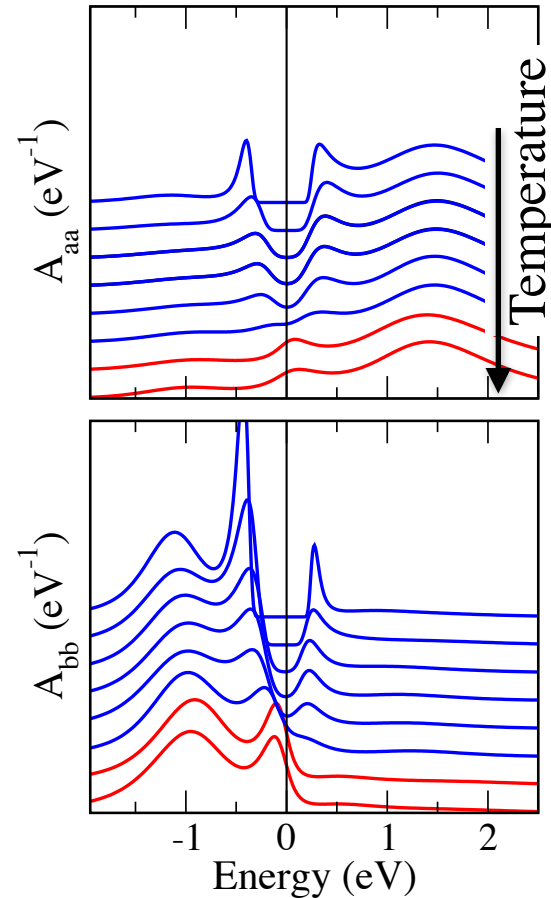
elliptic (E) $0 \neq |\phi^+| \neq |\phi^-| \neq 0$

Excitonic condensation (undoped) - L phase

order parameter $|\phi^+| = |\phi^-|$

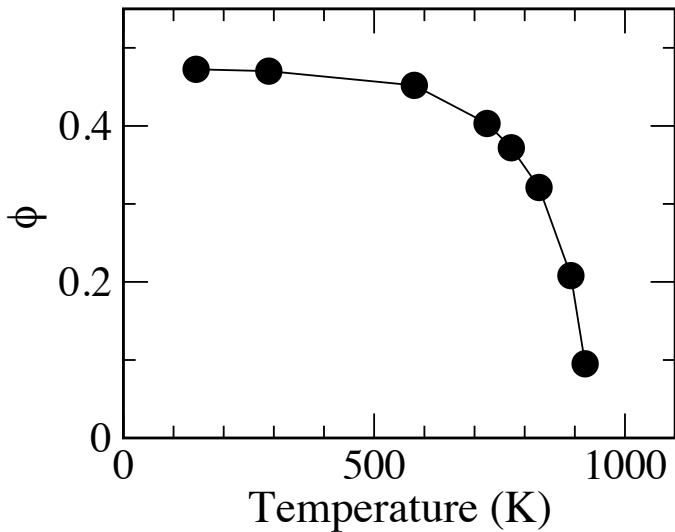


Spectral density (diagonal elements)

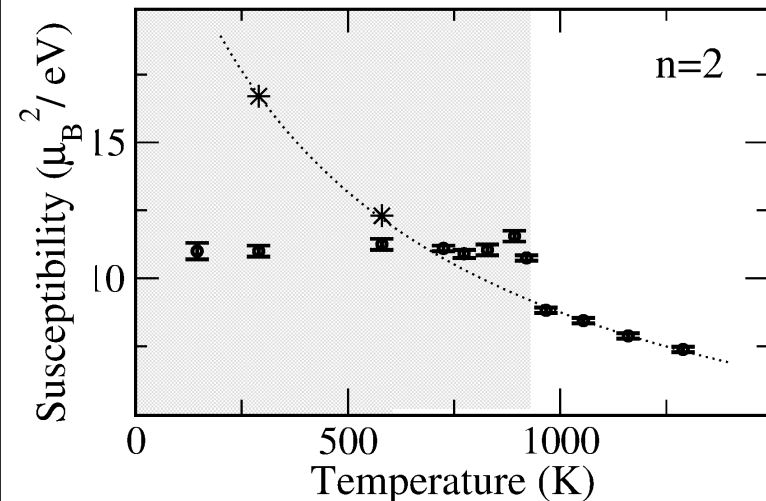


Excitonic condensation (undoped) - L phase

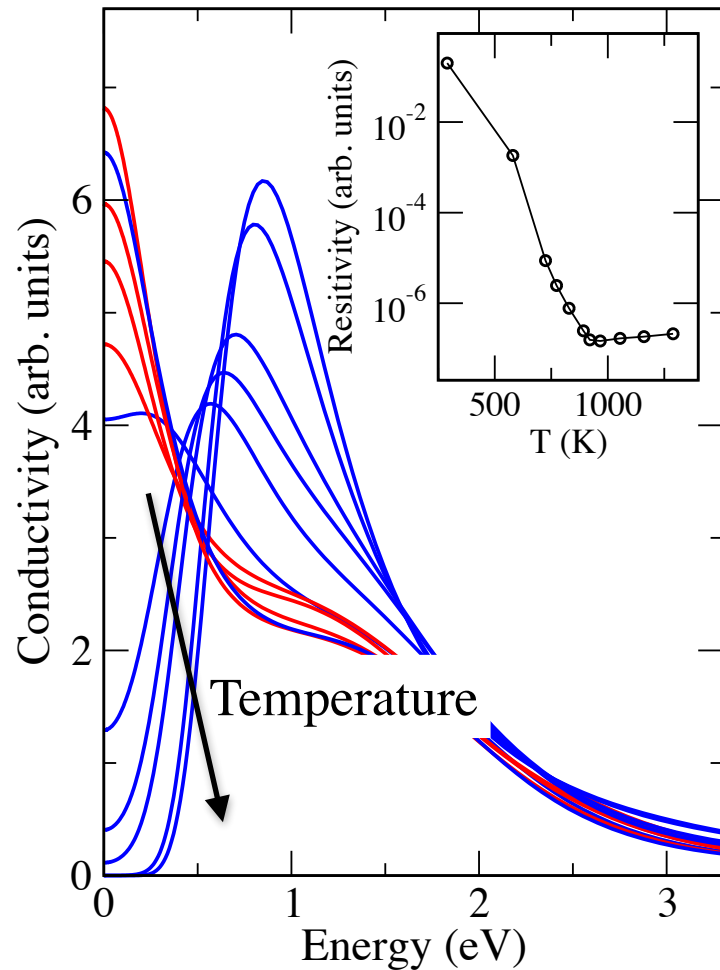
order parameter $|\phi^+| = |\phi^-|$



Uniform spin susceptibility

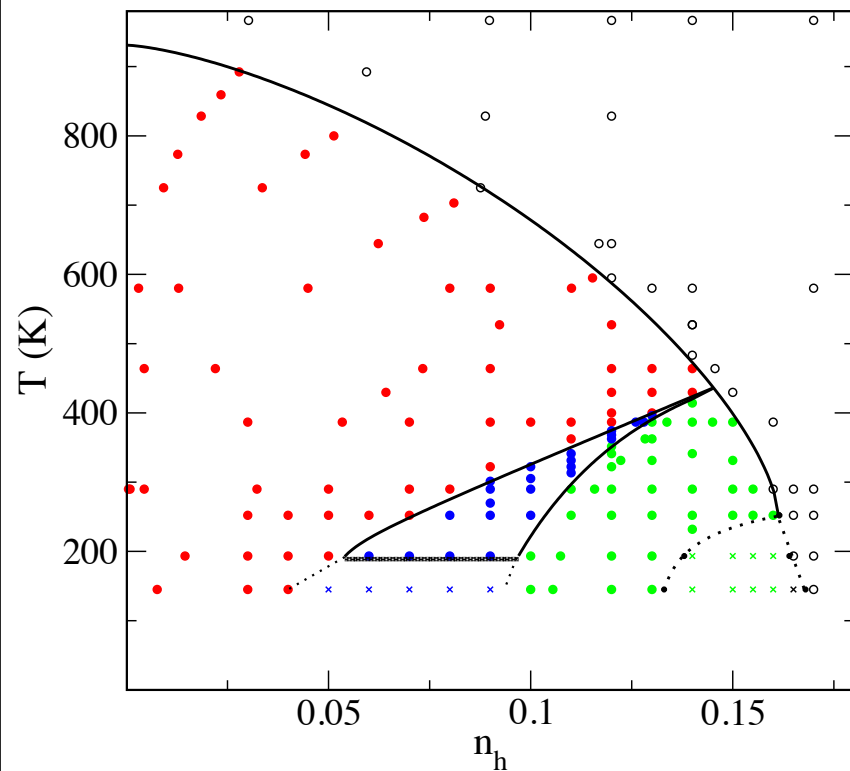


Optical conductivity (dc resistivity)

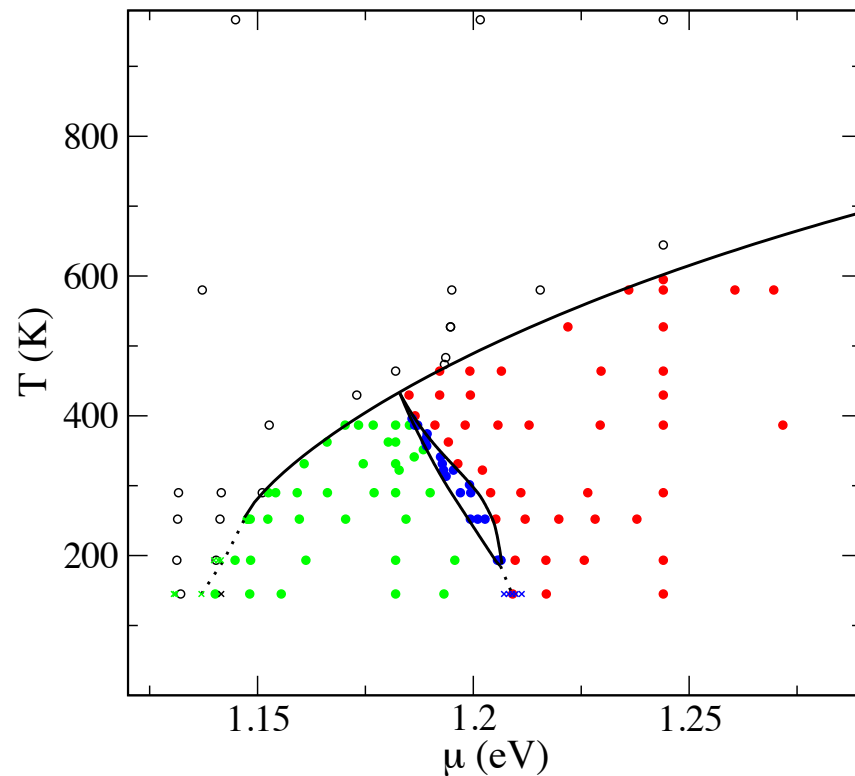


Excitonic condensation (doping) - all phases

n-T phase diagram



μ -T phase diagram

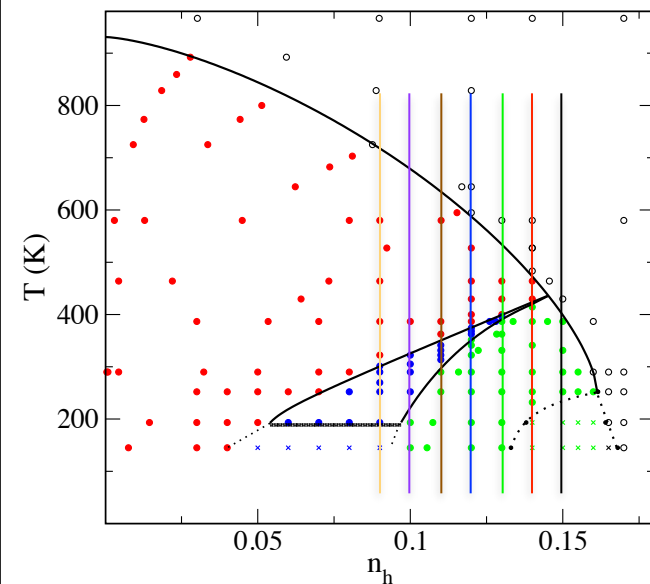


- L phase
- E phase
- C phase

n_h - hole concentration ($N=2-n_h$)

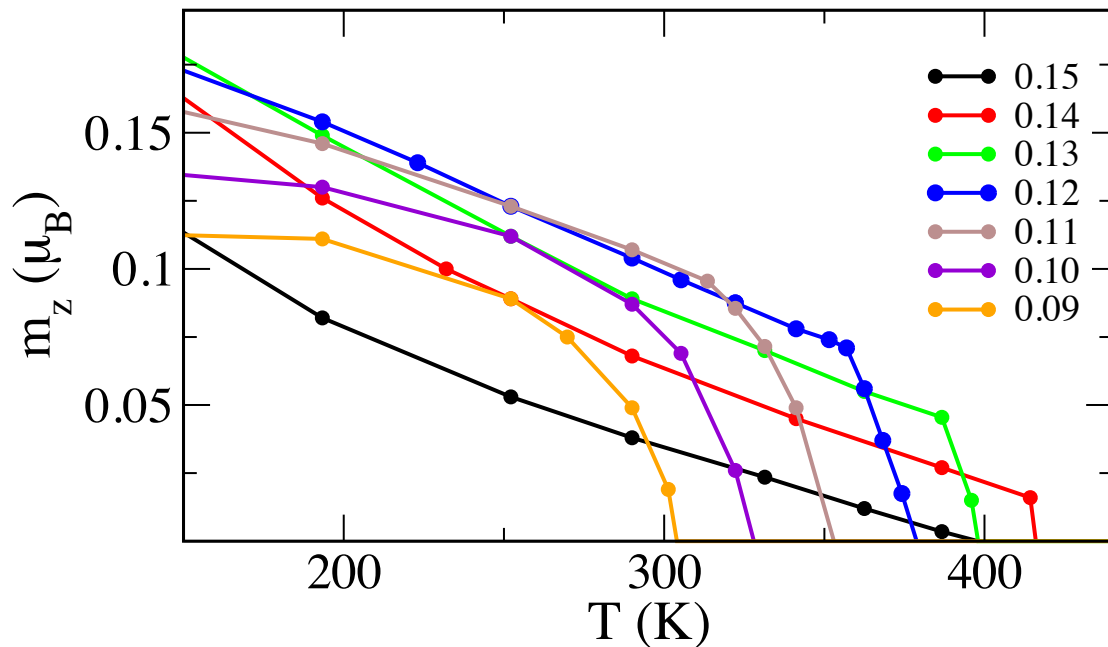
Excitonic condensation (doping) - all phases

n-T phase diagram



Ferromagnetic magnetization

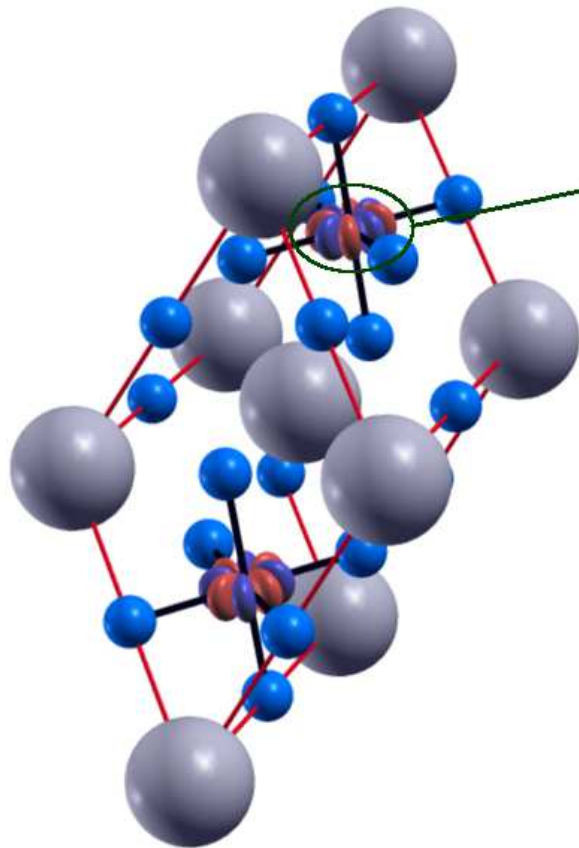
$$\langle m_z \rangle \parallel i(\bar{\phi} \wedge \phi)$$



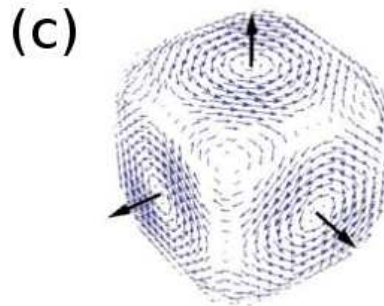
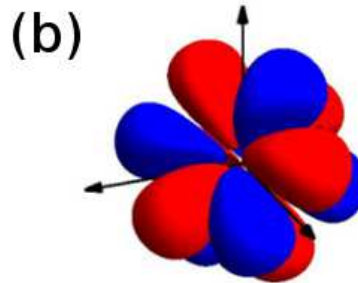
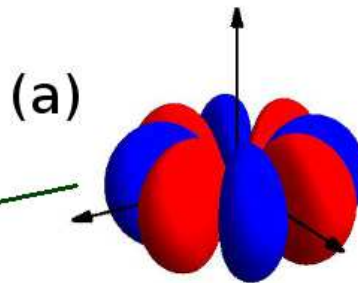
Orbital degeneracy - d⁶ perovskites

LDA+U (static mean-field) solutions with excitonic order for hypothetical cubic structure of LaCoO₃

LaCoO₃ AF-EC order



local spin density



ϕ_{β}^{α}

$$\begin{pmatrix} 0 & 0 & X \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & X \\ 0 & 0 & X \\ 0 & 0 & X \end{pmatrix}$$

$$\begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}$$

residual group

$$\tilde{D}_{4h} \times U(1)$$

$$\tilde{D}_{3d} \times U(1)$$

$$O_h$$

Conclusions

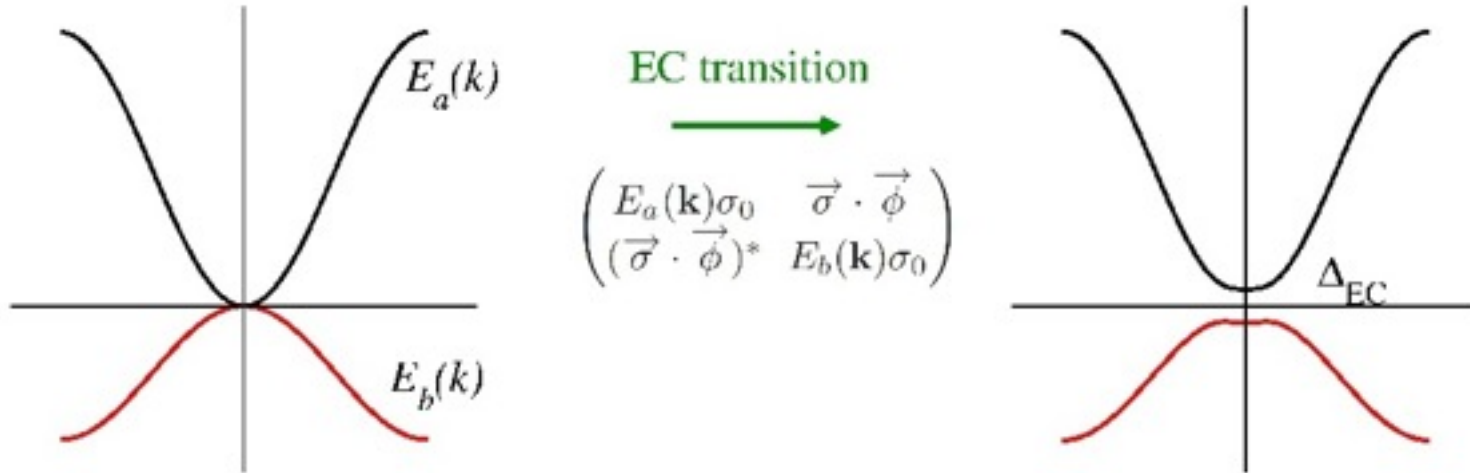
- Solids close to spin-state transition may be unstable towards condensation of spinful excitons.
- The excitonic order parameter has complex structure and allows multiple phases.
- The excitonic order may lead to a long-range order of magnetic multipoles or local spin currents, but can also induce ferromagnetic polarisation.

Phys. Rev. B **89**, 115134 (2014)

arXiv:1405.1191

arXiv:1410.5198

Excitonic insulator



A band insulator with a very narrow gap (positive or negative) is unstable towards opening of a gap due to electron-hole attraction - condensation of excitons.

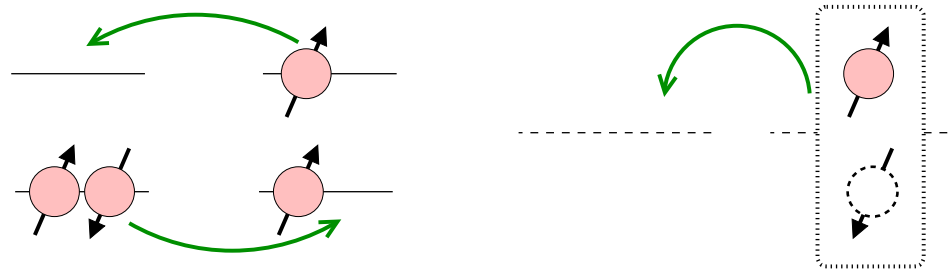
The gap can have spin-singlet or spin-triplet symmetry and be real or imaginary. Which of these options is realised depends on the interaction term and details of the band structure.

Mott, 1961

Halperin and Rice, 1968

Strong coupling picture of excitonic condensation

Strong coupling: HS states behave as hard-core bosons with the vacuum state $|\text{vac}\rangle \equiv |\text{LS}\rangle$



$$H_{\text{eff}} = \sum_{i,s} \mu n_{i,s} + J_{\perp} \sum_{i,j,s} d_{i,s}^{\dagger} d_{j,s} + J_{\parallel} \sum_{\langle ij \rangle, s, s'} n_{i,s} n_{j,s'} + J_0 \sum_{\langle ij \rangle} S_i^z S_j^z$$

$$d_1^{\dagger} = a_{\uparrow}^{\dagger} b_{\downarrow} \quad d_{-1}^{\dagger} = a_{\downarrow}^{\dagger} b_{\uparrow}$$

Bose-Einstein condensation = spontaneous hybridization between HS and LS states on the same site (breaks spin rotational symmetry)

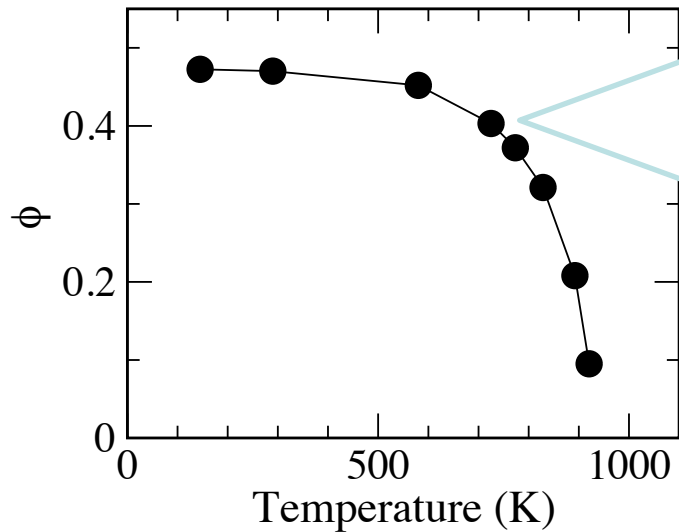
Batista, 2001

Balents, 2000

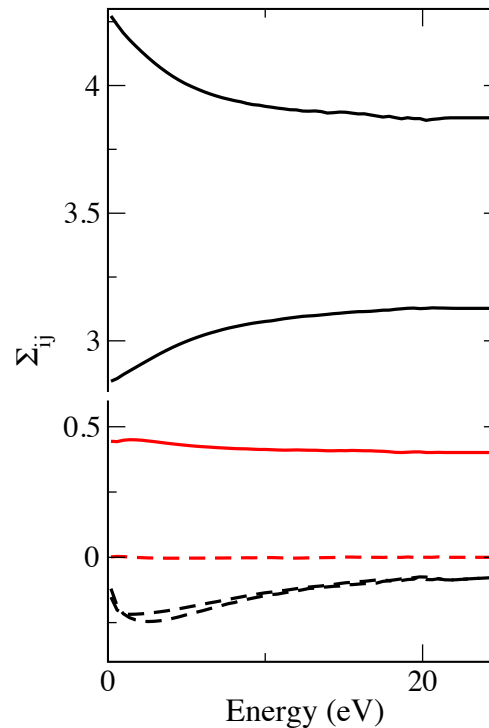
Excitonic condensation (undoped) - L phase

order parameter $|\phi^+| = |\phi^-|$

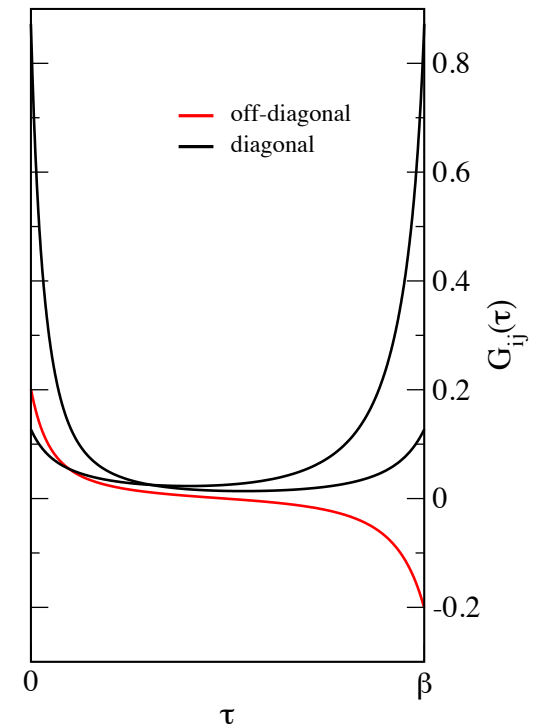
Spectral density (diagonal elements)



Self-energy

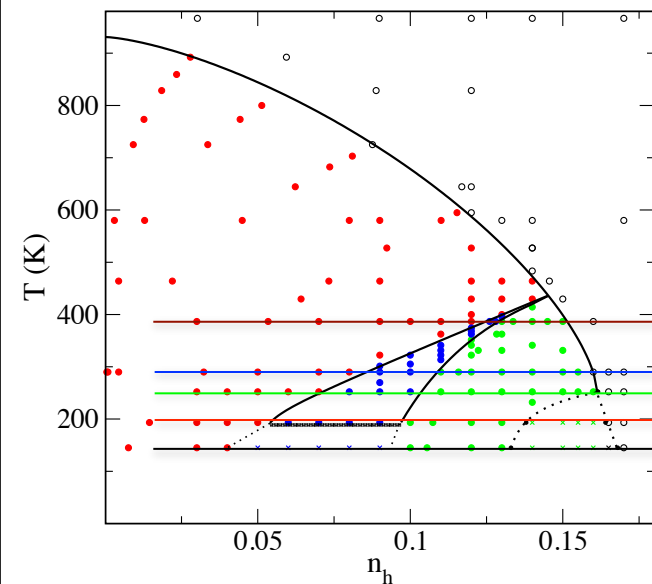


Green's function



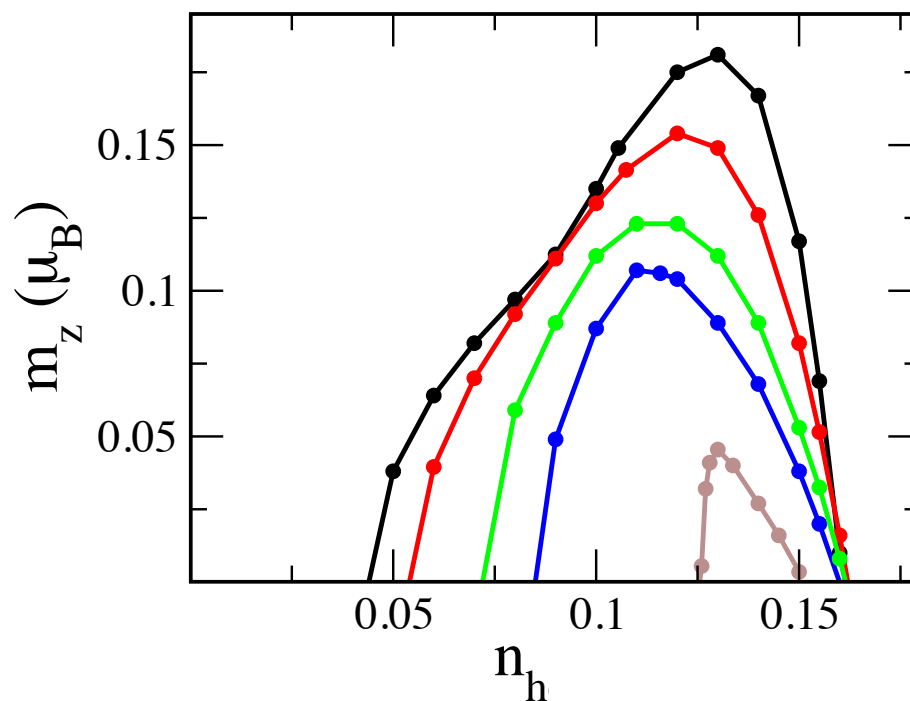
Excitonic condensation (doping) - all phases

n-T phase diagram



Ferromagnetic magnetization

$$\langle m_z \rangle \parallel i(\bar{\phi} \wedge \phi)$$



EC in cubic d⁶ perovskite

Exciton = bound pair of e_g electron and t_{2g} hole


How do we detect the EC order?

Local d-occupation matrix (10 × 10):

spin structure:

$$\mathbf{D} = \begin{pmatrix} \mathbf{D}_0 + \phi^z & \phi^x + i\phi^y \\ (\phi^x + i\phi^y)^* & \mathbf{D}_0 - \phi^z \end{pmatrix}$$

orbital structure:


$$\phi_{xy}^\alpha d_{x^2-y^2} \otimes d_{xy} + \phi_{zx}^\alpha d_{z^2-x^2} \otimes d_{zx} + \phi_{yz}^\alpha d_{y^2-z^2} \otimes d_{yz}$$

The order parameter has 9 components (or 18 real components)

ϕ_β^α

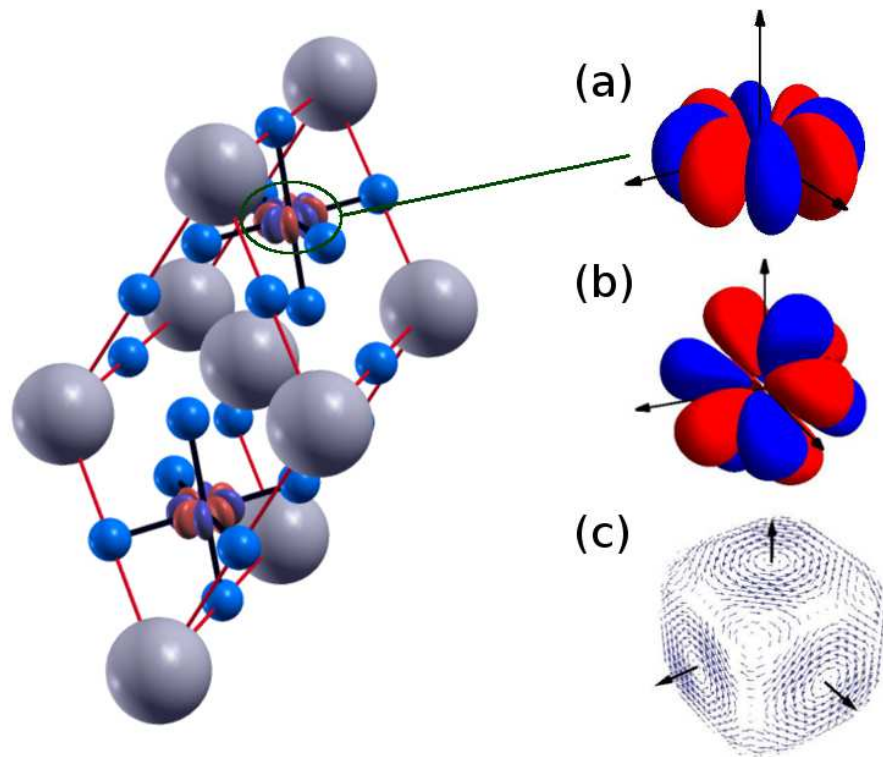
$\alpha = x, y, z$ transforms like a vector under spin rotations

$\beta = x, \hat{y}, \hat{z}$ transforms like a pseudovector under O_h operations

The spin and orbital symmetry does not specify the ordered phase uniquely, possible solutions can be classified by their residual symmetry.

Examples of LDA+U EC solutions

LaCoO₃ AF-EC order



ϕ_{β}^{α}

$$\begin{pmatrix} 0 & 0 & X \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

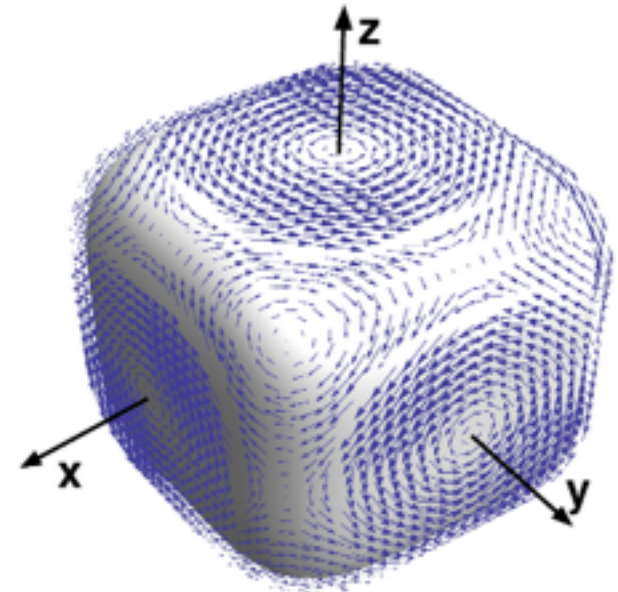
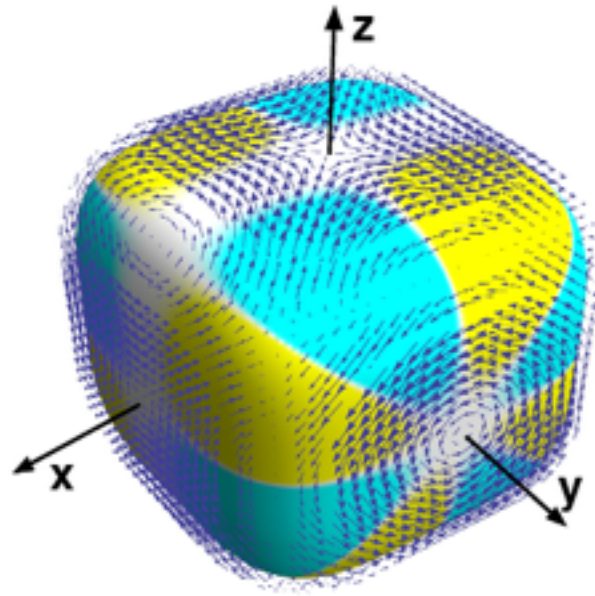
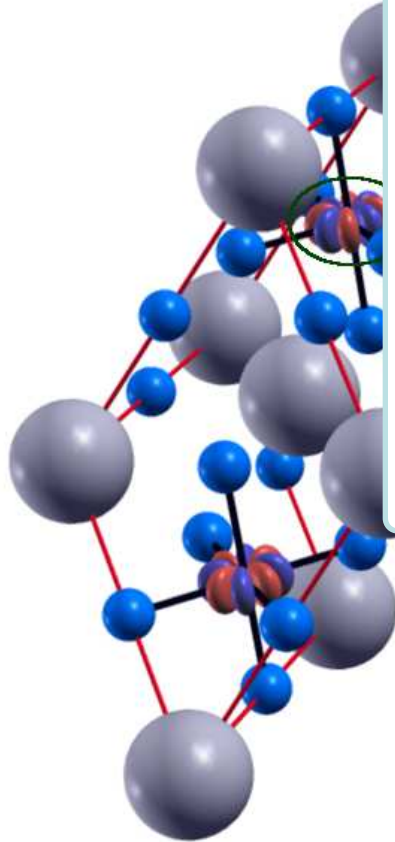
$$\begin{pmatrix} 0 & 0 & X \\ 0 & 0 & X \\ 0 & 0 & X \end{pmatrix}$$

$$\begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}$$

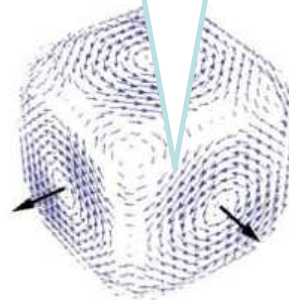
i	X	$E_{(i)}$ [meV/f.u.]
1	0.182	-43
2	0.134	-73
3	0.144	-82

Examples of EC solutions

LaCoO₃ AF-EC or



(c)



$$\begin{pmatrix} X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X \end{pmatrix}$$

O_h

up

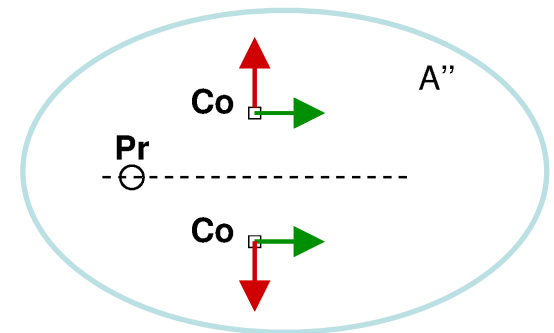
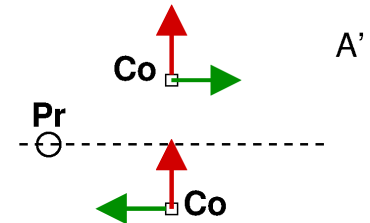
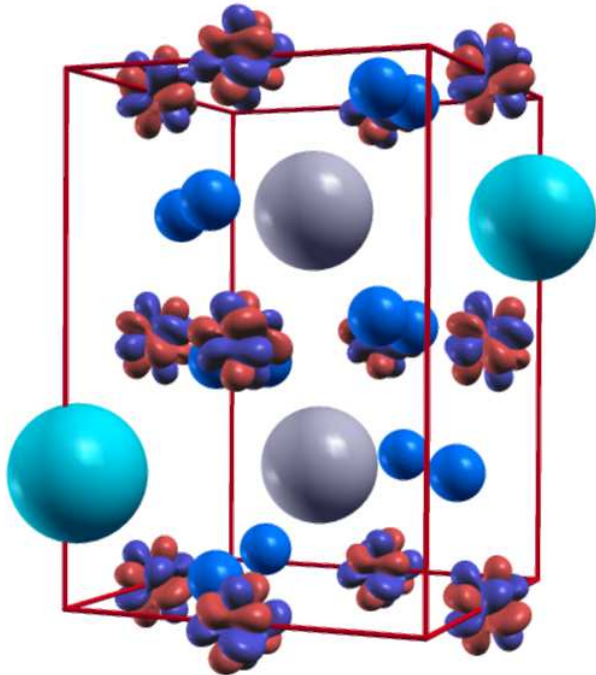
Examples of LDA+U for PCCO

orthorhombic structure: 4 Co atoms per f.u. two inequivalent Co positions

Product solution:

	1	2	3	4
ϕ_{yz}	0.182	0.182	0.216	0.216
ϕ_{zx}	0.228	0.228	-0.212	-0.212
ϕ_{xy}	-0.071	0.071	-0.093	0.093

Orbital pseudovectors on
sym. related Co atoms:

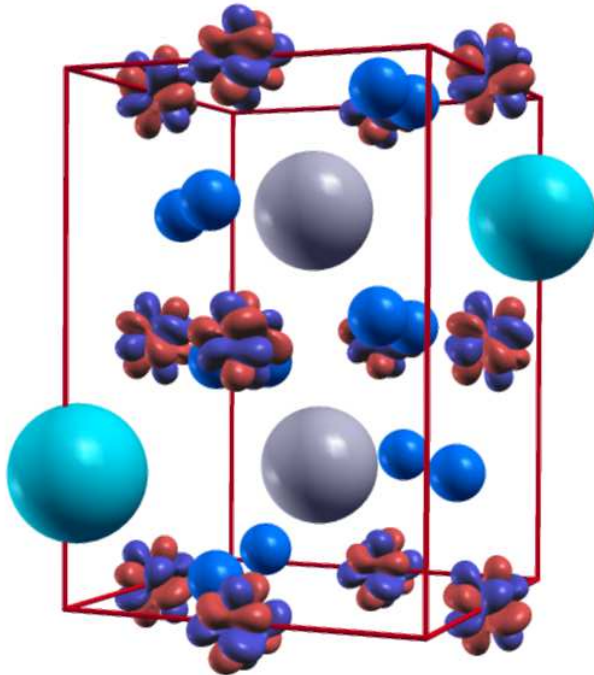


Origin of exchange splitting on Pr

Coupling of Pr 4f¹ spin to p-d orbitals: effective multi-channel Kondo Hamiltonian

$$H^{(n)} = \sum_{\alpha\alpha'} \sum_{mm'} \sum_i \mathbf{S} \cdot \boldsymbol{\sigma}_{\alpha\alpha'} J_{i,mm'}^{(n)} c_{im\alpha}^\dagger c_{im'\alpha'} + \text{c.c.}$$

Below T_c effective exchange field appears: $h_\gamma^{(n)} = \sum_{imm'} J_{i,mm'}^{(n)} \sum_{\alpha\alpha'} 2 \text{Re} \langle c_{im\alpha}^\dagger \sigma_{\alpha\alpha'}^\gamma c_{im'\alpha'} \rangle$



The site symmetry of the EC order parameter with respect to the Pr site decides whether contributions of from different Co site interfere constructively or destructively.

For the present EC solution $h=0$ in the absence of spin-orbit coupling in Pr 4f shell. With SOC splitting on 10 meV scale is obtained.